Renormalization Group beta function for SU(3) gauge-fermion systems

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Motivation

- Study properties of strongly coupled gauge-fermion systems
- Characterize nature of such systems
  → Where is the onset of the conformal window?
- Determine properties such as anomalous dimensions
  → Important for BSM model building
  Talk by Curtis T. Peterson Wed. 16:30 BSM
  Talk by Chris Monahan Wed. 16:50 SM Parameters

Renormalization Group β function

$$\beta(g^2) = \mu^2 \frac{dg^2}{d\mu^2}$$

- Encodes dependence of coupling g^2 on the energy scale μ^2
- β has no explicit dependence on μ^2, only implicit through $g^2(\mu)$
- Known perturbatively up to 5-loop order in the $\overline{\text{MS}}$ scheme (1- and 2-loop are universal)
  [Baikov et al. PRL118(2017)082002] [Ryttov and Shrock PRD94(2016)105015]
- Perturbative predictions reliable at weak coupling,
  nonperturbative methods needed for strong coupling

Step-Scaling β function

- Discretized β function determined using numerical lattice field theory calculations [Lüscher et al. NPB359(1991)221]
  - → Choose symmetric L^4 setup where the size L of the lattice is the only scale
  - → Determine β function by changing the scale $L \to s \cdot L$
- Gradient flow [Narayanan and Neuberger JHEP 0603 (2006) 064] [Lüscher CMP 293 (2010) 899][JHEP 1008 (2010) 071]
  - → Continuous smearing transformation which can be used to define a renormalized coupling

$$g_c^2(L) = \frac{128\pi^2}{3(N_c^2-1)} \frac{1}{C(c,L)} t^2 \langle E(t) \rangle$$

  - → Relate flow time t to scale L: $\sqrt{8t} = c \cdot L$ [Fodor et al. JHEP11(2012)007][JHEP09(2014)018]
  - → Calculate difference

$$\beta_s^c(g_c^2; L) = \frac{g_c^2(sL) - g_c^2(L)}{\log(s^2)}$$

  - → Extrapolate $L \to \infty$ to remove discretization effects and take the continuum limit

Setup

- Symanzik gauge action
- Möbius domain-wall fermions with three levels of stout smearing ($\varrho = 0.1$)
- Input quark mass $am_q = 0$, $L_s = 12$ or 16 such that $am_{res} < 10^{-5}$
- Fermions with anti-periodic boundary conditions in space and time

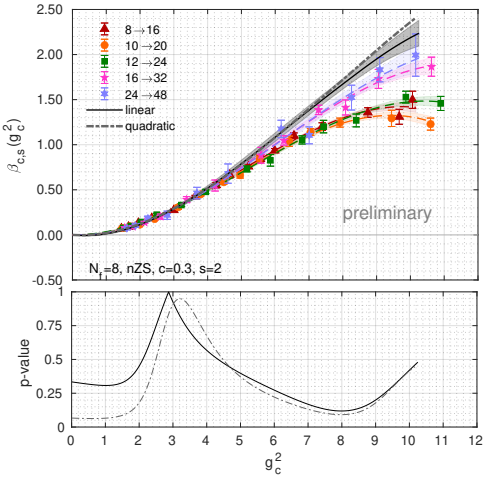
$N_f = 4$	$N_f = 6$	$N_f = 8$
6–9 bare couplings	8–12 bare couplings	11–18 bare couplings
$\beta = 8.50$ - $4.50(4.20)$	$\beta = 7.00$ - $4.30(4.02)$	$\beta = 7.00$ - $4.10(4.00)$

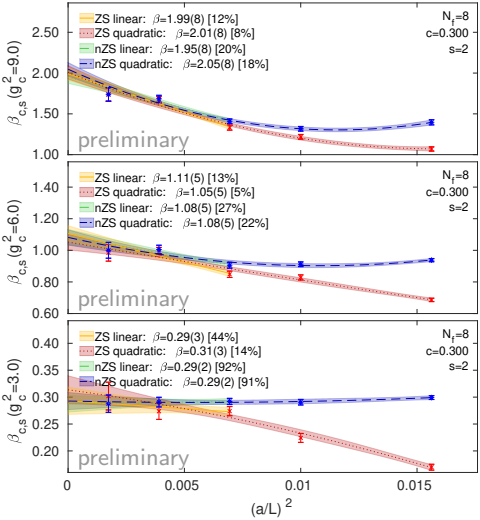
- 5 volume pairs with $s = 2$

40^4, 32^4, 12^4, 24^4, 20^4, 16^4, 10^4, 8^4 $\qquad$ 48^4, 32^4, 12^4, 24^4, 20^4, 16^4, 10^4, 8^4

