A REVIEW ON GLUEBALL HUNTING

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Introduction

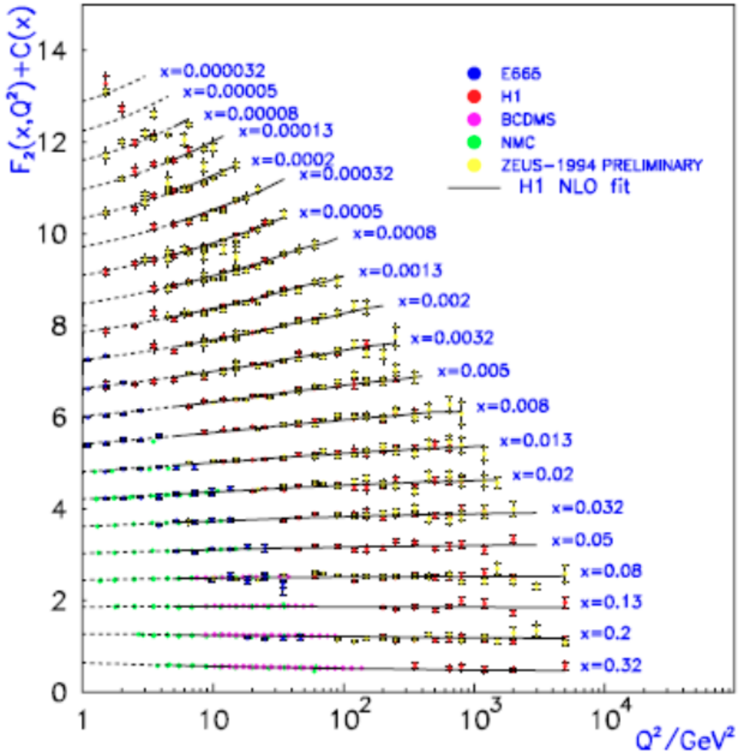
Quantum ChromoDynamics

$$\mathscr{L} = -\frac{1}{g_0^2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_{f=u,d,s,\ldots} \bar{q}_f(\imath \not{D} + m_f)q_f$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + {\color{red} f^{abc} G^b_\mu G^c_\nu}$$

It is a theory of quarks and gluons:

- → Very successfull at high-energies (DIS, Jets,...)
- → Running coupling: No perturbation theory (PT) at low-energies.



Introduction

Glueballs

- → Important footprint in literature (~ 1000)
- → Theoretical predictions mostly consistent.
- → Experimental detection challenging: overlapped resonances.
- → Essential for understanding of QCD.

Fritzsch and Gell-Mann, 1972; Fritzsch and Minkowski, 1975

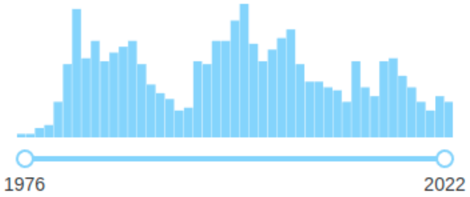


Figure: "find t glueballs OR t glueballs"

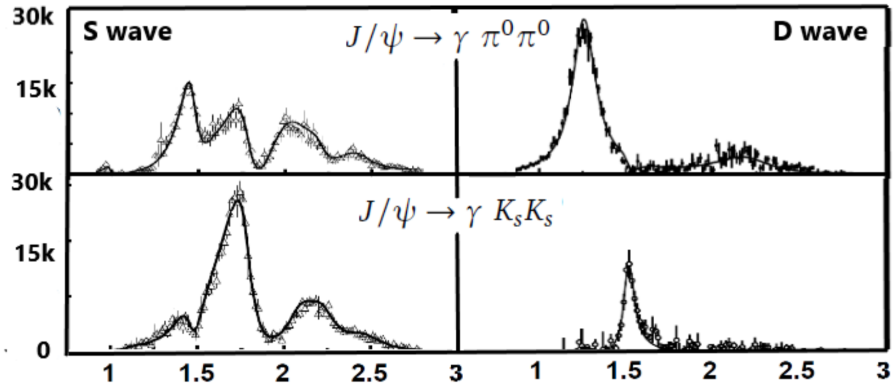


Figure: The $J/\psi \to \gamma\pi\pi, \gamma K\bar{K}$ as a function of the invariant $\pi\pi$ of KK energy. From Klempt, 2022.

Not only QCD

- → Important for general non-Abelian gauge theory.
- → Present in many scenario on BSM, Dark Matter[a],...

[a] Carenza et al., 2022.

Introduction

Glueball Hunting

The determination of

$$J^{PC}, \qquad m(J^{PC}), \qquad \Gamma(J^{PC})$$

from theory and experiment.

Outline of this talk

- → Brief overview of QCD inspired models: MIT bag model, QCD sum rules
- → Lattice status: spectrum and decays from quenched and unquenched simulations
- → Introduction to experiments: glue rich processes

Reviews

- → “The Physics of Glueballs” Mathieu, Kochelev, and Vento, 2009
- → “The Status of Glueballs” Ochs, 2013
- → “Glueballs as the Ithaca of meson spectroscopy: From simple theory to challenging detection” Llanes-Estrada, 2021
- → “The Experimental Status of Glueballs” Crede and C. A. Meyer, 2009

Examples of Constituent models

The simplest model

Let

$$|G\rangle = \hat{G}|\Omega\rangle$$

be a pure glueball state. Then $\hat{G}$ is:

- → a color-singlet (confinement)
- → Bose symmetric (gluon is spin-1)

and[a] for 2, 3, ... gluon states

$$(2g): \quad B^a_\mu B^a_\nu, \quad (3g): \quad f_{abc} B^a_\mu B^b_\nu B^c_\sigma, \, d_{abc} B^a_\mu B^b_\nu B^c_\sigma$$

Since B^a_μ is massless[b],

$$(2g): \quad J^{PC} = 0^{++}, 2^{++}, 0^{-+}, \ldots$$

$$(3g): \quad J^{PC} = 1^{+-}, 1^{--}, 3^{--}, \ldots$$

→ exotic states
→ $\hat{G}$ flavour-singlet: Γ flavour-agnostic

[a] Fritzsch and Minkowski, 1975.
[b] Landau, 1948; C.-N. Yang, 1950.

