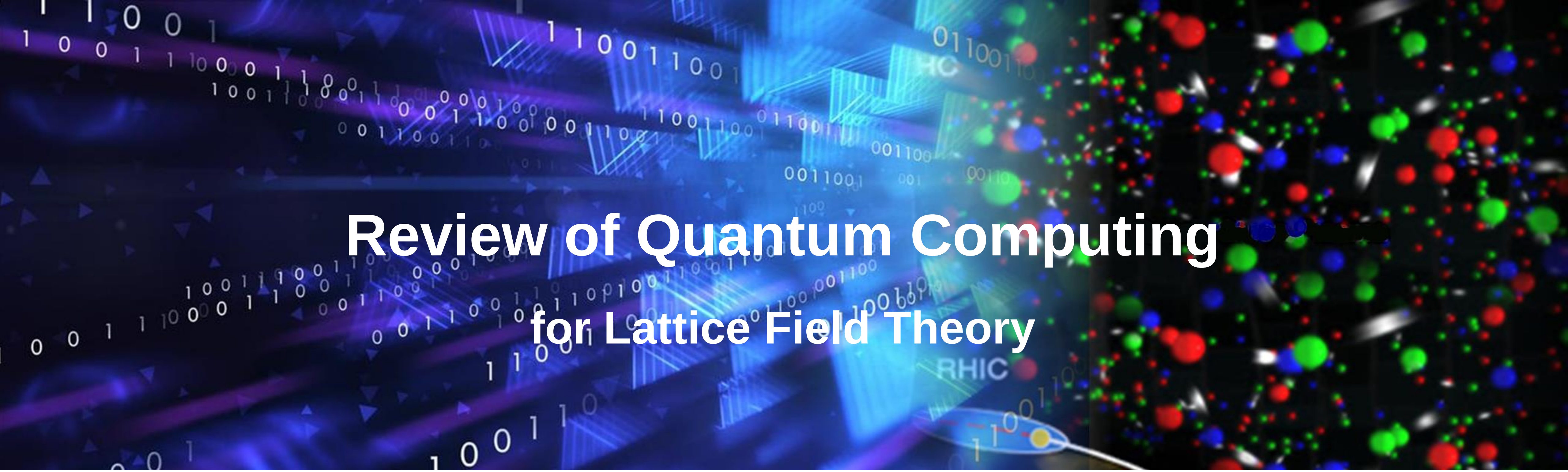

Review of Quantum Computing

for Lattice Field Theory

Lena Funcke

with Karl Jansen, Stefan Kühn, Tobias Hartung, et al.

Lattice 2022 Conference, Bonn, 8 August 2022

Why quantum computing?

Computational costs of lattice field theory	Computational challenges of lattice field theory

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Computational costs of lattice field theory | **Computational challenges of lattice field theory**

Supercomputer usage for different fields (INCITE 2019)

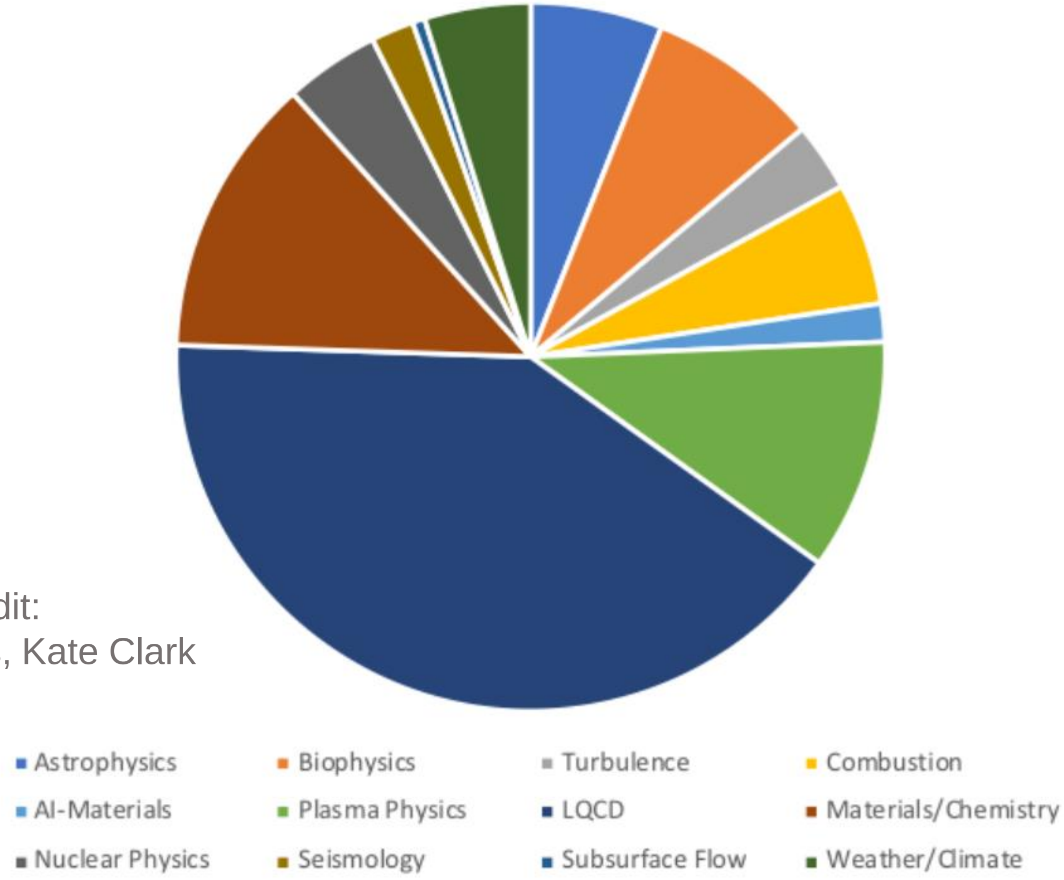

Figure credit:
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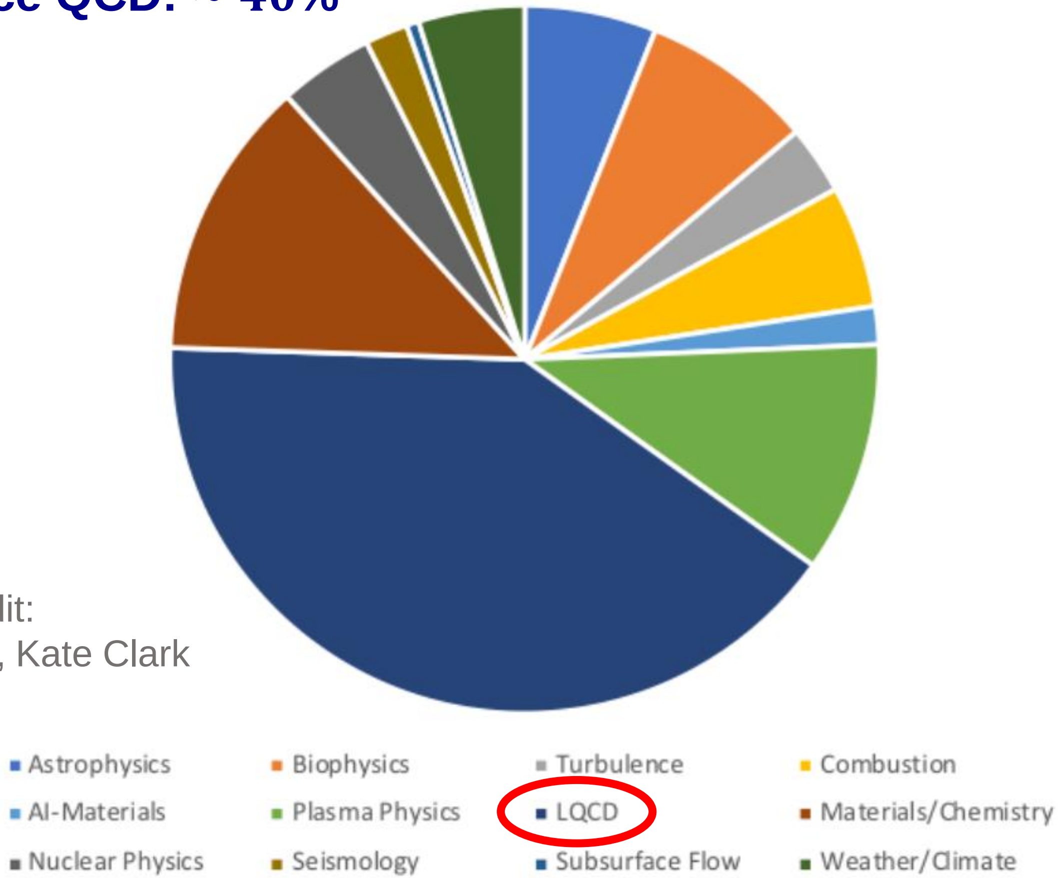
→ **Lattice QCD: $\sim$ 40%**



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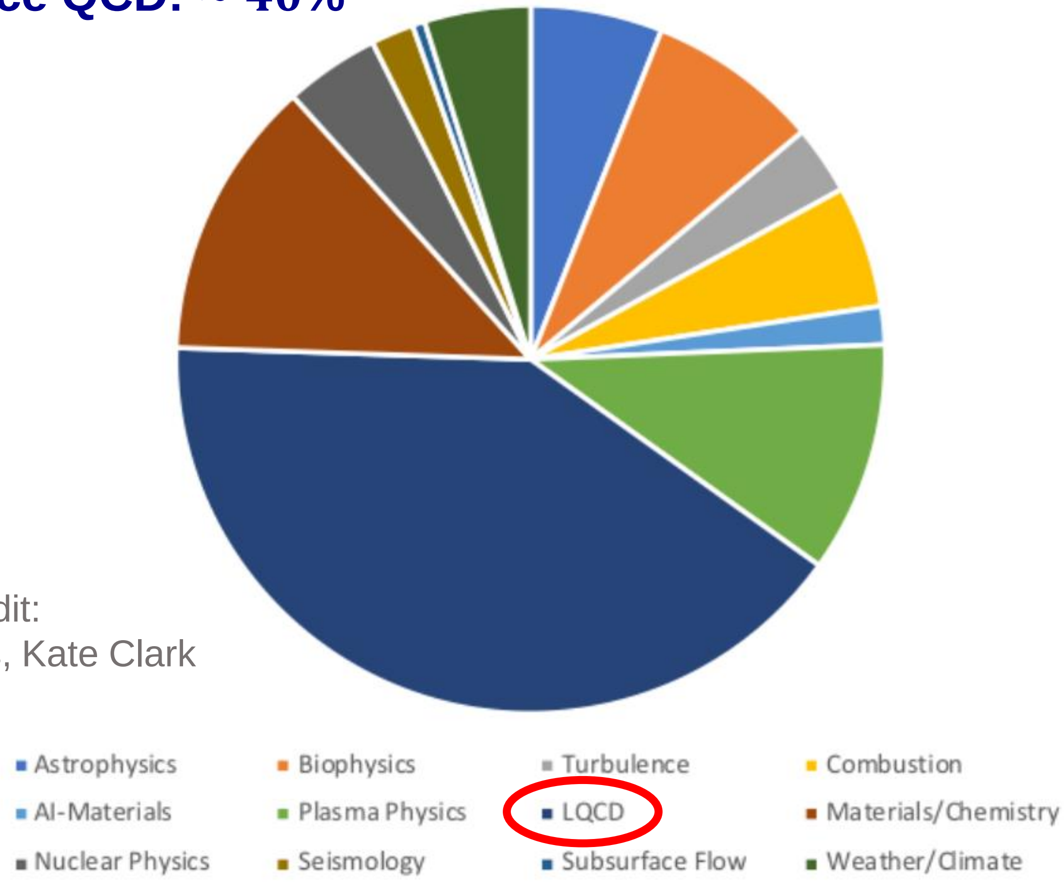

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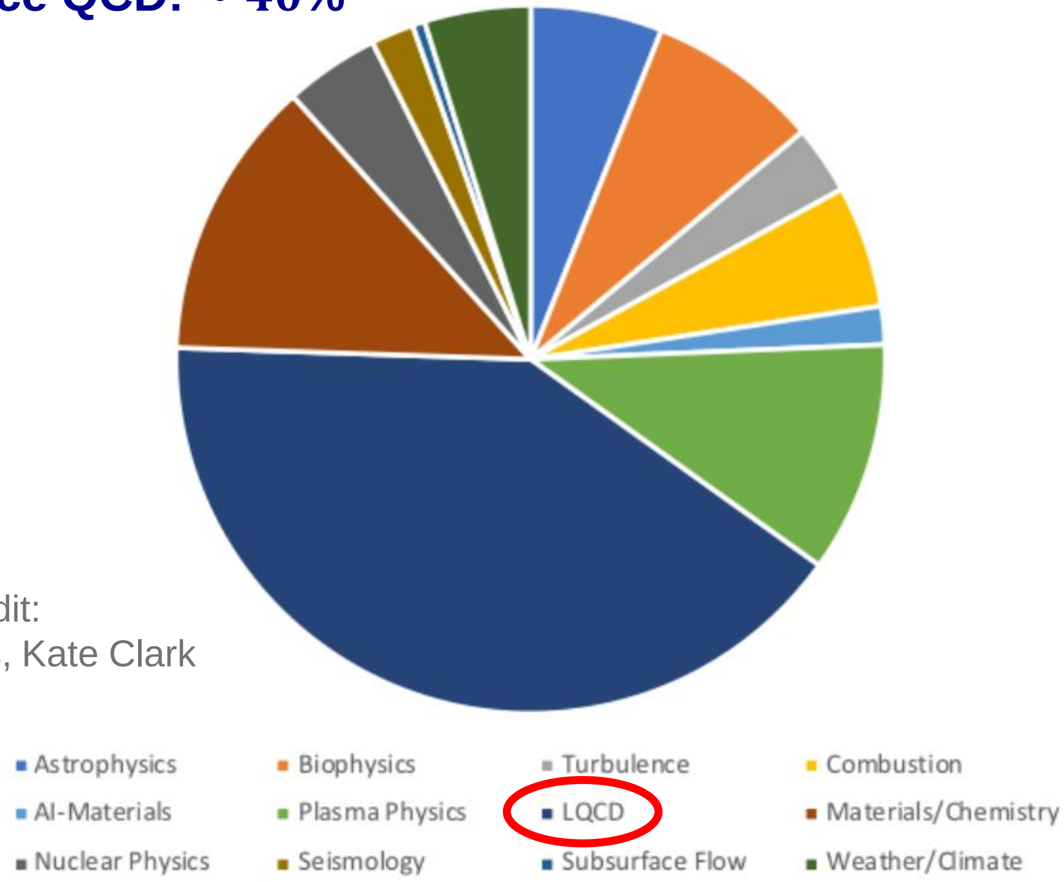

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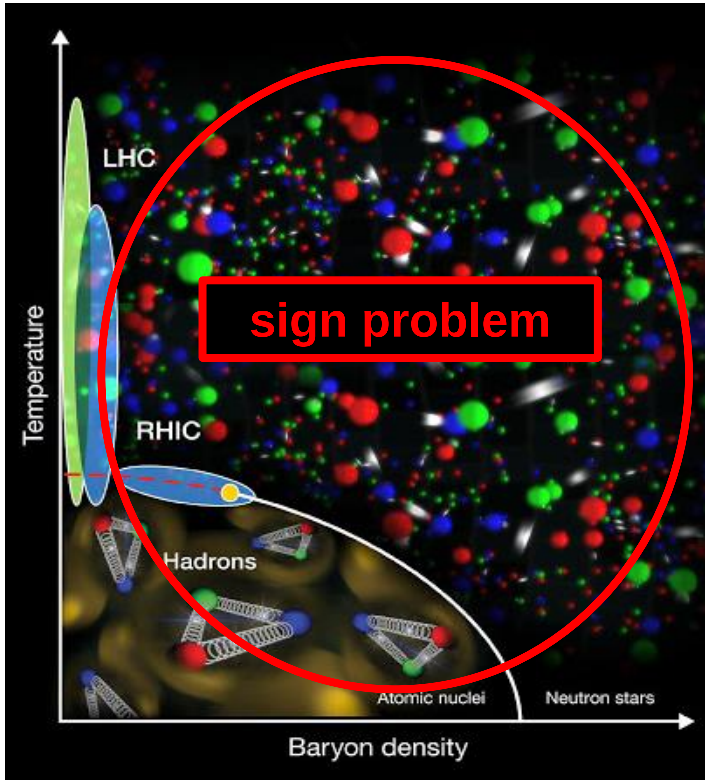

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BNL/RHIC

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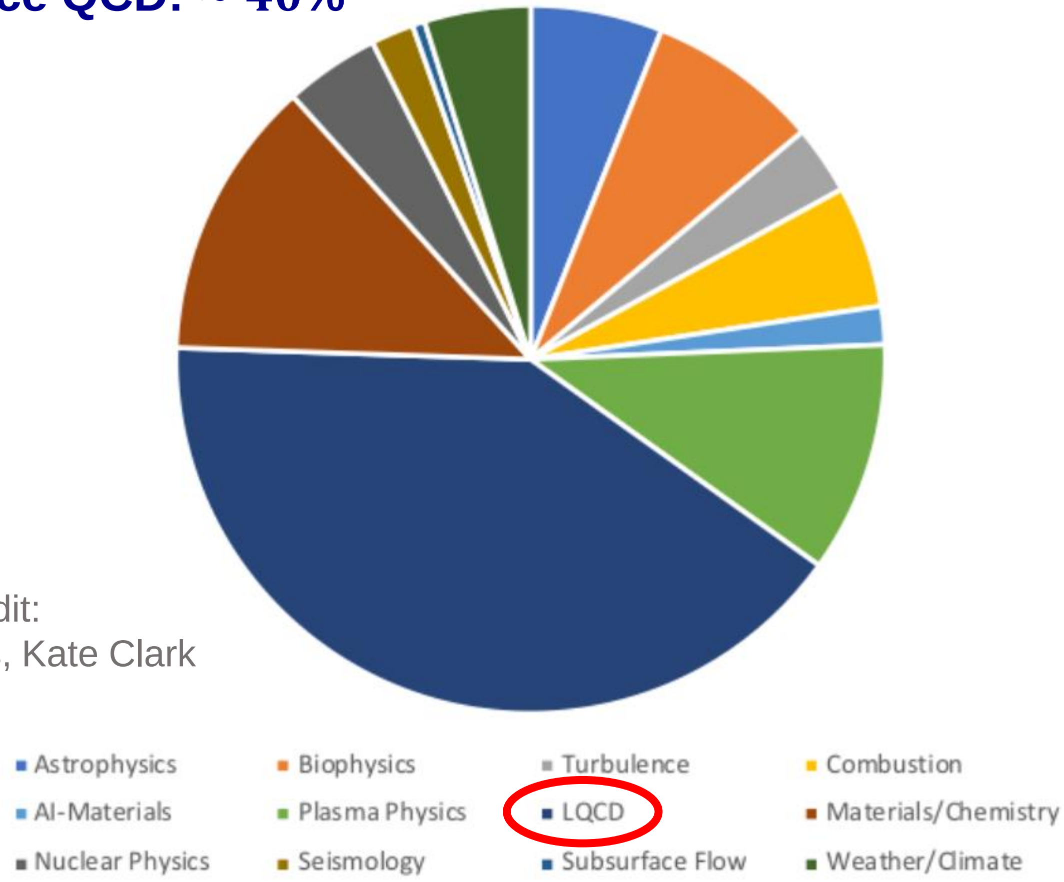

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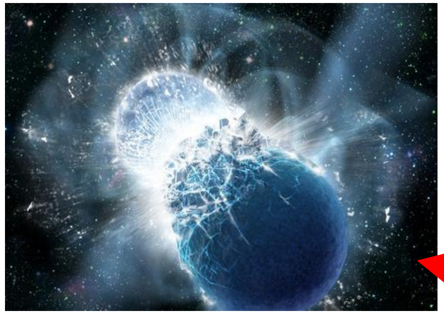

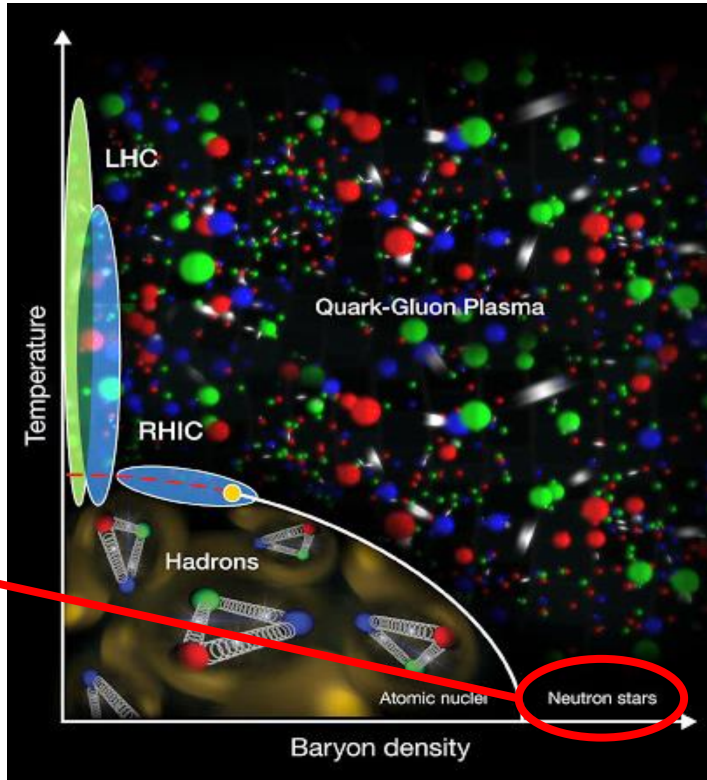

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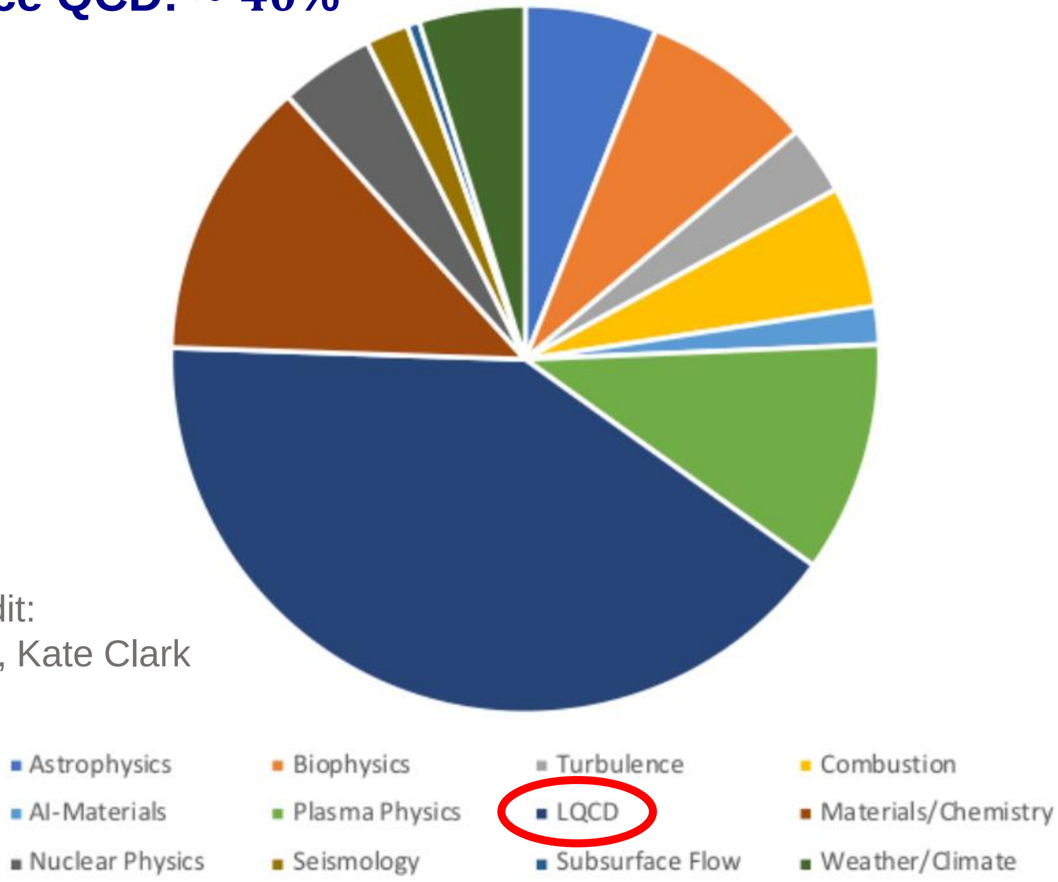

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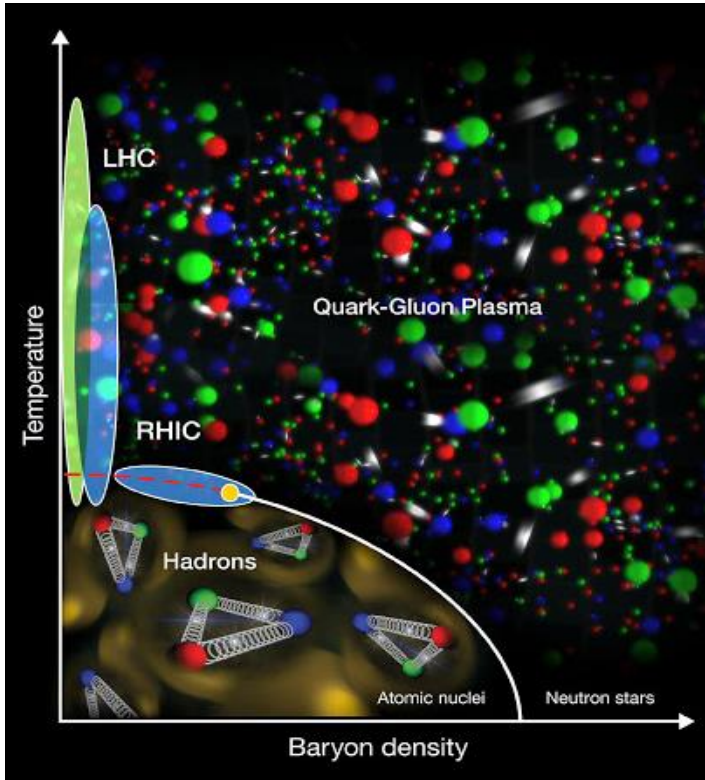

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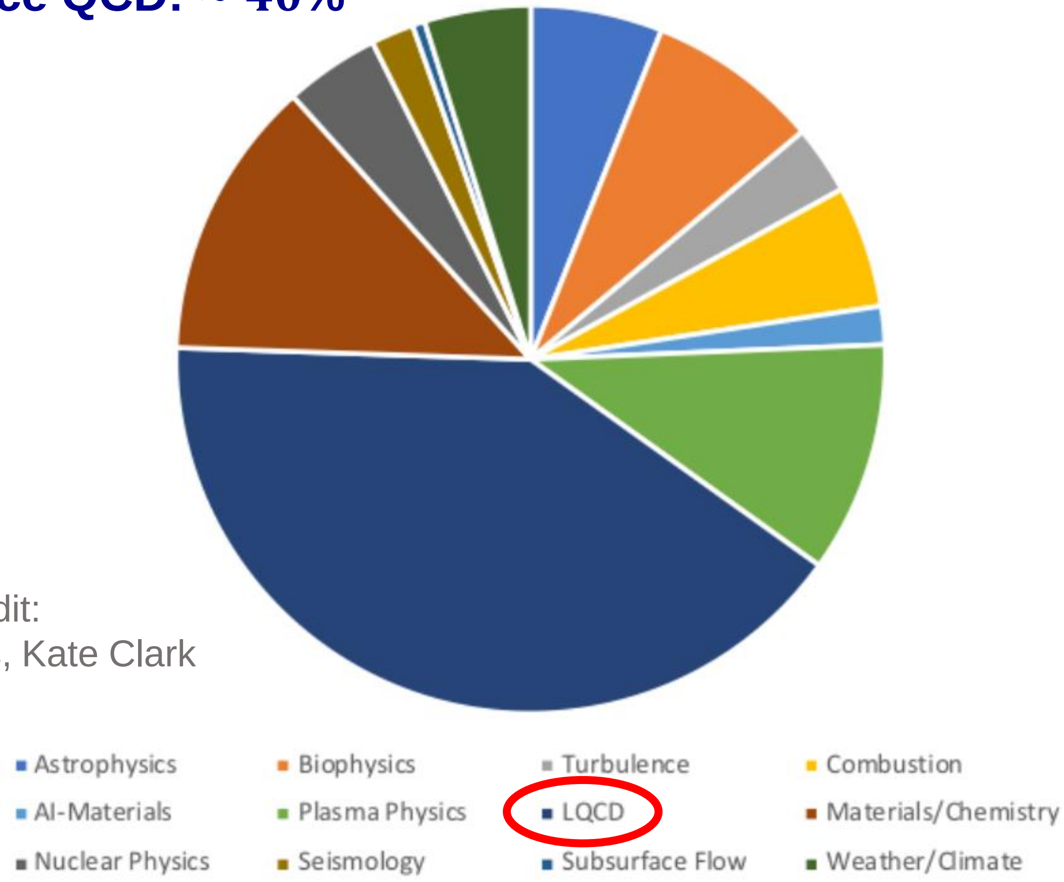

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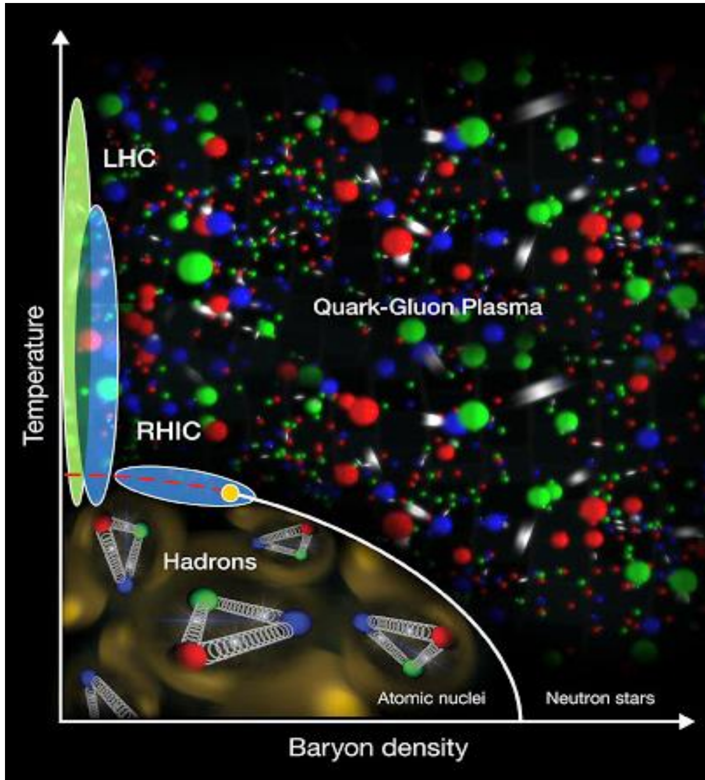

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Why is the sign problem exponentially hard?

Example: finite baryon chemical potential	Reweighting procedure

Why is the sign problem exponentially hard?

