Gluon Structure from Lattice QCD

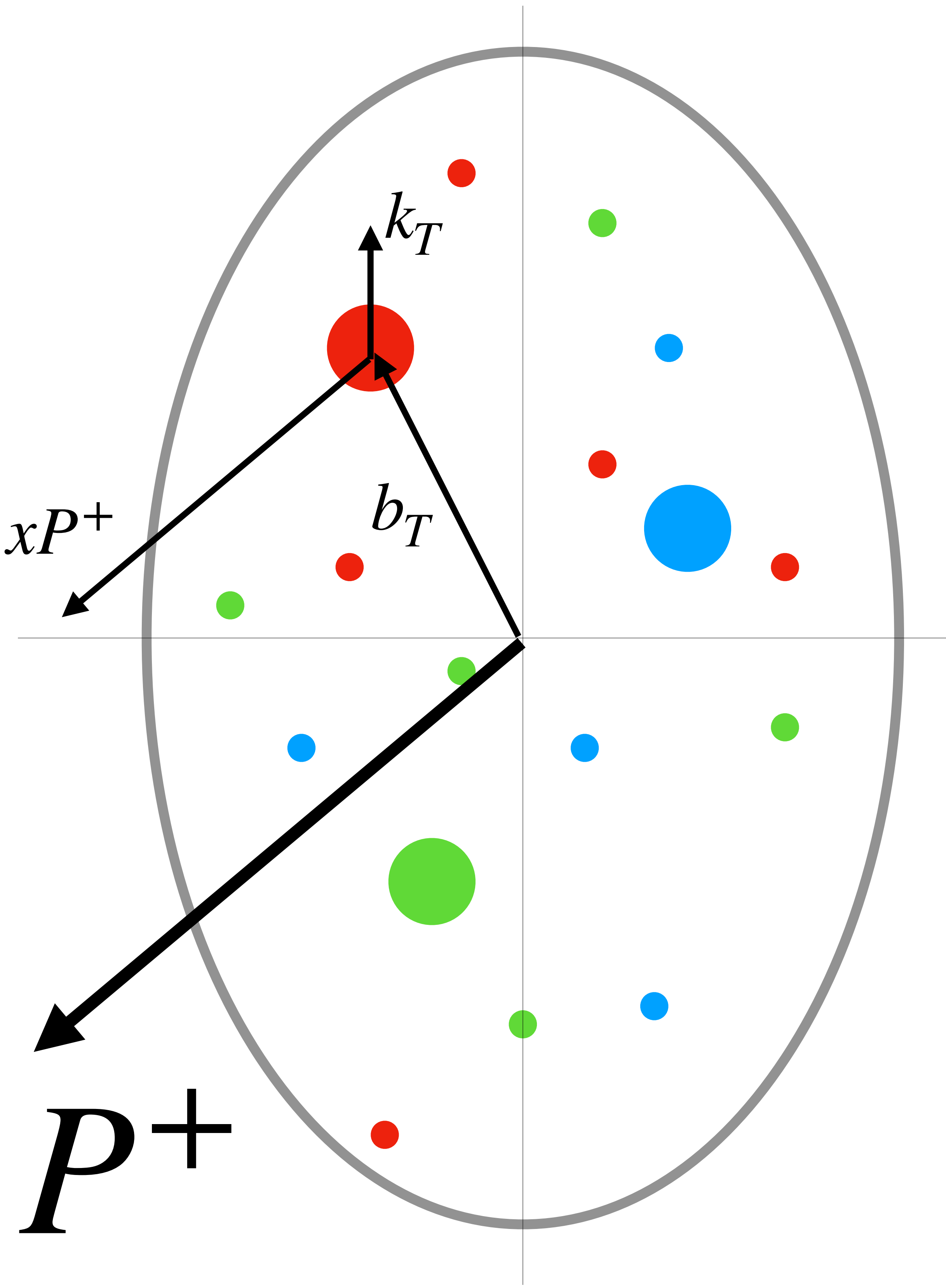
Joe Karpie (Columbia U) part of the HadStruc Collaboration

Outline

- Review Parton Structure
  - Positivity and phenomenological global analyses
- Lattice approaches to PDFs
  - Wilson line operators
  - Inverse problems
  - Techniques for reaching high momentum
- Numerical results
  - Unpolarized gluon distribution
  - Helicity gluon distribution

Parton Structure

- Internal structure of hadrons
- How are quarks and gluons distributed?
- How do they contribute to cross sections, hadron mass, spin, …..?
  - **Longitudinal** structure from momentum fraction x
  - **Transverse** structure from
    - Impact parameter b_T
    - Transverse momentum k_T



Parton Structure

For various flavors and spin combinations

Wigner Distribution/
Generalized Transverse Momentum Distribution (GTMD)

$F(x, b_T, k_T)$

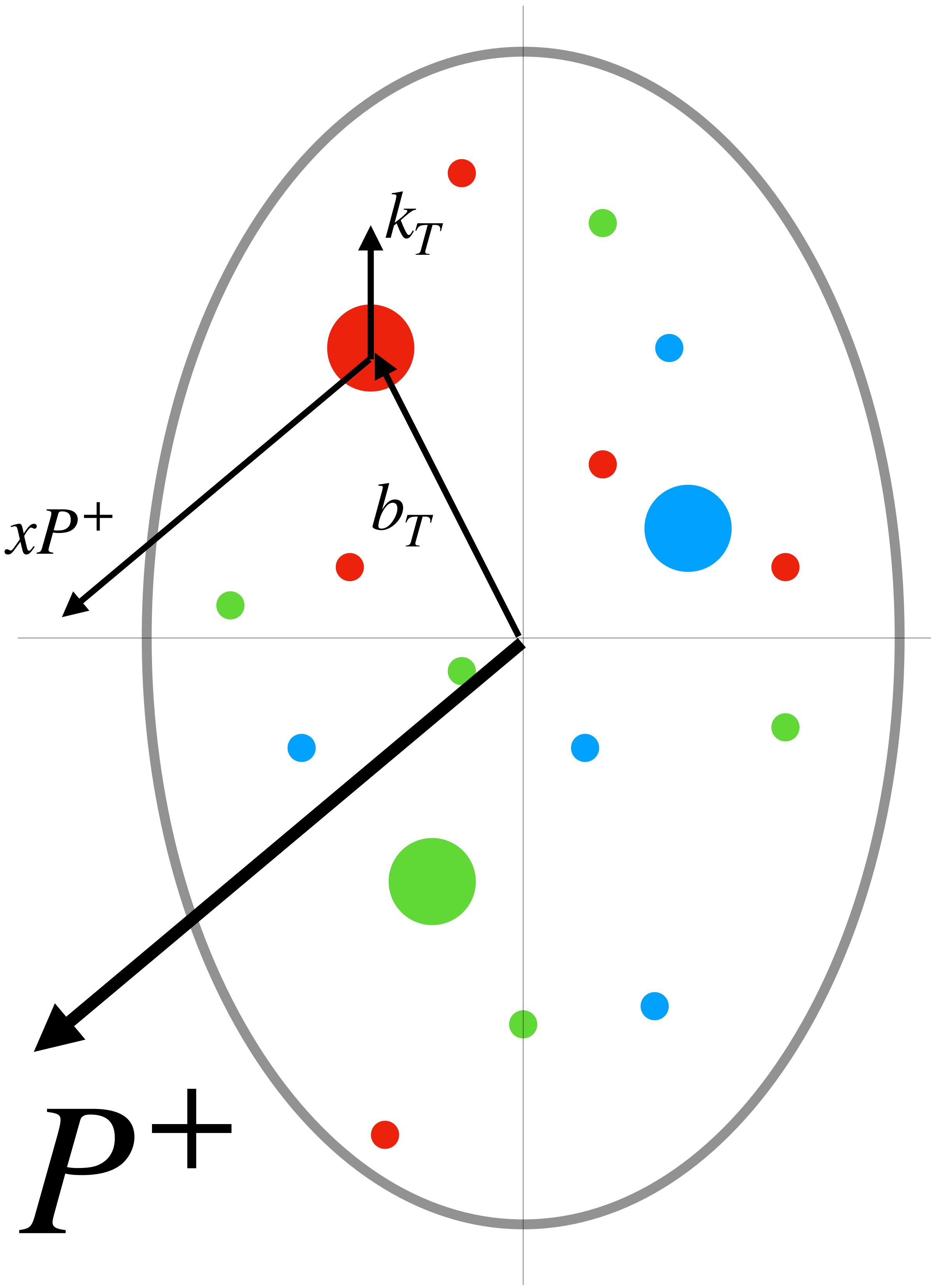
$\int d^2b_t$ $\int d^2k_t$

Transverse Momentum Distribution (TMD)
$f(x, k_T)$

Generalized Parton Distribution (GPD)
$f(x, b_T)$

$\int d^2k_t$ $\int d^2b_t$

Parton Distribution Function (PDF)
$f(x)$



Parton and Ioffe Time distributions

- Unpolarized Ioffe time distributions

Ioffe time: $\nu = p \cdot z$

V. Braun, P. Gornicki, L. Mankiewicz
Phys Rev D 51 (1995) 6036-6051

- $$I_q(\nu, \mu^2) = \frac{1}{2p^+} \langle p \,|\, \bar{\psi}_q(z^-) \gamma^+ W(z^-; 0) \psi_q(0) \,|\, p \rangle_\mu$$

$z^2 = 0$

- $$I_g(\nu, \mu^2) = \frac{1}{(2p^+)^2} \langle p \,|\, F_{+i}(z^-) W(z^-; 0) F^i_+(0) \,|\, p \rangle_\mu$$

