Use of Schwinger-Dyson equation in constructing an approximate trivializing map

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(RBRC)

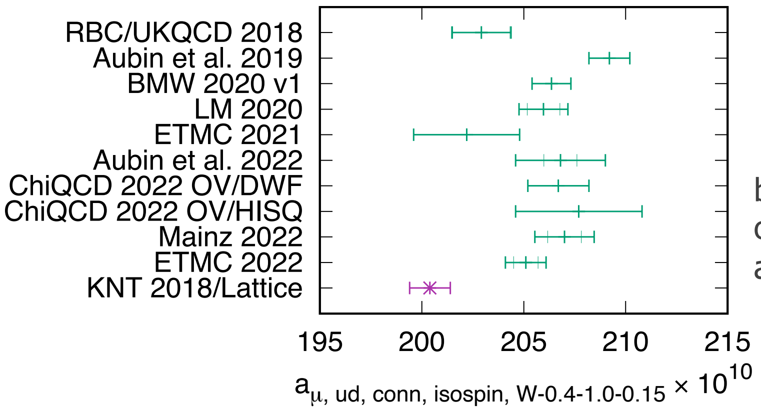
LATTICE2022 8/8/2022 @Bonn

work in collaboration with
P. Boyle, T. Izubuchi, L. Jin, C. Jung, C. Lehner, and A. Tomiya
BNL/Edinburgh BNL/RBRC UConn/RBRC BNL/RBRC BNL/Regensburg/RBRC IPUT Osaka

(work in progress)

QCD lattice calculation has been taking
an important and crucial role in the precision test of the standard model.

e.g., HVP contribution in muon g-2



Fri HS3 18:20 Christoph Lehner
"RBC/UKQCD update of the HVP contribution to the muon g-2"

borrowed from Christoph's slide;
overview plot of tentative lattice results
and the experimental R-ratio result

The required precision is getting in a magnificent order,
and calculation on fine lattices has become more demanding.

However, as we reach the continuum limit, we face the *critical slowing down*,
which is often characterized by very long autocorrelation of topological charge.

Goal of this study

Develop the idea of *trivializing map* Nicolai 1980, Lüscher 0907.5491
map between finite β theory and $\beta = 0$ theory

and utilize it to generate fine lattice configurations with reduced numerical cost.

g-2 will be extensively addressed in this conference:

Mon **HS2** 17:50 $\gamma^*\gamma^* \to \pi\pi$ R.Briceno
Tue **HS3** 15:00 ~~isospin~~ M.Hoferichter
15:20 ~~isospin~~ M.Bruno
15:40 coord space J.Parrino
16:20 twisted mass G.Gagliardi
16:50 improved Wilson S.Kuberski
17:10 disconnected S.Bacchio
17:30 EM coupling M.T.San Jose Perez
17:50 EM coupling H.Wittig, S.J.Perez
PosterA low-mode noise M.Lynch
PosterB strange&charm P.Tavella
Wed **HS6** 14:40 ~~isospin~~ A.Portelli
HS4 15:00 f_K, f_π F.Stokes
Thu **HS3** 9:00 C^* bc R.Gruber
9:20 C^* bc A.Altherr
9:40 FV correction S.Martins
10:00 lattice R-ratio A.De Santis
10:40 π pole to HLbL G.Kanwar
11:30 η pole to HLbL S.A.Burri
HS4 11:30 UV $\log a$ R.Sommer
Fri **HS7** 15:10 IR $\pi\pi$ S.Paul
HS3 15:30 overlap&staggered mixed A.Kotov
15:50 $F_{\pi^0/\eta/\eta' \to \gamma^*\gamma^*}$ W.Verplanke
16:40 q_l connected S.Gottlieb
17:00 BWM update B.Toth
17:20 QED correction G.Sinha Ray
17:40 overlap valence G.Wang
18:00 staggered finite a M.Golterman
18:20 RBC/UKQCD update C.Lehner

Please let me know if I missed your work or if I have a wrong description.

Introduction (2/3)

There are many promising approaches for solving the critical slowing down/topological freezing:

Trivializing/normalizing map

Mon HS7 14:00 *Flow-based density of states for complex actions* **Julian Urban**
14:20 *Stochastic normalizing flows for lattice field theory* **Elia Cellini**
15:00 *Gauge-equivariant flow models for sampling in lattice field theories with pseudofermions* **Fernando Romero-Lopez**
16:30 *Status update on flow models for gauge field generation* **Phiala Shanahan**
17:50 *Machine Learning Trivializing Maps* **Joe Marsh Rossney**
18:10 *Learning trivializing flows* **David Albandea**
Wed HS7 17:50 *Generative models for scalar field theories: how to deal with poor scaling?* **Javad Komijani**

Master field analysis

Mon HS5 16:40 *Translating topological benefits in very cold master-field simulations* **Anthony Francis**

Metadynamics

Mon HS7 17:30 *Topology changing update algorithms for SU(3) gauge theory* **Timo Eichhorn**
Poster B *Metadynamics Surfing on Topology Barriers in the Schwinger Model* **Philip Rouenhoff**

Fourier acceleration

Poster B *Fourier acceleration in strongly-interacting linear sigma models* **Joshua Swaim**

Parallel tempering

Tue HS5 14:20 *Towards glueball masses of large-N SU(N) Yang-Mills theories without topological freezing via parallel tempering on boundary conditions* **Claudio Bonanno**
HS1 15:20 *Parallel tempering algorithm applied for the deconfinement transition of quenched QCD* **Ruben Kara**

Multicanonical approach

Tue HS5 15:20 *Topological susceptibility in high temperature full QCD via staggered spectral projectors* **Francesco D'Angelo**

Skewed detailed balance

Thu HS7 10:40 *Towards the Application of Skewed Detailed Balance in Lattice Gauge Theories* **Joao C. Pinto Barros**

Fermion algorithms

Sat HS2 11:00 *Review on Algorithms for dynamical fermions* **Jacob Finkenrath**

Please let me know if I missed your work.

Also around BNL there are intense studies on

Proceedings of lattice 2021

- Gauged-fixed Fourier acceleration — **Sheta, Zhao, Christ 2108.05486**
- Riemannian manifold HMC — **Nguyen, Boyle, Christ, Jang, Jung 2112.04556**
- L2HMC — **Foreman, X.Y.Jin, Osborn 2201.01582**
- Machine learned trivializing map — **X.Y.Jin 2201.01862**

- Alternatively to Luscher's trivializing flow (flow kernel obtained as a t-expansion),
  Lüscher 0907.5491 flow time
  we design the approximate trivializing flow with the *Schwinger-Dyson* (SD) *equation*.
  Gonzalez-Arroyo, Okawa 1987, de Forcrand, Perez, Hashimoto, Hioki, Matsufuru, Miyamura, Nakamura, Stamatescu, Tagoi, Takaishij, Umeda hep-lat/9806008
- By extending L. Jin's code of field-transformed HMC to include generic flow kernels,
  Jin LATTICE 2021
  we perform the HMC with the resulting exact effective action after this flow.

