Magnetic catalysis in the (2+1)-dimensional Gross-Neveu model

(handout version)

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Handout version*

This handout is a slightly modified version of the talk given at Lattice22.
Some additional comments were added in order to give context to the figures shown.
Slides marked by an asterisk (*) were not part of the original talk.

Gross-Neveu (GN) model

$$\mathcal{L} = i\bar{\psi}\left(\not{\partial} + \mu\gamma_0 + ie\not{A}\right)\psi + \frac{g^2}{2N_f}(\bar{\psi}\psi)^2$$

- N_f flavors
- no mass term
- chemical potential μ
- external field A_μ

Gross-Neveu (GN) model

$$\mathcal{L} = i\bar{\psi}\left(\partial\!\!\!/ + \mu\gamma_0 + ieA\!\!\!/\right)\psi + \frac{g^2}{2N_f}(\bar{\psi}\psi)^2$$

or, equivalently,

$$\mathcal{L} = i\bar{\psi}\left(\partial\!\!\!/ + \sigma + \mu\gamma_0 + ieA\!\!\!/\right)\psi + \frac{N_f}{2g^2}\sigma^2$$

Ward identitiy	discrete chiral symmetry
$\langle\bar{\psi}\psi\rangle = \frac{iN_f}{g^2}\langle\sigma\rangle$	$\psi \to i\gamma_5\psi,\ \bar{\psi} \to i\bar{\psi}\gamma_5,\ \sigma \to -\sigma$

Gross-Neveu (GN) model*

One commonly gets rid of the $(\bar{\psi}\psi)^2$ term by introducing the auxiliary scalar field σ into the Lagrangian. These two Lagrangians are equivalent, as can be seen using the equations of motion for σ.
Notice that the GN model has a discrete $\mathbb{Z}_2$ symmetry.

Motivation & Goals

Why GN model?

- **Toy model for QCD**
- **Solid State Physics**
- . . .

Why magnetic field?

- **Heavy-ion collisions**
- **Neutron stars**
- **Early universe**

In this talk

- Study influence of magnetic field on GN (T, μ) phase diagram using Lattice Field Theory.
- 2+1 dimensions.

Motivation & Goals*

Variants of the Gross-Neveu model have been used successfully as toy models for QCD due to some intersting features they share with QCD, such as chiral symmetry and its spontaneous breakdown and in low dimensions renormalizability and asymptotic freedom.
In solid state physics they are used to describe planar and one-dimensional materials such as graphene, high-T_c superconductors or polymers.

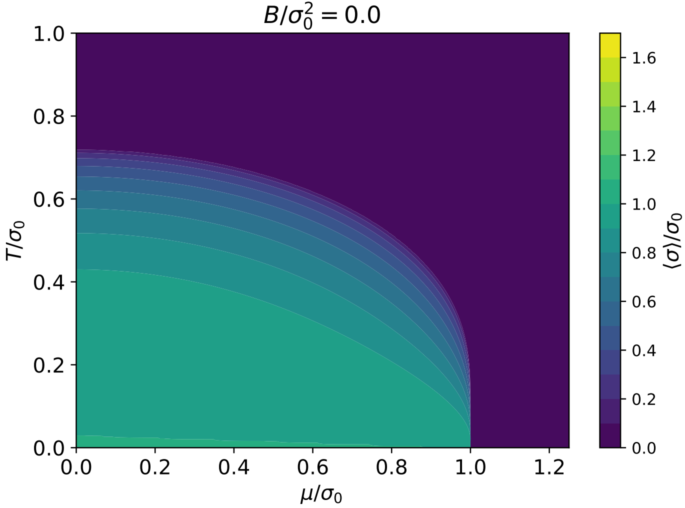
Very strong magnetic fields are present in heavy-ion collisions, nucleon stars and likely also were at the early stages of the universe. Thus it is important to understand their potential influence on the structure of matter.

