Quantum Computing for Open Systems

Jay Hubisz, **Bharath Sambasivam**, Judah Unmuth-Yockey (2021), "*Quantum algorithms for open lattice field theory*". Physical Review A, 104(5), 052420.

Erik Gustafson, Michael Hite, Jay Hubisz, **Bharath Sambasivam**, Judah Unmuth-Yockey *Work in Progress*

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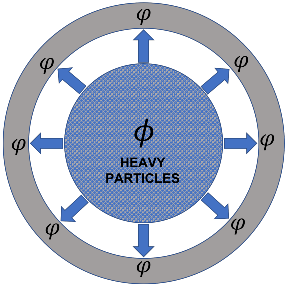
Motivating Open Quantum Systems

- Situations where only part of physical system is accessible/ is of interest
- Well described/ approxed. by EFTs (sometimes with NH Hamiltonians)
- e.g: FTs at complex couplings (μ, topological terms)
- Have rich phase structure: Lee-Yang edges, Fisher Zeros
- Classical simulation hard because of Sign problem



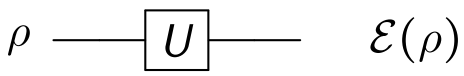
This work

Construct NISQ-era algorithms for Open quantum systems and apply it to the 1-D quantum Ising Model with an imaginary longitudinal magnetic field

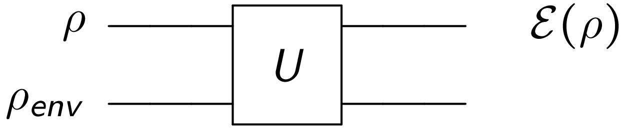
Quantum Operations

$$\mathcal{E} : \rho \longrightarrow \rho'$$

Closed system



Open system



- Trace out environment $\longrightarrow$ system evolves via $\mathcal{E}$
- QOs can be written in an operator-sum representation:

$$\begin{aligned}\mathcal{E}(\rho) &= \sum_k \langle e_k| \hat{U}\big[\rho \otimes |e_0\rangle\langle e_0|\big]\hat{U}^\dagger |e_k\rangle \\ &= \sum_k \hat{E}_k \rho \hat{E}_k^\dagger, \quad \text{where}\end{aligned}$$

$$\hat{E}_k = \langle e_k| \hat{U} |e_0\rangle, \quad (\textit{Kraus Operators})$$

Measurement and Probabilities

- QOs make system-environment entanglement transparent
- Measurement $\{|e_k\rangle\}$ of $\rho_{env} \equiv \rho \longrightarrow \frac{\hat{E}_k \rho \hat{E}_k^\dagger}{p_k}$, $k \in 0, \cdots, N$ with

$$p_k = \mathrm{Tr}\left(\hat{E}_k \rho \hat{E}_k^\dagger\right)$$

- ρ_{env} needs $\log_2(N+1)$ ancillary qubits
- Measuring ancillas $\longrightarrow$ which Kraus operator has acted on system
- Trace preserving condition $\sum_k \hat{E}_k^\dagger \hat{E}_k = \mathbb{1}$ guarantees
    1. $\sum_k p_k = 1$
    2. Unitarity of System + Environment evolution

General idea

The $\hat{E}_k$'s need not be Unitary! Make one of them align with desired evolution of Open system, while leaving environment unchanged

Example: $K_0 - \bar{K}_0$ system[1]

- $K_0 \longrightarrow \bar{K}_0$; $\bar{K}_0 \longrightarrow K_0$; $K_0 / \bar{K}_0 \longrightarrow$ light states $\equiv |1\rangle$ (weak)

$$\hat{H} = \hat{G} - i\frac{\hat{\Gamma}}{2}$$

- initial: $\rho = \rho_{K_0\bar{K}_0} \otimes |0\rangle\langle 0| + \left(1 - Tr\left(\rho_{K_0\bar{K}_0}\right)\right) \otimes |1\rangle\langle 1|$
- Evolution using a trace preserving Quantum Operation

$$\hat{E}_0 = \begin{pmatrix} e^{\left(-i\hat{G}-\frac{\hat{\Gamma}}{2}\right)t} & \vec{0} \\ \vec{0}^\dagger & 1 \end{pmatrix}, \hat{E}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{1-e^{-\Gamma_1 t}} & 0 & 0 \end{pmatrix}, \hat{E}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{1-e^{-\Gamma_2 t}} & 0 \end{pmatrix}$$

- $\hat{E}_1$, $\hat{E}_2 \equiv K_0$, $\bar{K}_0$ decaying

[1] H. Feshbach, Annals of Physics **19**, 287 (1962)

General Mapping

Claim

Any NH Hamiltonian ($\hat{H} = \hat{G} + i\hat{K}$) can be mapped onto a trace-preserving quantum operation.

