Symanzik Improvement of Non-Relativistic Field Theories

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The Big Picture:

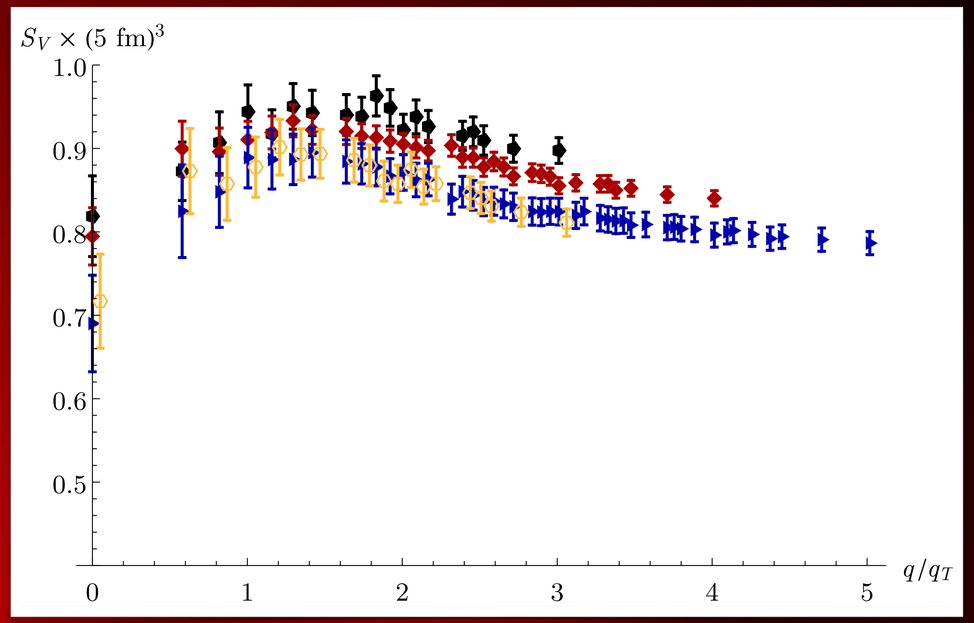
- Leading order pionless EFT

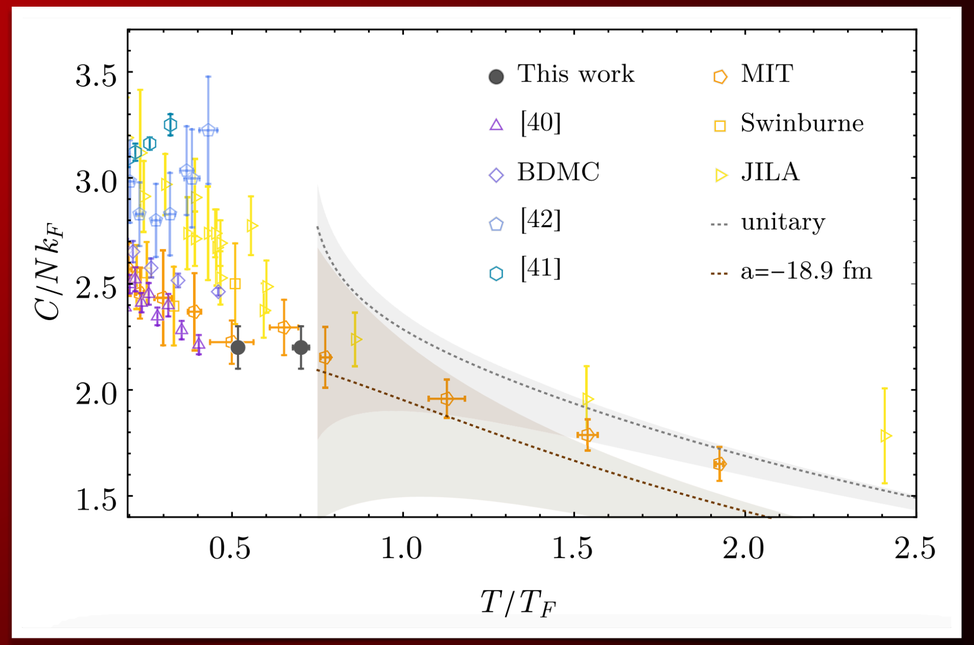
$$H = \sum_x \Delta x^3 \left\{ \frac{\nabla\psi^\dagger \cdot \nabla\psi}{2M} + C_0 \left(\psi^\dagger \psi\right)^2 \right\}$$