Large-N_f mean-field results*

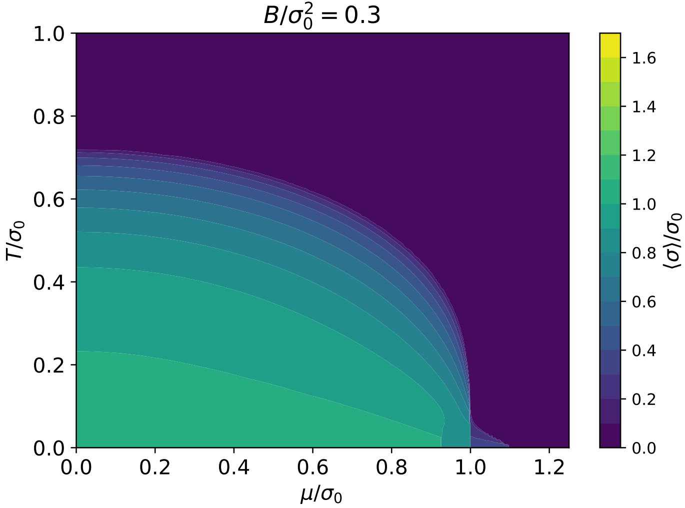
Going to the limit of an infinite flavor number, $N_\mathrm{f} \to \infty$, often gives a qualitatively correct picture of the phase structure of Four-Fermi theories even at finite N_f. In this case, computing the path integral reduces to a simple minimization problem (see the Appendix).

On the next few slides we show large-N_f phase diagrams in the (T, μ) plane of the $(2+1)$-dimensional GN model for various values of the magnetic field, which we assume constant, homogenous and orthogonal to the spatial plane. We assume σ to be homogeneous in space and time.