Example: finite baryon chemical potential | **Reweighting procedure**

Partition function

$Z = \int DUD\bar{\psi}\, D\psi e^{-S} = \int DUe^{-S_g} \det M$

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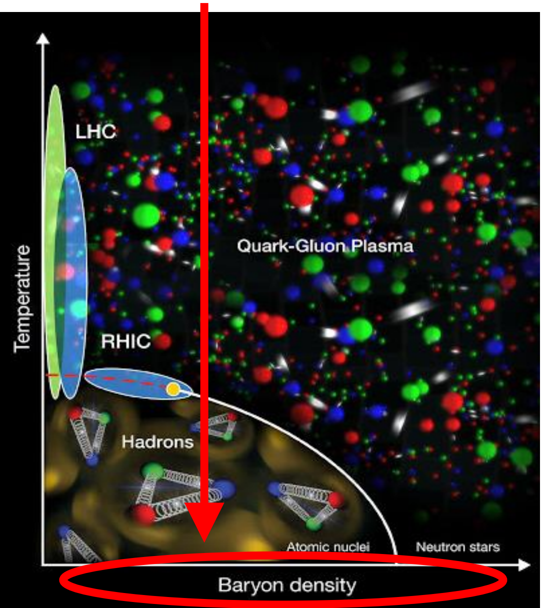

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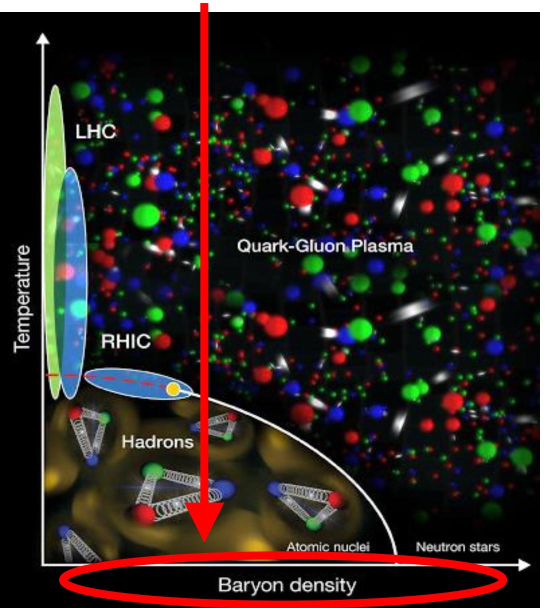

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Reweighting procedure

Example: phase quenched theory

$$\langle O \rangle = \frac{\int DU e^{-S_g} |\det M| e^{i\phi} O}{\int DU e^{-S_g} |\det M| e^{i\phi}} = \frac{\langle e^{i\phi} O \rangle_{pq}}{\langle e^{i\phi} \rangle_{pq}}$$

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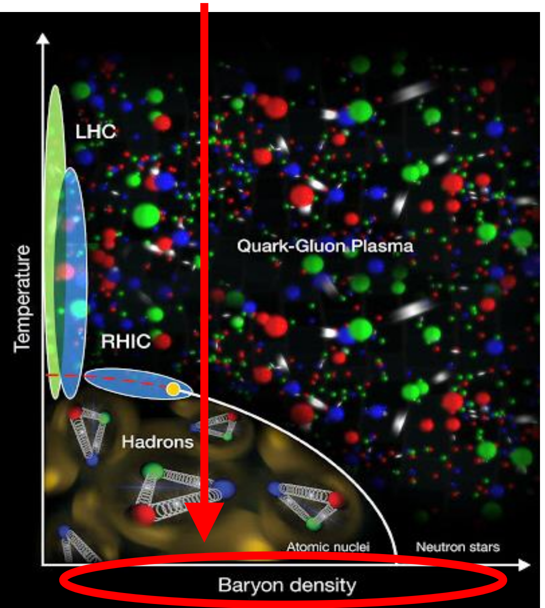

Figure credit: BNL/RHIC, de Forcrand

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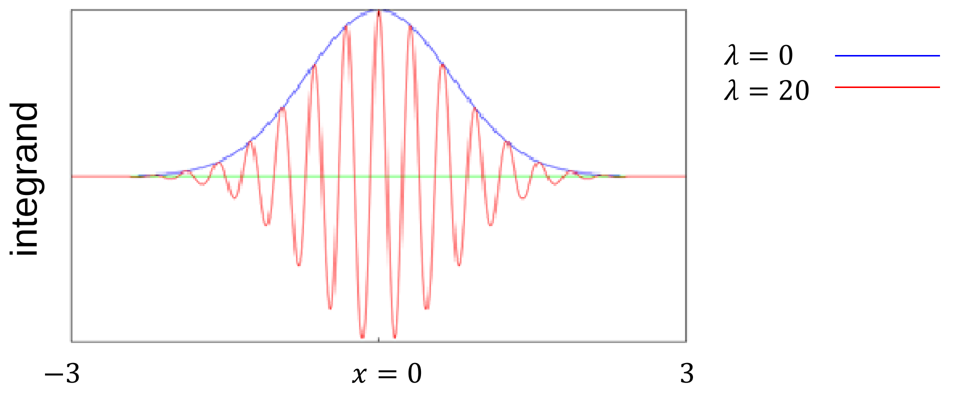
Highly oscillatory integrands

Near-cancellation of positive & negative contributions



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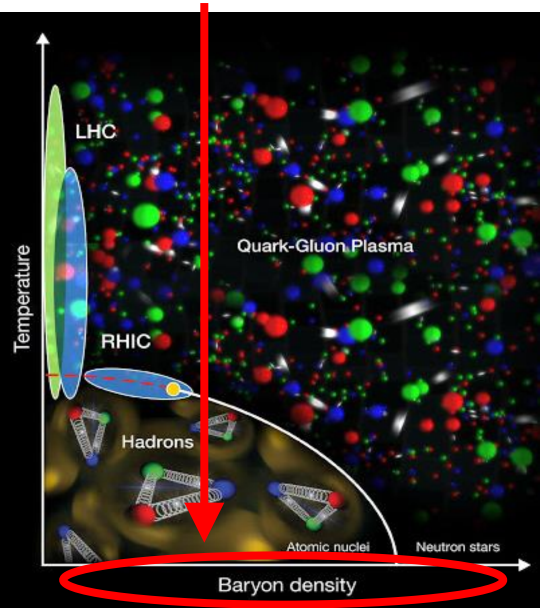

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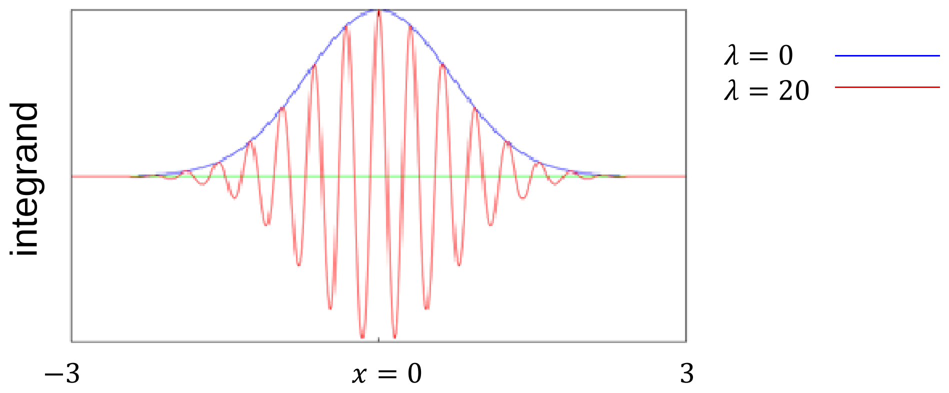
Near-cancellation of positive & negative contributions

Sample number grows *exponentially* with volume V



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Do we really need *quantum* computing?

Classical approaches to tackle the sign problem	Why is(n’t) classical computing enough?

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Classical approaches to tackle the sign problem	**Why is(n't) classical computing enough?**

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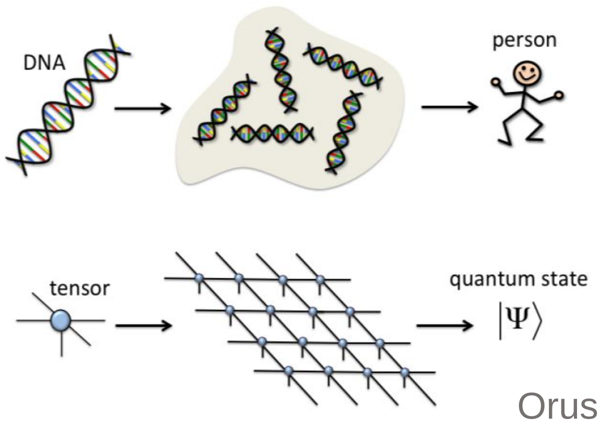

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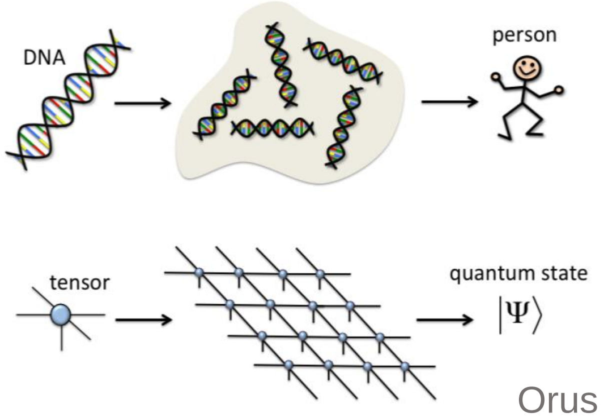

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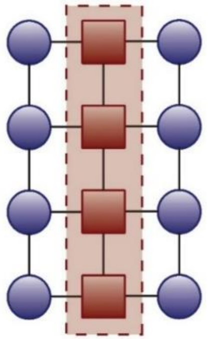

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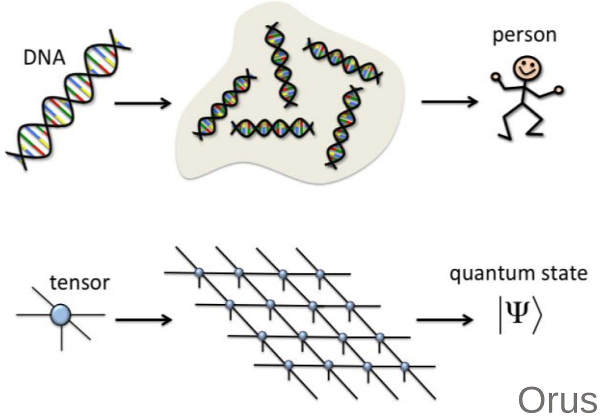

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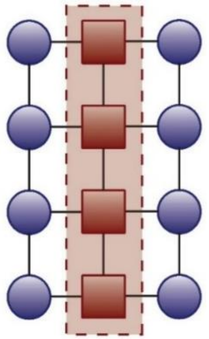

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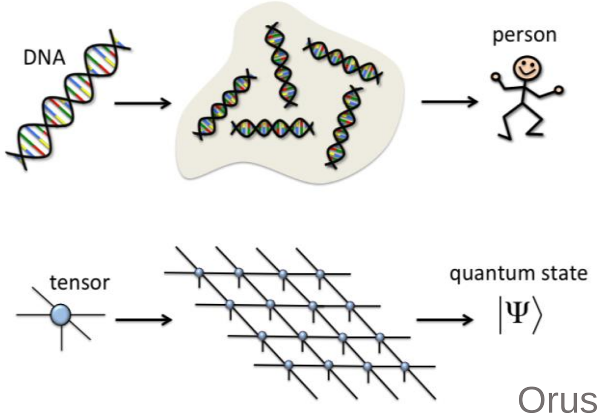

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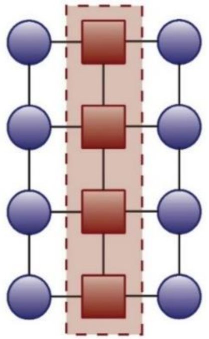
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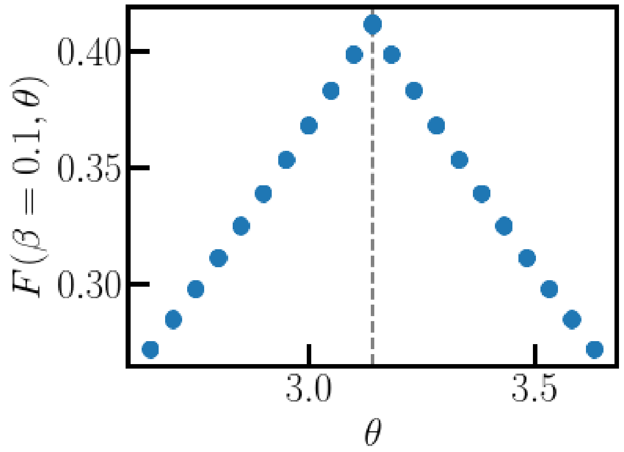

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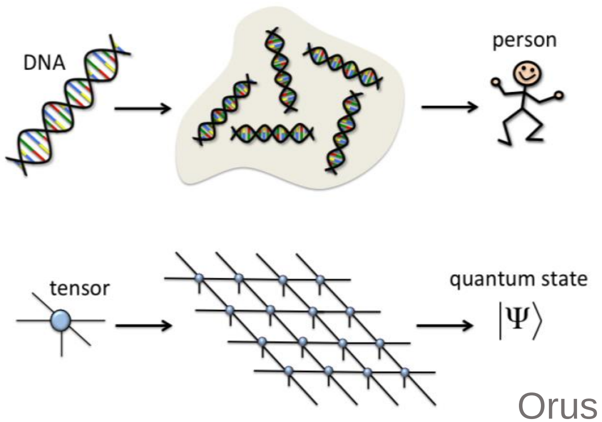

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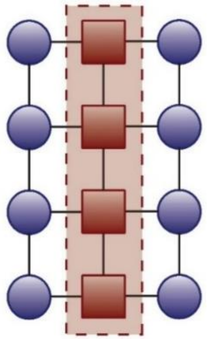

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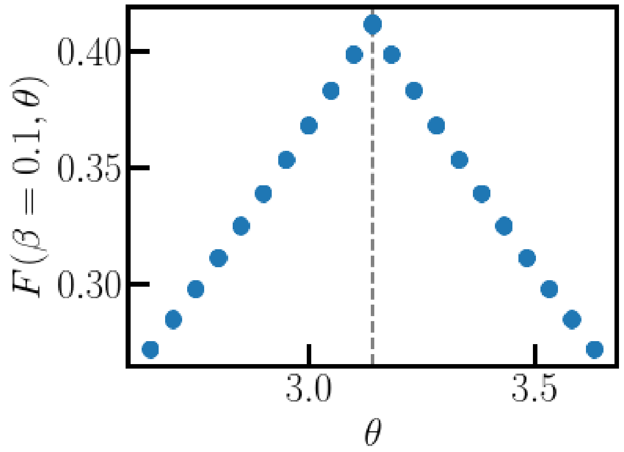

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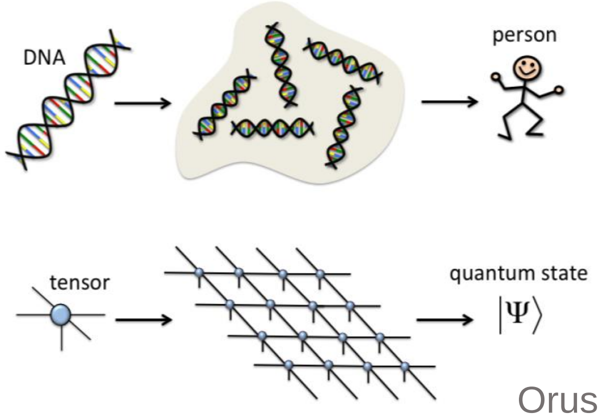

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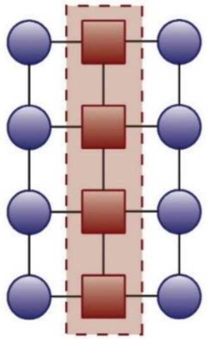

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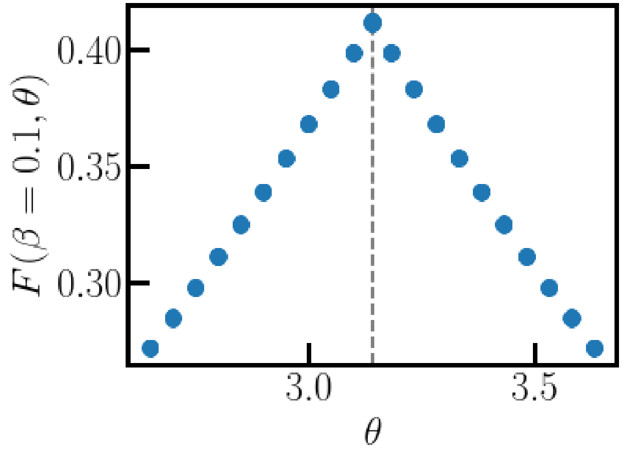

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No efficient parametrization of highly entangled states

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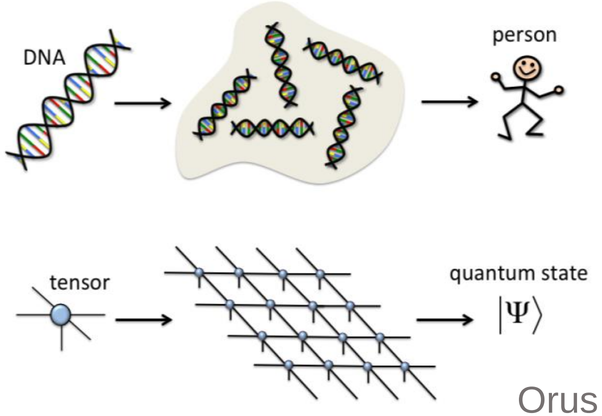

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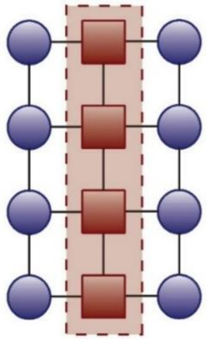

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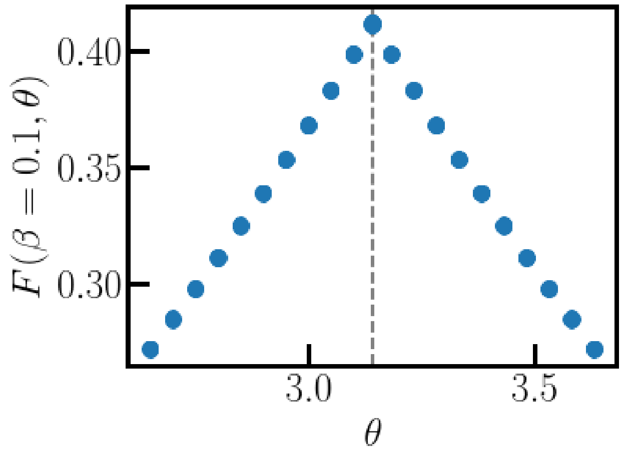

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In real-time evolution, tensor size can grow *exponentially*

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Example: 1+1D Bose-Hubbard model	**Why is(n't) classical computing enough?**

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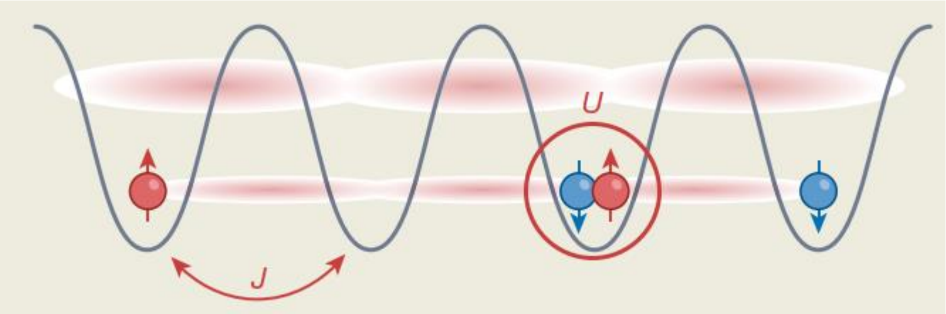

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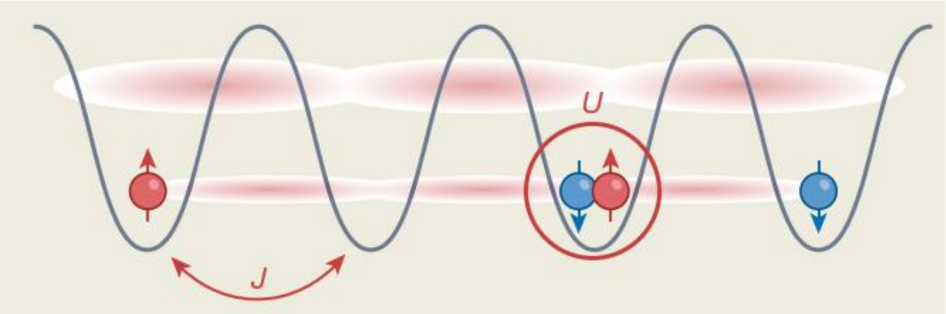

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Do we really need *quantum* computing?

Example: 1+1D Bose-Hubbard model | **Why is(n't) classical computing enough?**

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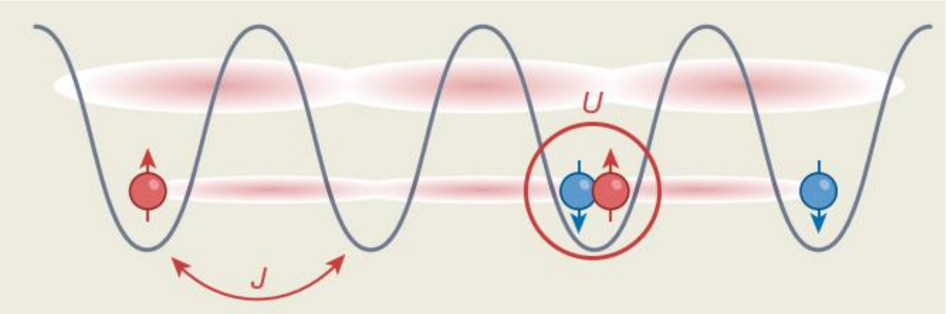

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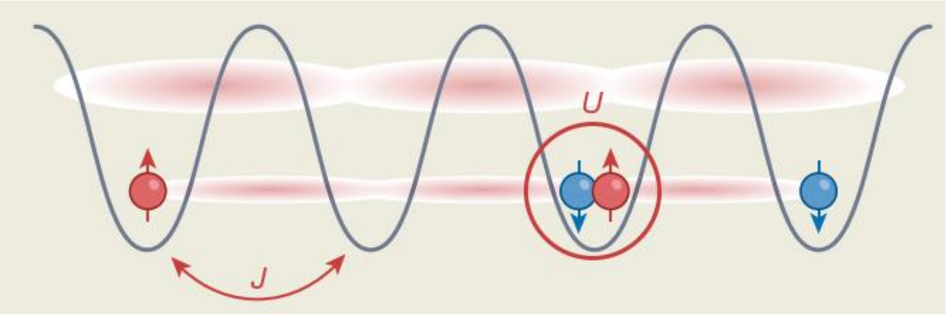

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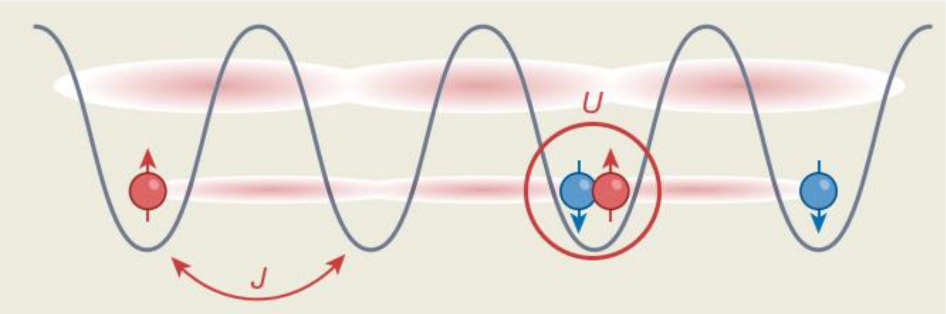

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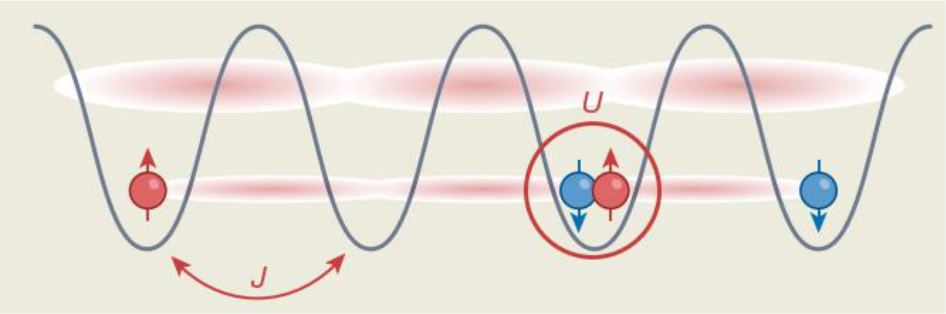

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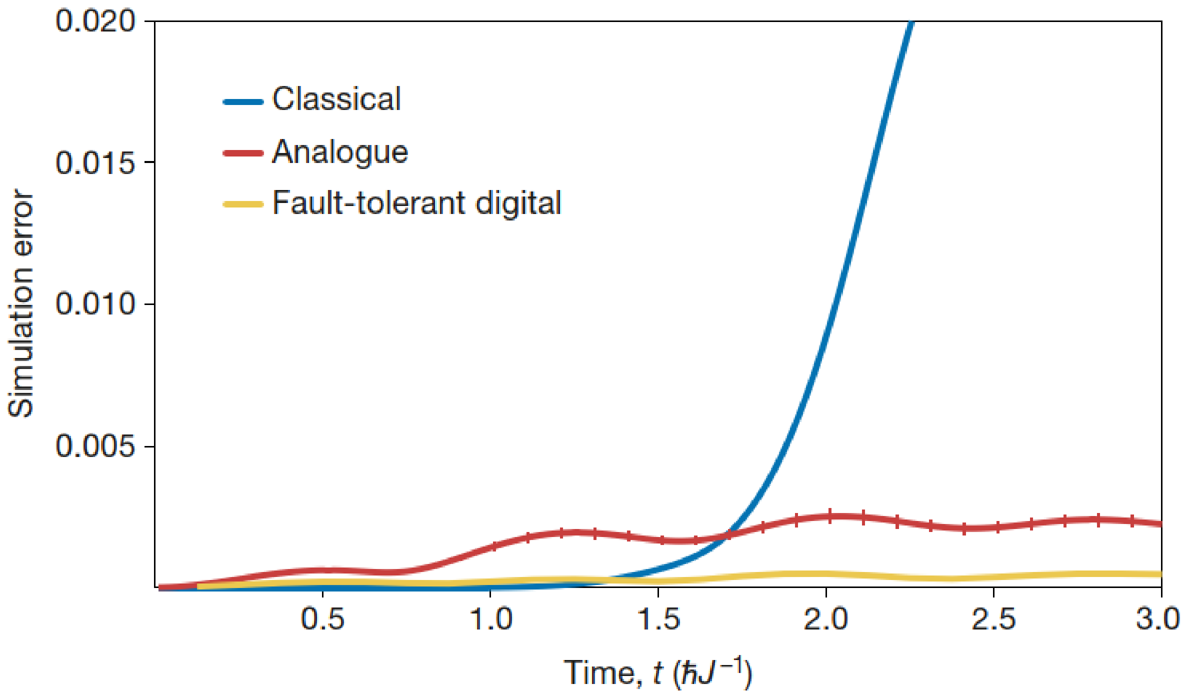

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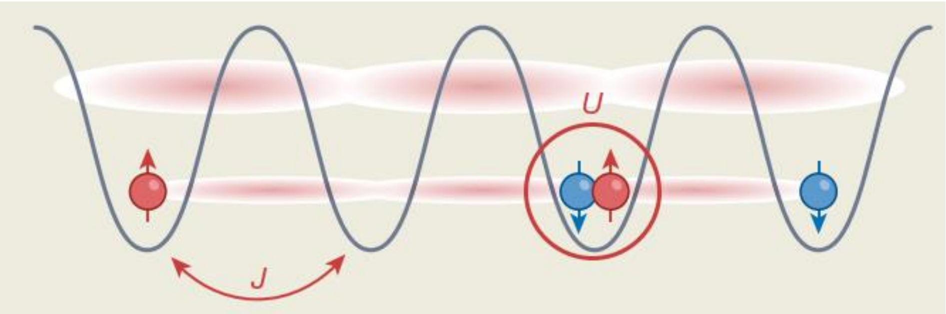

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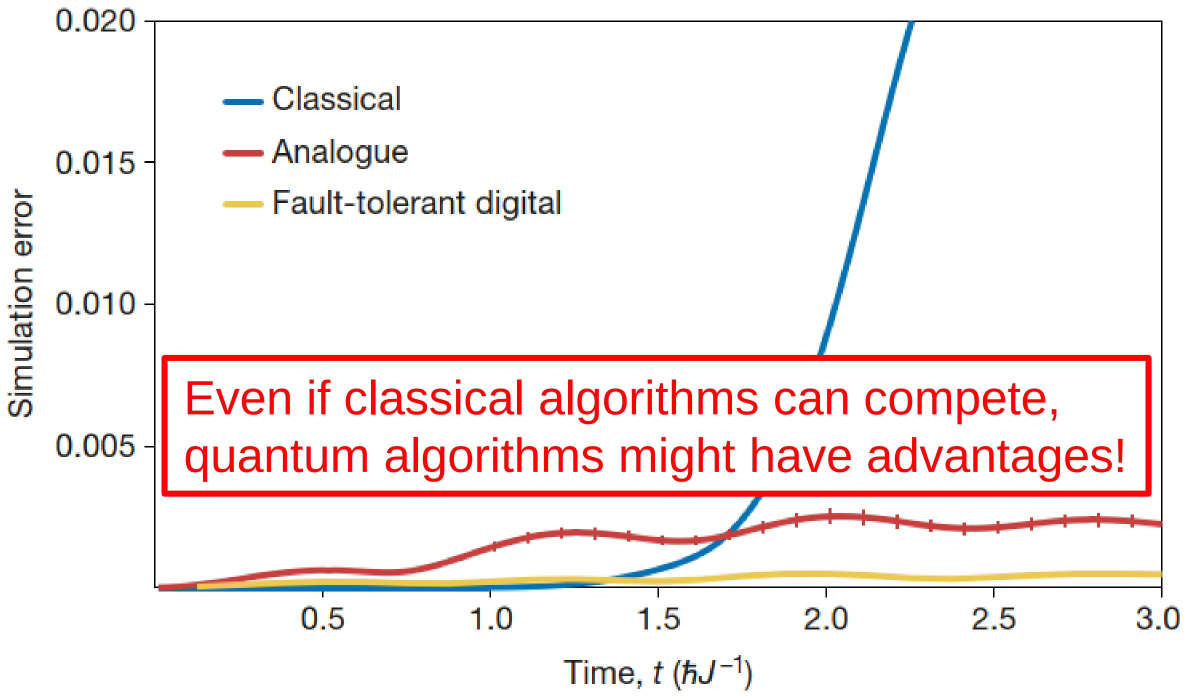