$i = x, y$

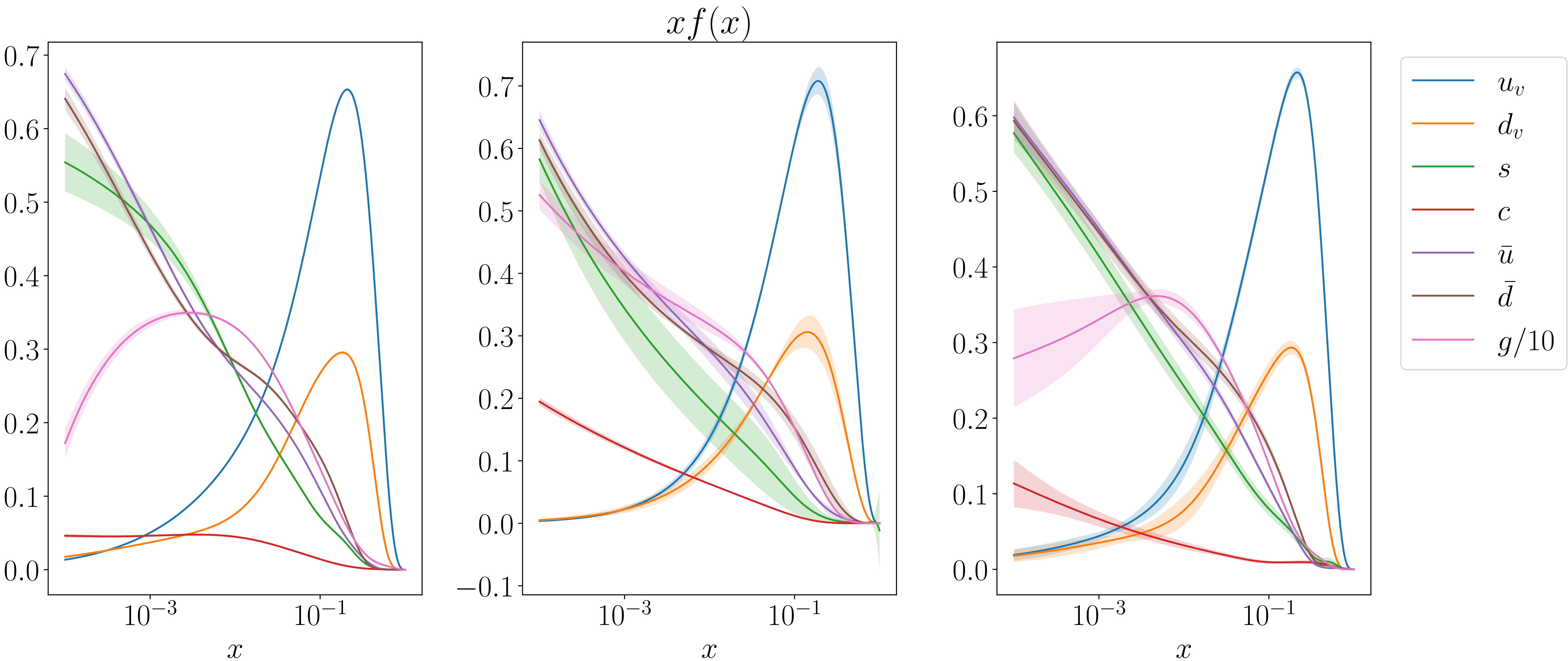
- Parton Distribution Functions

- $$I_q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} f_q(x, \mu^2)$$

- $$I_g(\nu, \mu^2) = \int_0^1 dx\, \cos(x\nu)\, x f_g(x, \mu^2)$$

Phenomenological Fits

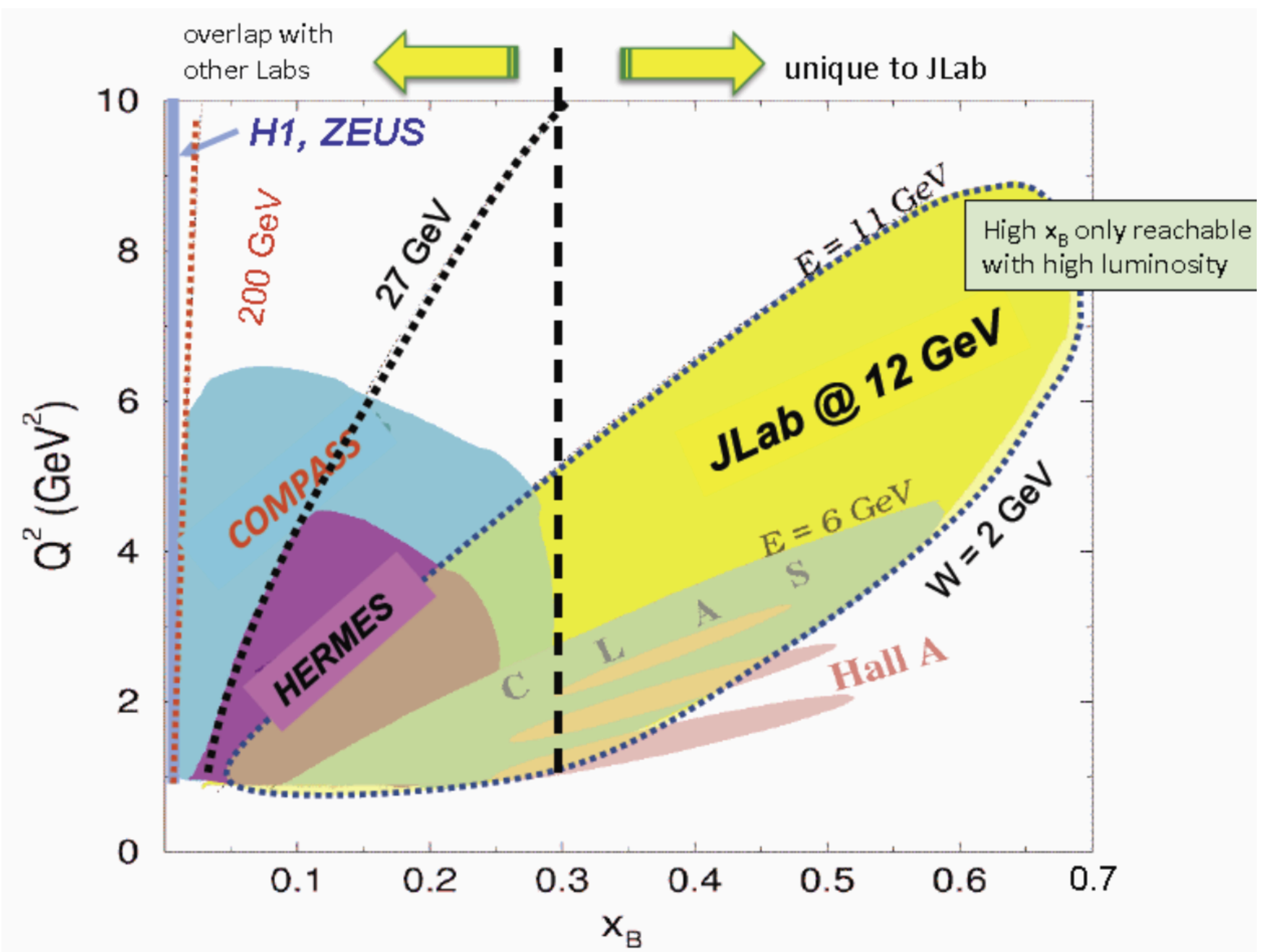
- **Nucleon unpolarized PDFs** from analysis of global experimental data
    - Best known distributions



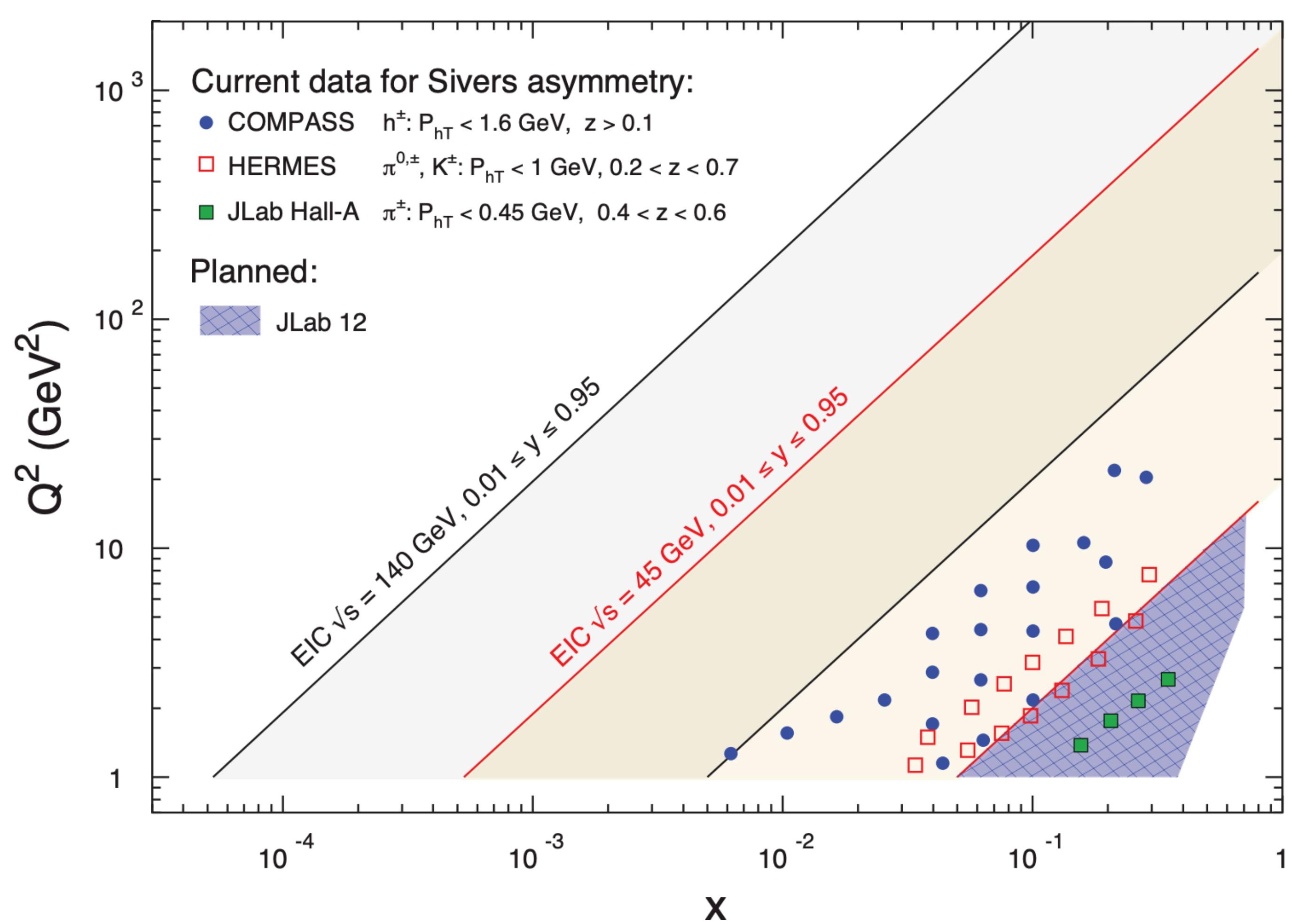
(Left) MSHT20 Eur. Phys. J. C 81 (2021) 4, 341. (Center) JAM20 Phys. Rev. D 104 (2021) 1, 016015.
(Right) NNPDF4.0 Eur. Phys. J. C 82 (2022) 5, 428