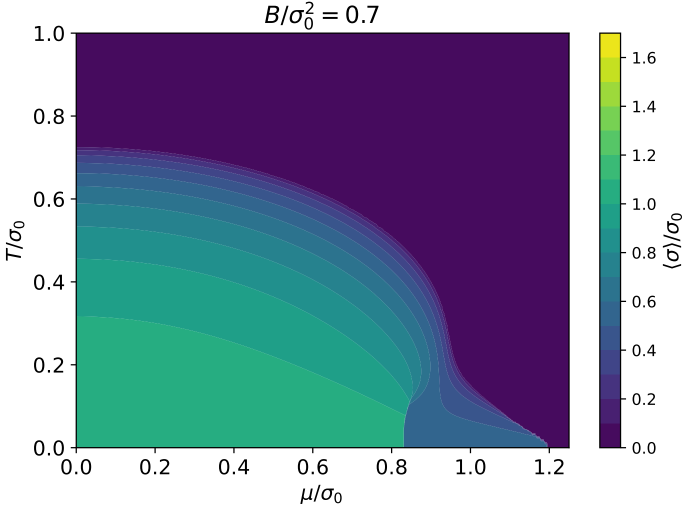
Large-N_f mean-field results



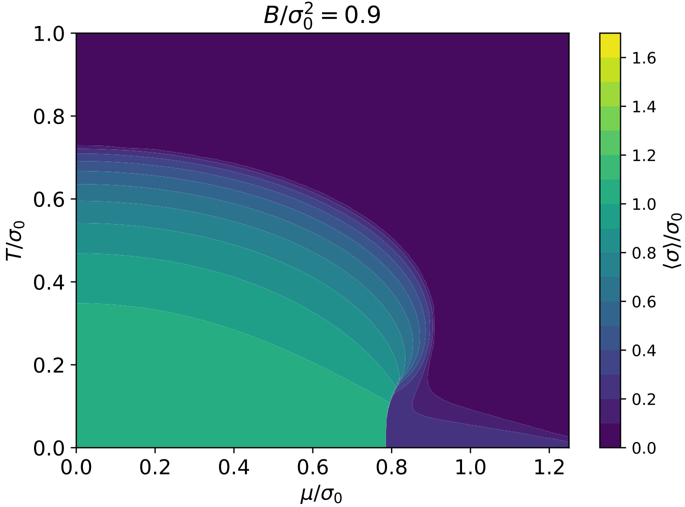
Large-N_f mean-field results



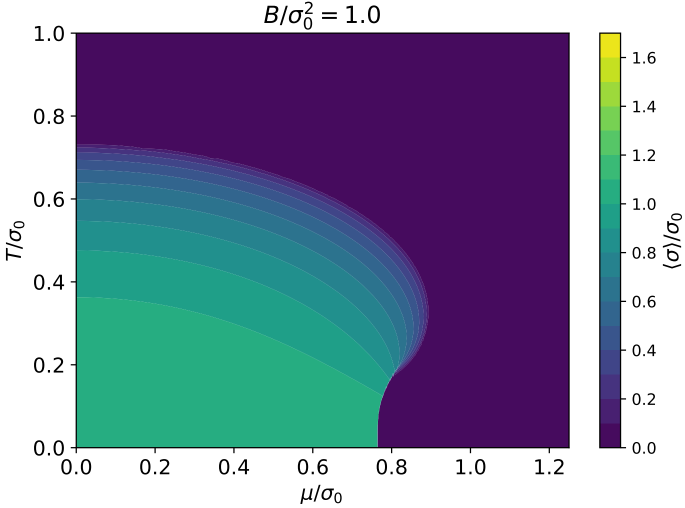
Large-N_f mean-field results



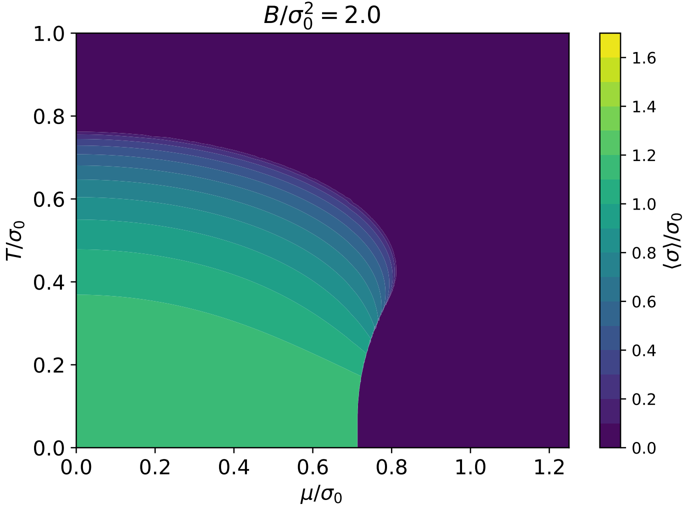
Large-N_f mean-field results



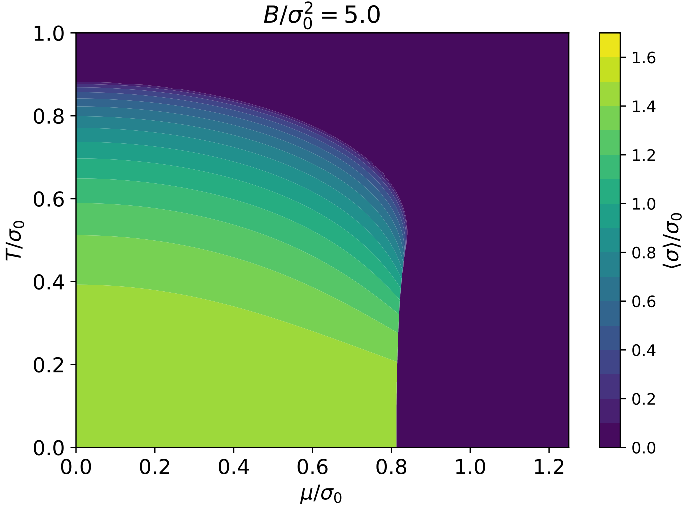
Large-N_f mean-field results



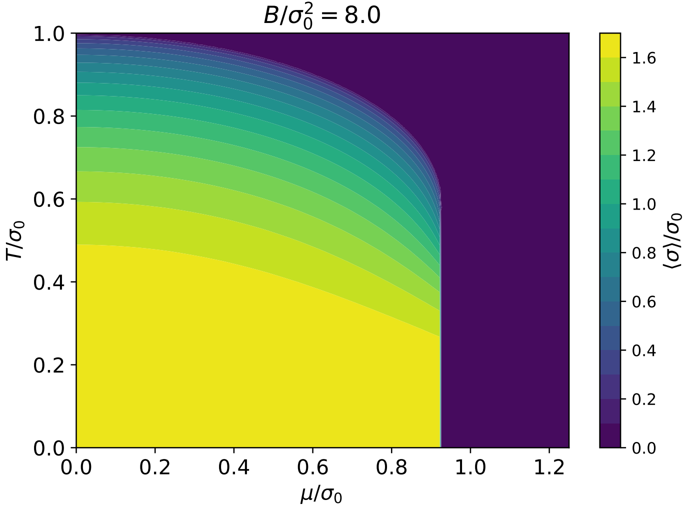
Large-N_f mean-field results



Large-N_f mean-field results



Large-N_f mean-field results



Large-N_f mean-field results*

We observe:

- At $\mu = 0$:
  - $\frac{\partial\langle\sigma\rangle}{\partial B} > 0$, i.e. magnetic catalysis.
  - Critical temperature grows with B.
- At $T = 0$:
  - $\frac{\partial\langle\sigma\rangle}{\partial B} \lessgtr 0$, i.e. both magnetic catalysis and inverse magnetic catalysis.
  - Multiple phase transitions for small B.
  - Critical chemical potential is non-monotonic in B.
- At large B:
  - $\frac{\partial\langle\sigma\rangle}{\partial B} > 0$, i.e. magnetic catalysis everywhere.
  - Critical T and μ grow with B.

