Nucleon-nucleon scattering from distillation

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Bonn, Germany

Nuclear physics

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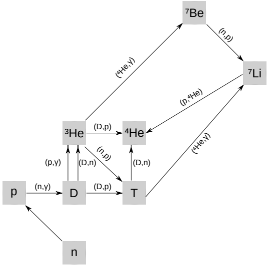

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Big Bang nucleosynthesis has *deuterium bottleneck*:
low deuteron binding energy 2.2 MeV delays onset of nucleosynthesis.
→ controls abundances of light elements.

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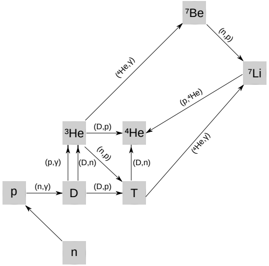

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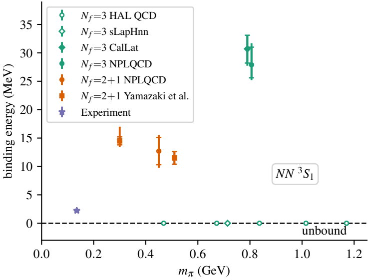
$\longrightarrow$ controls abundances of light elements.

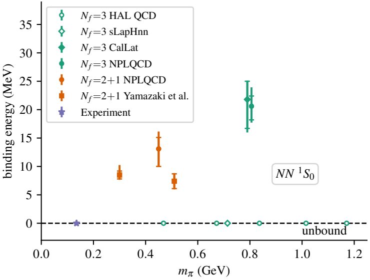
How strongly does deuteron binding depend on quark masses?
Could *pp* or *nn* bind?

Status: nucleon-nucleon scattering from LQCD

Controversy over presence of bound states at heavy quark masses.



deuteron



dineutron

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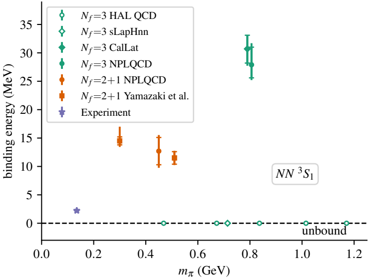

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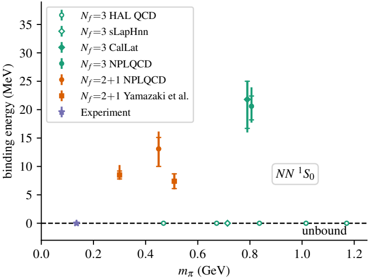

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No calculation performed using more than one lattice spacing.

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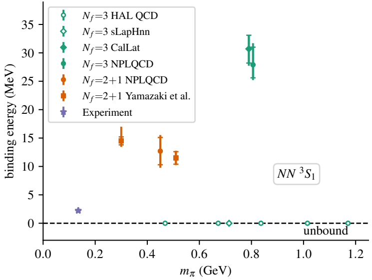

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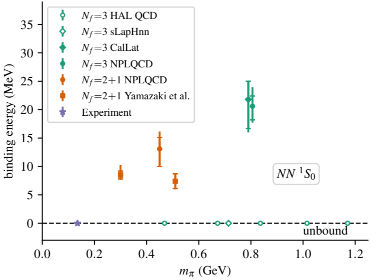

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All calculations that obtain bound states
use $\langle BB(t) H^\dagger(0) \rangle$ asymmetric correlation functions.

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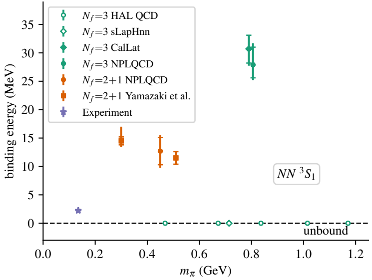

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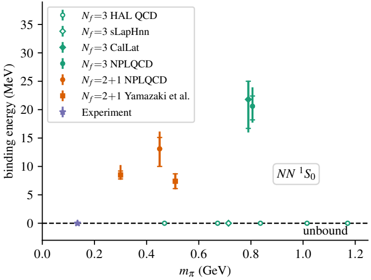

dineutron

See also recent perspective piece:

On the reliable lattice-QCD determination of multi-baryon interactions and matrix elements

