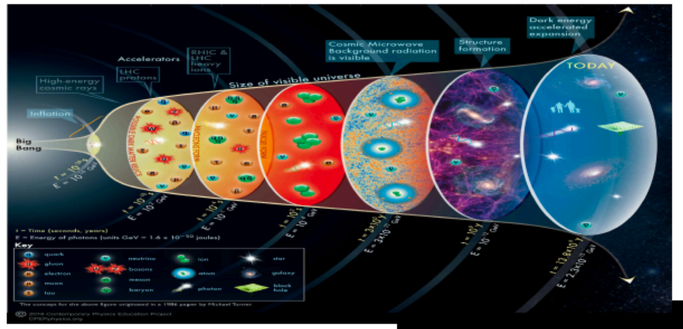
QCD thermodynamics: An overview of recent progress

Francesca Cuteri

Goethe Universität - Frankfurt am Main

The 39th International Symposium on Lattice Field Theory
August 08, 2022

Hot/dense/magnetized states of QCD matter - The sketch



Early Universe (QCD epoch)

$100\mathrm{MeV} \lesssim T \lesssim 200\mathrm{MeV}$

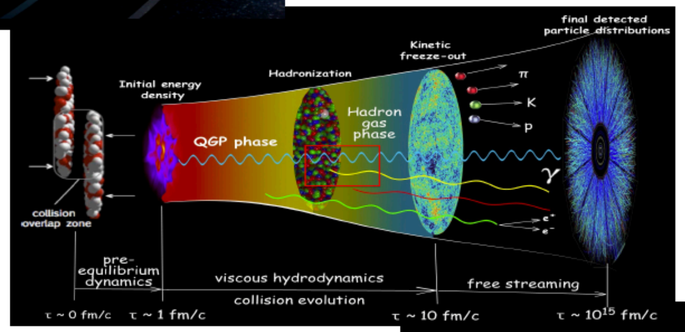
$b = (8.60 \pm 0.06) \times 10^{-11}$, $q = 0$

$|l| = |l_e + l_\mu + l_\tau| < 0.012$

$B? \longrightarrow 10^{-16} \lesssim B \lesssim 10^{-9}\,\mathrm{Gauss}$ (EGMFs)

Hotter

"diluter"



Heavy ion collisions (HIC)

$50\mathrm{MeV} \lesssim T \lesssim 200\mathrm{MeV}$

$n \lesssim 0.12\mathrm{fm}^{-3}$

$\delta = N - Z/(N+Z) \lesssim 0.25$

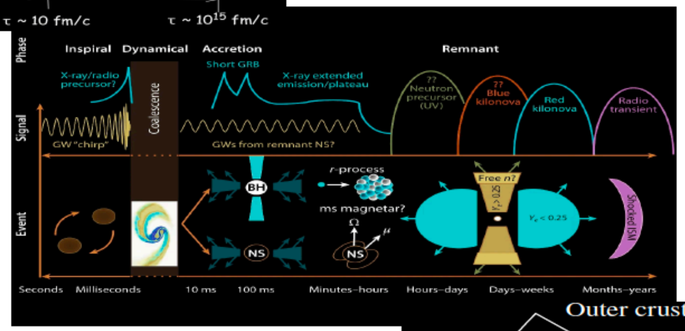
$B \lesssim 10^{19}\,\mathrm{Gauss} \sim 10^{20} B_{\mathrm{Earth}}$

$T \in [50, 80]\mathrm{MeV}$

$n \sim 2n_0, \quad n_0 = 0.16\mathrm{fm}^{-3}$

$n_{\mathrm{quark}} \neq 0$

$10^{10} \lesssim B \lesssim 10^{12}\,\mathrm{Gauss} \rightarrow B \gtrsim 10^{16}\,\mathrm{Gauss}$