MIT Bag model[1]

From[a] the spherical boundary conditions

$$(TE): P, C = (-1)^l, +, \quad (TM): P, C = (-1)^{l+1}, -$$

For n gluons

$$|G\rangle = |(TE)^{n_E}(TM)^{n_E}\rangle, \quad n_e + n_M = n$$

$$(2g): \quad J^{PC} = 0^{++}, 2^{++}, 0^{-+}, 2^{-+}, \ldots$$

$$(3g): \quad J^{PC} = 0^{+-}, 1^{+-}, 1^{--}, 3^{+-}, \ldots$$

From other hadrons: $B \sim 0.1$ GeV, thus,

$$\begin{aligned} m(\mathrm{TE})^2 &= \quad 960 \text{ MeV}, \\ m(\mathrm{TM})^2 &= \quad 1590 \text{ MeV}, \\ m(\mathrm{TETM}) &= \quad 1290 \text{ MeV}. \end{aligned}$$

[a] Johnson, 1975.

A summary of phenomenological models

Many other examples...

- → Potential models(Cornwall, 1982)
- → Flux-tube models(Isgur and Paton, 1983; Isgur and Paton, 1985)
- → ...

Summary

- → No quarks involved.
- → Each model can be refined “at will".
- → (almost) All agree that the lightest states are found for

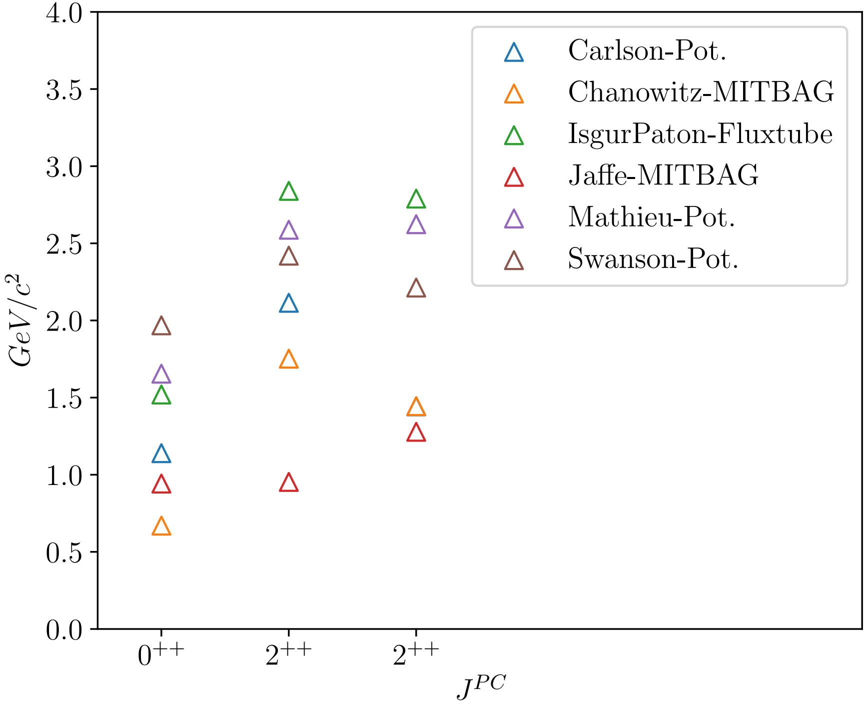
$$J^{PC} = 0^{++}, 2^{++}, 0^{-+}$$

with

$$m(0^{++}) \sim 1-2 \text{ GeV},$$

$$m(0^{-+}) \sim 1.5-2.5 \text{ GeV}$$

$$m(2^{++}) \sim 2.0-3.0 \text{ GeV}$$



Analytical techniques

QCD Sum Rules

$$\Pi(q^2) = \int \mathrm{d}^4x\, e^{\imath q\cdot x} \langle\Omega| T\{J(x)J(0)\} |\Omega\rangle \,,$$

where, depending on the desired quantum numbers

$$J_{\mathrm{S}}(x) = \alpha_s G^a_{\mu\nu} G^a_{\mu\nu}, \quad J_{\mathrm{PS}}(x) = \alpha_s G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

$$J_{\mathrm{T},\mu\nu}(x) = -G^a_{\mu\alpha} G^a_{\alpha\nu} + \frac{g^{\mu\nu}}{4} G^a_{\beta\alpha} G^a_{\alpha\beta}$$

Using the OPE at short distances and setting

$$\frac{1}{\pi}\mathrm{Im}\,\Pi(q^2) = \rho(s) = \sum_i f^2_{G_i}\,\delta(s - m^2_{G_i}) + \rho_{\mathrm{cont.}}(s)$$

f_{G_i} and m_{G_i} can be estimated[a]

$$m_{\mathrm{S}} \sim 1.780(170)\ \mathrm{GeV}, \quad m_{\mathrm{PS}} \sim 1.860(170)\mathrm{GeV},$$

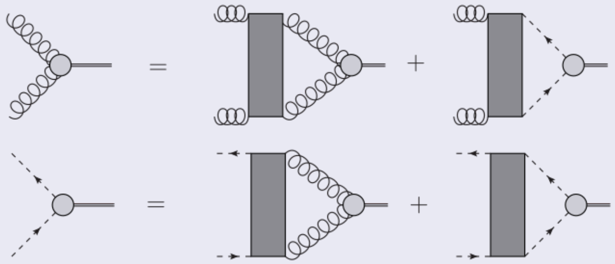
$$m_{\mathrm{T}} \sim 2.170(110)\ \mathrm{GeV}$$

and in a different setting[b], $\Gamma(G \to PS) \lesssim 100$ Mev.

[a]H.-X. Chen, W. Chen, and Zhu, 2021.
[b]Narison, 2009.