- Required evolution of system: $e^{-i\delta t\hat{H}}\rho e^{i\delta t H^\dagger} \approx e^{-i\delta t\hat{G}} e^{\delta t\hat{K}} \rho e^{\delta t\hat{K}} e^{i\delta t\hat{G}}$
- Add one state ≡ "decayed states" ≡ $|1\rangle$

$$\text{Expanded Initial state: } \rho_{\text{tot}}(0) = \begin{pmatrix} \rho & \vec{0} \\ \vec{0}^\dagger & 1 - \text{Tr}(\rho) \end{pmatrix}$$

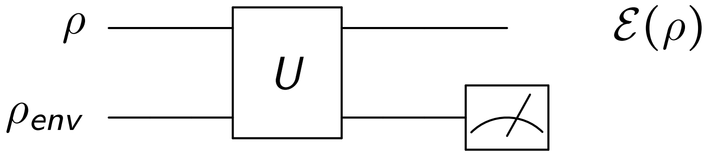
- w.l.o.g, $-\hat{K} > 0 \implies \hat{K} = -\text{diag}\left(\Gamma_1, \cdots, \Gamma_N\right)$

$$\hat{E}_0 = \begin{pmatrix} e^{-i\,\delta t\,\hat{G}} e^{\delta t\,\hat{K}} & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{E}_i = \begin{pmatrix} 0_{N\times N} & \vec{0}_N \\ \left[\sqrt{\mathbb{1} - e^{2\,\delta t\hat{K}}}\right]_i & 0 \end{pmatrix}$$

- $\sum_k \hat{E}_k^\dagger \hat{E}_k = \mathbb{1}$; $\hat{E}_0$ ≡ desired evolution $\hat{E}_i$'s ≡ System modes decaying
- $p_0 = \text{Tr}\left(\hat{E}_0 \rho \hat{E}_0^\dagger\right) = \text{Tr}\left(e^{2\delta t\,\hat{K}}\rho\right) < 1$. Exponentially small for large t

Quantum Channels- General idea

1. Construct $\hat{E}_0$ that effectively does $\mathcal{N} e^{-i\delta t \hat{H}} \rho\, e^{i\delta t \hat{H}^\dagger}$
2. Construct trace completing operators $\hat{E}_k$ s.t.: $\sum_k \hat{E}_k^\dagger \hat{E}_k = \mathbb{1}$
3. Understand nature of quantum jumps:
   a. Recoverable
   b. Non-recoverable
4. Expand system using ancillary qubits
5. Construct unitary (not-unique) on expanded system to mock-up quantum operation
6. Use post-selection on measurements on ancillas to minimize quantum jumps



Single qubit anti-hermiticity

Claim

Trotterized evolution according to any multi-qubit anti-Hermitian piece can be decomposed into a $\hat{\sigma}_z$ piece and a Unitary entangler

- Only need to consider $i\hat{k} = i\Theta(\hat{\sigma}_z - s\,\mathbb{1})$
- The relevant Evolution is

$$\exp\left\{\delta t \hat{k}\right\} = \begin{pmatrix} e^{(1-s)\delta t\,\Theta} & 0 \\ 0 & e^{-(1+s)\delta t\,\Theta} \end{pmatrix}$$

- Several ways to construct a quantum operation around this

Damping Channels (non-recoverable)

- Simple quantum operation:

$$\hat{E}_0^{\mathrm{DC}} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}, \quad \hat{E}_1^{\mathrm{DC}} = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix},$$

  where $\gamma = 1 - e^{-4\,\Theta\,\delta t}$

- A controlled y-rotation, with ancilla as the target can implement this:

$$|0\rangle\,|\psi\rangle \longrightarrow |0\rangle\,\hat{E}_0^{\mathrm{DC}}\,|\psi\rangle - |1\rangle\,\hat{E}_1^{\mathrm{DC}}\,|\psi\rangle$$

- The probability of success (measuring '0' on the ancilla) is maximal

$$p_s = \mathrm{Tr}\left(\hat{E}_0 \rho \hat{E}_0^{\dagger}\right) = 1 - \frac{\gamma}{2}(1 - r\cos\theta)$$

