Structure Factors of Neutron Matter

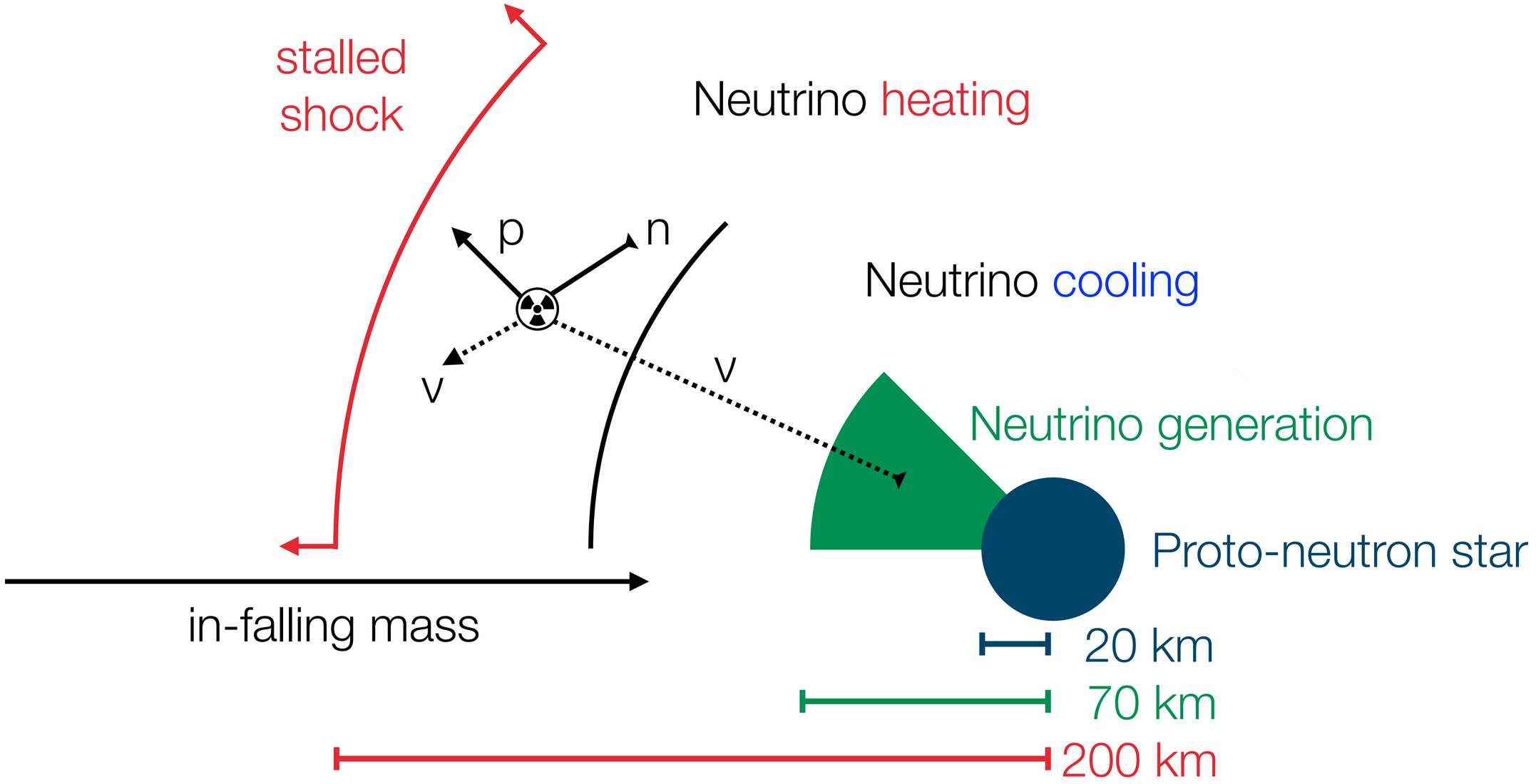
Alexandru, Bedaque, Berkowitz, & Warrington
PRL 126 (2021) 13, 132701
10.1103/PhysRevLett.126.132701
2008.02824

Evan Berkowitz
Forschungszentrum Jülich
11 August 2022
LATTICE 2022

See also: Symanzik Improvement of Non-Relativistic Field Theories
Neill Warrington
Next!

The Neutrinosphere

based on a figure by Dighe (2016)