R. Briceño, JRG, A. D. Hanlon, A. Nicholson, A. Walker-Loud,

Chapter 16 of Nuclear Forces for Precision Nuclear Physics – a collection of perspectives, arXiv:2202.01105

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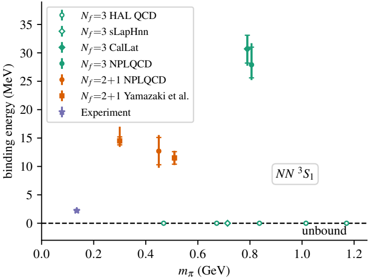

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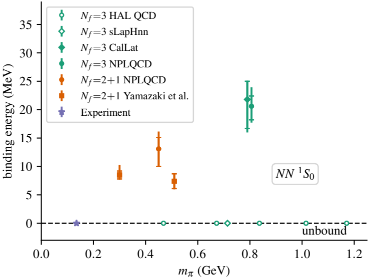


dineutron

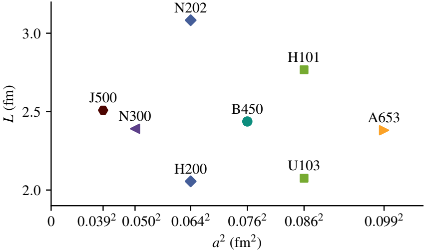
See also:

Michael Wagman, Thursday 11:30

Amy Nicholson, Friday 14:10

H dibaryon: discretization effects important!

Weakly bound H dibaryon from SU(3)-flavor-symmetric QCD
JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig, Phys. Rev. Lett. **127**, 242003 (2021)



Eight $N_f = 3$ clover ensembles from CLS.

SU(3)-symmetric point with physical $m_u + m_d + m_s$.

$m_\pi = m_K = m_\eta \approx 420$ MeV.

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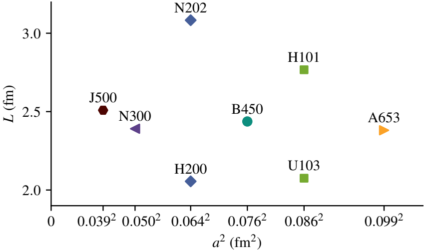

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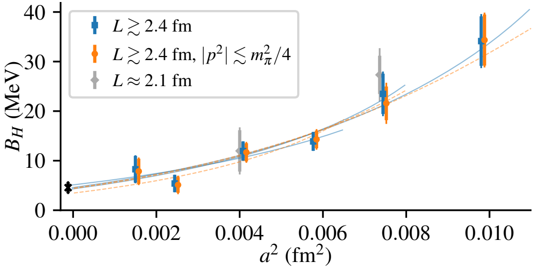
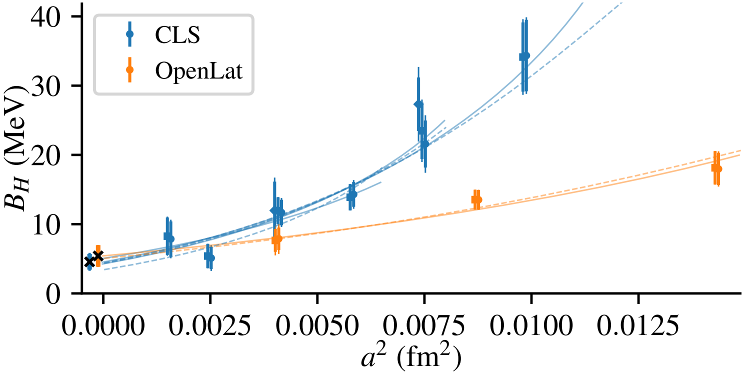
$m_\pi = m_K = m_\eta \approx 420$ MeV.

In continuum: $B_H = 4.56(1.13)(0.63)$ MeV.



H dibaryon: $a \to 0$ universality (PRELIMINARY)

BaSc: Baryon Scattering collaboration — combined effort of “Mainz” and sLapHnn



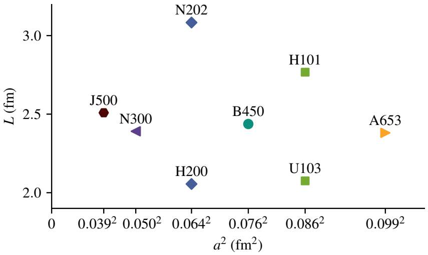
Three exp-clover ensembles with $L \approx 3$ fm.