JLab 12 GeV and the EIC

- Expand Kinematic Coverage
  - JLab extends to large x (also where lattice is most sensitive)
  - Unpolarized gluon PDF dominates low x and is goal of EIC
- Expand to polarized and 3D distributions (lattice goal as well)



JLab 12 GeV White Paper:
Eur. Phys. J. A 48 (2012) 187



EIC White Paper:
Eur. Phys. J. A 52 (2016) 9, 268

Positivity of the PDFs

- In parton model (LO without QCD interactions), $f_i^{\uparrow/\downarrow}(x) \geq 0$
  - Sometimes assumed for PDF analysis
  - For gluons implies,

$$2g(x) = g^{\uparrow}(x) + g^{\downarrow}(x) \qquad 2\Delta g(x) = g^{\uparrow}(x) - g^{\downarrow}(x)$$

$$|\Delta g(x)| \leq g(x)$$

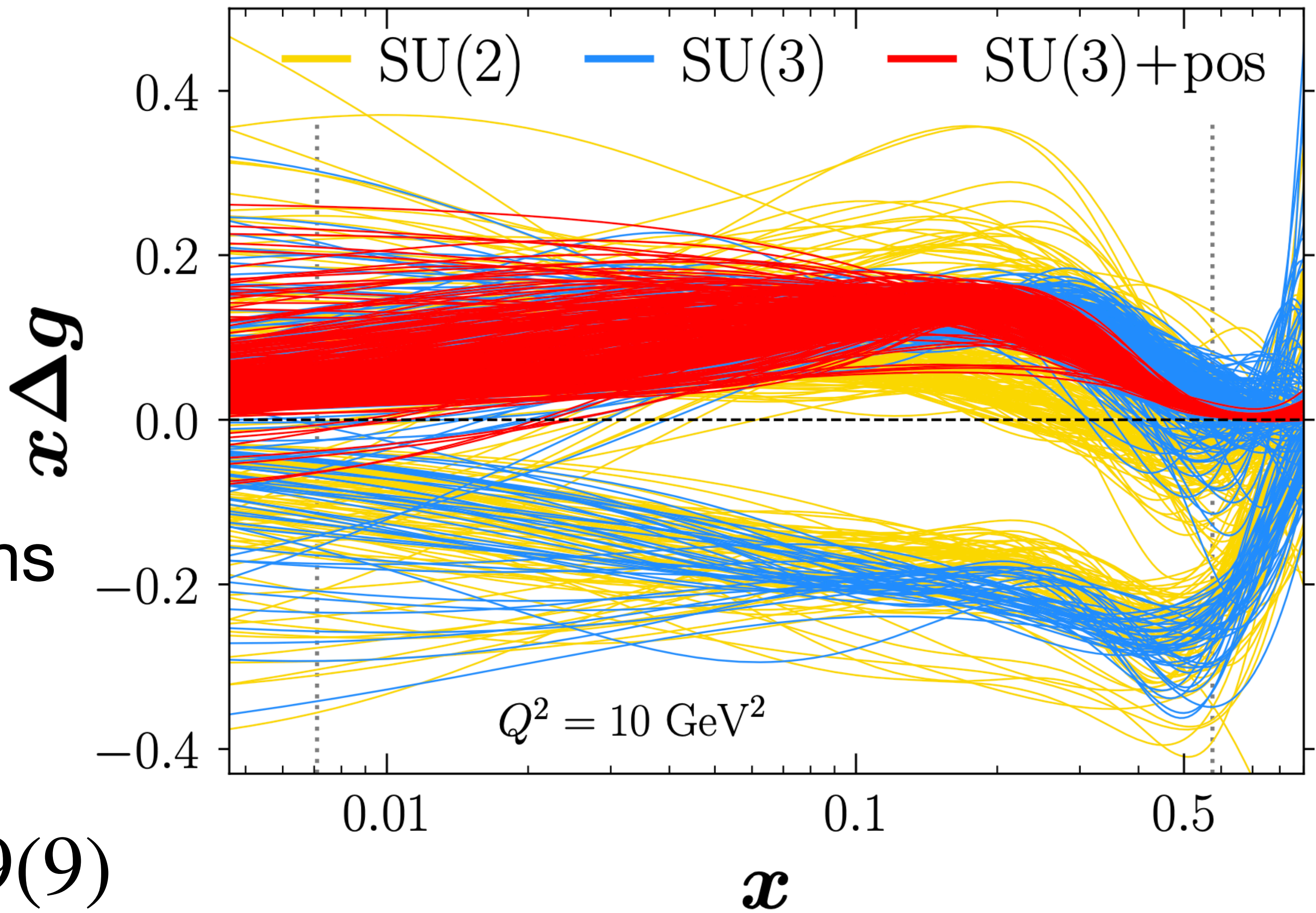
- With interactions, does not have to be true for $\overline{\mathrm{MS}}$ scheme

J. Collins, T. Rogers, N. Sato, Phys Rev D 105 (2022) 7,076010

- How will this effect analyses?

Spinning gluons

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$
- Without constraint: $\Delta G = 0.3(5)$
- Lattice: $\Delta G = 0.251(47)(16)$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

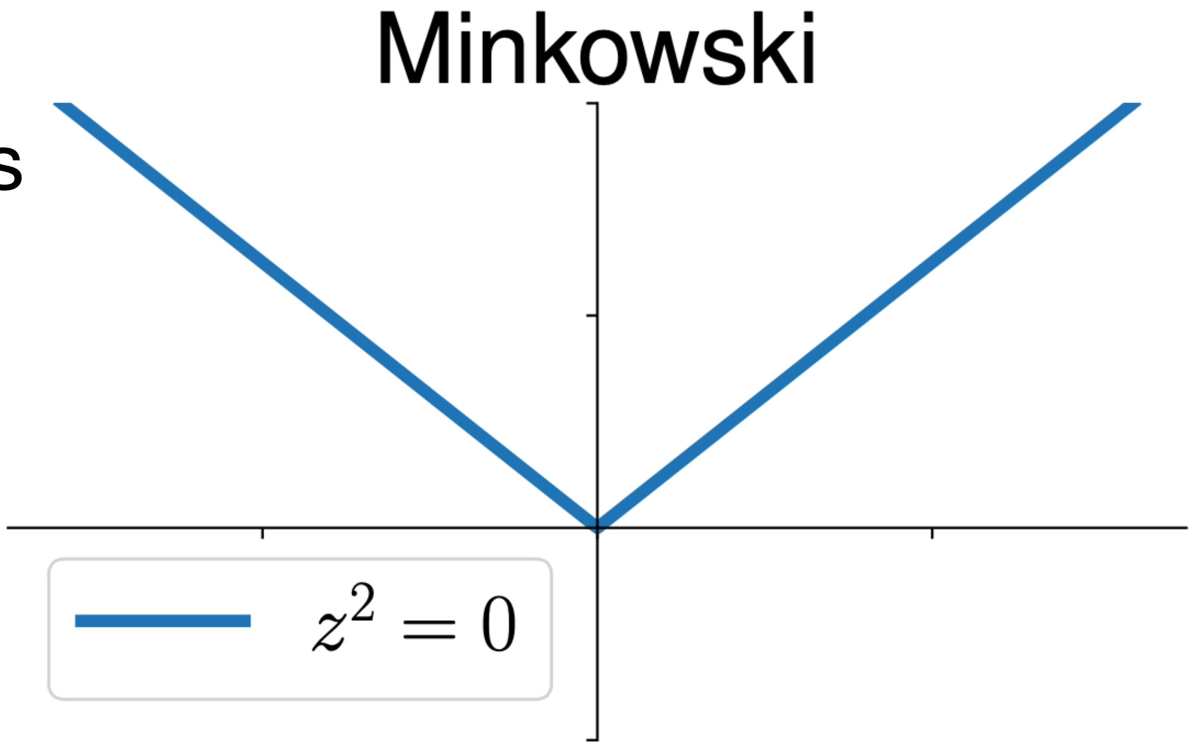
R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations



- Use space-like separations

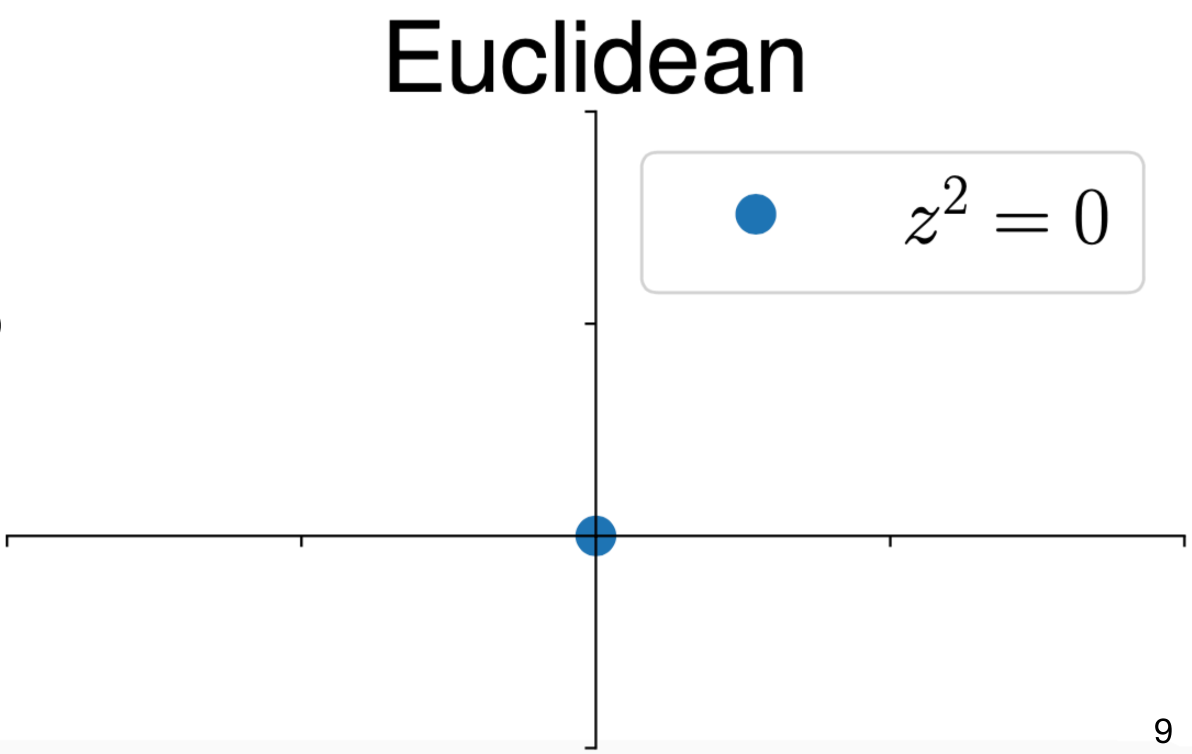
  X. Ji *Phys Rev Lett* 110 (2013) 262002