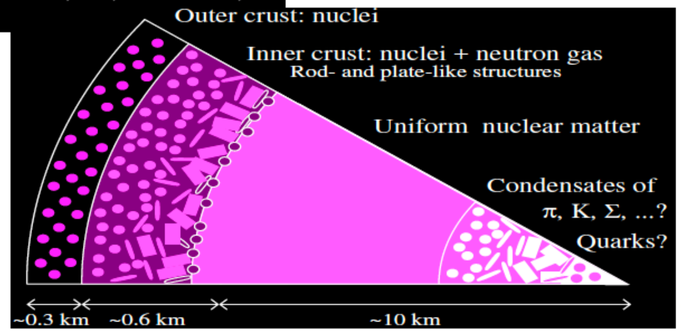
Binary Neutron Star (BNS) mergers

$T \lesssim 1\mathrm{KeV}$

$0.3 n_0 \lesssim n \lesssim 15 n_0, \quad n_0 = 0.16\mathrm{fm}^{-3}$

$n_p/n \sim 0.04$, at n_0

$10^8 \lesssim B \lesssim 10^{15}\,\mathrm{Gauss}$



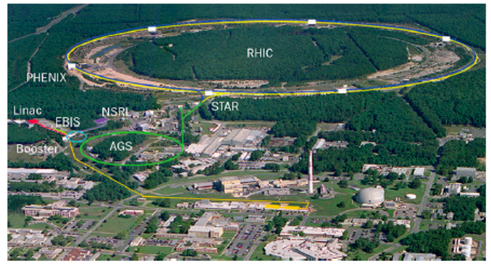
Colder

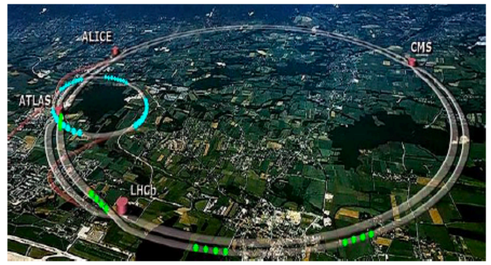
denser

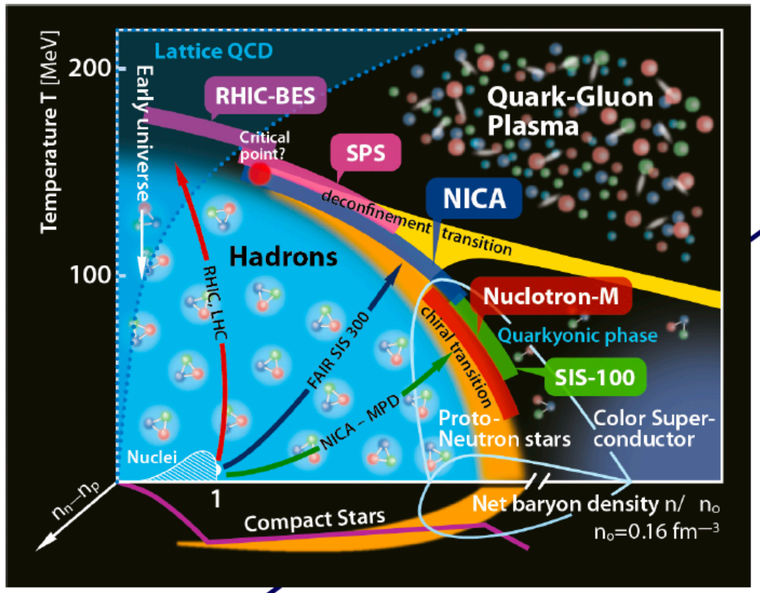
Neutron star interior

$1\mathrm{eV} \approx 12 \times 10^3 K; \quad B_{\mathrm{Earth}} \approx 0.5\mathrm{Gauss}$