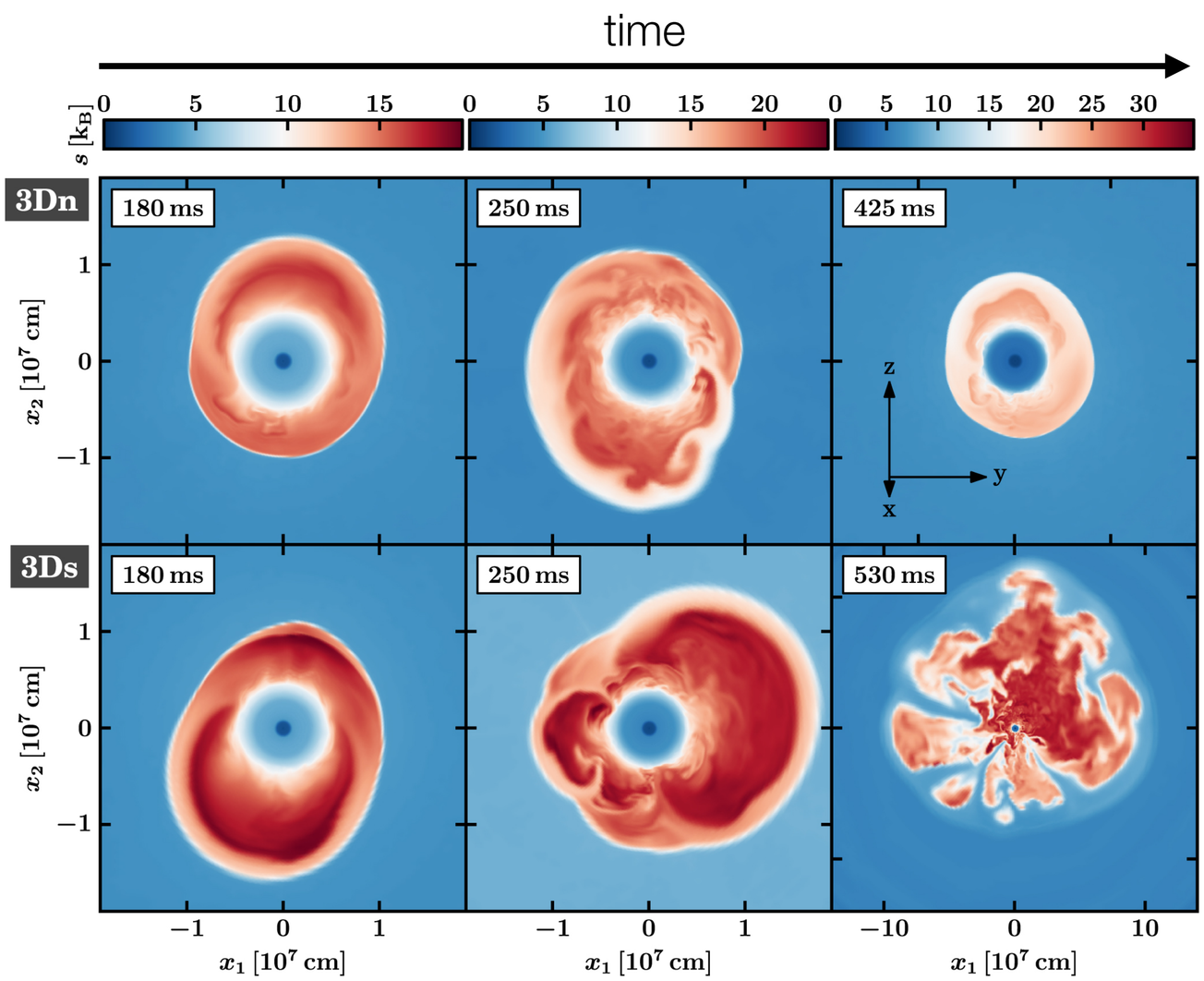


Pop, pop; Fizz fizz!

Melson, Janka, et al. ApJL 808 L42 (2015) 10.1088/2041-8205/808/2/L42

One-body effects

Additional effects (strangeness, many body, ...)



10 x bigger!

Pop, pop; Fizz fizz!

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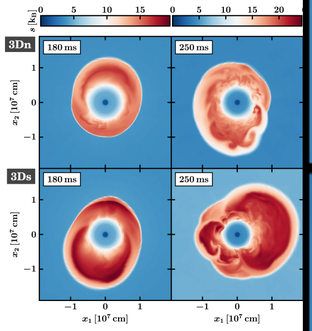
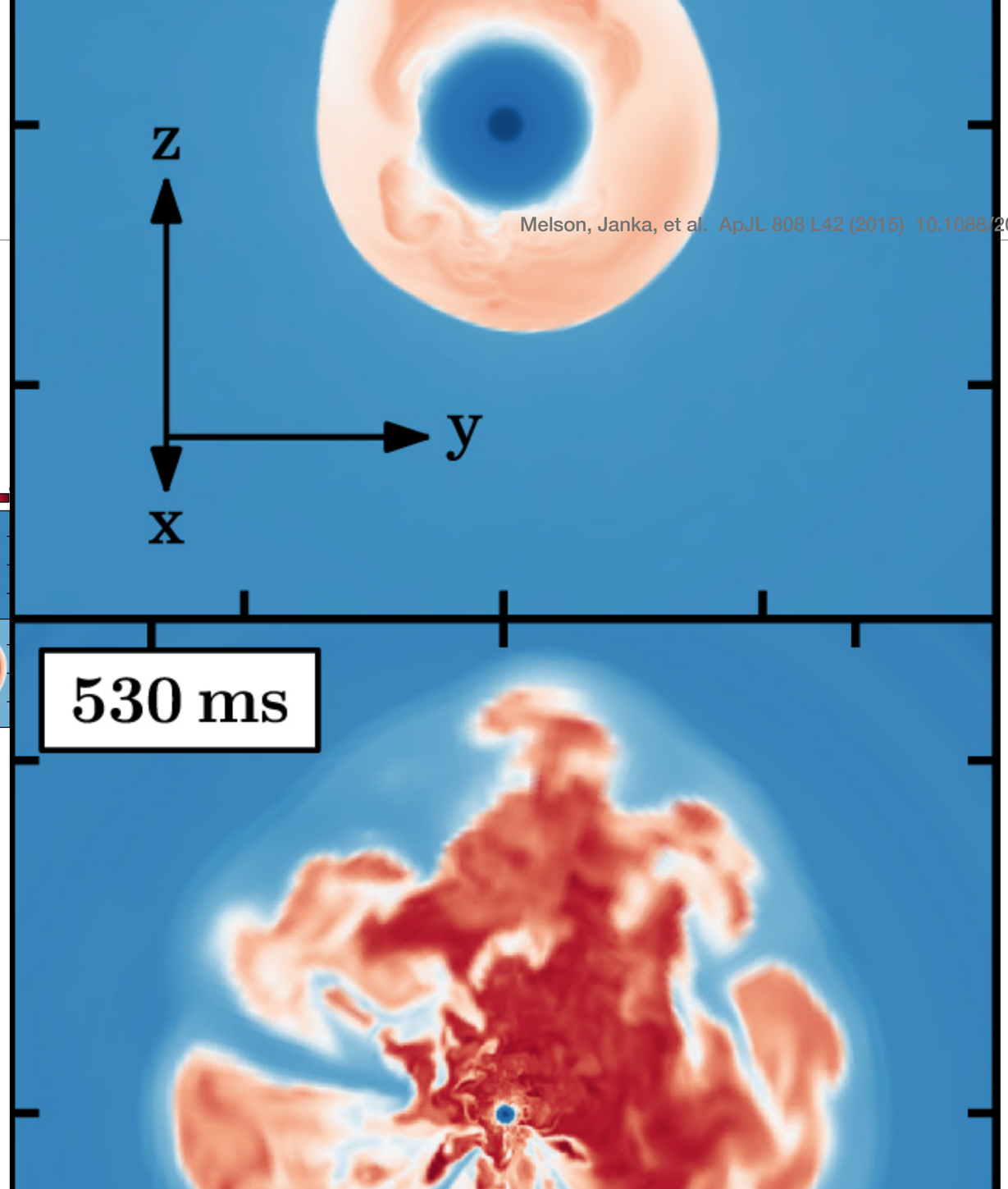
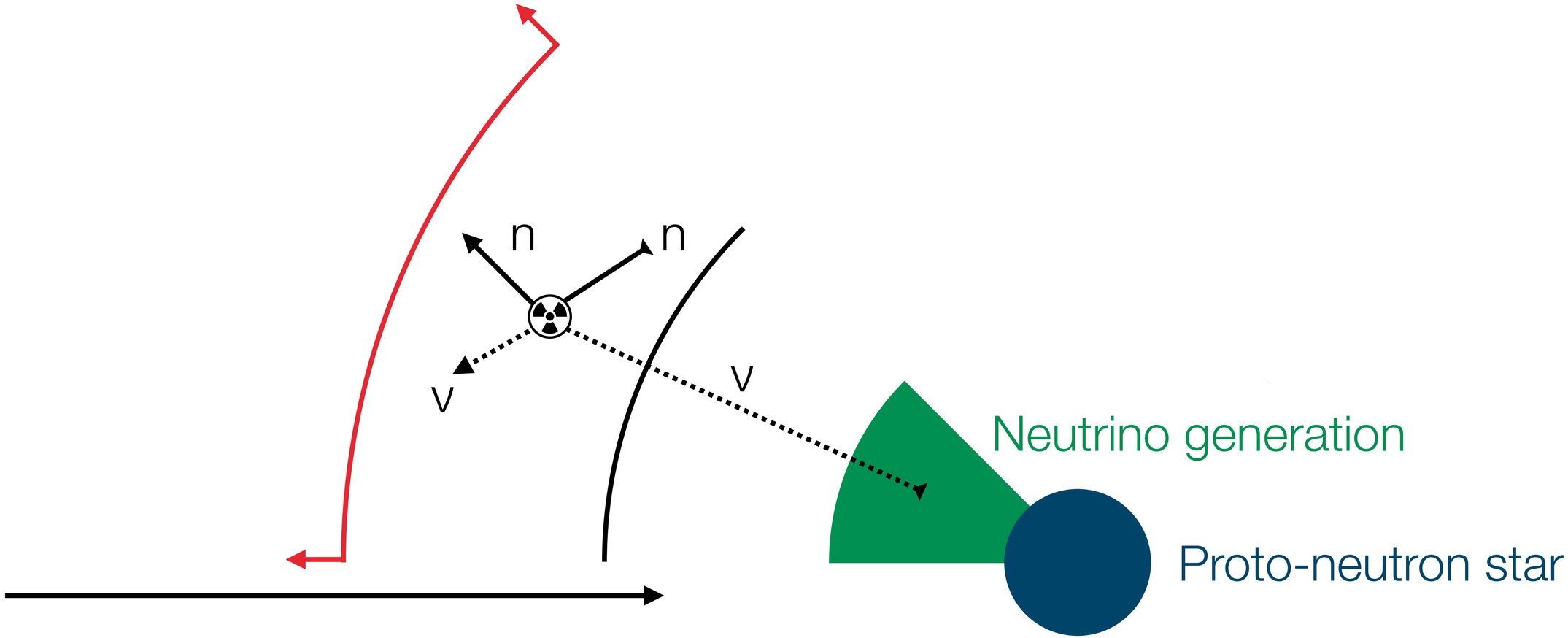




