Quantum computing for lattice supersymmetry

C. Culver
in collaboration with D. Schaich

Department of Mathematical Sciences
University of Liverpool

August 10, 2022

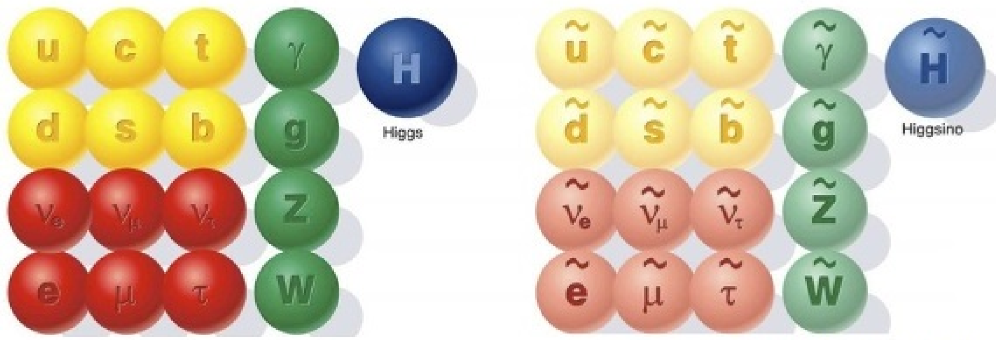
Outline

1. Motivation
2. Introduction
3. $0 + 1D$ supersymmetric quantum mechanics
4. $1 + 1D$ Wess-Zumino model
5. Conclusion

Motivation

Physics Motivation

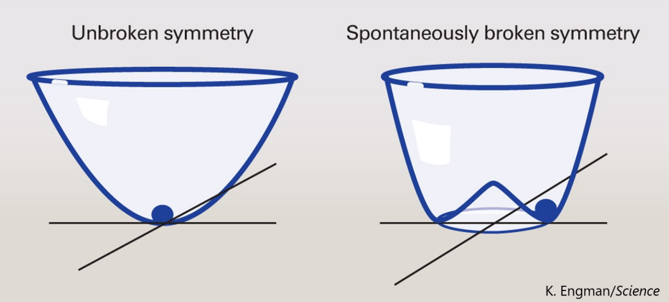
Supersymmetry as an extension to the standard model



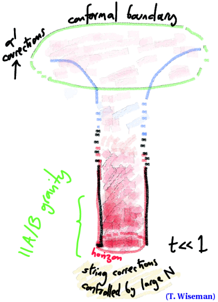
susarnat.canalblog.com

Physics Motivation

Dynamical symmetry breaking



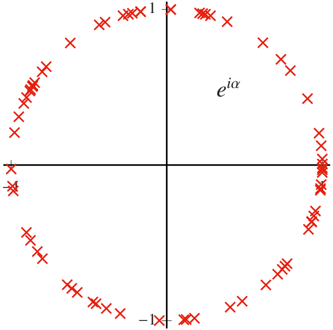
Holographic duality



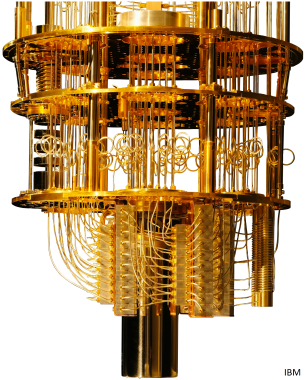
Practical Motivation

Sign problem

Lattice $\mathcal{N} = 4$ SYM

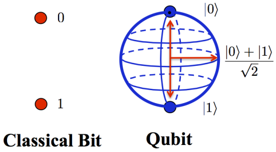


NISQ devices



Introduction

Quantum computing



- Map d.o.f of bosons/fermions to qubit d.o.f.
- Choose quantum computation
  - VQE - variational method to solve for lowest lying eigenvalues
  - Time evolution - study dynamics of Hamiltonian

Mapping degrees of freedom

- Bosonic
  - Harmonic oscillator basis with cutoff Λ excitations
  - n excitation to binary string of length N^q, $|n\rangle \rightarrow \left|\sum_i^{N^q-1} a_i 2^i\right\rangle$

Matrix Elements

$$|0\rangle\langle 1| = (X+iY)/2, \quad |1\rangle\langle 0| = (X+iY)/2,$$
$$|0\rangle\langle 0| = (1+Z)/2, \quad |1\rangle\langle 1| = (1-Z)/2$$

- Fermionic

Jordan-Wigner Transformation

$$\hat{b}^\dagger = \frac{1}{2}(X-iY),\ \hat{b} = \frac{1}{2}(X+iY)$$

Supersymmetry Facts

Hamiltonian

$H = Q^2$

- $E \geq 0$
- Supersymmetry conserved if $\langle \Omega \,|H|\, \Omega \rangle = 0$ [Witten, Nucl. Phys. B 188,513 (1981)]
- Non-zero E states appear in pairs