BS equations

By finding a self-consistent truncation to the BSE equations[a]



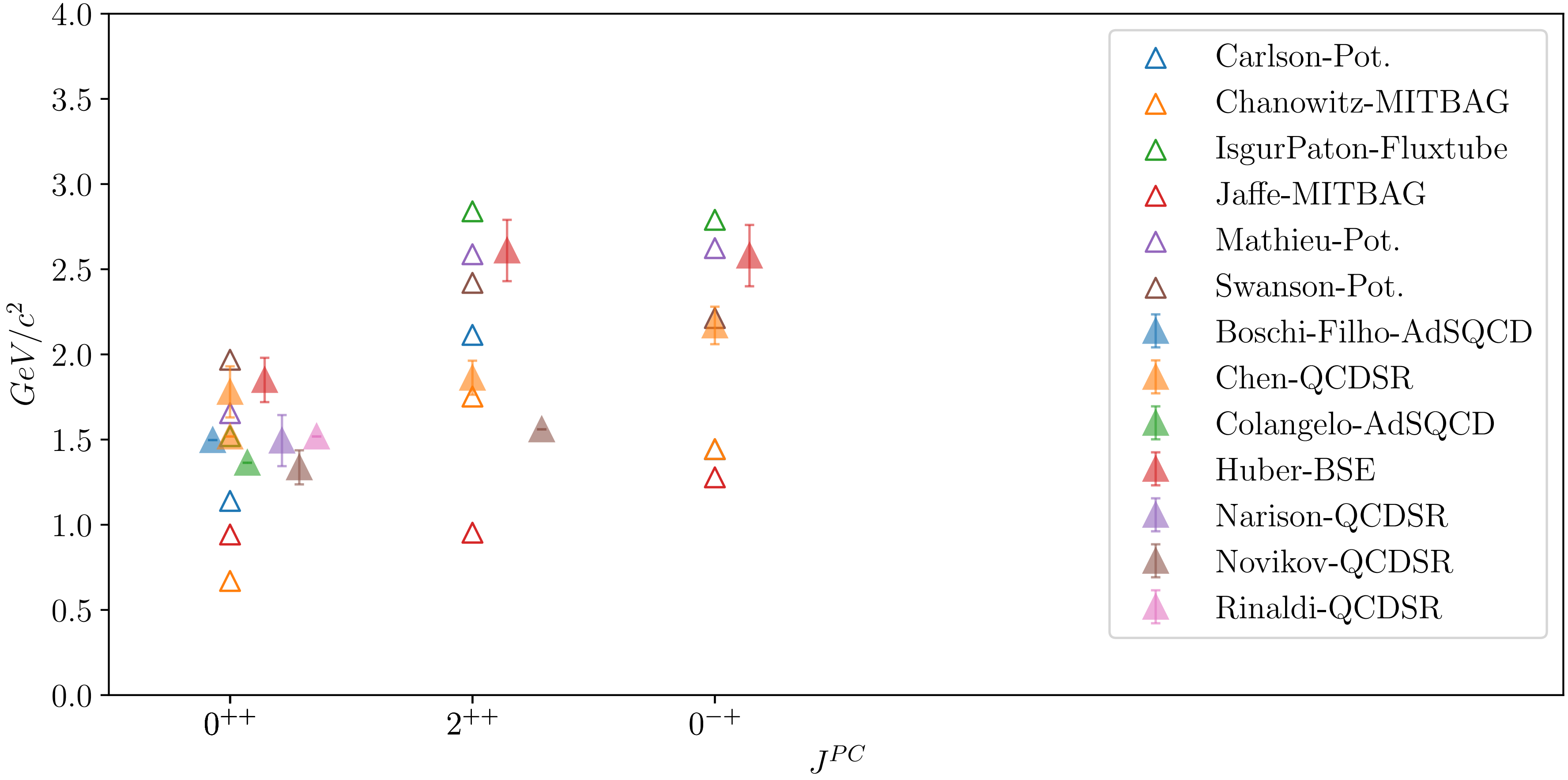
and to the Schwinger-Dyson equations for the gluon and ghost correlation functions, fixing an overall scale from lattice data allows to obtain

$$m_{\mathrm{S}} \sim 1.850(130)\ \mathrm{GeV}, \quad m_{\mathrm{PS}} \sim 2.610(180)\ \mathrm{GeV}$$

$$m_{\mathrm{T}} \sim 2.580(180)\ \mathrm{GeV}$$

[a]Huber, Fischer, and Sanchis-Alepuz, 2020.

Summary of analytical and phenomenological models



Wishlist for glueballs

→ The Spectrum of stable states: 2-point functions.

→ The decay width: phases shifts, 3-points functions

Lattice regularization

$$S = \sum_{x,\mu\nu} \beta \left(1 - \frac{1}{2N_c} \mathrm{Re\,Tr}\, U_{\mu\nu}(x)\right) + \sum_x \bar{\psi} M[U] \psi \quad (+\text{ impr.})$$

where $\beta = g_0^2/4N_c$,

$$U_{\mu\nu} = U_\mu(x)\, U_\nu(x+\hat{\mu})\, U_\mu^\dagger(x+\hat{\mu}+\hat{\nu})\, U_\nu^\dagger(x+\nu), \quad U_\mu(x) = e^{ia\int \mathrm{d}s A_\mu(s)}$$

and $M[U]$ is the fermionic matrix.

Observables

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}\, U)\ \det M[U] e^{-S_{\mathrm{YM}}},$$

where

$$Z = \int \mathcal{D}[U] \det M[U] e^{-S_{\mathrm{YM}}},$$

Space-time symmetries

Subduced representations of $T \rtimes O_h$

→ IRREPs of O_h:

$$A_1^\pm, A_2^\pm, E^\pm, T_1^\pm, T_2^\pm.$$

→ IRREPs of T: $\exp(\pm\imath \frac{2\pi}{L} n)$ with $n \in [0, L)$.

→ Correspondence with J

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

Estimation of the spectrum

Spectrum calculations

$$\mathscr{C}(t) = \langle \Phi(t)\Phi(0)\rangle = \sum_n |c_n|^2 e^{-tE_n} \longrightarrow |c_0|^2 e^{-E_0 t}, \qquad t \gg 1/E_0$$

- → Φ are $\vec{p} = 0$ color-singlet operators.
- → $c_n = \langle n|\Phi(0)|0\rangle$ are the overlaps.

Signal-to-noise ratio

At large euclidean time, signal/noise is exponentially decaying:

- → Maximize the overlap $|c_0|^2$: Variational method[a]
- → Use variance reduction at large-t: Multilevel[b].