- Simulations performed using `Grid` [Boyle et al. PoS Lattice2015 023]
- Measuring Zeuthen flow, Symanzik flow, and Wilson flow in `Qlua` [Pochinsky PoS Lattice2008 040]
- Apply tree-level normalization to reduce cutoff effects [Fodor et al. JHEP09(2014)018]
  → Poster Christian Schneider [A13], Talk Alberto Ramos Wed 17:30 Theo. Developments

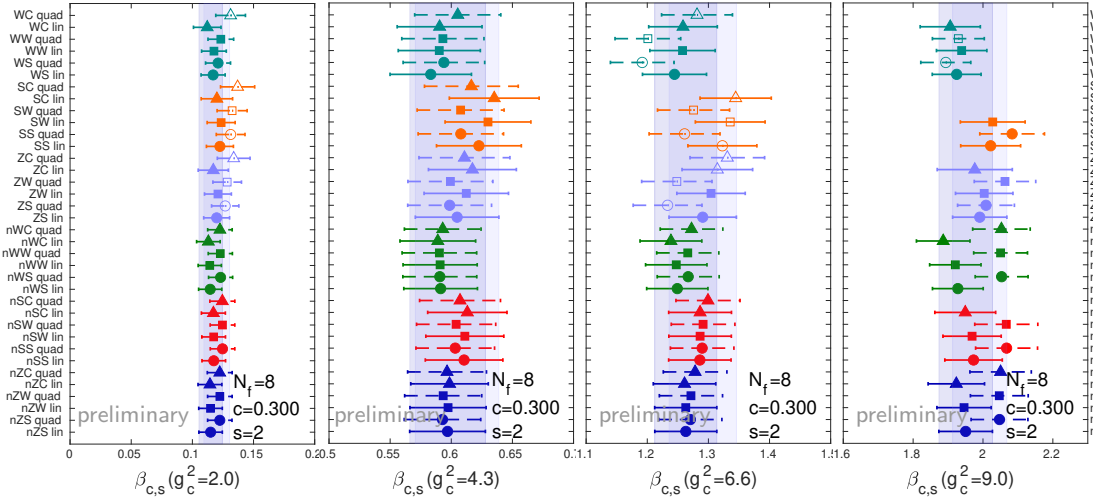
Analysis SU(3) with $N_f = 8$ fundamental flavors [Hasenfratz, Rebbi, OW in preparation]



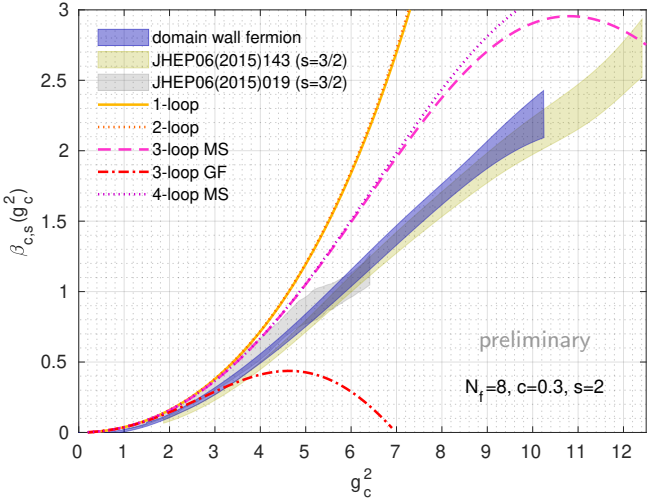


Systematic effects SU(3) with $N_f = 8$ fundamental flavors

[Hasenfratz, Rebbi, OW in preparation]



