Fourier acceleration in strongly-interacting linear sigma models

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Background - The Model

- We study the linear sigma model in Euclidean spacetime with four scalar fields

$$\mathcal{L}(x) = \frac{1}{2}\sum_{\nu,i}(\partial_\nu \phi_i(x))^2 - \frac{\mu^2}{2}\sum_i \phi_i(x)^2 + \frac{\lambda}{4!}\left(\sum_i \phi_i(x)^2\right)^2 + \alpha\phi_0(x).$$

- The SO(4) symmetry is both explicitly and spontaneously broken.
- We can use this model as an effective theory for two-flavor QCD.

Background - HMC Simulations

- Following the Hybrid Monte Carlo algorithm, we use the Hamiltonian

$$H = \frac{1}{2m}\Pi^2 + S(\phi),$$

  where $S(\phi)$ is the action of our linear sigma model.
- To generate an ensemble, we choose a random momentum field $\Pi(x) \sim e^{-\frac{1}{2m}\Pi^2}$ and evolve the system with H for one unit of time.
- We then repeat, using a different random momentum for each "trajectory."
- Optimally, we want little correlation between the initial and final fields.

Background - Fourier Acceleration

- Suppose that each mode $\phi_i(p)$ undergoes approximately harmonic oscillation about its equilibrium point.
- If this is the case, we can take advantage of this by introducing a momentum-dependent mass term $m(p)$.
- The kinetic term in the HMC Hamiltonian becomes

$$\frac{1}{2}\sum_{i,p}\frac{1}{2m_i(p)}|\Pi_i(p)|^2 = \sum_{i,x<y} G_i(x-y)\Pi_i(x)\Pi_i(y),$$

  where the form of $G(x-y)$ depends on the specific form of $m(p)$.
- We can choose each $m(p)$ so that the frequency of every mode is $\omega_p = \frac{\pi}{2}$.
- Assuming harmonic evolution, $\phi_i(p) = \text{Re}\left[Ae^{i\omega t}\right]$, and $P_i(p) = \text{Re}\left[i\omega m A e^{i\omega t}\right]$.
- We can show, over one unit of molecular dynamics time, that

$$P_{i,initial}(p) = \omega m \cos(\omega - \pi/2)\phi_{i,final}(p) - \sin(\omega - \pi/2)P_{i,final}(p).$$

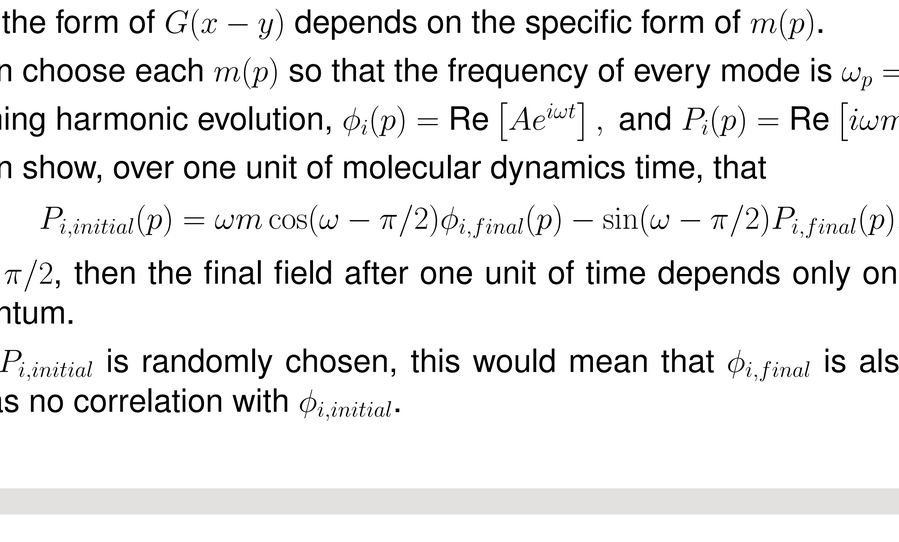
- If $\omega = \pi/2$, then the final field after one unit of time depends only on the initial momentum.
- Since $P_{i,initial}$ is randomly chosen, this would mean that $\phi_{i,final}$ is also random and has no correlation with $\phi_{i,initial}$.

Autocorrelation Length

- We find that Fourier acceleration can be effective at reducing the autocorrelation length.

$8^3 \times 16$ lattice, $\mu^2 = 0.4$, $\lambda = 1.0$, $\alpha = 0.1$

$m_\pi \approx 0.28 \pm 0.03$, $m_\sigma \approx 0.46 \pm 0.07$, $F_\pi \approx 1.67 \pm 0.25$



Choosing the Mass Terms

- We choose masses using $m_i(p) = \frac{1}{\omega^2}\frac{\langle|\dot{\Pi}_i(p)|\rangle}{\langle|\phi_i(p) - \sigma_{vev}\delta_{i,0}\delta_{p,0}|\rangle}$, where the averages are taken over the course of the trajectory, and σ_{vev} is the vacuum expectation value of $\phi_0(p=0)$ (all other modes have zero v.e.v).
- We run a set of trajectories, all with the same masses, and then update the masses with the average estimated masses over the previous trajectories.