  - Wilson line operators

$$O_{\Gamma}^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

  - Factorizations exist analogous to cross sections

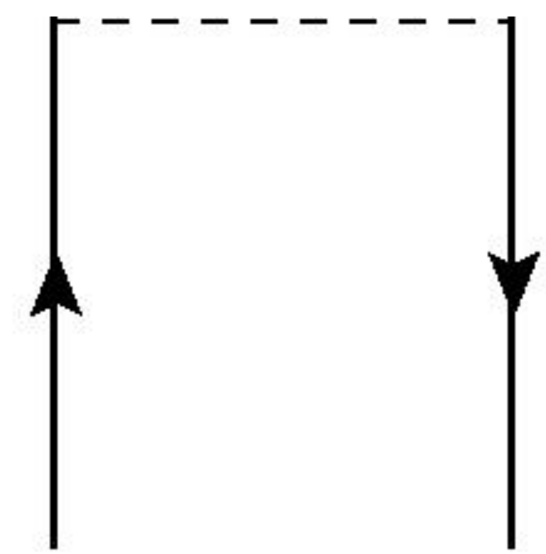


Many approaches

For Gluon Structure see:
J. Delmar Wed. 4:30 pm
R. Sufian Wed. 5:50 pm

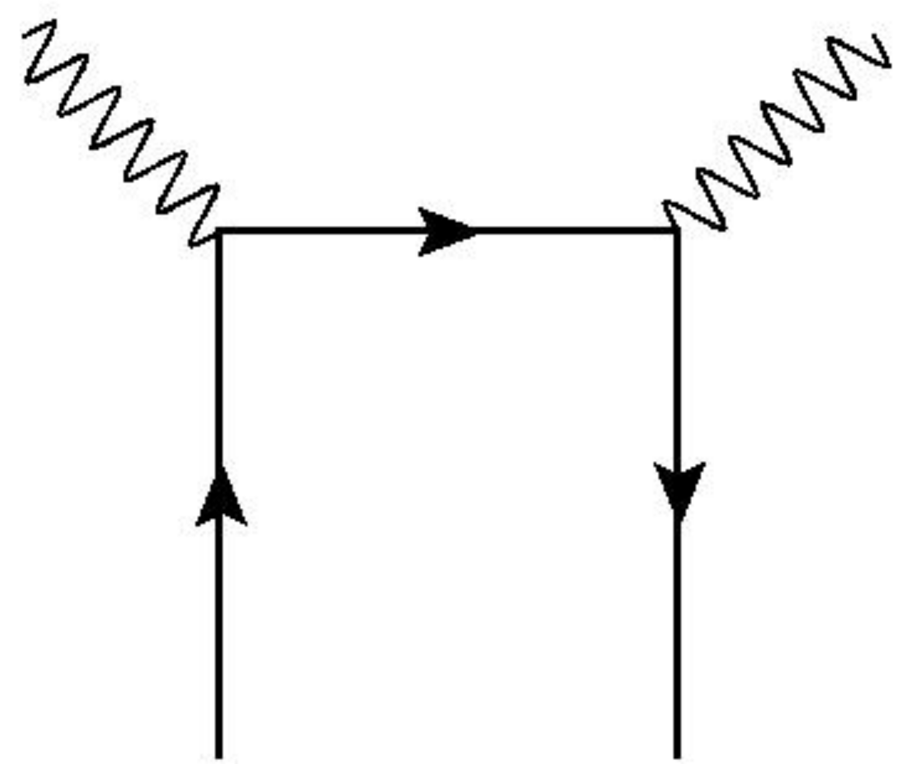
- **Wilson line operators**

$$O_{WL}(x;z) = \bar{\psi}(x+z)\Gamma W(x+z;x)\psi(x)$$



  - LaMET X. Ji *Phys. Rev. Lett.* 110 (2013) 262002
  - Pseudo-PDF A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025
- Two current correlators

$$O_{CC}(x,y) = J_{\Gamma}(x)J_{\Gamma'}(y)$$



  - Hadronic Tensor
    K.-F. Liu et al *Phys. Rev. Lett.* 72 1790 (1994)
    Phys. Rev. D 62 (2000) 074501
  - HOPE
    W. Detmold and C.-J. D. Lin, *Phys. Rev. D* 73 (2006) 014501
  - Short distance OPE
    V. Braun and D. Muller *Eur. Phys. J. C* 55 (2008) 349
  - OPE-without-OPE
    A. Chambers et al, *Phys. Rev. Lett.* 118 (2017) 242001
  - Good Lattice Cross Sections
    Y.-Q. Ma and J.-W. Qiu *Phys. Rev. Lett.* 120 (2018) 2, 022003

Wilson Line Matrix Elements

- Matrix element $M(p,z) = \langle p \,|\, \bar{\psi}(z)\gamma^\alpha W(z;0)\psi(0) \,|\, p\rangle$ $\qquad z^2 \neq 0$

$$= 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

- Quasi-PDF: $\tilde{q}(y, p_z^2) = \dfrac{1}{2p_\alpha} \displaystyle\int dz e^{iyp_z z} M(z, p_z)$

- Large Momentum Effective Theory: X. Ji *Phys. Rev. Lett.* 110 (2013) 262002

- $$\tilde{q}(y, p_z^2) = \int \frac{dx}{|x|} K\left(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}\right) q(x, \mu^2) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(xp_z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)p_z)^2}\right)$$

- Pseudo-ITD: A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

$$\mathcal{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda_{\mathrm{QCD}}^2 z^2)$$

$$= \int du C'(u, \mu^2 z^2) I_q(u\nu, \mu^2) + O(\Lambda_{\mathrm{QCD}}^2 z^2)$$

The Role of Separation and Momentum

- In quasi-PDF and pseudo-PDF, separation and momentum swap roles

Scale:

p_z^2 / z^2

Fourier variable:

z / p_z , or $\nu = p \cdot z$

- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ_{QCD}^2
- Technically only requires single value

- Variable for inverse Fourier Transform
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem

Inverse Problems for pseudo-PDFs

- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$xg(x) = \int_0^\infty d\nu \, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

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- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$xg(x) = \int_0^\infty d\nu \, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

- Forward integral to an ill posed matrix equation

$$\mathfrak{M}(\nu) = \int_0^1 dx \, C(x\nu) \, xg(x) \to [\mathbf{C}][\mathbf{xg}]$$

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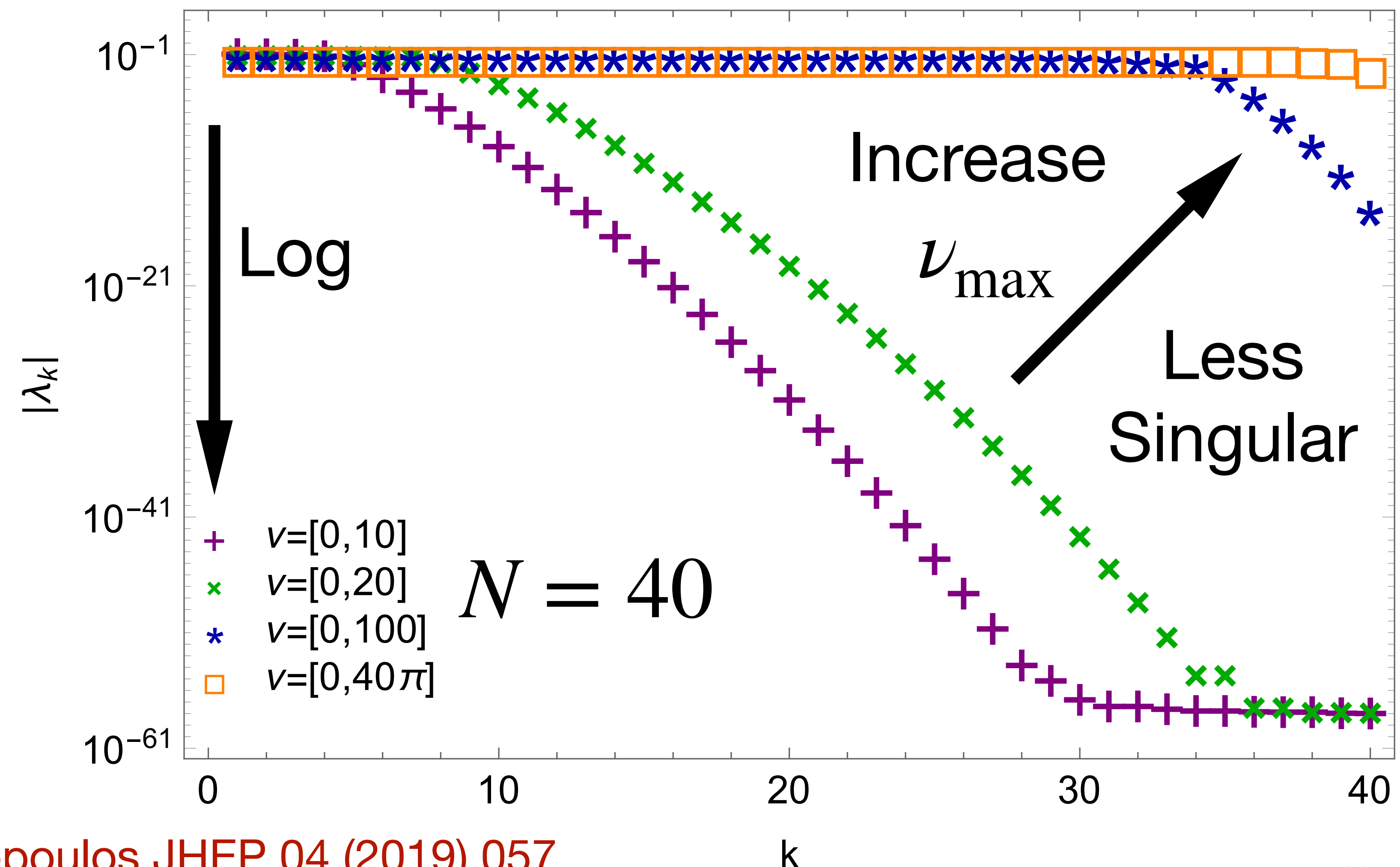