Target: response of neutrinosphere

Pethick & Iwamoto PRD 25 313 (1982) ,10.1103/PhysRevD.25.313
Horowitz & Schwenk PLB 638 (2006) 10.1016/j.physletb.2006.05.055



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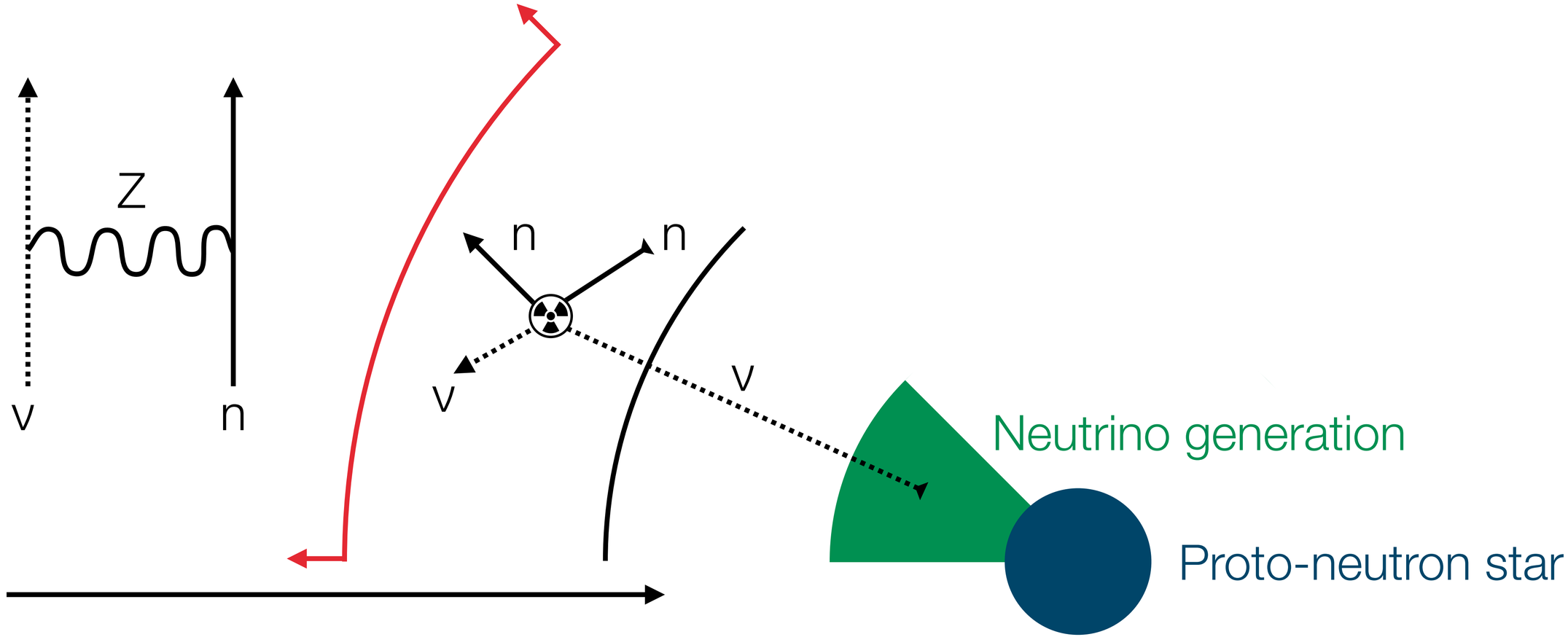

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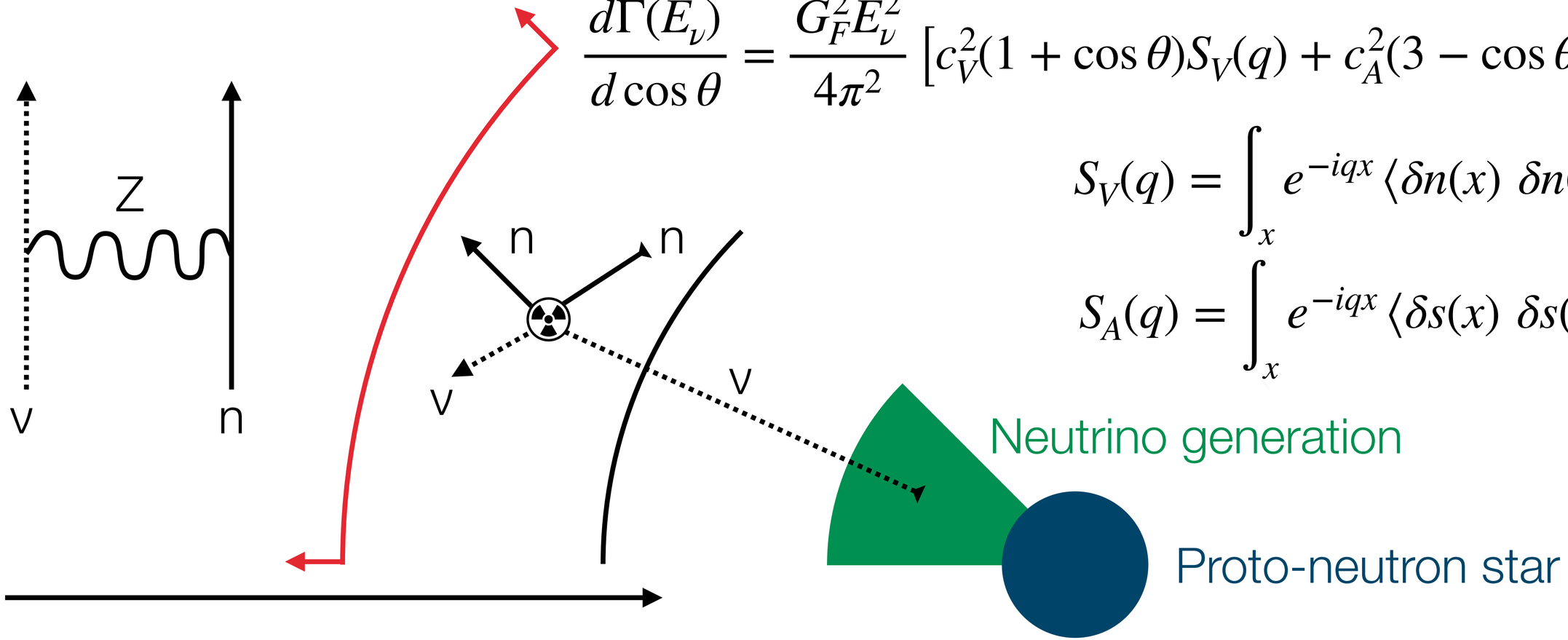
$$\frac{d\Gamma(E_\nu)}{d\cos\theta} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left[c_V^2(1+\cos\theta)S_V(q) + c_A^2(3-\cos\theta)S_A(q)\right]$$