Second action at SU(3) point: exponentiated clover / stabilized Wilson (OpenLat).
Smaller lattice artifacts than standard clover (CLS).

Outline: NN at SU(3) point on CLS lattices

1. Methodology: correlators and spectrum
2. Methodology: finite-volume quantization, $a \to 0$
3. $I = 1$, ${}^1S_0$ and dineutron
4. $I = 0$, ${}^3S_1$ and deuteron
5. uncoupled higher partial waves
6. $I = 0$, ${}^3S_1$–${}^3D_1$ mixing

Will use H200+N202 for analyzing data at single lattice spacing: $a = 0.064$ fm, $L = 2.1$ and 3.1 fm.



All results should be considered PRELIMINARY.

Correlators and spectrum

$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$, computed using distillation with baryon-baryon type operators:

$$\mathcal{O} \sim \sum_{\vec{x}_1,\vec{x}_2} e^{-i\vec{p}_1\cdot\vec{x}_1} e^{-i\vec{p}_2\cdot\vec{x}_2} qqq(\vec{x}_1)qqq(\vec{x}_2)$$

In each frame and irrep, systematically include one operator for every noninteracting level, up to a cutoff.

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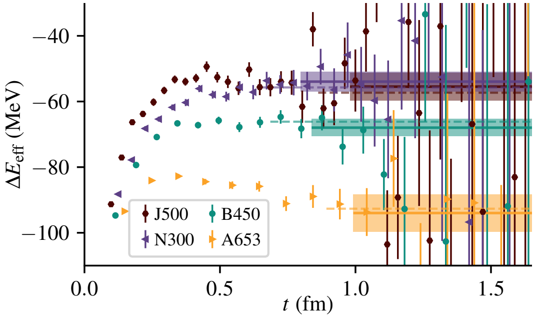
Solve GEVP and fit to ratio of diagonalized correlator,

$$R_n(t) \equiv \frac{\bar{C}_{nn}(t)}{C_N^{\vec{p}_1}(t) C_N^{\vec{p}_2}(t)},$$

to get energy difference from noninteracting level.



Quantization condition and continuum limit

$$\det\left[\tilde{K}^{-1}(p^2) - B(p^2)\right] = 0 \xrightarrow{\text{S wave}} p\cot\delta(p) = \frac{2}{\sqrt{\pi}L\gamma} Z_{00}^{\vec{P}L/(2\pi)}\left(1, \left(\frac{pL}{2\pi}\right)^2\right)$$

$B(p^2)$ computed using TwoHadronsInBox (C. Morningstar et al., 2017).

Fit to spectrum using ansatz for $p\cot\delta(p)$: e.g. $\sum_{i=0}^{N-1} c_i p^2$.

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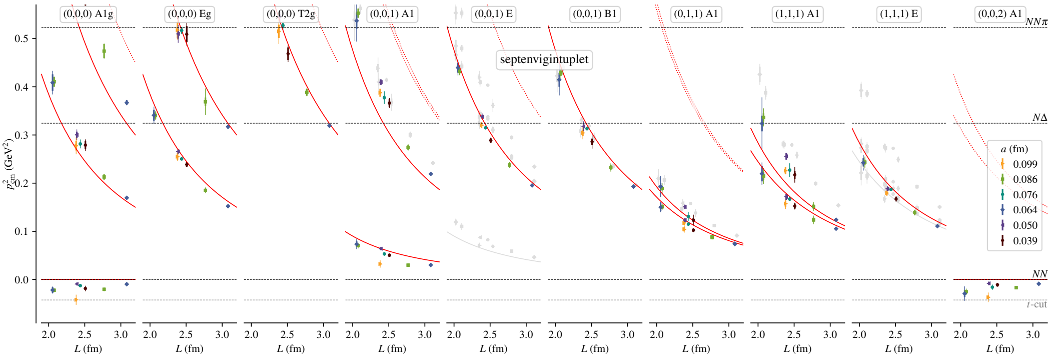
$$\begin{tikzcd}
E(L,a) \arrow[r, "\text{lattice quantization?}^*"] \arrow[d, "a \to 0"'] & \delta(p^2,a) \arrow[d, "a \to 0"] \\
E(L) \arrow[r, "\text{continuum quantization}"'] & \delta(p^2)
\end{tikzcd}$$

Continuum extrapolation:
follow blue path, applying continuum quantization condition at nonzero lattice spacing.