Advantages of this method

- the basis for the flow kernel can be chosen by hand
- can be applied to the general action of interest
- the coefficients in the kernel are determined by lattice estimates of the observables; no need for analytic calculation such as t-expansion
- truncation effects and goodness of the flow can be measured by the force norm

- We apply our method to Wilson and DBW2 actions and show that:
  - With the SD method, we can have a better control of the effective action than t-expansion
  - In particular cases, faster decorrelation (in MC step unit) is observed for long-ranged observables by adding rectangle and chair to the flow
    cf. Engel, Schaefer 1102.1852
  - However, we have large algorithmic overhead, and need to check the scaling with larger statistics to confirm the actual benefits at large β

Trivializing map

- For a field transformation $\mathcal{F}_t : V \mapsto U = \mathcal{F}_t(V)$, we have the effective action $S_{\mathrm{eff},t}(V)$:

$$\int dU\, e^{-S(U)} \mathcal{O}(U) = \int dV \det \mathcal{F}_{t*}(V)\, e^{-S(\mathcal{F}_t(V))} \mathcal{O}(\mathcal{F}_t(V))$$

$$\equiv \int dV\, e^{-S_{\mathrm{eff},t}(V)} \mathcal{O}(\mathcal{F}_t(V))$$

i.e.,

$$S_{\mathrm{eff},\,t}(V) \equiv S\big(\mathcal{F}_t(V)\big) - \ln\det \mathcal{F}_{t*}(V)$$

$\mathcal{F}_{t*}(V) = (\mathcal{F}_{t*}^{AB}(V))$: Jacobian matrix

$d\theta^A_{(U)} = \mathcal{F}^{AB}_{t*}(V)\, d\theta^B_{(V)}$ where $U'_{x,\mu} = e^{\theta^a_{x,\mu} T^a} U_{x,\mu}$

or $\partial^a_{x,\mu} U_{x,\mu} = T^a U_{x,\mu}$

T^a: su(3) generators, $\mathrm{tr}(T^a T^b) = -\frac{1}{2}\delta^{ab}$

Haar measure: $(dU) \propto \prod_A d\theta^A$

$A = (x, \mu, a)$ labels DOF

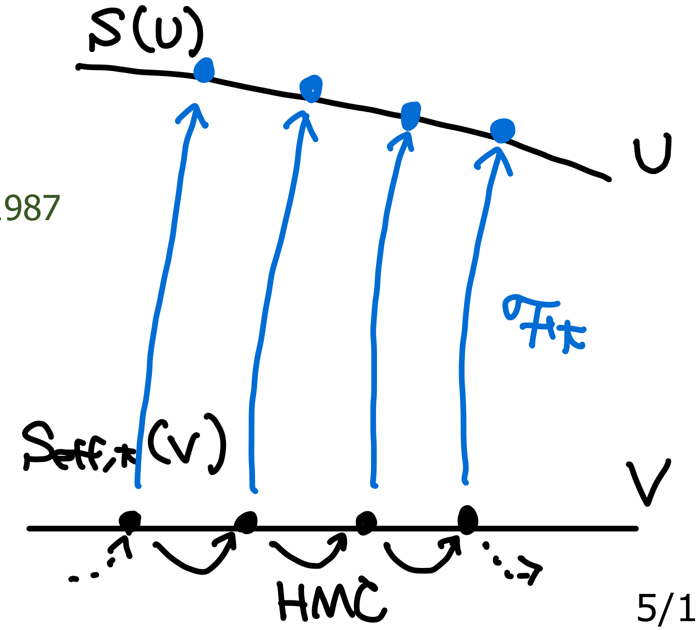
$\partial_{\theta^a_{x,\mu}} \equiv \partial^a_{x,\mu}$

- Lüscher proposed to use the *trivializing map* $\mathcal{F}_t$ in HMC that makes the finite β theory mapped to the strong coupling limit $\beta = 0$ at $t = 1$:

Duane, Kennedy, Pendleton, Roweth 1987

$$S_{\mathrm{eff},\,t=1}(V) = \mathrm{const.}$$

V will be decorrelated faster under the trivial action, and so does the configurations $U = \mathcal{F}_t(V)$



- Gradient flow ansatz:

$$\dot{\mathcal{F}}_{t,\epsilon}(U)_{x,\mu} = -T^a \partial^a_{x,\mu} \tilde{S}_t(U) \cdot U_{x,\mu}$$

- Require that $\mathcal{F}_t$ trivializes the theory at $t = 1$:

$$S_{\mathrm{eff},\,t}(V) \overset{\Delta}{=} S(\mathcal{F}_t(V)) - \ln\det \mathcal{F}_{t*}(V)$$

requirement

$$\overset{*}{=} (1-t)\, S(\mathcal{F}_t(V))$$

d/dt

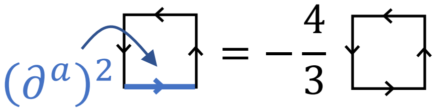
equation for the kernel:

$$-(\partial^A)^2 \tilde{S}_t + t\, \partial^A S\, \partial^A \tilde{S}_t \; = S$$

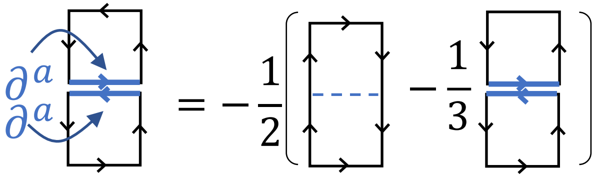
(up to irrelevant const)

from Jacobian   from action

$\partial^a_{x,\mu} \sim T^a$ insertion



$$\mathrm{tr}[(T^a)^2 \dots] = -\tfrac{4}{3}\mathrm{tr}[\dots]$$



$$\mathrm{tr}[T^a A]\,\mathrm{tr}[T^a B] = -\tfrac{1}{2}\left(\mathrm{tr}[AB] - \tfrac{1}{3}\mathrm{tr}A \cdot \mathrm{tr}B\right)$$

- Solution (for Wilson action $S = S_W$):

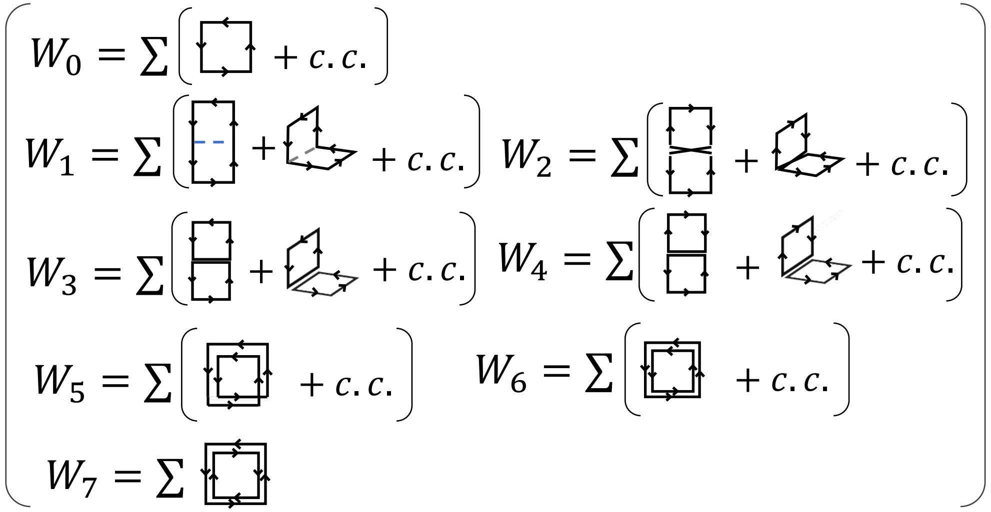
$$\tilde{S}_t = -\frac{\beta}{32} W_0$$

← LO: plaquette

$$+t\frac{\beta^2}{192}\left(-\frac{4}{33}W_1 + \frac{12}{119}W_2 + \frac{1}{33}W_3 - \frac{5}{119}W_4 + \frac{3}{10}W_5 - \frac{1}{5}W_6 + \frac{1}{9}W_7\right)$$

$$+O(t^2)$$

NLO: rectangle, chair, twisted rectangle …
"footprint 2 shapes"