$$S_V(q) = \int_x e^{-iqx}\langle\delta n(x)\ \delta n(0)\rangle$$

$$S_A(q) = \int_x e^{-iqx}\langle\delta s(x)\ \delta s(0)\rangle$$



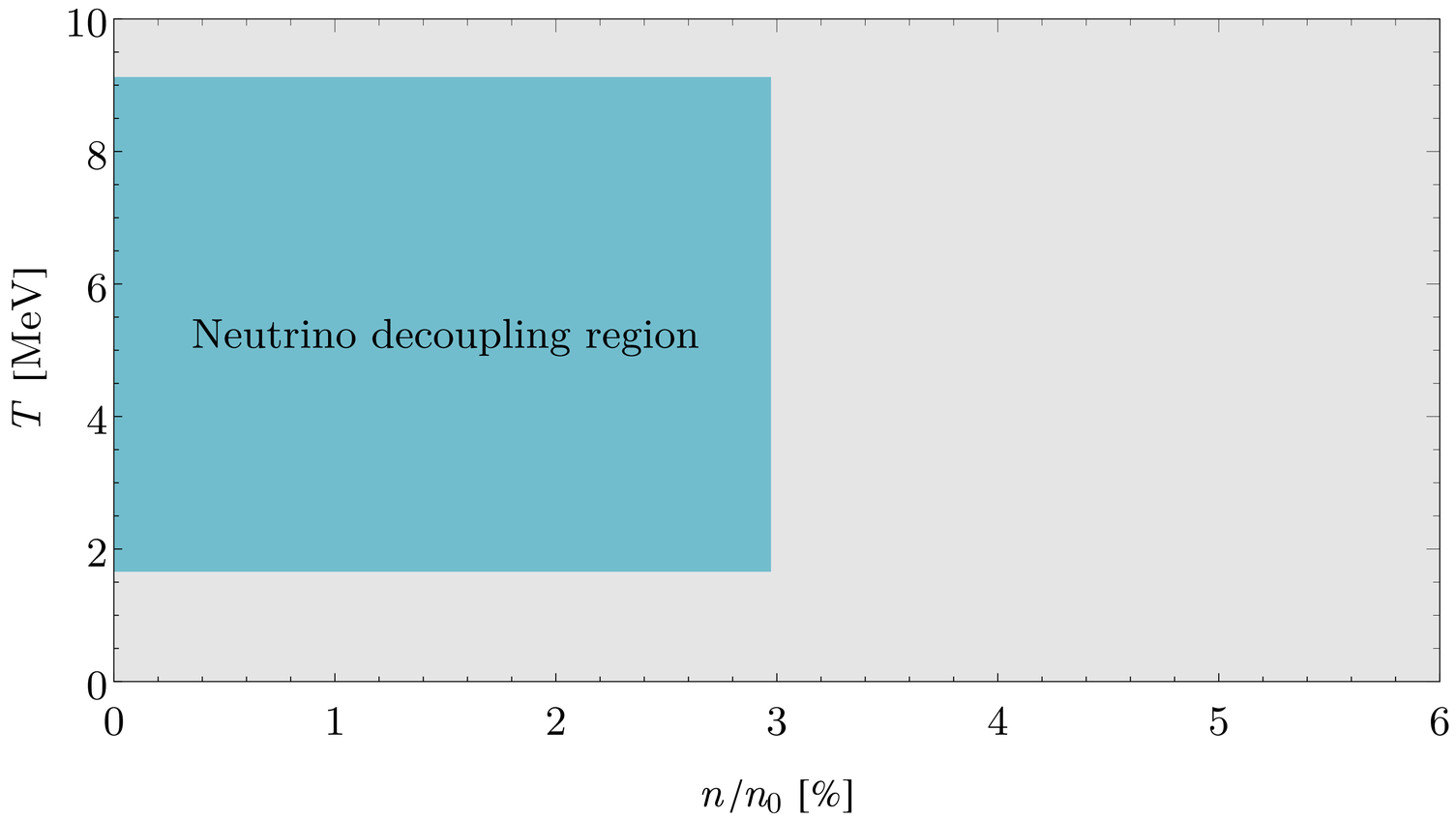
$\not{\pi}EFT$

Bedaque + van Kolck Ann. Rev. Nucl. Part. Sci. 52:339-396 (2002) 10.1146/annurev.nucl.52.050102.090637 nucl-th/0203055v1

$$H = \int_x \psi^\dagger \frac{\nabla^2}{2M}\psi + C_0(\psi^\dagger\psi)^2 + C_2(\psi^\dagger\psi)\,\nabla^2(\psi^\dagger\psi) + \cdots$$ converges when $k \lesssim m_\pi$

Also of interest for:

- cold atoms
- unitary gasses
- ...