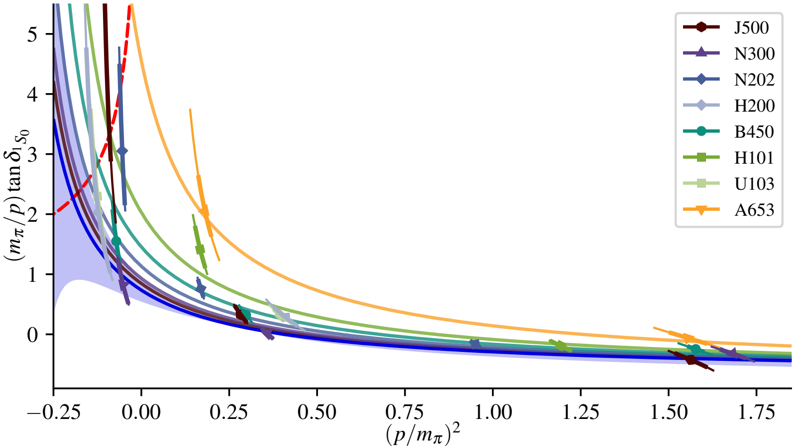
Combined fits to multiple lattice spacings:
let $c_i = c_{i0} + c_{i1}a^2$.

27-plet (NN $I = 1$): spin 0 spectrum



Operators constructed with definite spin. Spin-1 states (gray) identified via overlaps.
Quantization condition factorizes in spin. Here ${}^1S_0$ and ${}^1D_2$ are relevant.
Red curves: noninteracting levels.

$NN\ I = 1\ {}^1S_0$ phase shift



Levels from rest frame and first moving frame.

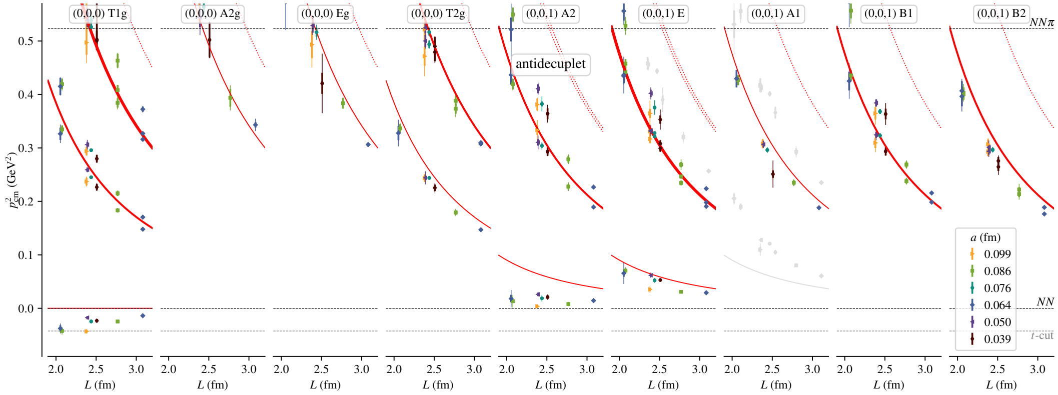
Fit using rational function

$$p\cot\delta(p) = \frac{c_0 + c_1 p^2}{1 + c_2 p^2}.$$

Virtual bound state.

Phase shift decreases in continuum limit.

Antidecuplet (NN $I = 0$): spin 1 spectrum (1)



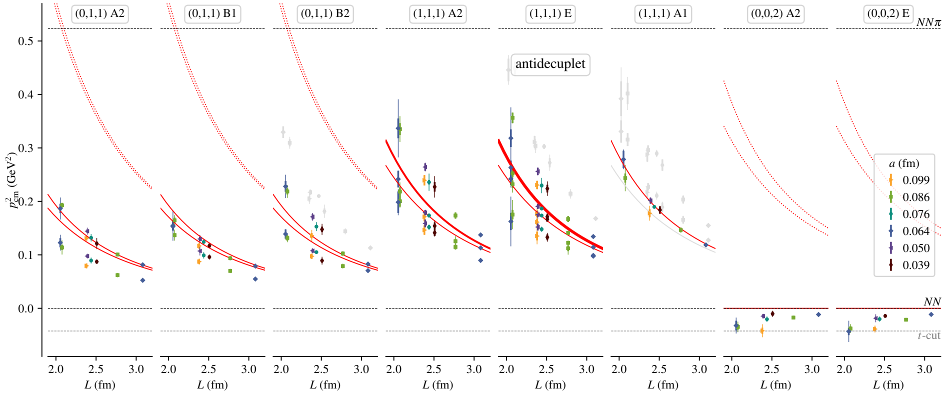
Spin-zero states shown in gray.