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Quantum hardware	Quantum algorithms

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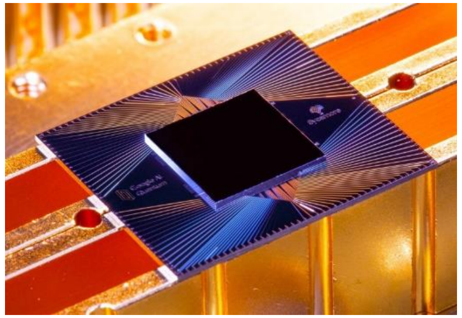

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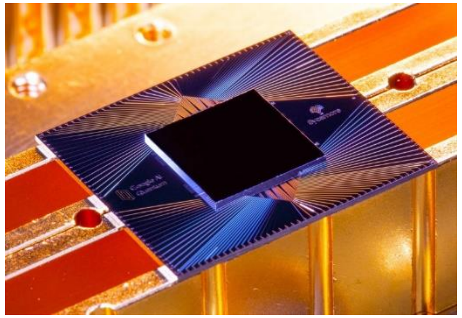

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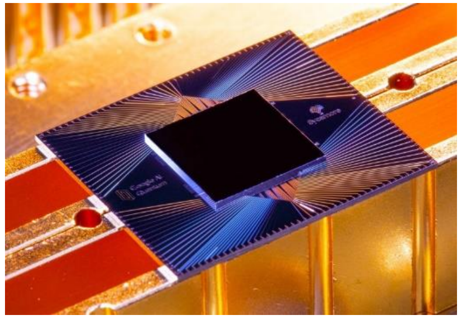

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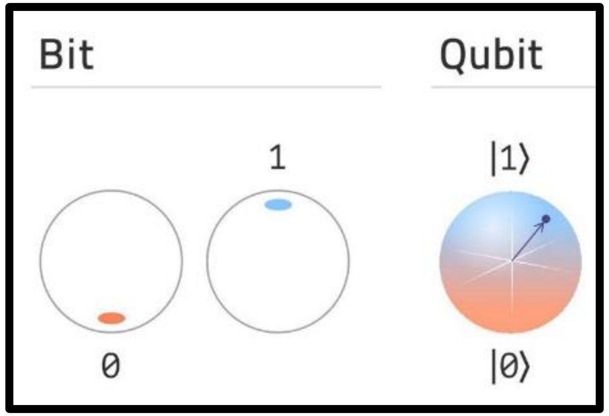

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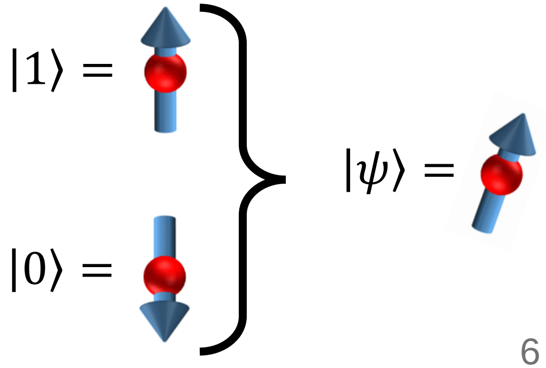

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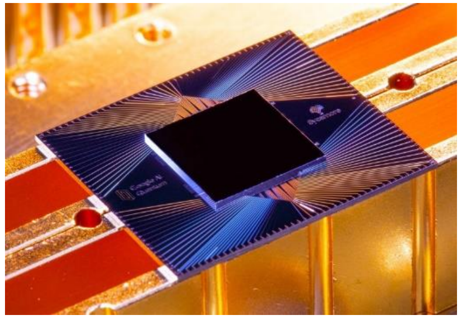

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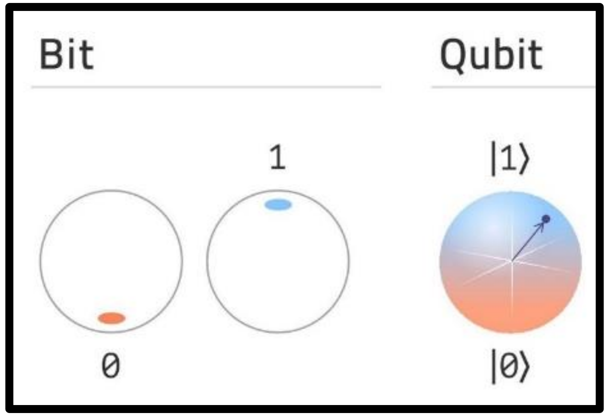

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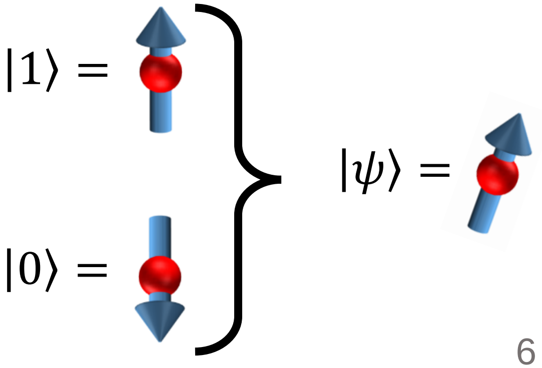

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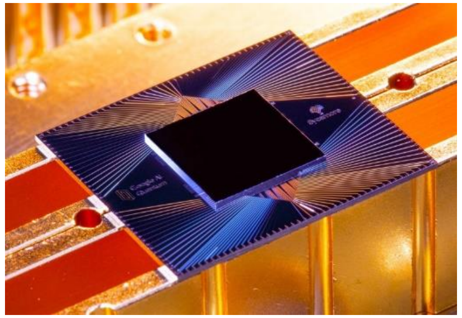

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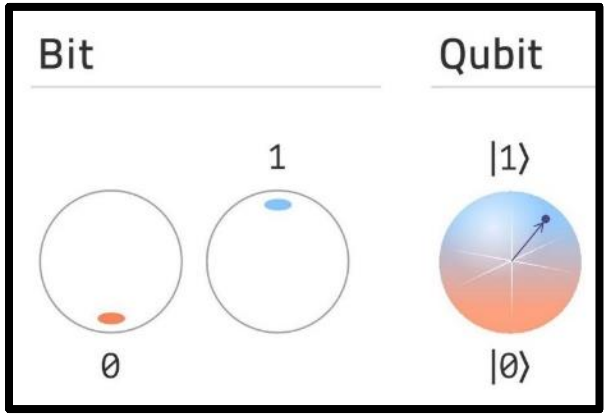

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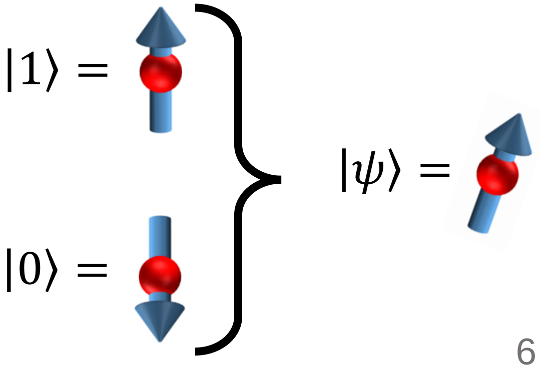

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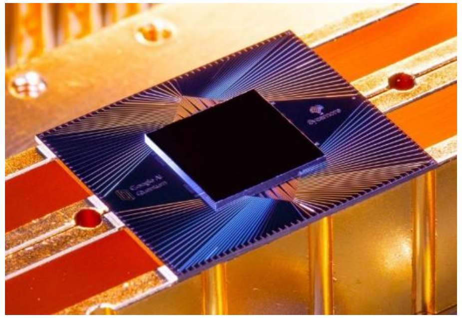

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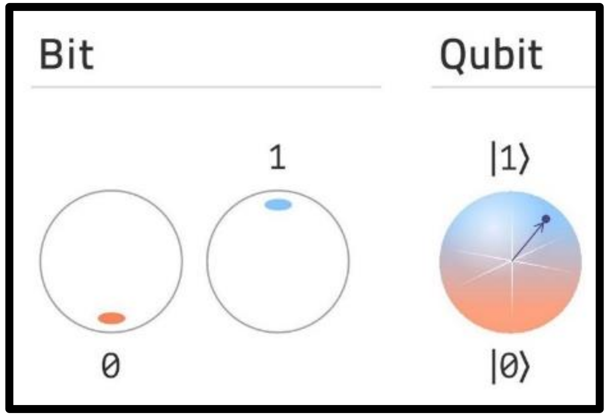
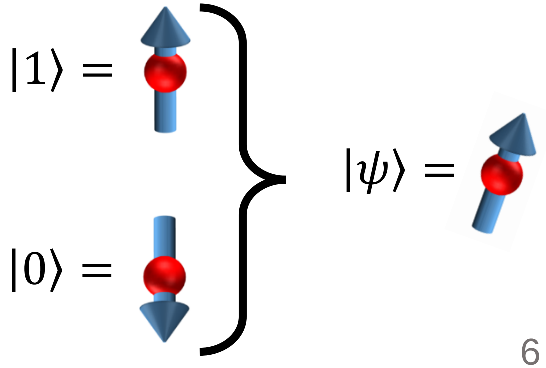
cryptography, optimization problems, …





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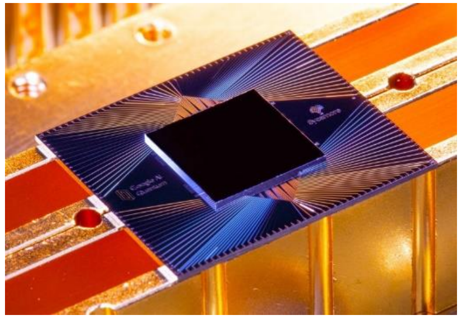

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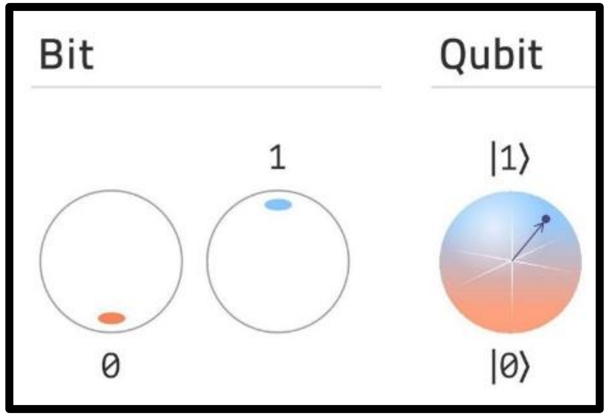

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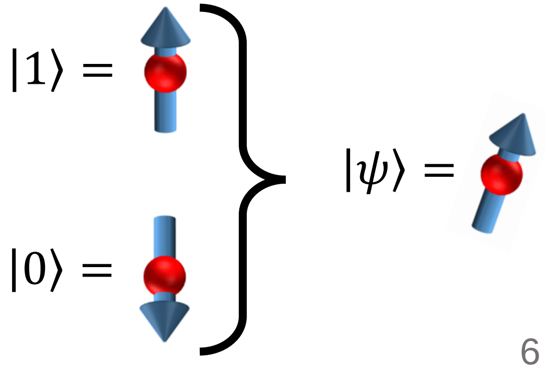

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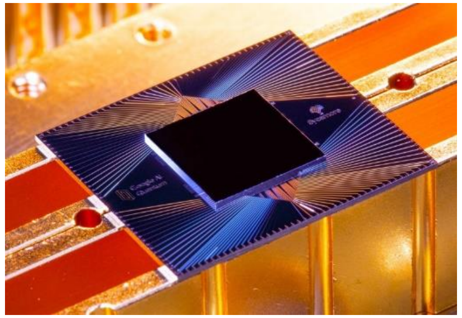

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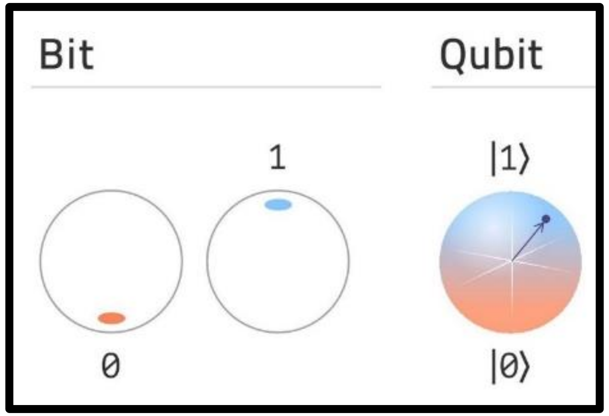

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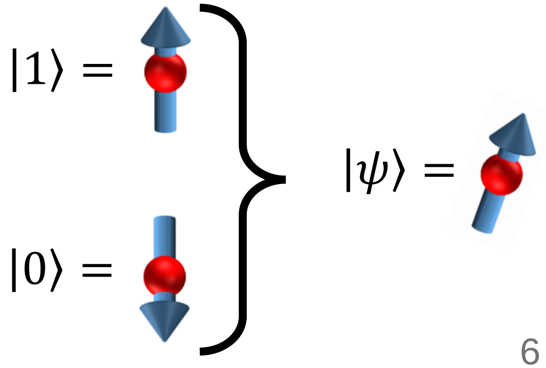

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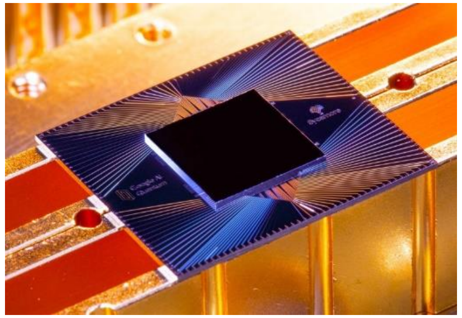

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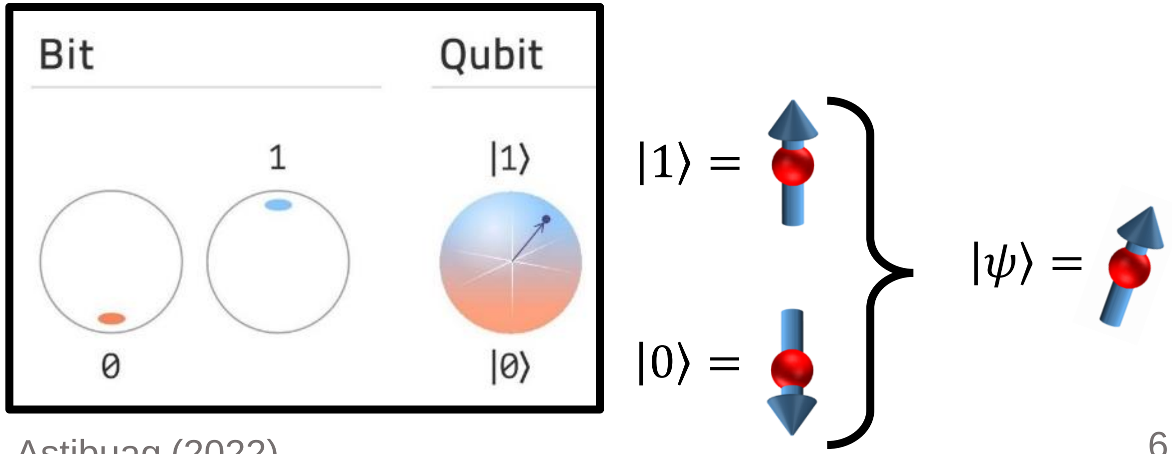

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Example: “quantum advantage” (2019)	Example: “classical advantage” (2021)

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We develop a high-performance tensor-based simulator for random quantum circuits(RQCs) on the new Sunway supercomputer. Our major innovations include: (1) a near-optimal slicing scheme, and a path-optimization strategy that considers both complexity and compute density; (2) a three-level parallelization scheme that scales to about 42 million cores; (3) a fused permutation and multiplication design that improves the compute efficiency for a wide range of tensor contraction scenarios; and (4) a mixed-precision scheme to further improve the performance. Our simulator effectively expands the scope of simulatable RQCs to include the 10×10(qubits)$\times(1+40+1)$(depth) circuit, with a sustained performance of 1.2 Eflops (single-precision), or 4.4 Eflops (mixed-precision)as a new milestone for classical simulation of quantum circuits; and reduces the simulation sampling time of Google Sycamore to 304 seconds, from the previously claimed 10,000 years.

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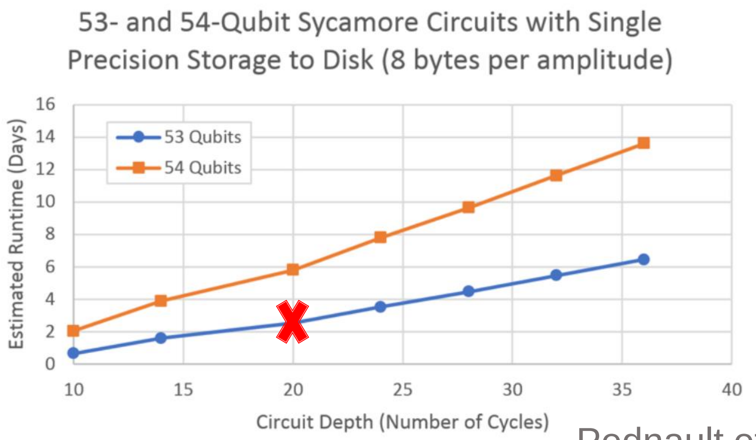

Note: classical runtime improved from 2.5 days (2019) to 304 seconds (2021)

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→ **Quantum-classical race:**
algorithms and hardware quickly advance

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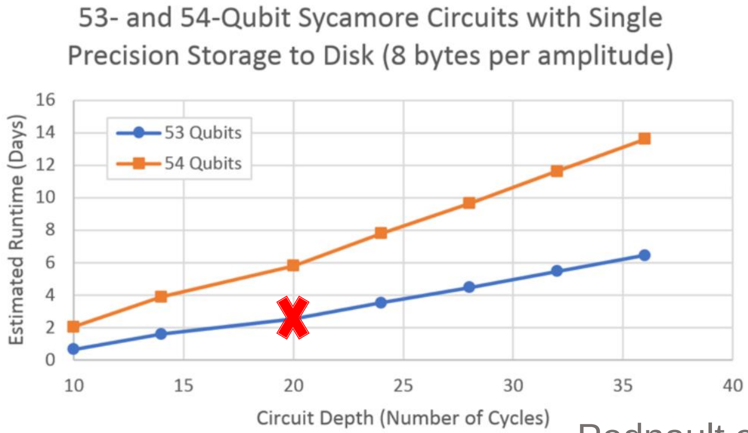

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→ **Quantum-classical race:**
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→ **For exponentially hard problems:**
small quantum step ↔ giant classical leap

Example: “classical advantage” (2021)

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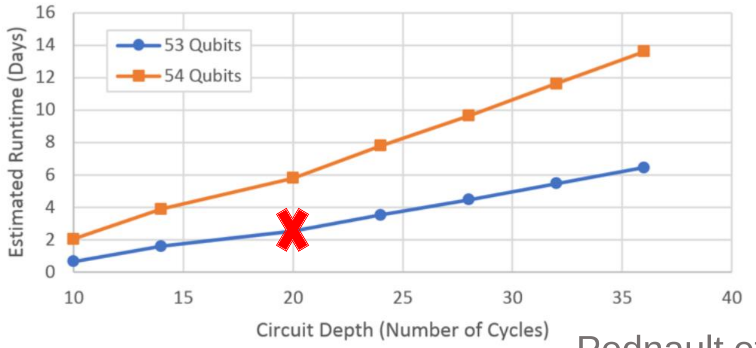
We develop a high-performance tensor-based simulator for random quantum circuits(RQCs) on the new Sunway supercomputer. Our major innovations include: (1) a near-optimal slicing scheme, and a path-optimization strategy that considers both complexity and compute density; (2) a three-level parallelization scheme that scales to about 42 million cores; (3) a fused permutation and multiplication design that improves the compute efficiency for a wide range of tensor contraction scenarios; and (4) a mixed-precision scheme to further improve the performance. Our simulator effectively expands the scope of simulatable RQCs to include the 10×10(qubits)×(1+40+1)(depth) circuit, with a sustained performance of 1.2 Eflops (single-precision), or 4.4 Eflops (mixed-precision)as a new milestone for classical simulation of quantum circuits; and reduces the simulation sampling time of Google Sycamore to 304 seconds, from the previously claimed 10,000 years.

53- and 54-Qubit Sycamore Circuits with Single Precision Storage to Disk (8 bytes per amplitude)



Note: classical runtime improved from 2.5 days (2019) to 304 seconds (2021)

Pednault et al. (2019)

Quantum computing: where will we go?

The Path to Go…	A Rough Sketch…

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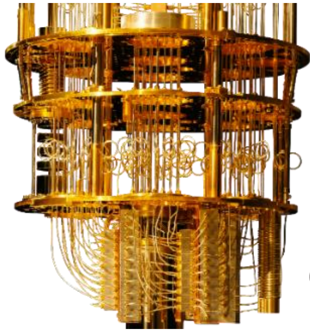

Carlow (2018)

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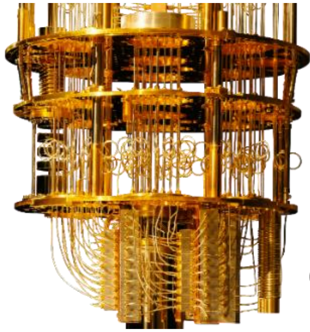

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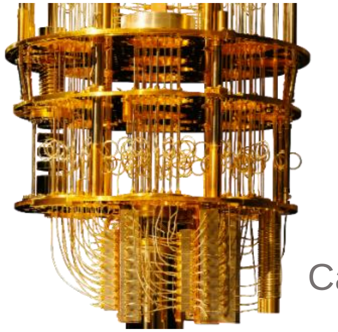

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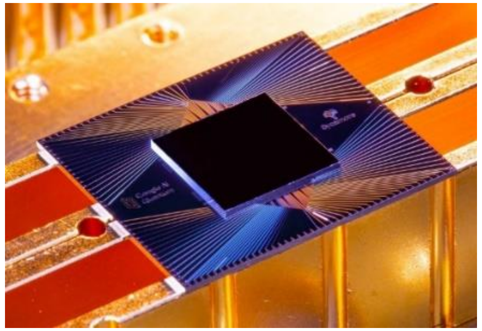

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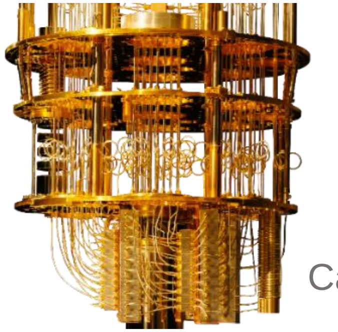

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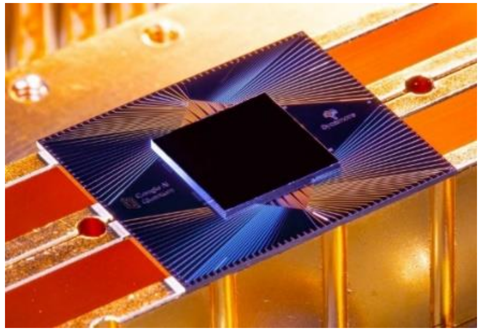

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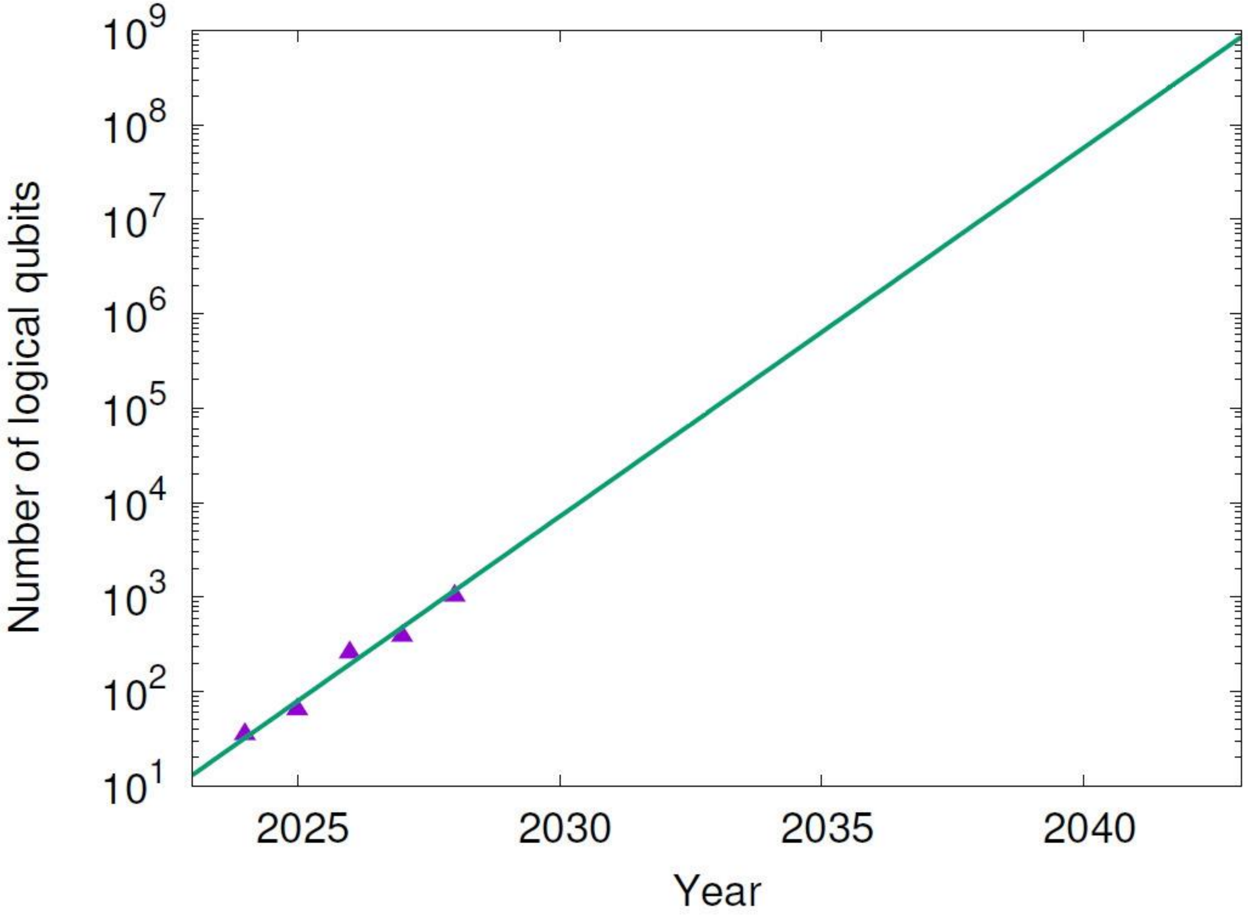

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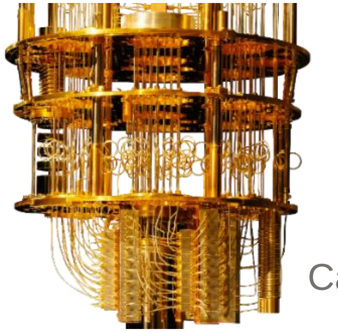

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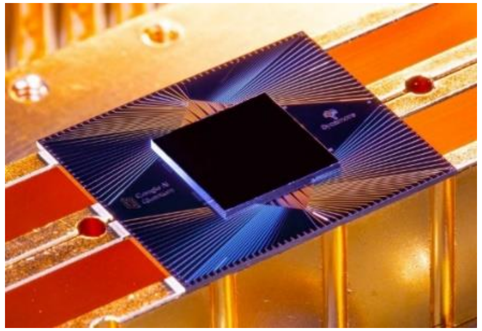

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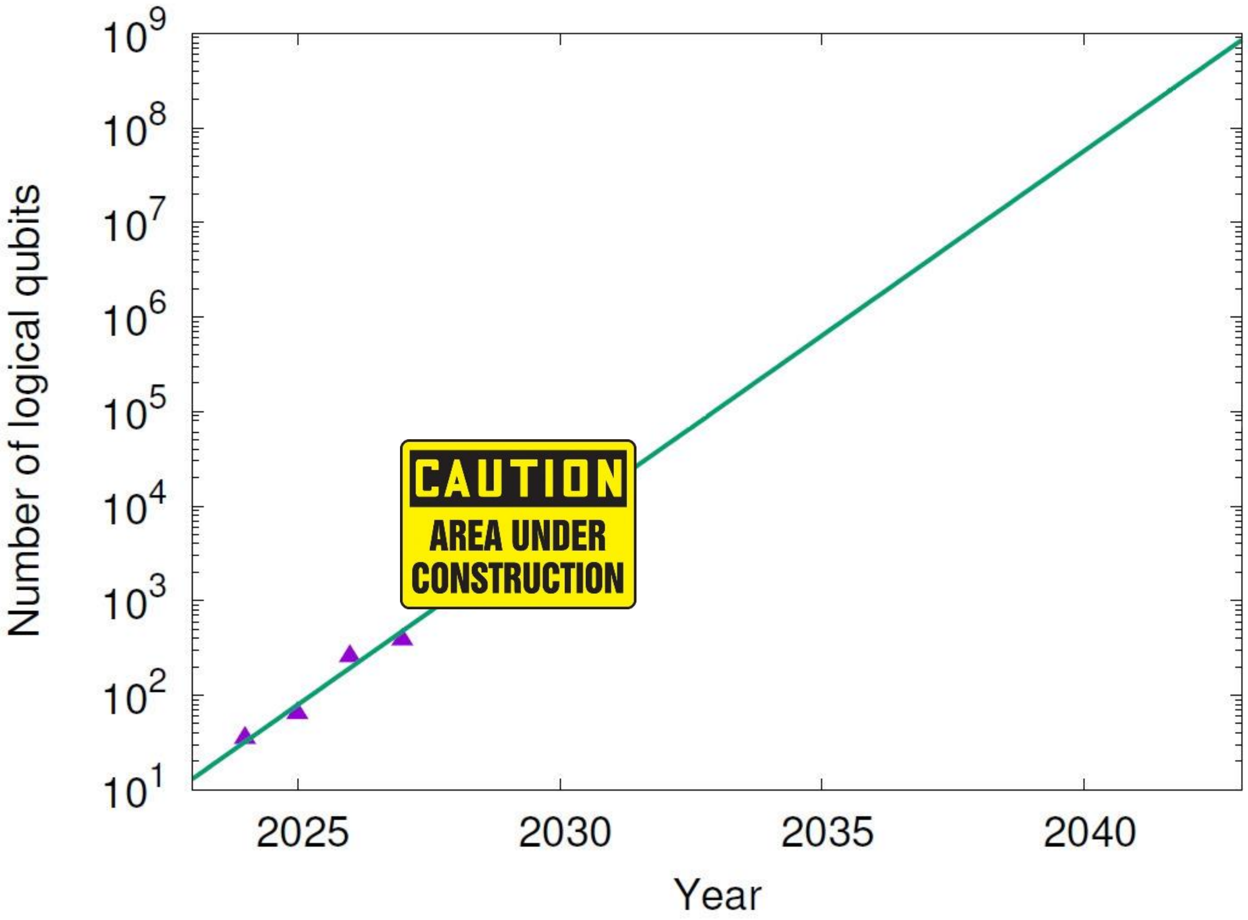

