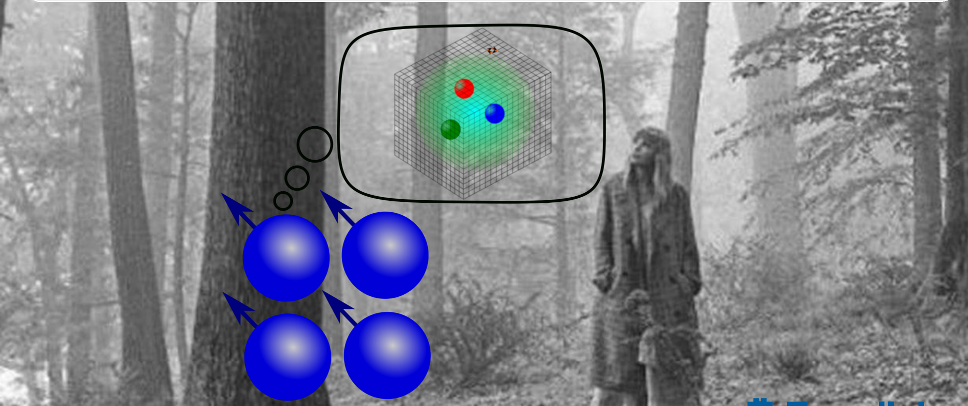
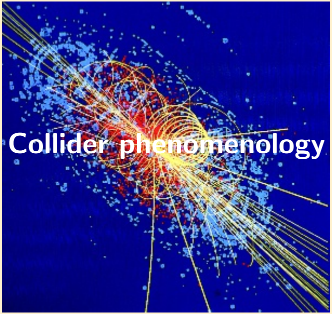
Improving Quantum Simulations towards lattice SU(3)

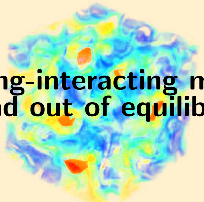
Hank Lamm

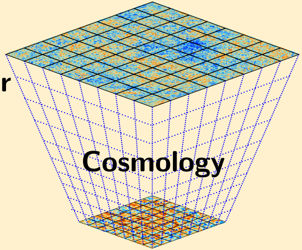


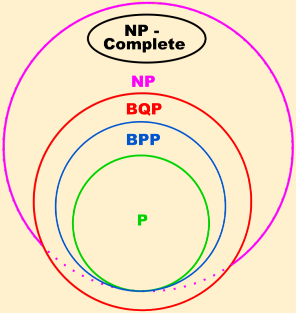
Fermilab

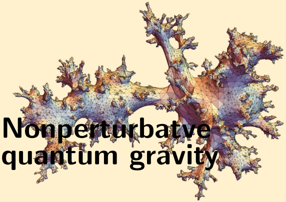
Fundamentally, HEP requires QC[1]











[1] Bauer, C. W. et al. In: (Apr. 2022). arXiv: 2204.03381 [quant-ph].

What might a galactic algorithm look like?

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Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1*] Keith S. M. Lee,[2] John Preskill[3]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.

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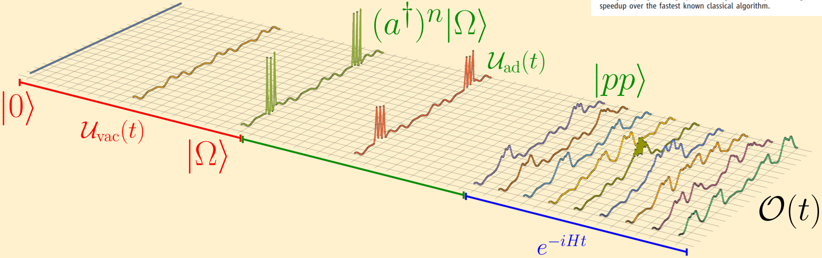

Vacuum Prep+**Adiabatic evolution**+**Trotterization**+Measurements[2]

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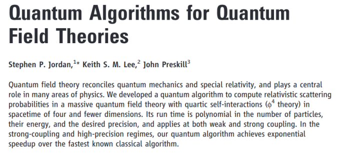
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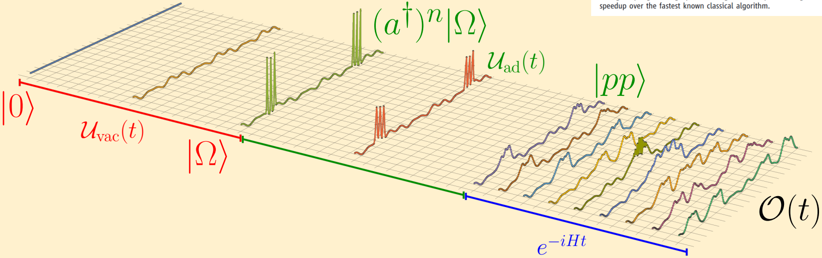