Thickness of red curves proportional to degeneracy of noninteracting level.

(39 levels) × (8 ensembles) = 312, although some lie above $NN\pi$ threshold.

${}^3S_1$, ${}^3D_1$, ${}^3D_2$, ${}^3D_3$ can be relevant.

Antidecuplet (NN $I = 0$): spin 1 spectrum (2)



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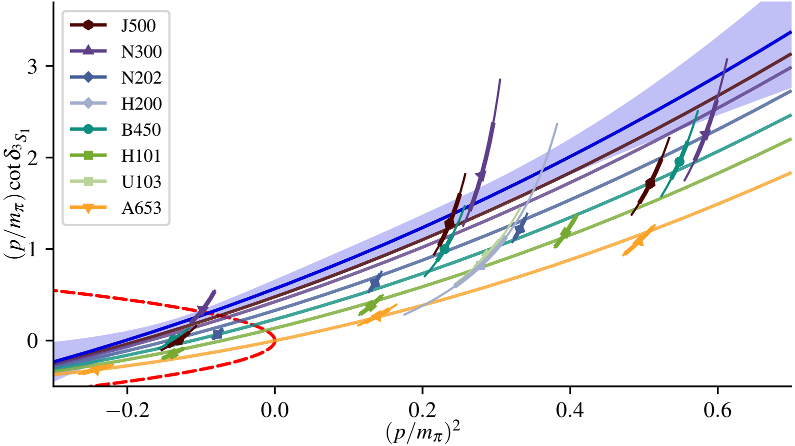
NN $I = 0$ $^3S_1$ phase shift



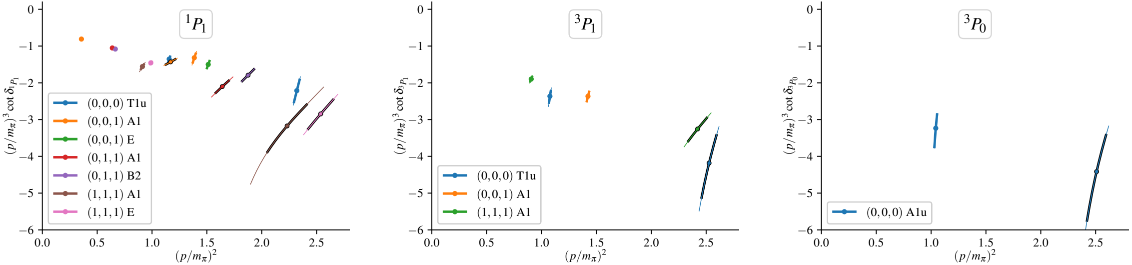
Helicity-averaged levels from first two moving frames. Neglect mixing with $^3D_1$.

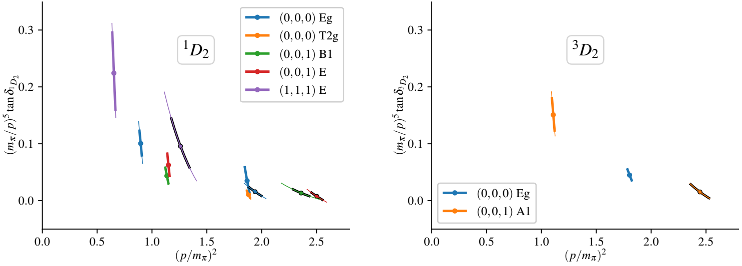
R. A. Briceño *et al.*, PRD **88**, 114507 (2013)

Virtual bound state.

Phase shift decreases in continuum limit.