- p_s becomes exponentially small for large evolution times

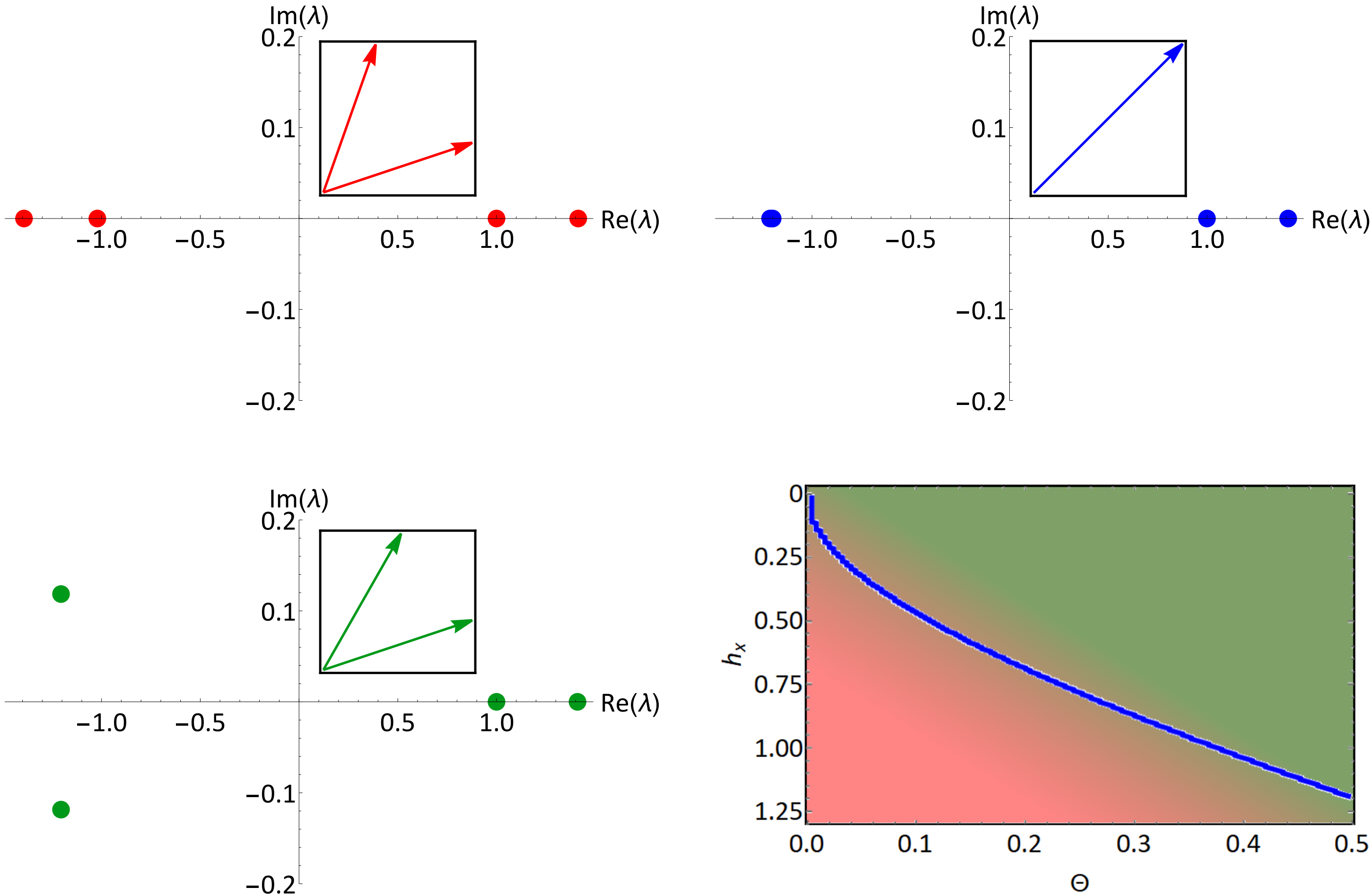
Application to a simple Lattice FT

- Apply to the 1-dimensional quantum Ising model with an imaginary longitudinal magnetic field:

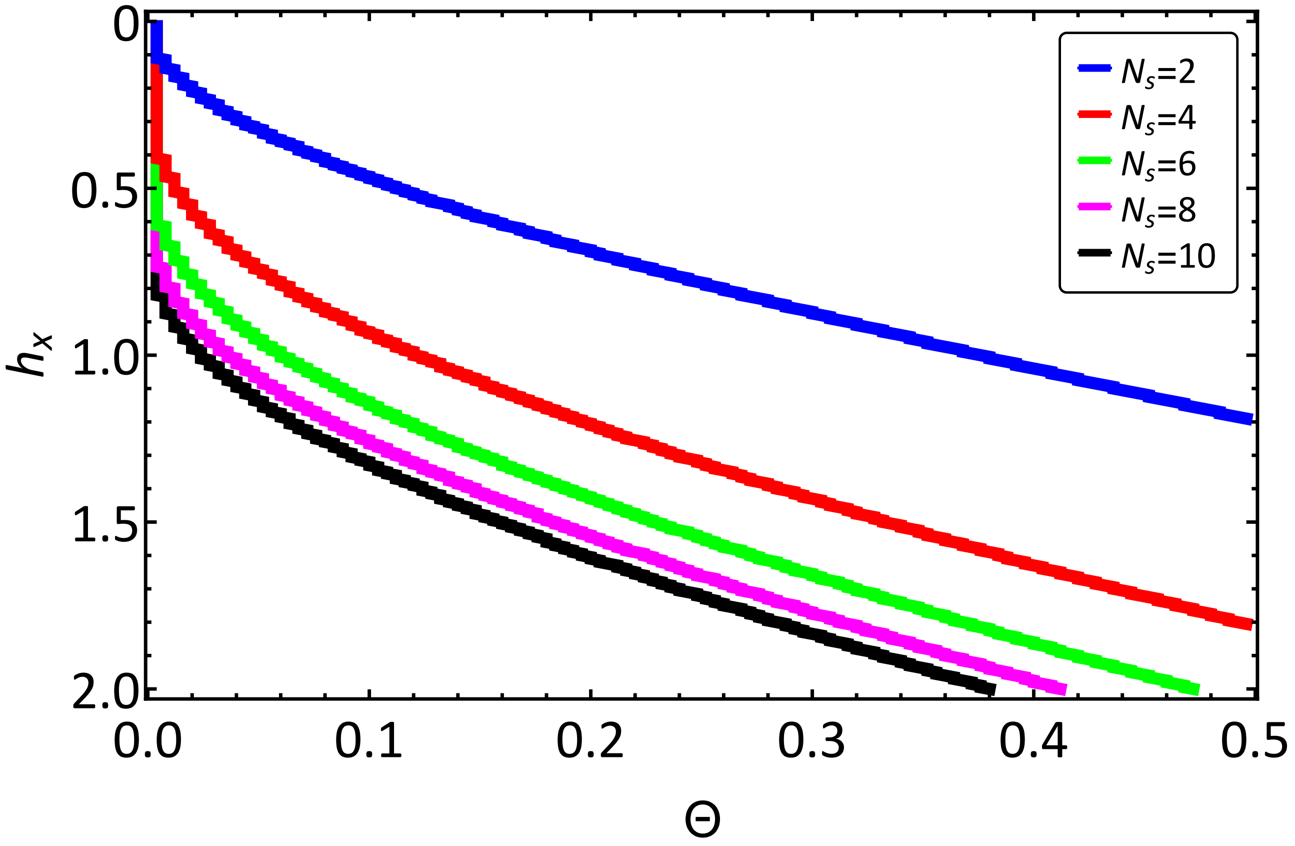
$$\hat{H} = \underbrace{-\sum_{<i,j>} \hat{\sigma}_i^z \hat{\sigma}_j^z - h_x \sum_i \hat{\sigma}_i^x}_{\hat{G}} + \underbrace{i\,\theta \sum_i \hat{\sigma}_i^z}_{\hat{K}} \,.$$

- This is a well studied model, and is a good benchmark
- Non-Hermitian part is simple and acts on individual sites