Lattice results*

To investigate how much of the large-N_f phase structure survives when going beyond mean-field, we have performed lattice simulations along the T and μ axes of the phase diagram at finite magnetic field.

After outlining our lattice setup on the next slide, we show on the slides that follow the behavior of the chiral condensate $\langle|\sigma|\rangle$ in various scenarios:

(6) ... infinite-volume extrapolation of the T-dependence at $B = 0 = \mu$.

(7) ... infinite-volume extrapolation of the μ-dependence at $B = 0 \approx T$.

(8) ... infinite-volume and continuum extrapolations of the B-dependence at $\mu = 0 \approx T$.

(9) ... B-dependence at various T and $\mu = 0$ for one lattice size.

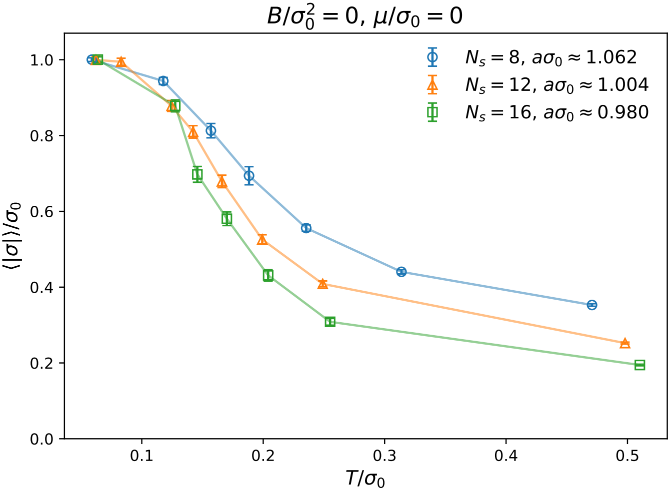
(10) ... μ-dependence at various B and $T \approx 0$ for one lattice size.

Lattice results

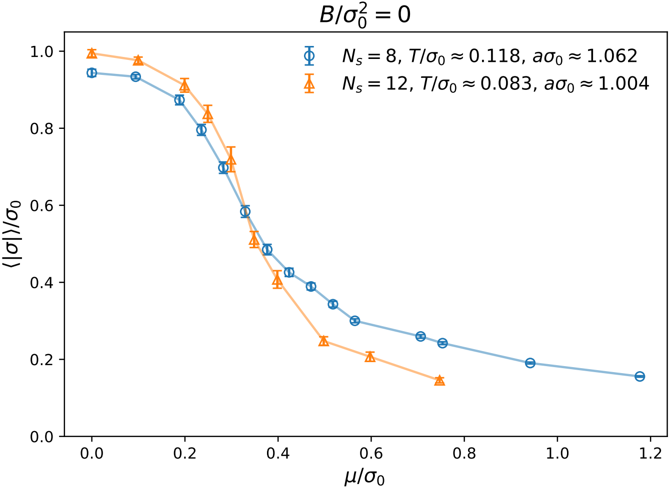
Simulation details:

- $N_\mathrm{f} = 1$ overlap fermions.
- Measure $|\sigma|$ to avoid cancellations.
- 8^3, 12^3 and 16^3 lattices with different lattice spacings.
- Change temperature by varying N_t.
- RHMC algorithm.
- Complex-action problem for $\mu \neq 0 \neq B$, probably mild.
  For now: phase quenching, $\det(D) \to |\det(D)|$.