- Lagrangian formalism

Path integral

$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathrm{d}\hat{q}\, \mathrm{d}\hat{b}\, \mathrm{d}\hat{b}^\dagger\, \mathcal{O}\, e^{iS(\hat{q},\hat{b},\hat{b}^\dagger)}$

- Condition for symmetry breaking

Witten index

$\mathcal{W} = \mathrm{Tr}\left[(-1)^F e^{-iHt}\right] = 0$

$0 + 1D$ supersymmetric quantum mechanics

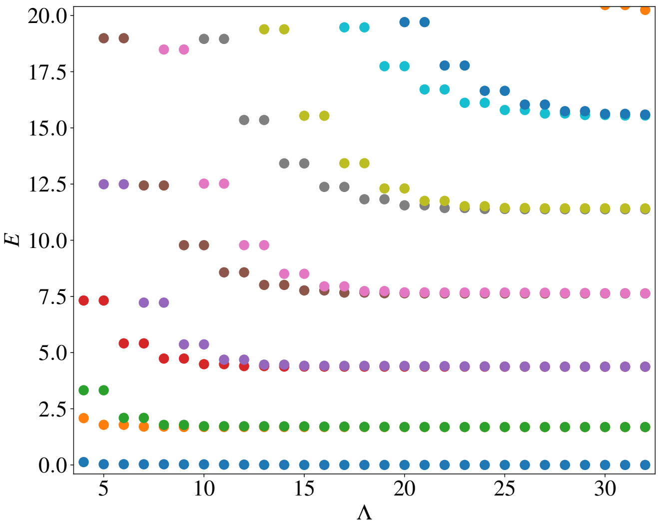
$0+1$ dimensional quantum mechanics

Hamiltonian

$$H = \frac{1}{2}\left(\hat{p}^2 + [W'(\hat{q})]^2 - W''(\hat{q})\left[\hat{b}^\dagger, \hat{b}\right]\right)$$

- Superpotentials W studied with complex Langevin [Joseph-Kumar, arxiv:2011.08107]:
  - HO: $\frac{1}{2}m\hat{q}^2$ preserves supersymmetry
  - AHO: $\frac{1}{2}m\hat{q}^2 + \frac{1}{4}g\hat{q}^4$ preserves supersymmetry
  - DW: $\frac{1}{2}m\hat{q}^2 + g(\frac{1}{3}\hat{q}^3 + \hat{q}\mu^2)$ breaks supersymmetry
- For HO with Λ=2, $H = 1.5I^0I^1 + I^0Z^1 - 0.5Z^0I^1$

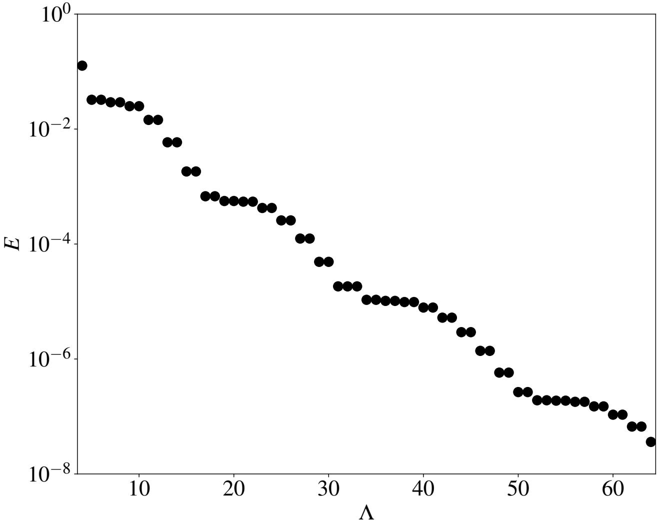
Supersymmetric anharmonic oscillator spectrum



$m = 1,\ g = 1$

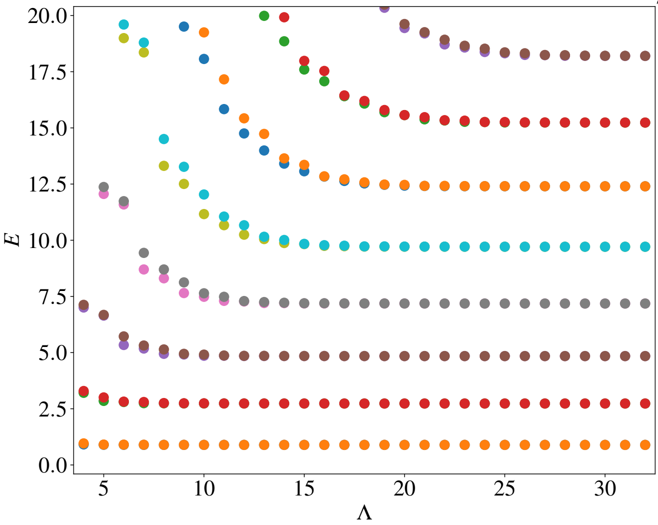
Supersymmetric anharmonic oscillator spectrum