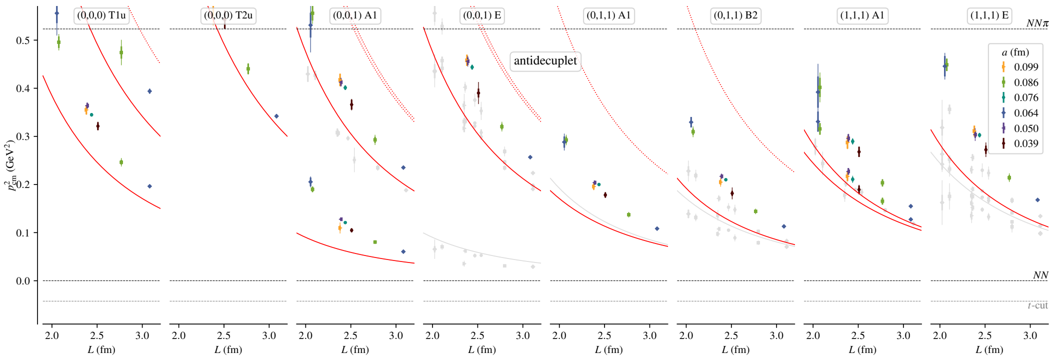
Uncoupled P and D waves





Two ensembles with
$a = 0.064$ fm:
$L = 48$ (solid)
$L = 32$ (black outline)

Antidecuplet (NN $I = 0$) spin 0 spectrum



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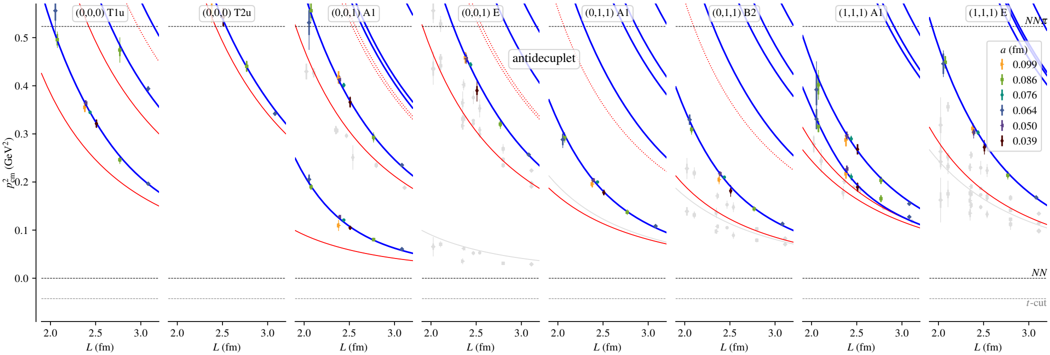

Fit 70 levels with 4 parameters:

$$p^3 \cot \delta_{^1P_1} = c_0 + c_1 p^2, \qquad p^7 \cot \delta_{^1F_3} = c_2 + c_3 p^8.$$

Good fit quality assuming no discretization effects.

Coupled partial waves

E.g. ${}^3S_1$–${}^3D_1$. Quantization condition:

$$0 = \det\left(\tilde{K}^{-1} - B\right).$$

Blatt-Biedenharn parametrization:

$$\tilde{K}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & -\sin\epsilon_1 \\ \sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} p\cot\delta_{1\alpha} & 0 \\ 0 & p\cot\delta_{1\beta} \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & \sin\epsilon_1 \\ -\sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix}.$$

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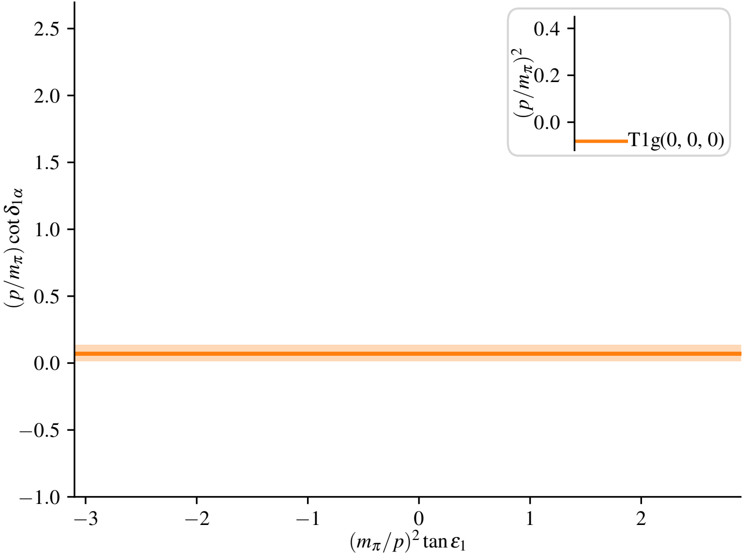
Each energy level imposes constraint