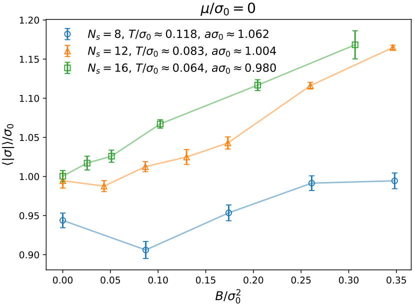
Lattice results

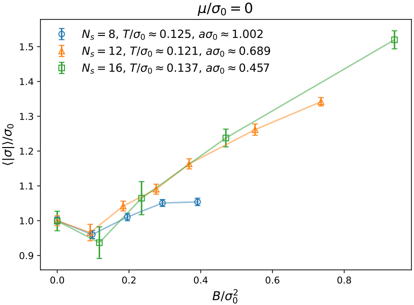


Lattice results

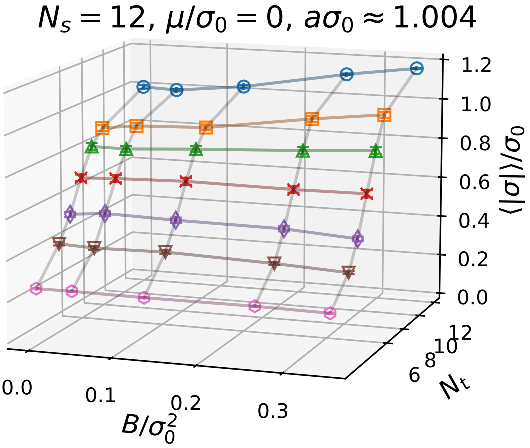


Lattice results

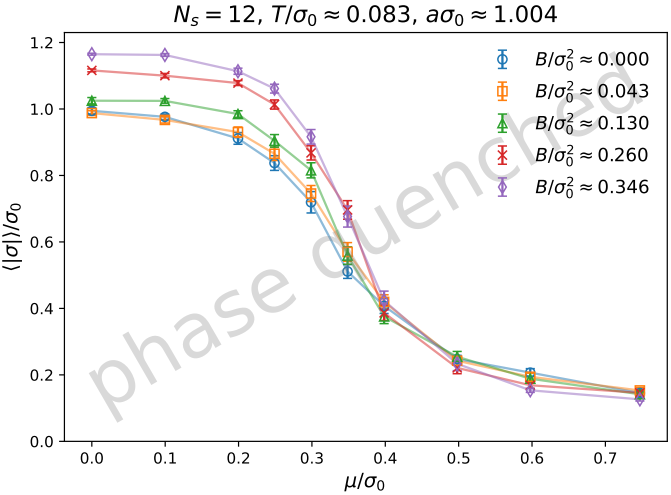




Lattice results



Lattice results



Lattice results*

Notice that due to taking the absolute value $|\sigma|$ we cannot observe a vanishing order parameter beyond the ”phase transition”.

Our results indicate that in the infinite-volume limit a proper phase transition is recovered. The $B = 0$ results are furthermore qualitatively consistent with existing literature.

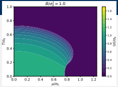
On slide (8) we see that the non-monotonic behavior at small B is a finite-size effect.

Apart from uncertainties due to the complex-action problem at $\mu \neq 0 \neq B$ our results suggest that there is magnetic catalysis everywhere below the phase transition.

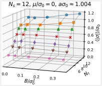
Summary & Outlook

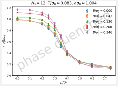
Summary

- Magnetic catalysis and inverse magnetic catalysis in large-N_f.



- Only magnetic catalysis (?) for $N_\mathrm{f} = 1$.





- Contradicts OPT calculations [J.-L. Kneur, M. B. Pinto, R. O. Ramos; Phys. Rev D **88** (2013)].

Outlook

- Inhomogeneous phases?
- More sophisticated models.

Summary & Outlook*

It is known that magnetic fields can induce the presence of inhomogeneous phases in Four-Fermi theories. We wish to investigate this on the lattice in the future.

Furthermore, in order to make predictions for QCD, we would like to study more sophisticated models that more closely resemble QCD, both in 3D and in 4D.