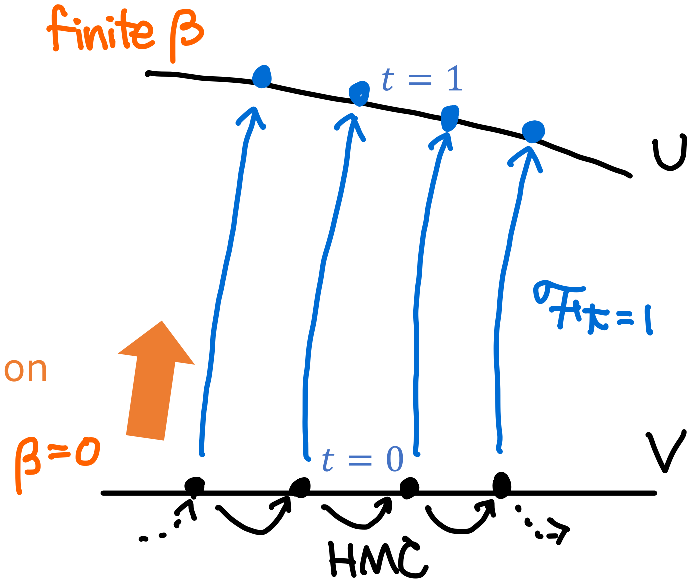
Boyle, Izubuchi, L. Jin, Jung, NM, Lehner, Tomiya, work in progress

- Note that the t-expansion is performed around $t=0$;
  this corresponds to expanding *around the trivialized theory*.

  In fact, the expansion parameter t appears in the combination βt
  ∴ approximation is better for small β.

  However, our primary target is the large β theory;
  ∴ we rather want to approximate the map
  based on the information at large β.

direction of
construction
in the t-expansion



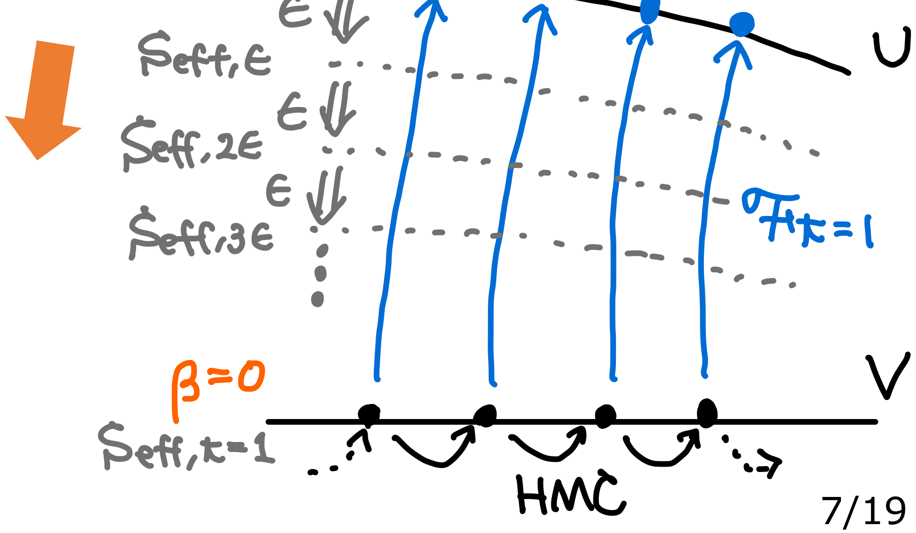
- In the Schwinger-Dyson (SD) method below,
  we sequentially determine the flow *from the finite β theory*.

  - At each intermediate step,
    we determine the effective couplings by the SD equation.

  - We then design the flow to decrease the couplings
    from the lattice estimates of Wilson loops.

  This determination of the flow can be seen as
  iteratively reexpanding the flow kernel at each step.

direction of
construction
in the SD method



Schwinger-Dyson equation

Schwinger-Dyson tomography (1/2)

Gonzalez-Arroyo, Okawa 1987, de Forcrand, Perez, Hashimoto, Hioki, Matsufuru, Miyamura, Nakamura, Stamatescu, Tagoi, Takaishij, Umeda hep-lat/9806008

- Suppose that

$$S_{\mathrm{eff}}(V) = \sum_j \beta_j W_j$$

W_j: Wilson loops

we omit the subscript t momentarily

- The couplings β_j obey the linear equation:

$$\sum_j \beta_j \left\langle \partial^A W_j \, \partial^A W_i \right\rangle_{S_{\mathrm{eff}}} = \langle (\partial^A)^2 W_i \rangle_{S_{\mathrm{eff}}}$$

$\langle \cdot \rangle_{S_{\mathrm{eff}}}$: expectation value with respect to S_{eff}

$\because$

- Consider a variation using W_i as the flow kernel:

$$\delta V = -\epsilon T^A \partial^A W_i \cdot V$$

- The path integral is invariant under this variation (Schwinger-Dyson equation):

$$0 = \delta \int (dV)\ e^{-S_{\mathrm{eff}}(V)} = \int (dV)\ e^{-S_{\mathrm{eff}}(V)} \epsilon \left[\underbrace{-(\partial^A)^2\, W_i}_{\text{from Jacobian}} + \underbrace{\partial^A S_{\mathrm{eff}}\, \partial^A W_i}_{\text{from action}} \right]$$

- Combining this formula with the expansion of $S_{\mathrm{eff}}(V)$:

$$\sum_j \beta_j \left\langle \partial^A W_j \, \partial^A W_i \right\rangle_{S_{\mathrm{eff}}} = \langle (\partial^A)^2 W_i \rangle_{S_{\mathrm{eff}}} \quad \square$$

cf. Gonzalez-Arroyo, Okawa 1987, de Forcrand, Perez, Hashimoto, Hioki, Matsufuru, Miyamura, Nakamura, Stamatescu, Tagoi, Takaishij, Umeda hep-lat/9806008

- Generically, we need infinite number of couplings to exactly parametrize $S_{\text{eff}}(V) = \sum_j \beta_j W_j$.

$$\therefore \quad \sum_j \beta_j \left\langle \partial^A W_j \, \partial^A W_i \right\rangle_{S_{\text{eff}}} = \langle (\partial^A)^2 W_i \rangle_{S_{\text{eff}}} \quad \text{: infinite-dimensional matrix}$$

- Instead, we can try to mimic $S_{\text{eff}}(V)$ with a finite basis:

$$S'_{\text{eff}}(V) = \sum'_j \beta'_j W_j .$$

prime symbols indicate truncation
(j runs a finite range)

We determine β'_j by:

$$\sum'_j \beta'_j \left\langle \partial^A W_j \, \partial^A W_i \right\rangle_{S_{\text{eff}}} = \langle (\partial^A)^2 W_i \rangle_{S_{\text{eff}}}$$

: finite-dimensional matrix
(i is also restricted to the finite range)

Such β'_j give the best approximation of $S_{\text{eff}}(V)$
in the sence that it minimizes the norm

$$\| S_{\text{eff}} - S'_{\text{eff}} \|_{S_{\text{eff}}} , \quad \text{where} \quad \underline{\| S \|^2_{S_{\text{eff}}} \equiv \langle (\partial^A S)^2 \rangle_{S_{\text{eff}}}}$$

L2 norm of the force
∴ The truncation error is calculable.