Phase structure and Exceptional points

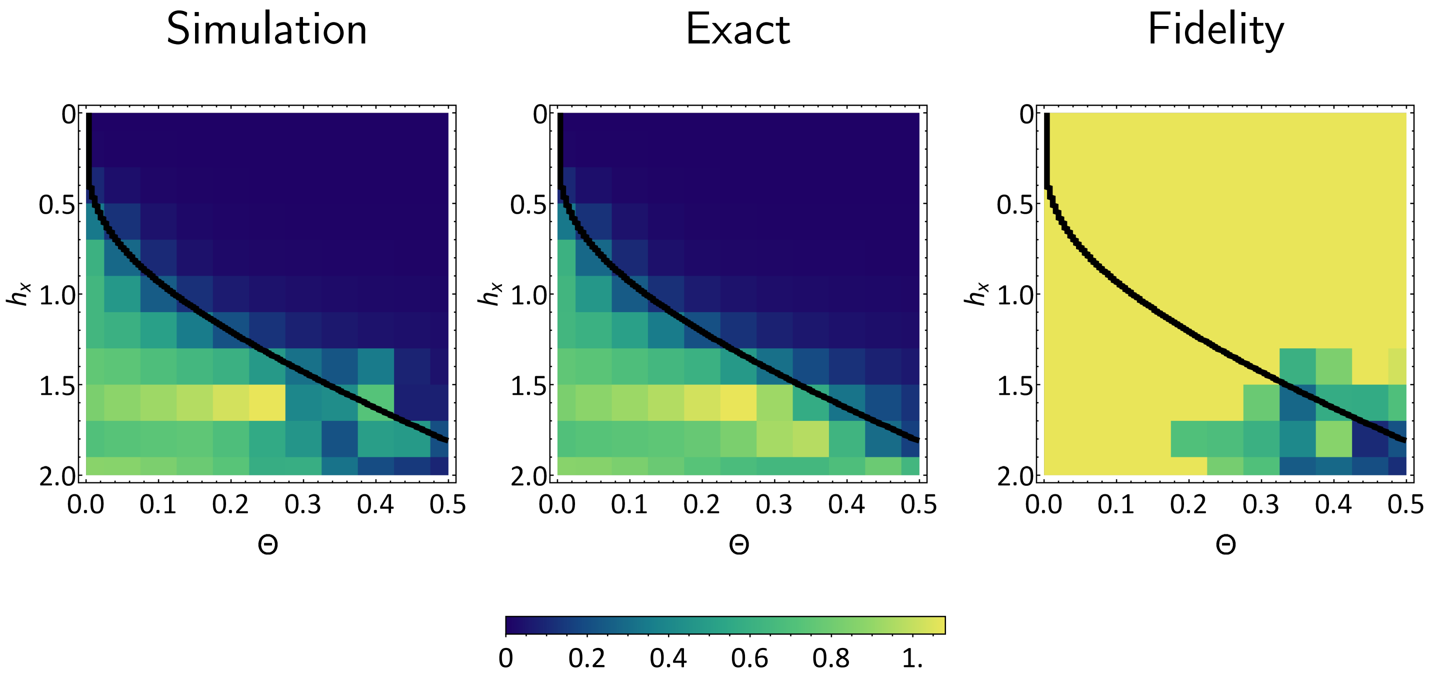


Lee-Yang edge



Rényi entropy plots

Plot of S_2 measured after 350 time-steps, with $N_s = 4$, $\delta t = 0.01$

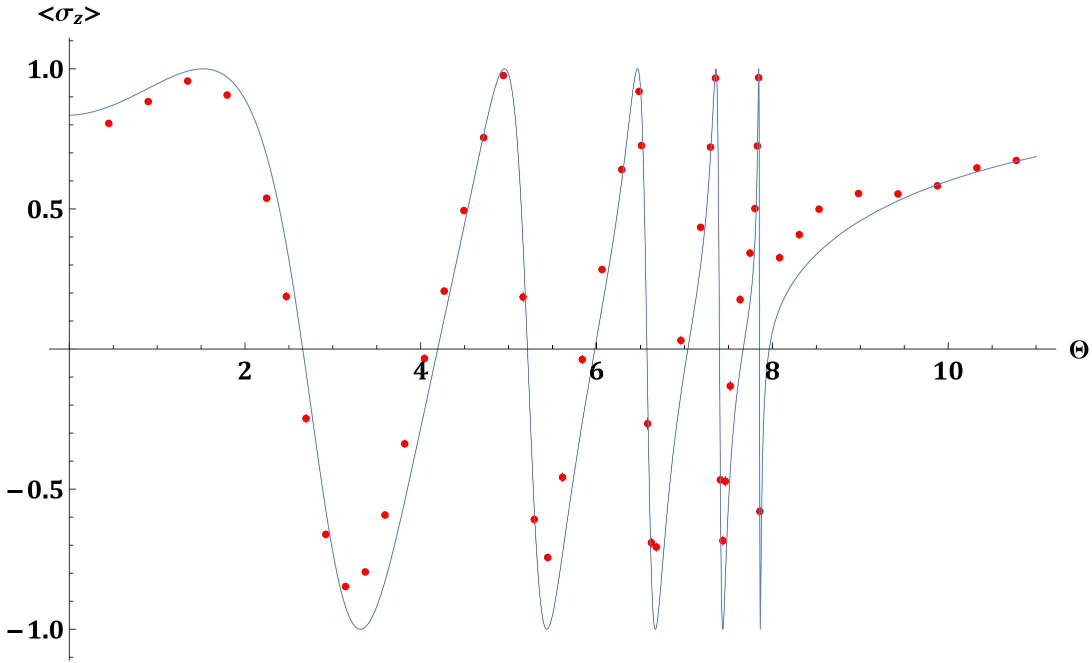


Rényi entropy is measurable using Parity measurements[2]

[2] S. Johri, D. S. Steiger, and M. Troyer, Phys. Rev. B 96, 195136 (2017)