Major experimental and observational campaigns, besides theoretical motivations

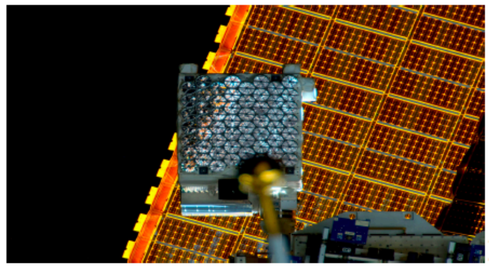




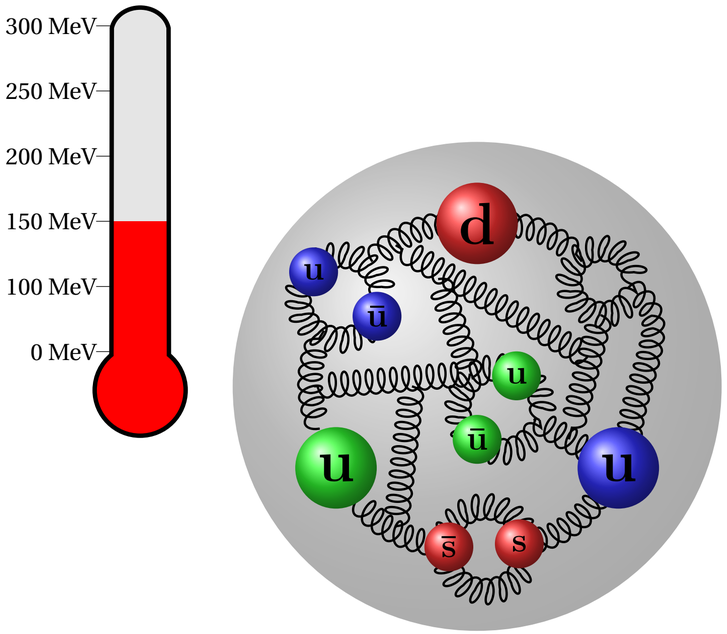


Observational astronomy

Heavy ion collisions



Nonzero temperature

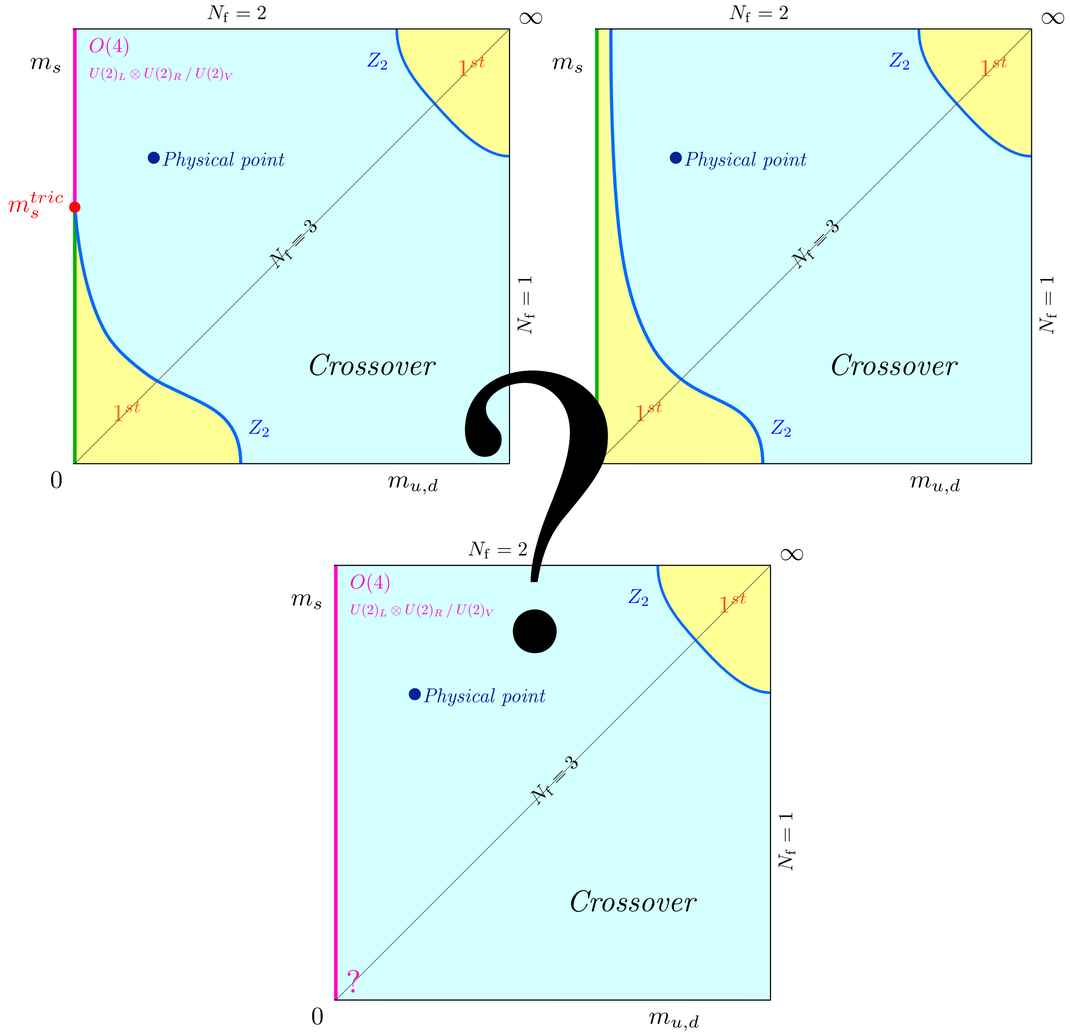


- The thermal crossover/transition
- Thermal effects on hadrons
- From the hadronic to the QGP phase: correlators, screening masses, spectral functions
- Transport properties
- Topological features