∵ Subtracted equation

$$\sum'_j \, (\beta_j - \beta'_j) \left\langle \partial^A W_j \, \partial^A W_i \right\rangle_{S_{\text{eff}}} = 0$$

is the stationary condition:

$$\frac{\partial}{\partial \beta'_i} \| S_{\text{eff}} - S'_{\text{eff}} \|^2_{S_{\text{eff}}}$$
$$= \frac{\partial}{\partial \beta'_i} \langle [\partial^A (S_{\text{eff}} - S'_{\text{eff}})]^2 \rangle_{S_{\text{eff}}}$$
$$= -2 \sum'_j \, (\beta_j - \beta'_j) \left\langle \partial^A W_j \, \partial^A W_i \right\rangle_{S_{\text{eff}}}$$
$$\equiv 0$$

This Schwinger-Dyson method
gives us a way to parametrize effective actions.

Design the flow with the Schwinger-Dyson equation (1/1)

Boyle, Izubuchi, L. Jin, Jung, NM, Lehner, Tomiya
work in progress

- We parametrize $\underline{\tilde{S}_t}$ in the same truncated space:
  kernel function

$$\tilde{S}_t(V) = \textstyle\sum_k' \gamma_{k,t} W_k$$

- Differentiating the equation for $\beta'_{j,t}$:

$$\left\langle -(\partial^A)^2 W_i + \textstyle\sum_j' \beta'_{j,t}\, \partial^A W_j\, \partial^A W_i \right\rangle_{S_{\mathrm{eff},\,t}} = 0$$

d/dt

$$\textstyle\sum_k' \gamma_{k,t} \left\langle \partial^B W_k \partial^B [-(\partial^A) W_i + \partial^A S'_{\mathrm{eff},\,t}\, \partial^A W_i] \right\rangle_{S_{\mathrm{eff},\,t}} = -\textstyle\sum_j' \dot{\beta}'_{j,t} \left\langle \partial^A W_j\, \partial^A W_i \right\rangle_{S_{\mathrm{eff},\,t}}$$

(d/dt) acting on the Boltzmann weight $\qquad (d/dt)$ acting on $\beta'_{j,t}$

This equation gives the coefficients $\gamma_{k,t}$ for a given $\dot{\beta}'_{j,t}$ (thus a trajectory of $S'_{\mathrm{eff},\,t}$)

- We here require $\dot{\beta}'_{j,t} = -\frac{\beta'_{j,t}}{1-t}$ so that $\beta'_{j,t} = (1-t)\beta'_{j,t=0} = (1-t)\beta_{j,t=0}$ (linear decrease)

Linear equation for $\gamma_{k,t}$:

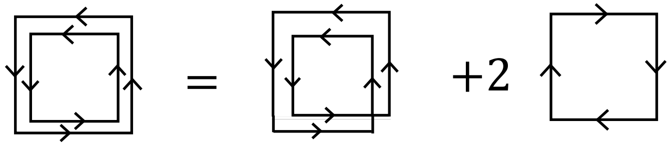
$$\textstyle\sum_k' \gamma_{k,t} \underline{\left\langle \partial^B W_k \partial^B [-(\partial^A) W_i + \partial^A S'_{\mathrm{eff},\,t}\, \partial^A W_i] \right\rangle}_{S_{\mathrm{eff},\,t}} = \frac{1}{1-t} \left\langle \partial^A S'_{\mathrm{eff},\,t} \partial^A W_i \right\rangle_{S_{\mathrm{eff},\,t}}$$

In practice, we use the numerical derivative with the five-point formula to calculate this matrix

Remarks (1/1)

- Some of the basis functions are not linearly independent (*"Mandelstam constraints"*) Mandelstam 1979

Relevant example:



$$\left(W_6 = W_5 + 2W_0 \right)$$

Further relation can be obtained by the Cayley–Hamilton eq:

$$U^3 = (\mathrm{tr}\, U)U^2 - \frac{1}{2}[(\mathrm{tr}\, U)^2 - \mathrm{tr}\, U^2]U + \mathbb{I}$$

$$\because\ (\mathrm{tr}U)^2 = \mathrm{tr}(U^2) + 2\mathrm{tr}U^\dagger \quad \left(U \in SU(3) \right)$$

We need to choose linearly independent basis in the inversion.

- We use the HMC algorithm with the exact transformed action $S_{\mathrm{eff}}(V)$ developed by Luscher.

Luscher 0907.5491

<u>Advantages of designing the trivializing map with the Schwinger-Dyson equation</u>

- the basis for the flow kernel can be chosen arbitrarily by hand
- can be applied to the general action of interest
- the coefficients in the kernel, $\gamma_{k,t}$, are determined by lattice estimates of the observables; no need for analytic calculation such as t-expansion
- truncation effects and goodness of the flow can be measured by the force norm

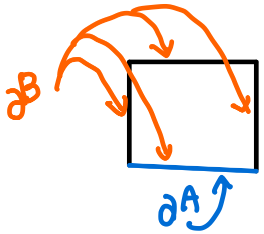
Boyle, Izubuchi, L. Jin, Jung, NM, Lehner, Tomiya work in progress

- Fully parallelized code based on qlat software (C++ codebase)
  https://github.com/waterret/Qlattice

  We extended L. Jin's code of field-transformed HMC
  Jin LATTICE 2021
  to include generic flow kernels including all the footprint 2 loops.

- Most costly part is the multiplication of a matrix including the Hessian $\partial^A \partial^B \tilde{S}_t$ in propagating the force.

E.g., plaquette

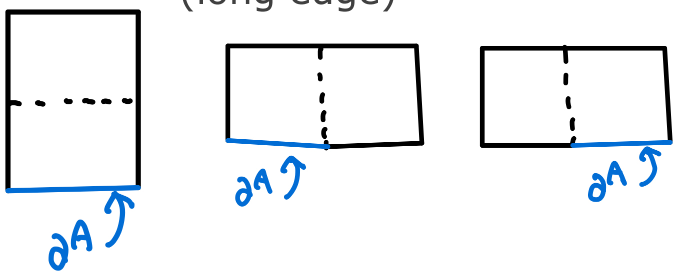


about
$3 \times 6/4 = 4.5$
cost difference

different pattern of derivatives
= different T^a insertions

rectangle
(short edge)

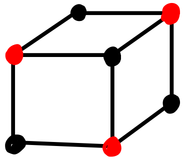
(long edge)



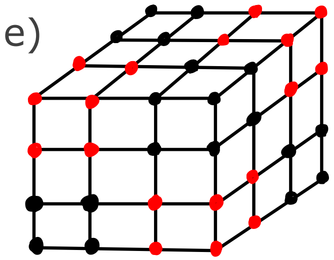
∂B: 6 patterns each

- By dividing the directions of the flowed links and appropriately coloring/masking the lattice for each type of loops, we can run the multiplication in parallel. Lüscher 0907.5491, Boyda, Kanwar. Pacaniere, Rezende, Albergo, Cranmer, Hackett, Shanahan 2008.05456

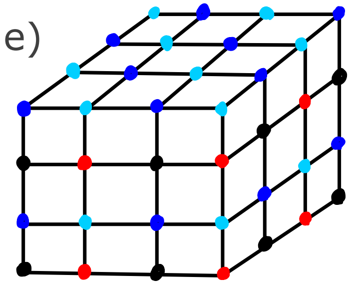
E.g., plaquette



rectangle
(short edge)



(long edge)



direction of the flowed links

Volume scaling is basically linear $O(\mathrm{vol})$ for local Wilson loop bases

Results

Computation resources (1/1)