[a] Wilson, 1974; Ishikawa, M. Teper, and Schierholz, 1982.
[b] H. B. Meyer, 2003; Della Morte and Giusti, 2011.

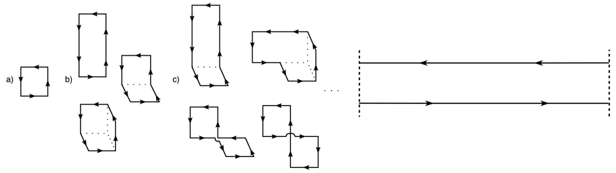


Figure: Example of contractible(left) and non-contractible(right) paths. From Biagio Lucini, Rago, and Rinaldi, 2010

Variational method

- → Define a *variational basis* of N operators $\{\Phi_i\}$ in each symmetry channel.
- → Compute for all t the $2N \times 2N$ matrix $\mathscr{C}_{ij}(t) = \langle \Phi_i(t)\Phi_j(0)\rangle$
- → Find $\{v_i\}$ that minimize the quantity

$$m_{\text{eff}}(\tau) = \ln \frac{v_i \mathscr{C}_{ij}(\tau) v_j}{v_i \mathscr{C}_{ij}(\tau - 1) v_j}$$

  a $\tau = 1$.
- → Fit

$$\mathscr{C}_{ii}(t) = |c_i|^2 \cosh(m_i t - N_L/2)$$

  to the data, using m_i and c_i as fitting parameters.

Continuum and large volume limits

After reaching the infinite volume limit, the continuum spectrum can be extrapolated to $a = 0$,

$$\frac{m}{\mu}(a) = \frac{m}{\mu}(0) + ca^2\mu + \ldots$$

where μ is come physical scale.

Quenched calculations

Quenched systems

- → Center symmetry. → No sea quarks, $\det M[U] = 1$.
- → Very different from physical Glueballs: no mixing, no decays
- → The spectrum was calculated in all the available channels in the infinite volume and continuum limits.

Technical details

- → HB+OR algorithms: large statistic, high precision.
- → Pure-glue variational basis
- → Poor overlap without blocked and smeared operators[a]
- → Anisotropic lattice, improved actions[b]
- ⚠ All the states in the channel propagate[c]: scattering, di-torelons, ...
- ⚠ The effects of topological freezing[d] as $a \to 0$. (talk by C. Bonanno today @14.20).

[a] M. Teper, 1987.
[b] Morningstar and Peardon, 1997.
[c] Biagio Lucini, Rago, and Rinaldi, 2010.
[d] Bonanno et al., 2022.

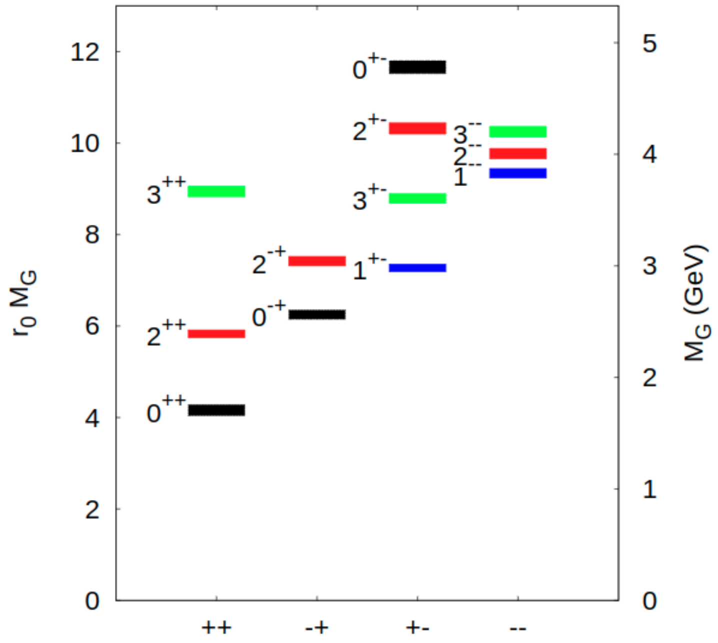


Figure: The quenched spectrum, taken from F. Chen et al., 2021. It is consistent with Morningstar and Peardon, 1999

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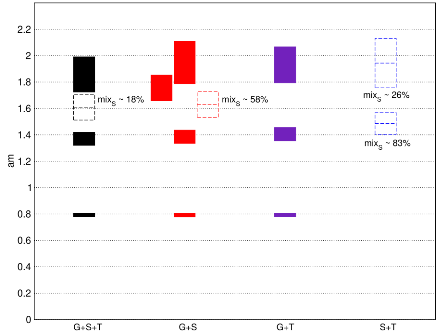

Figure: The effect of including different kinds of operators in the variational basis. From Biagio Lucini, Rago, and Rinaldi, 2010.

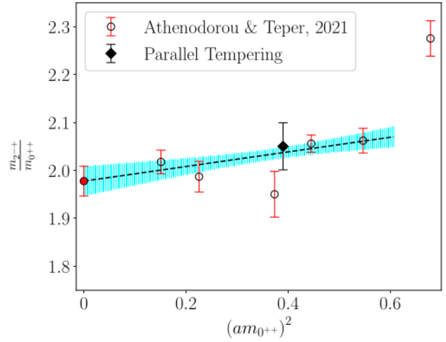

Figure: Effect of top. freezing on m_T/m_S. From Bonanno et al., 2022.

Results

- → Estimates obtained with different techniques are compatible.
- → The lightest states are the scalar, tensor, pseudo-scalar, with[a]

$$
\begin{aligned}
m(0^{++}) &= 1475 - 1730 \text{ MeV}, \\
m(2^{++}) &= 2150 - 2400 \text{ MeV}, \\
m(0^{-+}) &= 2250 - 2590 \text{ MeV}
\end{aligned}
$$

[a] H.-X. Chen, W. Chen, Liu, et al., 2022.