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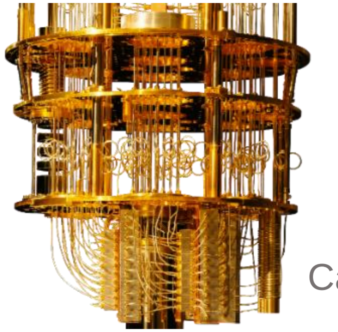

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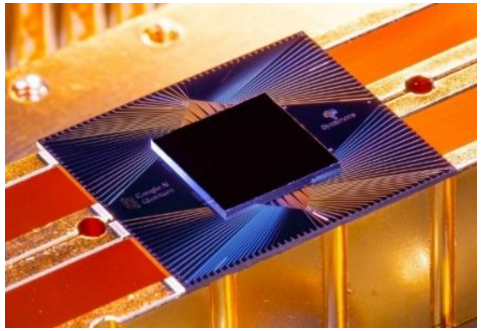

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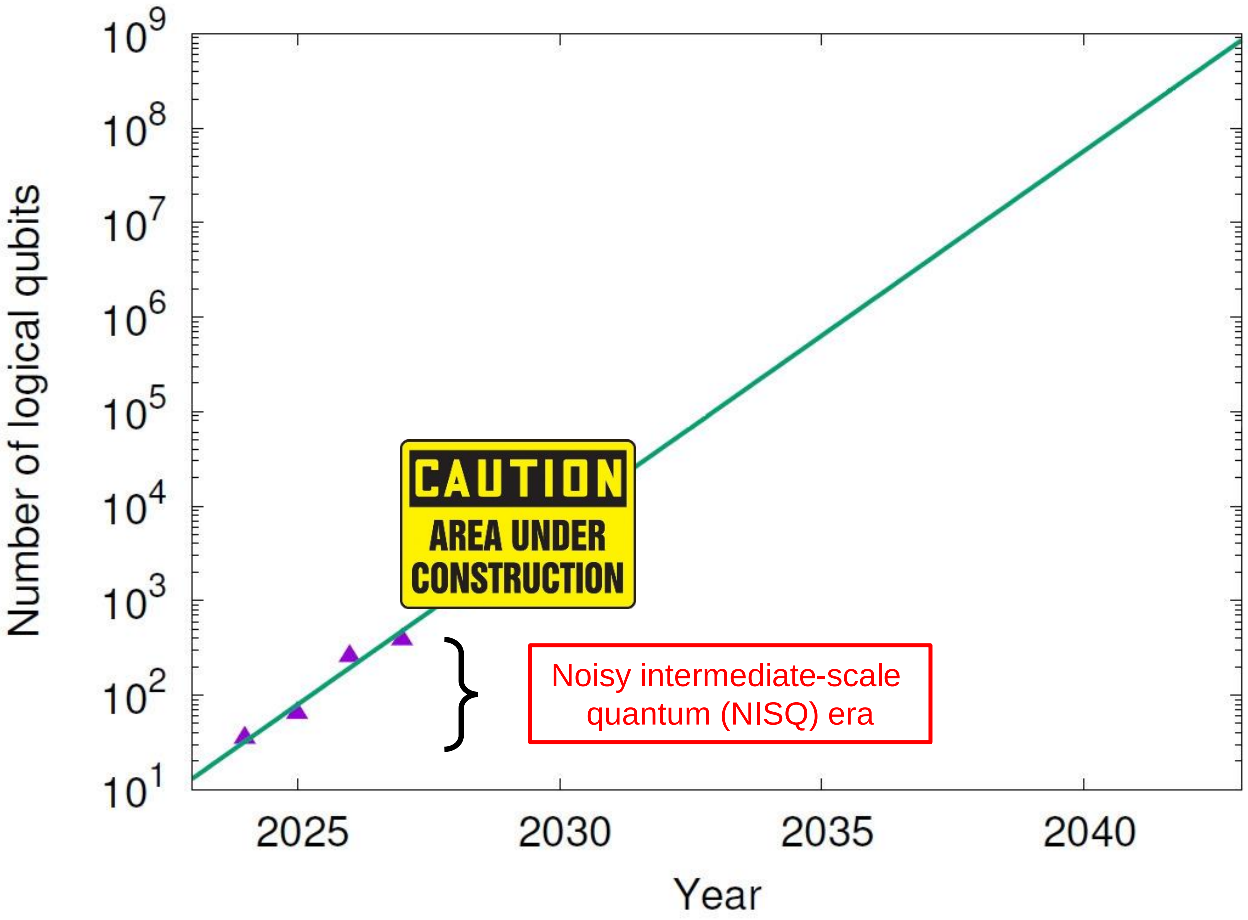

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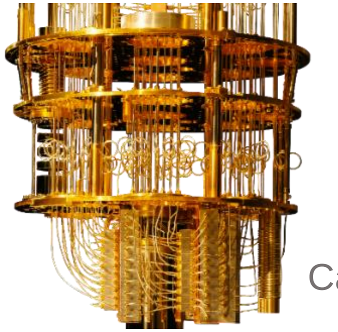

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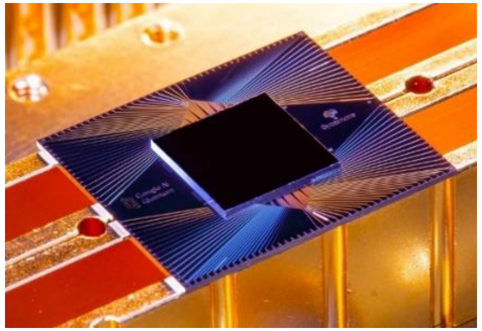

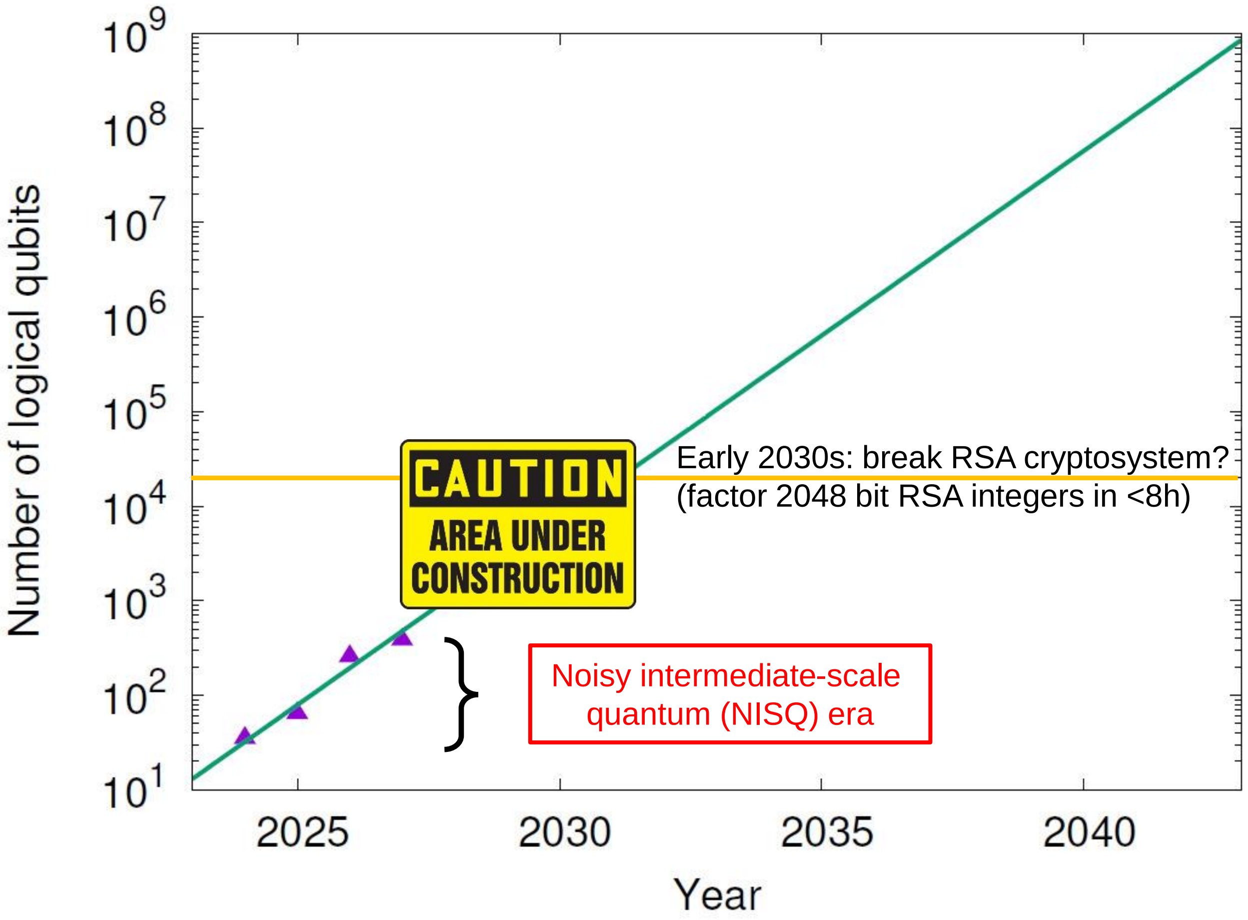

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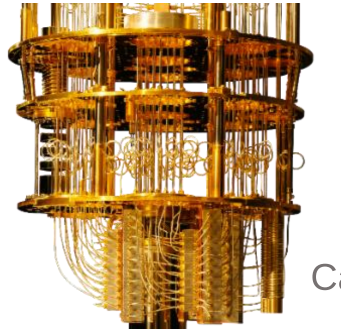

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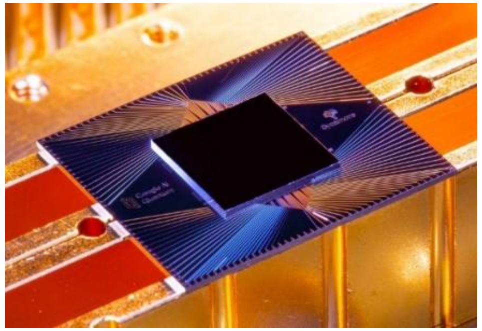
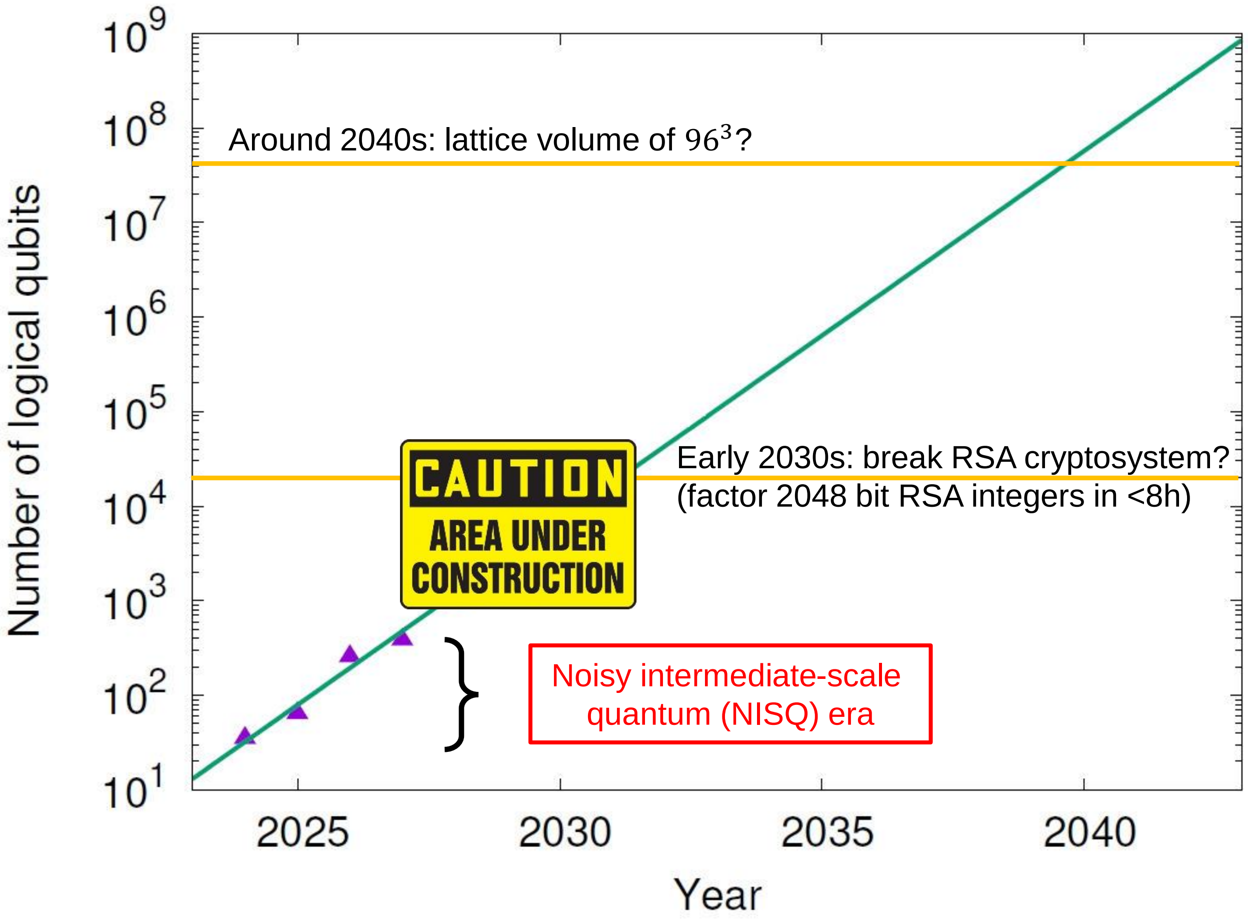


Far Future

Need $\mathcal{O}(10^7 - 10^8)$ logical qubits for lattice volume of 96^3



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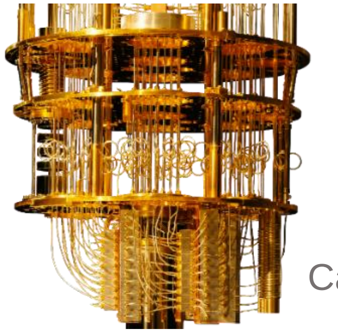

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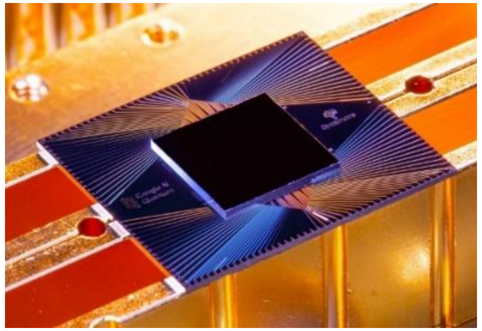

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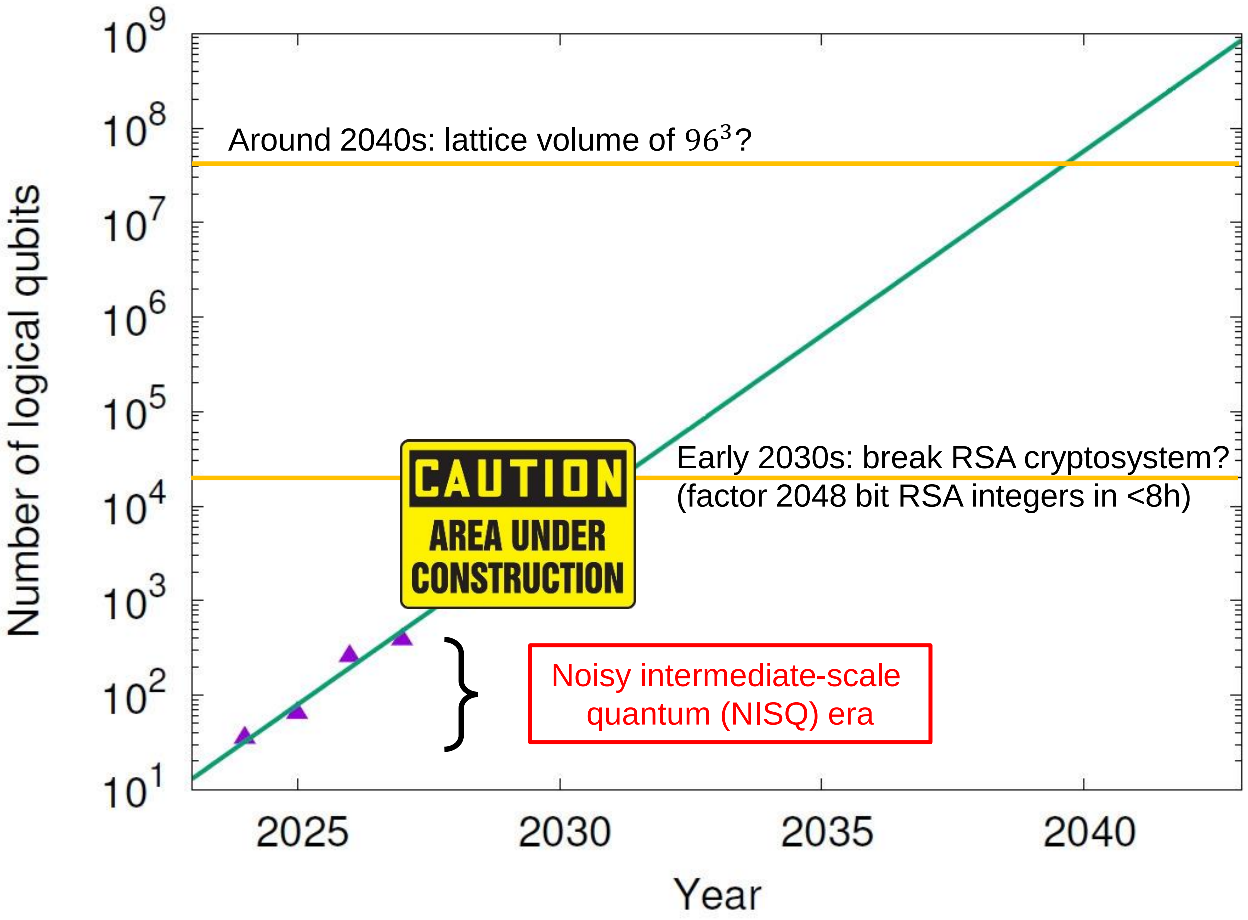
→ Analogy: lattice QCD from 1980s to 2020s?



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How can we reduce the noise?

Noisy quantum circuit	Error mitigation versus error correction

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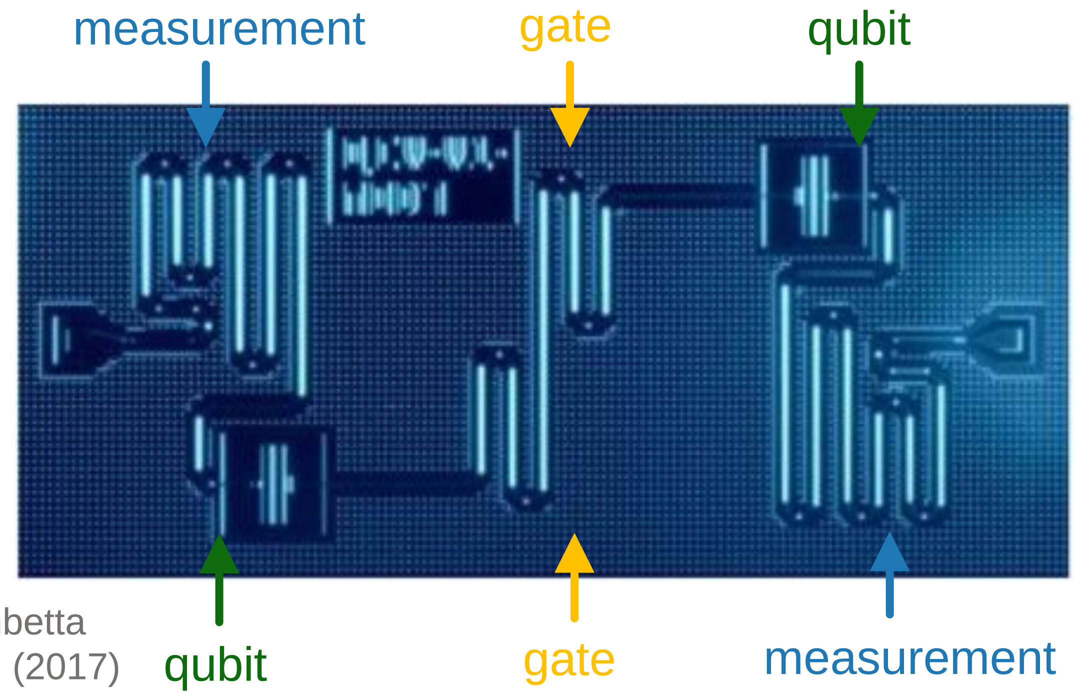

Gambetta et al. (2017)

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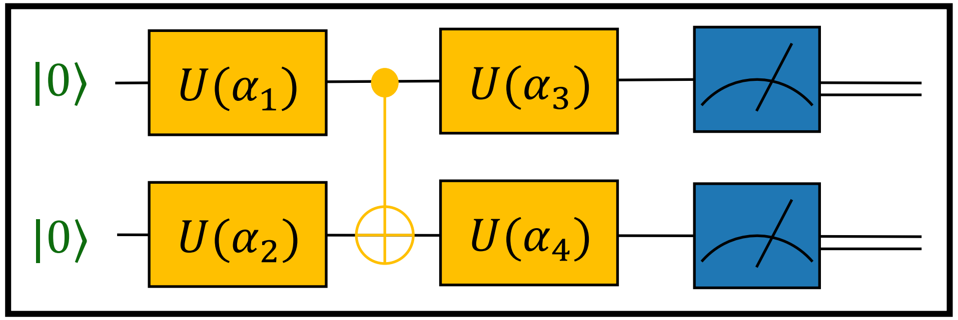

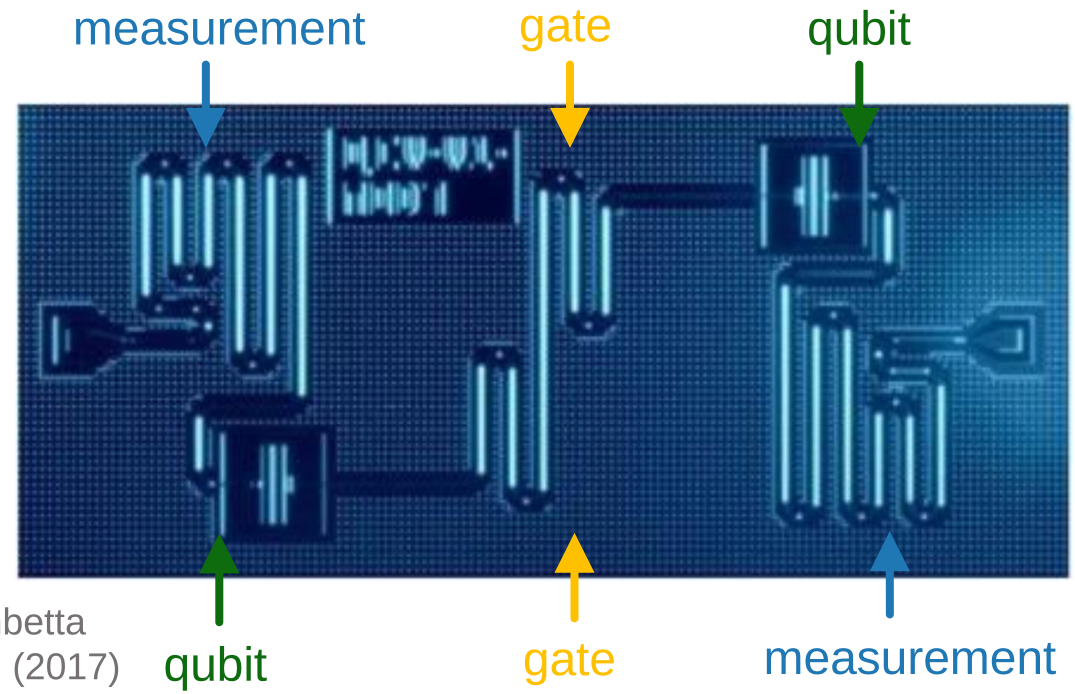

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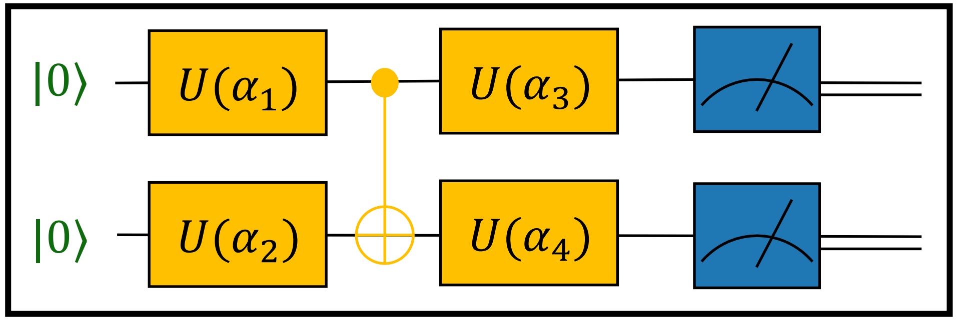
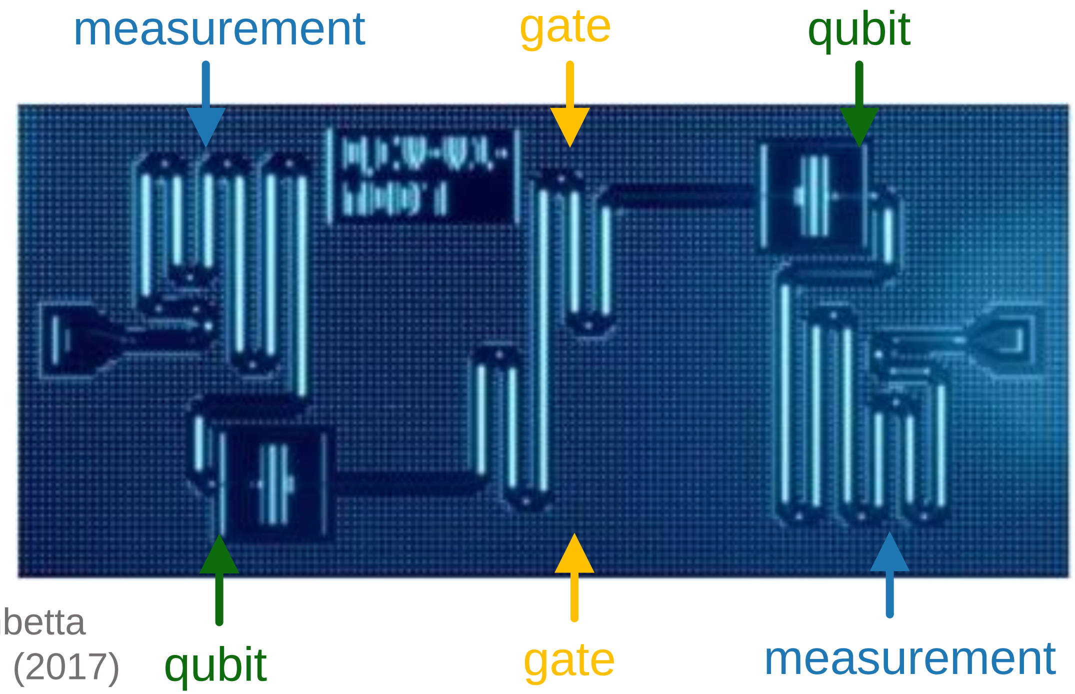
$|\psi(\vec{\alpha})\rangle =$ 



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Quantum noise: affecting qubits, gates, measurement

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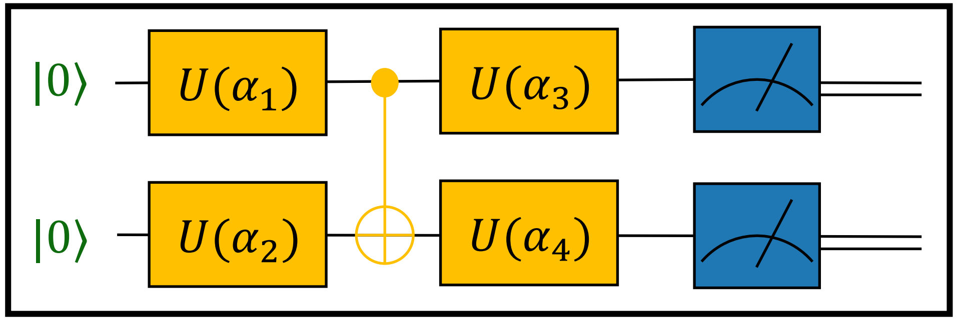

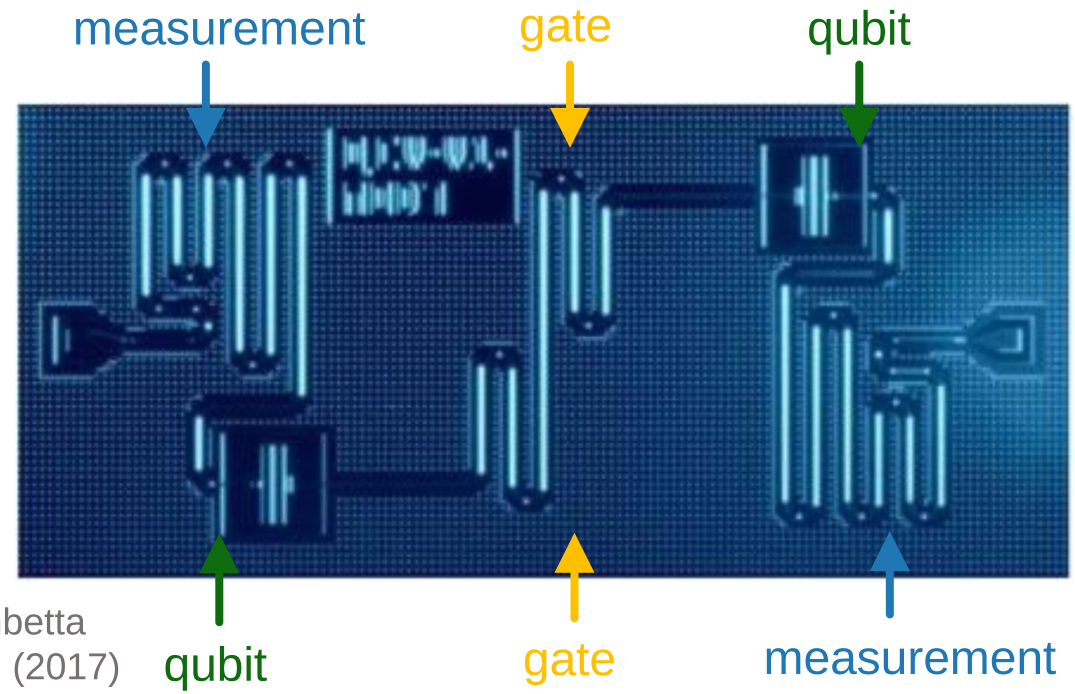

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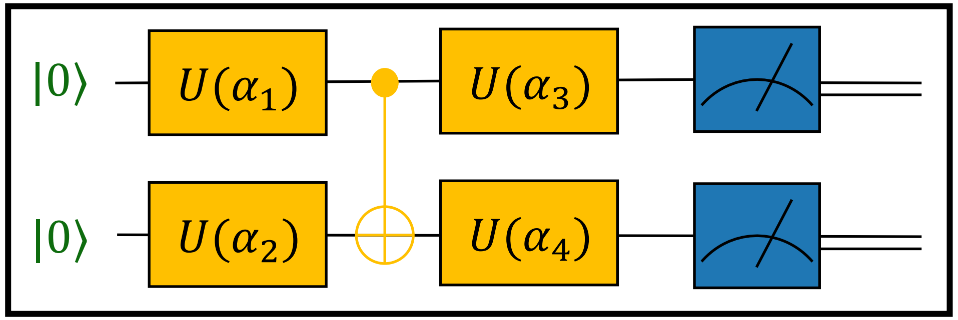