Lattice Formulation

Alexandru, Bedaque, Warrington PRC 101 045805 (2020) 10.1103/PhysRevC.101.045805 1907.03914
Alexandru, Bedaque, Berkowitz, & Warrington PRL 126 (2021) 13, 132701 10.1103/PhysRevLett.126.132701 2008.02824

$$H = \int_x \psi^\dagger \frac{\nabla^2}{2M}\psi + C_0(\psi^\dagger\psi)^2 + C_2(\psi^\dagger\psi)\,\nabla^2(\psi^\dagger\psi) + \cdots$$

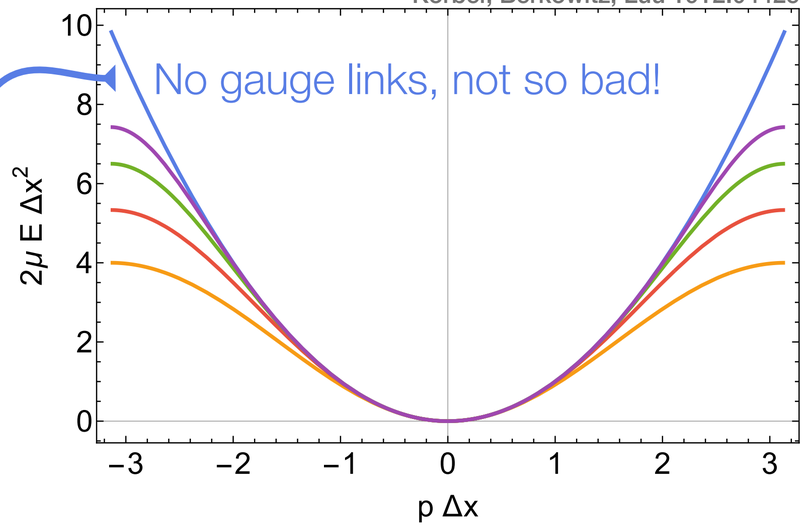
● C_0

See Neill's talk (next)

Kronecker delta ○ ○ ○ ○ ○ ○

Choice in dispersion,
pick the "perfect" p^2 one.

Körber, Berkowitz, Luu 1912.04425



$$\mathcal{Z} = \mathrm{tr}\,\exp - \beta(H - \mu N) \longrightarrow \propto \int \mathcal{D}A\; e^{-A^2/2\Delta t C_0}\; \det D^2$$

Hubbard-Stratanovich, integrate the ψs

No sign problem with finite μ!

Tuning $\not{\pi}EFT$ to the ERE

Kaplan, Savage + Wise Nucl. Phys. B534 329-355 (1998) 10.1016/S0550-3213(98)00440-4 nucl-th/9802075
Bedaque + van Kolck Ann. Rev. Nucl. Part. Sci. 52:339-396 (2002) 10.1146/annurev.nucl.52.050102.090637 nucl-th/0203055v1

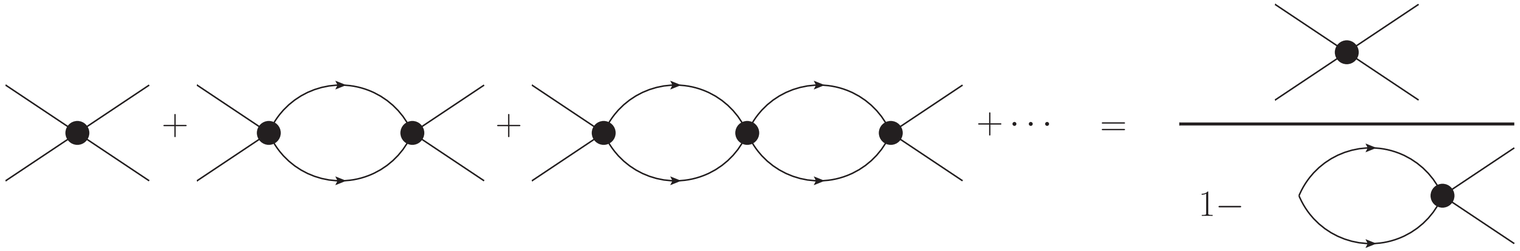
$$p\cot\delta = -\frac{1}{a_0} + \frac{1}{2}rp^2 + \cdots$$

Fix ERE parameters from experiment

Neutrons: $a_0 = -18.9$ fm, $r = 2.8$ fm ...

Unitary gas: $p \cot \delta = 0$

$$H = \int_x \psi^\dagger \frac{\nabla^2}{2M}\psi + C_0(\psi^\dagger\psi)^2 + C_2(\psi^\dagger\psi)\nabla^2(\psi^\dagger\psi) + \cdots$$



$$C_0 = -\frac{\Delta x}{M}\left(\alpha - \frac{\Delta x}{4\pi a}\right)^{-1}$$

known #

No Monte Carlo required!

Tuning $\not{\pi}EFT$ to the ERE

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Endres, Kaplan, Lee + Nicholson Phys. Rev. A84 043644 (2011) 10.1103/PhysRevA.84.043644 1106.5725

$$p\cot\delta = -\frac{1}{a_0} + \frac{1}{2}rp^2 + \cdots$$

0 when Δx → 0

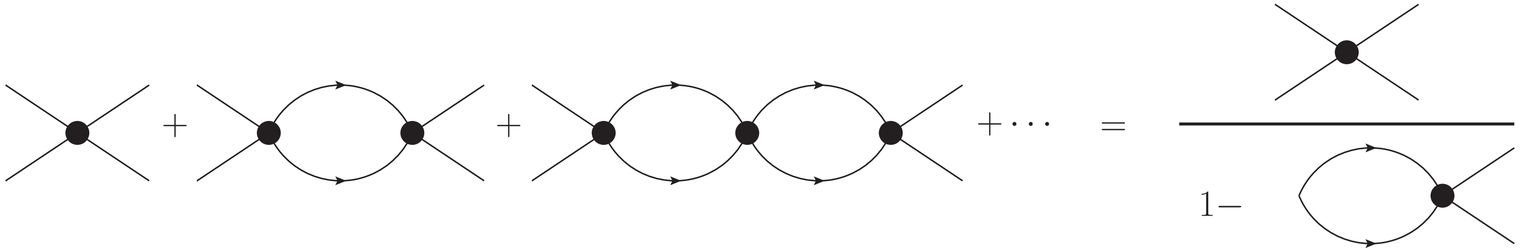
Fix ERE parameters from experiment

Neutrons: $a_0 = -18.9$ fm, r = ~~2.8~~ fm ...

Unitary gas: p cot δ = 0

$$H = \int_x \psi^\dagger \frac{\nabla^2}{2M}\psi + C_0(\psi^\dagger\psi)^2 + C_2(\psi^\dagger\psi)\nabla^2(\psi^\dagger\psi) + \cdots$$