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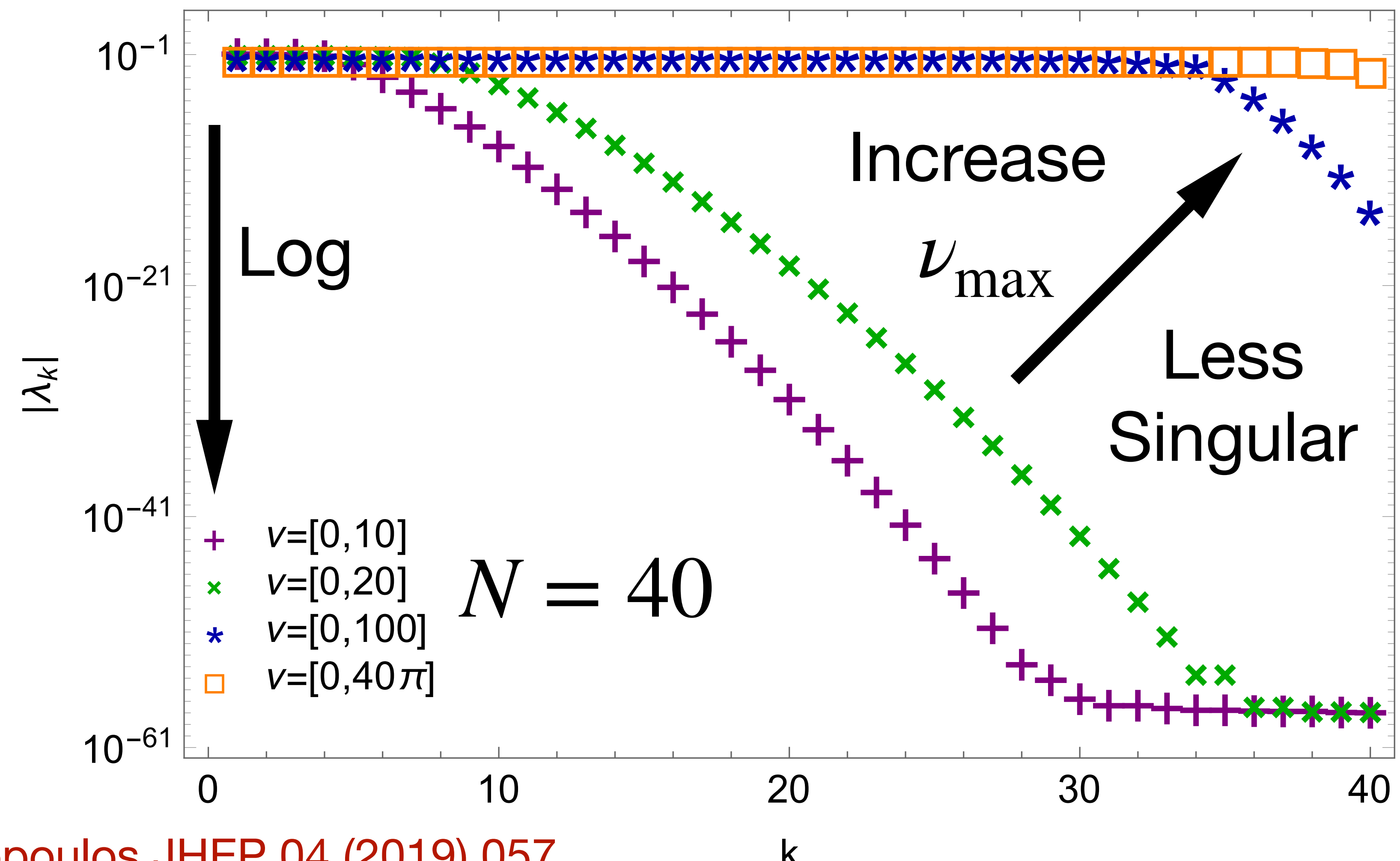
$$\mathfrak{M}(\nu) = \int_0^1 dx\, C(x\nu)\, xg(x) \to [\mathbf{C}][\mathbf{xg}]$$

- Must be regulated by additional information
  - Restricted functional form
  - Constraints on the PDF or parameters
  - Assumptions of smoothness, continuity, ….



JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Inverse Problems for Parton Physics

- **Structure Functions**

$$F_2(x, Q^2) = \sum_i \int_x^1 d\xi \, C(\xi, \frac{\mu^2}{Q^2}) \, q(\frac{x}{\xi}, \mu^2)$$

- **LaMET (on the lattice)**

$$M(p_z, z) = \int_{-\infty}^{\infty} dy \, e^{iyp_z z} \, \tilde{q}(y, p_z^2)$$

- **pseudo-Distributions / Good Lattice Cross Sections**

$$\mathfrak{M}(\nu, z^2) = \int_{-1}^{1} dx \, C(x\nu, \mu^2 z^2) \, q(x, \mu^2)$$

- **Local Matrix elements / HOPE / OPE-without-OPE**

$$a_n(\mu^2) = \int_{-1}^{1} dx \, x^{n-1} \, q(x, \mu^2)$$

- **Hadronic Tensor**

$$\tilde{W}_{\mu\nu}(\tau) = \int d\nu \, e^{-\nu\tau} \, W_{\mu\nu}(\nu)$$

Difficulty Reaching High Momentum

- **Creating operator with high overlap and signal**
  - Momentum smearing improves overlap with moving states
    G. Bali et al Phys. Rev. D 93 (2016) 9, 094515
  - Distillation from all time slices improves signal
  - GEVP optimizes the overlap with ground state

M. Peardon, et al,
Phys. Rev. D 80 (2009) 054506
C. Egerer et al
Phys. Rev. D 103 (2021) 3, 034502

- **Excited state energy gap shrinks**
  - Larger times needed for ground state
  - Summed GEVP techniques can remove lowest states and suppress remaining

    J. Bulava, M. Donnellan, R. Sommer JHEP 01 (2012) 140

See R. Sufian
Wed. 5:50 pm
for implementation

- **Exponentially suppressed signal-to-noise ratio**
  - No current solutions

Gluon Matrix Elements

- **General Matrix Element**

$$M^{\mu\alpha;\nu\beta}(z,p,s) = \langle p,s\,|\,\mathrm{Tr}\Big[F^{\mu\alpha}(z)W(z;0)F^{\nu\beta}(0)\Big]\,|\,p,s\rangle$$

  - Assume z is along cardinal direction (eventually lattice axis)

- **Renormalization** Z-Y. Li, Y-Q. Ma, J-W. Qiu. *Phys. Rev. Lett.* 122 (2019) 6, 062002

  - Multiplicatively renormalizable

  - Depends on how many of μ, ν, ρ, σ are in z direction.

- Matrix element has complicated Lorentz decomposition in terms of p^μ, z^μ, s^μ

  - Need to isolate amplitudes with leading twist contributions

Spin Averaged matrix element

- Spin averaged combination $\mathscr{M}(\nu, z^3) = \frac{1}{2p_0^2}\left[M_{ti;it} + M_{ij;ij}\right]$
  - Gives **one** amplitude with leading twist contribution

$i, j = x, y$

I. Balitsky, W. Morris, A. Radyushkin Phys Rev D 105 (2022) 1, 014008
T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

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- Use **ratio** with finite continuum limit

$$\mathfrak{M}(\nu, z^2) = \frac{\mathscr{M}(\nu, z^2)\ \mathscr{M}(0,0)\,|_{p=0,z=0}}{\mathscr{M}(\nu,0)\,|_{z=0}\,\mathscr{M}(0,z^2)\,|_{p=0}}$$

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- Relation to **gluon** and **quark singlet** ITD

$$\langle x \rangle_g \mathfrak{M}(\nu, z^2) = \int_0^1 C^{gg}(u, \mu^2 z^2) I_g(u\nu, \mu^2) + C^{qg}(u, \mu^2 z^2) I_s(u\nu, \mu^2)$$