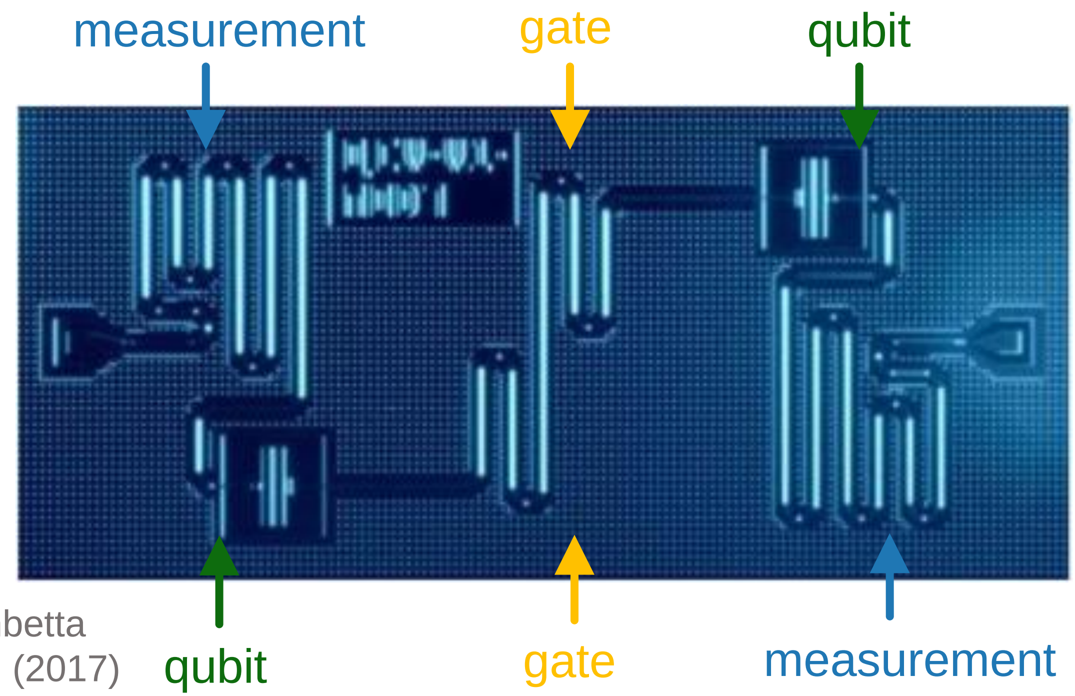

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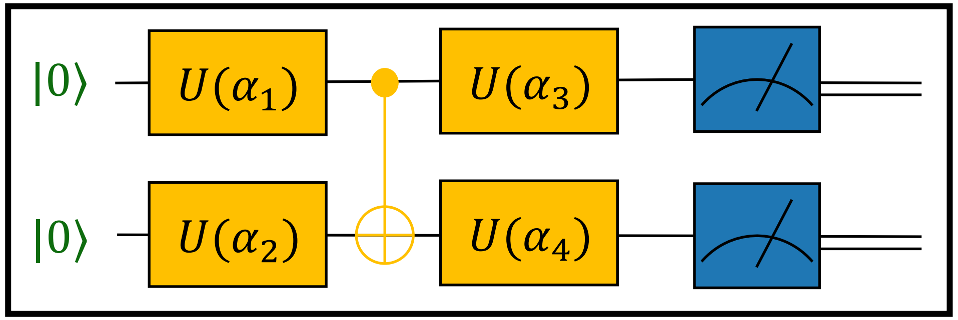

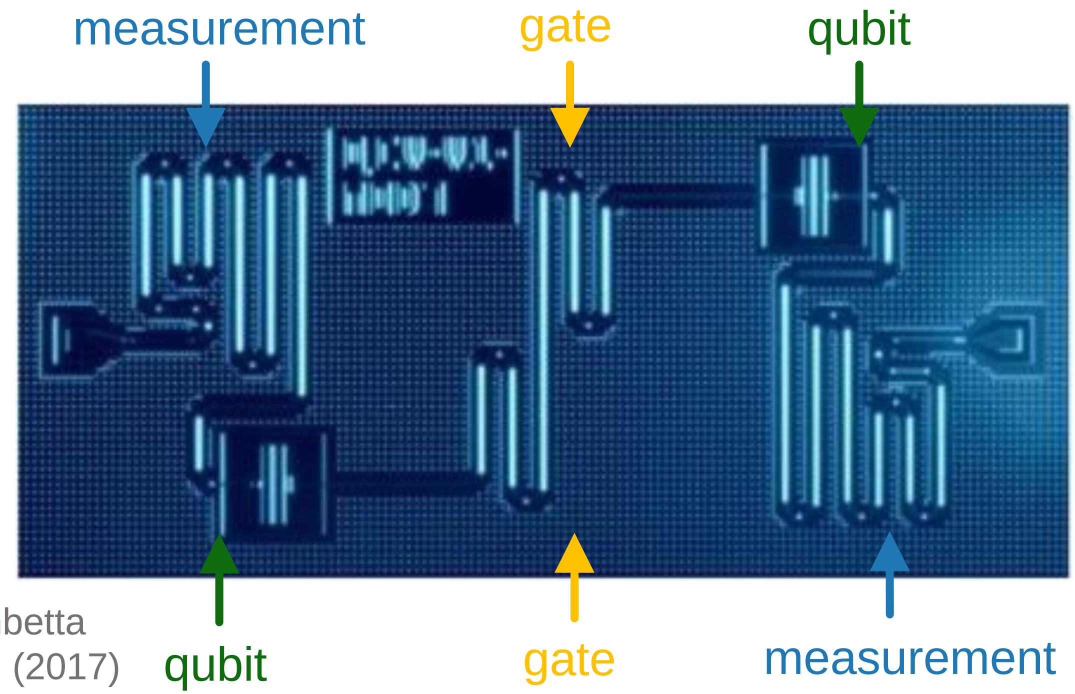

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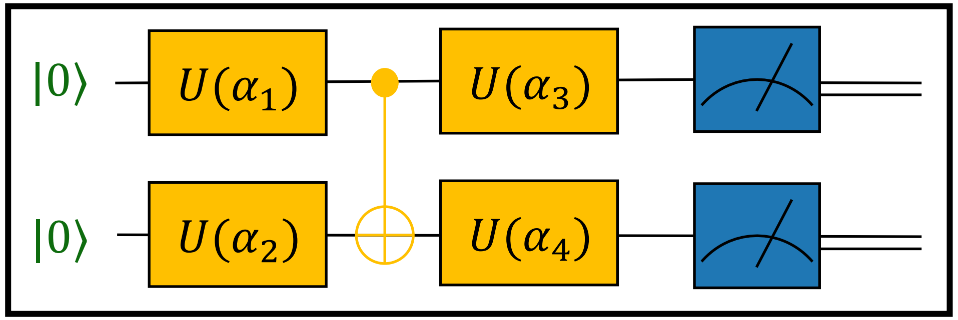

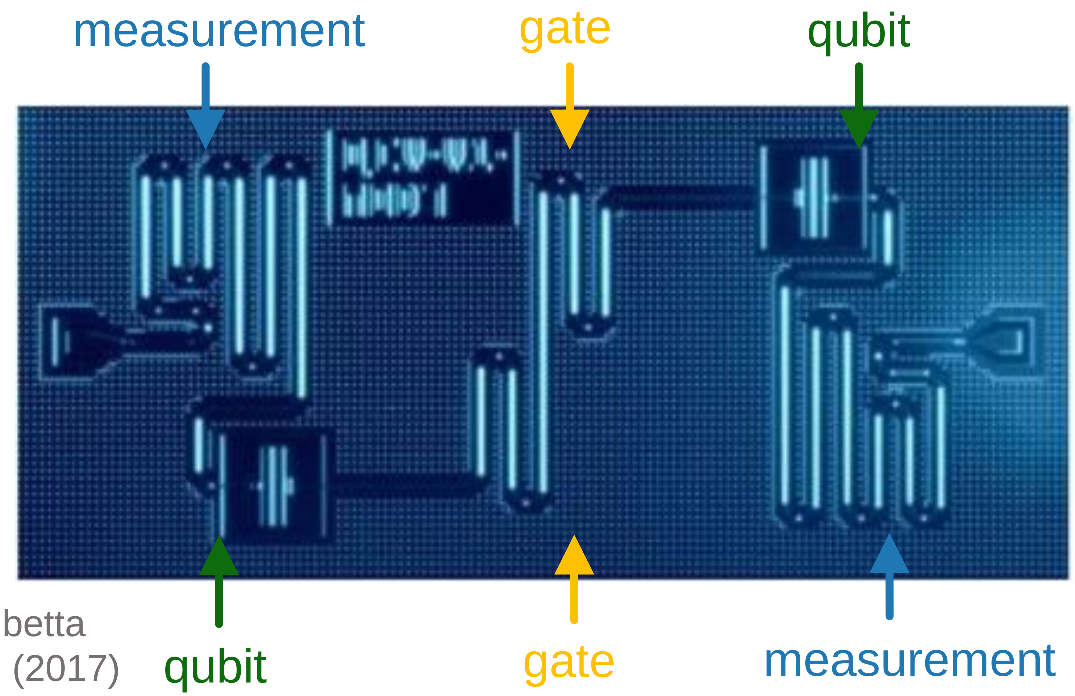

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For EC, need extra qubits and noise below *threshold*[5]

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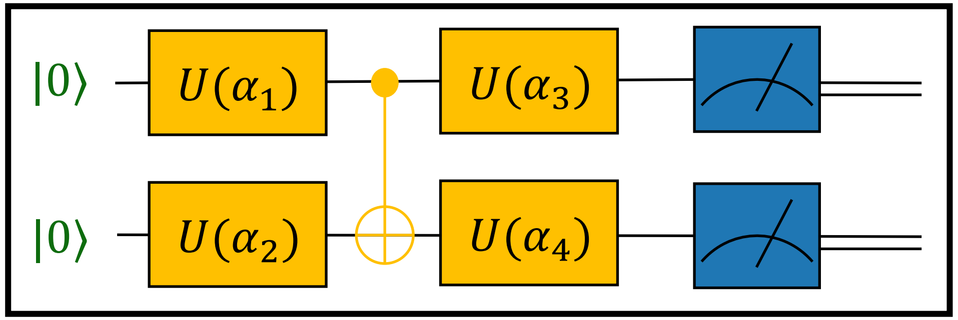

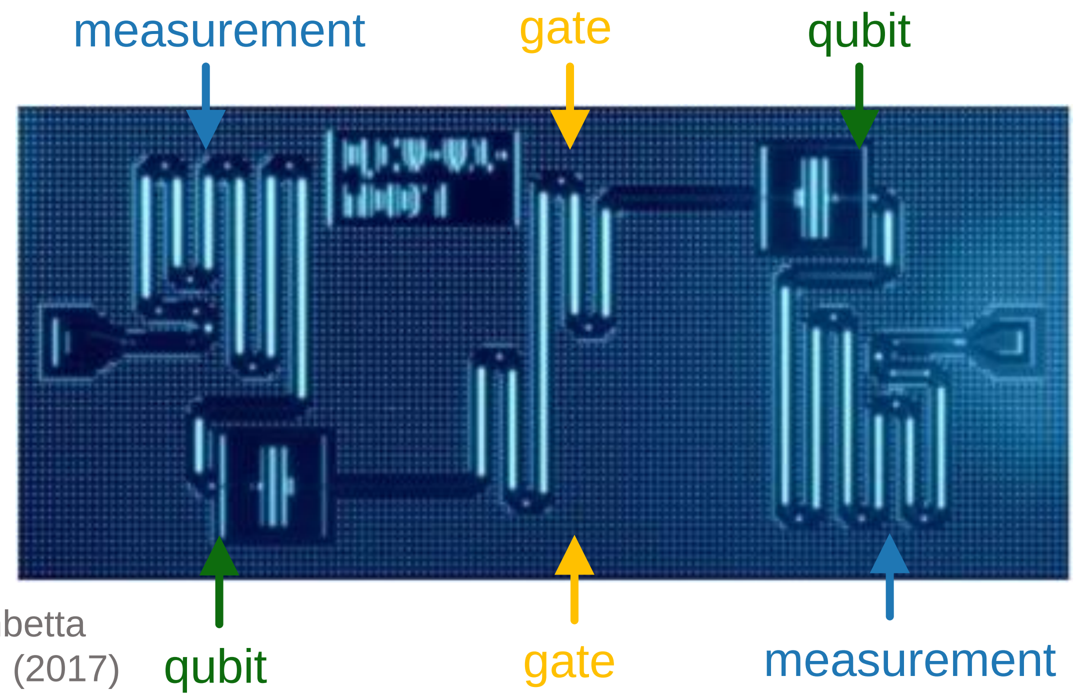

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E.g. surface code needs > 1000 extra qubits for $p < 0.1\%$

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Error mitigation: how can we reduce the noise?

Example: measurement error mitigation	Operator rescaling method

Error mitigation: how can we reduce the noise?

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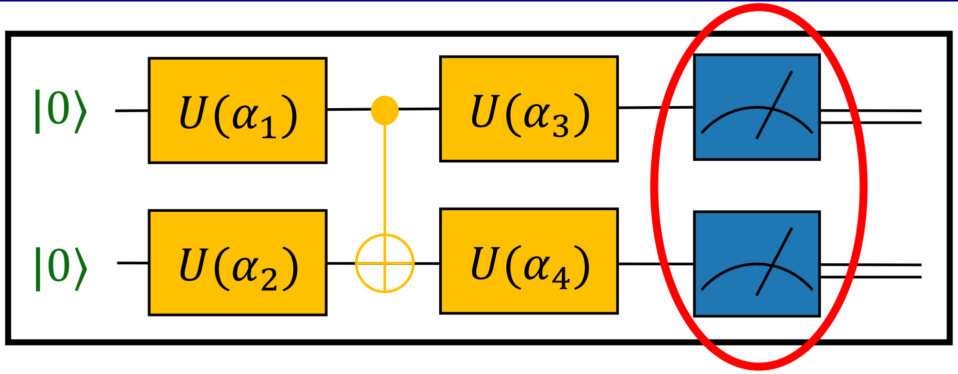

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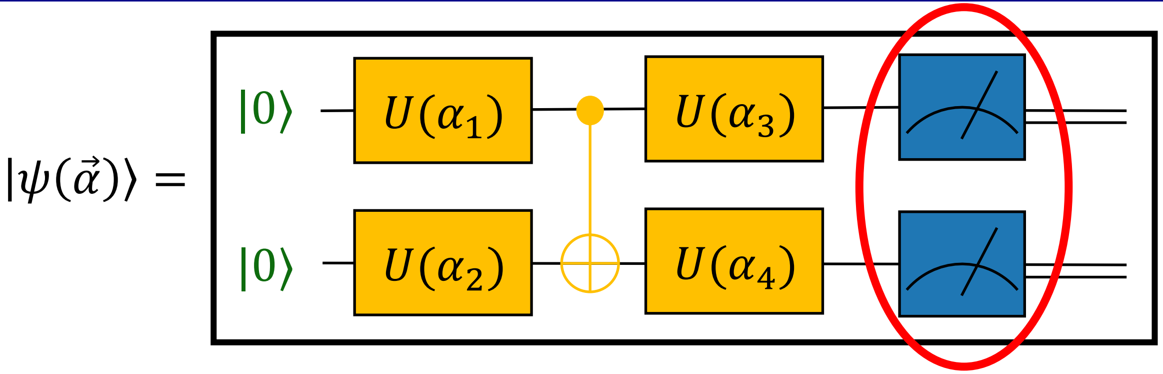

Operator rescaling method

Goal

mitigate bit-flip errors during readout: $0 \xrightarrow{p_0} 1$ or $1 \xrightarrow{p_1} 0$

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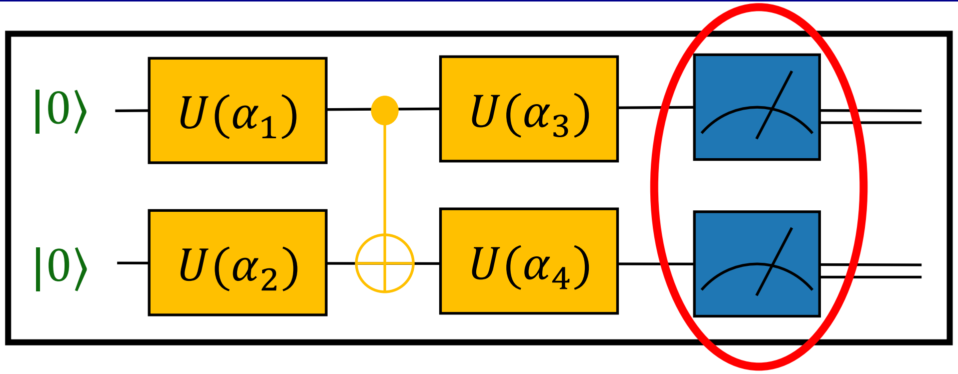

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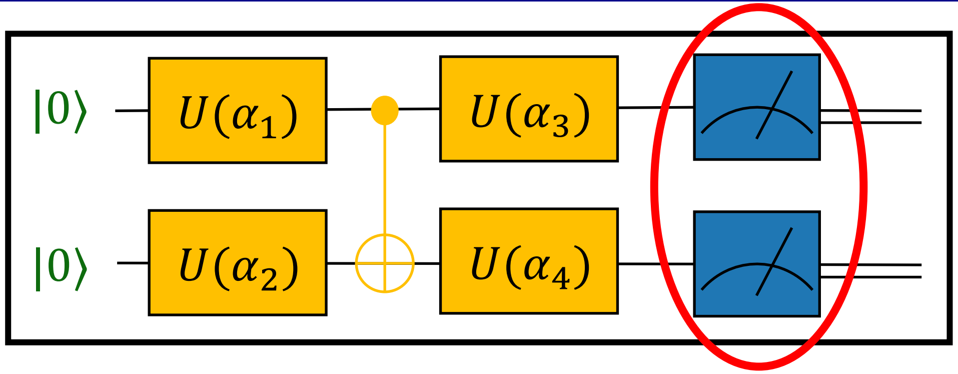

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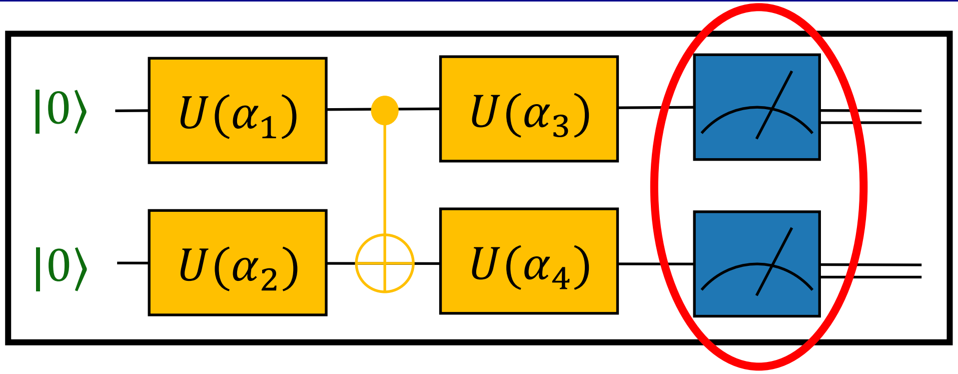

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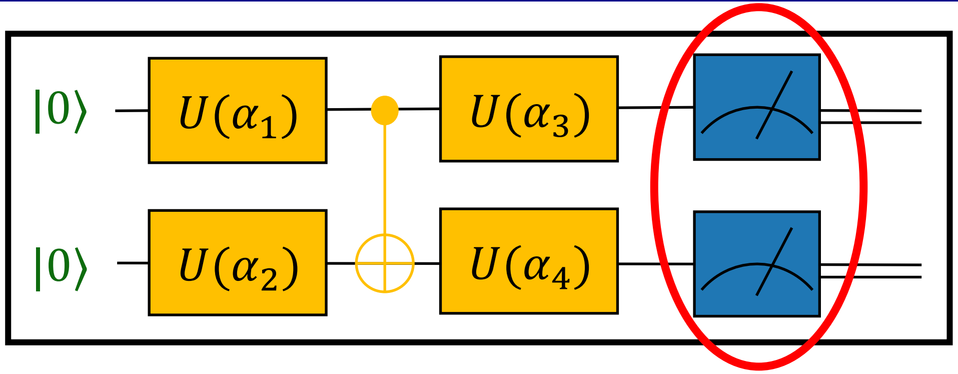

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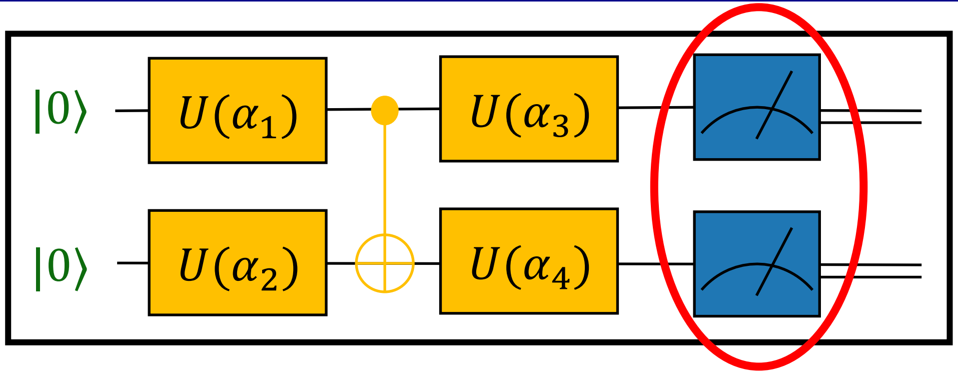
Error mitigation: how can we reduce the noise?

Example: measurement error mitigation

$|\psi(\vec{\alpha})\rangle =$



Operator rescaling method

Goal

mitigate bit-flip errors during readout: $0 \xrightarrow{p_0} 1$ or $1 \xrightarrow{p_1} 0$

Method [1]

replace operators by noisy operators: $\langle\tilde{\psi}|\boldsymbol{O}|\tilde{\psi}\rangle \to \langle\psi|\tilde{O}|\psi\rangle$

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Total noisy operator: $\tilde{O}$

$$= (1-p_0)(1-p_1)\boldsymbol{Z} + p_0 p_1(-Z) + p_0(1-p_1)(-\mathbb{I}) + (1-p_0)p_1\mathbb{I}$$

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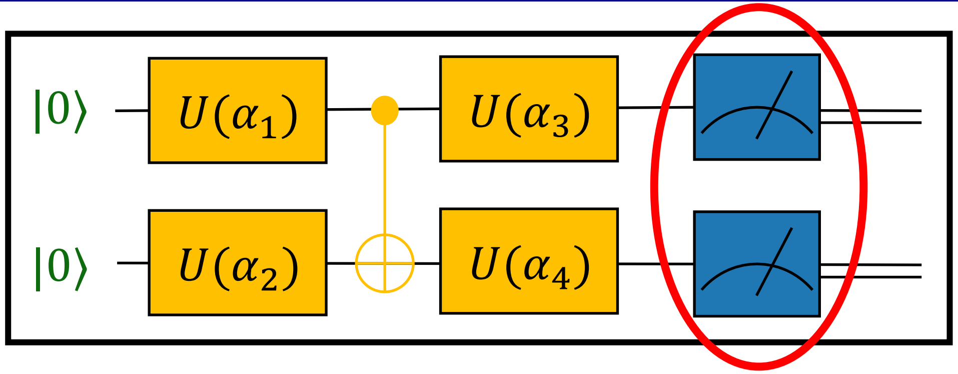

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Rescaled (zero-noise) operator:

$$\boxed{\rightarrow \boldsymbol{Z} = \frac{1}{1-p_0-p_1}\tilde{O} - \frac{p_1-p_0}{1-p_0-p_1}\mathbb{I}}$$

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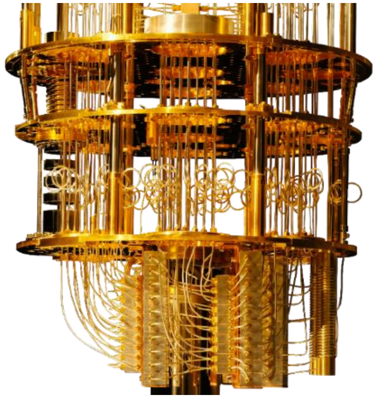

Carlow (2018)

[1] Kandala et al. (2017), Yeter-Aydeniz et al. (2019), LF et al. (2020)

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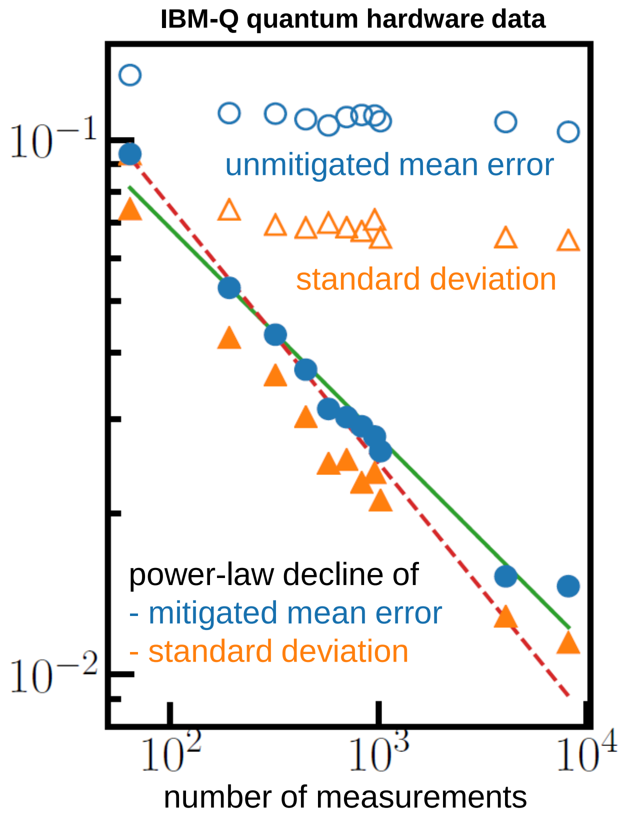
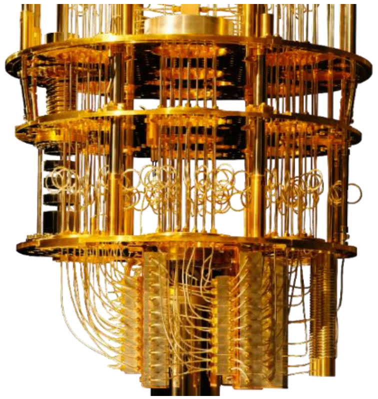


LF et al. (2020)



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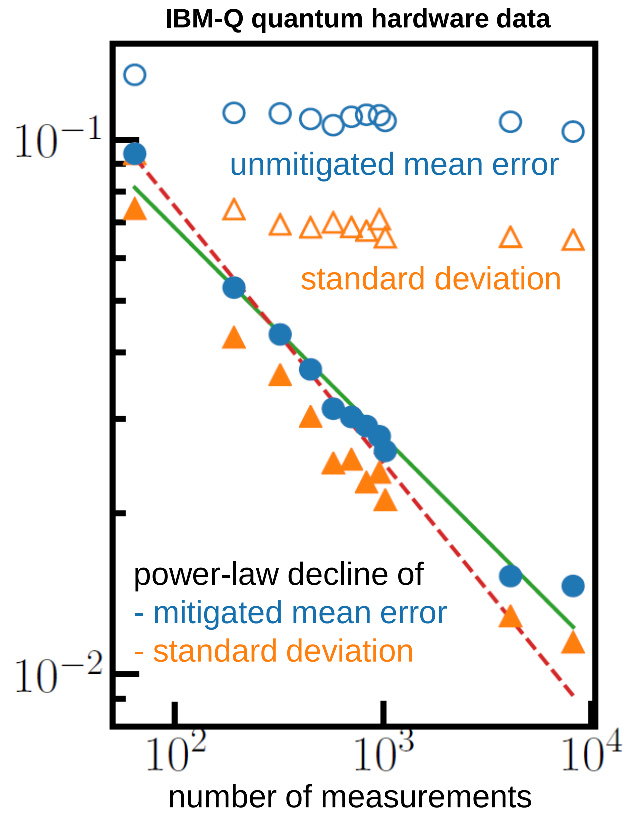

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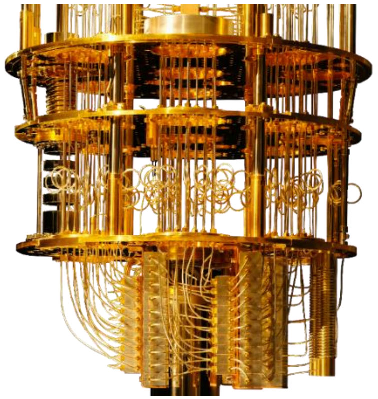

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Other mitigation techniques

Zero-noise extrapolation,[2] randomized compiling,[3] quasi-probability decomposition,[4] …

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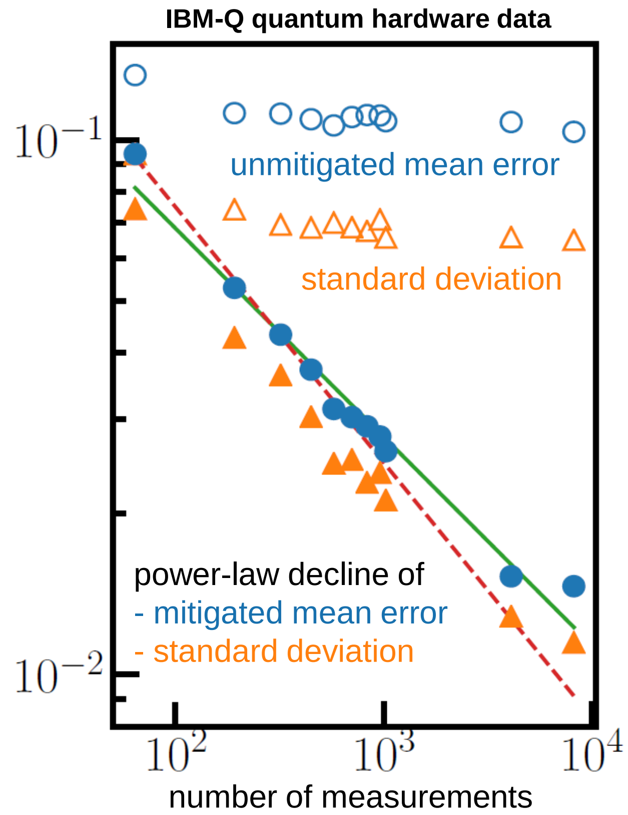

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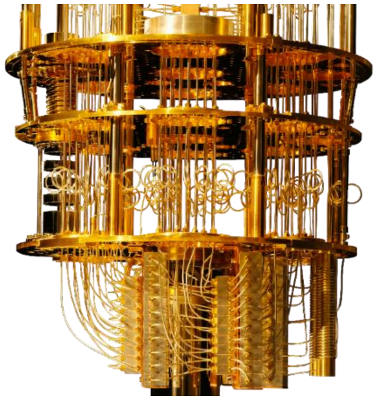

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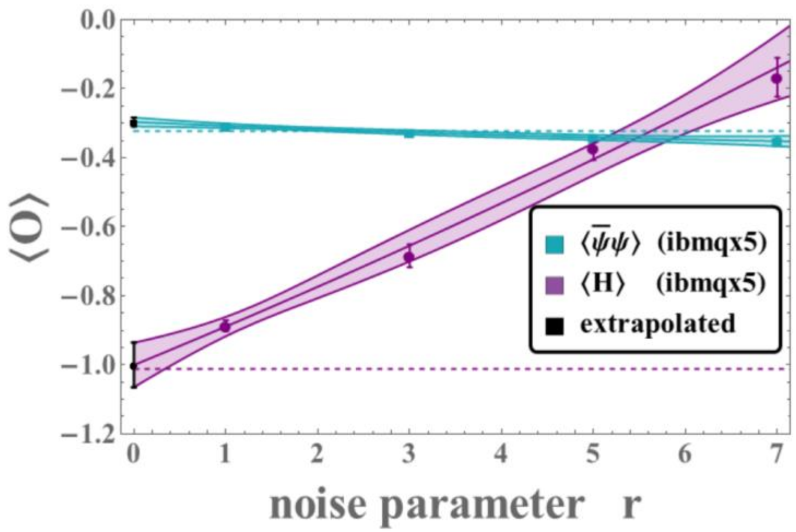
Lattice field theory applications

Zero-noise extrapolation for lattice Schwinger model:



Klco et al. (2018)

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Which field theories have already been simulated?