$$\Longrightarrow \quad k \cot\delta(k) = -\frac{1}{a}$$

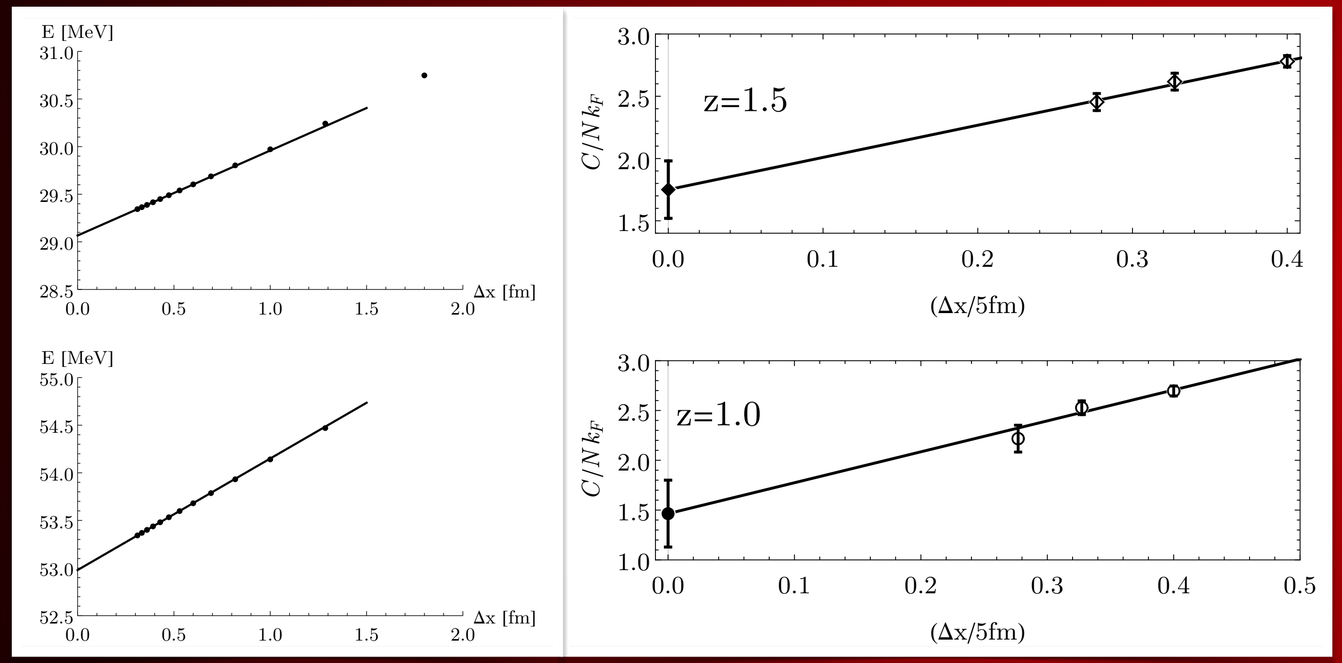
- Large artifacts in current simulations

- Finite-volume improvement scheme

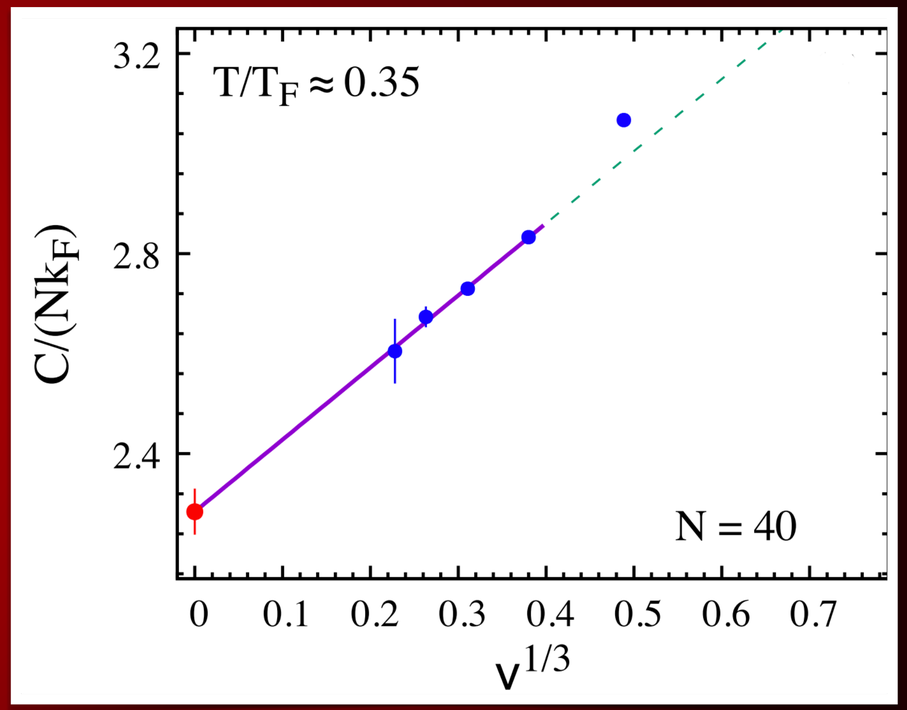




Linear Convergence:



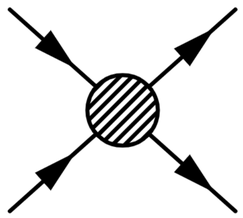
Berkowitz, NCW, et. al. PRL (2022)



Alhassid & Gilbreth PRL (2020)

Linear Convergence:

Kaplan, Savage & Wise (1998), Bedaque & van Kolck (2002)



$$= \left(\frac{\Delta x}{MC_0} + \mathcal{P} \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 - EM\Delta x^2} + i\frac{\sqrt{EM\Delta x^2}}{4\pi} \right)^{-1}$$

$$= \left(\frac{\Delta x}{MC_0} + \alpha + \beta\left(EM\Delta x^2\right) + i\frac{\sqrt{EM\Delta x^2}}{4\pi} + \mathcal{O}\left((EM\Delta x^2)^2\right) \right)^{-1}$$

$$iA = \frac{4\pi/M}{k\cot\delta(k) - ik}$$

$$= \frac{4\pi/M}{(-1/a + rk^2/2 + \ldots) - ik}$$

$$\Longrightarrow$$

$$C_0(\Delta x) = -\frac{\Delta x}{M}\left(\alpha - \frac{\Delta x}{4\pi a}\right)^{-1}$$

$$r(\Delta x) = 0.337\Delta x$$

Symanzik Improvement :

K. Symanzik, NPB 226, 187 (1983)

$$H \to H + \underbrace{\sum_i C_i \mathcal{O}_i}$$

Tuned to cancel low order Δx dependence

Symanzik Improvement :

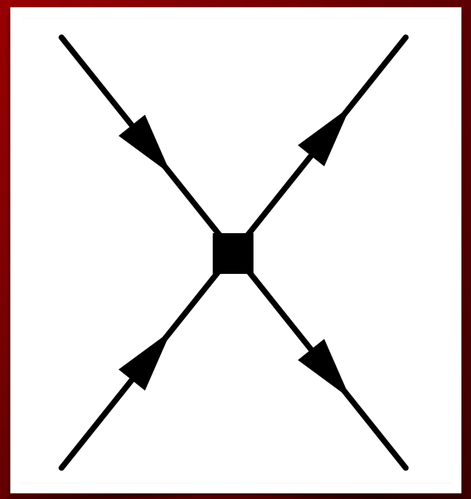
K. Symanzik, NPB 226, 187 (1983)

$$\delta H = C_2 \sum_{xi} \Delta x^3 \, n(x) n(x + \delta x_i) \quad \textbf{where} \quad n(x) = \psi^\dagger(x)\psi(x)$$

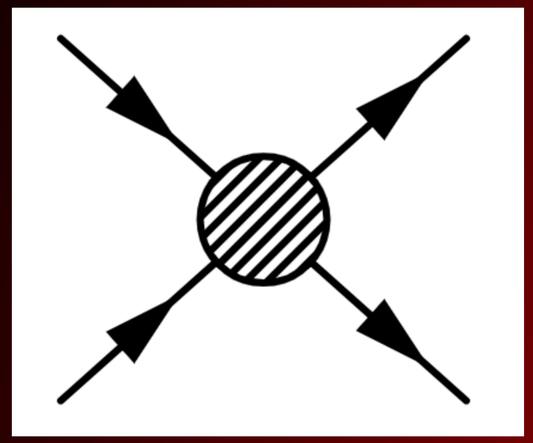
Intuition:

$$\sum_i n(x) n(x + \delta x_i) =$$

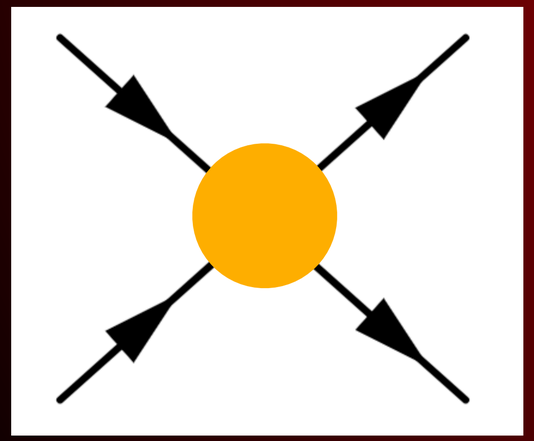
$$n(x)^2 + \Delta x^2 \, n(x) \nabla^2 n(x) + \ldots$$