- RIKEN HOKUSAI

- Univ of Tokyo Oakforest-PACS (retired)

picture taken from HP of CCS

- USQCD facility at BNL (KNL)
  funded by US DOE

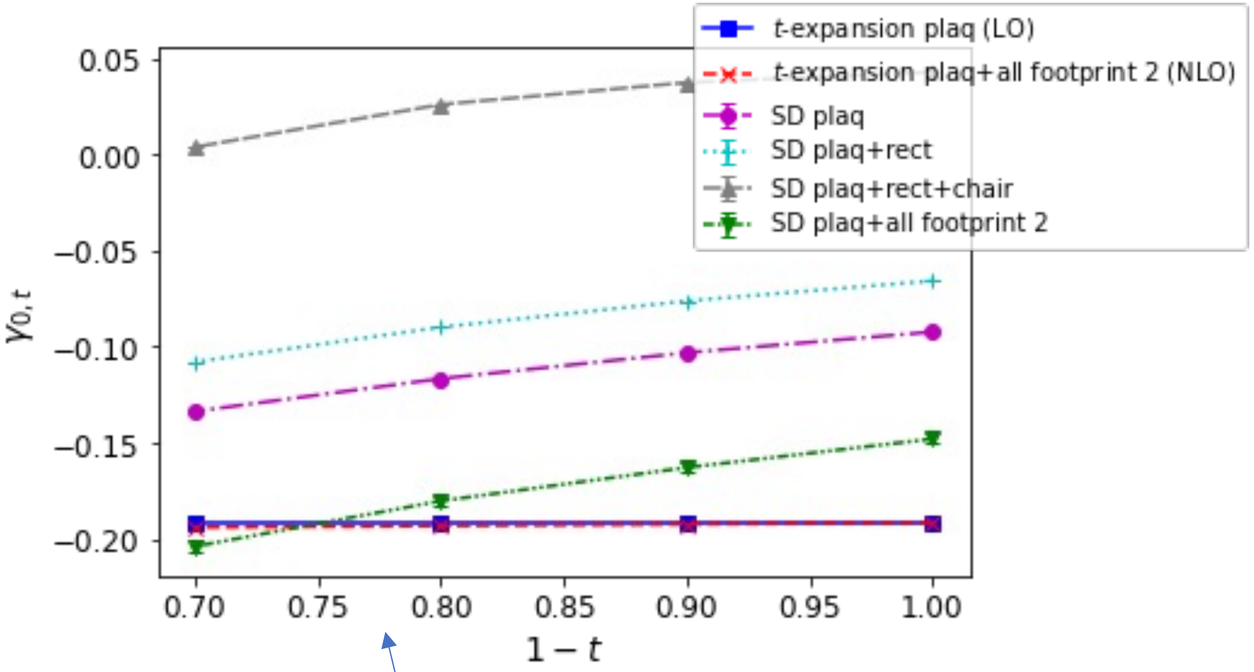
We are grateful for these resources.

$8^4, \beta = 6.13$ Wilson $(a^{-1} = 2.56\ \mathrm{GeV})$

Ce-Consonni-Engel-Giusti 1506.06052

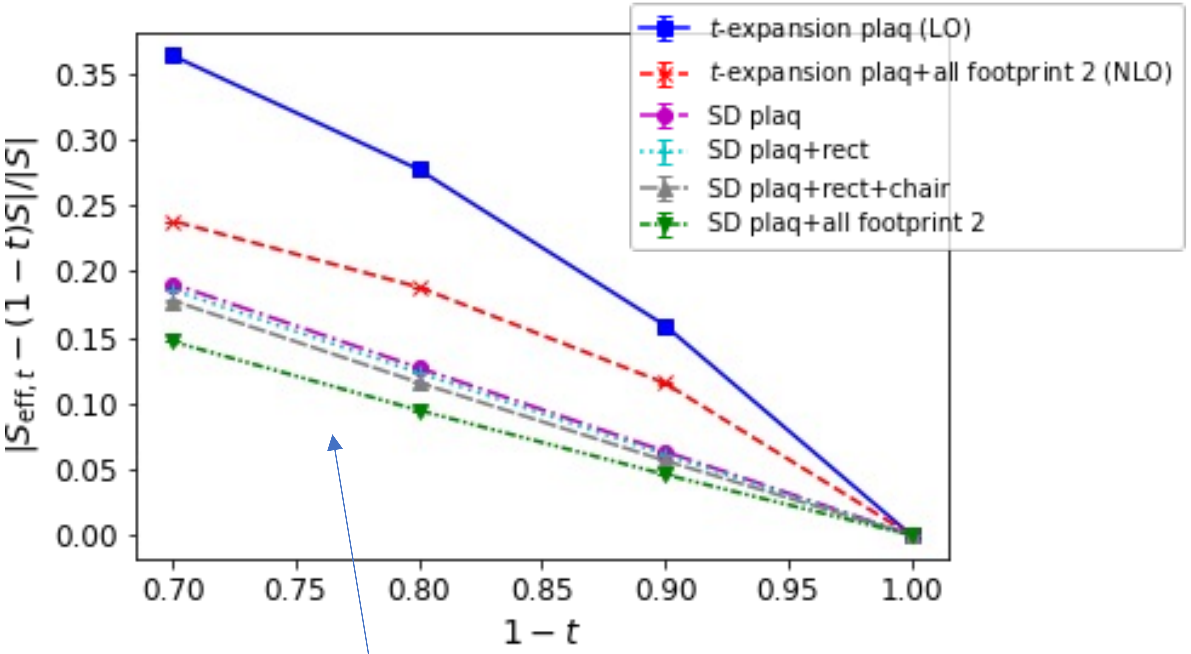
Boyle, Izubuchi, L. Jin, Jung, NM, Lehner, Tomiya
work in progress

Determined $\gamma_{0,t}$(plaqutte coefficient)



flow obtained by Schwinger-Dyson (SD)
is quite different from that by t-expansion

Difference from the target trajectory of effective action



With the SD method,
we can have better control of the effective action

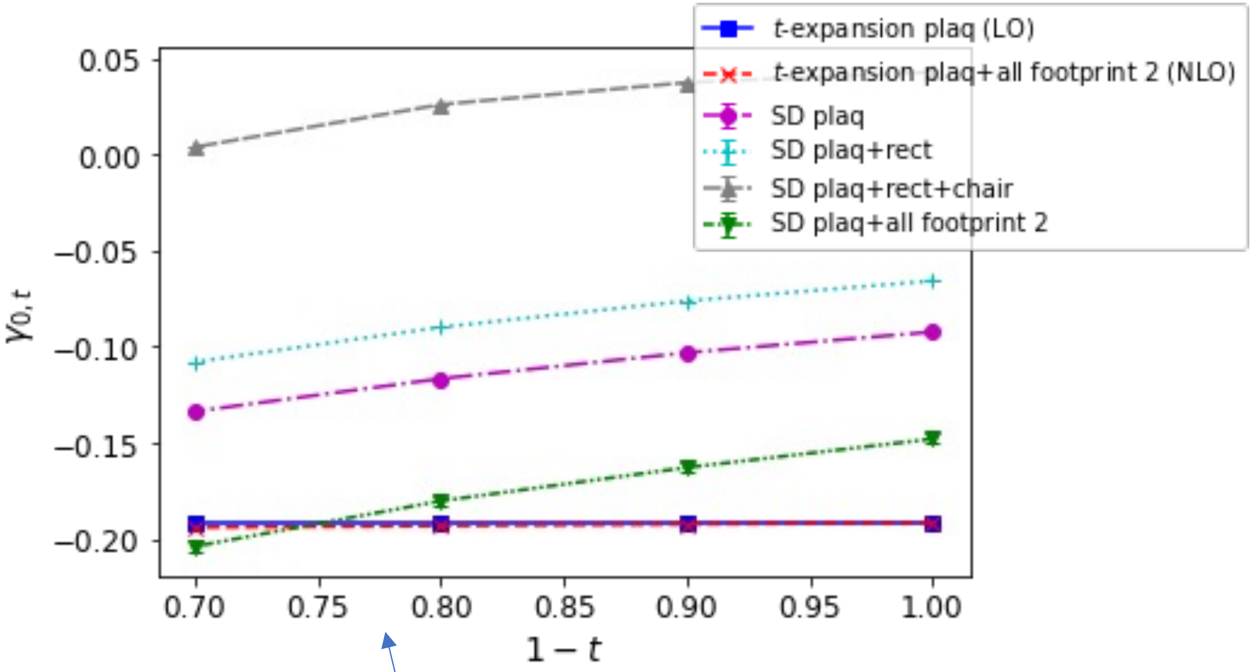
Evolution of effective action [Wilson action] (1/1)