Experimental results on “public” QC	Experimental results on “private” QC

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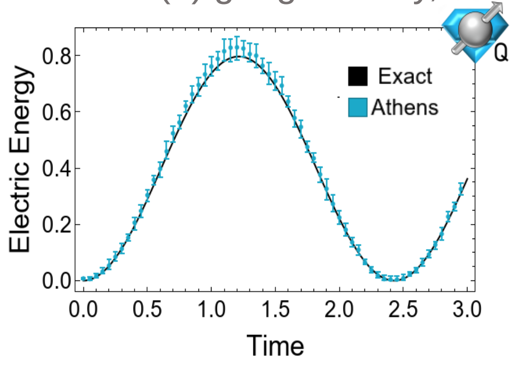
IBM-Q’s superconducting qubits

Real-time evolution: Schwinger model,[1] SU(2),[2] SU(3),[3] ...

1+1D SU(3) gauge theory, one plaquette [3]



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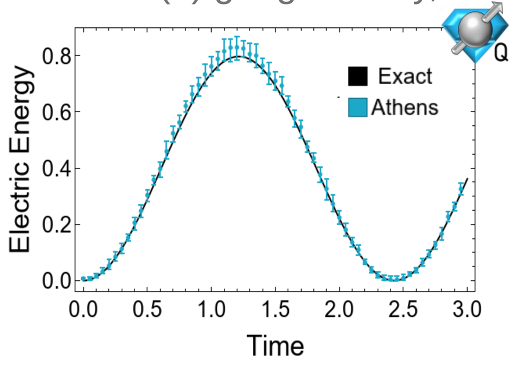
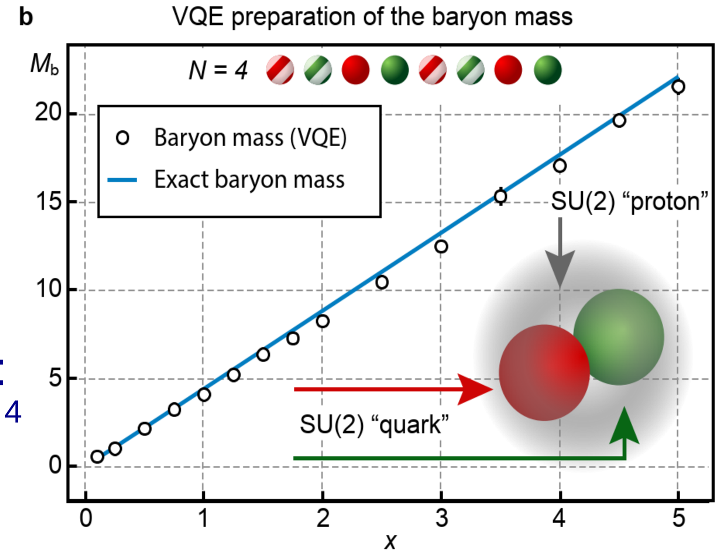


Variational computation:
SU(2) "hadron" masses [4]



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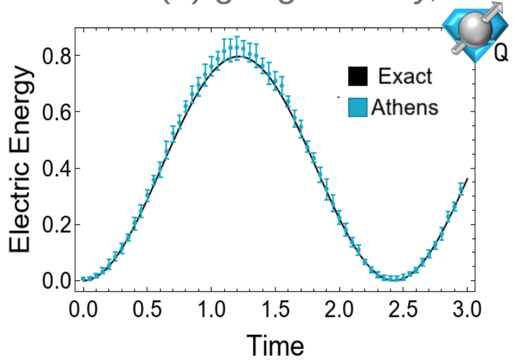

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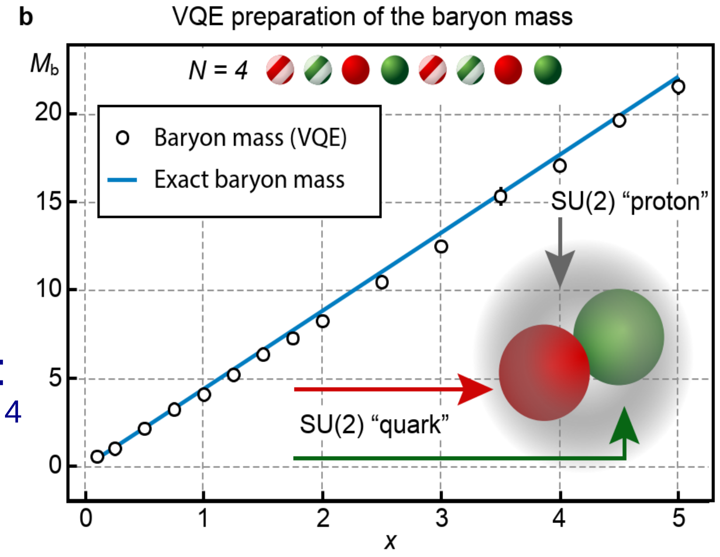
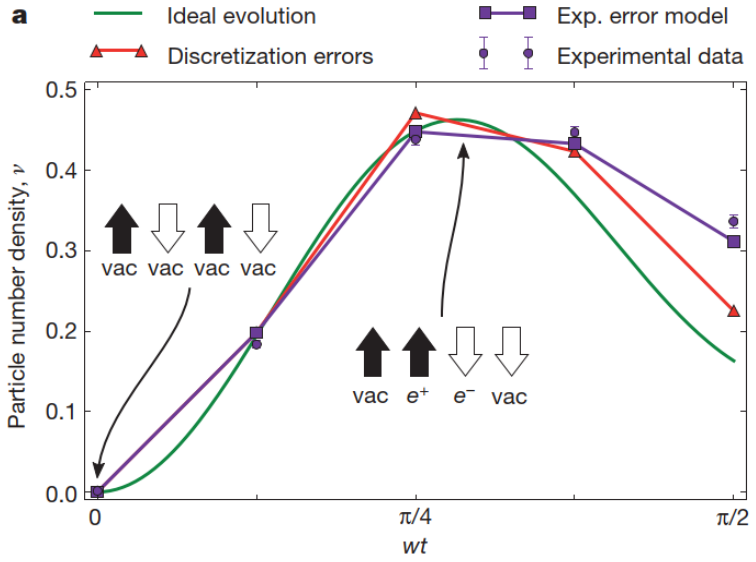


Experimental results on "private" QC

Trapped ions

Real-time evolution: Schwinger model, [5]…



Martinez et al. (2016)

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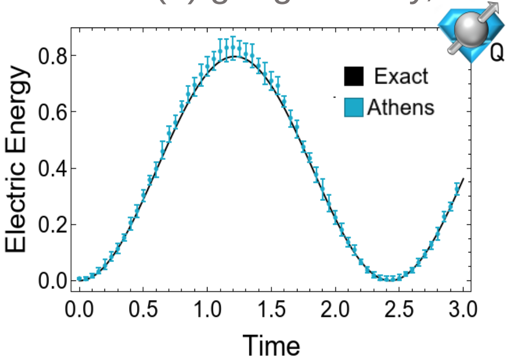

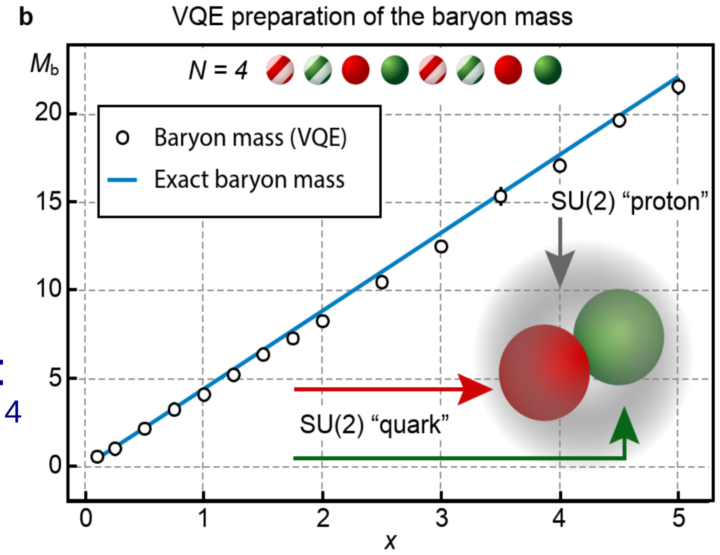

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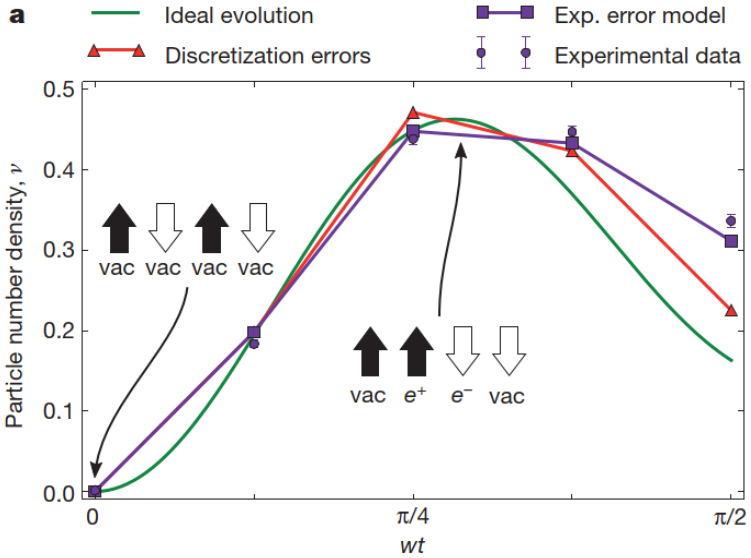

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Cold atoms

Analog simulation: Bose-Hubbard,[6] Schwinger model,[7] ...

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How to simulate these field theories?

Example: hybrid quantum-classical algorithms	**Variational Quantum Eigensolver (VQE)** [1]

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Key concept

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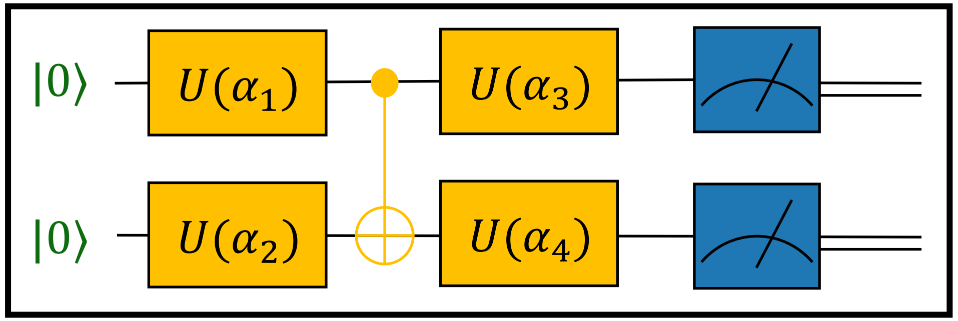

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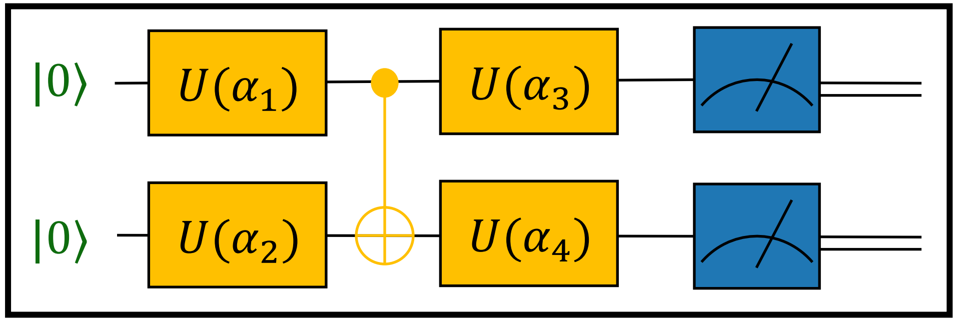
$|\psi(\vec{\alpha})\rangle =$ 

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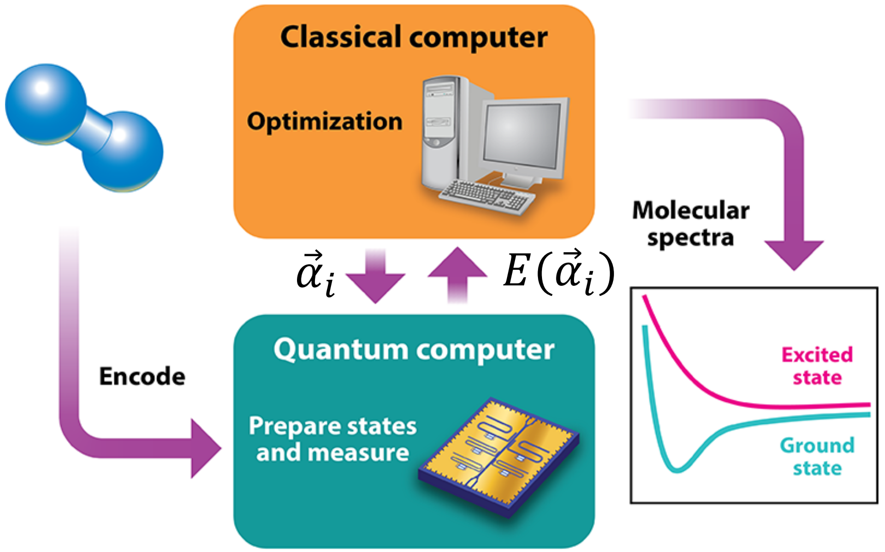

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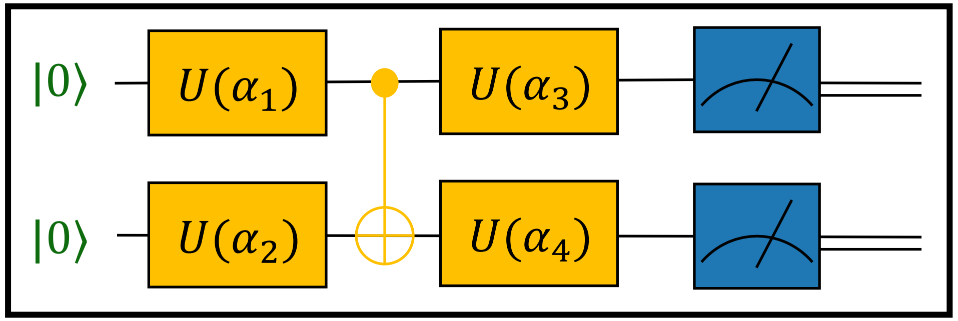

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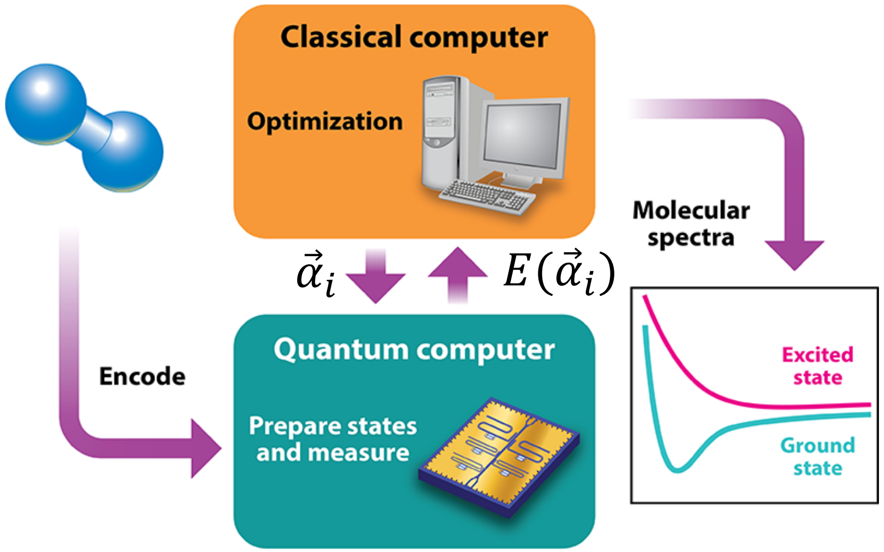

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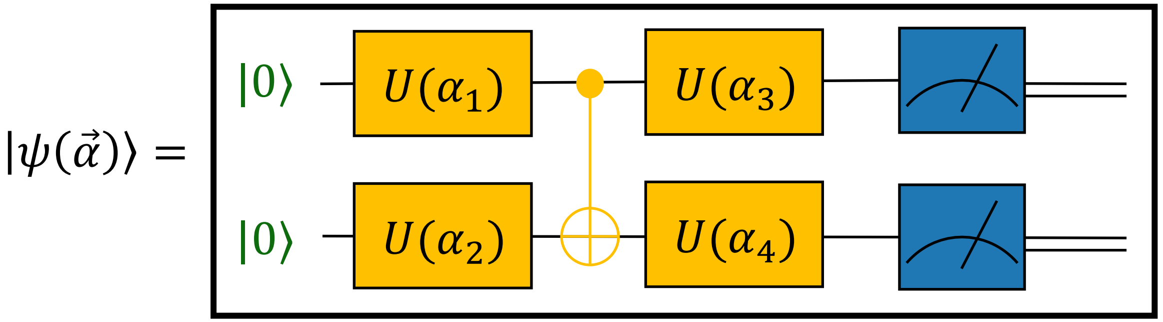

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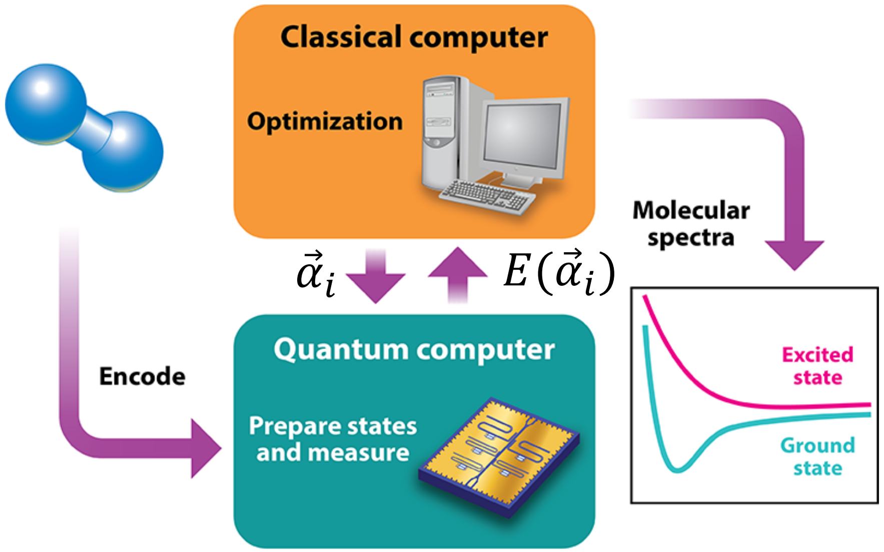

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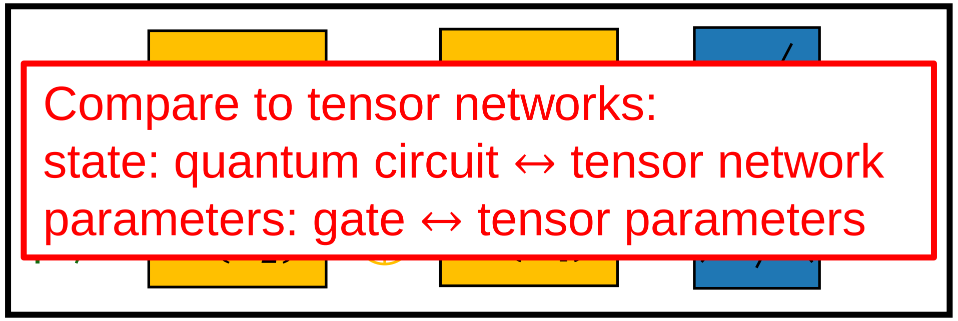

Compare to tensor networks:
state: quantum circuit ↔ tensor network
parameters: gate ↔ tensor parameters

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Quantum circuit design [1]	**Example: geometrical method** [2]

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Quantum circuit design [1] | **Example: geometrical method** [2]

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$|\psi(\vec{\alpha})\rangle$ should reach all physical states in Hilbert space

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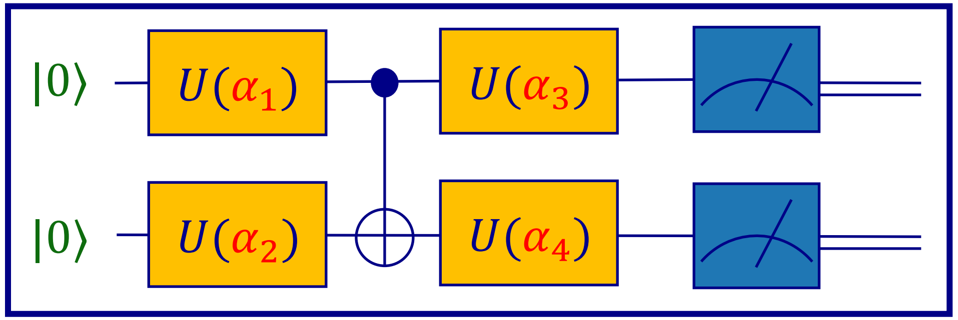

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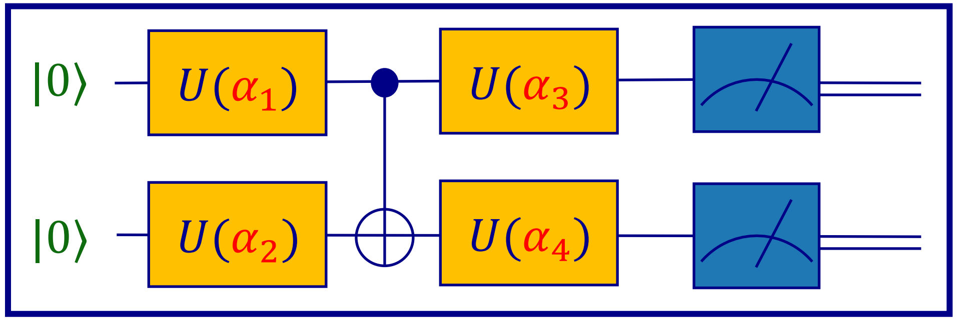

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$|\psi(\vec{\alpha})\rangle$ should include physical symmetries

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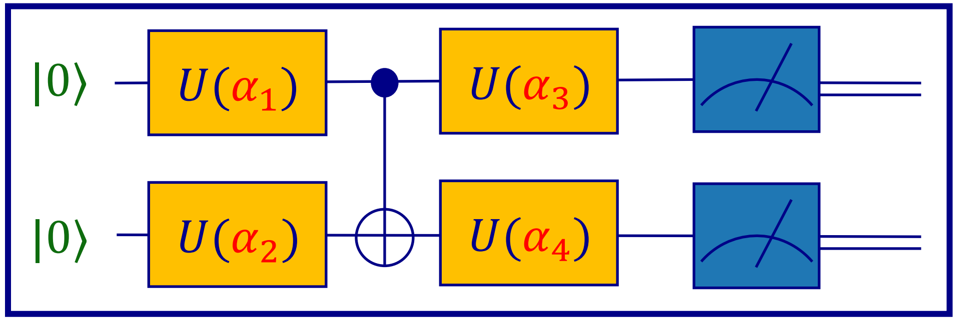

[1] Martinez et al. (2016), Klco et al. (2018,2019), Ciavarella et al. (2019a,b), Schweizer et al. (2019), Yang et al. (2020), Mil et al. (2020), de Jong et al. (2021), Nguyen et al. (2021), Atas et al. (2021), ..., [2] LF, Hartung, Jansen, Kühn, Stornati, *Quantum* (2021), *IEEE ICWS* (2021)

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Quantum circuit design [1]

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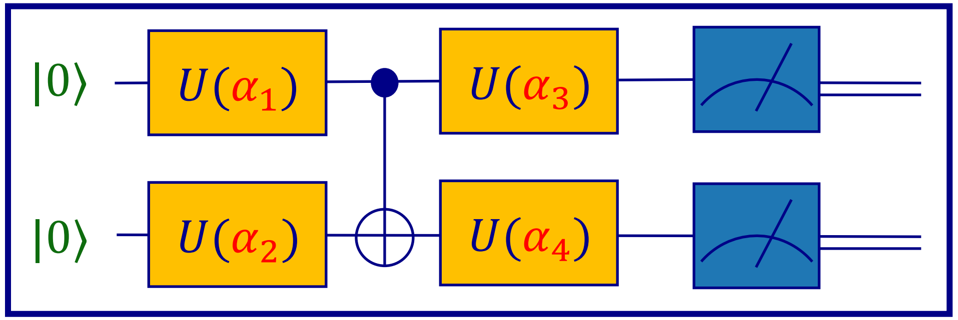

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Circuit manifold M: states $|\psi(\vec{\alpha})\rangle$ reachable by circuit

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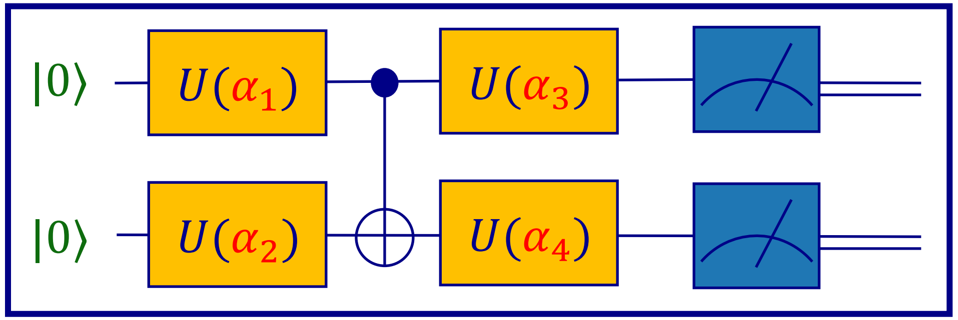

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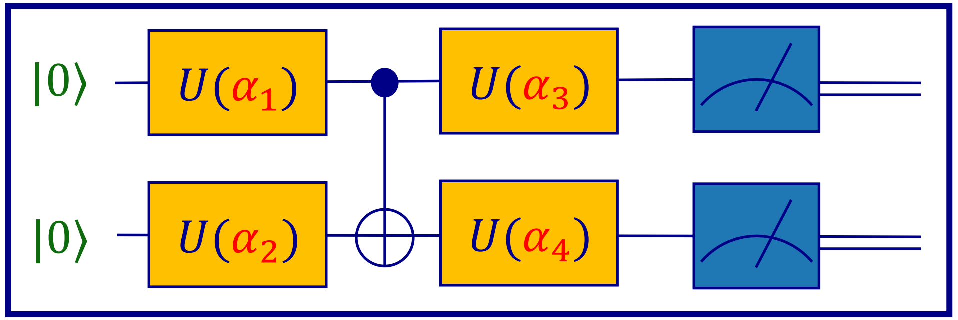
$|\psi(\vec{\alpha})\rangle =$ 

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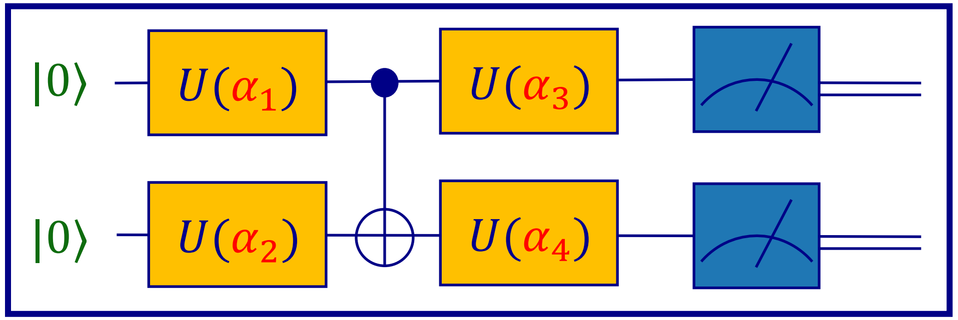

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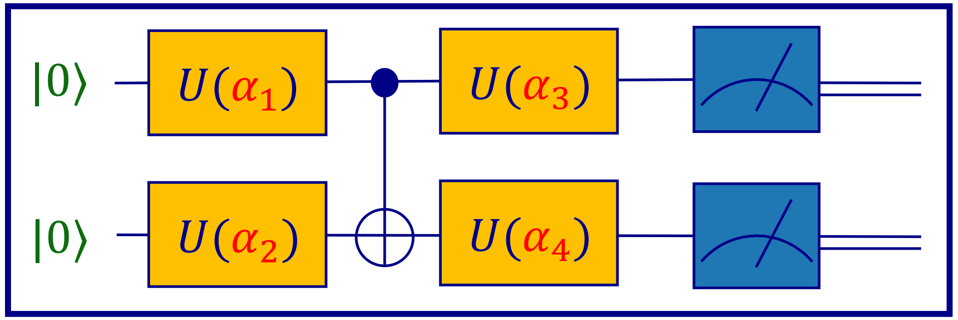

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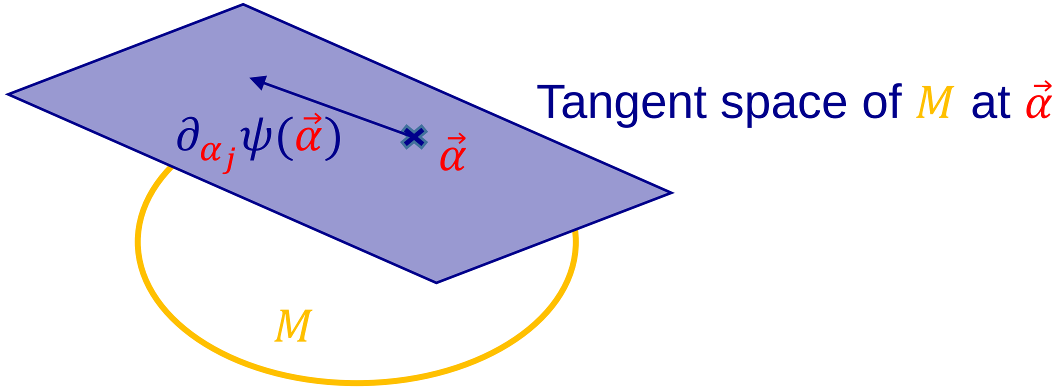

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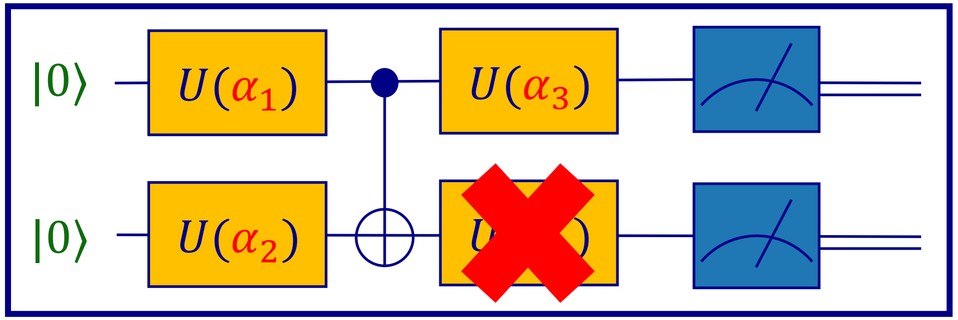

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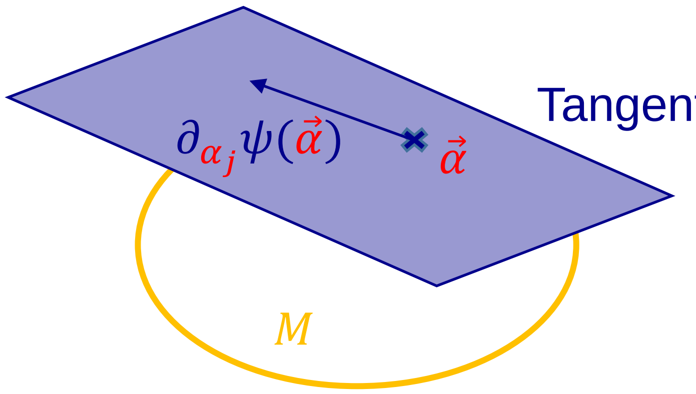

Tangent space of M at $\vec{\alpha}$

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How to deal with gauge fields?