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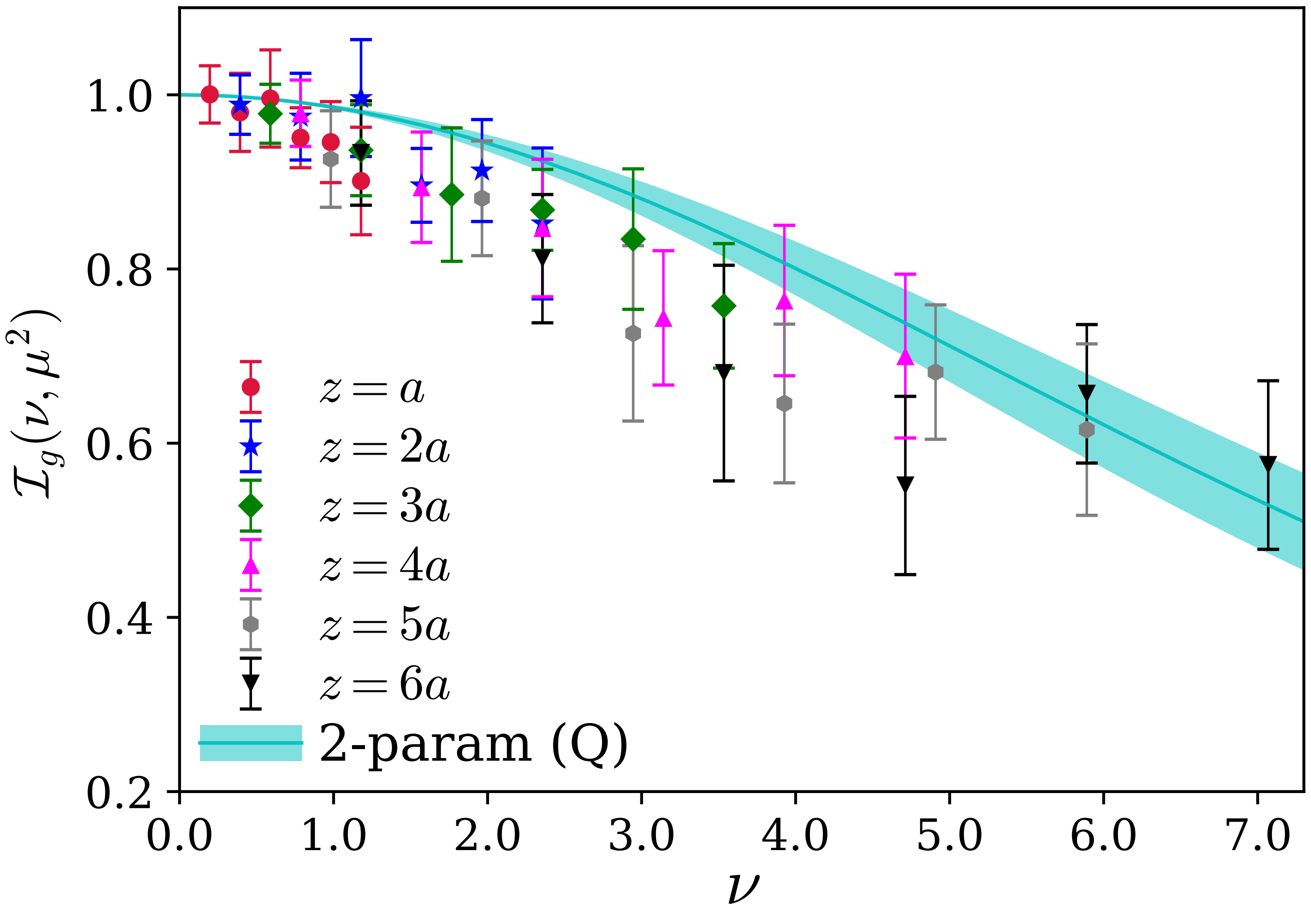
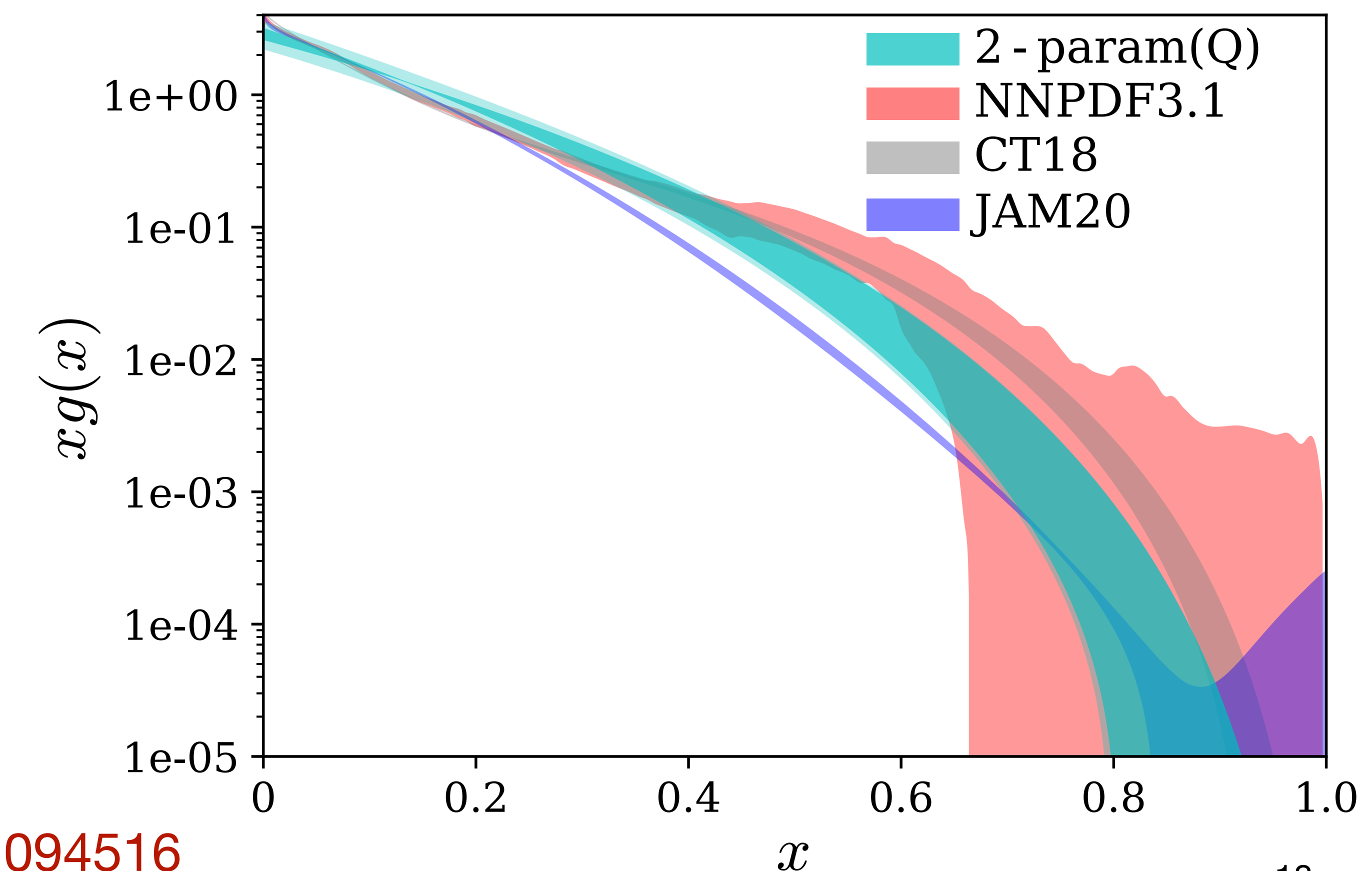
Unpolarized Gluon PDF



- Extrapolated to $\tau \to 0$ flow time
- Modified by NLO formula

$a = 0.094$ fm
$m_\pi = 358$ MeV

- ITD fit to cosine transform of
  $xg(x) = x^a(1-x)^b / B(a+1, b+1)$
- Qualitative agreement with global analysis



T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution

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- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathscr{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

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Neglected for now

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \cancel{\widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)}$$

Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
JHEP 02 (2022) 193

$$\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = (sz)\big(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{pp}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{zz}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{zp}$$
$$+(sz)\big(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{pz}$$
$$+(sz)\big(p_\mu z_\alpha - p_\alpha z_\mu\big)\big(p_\lambda z_\beta - p_\beta z_\lambda\big)\widetilde{\mathcal{M}}_{ppzz}$$
$$+(sz)\big(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\big)\widetilde{\mathcal{M}}_{gg}$$

$$\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = \Big(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\Big)\widetilde{\mathcal{M}}_{sp}$$
$$+\Big(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{ps}$$
$$+\Big(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\Big)\widetilde{\mathcal{M}}_{sz}$$
$$+\Big(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{zs}$$
$$+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps}$$
$$+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}$$

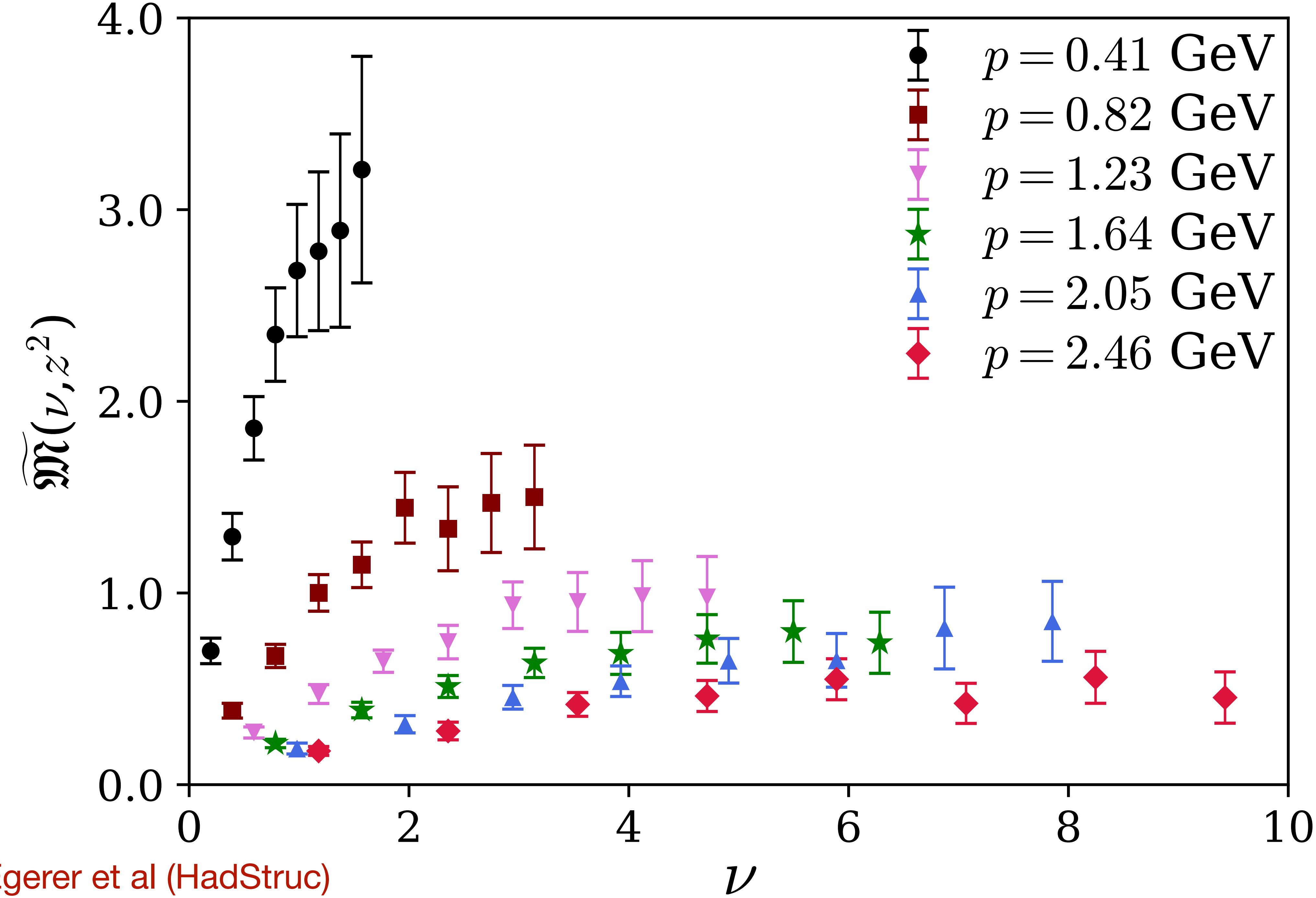
Want: $M_{\Delta g}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\,\widetilde{\mathcal{M}}_{pp}\right]$

Can get: $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$

$$= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp}$$
$$= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\,\widetilde{\mathcal{M}}_{pp}$$