$8^4, \beta = 6.13$ Wilson $(a^{-1} = 2.56\ \mathrm{GeV})$

Ce-Consonni-Engel-Giusti 1506.06052

Boyle, Izubuchi, L. Jin, Jung, NM, Lehner, Tomiya
work in progress

Determined $\gamma_{0,t}$(plaqutte coefficient)



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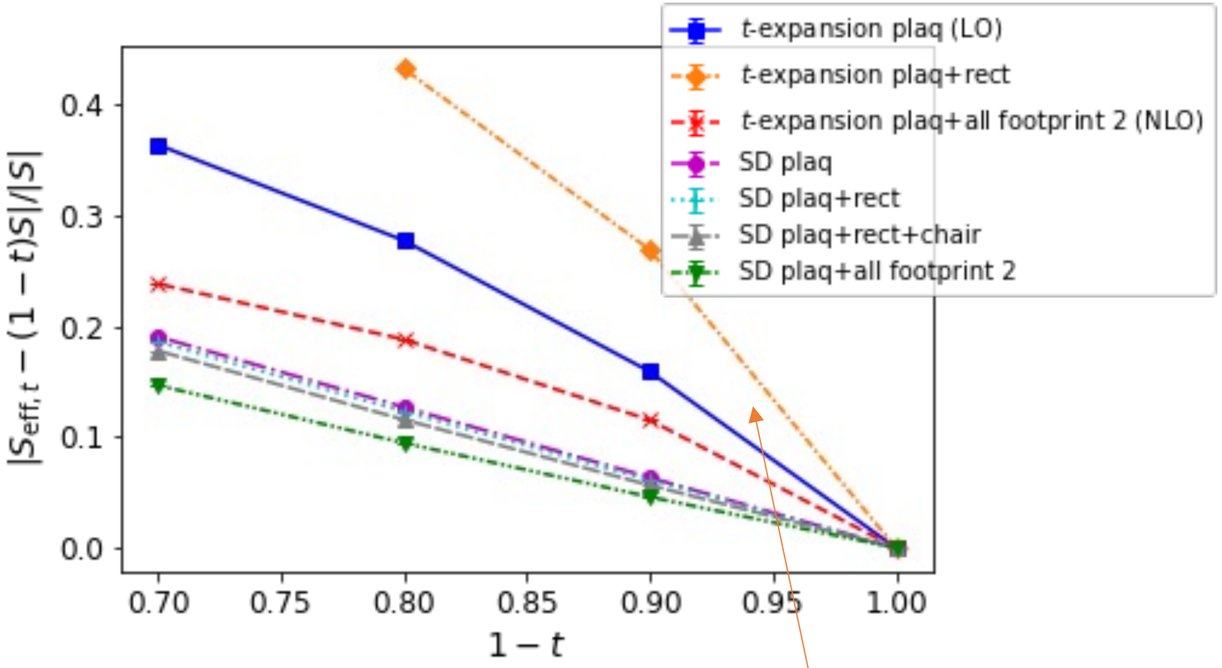
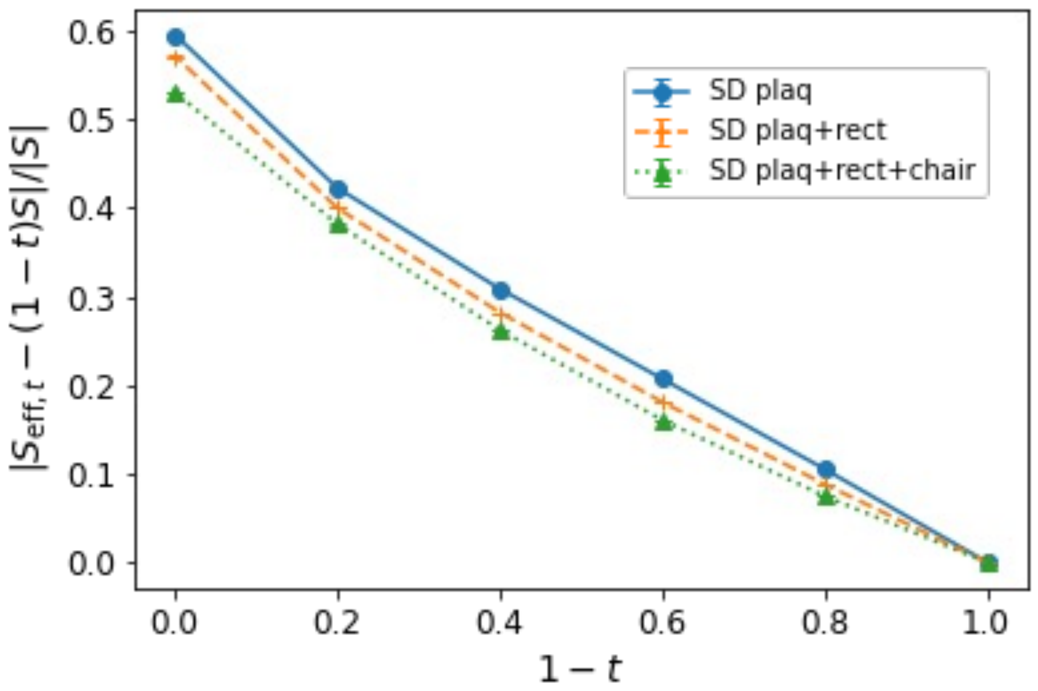


Naively adding the rectangle term
to the LO t-expansion makes
the deviation more significant

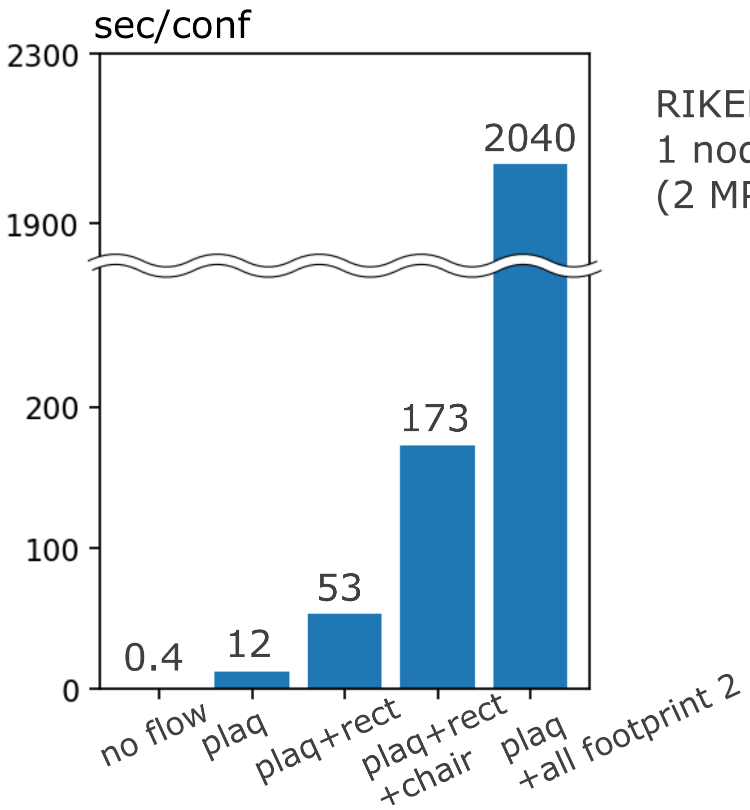
$8^3 \times 16, \beta = 0.89$ DBW2 $(c_1 = -1.4008)$ $(a^{-1} = 1.49\ \mathrm{GeV})$ Necco hep-lat/0309017