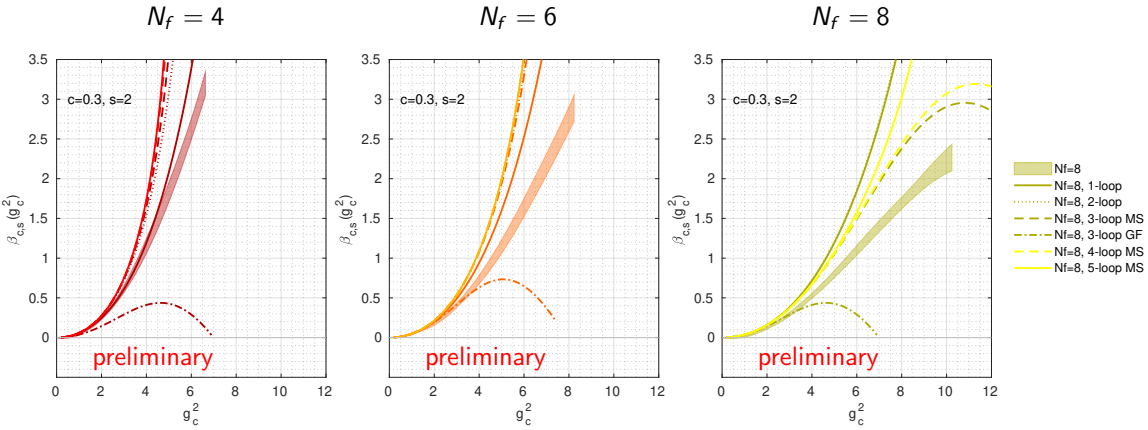
Comparison SU(3) with $N_f = 8$ fundamental flavors [Hasenfratz, Rebbi, OW in preparation]



- 3-loop GF
  [Harlander, Neumann JHEP06(2016)161]

[Hasenfratz, Schaich, Veernala JHEP06(2015)143] [Fodor, Holland, Kuti, Mondal, Nogradi, Wong JHEP06(2015)019]

SU(3) with N_f fundamental flavors



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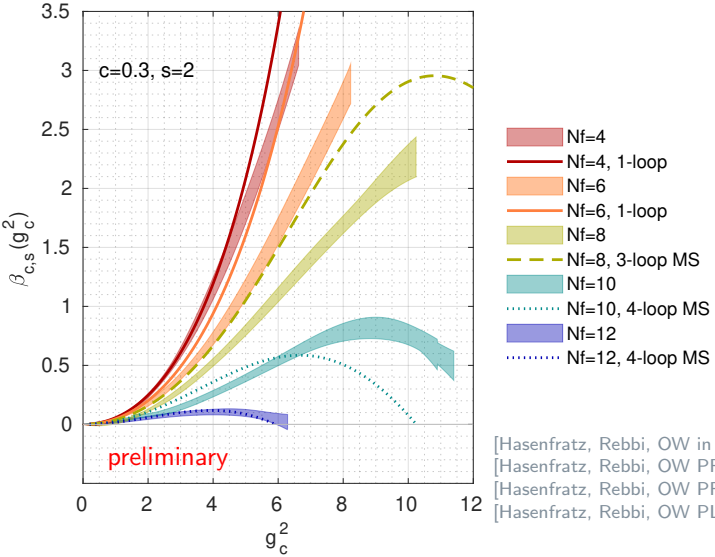
SU(3) with N_f fundamental flavors



[Hasenfratz, Rebbi, OW in preparation]
[Hasenfratz, Rebbi, OW PRD 101(2020)114508]
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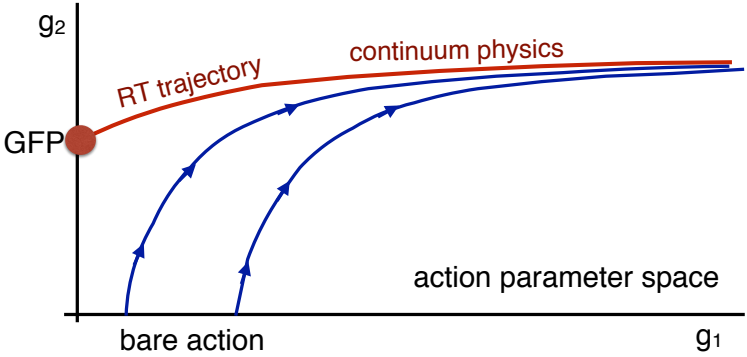
[Hasenfratz, Rebbi, OW PLB 798(2019)134937]