Infinite Hilbert space	Gauge invariance

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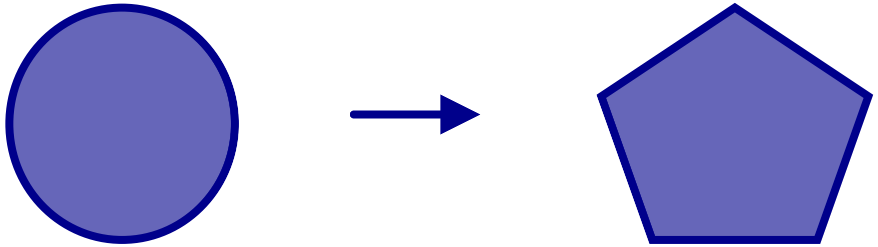

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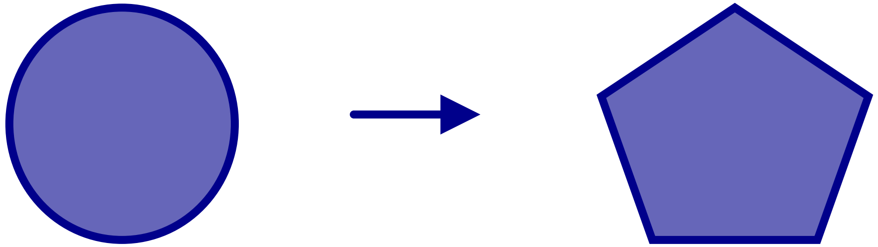

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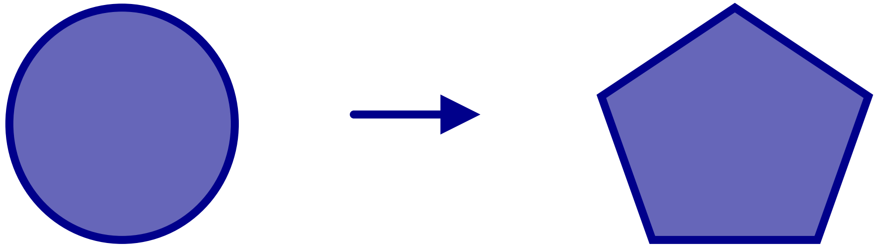

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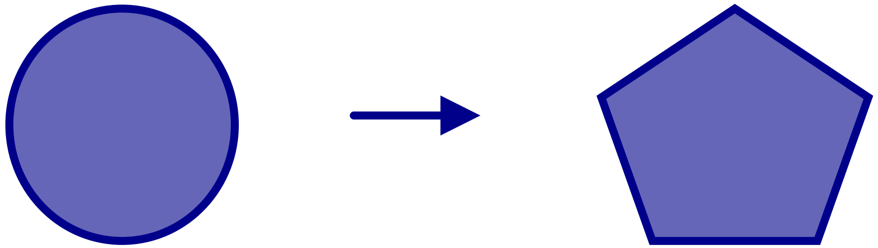

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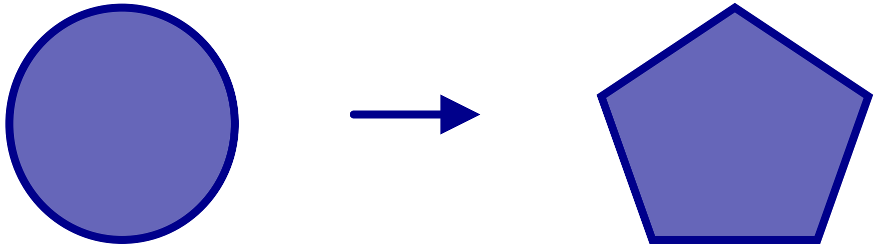

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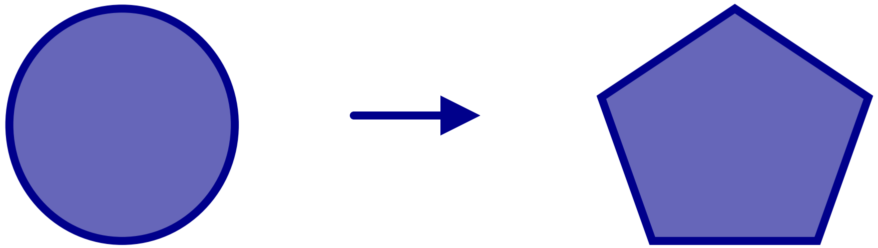

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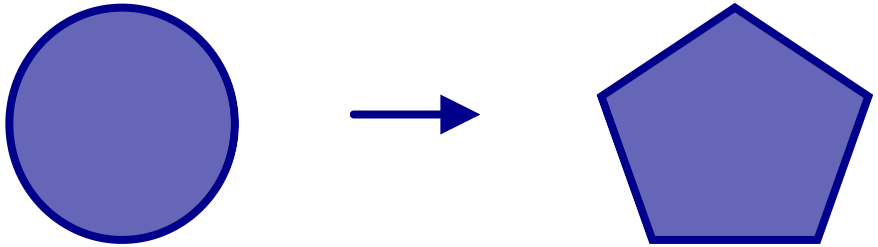

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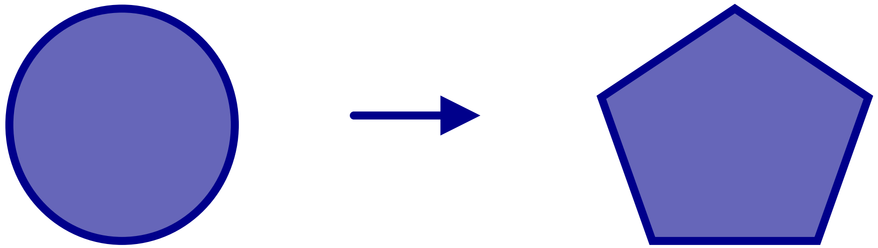

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Comparing Wilson and staggered fermions	Finding a resource-efficient implementation
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Developing efficient digitization & improved Hamiltonians

→ talk by Henry Lamm tomorrow at 5:50pm

1+1D Ising model

Studying NISQ algorithms for open quantum systems

→ talk by Bharath Sambasivam tomorrow at 6:10pm

1+1D extended O(2) model

Studying discrete approximations of continuous groups

→ talk by Leon Hostetler on Wednesday at 3:40pm

More details: parallel talks @ this conference

SU(N) lattice gauge theories

1+1D SU(2) gauge theory

Implementing real-time evolution on IBM-Q hardware

→ talk by Emanuele Mendicelli tomorrow at 4:30pm

Comparing Schwinger-boson & loop-string hadron form.

→ talk by Jesse Stryker on Thursday at 11:50am

SU(2) gauge theories

Developing efficient digitization via finite subgroups

→ talk by Timo Jakobs tomorrow at 2:40pm

Defining canonical momenta for discretized gauge fields

→ talk by Carsten Urbach tomorrow at 3pm

SU(3) gauge theories

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Lattice field theories

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1+1D Wess-Zumino model

Studying dynamical breaking of supersymmetry

→ talk by Christopher Culver tomorrow at 2:20pm

Summary: where do we stand, where will we go?

The Path to Go…	A Rough Sketch…

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Present

Hardware: $\mathcal{O}(10-100)$ noisy qubits with error mitigation

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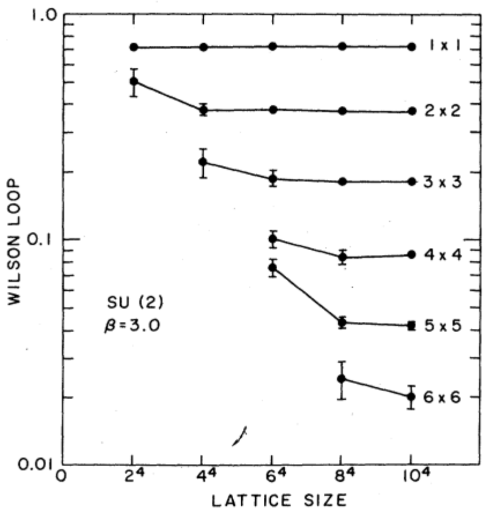

Creutz (1980)

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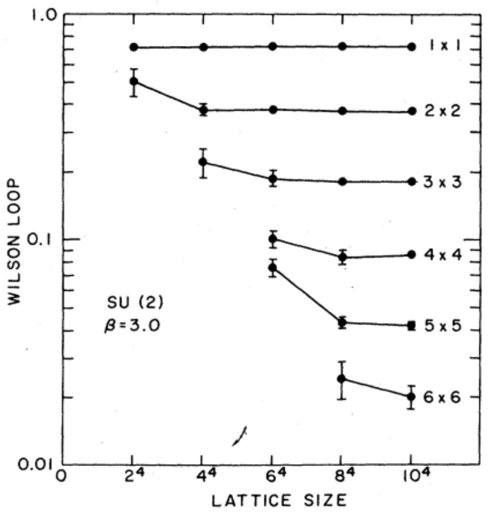

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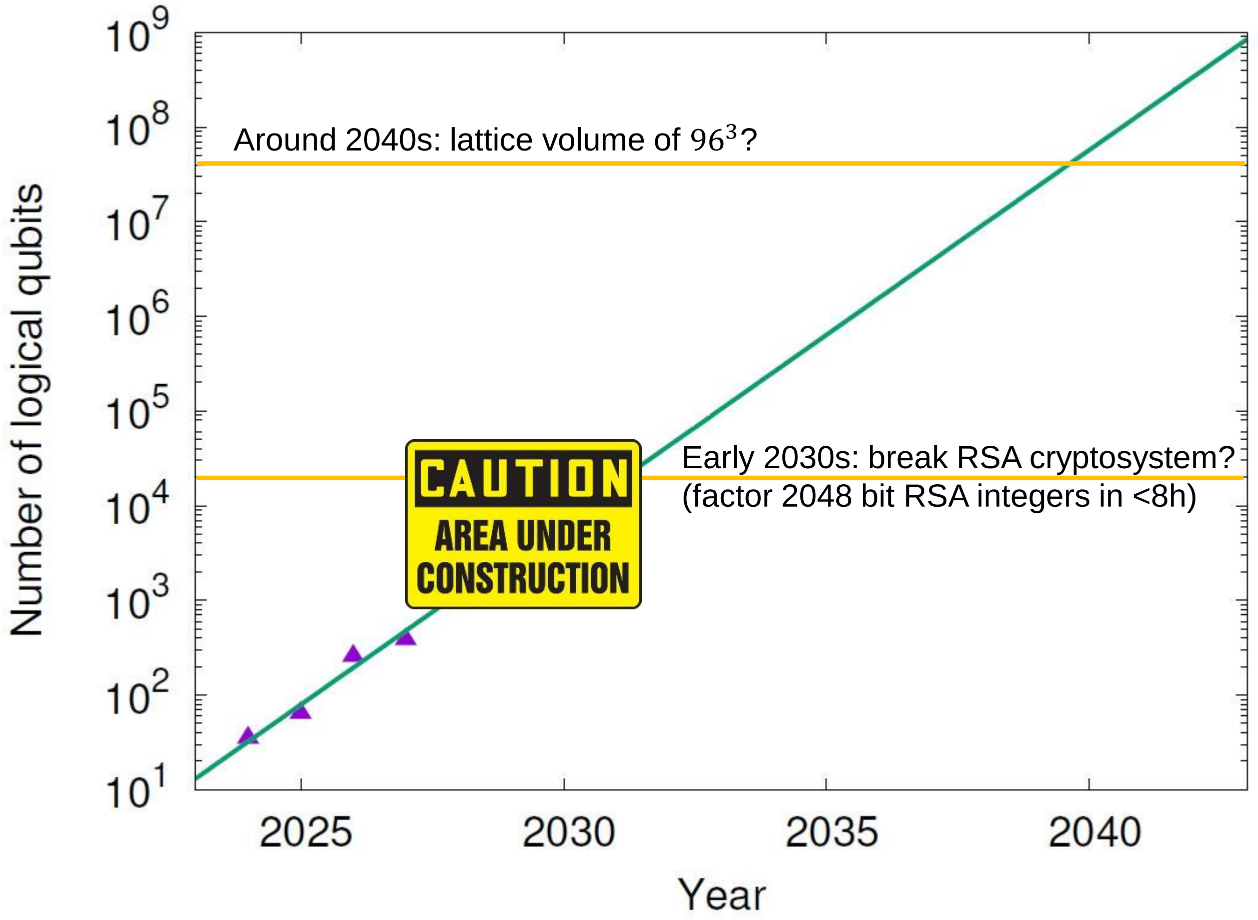

Gidney, Ekera (2019);
Kan, Nam (2021)

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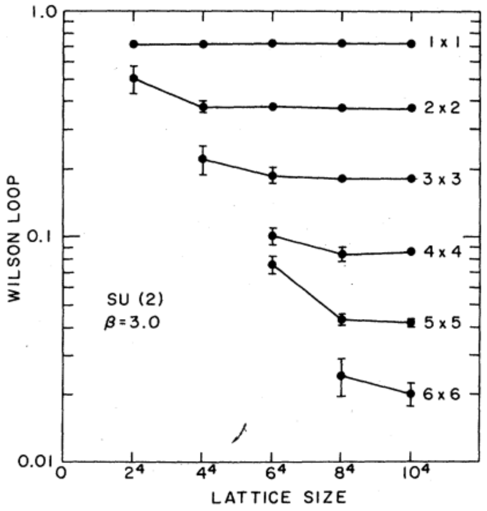

Creutz (1980)

Future

Hardware: $\mathcal{O}(1000)$ error-corrected qubits by 2029?

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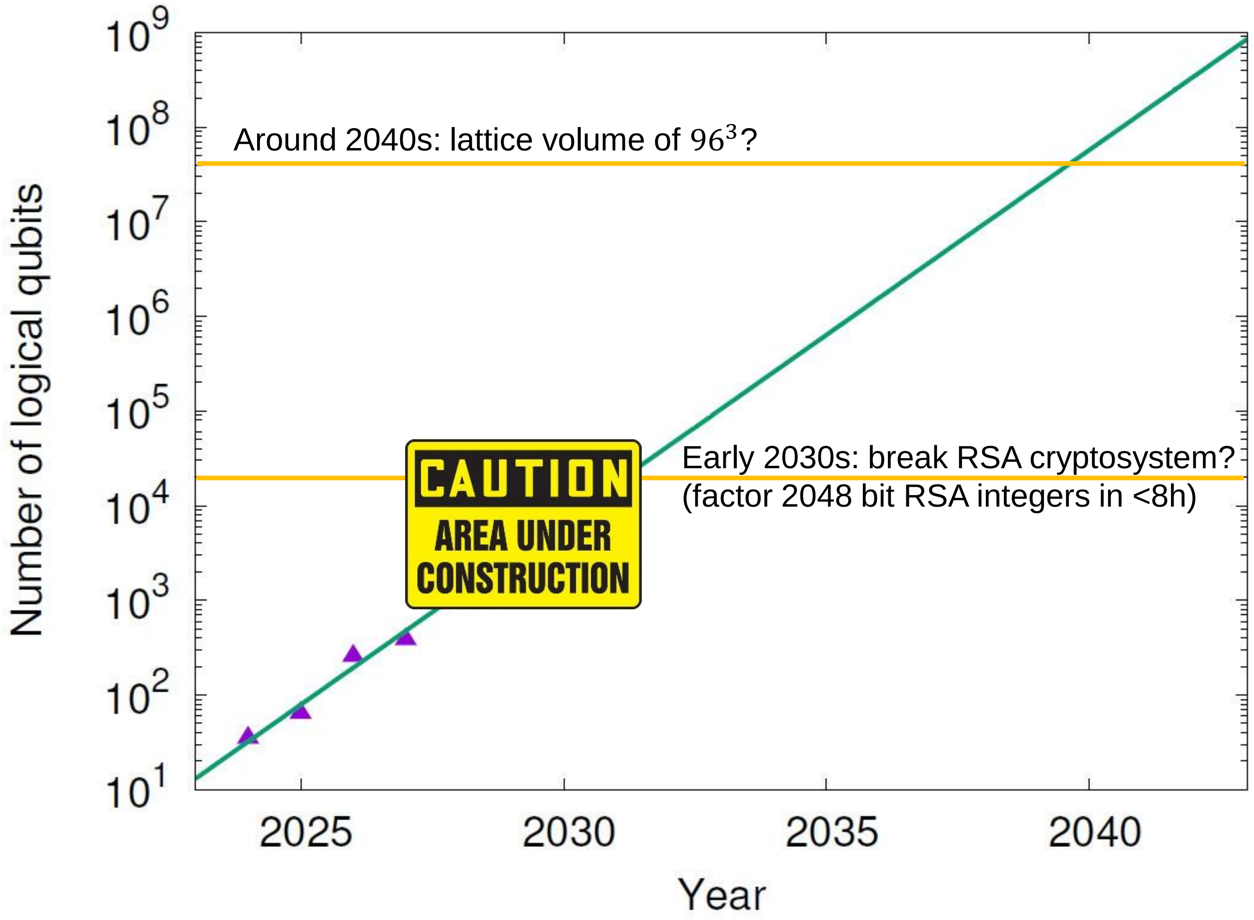

Gidney, Ekera (2019); Kan, Nam (2021)

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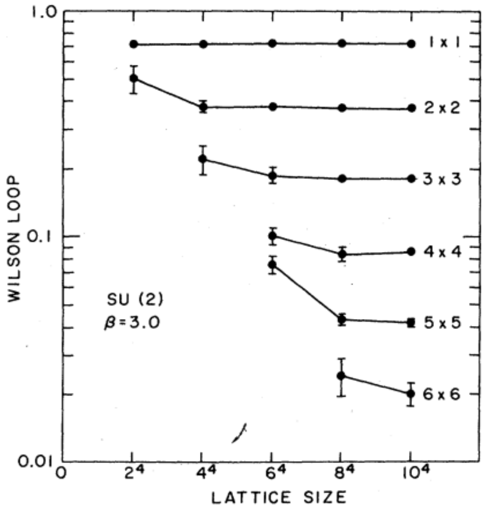
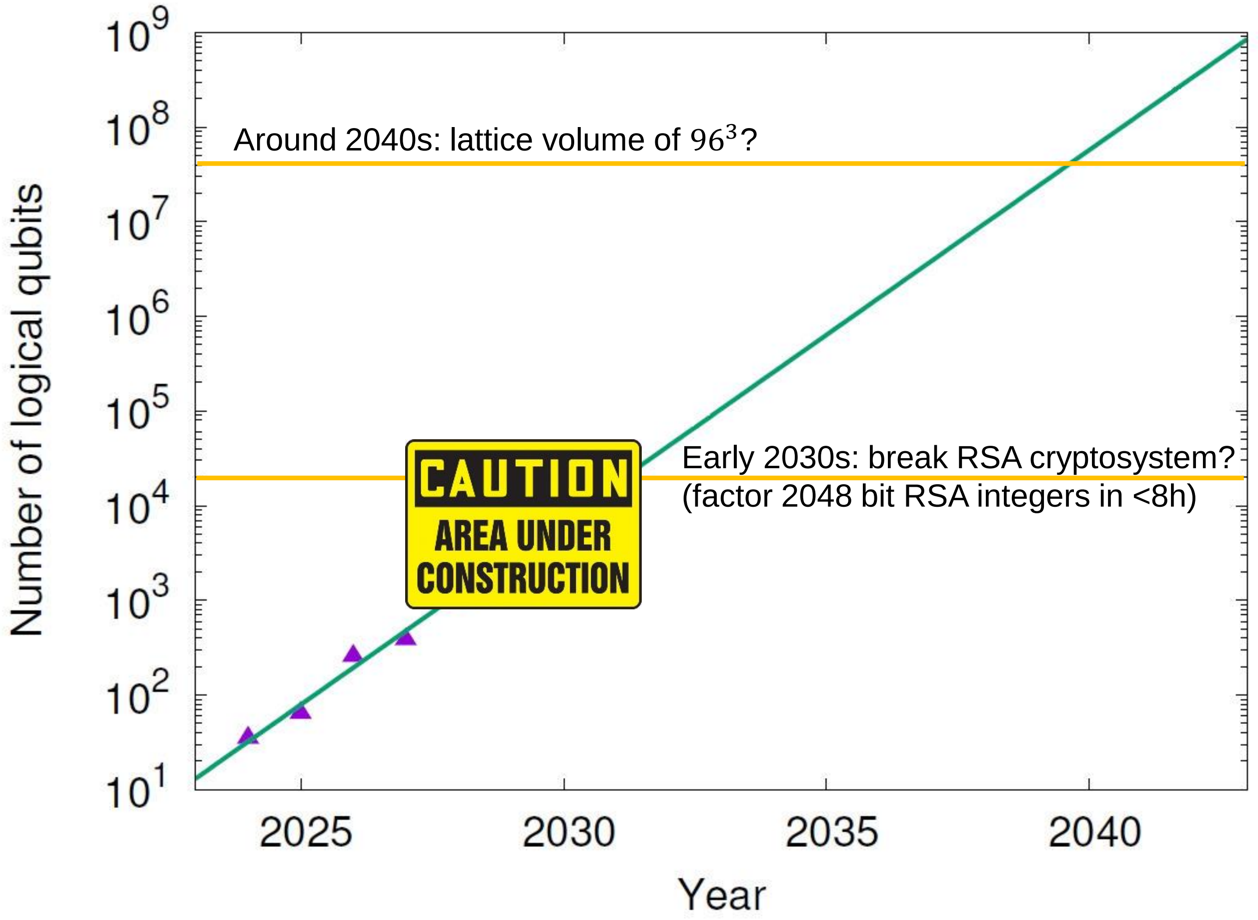


Creutz (1980)

Future

Hardware: $\mathcal{O}(1000)$ error-corrected qubits by 2029?

Algorithms: improved Hamiltonians, resource-efficient formulations of 3+1D LGTs, …



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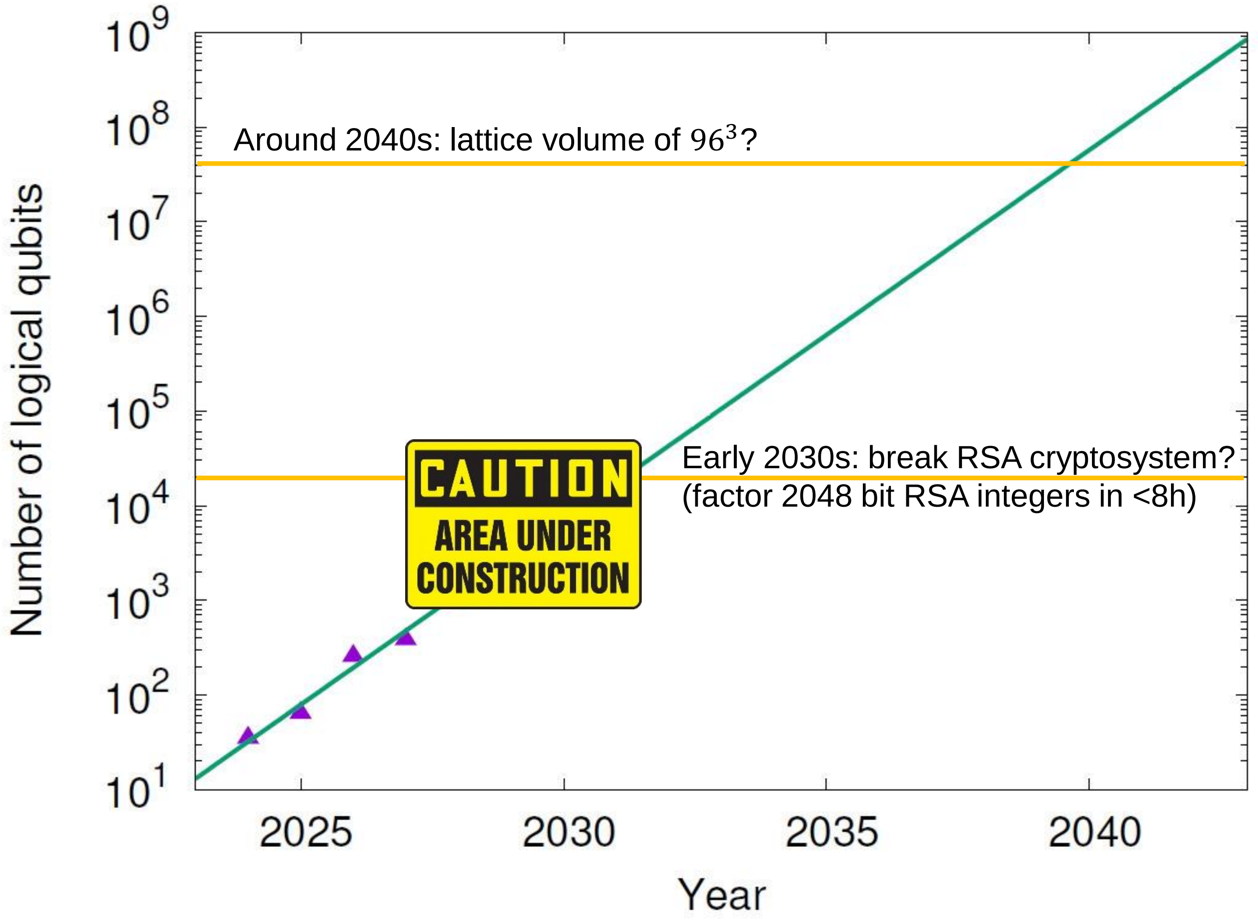