Boyle, Izubuchi, L. Jin, Jung, NM, Lehner, Tomiya
work in progress

Difference from the target trajectory of effective action



Computational cost (1 step flow)



RIKEN HOKUSAI
1 node
(2 MPI x 40 OpenMP)

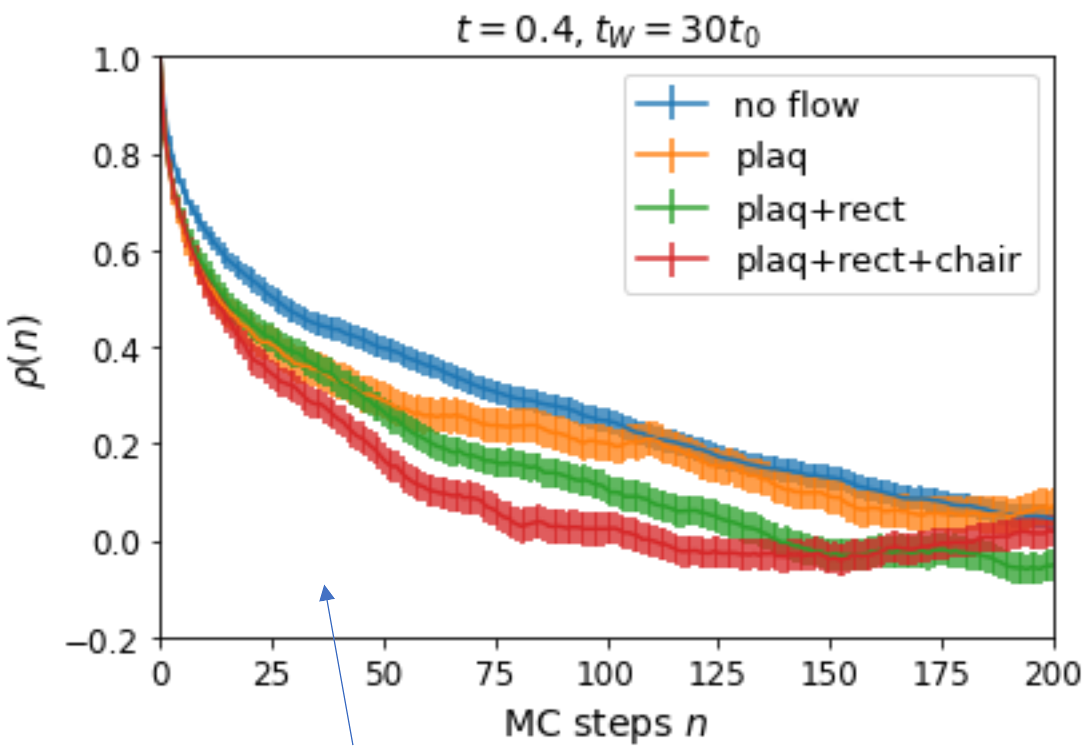
The increase is due to the increase of the nonzero matrix elements in $\partial^A \partial^B \tilde{S}_t$.

It seems essential to design the flow with a few types of loops to circumvent large algorithm overhead.

Boyle-Izubuchi-L. Jin-Jung-NM-Lehner-Tomiya, work in progress

$8^3 \times 16, \beta = 0.89$ DBW2 $\quad (a^{-1} = 1.49 \text{ GeV})$

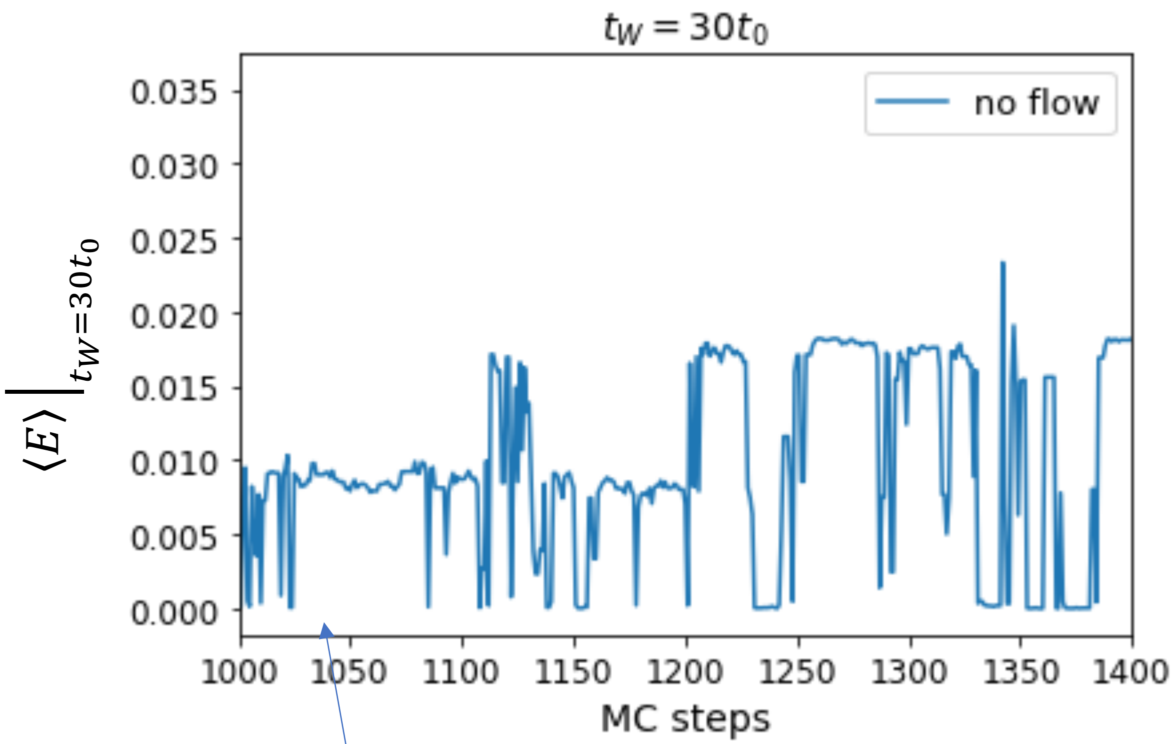
Normalized autocorrelation function $\rho(n)$
for the smeared energy density $(t_w = 30t_0)$