Beyond Step-Scaling: real-space Renormalization Group (RG) flow



Beyond Step-Scaling: real-space Renormalization Group (RG) flow

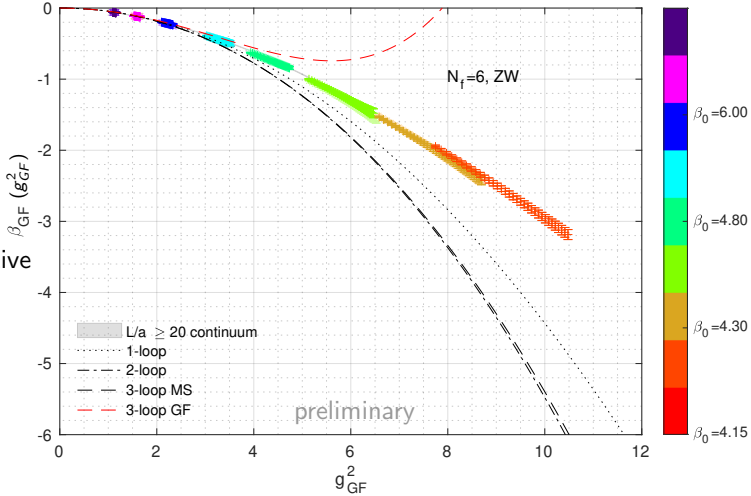
- RG flow: change of action parameters under RG transformation
- Gradient flow is a continuous transformation
  - → Defines real-space RG blocked quantities
  - → By incorporating coarse graining as part of calculating expectation values, it is turned into an RG transformation [Carosso, Hasenfratz, Neil PRL 121 (2018) 201601]
- Relate GF time t/a^2 to RG scale change $b \propto \sqrt{t/a^2}$
  - → Quantities at flow time t/a^2 describe physical quantities at energy scale $\mu \propto 1/\sqrt{t}$
  - → Local operator with vanishing anomalous dimension can be used to define running coupling
    - ⇝ Simplest choice: $t^2\langle E(t)\rangle$ [Lüscher JHEP 1008 (2010) 071]
- Continuous RG β function

$$\beta(g^2_{GF}) = \mu^2 \frac{dg^2_{GF}}{d\mu^2} = -t\frac{dg^2_{GF}}{dt}$$

[Fodor et al., EPJ Web Conf. 175(2018)08027] [Hasenfratz, OW PRD101(2020)034514]

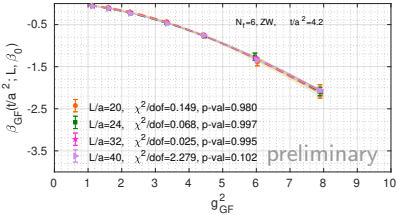
Example: SU(3) with $N_f = 6$ [Hasenfratz, OW in preparation]

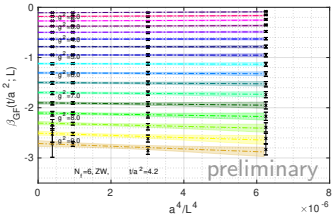
- "Raw" data overlayed on continuum result
- Fast "running" coupling
  $\rightsquigarrow$ Confinement
- Plot: Comparison of non-perturbative and perturbative determinations
- 3-loop GF
  [Harlander, Neumann JHEP06(2016)161]

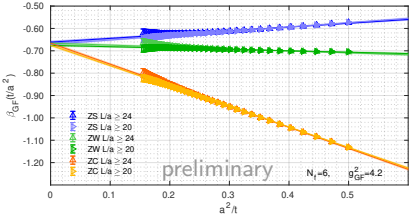


Analysis steps continuous β function [Hasenfratz, OW PRD101(2020)034514][in preparation]

- Calculate $g^2_{GF}(t; L, \beta_0)$ and derivative $\beta_{GF}(t; L, \beta_0)$ for **all** flow times t, volumes L, bare coupling β_0