$$p\cot\delta_{1\alpha} = \frac{B_{00} + (B_{01} + B_{10})x + B_{11}x^2}{1 + p^4x^2}, \qquad x = p^{-2}\tan\epsilon_1.$$

$\delta_{1\alpha}$ and ϵ_1 on N202



Assume $\delta_{1\beta} = 0$.
Also neglect ${}^3D_2$, ${}^3D_3$.

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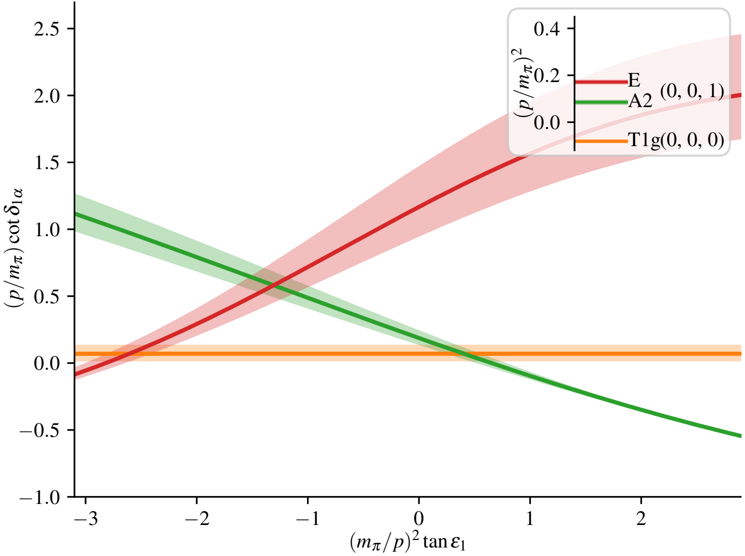

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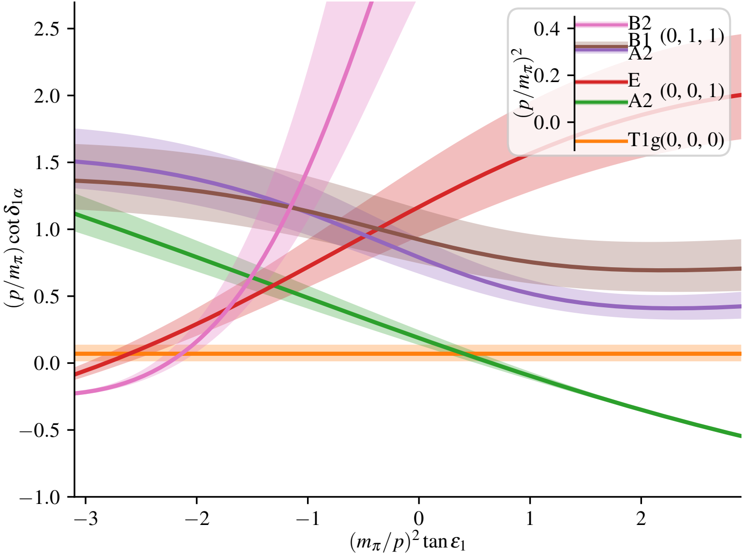