Faster decorrelation (in MC steps)
by including extended loops

t_W: Wilson smearing flow time

$t_W^2 \langle E \rangle|_{t_W = t_0} = 0.3$ Luscher 1006.4518



Smearing is performed so a long time
that the energy reflects the instantons

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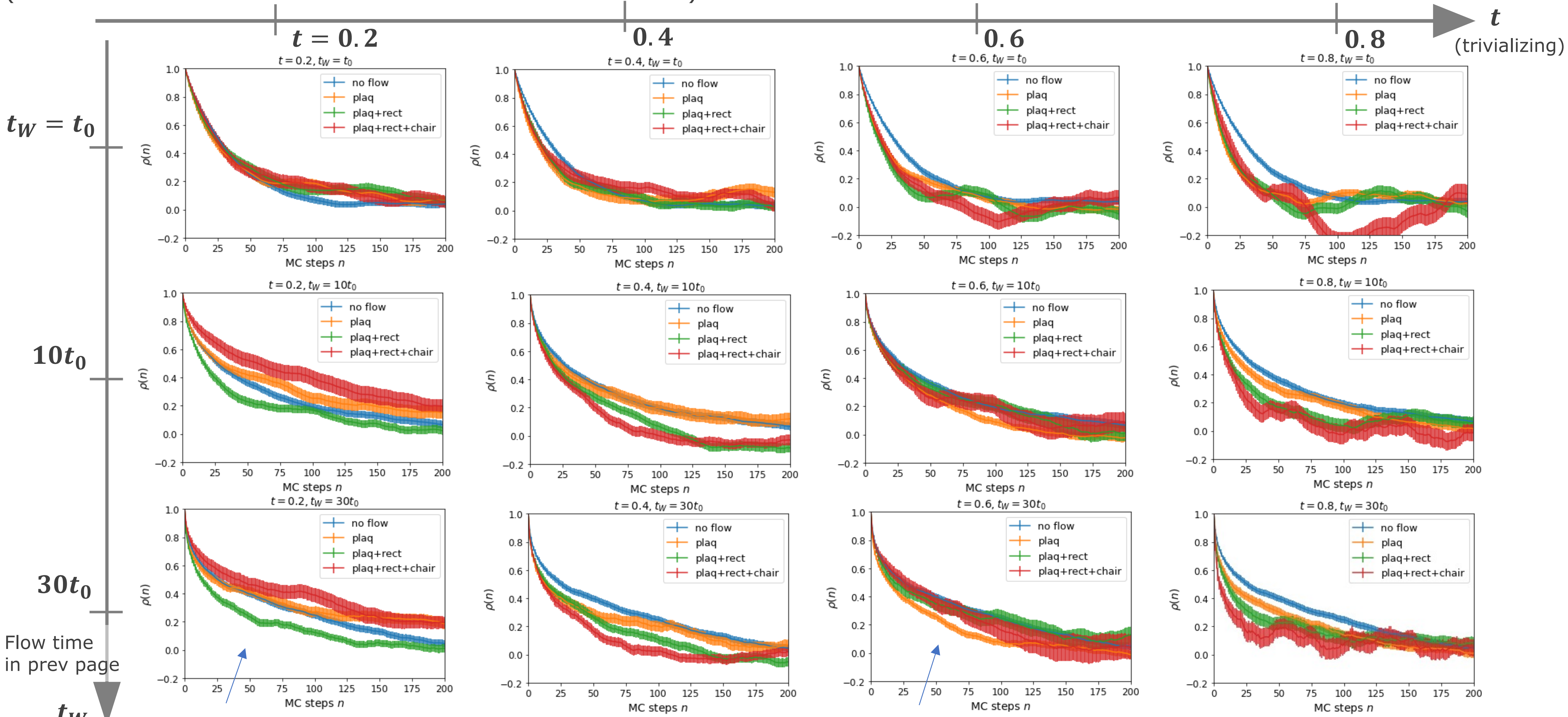
Faster decorrelation is observed by the extended loops, but the autocorrelation is not controlled completely:
(and statistics should be increased for definite conclusions)



plaq+rect is the best
(presumably ∵DBW2)

decorrelation by large kernels
not visible for this flow time

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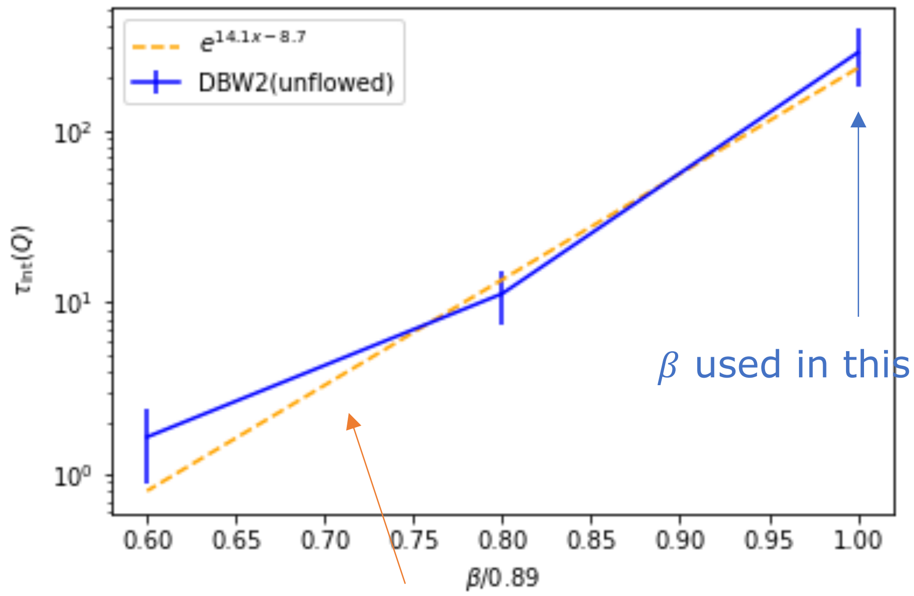
- Integrated autocorrelation time of the topological charge increases exponentially with β ($\because$ with a power of a^{-1}).

  If not complete trivialization,
  we can have large benefits by changing the exponent.



Possible strategies?

- SD method itself has a large room for optimization

  choose a different path of β_t' (e.g., exponential decrease, decrease c_1), t-dependent step sizes, change basis at each step …

- On the other hand, the relatively small improvements show that the truncated large loops contribute to increasing the autocorrelation.

  In fact, we need an *infinite* number of loops to obtain the exact trivializing flow, with which we expect to decrease the autocorrelation of *all modes* of the system;

  Lüscher 0907.5491

  since we need to deal with finite number of loops,
  we can be more specific to particular slow observables.

  e.g., change instanton potential to stimulate tunneling

  cf. Smith, Teper hep-lat/9801008
  DeGrand, Hasenfratz, Kovacs hep-lat/9801037
  De Forcrand, Garcia Perez, Hetrick, Stamatescu hep-lat/9802017
  Hasenfratz-Nieter hep-lat/9806026

Summary and Outlook

Summary

- We proposed a way to design an approximate trivializing map with the Schwinger-Dyson equation

  Advantages of this method

  - the basis for the flow kernel can be chosen arbitrarily by hand
  - can be applied to the general action of interest
  - the coefficients in the kernel are determined by lattice estimates of the observables; no need for analytic calculation such as t-expansion
  - truncation effects and goodness of the flow can be measured by the force norm

- We showed that
  - With the SD method, we can have a better control of the effective action
  - We have positive effects in autocorrelation of long-ranged objects by adding rectangle and chair to the flow

Outlook

- Improve statistics for definite conclusions & confirm the scaling
- Develop more efficient strategies

  E.g., - Choose different trajectory of β_t' (exponential decrease, decrease c_1, …)

  - Be specific to particular slow modes (change instanton potential, …)

- Include fermion (Do we need more extended Wilson loops in the flow?)

cf. Grid: https://github.com/paboyle/Grid

GPT: https://github.com/lehner/gpt

Thank you.

Integrated autocorrelation time [DBW2 action] (1/1)

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