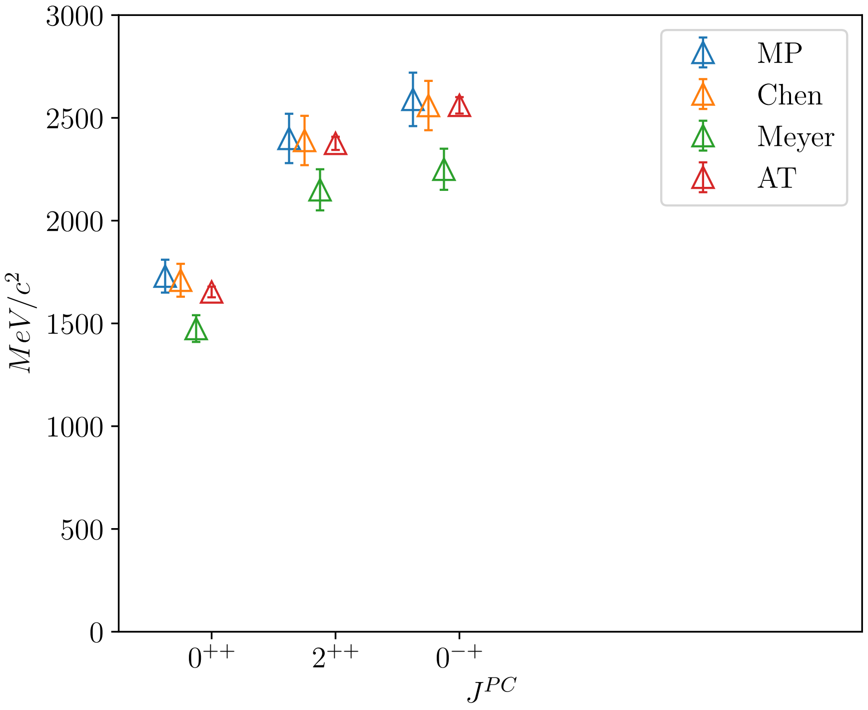

Figure: The full quenched spectrum, collection of data from H.-X. Chen, W. Chen, Liu, et al., 2022.

Quenched decays

- → Phase shifts: finite volume study[a]
- → *simplified method* $0^{++} - 2PS$ coupling λ from 3 point functions[b],

$$\Gamma_{tot}(0^{++} \to 2PS) \sim 108(29)\ \text{MeV},$$

  but potentially large systematics.
- → The decay of J/ψ to pseudo-scalar[c] and tensor[d] glueballs were studied and the form factor was computed as a function of Q^2.

[a] Luscher, 1986.
[b] Sexton, Vaccarino, and Weingarten, 1995.
[c] Gui et al., 2019.
[d] Y.-B. Yang et al., 2013.

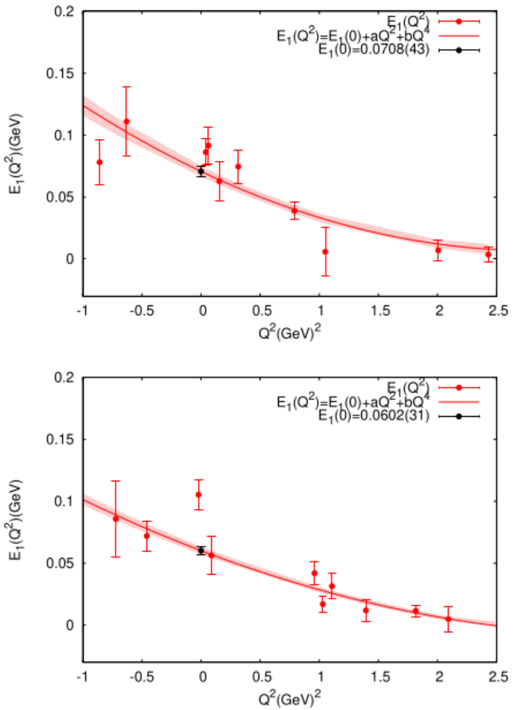

Figure: The form factor for $J/\psi \to \gamma G$ decay. From Gui et al., 2019.

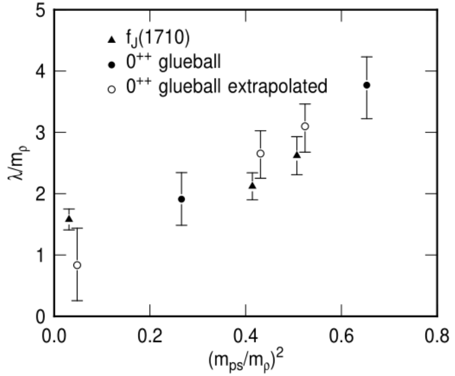

Figure: The $G - 2PS$ coupling as a function of $(m_{PS}/m_\rho)^2$ extrapolated to $PS = \pi$, K,

Quenched calculations

Other directions

- → The spectrum was also studied[a] at large-N_c
- → for $Sp(2N)$ groups[b],
- → for trace deformed actions[c]
- → Its universal features were investigated[d]

[a] B. Lucini and M. Teper, 2001; Athenodorou and Michael Teper, 2021.
[b] Bennett et al., 2021.
[c] Athenodorou, Cardinali, and D'Elia, 2021.
[d] Bennett et al., 2020; Hong et al., 2017.

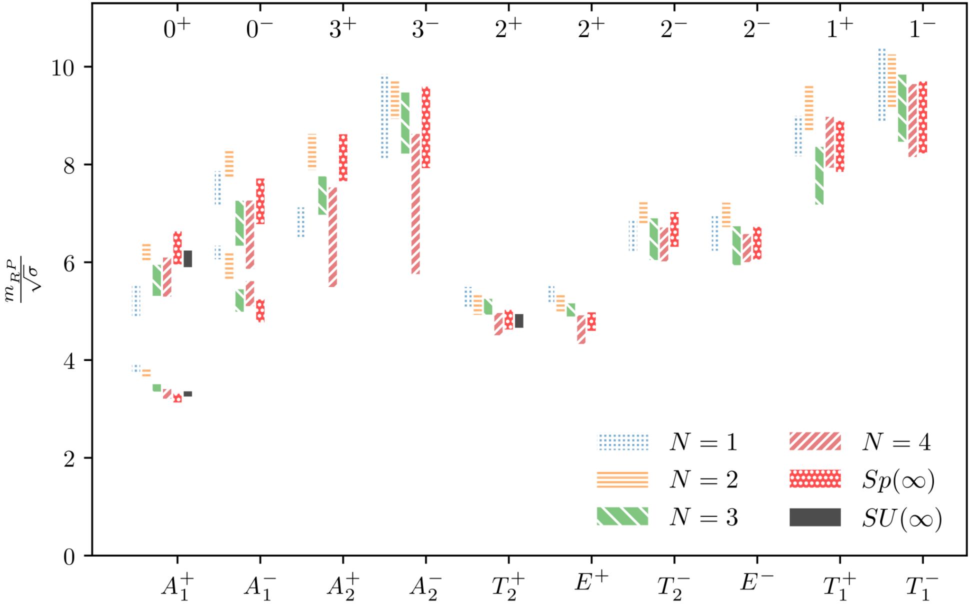

Figure: The spectrum of $Sp(2N)$ for $N = 1, 2, 3, 4$ and its large-N limit. From Bennett et al., 2021.