Vacuum Prep+**Adiabatic evolution**+**Trotterization**+Measurements[2]

Example: $|\langle p\bar{p}|U(t)|\pi\pi\pi\pi\rangle|^2$ needs $\mathcal{O}(10^8)$ **logical qubits**

$$\approx \left(\frac{4\text{ fm}}{0.05\text{ fm}}\right)^3 \times (3\text{ links} \times 11\text{ qubits} + 3\text{ colors} \times 2\text{ flavors} \times 2\text{ spins} \times 1\text{ qubit})$$

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Today's estimate: $\mathcal{O}(10^8)$ q & $\mathcal{O}(10^{55})$ T-gates[3]

"...99.998% of the gate counts stem from **QFOPs**...The SU(3) *HI collision* problem is...> 3 **yrs** of runtime on an **exa-scale** quantum supercomputer."

Cracking RSA and Quantum Chemistry need $\mathcal{O}(10^7)$ q & $\mathcal{O}(10^{20})$!

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What will it take for practical quantum advantage?

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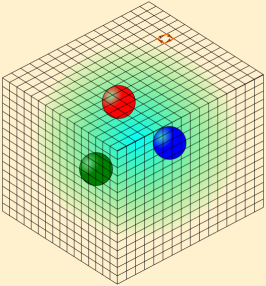

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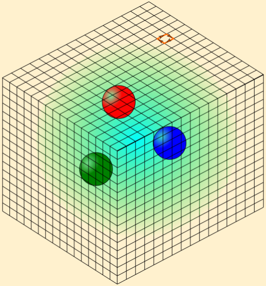

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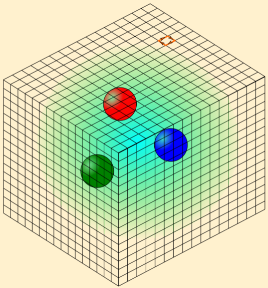

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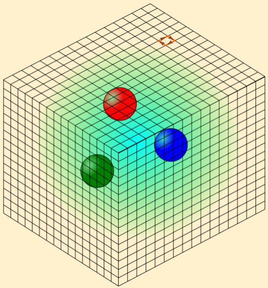

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Using discrete subgroups to digitize gluons[6][7][8][9]

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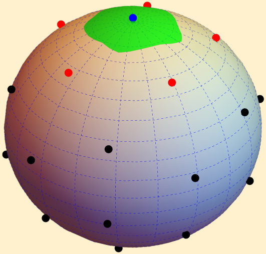
Replace $G \to H$ in $e^{-S}, e^{-i\mathcal{H}}$



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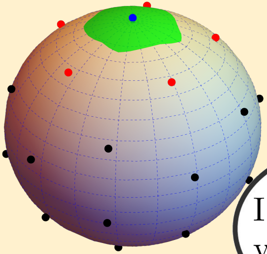

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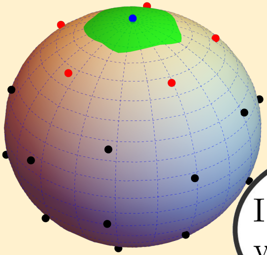

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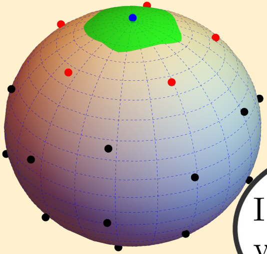

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- I **believe** endgame will be **3x3** matrices

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$a \approx 0.07$ fm ≈ 2 GeV^{-1} seems possible[10][11]

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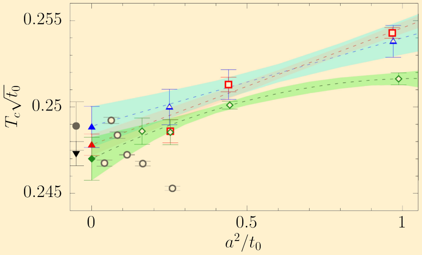

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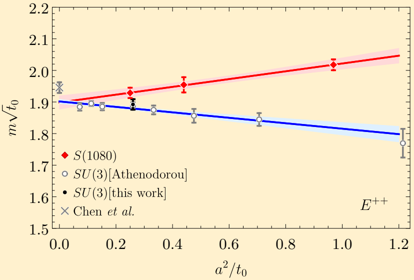

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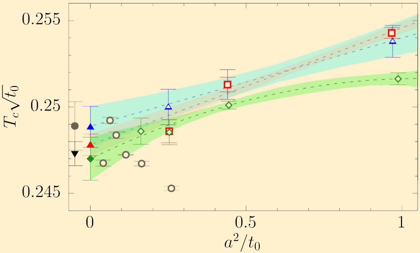

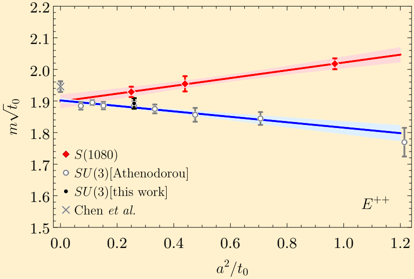

10$\times$ increase in aE without observing discrepancy

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What is trotterization?