Helicity Gluon Matrix Element

- Large contamination from $\frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$ will need to be removed

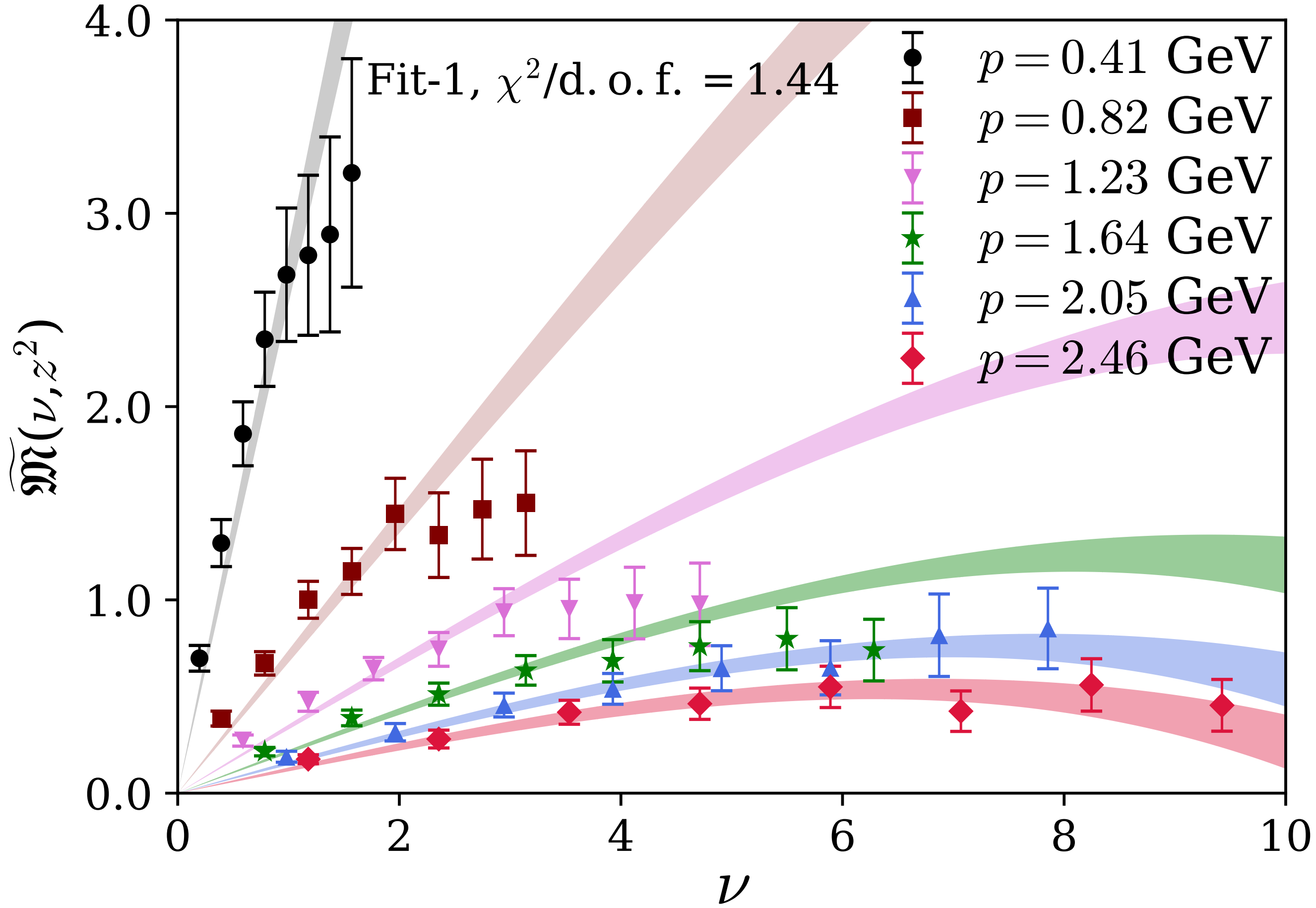


$a = 0.094$ fm
$m_\pi = 358$ MeV

C. Egerer et al (HadStruc)
arXiv: 2207.08733

Correcting Helicity Gluon Results

- Model both terms

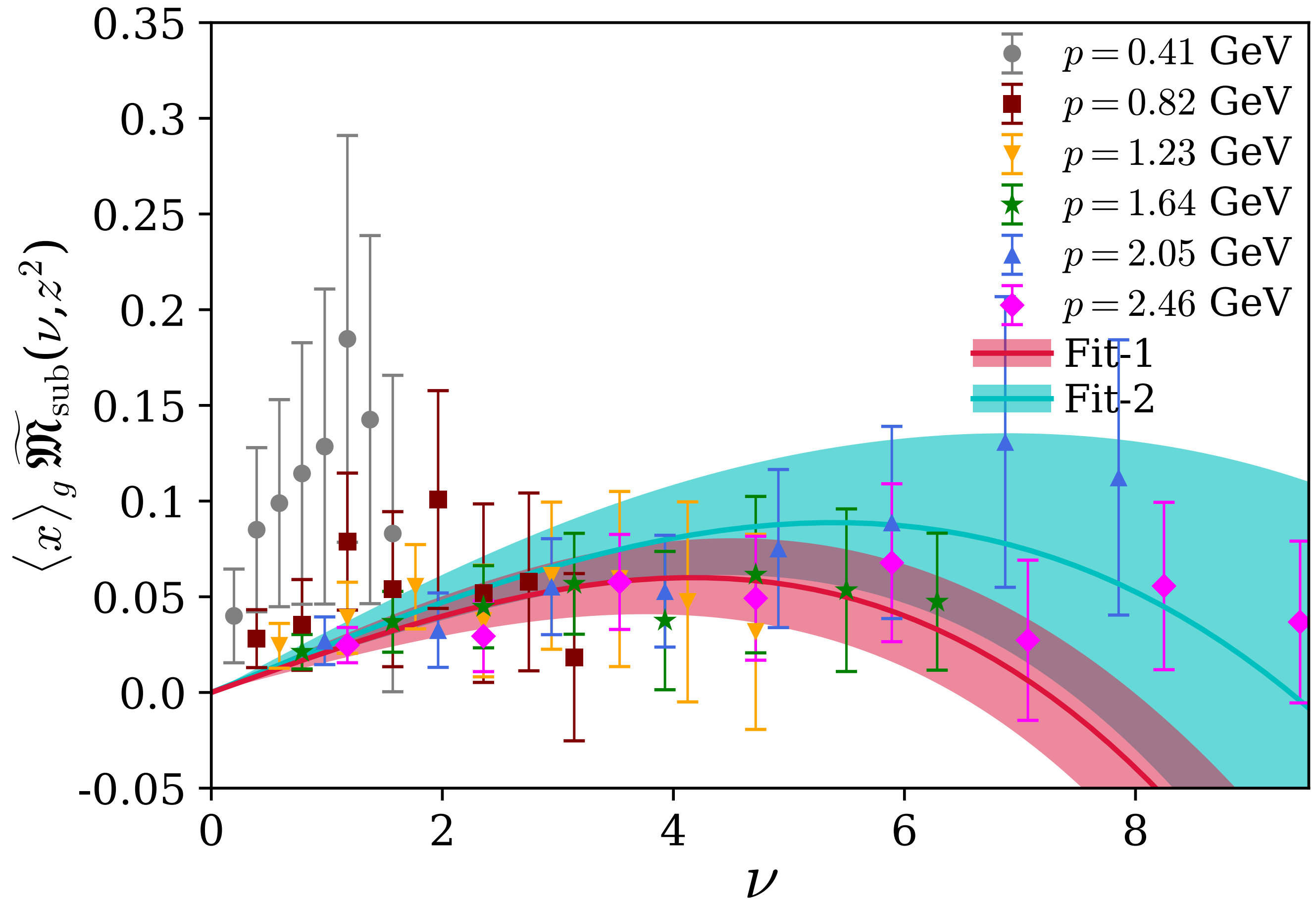


$a = 0.094$ fm

$m_\pi = 358$ MeV

C. Egerer et al (HadStruc) arXiv: 2207.08733

- Subtract rest frame

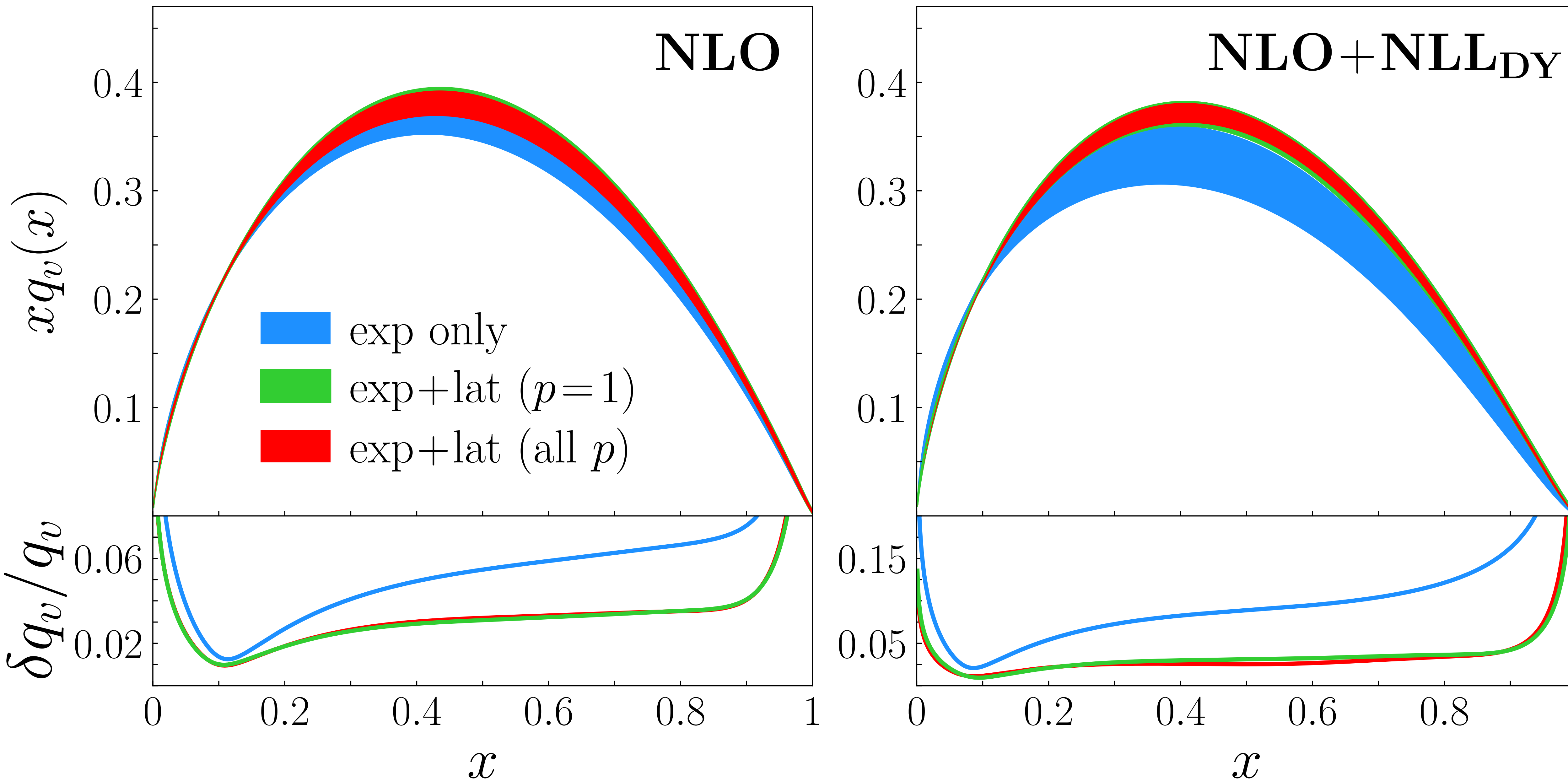


See R. Sufian
Wed. 5:50 pm
for details

What can we do with this new data?