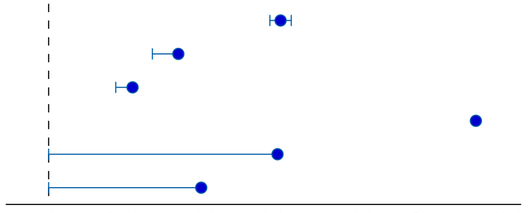
Reka Agnes Vig, Tue 15:40

Tamas G. Kovacs, Wed 17:30

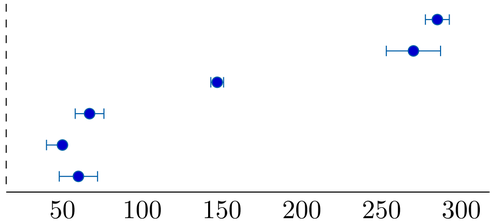
The thermal crossover/transition as a function of the flavor content (light masses)



N_f	Action	N_τ	$m_\pi^{Z_2}$ [MeV] at $\mu = 0$	Ref.
3	clover	6-8		Jin et al. (2015)
	clover	8-10		Jin et al. (2017)
	clover	10-12		Kuramashi et al. (2020)
2	std	4		Philipsen, Pinke (2016)
	tm	12		tftM coll. (2013)
	clover	16		Brandt et al. (2017)