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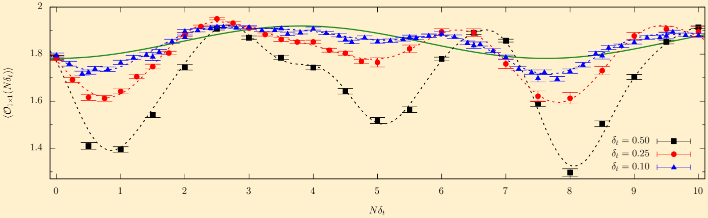

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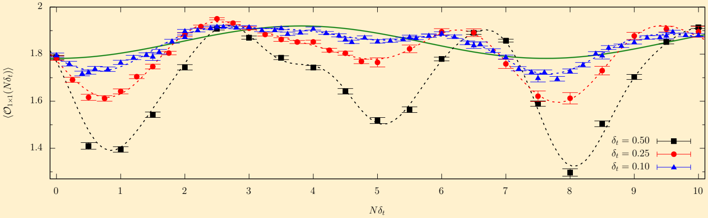

- Introduces **higher dimension operators** and a_t

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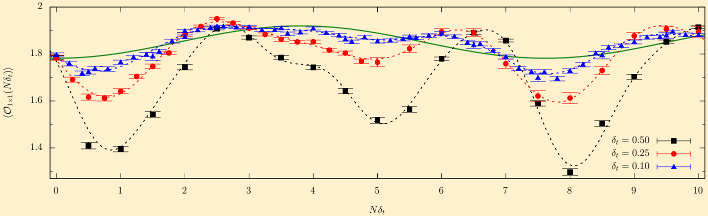

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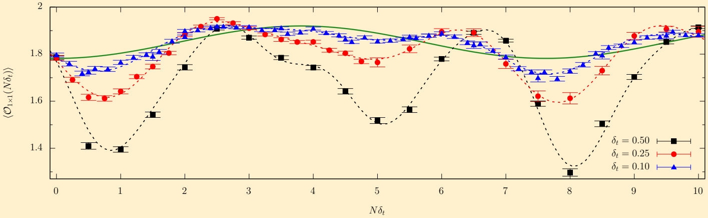

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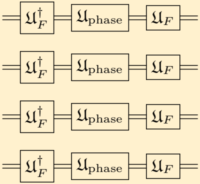
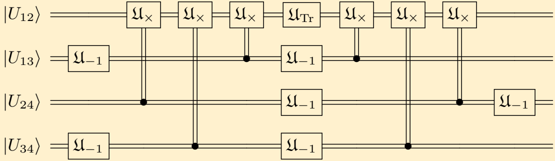
What low-level primatives are required for LGT?[14]

How do we build $U_K = e^{iH_K}$ **and** $U_V = e^{iH_V}$?





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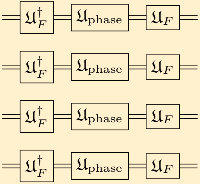

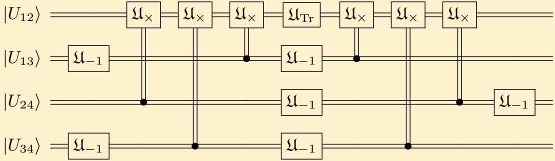

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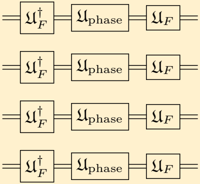

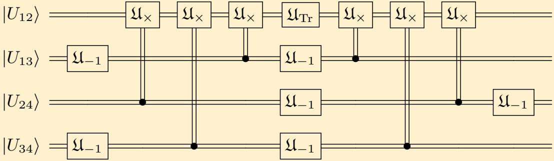

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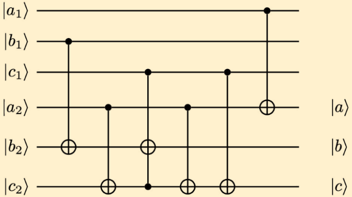

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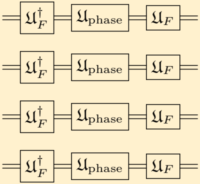