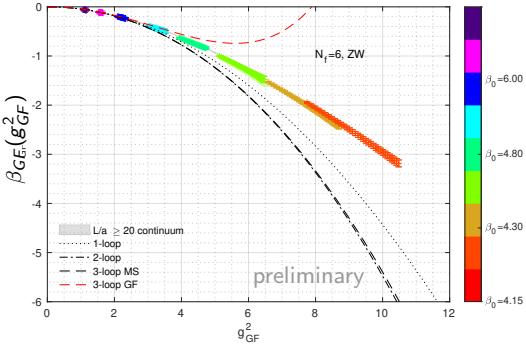


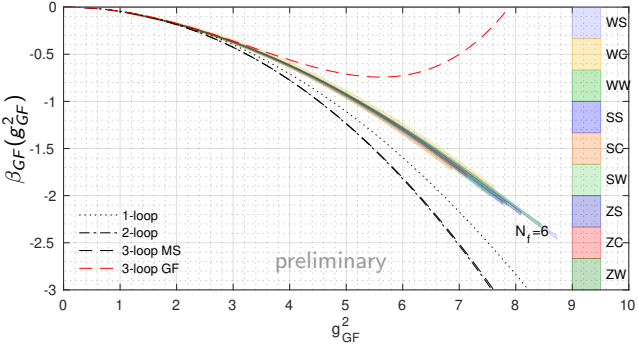




- Interpolate for fixed flow time and L in g^2_{GF}
- Take infinite volume limit keeping t and g^2_{GF} fixed
  → Vary extrapolation to test stability
- Take continuum limit $t/a^2 \to \infty$ for fixed g^2_{GF}
  → Check for systematic effects varying range of flow times, operators, gradient flows, ...

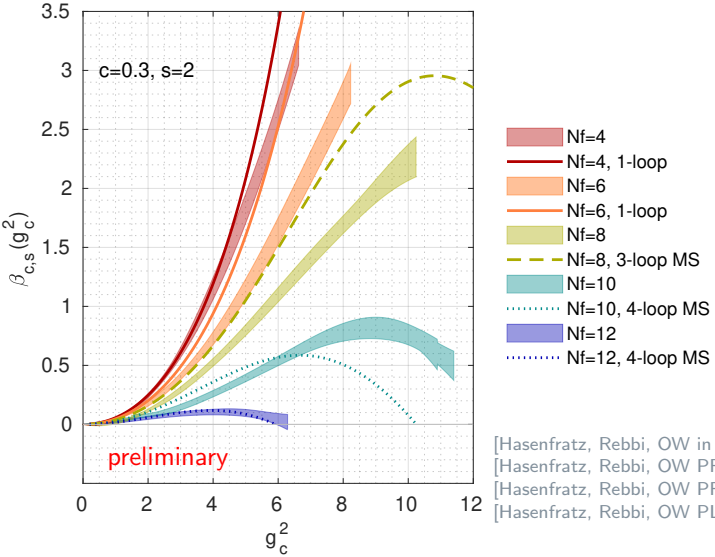
Continuous β function for SU(3) with $N_f = 6$ [Hasenfratz, OW in preparation]





- Analysis for $N_f = 4$ and 8 in progress

SU(3) with N_f fundamental flavors (step-scaling)