0 Tuning further terms: Neill's talk (next)



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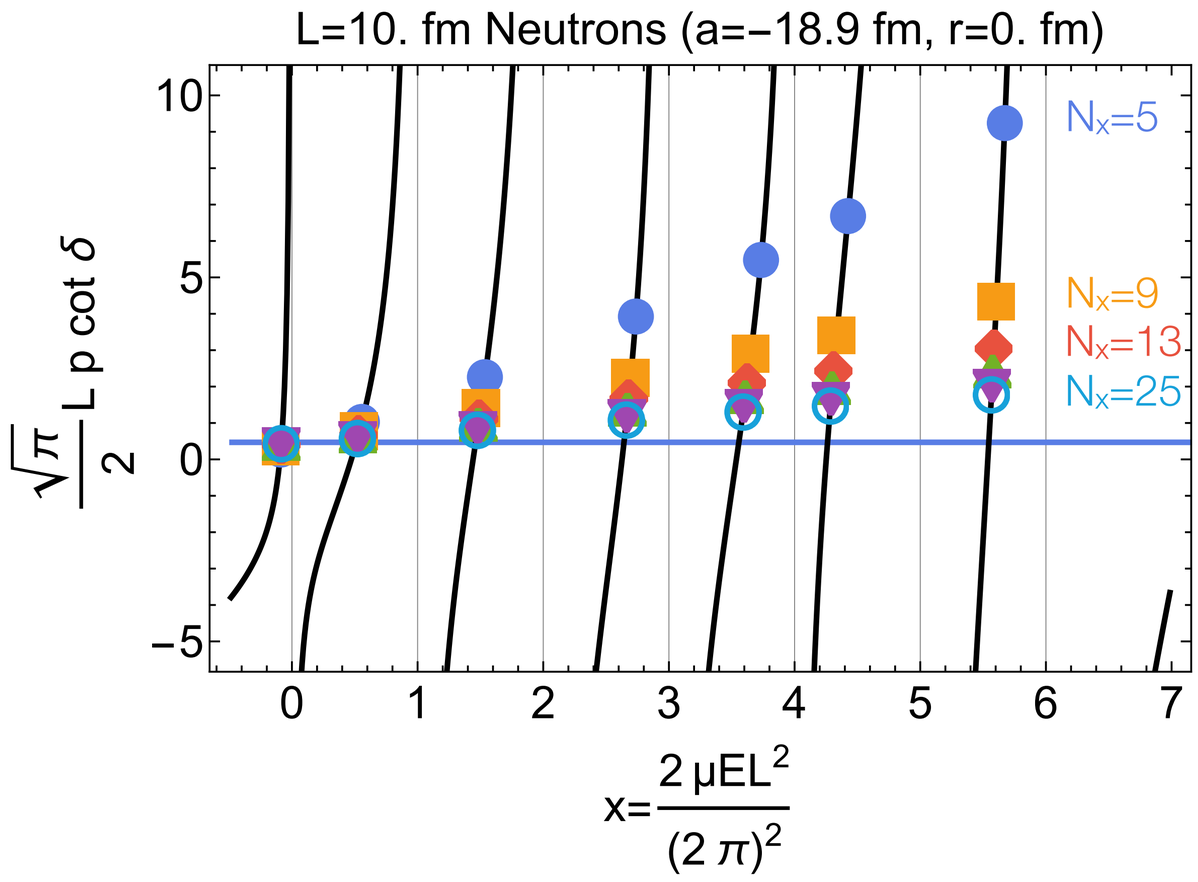
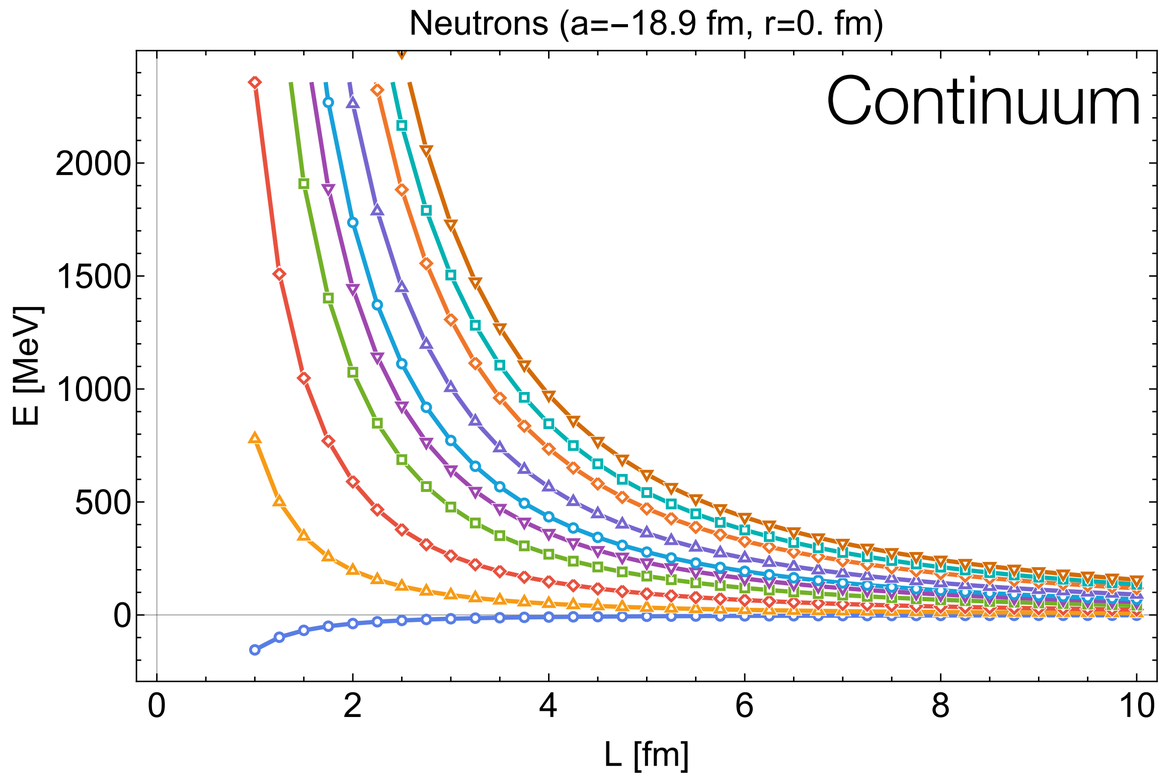
Two-Body A_{1g} Spectrum + Lüscher's Method





$$C_0 = -\frac{\Delta x}{M}\left(\alpha - \frac{\Delta x}{4\pi a}\right)^{-1} \qquad r \approx 0.33\Delta x$$

Observables

Drut, Lähde, + Ten PRL 106 205302 (2011) 10.1103/PhysRevLett.106.205302 1012.5474
Jensen, Gilbreth, + Y. Alhassid PRL 125 043402 10.1103/PhysRevLett.125.043402 1906.10117
Alexandru, Bedaque, Warrington PRC 101 045805 (2020) 10.1103/PhysRevC.101.045805 1907.03914
Alexandru, Bedaque, Berkowitz, + Warrington PRL 126 (2021) 13, 132701 10.1103/PhysRevLett.126.132701 2008.02824