Technical aspects

- → Computationally more expensive
- → Many fermion discretizations: Wilson, Staggered, Twisted mass,...
- → Anisotropic lattices[a] and improved actions[b].
- → Pure-glue variational bases.

[a] Sun et al., 2018.
[b] Hart and M. Teper, 2002; Gregory et al., 2012.

The pure glueball component

- → The spectrum was obtained in $N_f = 2$ with Wilson[a] and clover improved[b] actions at $m_{PS}/m_\rho \geq 0.5$ and $a \simeq 0.1 fm$.
- → The effects of scattering states was studied[c] on $N_f = 2+1$ AQSTAD improved staggered fermions.
- → First computation at physical point, CDER at two lattice spacings F. Chen et al., 2021, $N_f = 2+1$

[a] Bali et al., 2000.
[b] Hart and M. Teper, 2002.
[c] Gregory et al., 2012.

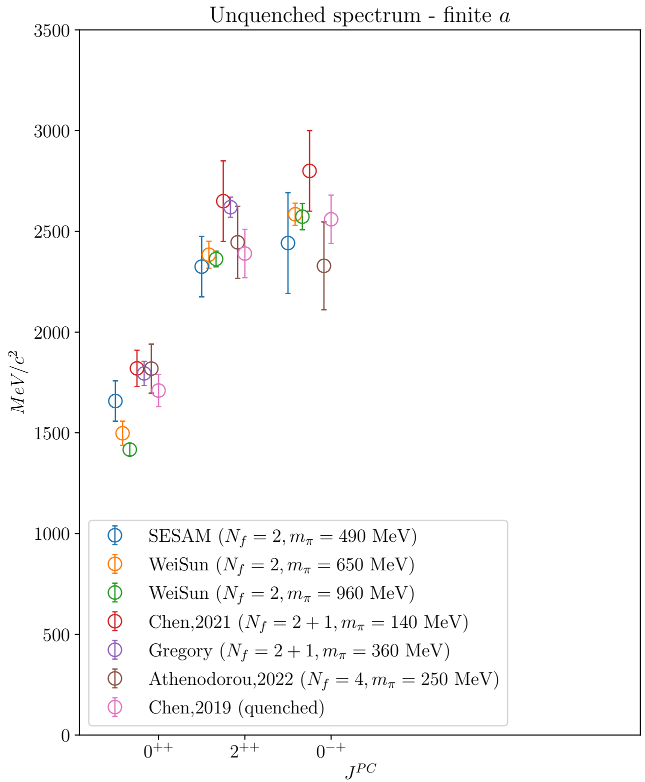

Figure: Quenched and Unquenched spectrum of glueball component at finite a.

The physical glueball

- → Fermion operators in the variational basis. bases.
- → Spectrum and mixing studied in $N_f = 2$ clover improved[a] at $a \simeq 0.13$ fm.
- → Mass suppression seems[b] to be smaller for smaller a, but still present.
- → Light scalar meson spectrum seems to be independent on the presence of pure-glue operators[c].

[a]Craig McNeile and Michael, 2001.
[b]Hart, C. McNeile, et al., 2006.
[c]Brett et al., 2020.

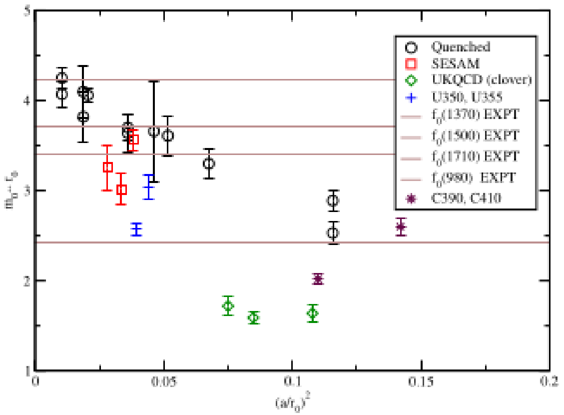

Figure: The masses of physical glueballs as a function of $(a/r_0)^2$. The value of m_π for each ensemble is not indicated. From Craig McNeile and Michael, 2001.

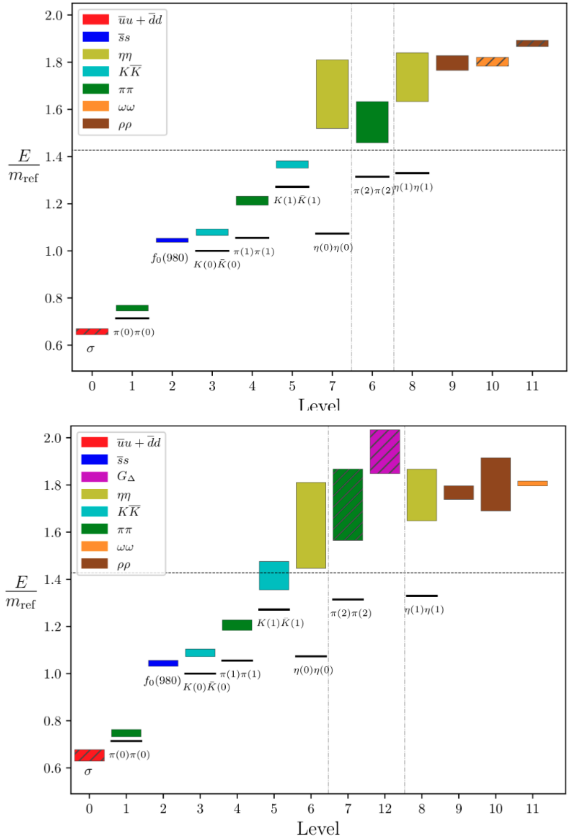

Figure: The spectrum of light hadrons with(below) and without(above) glueball operators in the variational basis. From Brett et al., 2020.