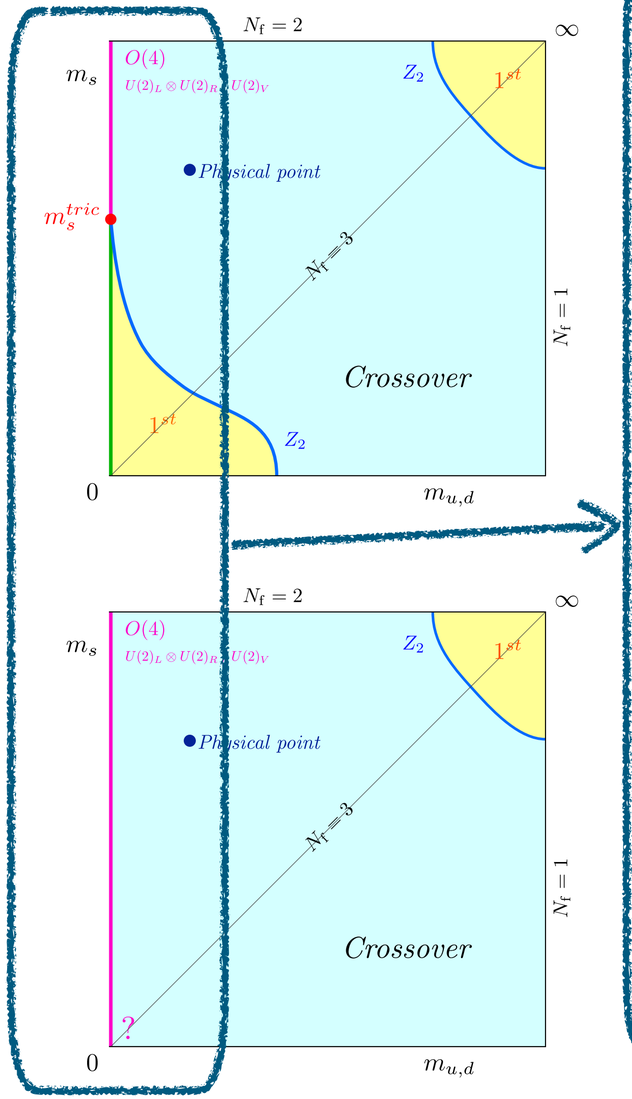


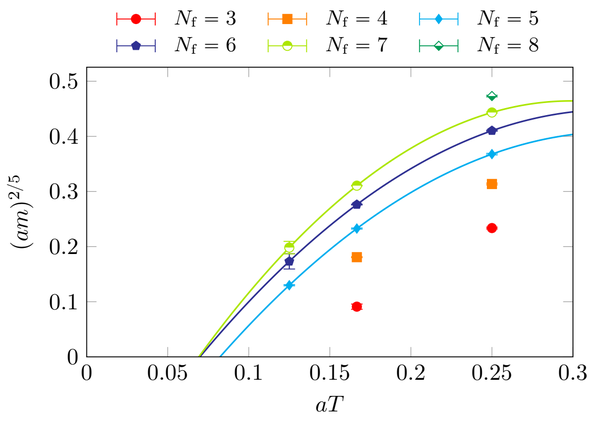
N_f	Action	N_τ	$m_\pi^{Z_2}$ [MeV] at $\mu = 0$	Ref.
	std	4		Karsch, Laermann, Schmidt (2001)
	std	4		Christ, Liao (2003)
3	std	6		de Forcrand, Kim, Philipsen (2007)
	p4	4		Karsch et al. (2004)
	HISQ	6		Bazavov et al. (2017)
2	std-μ_i	4		Bonati et al. (2014)

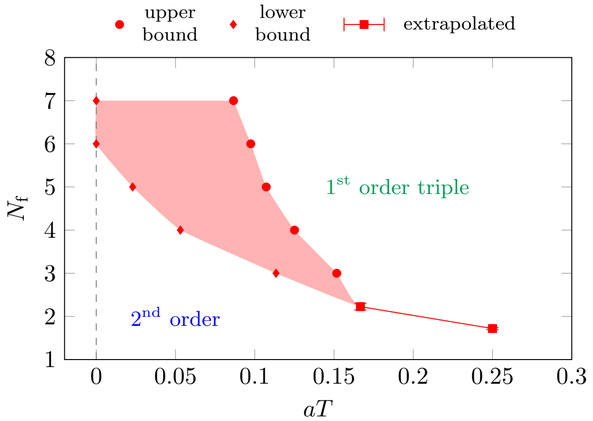


The thermal crossover/transition as a function of the flavor content

Columbia plot: Light masses







staggered unimproved $N_\tau \in [4-8]$ & reanalysis of $N_f = 3$ clover Wilson at $N_\tau \in [4-12]$

Cuteri, Philipsen, Sciarra '21 (JHEP)

Evidence for continuum chiral limit to feature 2nd order transitions for $N_f \in [2-7]$

Thermal transition with Möbius domain wall fermions

- $N_f = 3$, to locate Z_2 boundary
  - Crossover at $m_q^{\overline{MS}}(2\text{ GeV}) \sim 40$ MeV, for $T_{pc} \sim 196$ MeV
  - Transition at $m_q^{\overline{MS}}(2\text{ GeV}) \sim 4$ MeV, for $T_{pc} \sim 131$ MeV ??
- $N_f = 2+1$, near the physical point
  - Dealing with residual chiral symmetry breaking by $m_{res} > \bar{m}_{ud}$
  - First results at $N_\tau \in \{12, 16\}$

Yu Zhang, Tue 14:20, Yasumichi Aoki, Fri 16.40

Issaku Kanamori, Fri 17.00

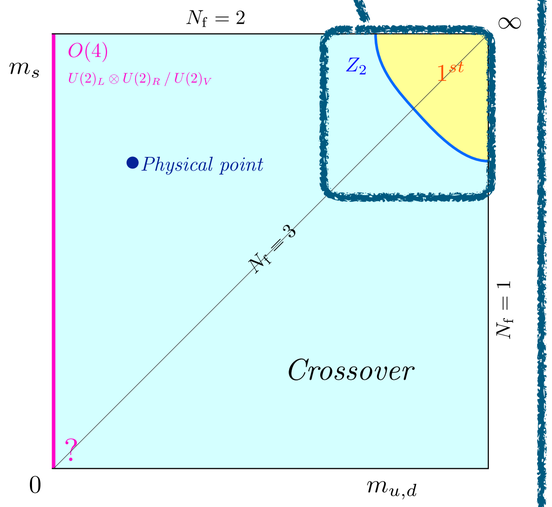
Nakamura et al '22 (PoSLAT21)

Larger 1st order triple regions in the Roberge-Weiss plane

- HISQ fermions with $N_\tau = 4$ — Cuteri et al '22 (PRD)
  - RW endpoint 2nd order for $m_{ud} \geq m_s/320$
  - In the chiral limit $T_\chi = T_{RW} = 195(1)$ MeV
- Unimproved staggered with $N_\tau = 4$ at nonzero imaginary μ_I, μ_B
  - Even larger 1st order region at $N_f = 2$ — Brandt et al '22
- Localization properties of Dirac modes — Cardinali et al '22 (PRD)
- Larger N_f values — Alfredo D'Ambrosio, Tue 15:00