Form Factors

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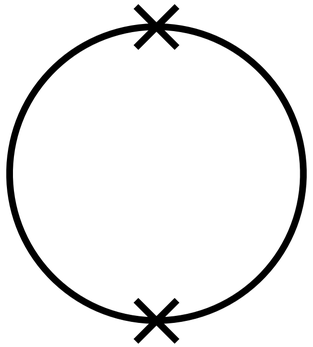
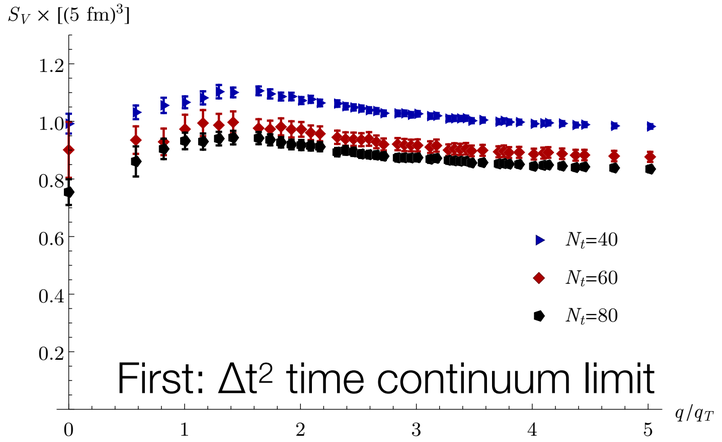




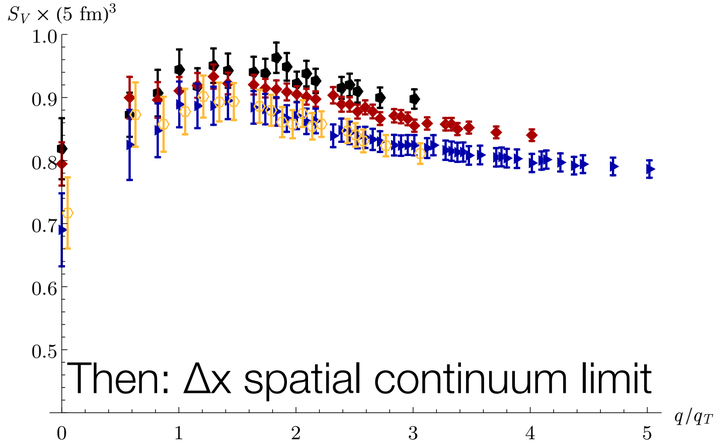
Contact

Tan Annals of Phys. 323 2952 (2008) 10.1016/j.aop.2008.03.004 cond-mat/0505200
Braaten + Platter PRL 100 205301 (2008) 10.1103/PhysRevLett.100.205301 0803.1125

$$C = \lim_{q\to\infty} q^4 n(q) \propto \left\langle -\frac{\partial H}{\partial a_0^{-1}} \right\rangle$$

~ counts how often two fermions share a site

$$S_{V/A}(q) = \langle n \rangle \pm \frac{C}{8Vq} + \mathcal{O}\left(q^{-2}\right)$$



Finally: infinite volume limit

Thermodynamic Results

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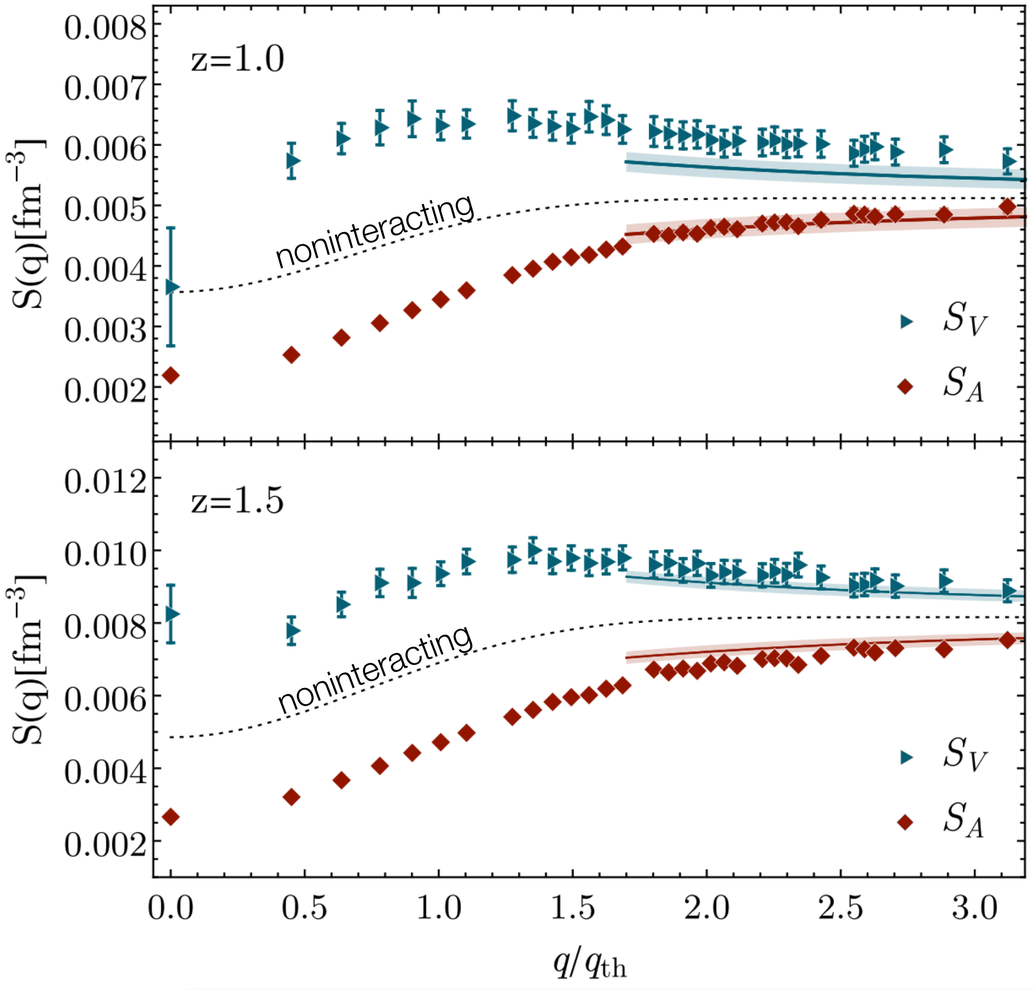

$$\langle n \rangle \pm \frac{C}{8Vq}$$

$$\langle n \rangle \pm \frac{C}{8Vq}$$

Nontrivial check!