1-qubit plot (IBM Yorktown)

$$\hat{H} = \underbrace{-h_x \hat{\sigma}^x}_{\hat{G}} + i \underbrace{\theta \hat{\sigma}^z}_{\hat{K}}$$

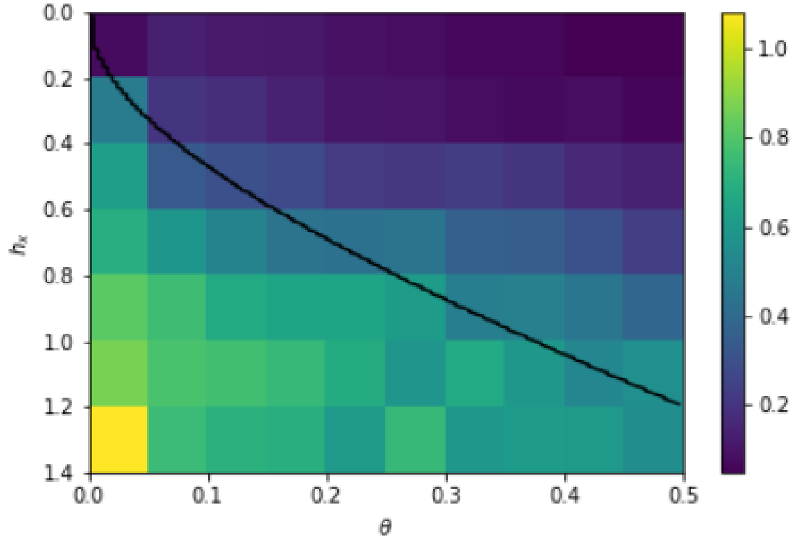


Coutresy: Erik Gustafson

2-qubit example (IBM Lima)

$$\hat{H} = -\left(\hat{\sigma}_1^z \otimes \hat{\sigma}_2^z\right) - h_x\left(\hat{\sigma}_1^x \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\sigma}_2^x\right) + i\Theta\left(\hat{\sigma}_1^z \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\sigma}_2^z\right)$$