$m = 1,\ g = 1$



Double well spectrum

$m = 1,\ g = 1,\ \mu = 1$



Λ	Exact	VQE
2	9.38e-01	9.38e-01
4	1.27e-01	1.27e-01
8	2.93e-02	2.93e-02
16	1.83e-03	6.02e-02
32	1.83e-05	6.63e-01

(a) Anharmonic oscillator.

Λ	Exact	VQE
2	1.08e+00	1.08e+00
4	9.15e-01	9.15e-01
8	8.93e-01	8.93e-01
16	8.92e-01	8.94e-01
32	8.92e-01	8.95e-01

(b) Double well.

VQE

Λ	Exact	VQE
2	9.38e-01	9.38e-01
4	1.27e-01	1.27e-01
8	2.93e-02	2.93e-02
16	1.83e-03	6.02e-02
32	1.83e-05	6.63e-01

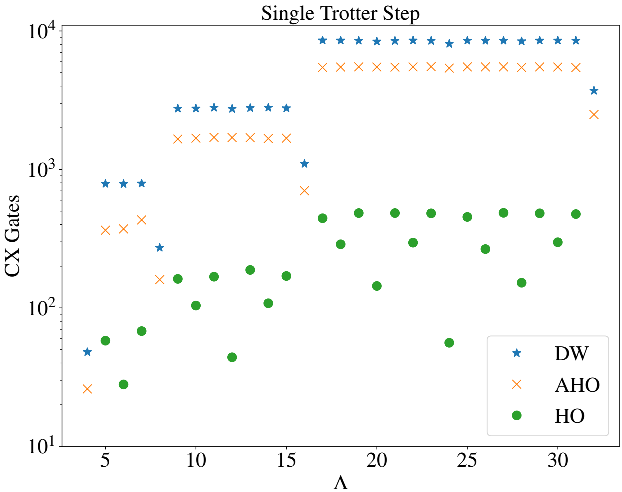
(a) Anharmonic oscillator.

Λ	Exact	VQE
2	1.08e+00	1.08e+00
4	9.15e-01	9.15e-01
8	8.93e-01	8.93e-01
16	8.92e-01	8.94e-01
32	8.92e-01	8.95e-01

(b) Double well.

- Similar problems to (a) in BMN matrix model [Rinaldi et al, PRX Quantum 3 (2022)]

Gate costs for 0+1



$1+1D$ Wess-Zumino model

$1+1$D Wess-Zumino model

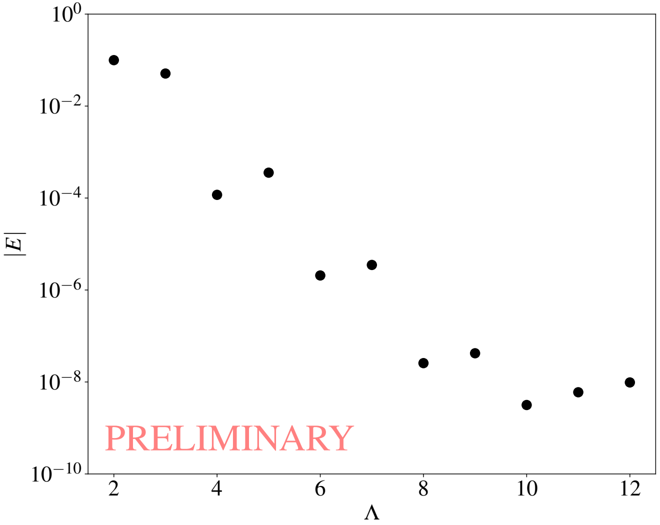
Lattice Hamiltonian

$$H = \sum_{n=0}^{N-1} \left[\frac{p_n^2}{2a} + \frac{a}{2}\left(\frac{\phi_{n+1}-\phi_{n-1}}{2a}\right)^2 + \frac{a}{2}V(\phi_n)^2 + aV(\phi_n)\frac{\phi_{n+1}-\phi_{n-1}}{2a} \right.$$
$$\left. +(-1)^n V'(\phi_n)\left(\chi_n^\dagger \chi_n - \frac{1}{2}\right) + \frac{1}{2a}\left(\chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n\right)\right], \quad (1)$$