Unquenched Glueballs

The decays

- → Should use FV formalism[a].
- → A simpler method can be used when near threshold[b] but large systematics[c]

[a]Luscher, 1986.
[b]Sexton, Vaccarino, and Weingarten, 1995.
[c]Craig McNeile and Michael, 2001.

Recent studies

- → Distillation profiles were optimized for glueballs[a](see talk by J.A.U Nino today @15.40)
- → The $0^{-+} - \eta/\eta_c$ mixing energy in $N_f = 2$ energy[b].
- → Interglueball potential from BSE[c]
- → Scattering cross section of $SU(2)$ glueballs using HAL QCD method[d].
- → Recently, $N_f = 4$ and $N_f = 2+1+1$ were explored (see talk by A. Athenodorou on 11Aug@9.00)

[a]Knechtli et al., 2022.
[b]Jiang, Sun, et al., 2022; Jiang, F. Chen, et al., 2022.
[c]Yamanaka, Nakamura, and Wakayama, 2022.
[d]Yamanaka, Iida, et al., 2020.

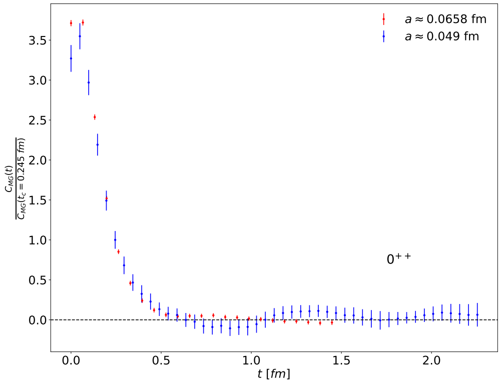

Figure: The $C_{\mathrm{MG}}(t)/C_{\mathrm{MG}}(t_c)$ correlation function where using distillation for the meson operators. From Knechtli et al., 2022.

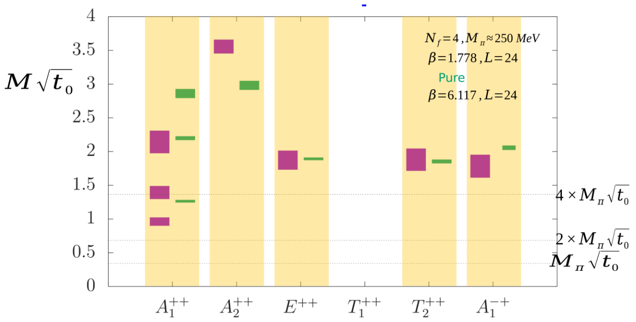

Figure: Spectrum of glueball component at $N_f = 4$, $m_{\mathrm{PS}} \sim 250$ MeV. From Athenodorou, This conference.

A summary of results from models and lattice

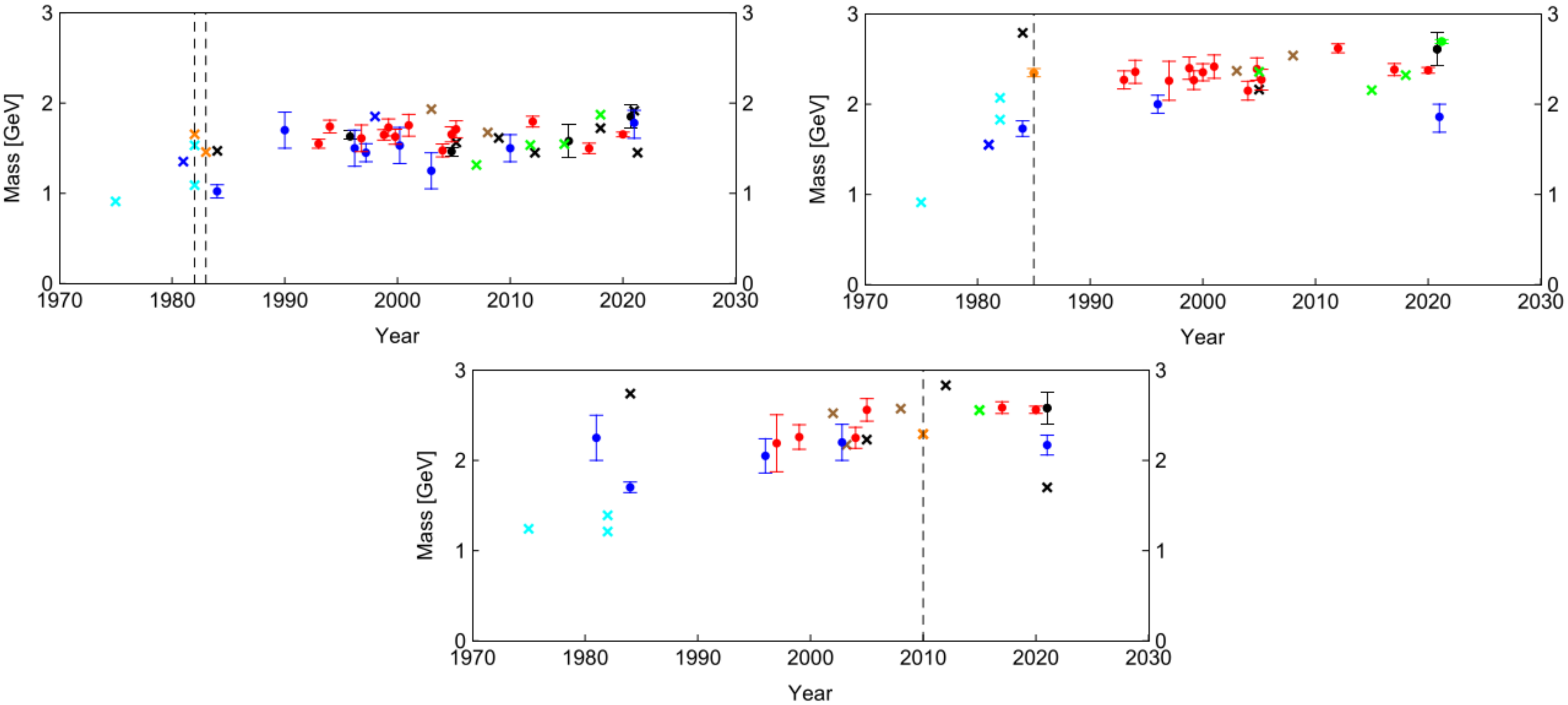

Figure: The value of the scalar(left), tensor(right) and pseudoscalar(center) masses as a function of time. From H.-X. Chen, W. Chen, Liu, et al., 2022.