Combining Lattice and Experiment

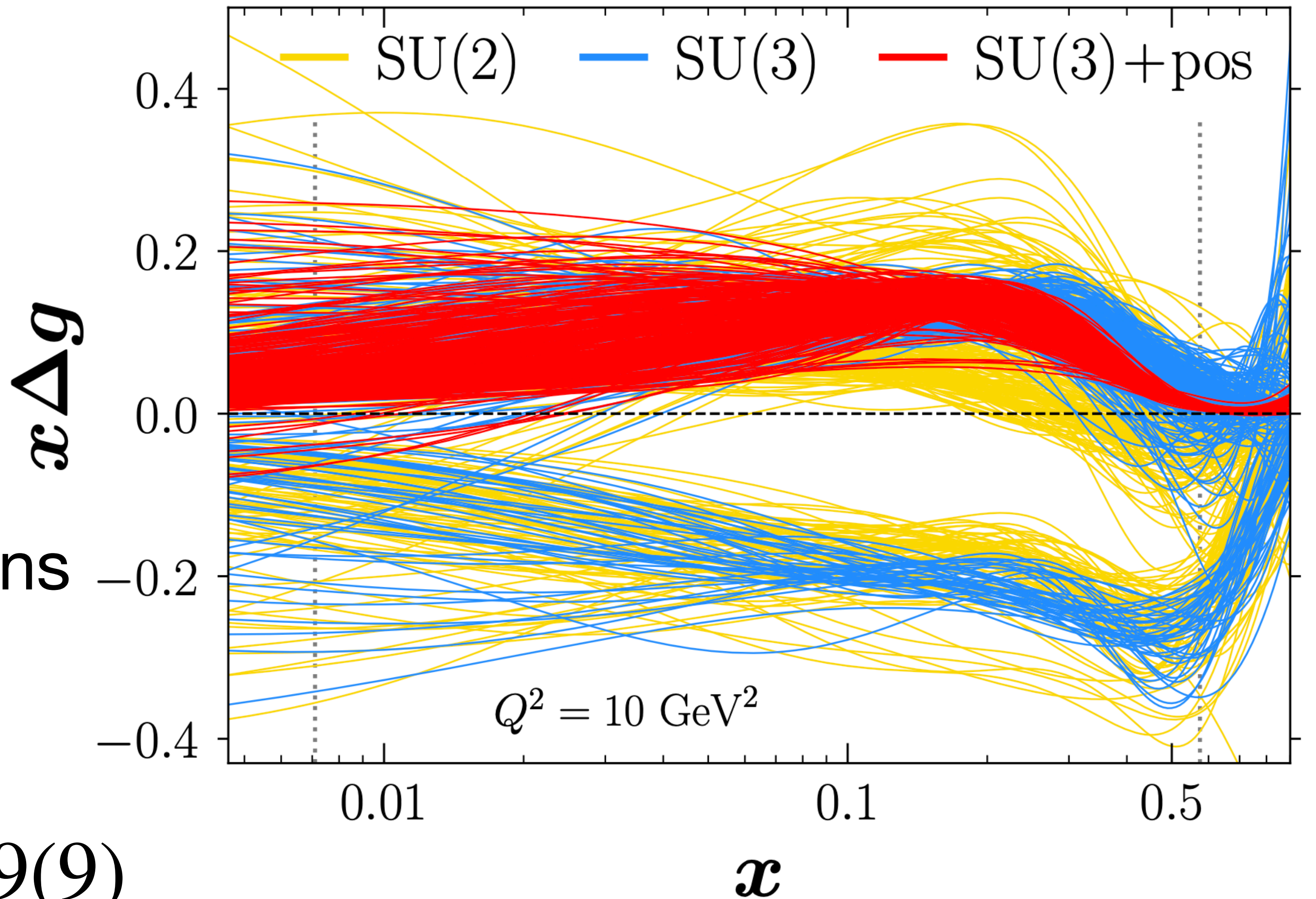
- Simultaneously fit Lattice and Experimental pion PDF data
- Each gives unique information complementing each other



P. Barry et al (HadStruc and JAM), *Phys. Rev. D* 105 (2022) 11, 114051

Spinning gluons (revisited)

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g(x)$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$
- Without constraint: $\Delta G = 0.3(5)$
- Lattice: $\Delta G = 0.251(47)(16)$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

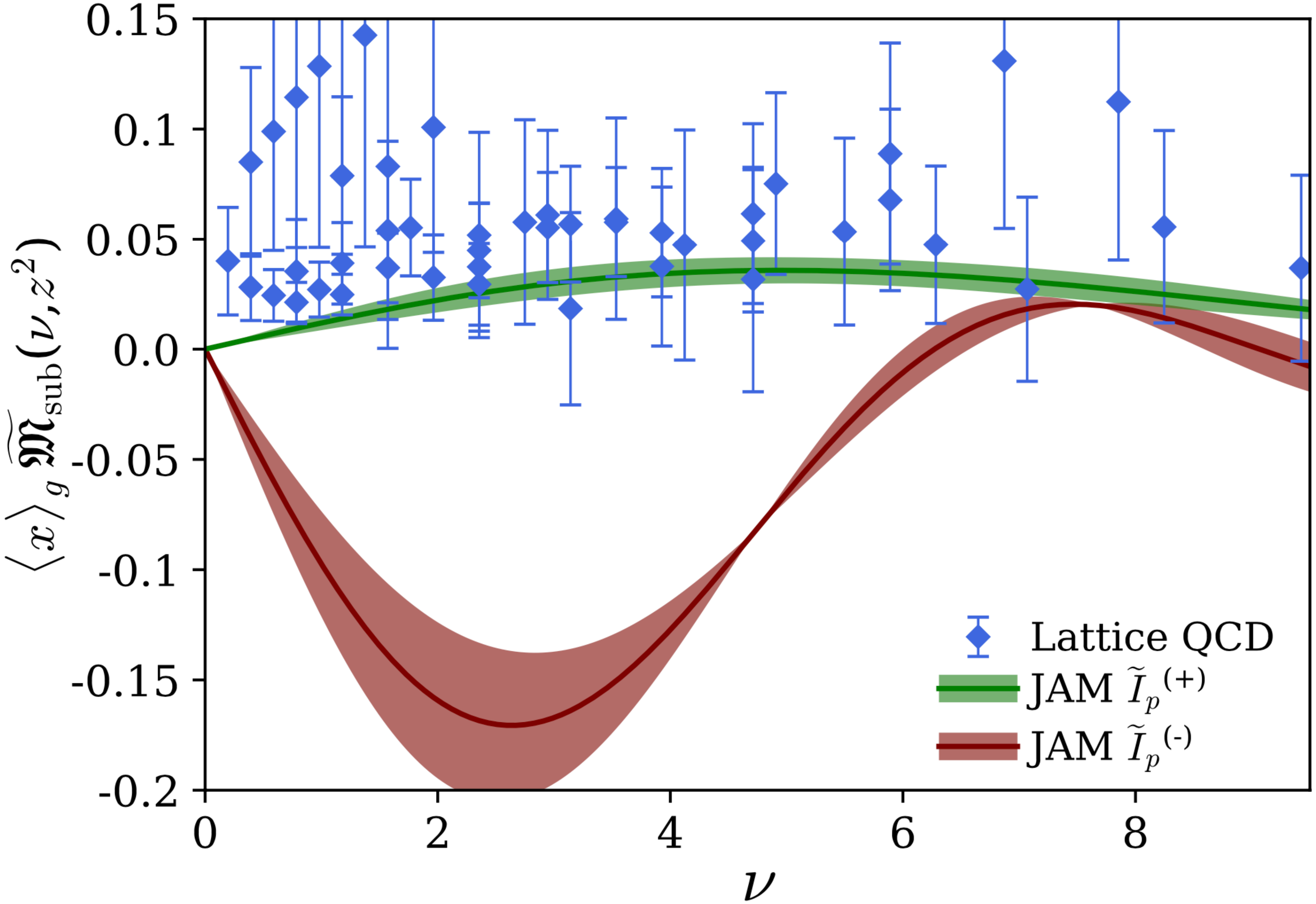
R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

Spinning gluons (revisited)

Can lattice data affect phenomenological polarized gluon analysis?



$$\Delta G = \int d\nu \, I_g(\nu)$$

- The positive and negative solutions without positivity constraints plotted in ν space

Y. Zhou et al Phys. Rev. D 105, 074022 (2022)
C. Egerer et al (HadStruc) arXiv:2207.08733

- Only positive band consistent with lattice data

Conclusions

- Gluon x or ν dependent structure requires state-of-the-art calculations
  - Use of **distillation** with large number of configurations
  - **Summed GeVP** to control excited states
  - **Wilson flow** to improve signal
- Future work towards gluon 3D distributions, higher twist PDFs,…. is possible given enough resources to find signal
- Possibly impact phenomenological PDF analyses
  - **Today:** with polarized gluon PDF
  - **Future:** JLab 12GeV and EIC data on PDF and on new distributions

Thank you and the organizers!