The thermal crossover/transition as a function of the flavor content

Columbia plot: Heavy masses



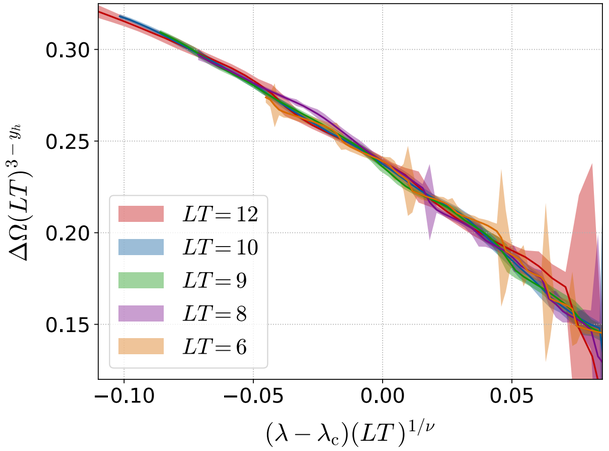
Deconfinement Z_2 boundary

- $N_f = 2$ unimproved staggered at $N_\tau \in \{8,10\}$
  Kaiser, Philipsen, Sciarra '22 (PoSLAT21)
  - Exploring possibility to develop a Ginzburg-Landau effective theory around the $Z2$-critical point

    Reinhold Kaiser, Tue 14:40
- $N_f \in \{1,2,3\}$ Wilson with hopping parameter expansion
  - LO reliable for $N_\tau = 4$, method to include needed higher-order terms for $N_\tau \geq 6$
  - Distribution function of the real part of the Polyakov loop consistent with Z_2 FSS at smaller N_σ/N_τ than B_4

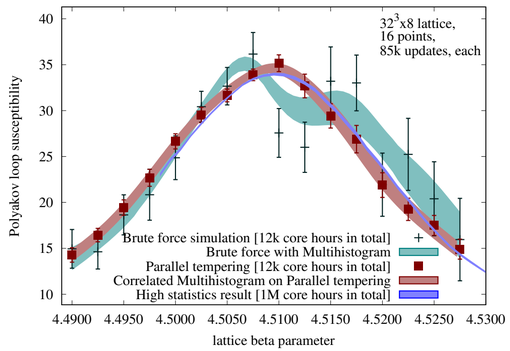
    Kazuyuki Kanaya, Wed 14:00

Kiyohara et al. '21 (PRD)



Reducing autocorrelations with parallel tempering

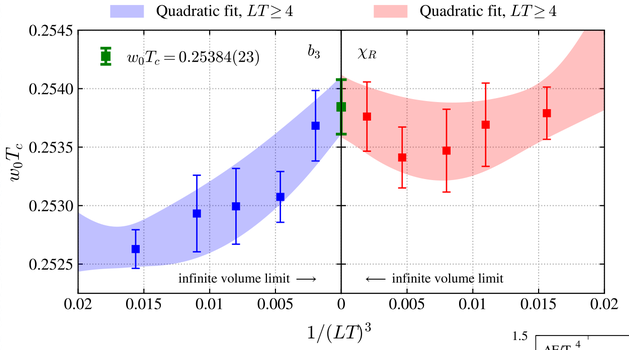
Borsanyi et. al '22 (PRD)



- Allows swap updates between simultaneous calculations at different T
- Can be extended to m, $\imath\mu$ or other parameters of the fermionic action
- Employed for ergodic sampling of Q_{top} using the density of states

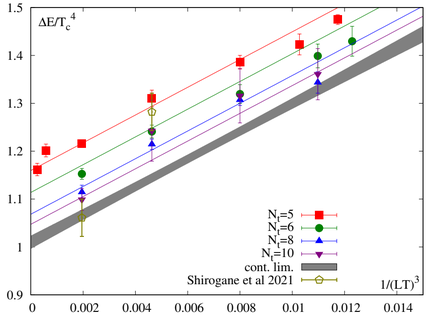
  Cossu et. al '21 (EPJc)

- Quenched QCD: 1st order transition in the continuum



- $w_0 T_c = 0.25384(23)$ first per-mille accurate result in thermodynamics

- latent heat understood as the discontinuity of the trace anomaly, $(\epsilon - 3p)/T^4$, as the transition is crossed