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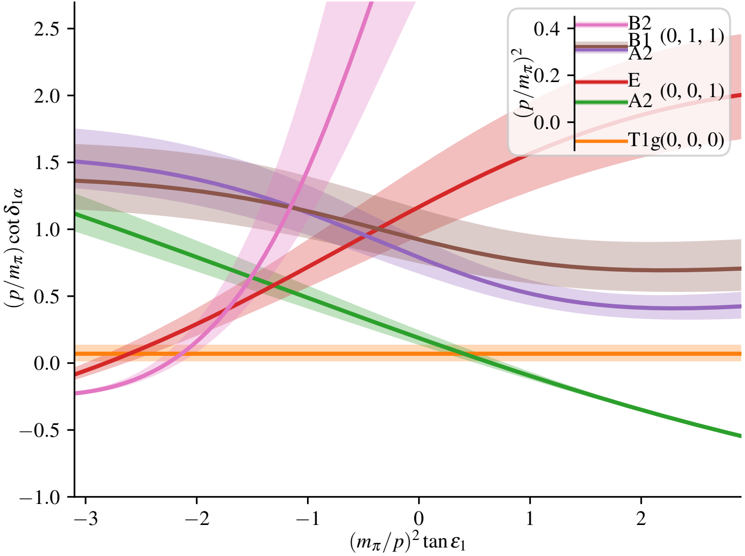
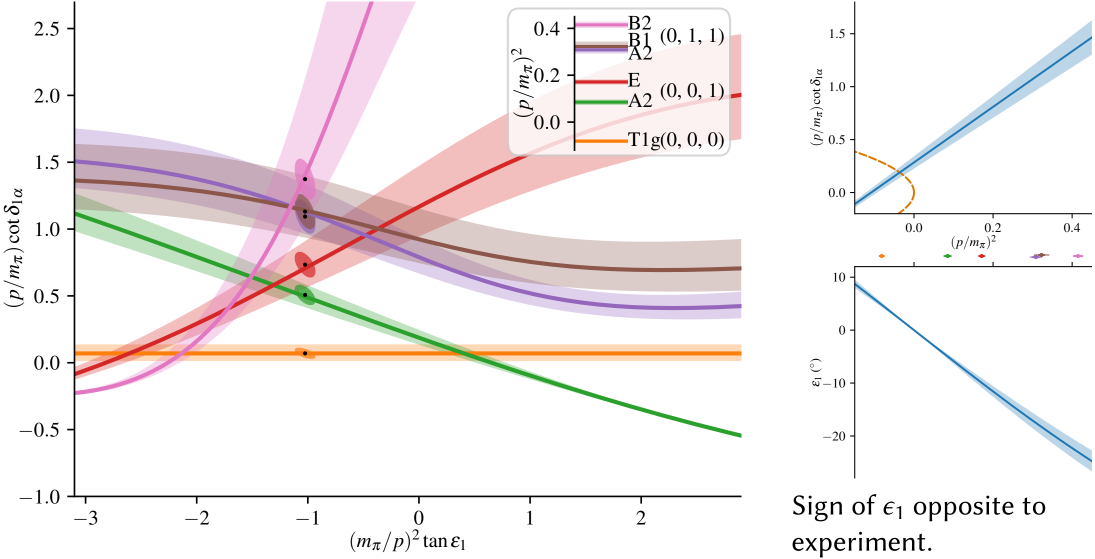


Assume $\delta_{1\beta} = 0$.
Also neglect ${}^3D_2$, ${}^3D_3$.

Fit spectrum using

$$p\cot\delta_{1\alpha} = c_1 + c_2 p^2,$$
$$p^{-2}\tan\epsilon_1 = c_3.$$

$\delta_{1\alpha}$ and ϵ_1 on N202



Sign of ϵ_1 opposite to experiment.

Summary / outlook

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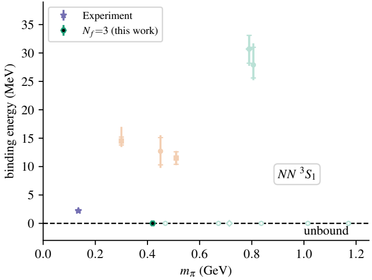
Summary / outlook

- Variational method with baryon-baryon operators yields many *NN* energy levels.
- Continuum limit important for *NN*. Physical $B_d = 2.2$ MeV will be challenging.
- With CLS action, lattice artifacts strengthen *S*-wave baryon-baryon interactions.
- No *NN* bound state at $m_\pi = m_K \approx 420$ MeV.



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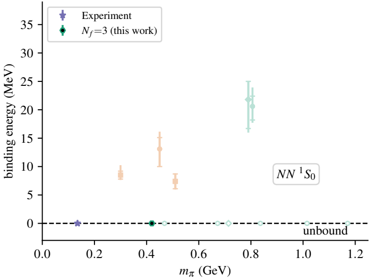

dineutron

Octet baryon and H dibaryon: $a \to 0$ on exp-clover