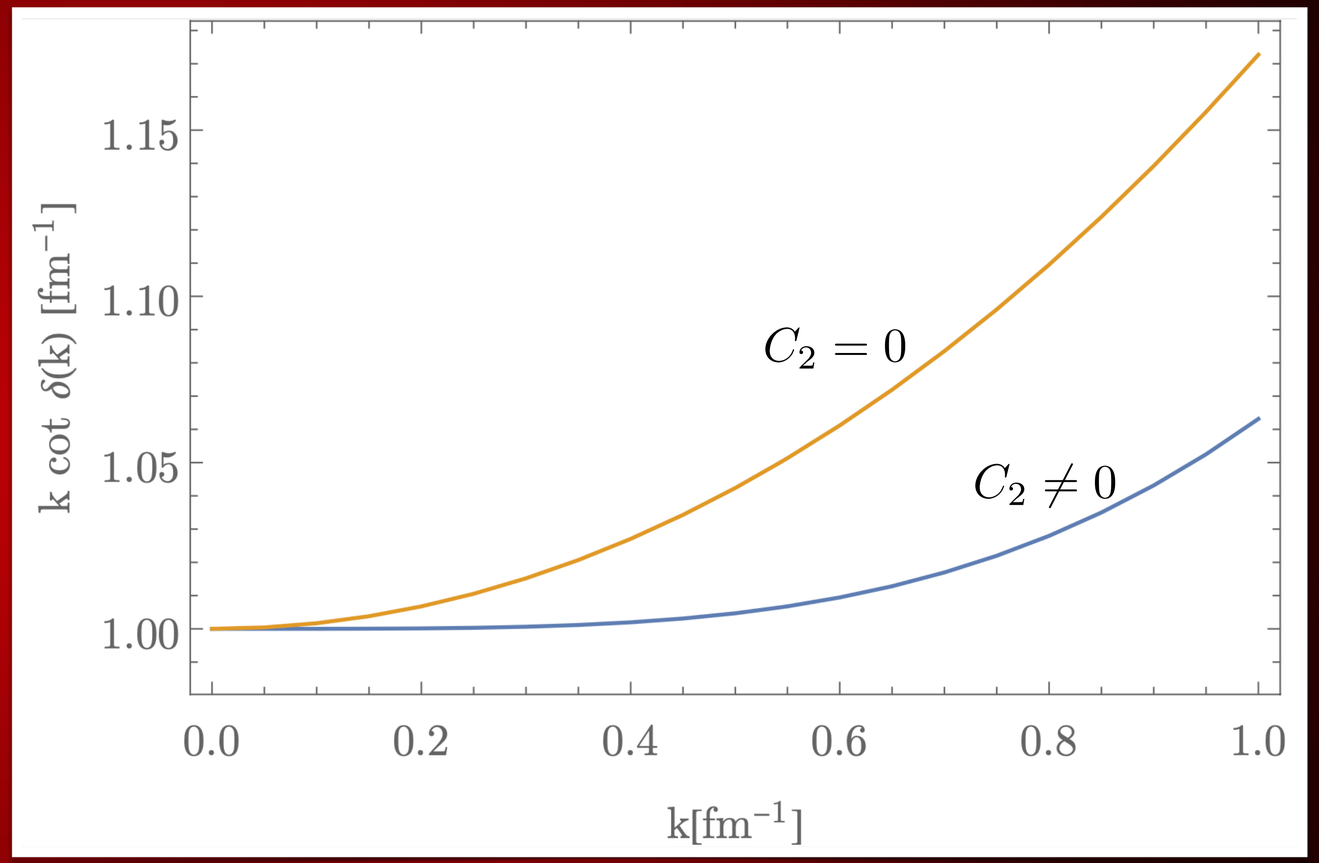
$$\sim \# \, k^2$$

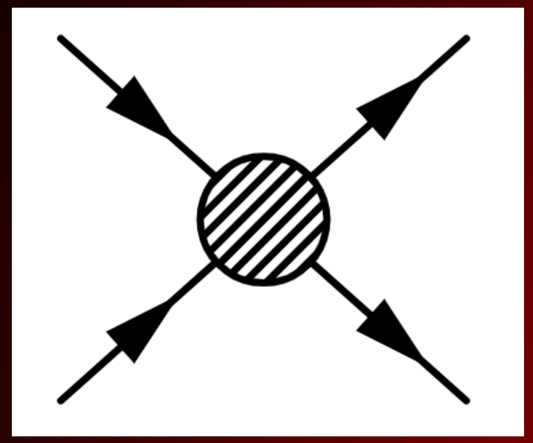


$H(C_0)$

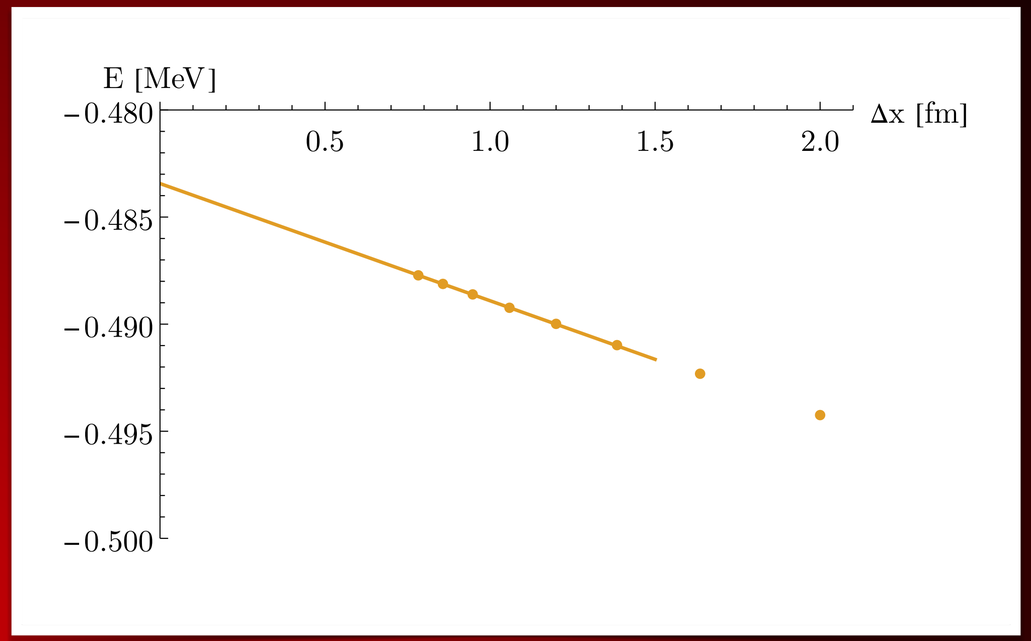


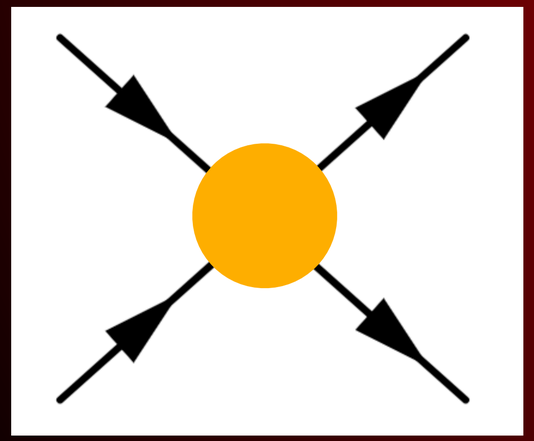
$H(C_0, C_2)$



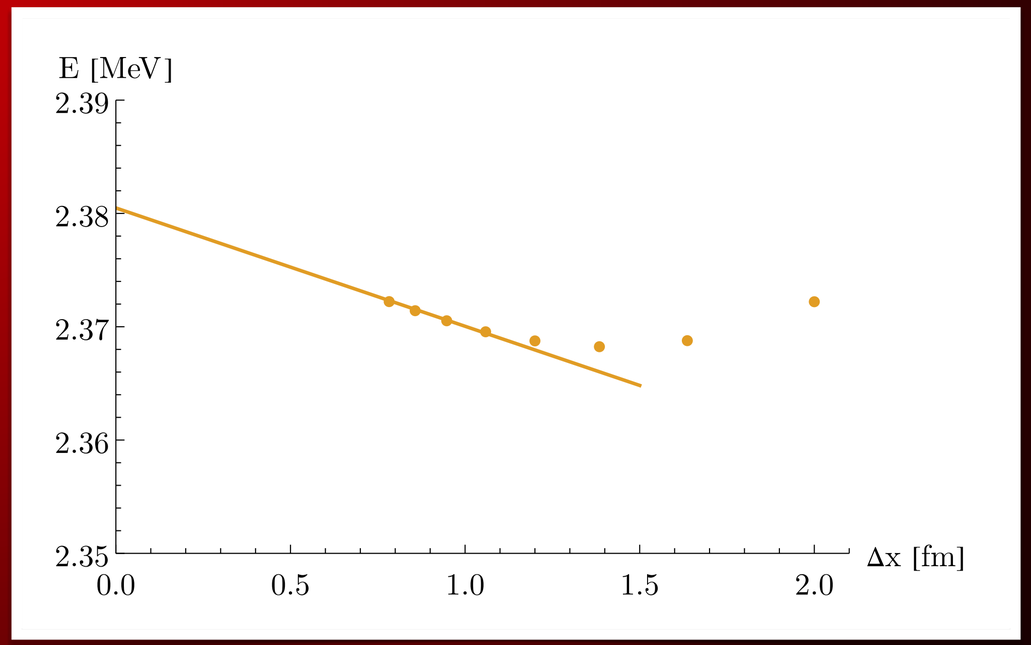


$H(C_0)$


E [MeV]
−0.480
−0.485
−0.490
−0.495
−0.500
0.5
1.0
1.5
2.0
Δx [fm]



$H(C_0, C_2)$


E [MeV]
2.39
2.38
2.37
2.36
2.35
0.0
0.5
1.0
1.5
2.0
Δx [fm]

Finite-Volume Scheme:

Idea: Tune to finite-volume observables

$$\frac{d}{d\Delta x}\begin{bmatrix} E_1(C_0, C_2, \Delta x\,;\, V) \\ E_2(C_0, C_2, \Delta x\,;\, V) \end{bmatrix} = 0$$

Lüscher formula:

$$k\cot\delta(k) = -\frac{1}{a} + \frac{1}{2}rk^2 + \ldots$$
$$= \frac{1}{\pi L} S\Big(\frac{k^2 L^2}{4\pi^2}\Big)$$

w/ $S(x) = \lim_{\Lambda\to\infty}\Big(\sum_{|\mathbf{j}|<\Lambda} \frac{1}{\mathbf{j}^2 - x} - 4\pi\Lambda\Big)$

Lüscher CMP (1986)

Workflow:

$H(C_0, C_2, \Delta x)$

↓

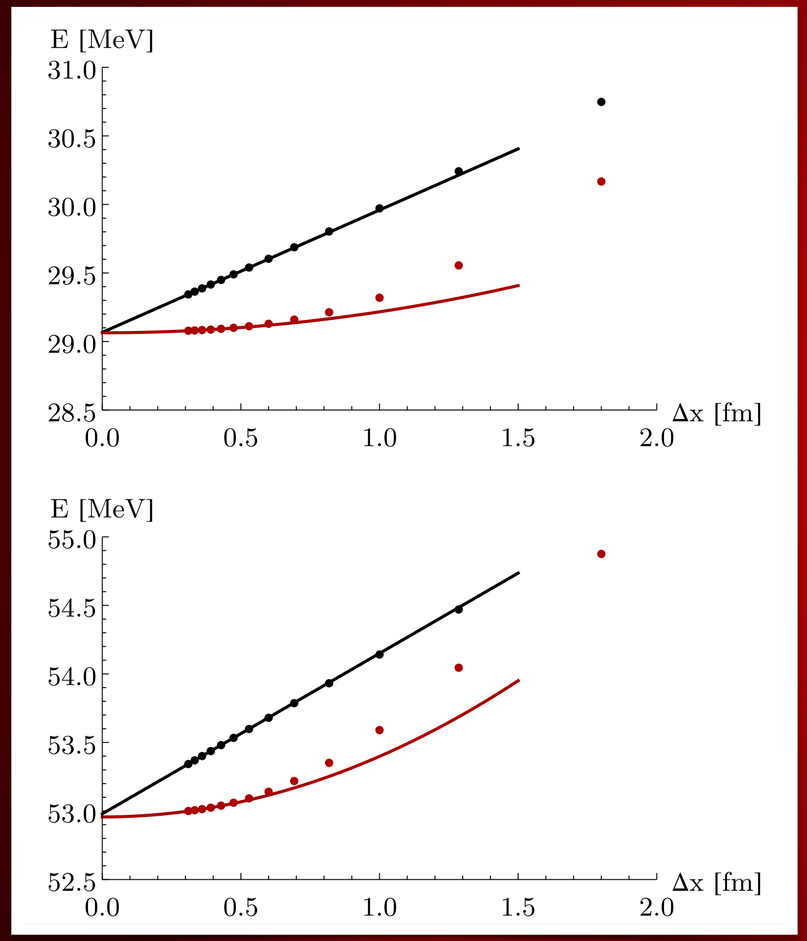
$E_i(C_0, C_2, \Delta x)$

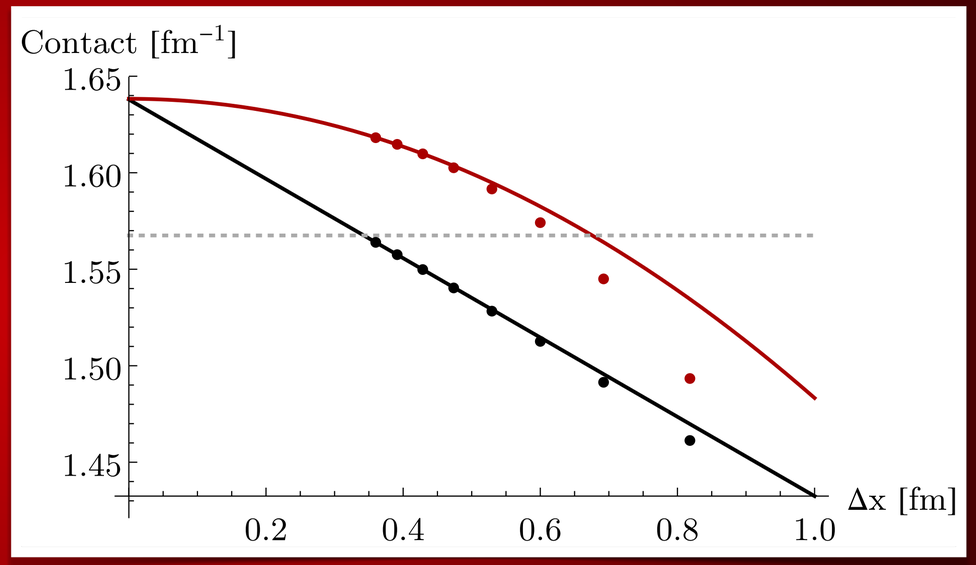