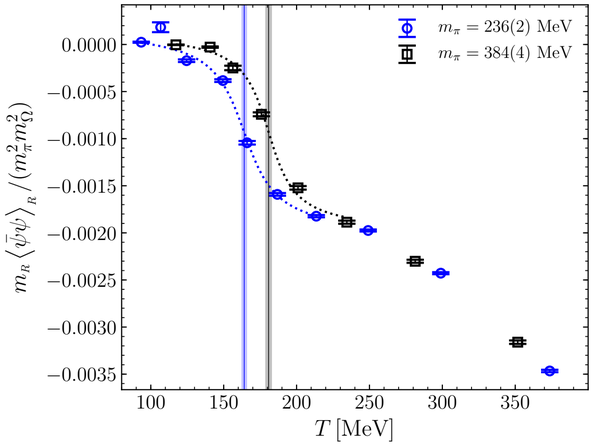


Ruben Kara, Tue 15:20

Thermal effects on hadrons

"Generation 2L" FASTSUM anisotropic, clover $N_f = 2+1$

a_τ [fm]	a_τ^{-1} [GeV]	$\xi = a_s/a_\tau$	a_s [fm]	m_π [MeV]	$T_{pc}^{\bar\psi\psi}$ [MeV]
0.03246(7)	6.079(13)	3.453(6)	0.1121(3)	239(1)	167(2)(1)



- T varied in fixed-scale approach
- T_c shifted by ~ 20 MeV when m_π goes from 390 to 240 MeV

Aarts et al. '22 (PRD)

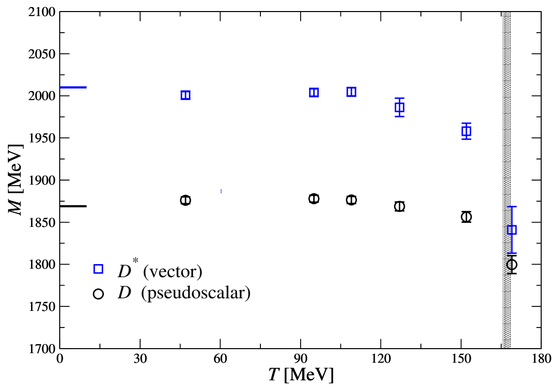
Bottomonium & interquark bottomonium potential

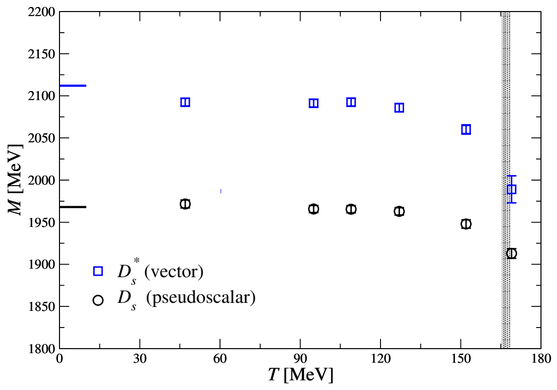
- Backus-Gilbert method for reconstructing spectra of NRQCD bottomonia
  - Results for the η_b, Υ and χ_{b1} generated from Tikhonov-regularized Backus-Gilbert coefficient sets.
- Thermal interquark potential of bottomonium using the HAL QCD method with NRQCD quarks Spriggs et al. '22 (PoS LAT21)

Ben Page, Wed 14:20, Thomas Spriggs, Wed 14:40

Charmed baryons & mesons at $T > 0$ in the hadronic phase

- First systematic study throughout hadronic phase of D and D_s masses (PS & V channels)
  - Groundstate mass at T_{min}
  - T effects, by ratios of correlators (no fitting/spectral reconstruction), at the percentage level