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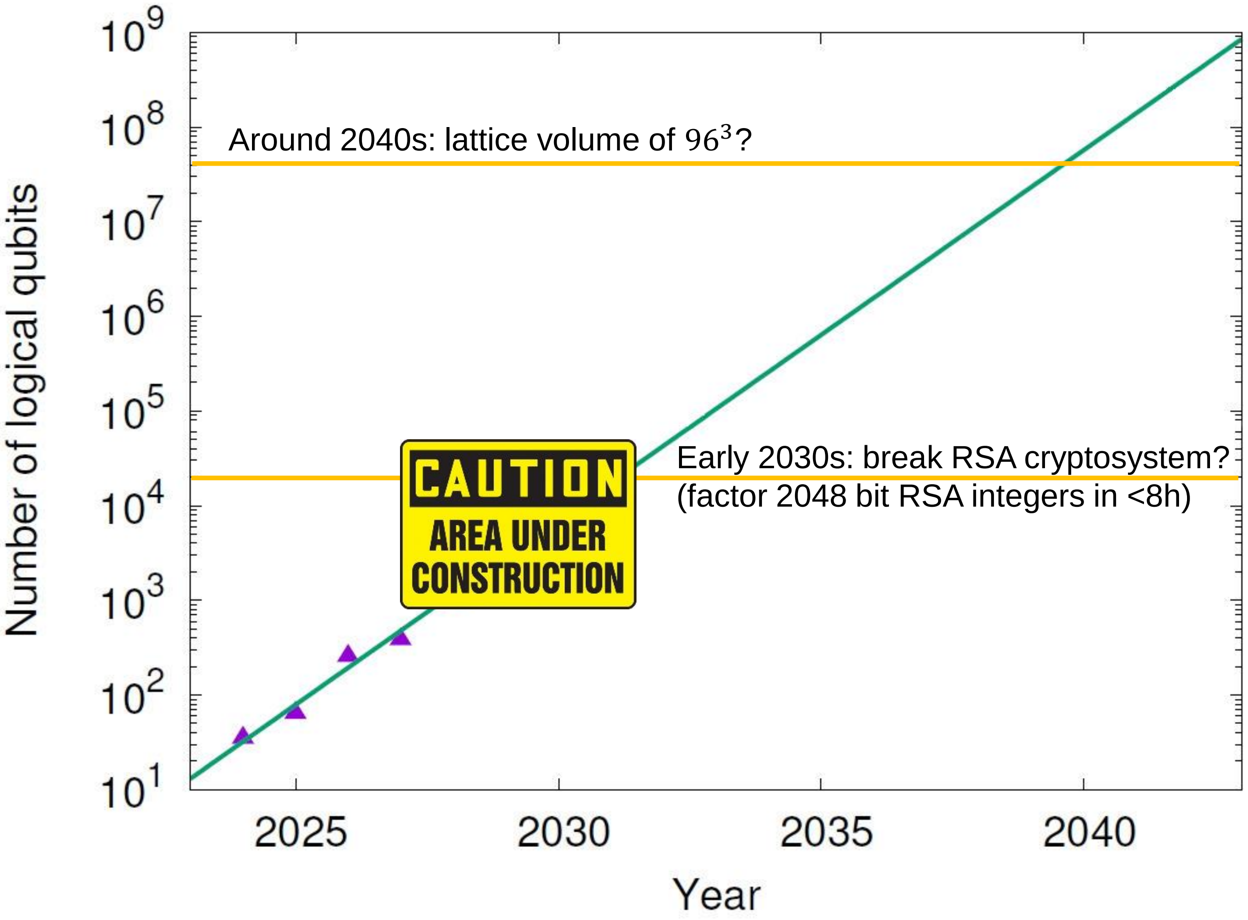
Quantum

new and quickly progressing

→ high risk, high gain



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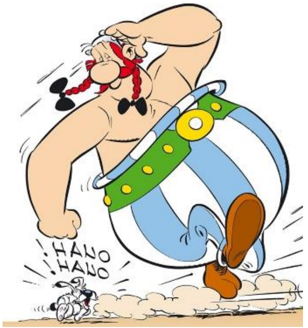
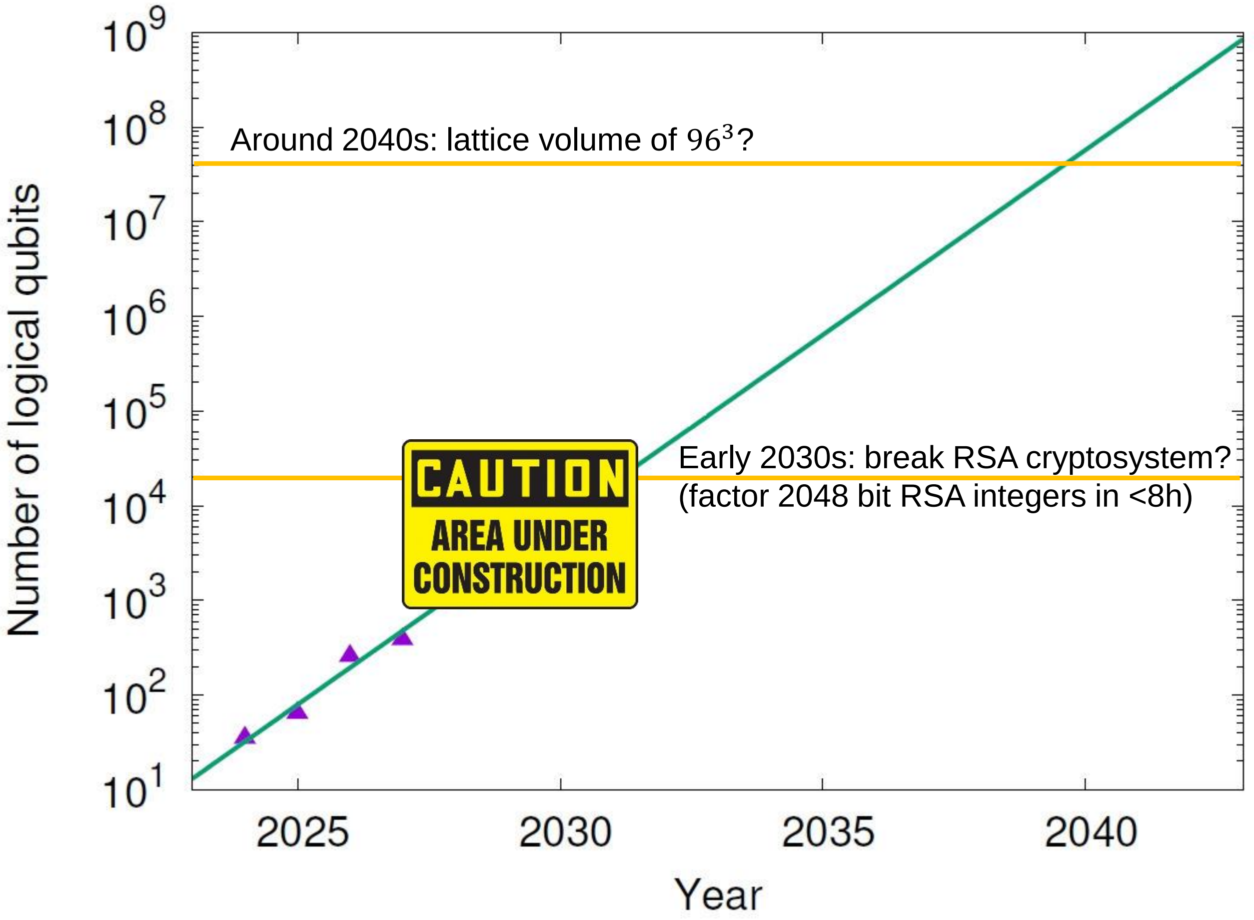
Classical

more established but limited

→ solid, promising





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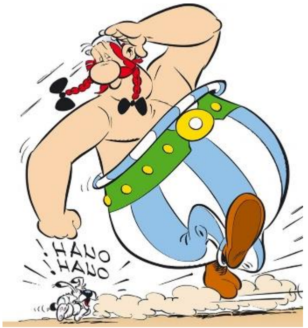

Combination

→ quantum-classical algorithms

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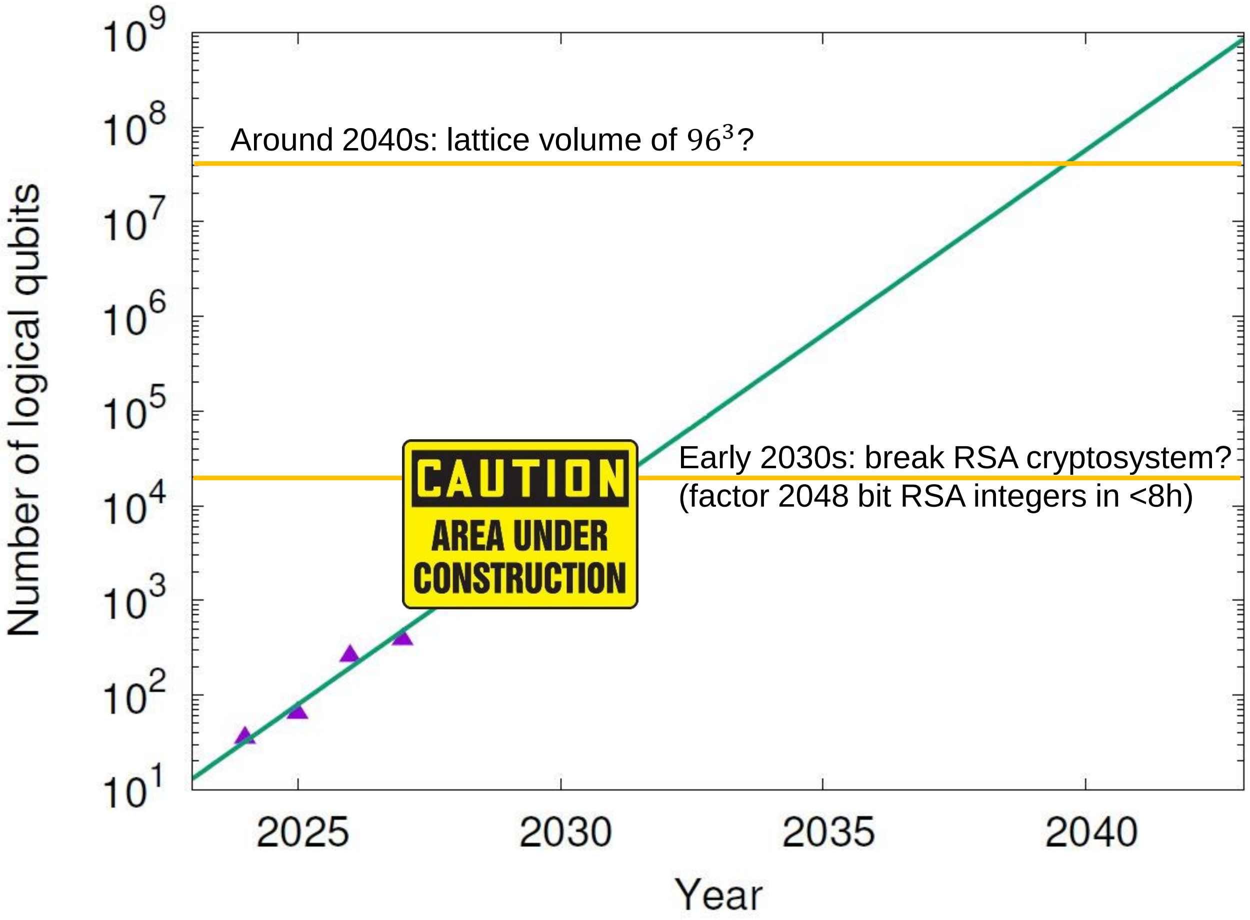

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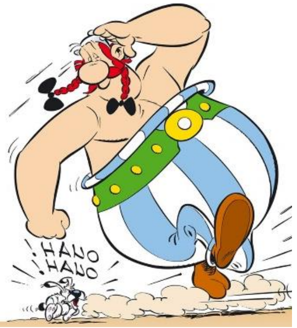

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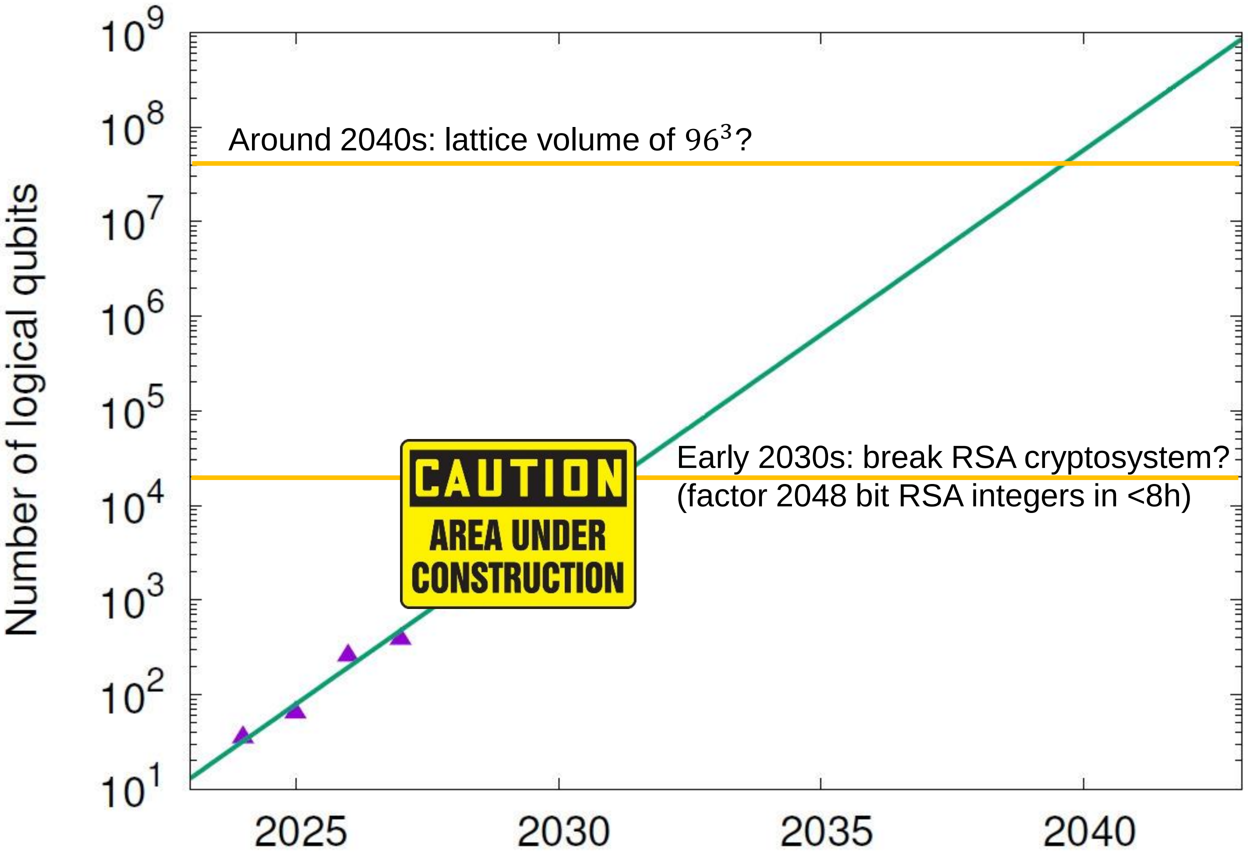


Thank you for listening!

Do you have any questions?

Backup: details of Bose-Hubbard quantum simulation

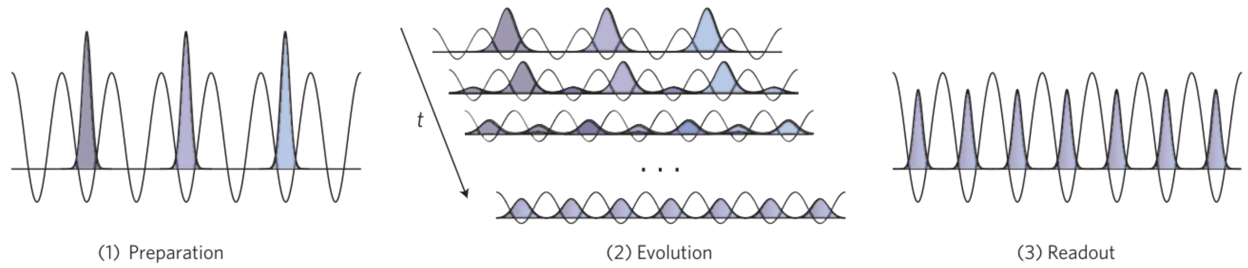
Classical versus quantum simulation | **Why is(n't) classical computing enough?**

Bose-Hubbard Hamiltonian

$$\mathcal{H} = \sum_j -J\left(\hat{a}_j^\dagger \hat{a}_{j+1} + h.c.\right) + \frac{U}{2}\hat{n}_j(\hat{n}_j - 1) + \frac{K}{2}\hat{n}_j j^2$$

Experimental goal [1]

Simulate quantum *tunneling* from even to odd sites



Experimental setup

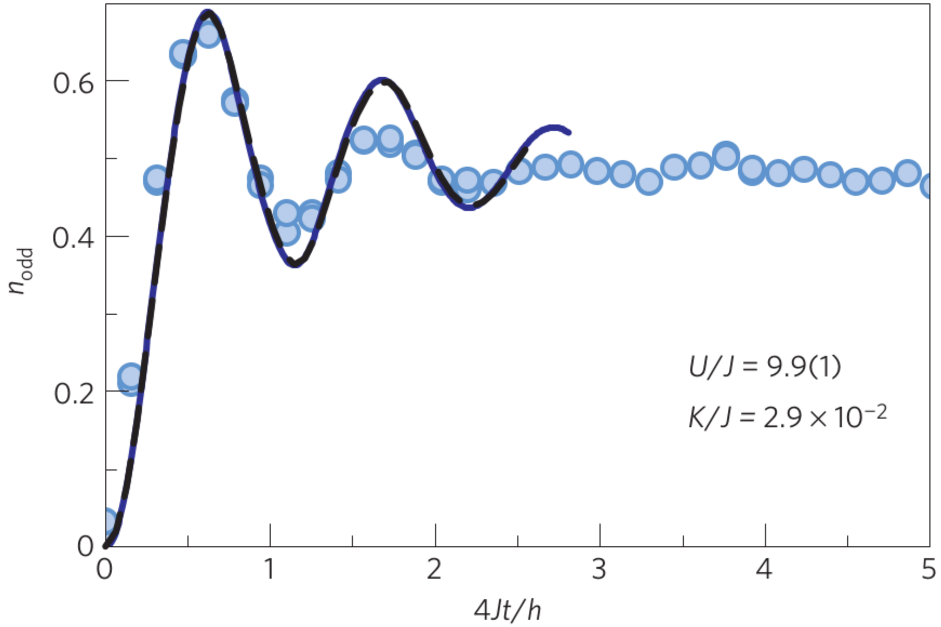
Quantum simulation: ultracold atoms in optical lattice

Classical benchmark: tensor networks (MPS-based)

[1] Trotzky et al. (2012)

Experimental results

Classical (line) vs. quantum (circles) simulation



this dynamical quantum simulator outperforms any continuous-time numerical simulation, for which the calculational effort increases with t. Simulation methods on classical computers, such as matrix-product state based t-DMRG used here, suffer from an extensive increase in entanglement entropy which limits the relaxation times accessible in the calculations[24,25].

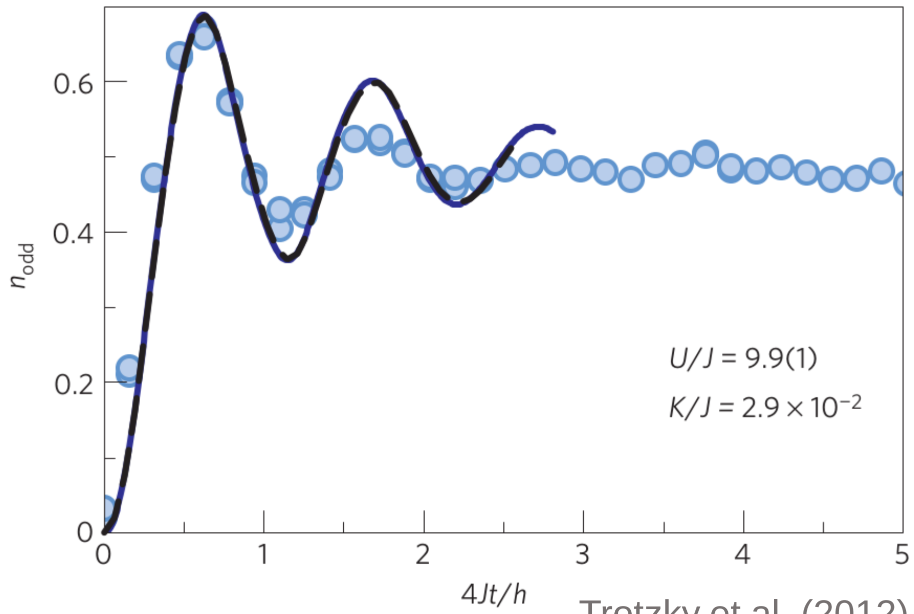
Backup: analog versus digital quantum computers

Analog	Digital

Analog

Concept

- Use controllable quantum system to simulate the behavior of another quantum system
- Continuous time evolution
- Usually non-universal

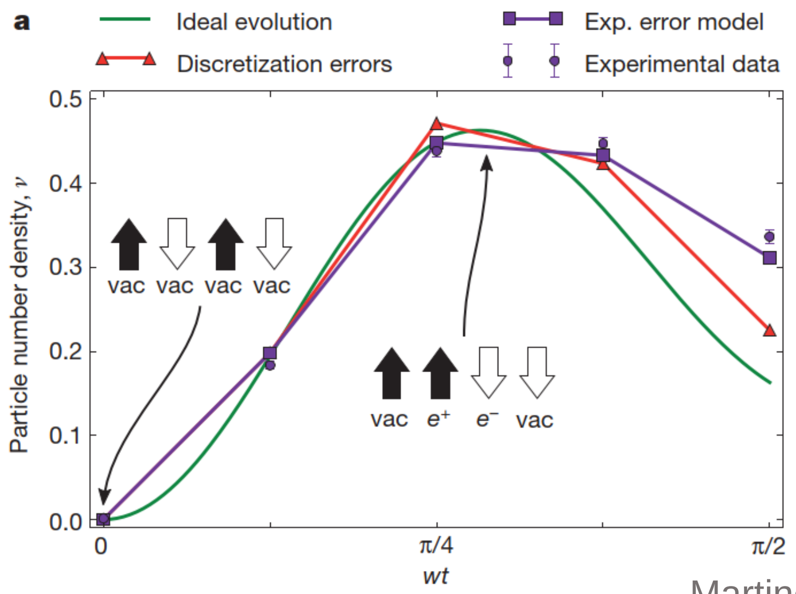


Trotzky et al. (2012)

Digital

Concept

- Construct set of logical gates onto qubits
- Discrete time evolution
- Usually universal



Martinez et al. (2016)

Backup: how to measure the energy in VQE?

Example: massless Schwinger model

Original Hamiltonian

$$\mathcal{H} = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \mathrm{h.c.}\right) + \frac{ag^2}{2}\sum_{n=0}^{N-2} F_n^2$$

with $\theta_n = -aqA_n^1$, $gF_n = E_n$, $[\theta_n, L_m] = i\delta_{nm}$, $\theta_n \in [0,2\pi]$

Eliminate $\boldsymbol{\theta_n}$

$\phi_n^\dagger e^{i\theta_n}\phi_{n+1} \to \phi_n^\dagger \phi_{n+1}$ from gauge transformation:

$\phi_n \to \left(\prod_{k=0}^{n-1} e^{-i\theta_n}\right)\phi_n$ and $\phi_n^\dagger \to \phi_n^\dagger\left(\prod_{k=0}^{n-1} e^{i\theta_{n-k}}\right)$

Eliminate $\boldsymbol{F_n}$

$F_n = \sum_{k=0}^{n} Q_k$ from solving Gauß law (for OBC):

$F_n - F_{n-1} = Q_n\ \forall n$, where $Q_n = \phi_n^\dagger\phi_n - \frac{1}{2}[1-(-1)^n]$

[1] Banks et al. (1976), Hamer et al. (1997)

Mapping the model to qubits

Dimensionless spin Hamiltonian [1]

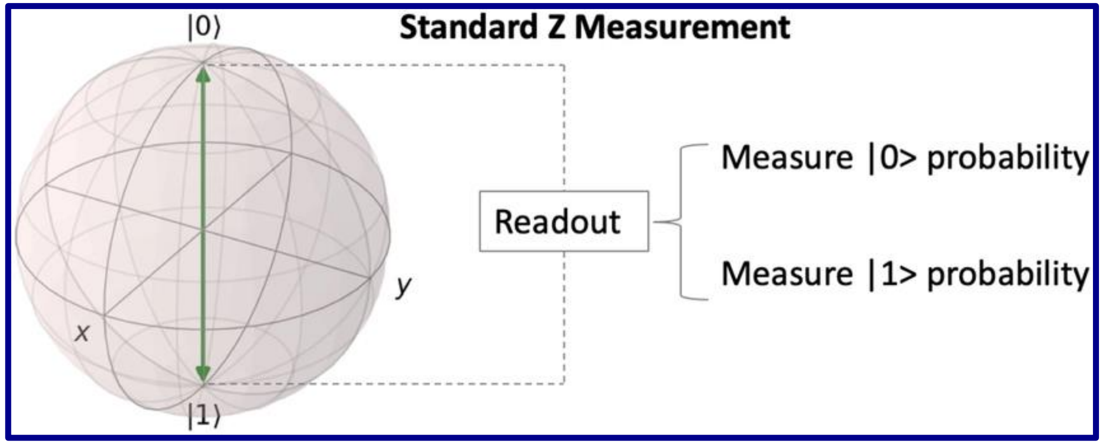
$$\mathcal{H} = x\sum_{n=0}^{N-2}\left(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right) + \frac{1}{2}\sum_{n=0}^{N-2}\left\{\sum_{k=0}^{n}\left[(-1)^k + \sigma_k^z\right]\right\}^2$$

from mapping $\phi_n^\dagger\phi_{n+1} \to \sigma_n^+\sigma_{n+1}^-$ and $\phi_n^\dagger\phi_n \to \frac{1}{2}(\sigma_n^z + \mathbb{I})$

Quantum computer

Measurement of $\langle\psi|\boldsymbol{O}|\psi\rangle$ with $\boldsymbol{O} \in \{\mathbb{I}, \sigma^z\}^{\otimes N}$

$\mathcal{H} = \sum_k h_k U_k^* \boldsymbol{O_k} U_k$ with $U_k^* \boldsymbol{O_k} U_k \in \{\mathbb{I}, \sigma^x, \sigma^y, \sigma^z\}^{\otimes N}$



Gokhale et al. (2020)

Backup: quantum volume

Concept	Timeline

Motivation

Number of *noisy* qubits: no good performance measure

New performance measure

Measure capabilities and error rates of quantum device

IBM's definition

$$\log_2 V_Q = \arg\max_{n \leq N}\{\min[n, d(n)]\}$$

Example

Successfully run circuit of depth $d = 8$ on $n = 8$ qubits: quantum volume is $V_Q = 2^8 = 256 \rightarrow$ size of state space

"Success"

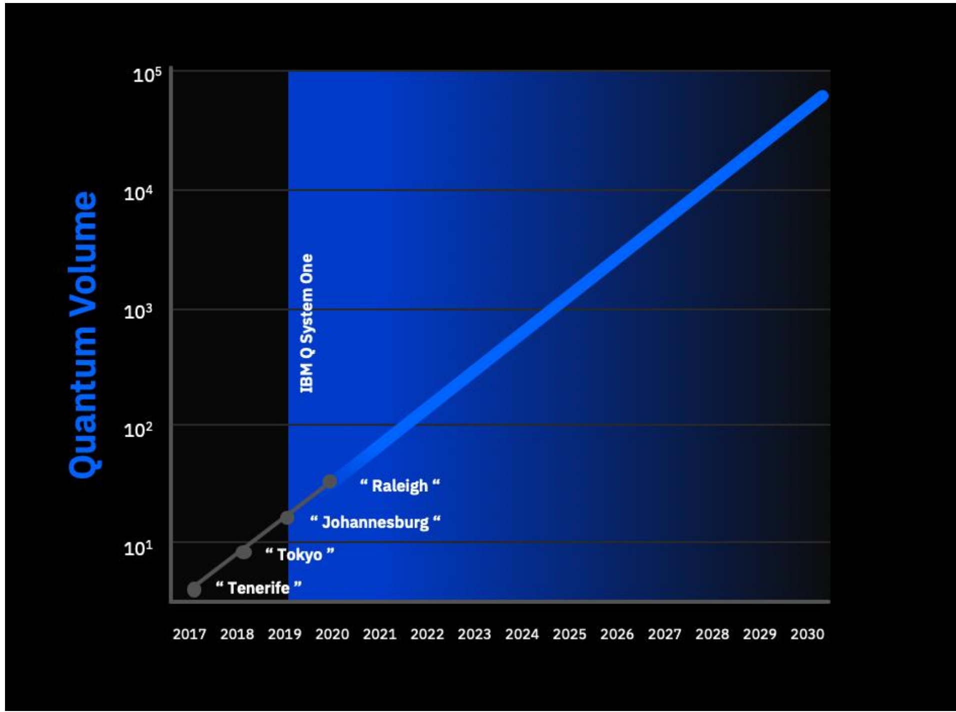
Most likely outputs of the circuit are computed correctly 67% of the time with a 2σ confidence interval

Last three years

Early 2020: $V_Q = 32$ (IBM) for $d = 5$, $n = 5$

Early 2021: $V_Q = 512$ (Honeywell) for $d = 9$, $n = 9$

Early 2022: $V_Q = 4096$ (Quantinuum) for $d = 12$, $n = 12$



Chow, Gambetta (2020)

Backup: preparing for overcoming sign problems in 3+1D

Example: 3+1D compact U(1) theory with θ-term

Goal

Simulate phase transition at $\theta = \pi$ and large $g = \beta^{-1/2}$

Analytical results

Derivation of Hamiltonian lattice θ-term:

$$\theta Q = -\frac{ig^2\theta}{8\pi^2 a} \sum_{n,i,j,k,b} \varepsilon_{ijk} \mathrm{Tr}\left[E^b_{n,i}\lambda^b \left(U_{n,jk} - U^\dagger_{n,jk}\right)\right]$$

Numerical results

Unlike in QCD, transition in U(1) might be not of first order

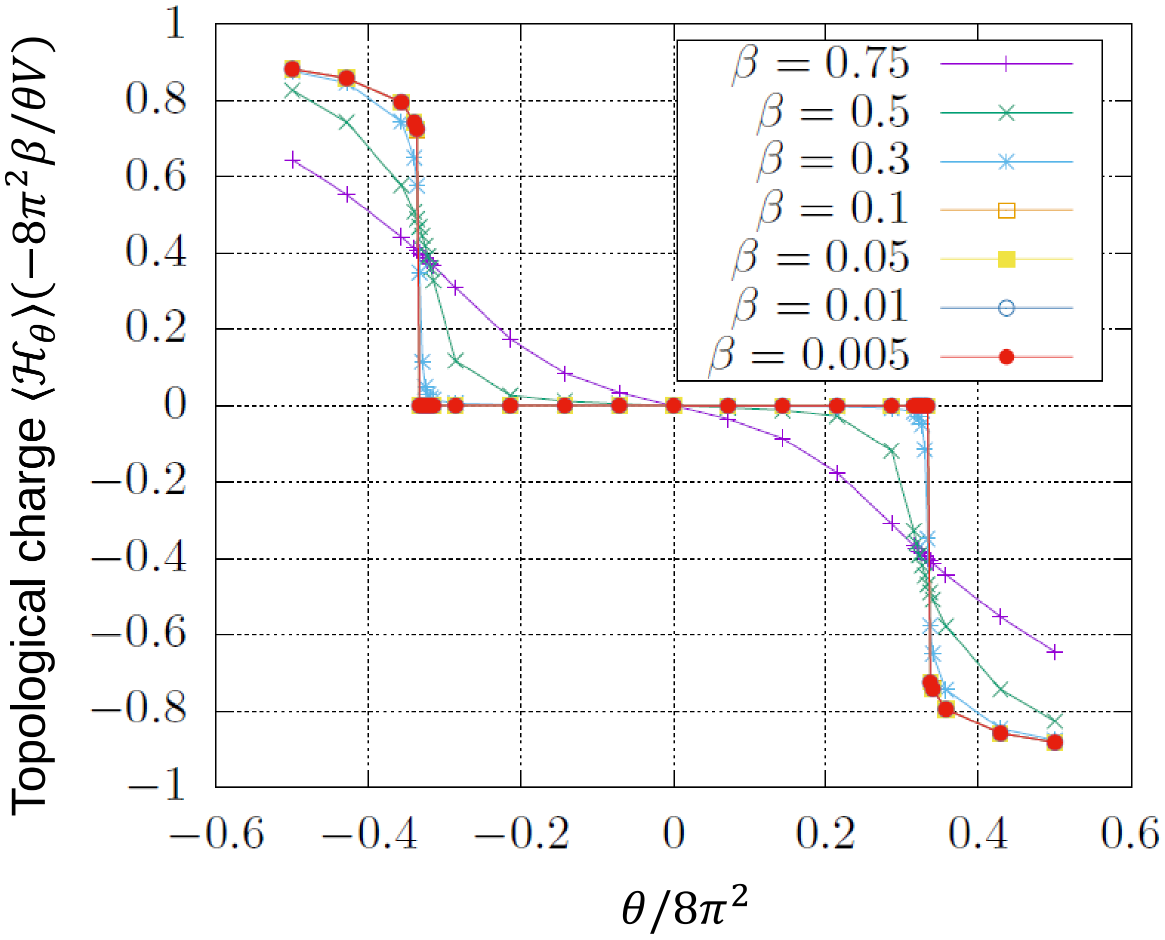
Near-future outlook

Larger-volume simulation with 3+1D tensor networks

Far-future outlook

First quantum simulation of 3+1D θ-term

Details: numerical ED results for single cube



Kan, LF, Kühn, Dellantonio, Zhang, Haase, Muschik, Jansen (2021a,2021b)