- Potentials
  - Linear: $m\phi$
  - Quadratic: $c+\phi^2$ [Beccaria, Campostrini, Feo, hep-lat/0109005 (2001)]
  - Cubic: $\frac{1}{3}g\phi^3 - \frac{m^2}{4g}\phi$ [Steinhauer, Wenger, PRL 113, 231601 (2014)]

$$\dim(H) = \Lambda^N \times 2^N$$

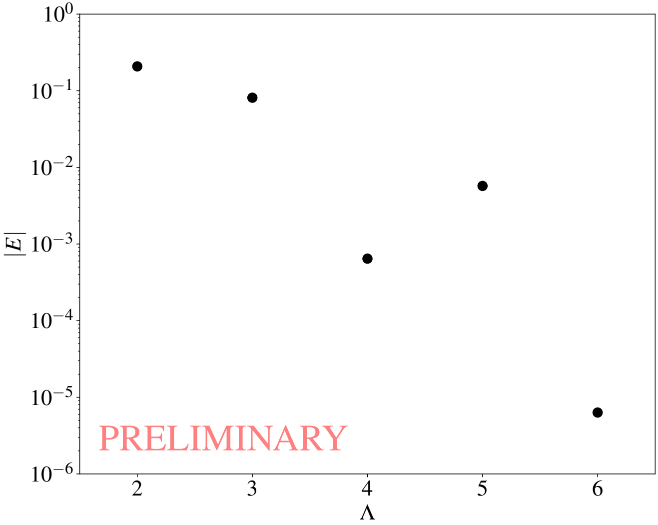
3 site linear potential



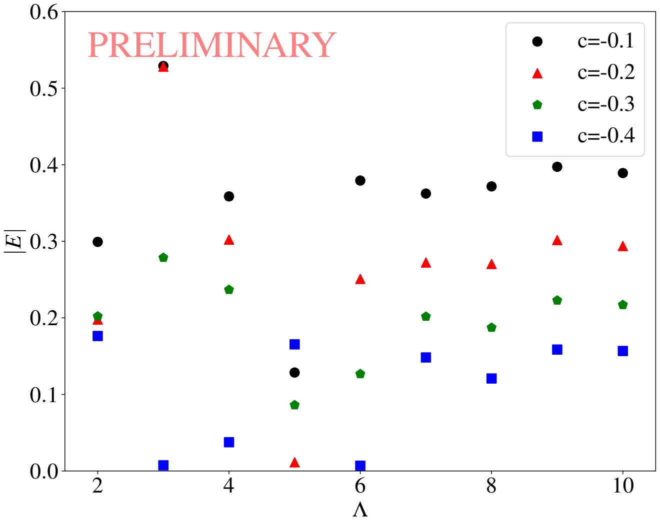
$m = 1$

4 site linear potential

$m = 1$



3 site quadratic potential



Conclusion

Conclusion

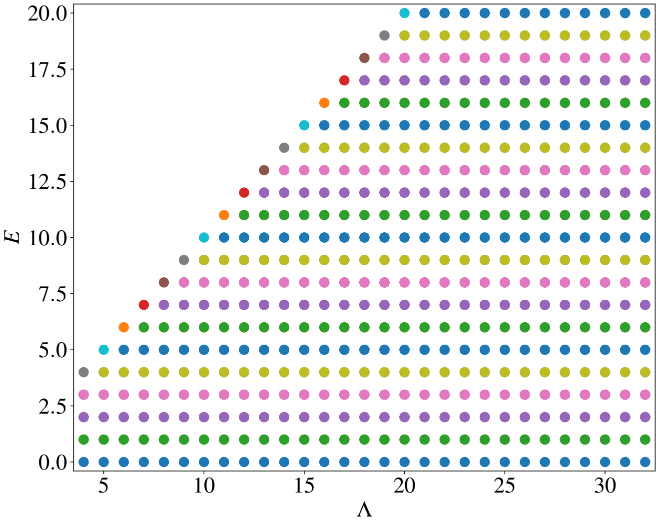
Results

- $0 + 1D$
  - Classical diagonalization works as expected
  - VQE struggles with AHO
  - Trotter step gate counts appropriate for NISQ
- $1 + 1D$
  - Classical preparation is severly limited
  - Bosonic cutoff introduces more error?
  - Energies trending in the correct direction

Ongoing

- Cubic potential for WZ
- Quantum approach to WZ
- Optimizing the quantum approach

0+1 harmonic oscillator spectrum



$m = 1.0$

0+1 harmonic oscillator spectrum

Λ	Exact	VQE
2	0.00e+00	5.34e-10
4	0.00e+00	1.07e-09
8	0.00e+00	4.06e-09
16	0.00e+00	1.13e-08
32	0.00e+00	4.81e-08

(a) Harmonic oscillator.