Contact*

For questions/discussion please do not hesitate to contact the author of this talk via
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Backup

The Large-N_f limit

The GN Lagrangian reads

$$\mathcal{L} = i\bar{\psi}\mathbb{1}_{N_\mathrm{f}}\left(\not{\partial} + \sigma + \mu\gamma_0 + ie\not{A}\right)\psi + \frac{N_f}{2g^2}\sigma^2 \ .$$

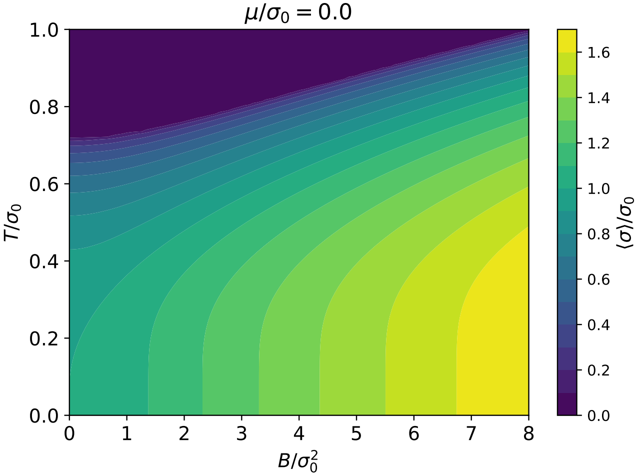
In the limit $N_\mathrm{f} \to \infty$, after integrating out the fermions in the path integral, the chiral condensate $\langle\sigma\rangle \propto \langle\bar{\psi}\psi\rangle$ is given by the minimum of

$$S_\mathrm{eff}[\sigma] = -\ln\det(D[\sigma]) + \frac{1}{2g^2}\int d^3x\,\sigma^2(x) \ ,$$

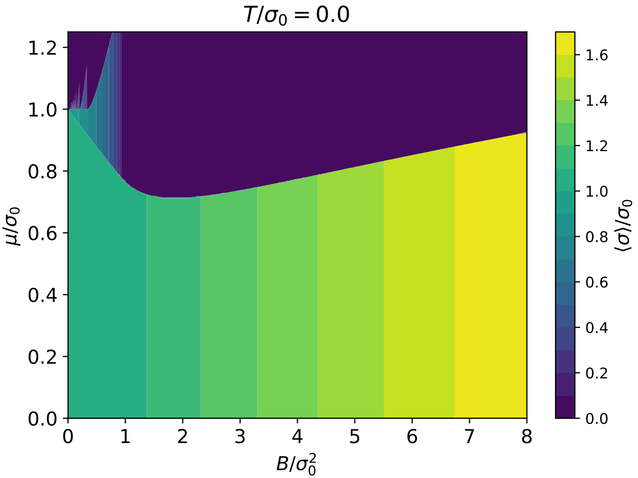
with

$$D[\sigma] = \not{\partial} + \sigma + \mu\gamma_0 + ie\not{A} \ .$$

Mean-field phase diagrams



Mean-field phase diagrams



Reducible representation of γ_μ

To allow for a notion of chirality in $(2+1)$ dimensions, we combine the two irreducible representations of the Dirac algebra into a reducible one:

$$\gamma_0 = \begin{pmatrix} \tau_2 & 0 \\ 0 & -\tau_2 \end{pmatrix} , \quad \gamma_1 = \begin{pmatrix} \tau_3 & 0 \\ 0 & -\tau_3 \end{pmatrix} , \quad \gamma_2 = \begin{pmatrix} \tau_1 & 0 \\ 0 & -\tau_1 \end{pmatrix} ,$$

where τ_μ are the usual Pauli matrices.

There are now two γ matrices which anti-commute with all the others:

$$\gamma_4 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} 0 & i\mathbb{1}_2 \\ -i\mathbb{1}_2 & 0 \end{pmatrix} .$$

Chiral symmetry in the continuum

The 1-flavor massless $(2+1)$-dimensional GN model in a reducible representation of the Dirac algebra has the following symmetries:

$$U_{\mathbb{1}}(1) : \psi \to e^{i\alpha}\psi \ ,$$
$$U_{\gamma_{45}}(1) : \psi \to e^{i\alpha\gamma_{45}}\psi \ , \quad \gamma_{45} = i\gamma_4\gamma_5 \ ,$$
$$\mathbb{Z}_2 : \psi \to i\gamma_5\psi.$$

The $\mathbb{Z}_2$ symmetry generated by γ_4 is not independent.