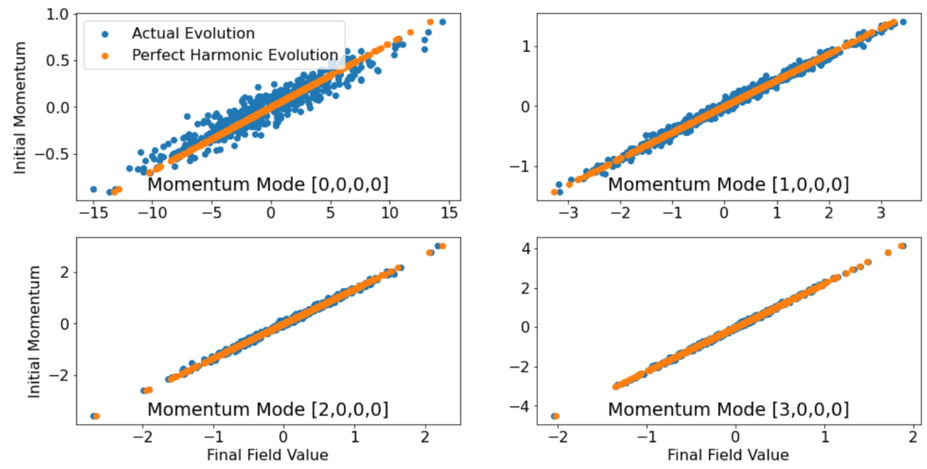
Evolution of Modes

- We find that the assumption of harmonic evolution is approximately valid for high momentum modes when λ is not too large.

Pion Field at End of Trajectory versus Momentum at Start

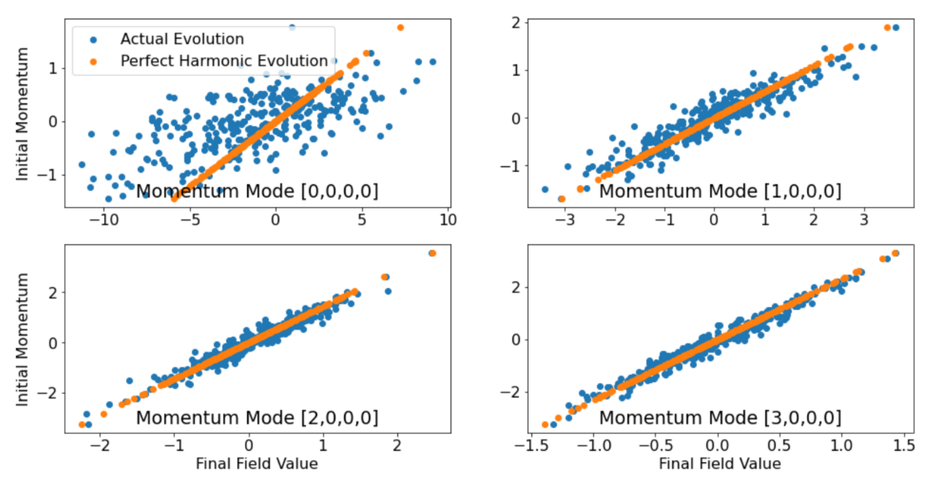
$8^3 \times 16$ lattice, $\mu^2 = 0.4$, $\lambda = 1.0$, $\alpha = 0.1$

$m_\pi \approx 0.28 \pm 0.03$, $m_\sigma \approx 0.46 \pm 0.07$, $F_\pi \approx 1.67 \pm 0.25$



$8^3 \times 16$ lattice, $\mu^2 = 2.0$, $\lambda = 5.0$, $\alpha = 0.1$

$m_\pi \approx 0.34 \pm 0.04$, $m_\sigma \approx 0.86 \pm 0.22$, $F_\pi \approx 1.22 \pm 0.27$



Error Reduction Method

- If we assume perfect harmonic evolution, for a given trajectory, we can predict final field values given only the initial momentum.
- On each trajectory, we can measure the difference between the actual final field values and the predicted ones. This difference may be less noisy than the field values themselves.
- Then, we can quickly generate many additional "predicted" fields by sampling additional "initial" momenta.
- We can take measurements on these new "predicted" fields and correct them using the calculated difference between predicted and actual fields.
- e.g. On a $8^3 \times 16$ lattice with $\mu^2 = 0.4$, $\lambda = 1.0$, and $\alpha = 0.1$, we get

$$m_\pi \approx 0.30 \pm 0.04$$

  measuring on 20 samples, and

$$m_\pi \approx 0.29 \pm 0.02$$

  measuring on 20 samples with 1000 additional "predicted" measurements.
- e.g. On a $8^3 \times 16$ lattice with $\mu^2 = 2.0$, $\lambda = 5.0$, and $\alpha = 0.1$, we get

$$m_\pi \approx 0.32 \pm 0.07$$

  measuring on 20 samples, and

$$m_\pi \approx 0.31 \pm 0.05$$

  measuring on 20 samples with 1000 additional "predicted" measurements.

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