Glueballs?

- The searches focus mainly on the lightest states: scalar, tensor, pseudo-scalar.
- Promising signals should come from the analysis of glue-rich processes,
- Glueballs should be supernumerary states with respect to $q\bar{q}$ multiplets.
- A priori, mixed with $q\bar{q}$ of similar mass
- Exotics: 0^{--}, 0^{+-}, 1^{-+},...

Glue rich processes

Data from multiple sources are combined to map the landscape of resonances with $I^G = 0^+$

- radiative decays of J/ψ: BESIII, ...

$$J/\psi \to \gamma hh, \quad J/\psi \to Vhh$$

- $p\bar{p}$ annihilations: Crystal Barrell, PANDA,...
- pp double Pomerons exchange: WA102,...

The peaks in $J^{PC} = 0^{++}, 2^{++}, 0^{-+}$ channels are then obtained with partial wave analysis.

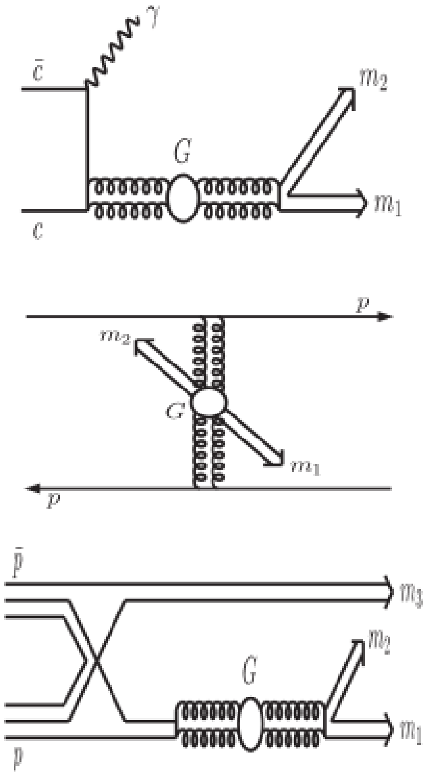

Figure: Gluon rich processes. From Mathieu, Kochelev, and Vento, 2009

The Scalar glueball

Glueball identification

- → Glueball candidates: resonances with mass is close to theoretical prediction.
- → Id: Coupled channel analysis to obtain G fraction.

The Scalar resonances

name	m(MeV)	Γ(MeV)
$f_0(500)$	$400 \to 500$	480(30)
$f_0(980)$	990(1)20	71(10)
$f_0(1370)$	$1200 \to 1500$	$200 \to 500$
$f_0(1500)$	1506(6)	112(9)
$f_0(1710)$	1704(12)	123(18)
$f_0(1770)$	1765(15)	180(20)
$f_0(2020)$	1992(16)	442(60)
$f_0(2100)$	2086^{20}_{-24}	284^{60}_{32}
$f_0(2200)$	2187(14)	~ 200
$f_0(2330)$	~ 2330	250(20)

Mixing scenarios

- → Assume one of $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ supernumerary[a],

$$M = \begin{bmatrix} M_G & f & \sqrt{2}f \\ f & M_S & 0 \\ \sqrt{2}f & 0 & M_N \end{bmatrix}, \quad f = \langle s\bar{s}|V|G\rangle = \langle n\bar{n}|V|G\rangle/\sqrt{2}, \quad |n\bar{n}\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$$

  based on branching ratio calculation yielded a scalar glueball at $m \sim 1490(30)$ MeV.

- → If a new isoscalar $f_0(1770)$ is introduced[b] then

res.	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(1770)$	$f_0(2020)$	$f_0(2100)$
G content	5(4)%	$< 5\%$	12(6)%	25(10)%	16(9)%	17(8)%

  and argued for a distributed scalar glueball at $m \sim 1865$ MeV and $\Gamma \sim 370$ MeV.

[a] Amsler and Close, 1995; Amsler and Close, 1996; Close and Kirk, 2000.
[b] Klempt and Andrey V. Sarantsev, 2022; A. V. Sarantsev et al., 2021.

The Tensor/Pseudo-scalar glueball

- → 2^{++}: Many resonances in the tensor channel, but branching ratios much lower than (quenched) lattice predictions[a].
- → 0^{-+}: Beside the resonances in PDG, the $X(1835)$, $X(2120)$, $X(2370)$, $X(2500)$ and $X(2600)$ recently observed at BES3[b].

[a] Ablikim et al., 2016.
[b] Group et al., 2020.

Conclusion and Outlook

- Quenched lattice results consistent with the predictions of phenomenological models.
  - → The lightest glueballs are the 0^{++}, 2^{++}, 0^{-+} channels.
  - → The results obtained in models and lattice seem spreaded around the values

$$m(0^{++}) \sim 1600 \text{ MeV}, \quad m(2^{++}) \sim m(0^{-+}) \sim 2400 \text{ MeV}$$

- Unquenched lattice results still at an exploratory level:
  - → Mass of the glueball components consistent with quenched results.
  - → No continuum limits and first principle calculations not at physical point.
  - → Mixing and decay to be studied with finite volume analysis.
  - → Distillation techniques and the “simplified" method could provide further results on mixing and decays.
- Despite years of searches, currently no undisputed experimental identification of the 0^{++}, 2^{++} and 0^{-+} glueballs
  - → Several different mixing scenarios seem to be simultaneously plausible
  - → Additional data data from the BES3 and PANDA experiments will lift ambiguity on several resonances and help to discriminate between these scenarios.
  - → Input from (unquenched) lattice of the decay width and mixing energy essential.

Thank you for your attention