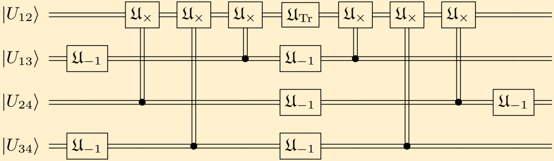

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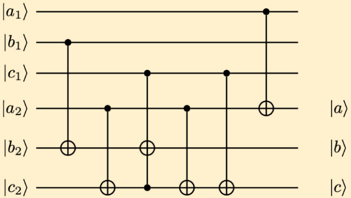

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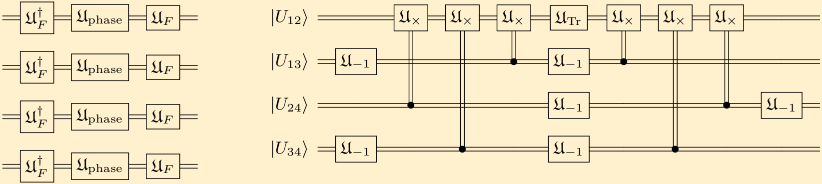

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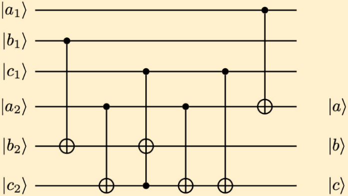

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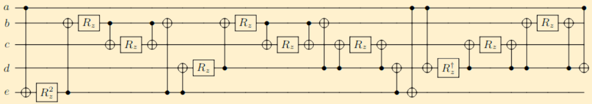

FIG. 4. Trace gate for BT

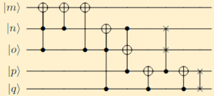

FIG. 2. Inversion Gate for the Binary Tetrahedral Group.

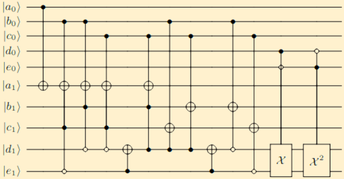

FIG. 3. Multiplication gate

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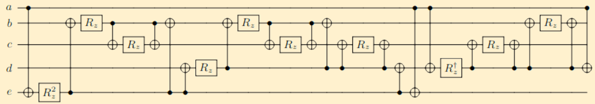

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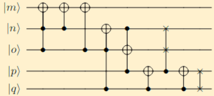

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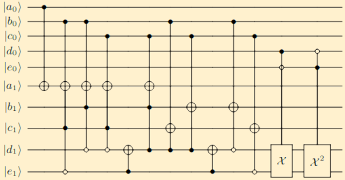

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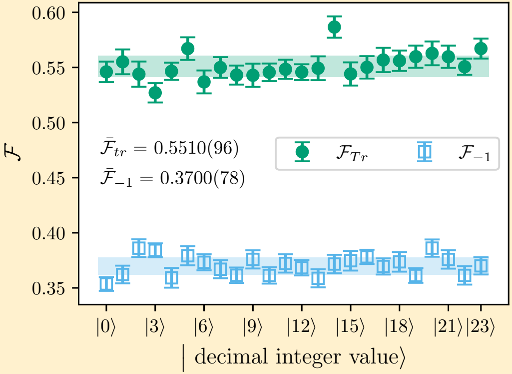

Kogut-Susskind[15] is not only Hamiltonian[16]

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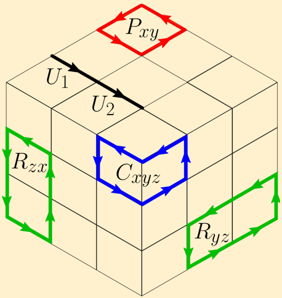

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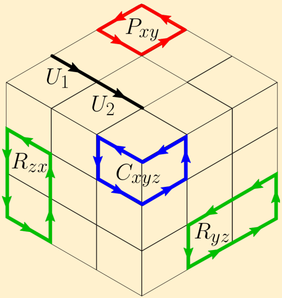

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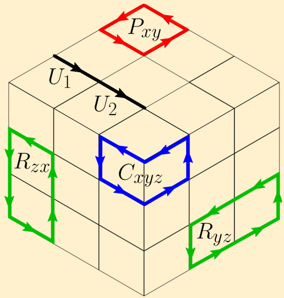

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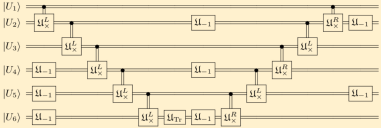

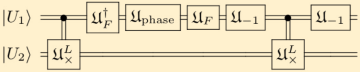

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It's one calculation, what could it cost?

A lot has been solved...and lots more to do

- **Digitizing** Field Theory
  - S(1080) seems viable
  - $\mathbb{BT}$ done, $\mathbb{BO}$ and $S(108)$ soon.
- Formulating **state preparation**
- Performing **Time Evolution**
  - Improved Hamiltonians
  - Theory of Trotterization
- **Measurements and Observables**
  - Viscosity?
- HEP-specialized **QEC/QEM**