Comparison

Outstanding Questions:
- Low temperature region
- Reduce computational cost
- Go beyond a_0

Neill's talk (next)

- Low-dimensional systems

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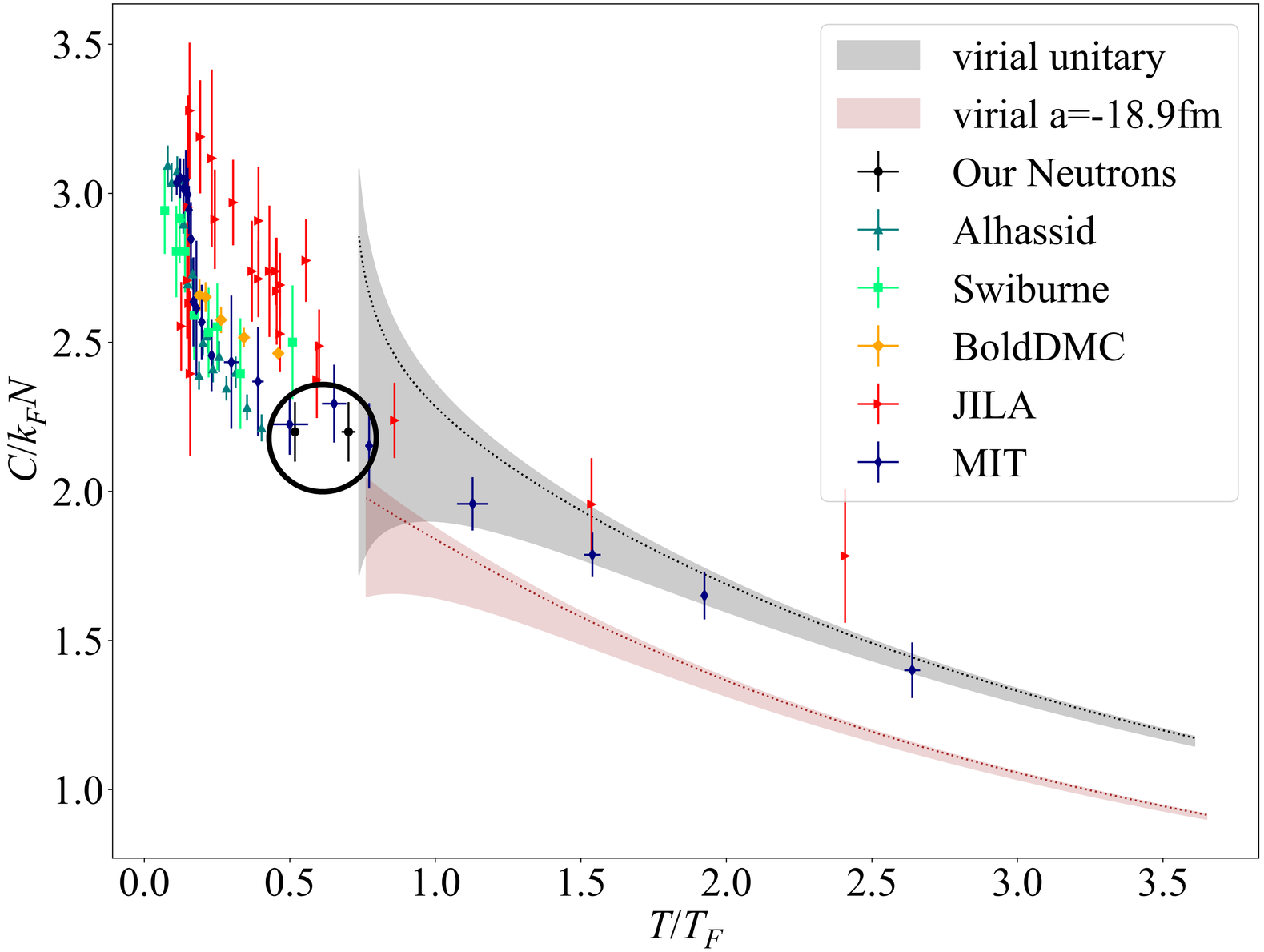
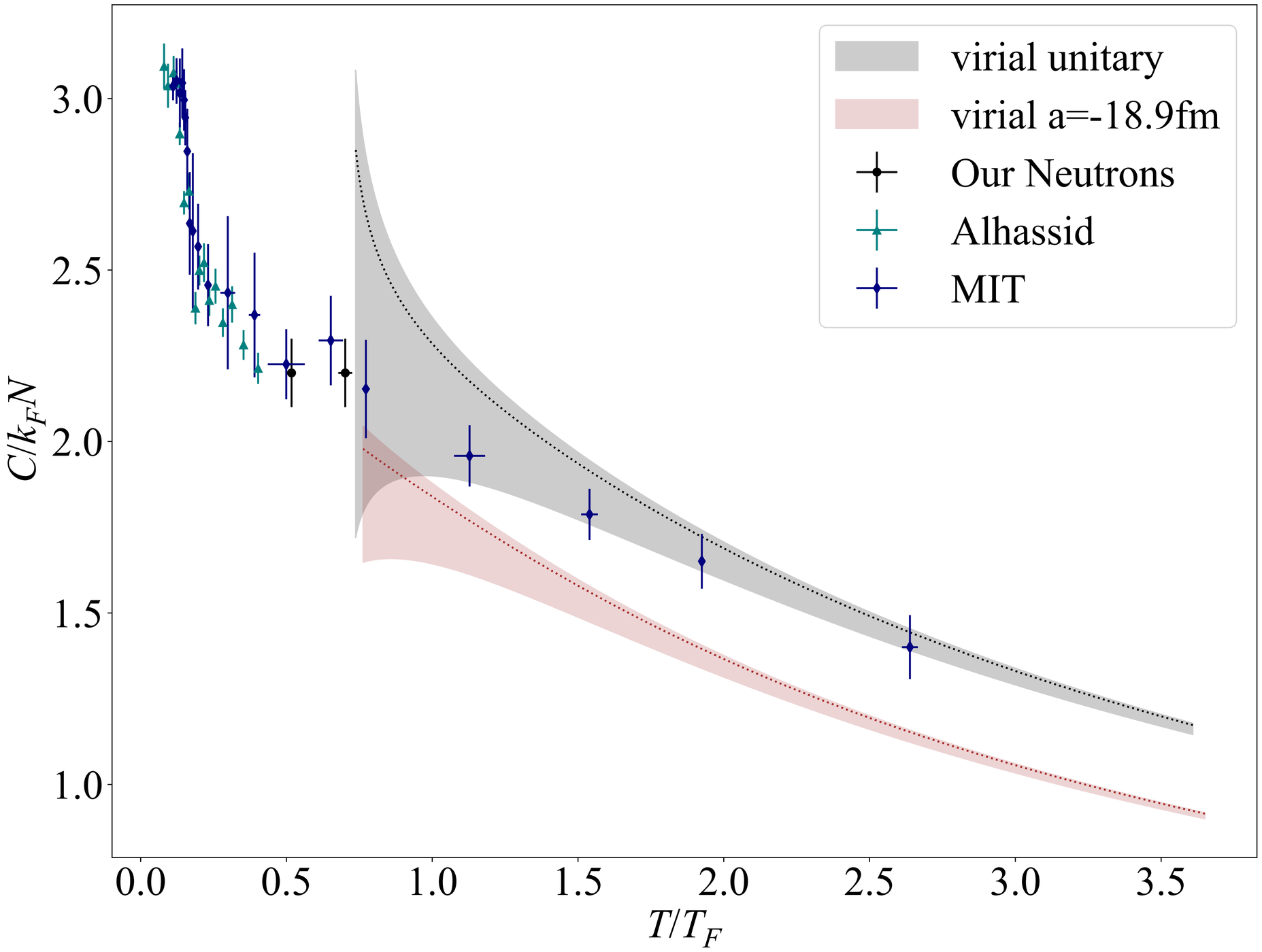


Backup slides



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