A mass term induces the breaking pattern

$$U_{\mathbb{1}}(1) \times U_{\gamma_{45}}(1) \times \mathbb{Z}_2 \to U_{\mathbb{1}}(1) \times U_{\gamma_{45}}(1) \ .$$

Chiral symmetry on the lattice

On the lattice the symmetries look as follows:

$$\begin{aligned} U_{\mathbb{1}}(1) &: \psi \to e^{i\alpha}\psi \ , \\ U_{\gamma_{45}}(1) &: \psi \to e^{i\alpha\gamma_{45}}\psi \ , \\ \mathbb{Z}_2 &: \psi \to i\gamma_5(1 - D_{\text{ov}})\psi \ , \quad \bar{\psi} \to i\bar{\psi}\gamma_5 \ , \end{aligned}$$

where D_{ov} is the massless overlap operator, in our case.

There is another $\mathbb{Z}_2$ generated by γ_4, which is, again, independent.

A mass term $\bar{\psi}\left(1 - \frac{D_{\text{ov}}}{2}\right)\psi$ again breaks the $\mathbb{Z}_2$ symmetry, but leaves both $U(1)$'s intact.

Overlap operator in the GN model

We use Neuberger's overlap operator [H. Neuberger; Phys. Lett B **417** (1998)]

$$D_{\mathrm{ov}} = \mathbb{1} + A/\sqrt{A^\dagger A}\,, \quad A = D_{\mathrm{W}} - \mathbb{1}\,,$$

where D_W is the standard Wilson operator. The full operator, including σ and μ, reads [R. V. Gavai, S. Sharma; Phys. Lett B **716** (2012)]

$$D_{\mathrm{full}} = \left(1 - \frac{\sigma + \mu\gamma_0}{2}\right) D_{\mathrm{ov}} + \sigma + \mu\gamma_0\,.$$

With the Ginsparg-Wilson chiral condensate $\Sigma_{\mathrm{GW}} = \left\langle \bar{\psi} \left(\mathbb{1} - \frac{D_{\mathrm{ov}}}{2}\right) \psi \right\rangle$ we have a Ward identity in analogy to the continuum theory:

$$\langle \sigma \rangle = \Sigma_{\mathrm{GW}}\,.$$

The observable

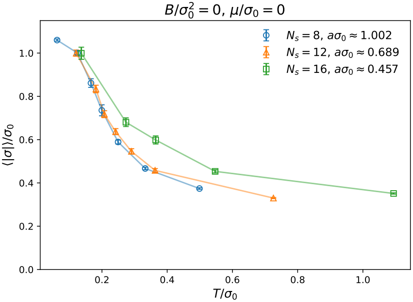
In order to avoid cancellation of contributions from the two minima of the effective action ($\pm\sigma$) in the broken phase we measure the absolute value

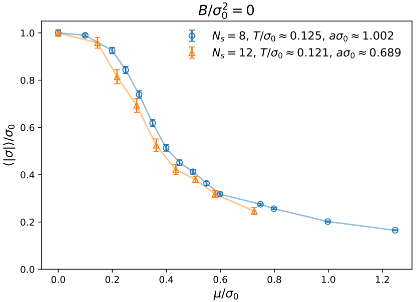
$$\langle|\sigma|\rangle = \langle|\sum_{x\in\Lambda}\sigma(x)|\rangle \ ,$$

where the sum runs over the whole lattice.

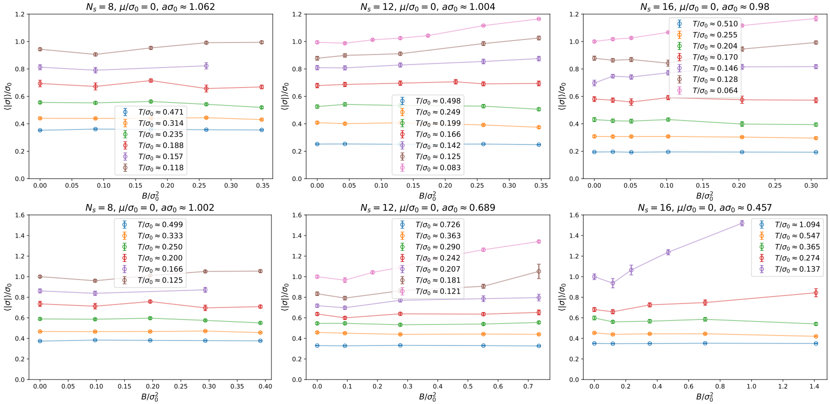
As a caveat, this definition makes it harder to determine a phase transition, as we cannot measure $\langle|\sigma|\rangle = 0$.

Lattice results at $B = 0$





Lattice results at $\mu = 0$



Lattice results at $T \approx 0$