↓

$k\cot\delta(k)$ correct?

adjust

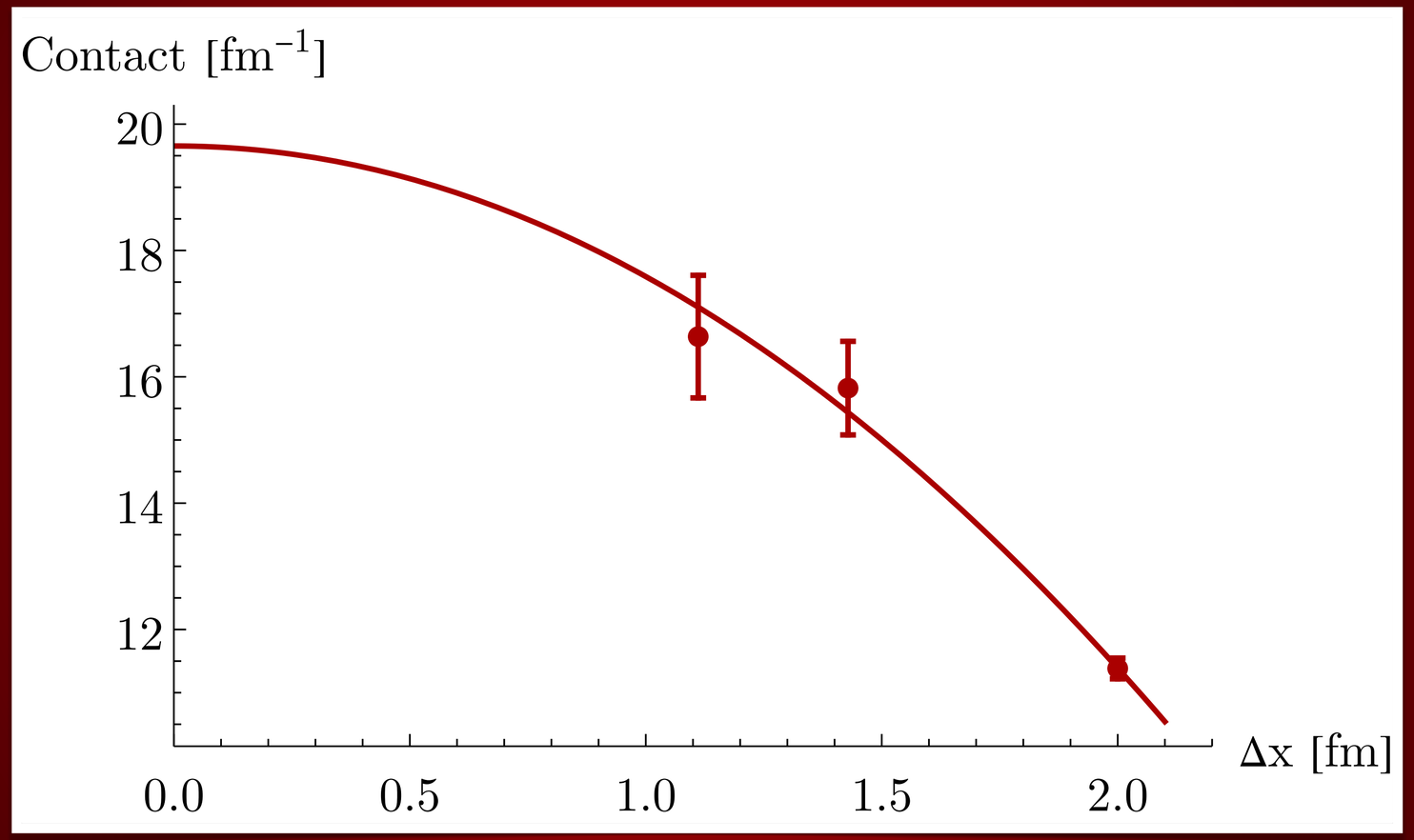
Proof of Principle:





Two particles

Proof of Principle:



~ 11 particles

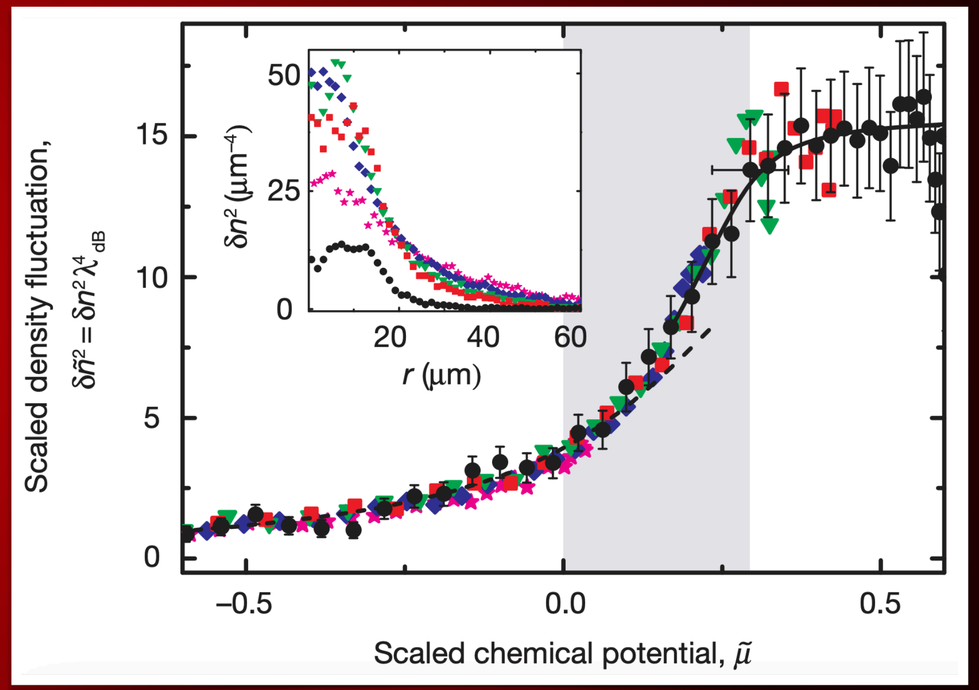
Wider applications

- Renormalization condition is general

$$\frac{d}{d\Delta x}\begin{bmatrix} E_1(C_0, C_2, \Delta x\,;\, V) \\ E_2(C_0, C_2, \Delta x\,;\, V) \end{bmatrix} = 0$$

- Only need short range interactions

- Higher-order improvement is trivial



Hung et. al. Nature (2011)

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THANK YOU
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Backup Slides

Modification of Tan Contact

Tan Energy Relation: $$-\frac{C}{4\pi M} = \frac{dE}{da^{-1}}$$

Tan, Annals of Physics (2008)

Braaten showed: $$-\frac{C}{4\pi M} = \langle \frac{\partial H}{\partial a^{-1}} \rangle = \langle \frac{\partial H}{\partial C_0} \frac{\partial C_0}{\partial a^{-1}} \rangle$$

Braaten & Platter PRL (2008)

$$= C_0^2 M^2 \, \langle \int d^3x \; n_1(x) n_2(x) \rangle$$

Modification of Tan Contact

Contact in improved theory:

$$-\frac{C}{4\pi M} = \langle \frac{\partial H}{\partial a^{-1}} \rangle = \langle \frac{\partial H}{\partial C_0}\frac{\partial C_0}{\partial a^{-1}} + \frac{\partial H}{\partial C_2}\frac{\partial C_2}{\partial a^{-1}} \rangle$$

Braaten operator

new operator