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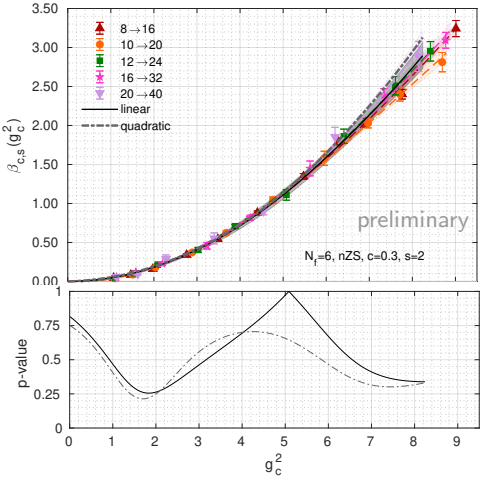
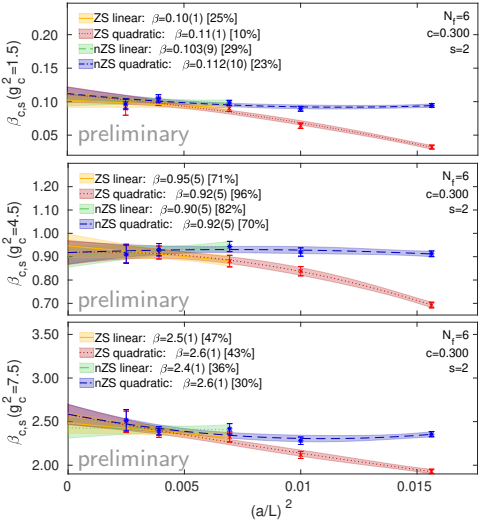
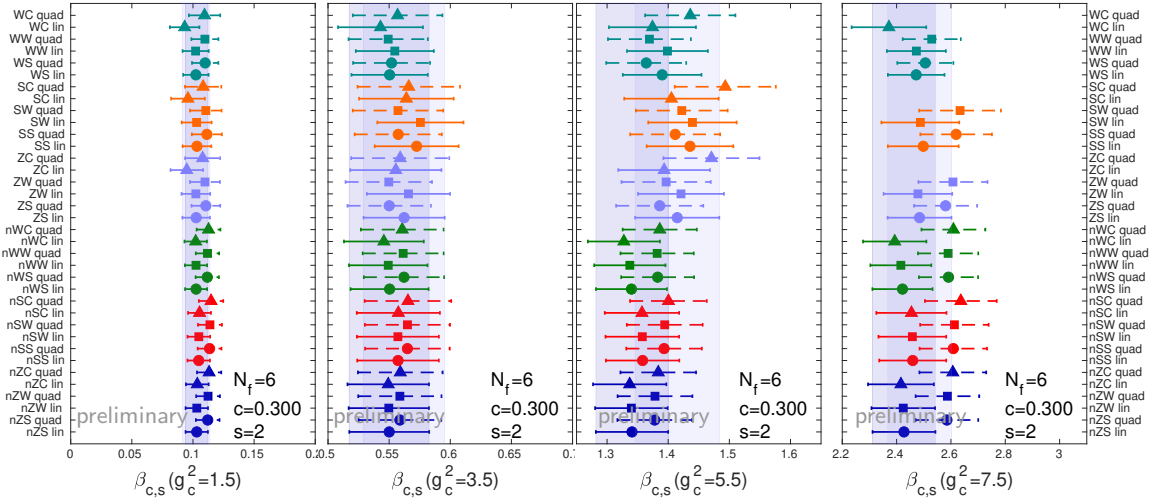
extra

SU(3) with $N_f = 6$ fundamental flavors: analysis



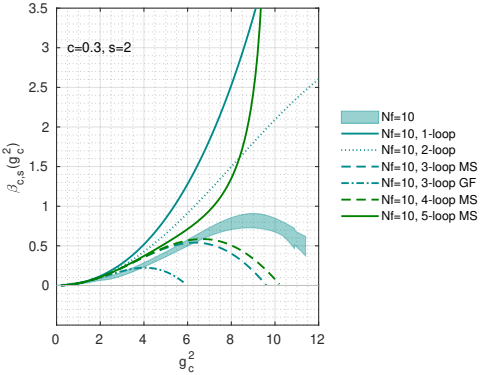


SU(3) with $N_f = 6$ fundamental flavors: systematic effects

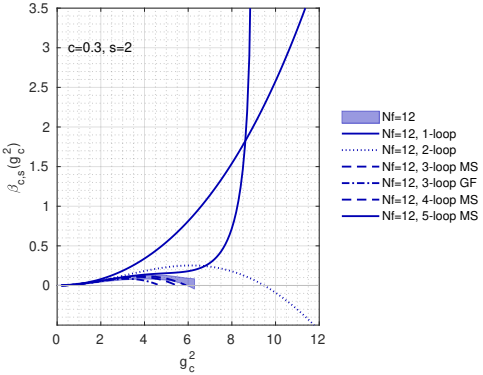


SU(3) with $N_f = 10$, 12 fundamental flavors

$N_f = 10$



$N_f = 12$



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