4 trotter steps, $\delta t = 0.5$

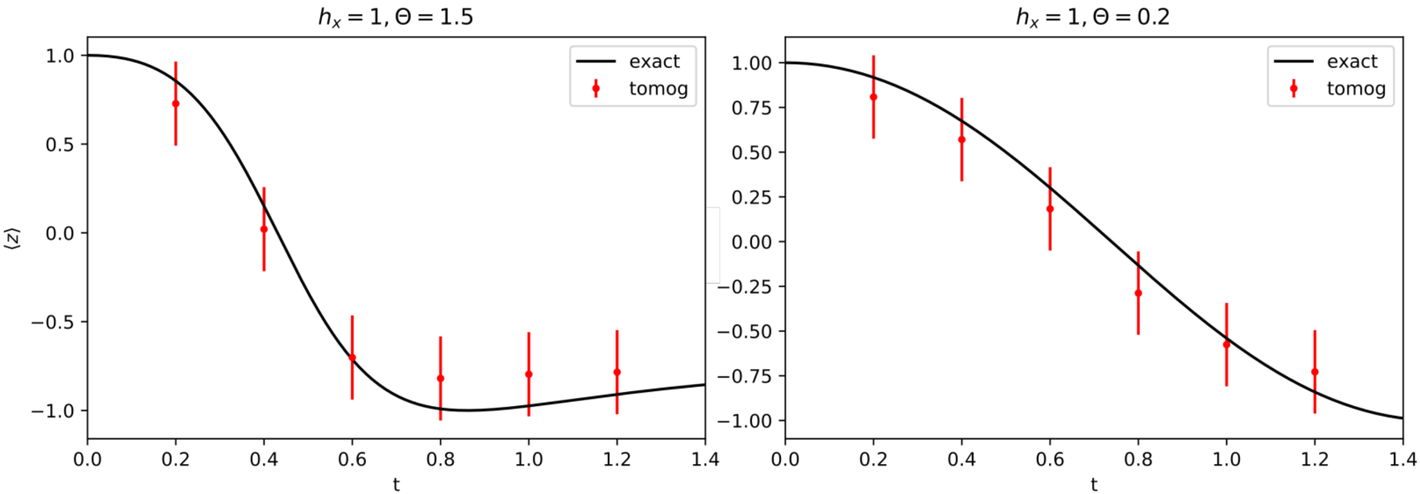


Coutresy: Michael Hite

1 qubit State Tomography and QITE

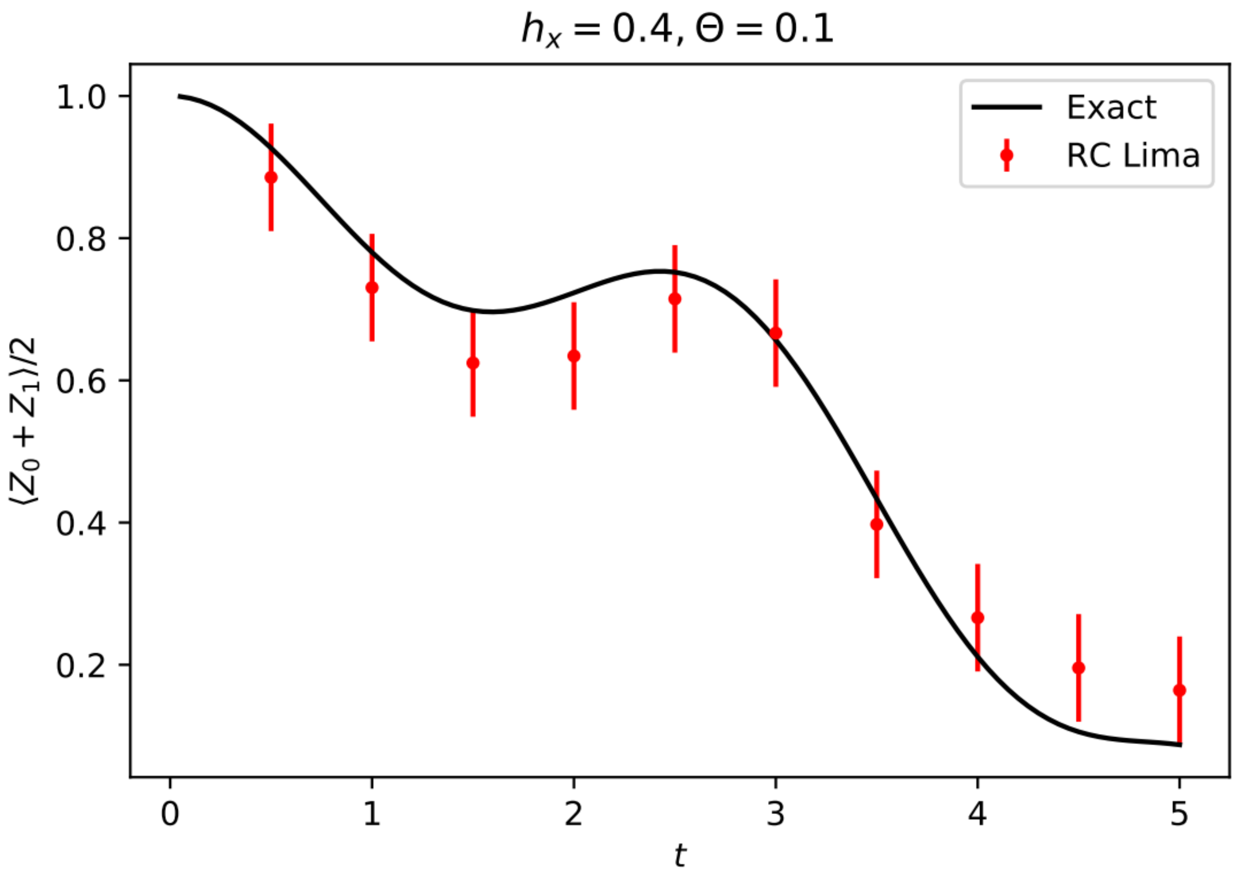
Ongoing work

Use tomography to create *save points* in evolution to minimize effects of quantum jumps, to enable simulation for longer time



Coutresy: Michael Hite

2 qubit State Tomography



Coutresy: Michael Hite

Conclusion

- NH Hamiltonians describe some effective theories, many have sign problems. Quantum computers could solve this, but we are in the NISQ era
- Constructed quantum operations for NH Hamiltonians and tested it on a 1-D quantum Ising chain with an imaginary longitudinal magnetic field
- Errors are $\mathcal{O}(1)$ at long times, but quantum phase structure can be probed for small system sizes, at small circuit depth
- Ongoing work: Scaling up further
- Also considering $\mathcal{Z}_N$ models with $N > 2$

Thank you ☺

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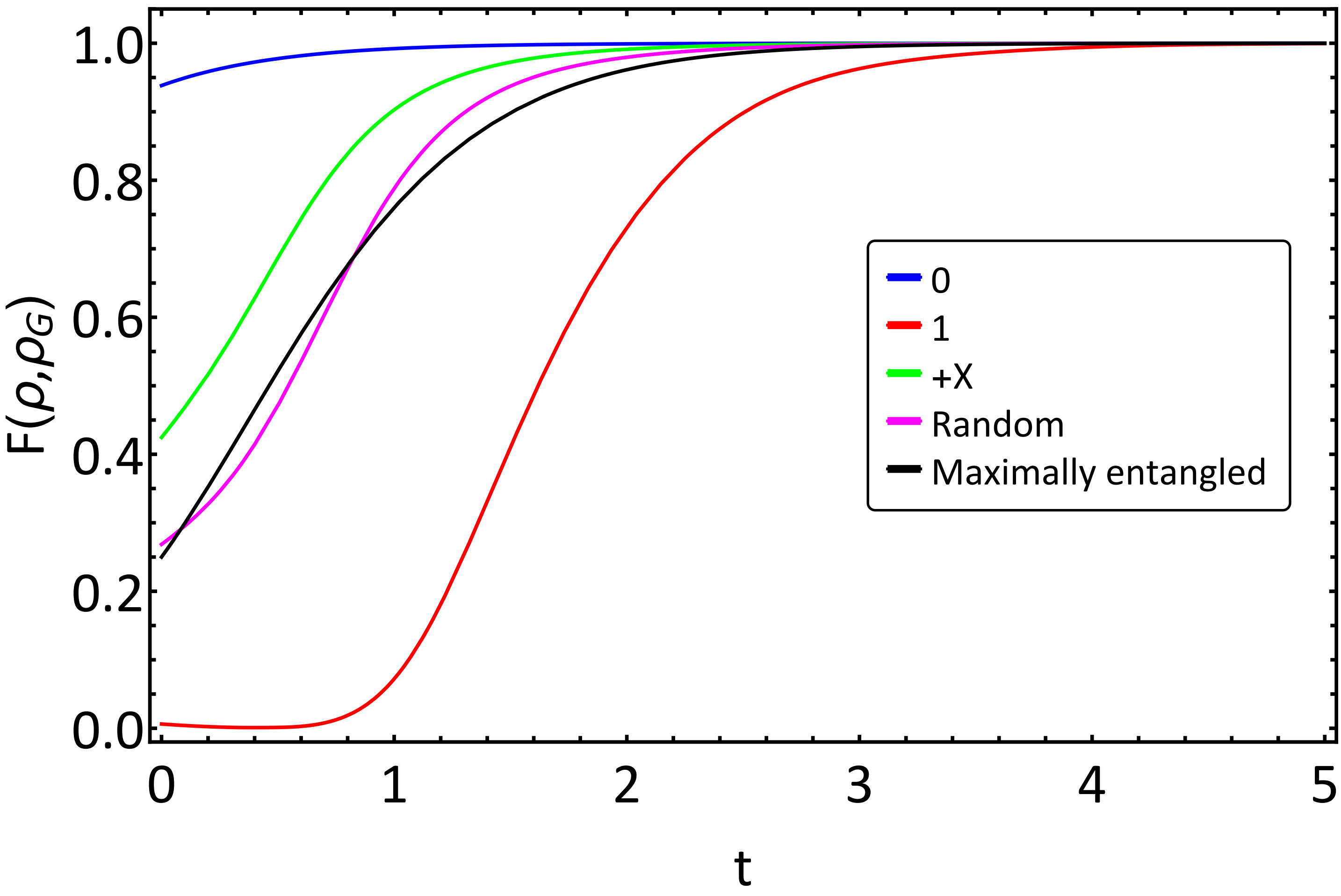
Initial state plot



State Tomorgraphy and QITE[3]

- Minimize $\Delta = \| \, |\psi'\rangle - e^{-i\delta t \hat{A}} |\psi\rangle \, \|^2$, with $|\psi'\rangle = \frac{e^{-\delta t \hat{K}}}{\sqrt{\langle\psi| e^{-2\delta t \hat{K}} |\psi\rangle}} |\psi\rangle$
- Expand to $\mathcal{O}(\delta t)$:

$$\Delta \approx \| \left(\hat{K} - \langle \hat{K} \rangle - i\hat{A} \right) |\psi\rangle \, \|^2$$

- Pauli expand $\hat{A}$

$$\Delta = \langle\psi| \left(\hat{K} - \langle \hat{K} \rangle \right)^2 |\psi\rangle + 2\Re \left(i \langle\psi| \sum_I a_I \sigma_I \left(\hat{K} - \langle \hat{K} \rangle \right) |\psi\rangle \right)$$

- Solve for coefficients a_I, and evolve by $\hat{A}$

[3] Motta, M., Sun, C., Tan, A.T.K. et al. Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution. Nat. Phys. 16, 205–210 (2020). https://doi.org/10.1038/s41567-019-0704-4