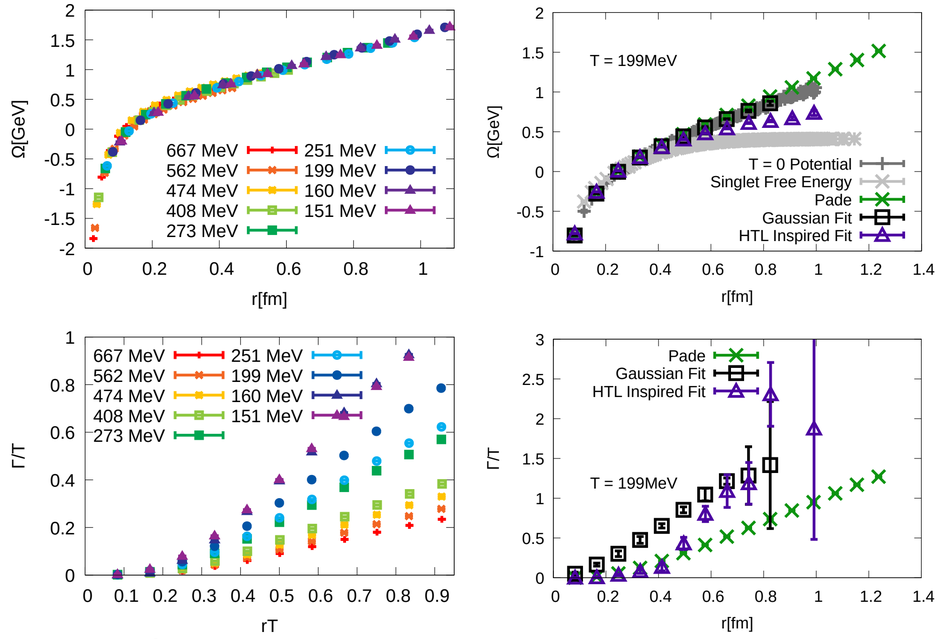
- Singly-, doubly- and triply-charmed baryons: Spectrum in both parity channels and investigating parity doubling due to the restoration

Chris Allton, Wed 16:30, Ryan Bigneli, Wed 16:50

From the hadronic to the QGP phase: correlators, screening masses, spectral functions

Complex static potential at nonzero T, with $N_f = 2+1$ HISQ

- Wilson line correlators in Coulomb gauge — Bala et al. '22 (PRD)
- Spectral functions with different methods: fits, HTL inspired fit, Padé rational approximation, Bayesian reconstruction, …
  - General qualitative feature: Dominant peak in spectral function
    - Position Ω is $\Re V_S^{(0)}$: It is T-independent, shows no screening
    - Effective width Γ/T is $\Im V_S^{(0)}$: It has strong T dependence



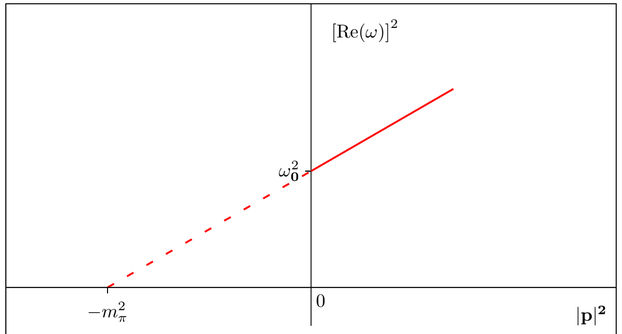
UPDATE: $m_s^{\text{phys.}}$, m_l such that $m_\pi = 310$ MeV on $96^3 \times N_\tau$ lattices with $1/a = 7.1$ GeV and $T \in [127 - 354]$ MeV — *Gaurang Parkar, Tue 17:50*

Charm and beauty in QGP, with $N_f = 2+1$ $\mathcal{O}(a)$-impr. Wilson on HISQ

Correlators & spectral functions in the PS channel using physical valence quark masses

Sajid Ali, Wed 15:40

Pion quasiparticle, with $N_f = 2+1$ $\mathcal{O}(a)$-impr. Wilson



Brandt et al. '15 (PRD)

Brandt et al. '18 (EPJ Web Conf.)

$m_\pi(T=0)$ 'splits' at nonzero T

- Lower pion quasiparticle mass ω_0: Real part of pole of $G_R(\omega, |\vec{p}| = 0)$ of the PS density in ω
- Higher static screening mass (inverse spatial correlation length): Pole of $G_R(\omega = 0, \vec{p})$ of the PS density in $|\vec{p}|$

UPDATE: $N_f = 2+1$, physical quark masses, lower $T = 128$ MeV

- $\omega_0 = 111(3)$ MeV sensibly reduced w.r.t. $m_\pi(T=0) = 130(1)$ MeV
- f_π^t changes little w.r.t. $T = 0$ decay constant
- static screening mass $m_\pi = 143(3)$ MeV increases with T
- chiral symmetry breaking/restoration via difference of the vector- and axialvector spectral functions — *Ardit Krasniqi, Fri 15:50*

From the hadronic to the QGP phase: correlators, screening masses, spectral functions

Exploring QCD up to the electroweak scale

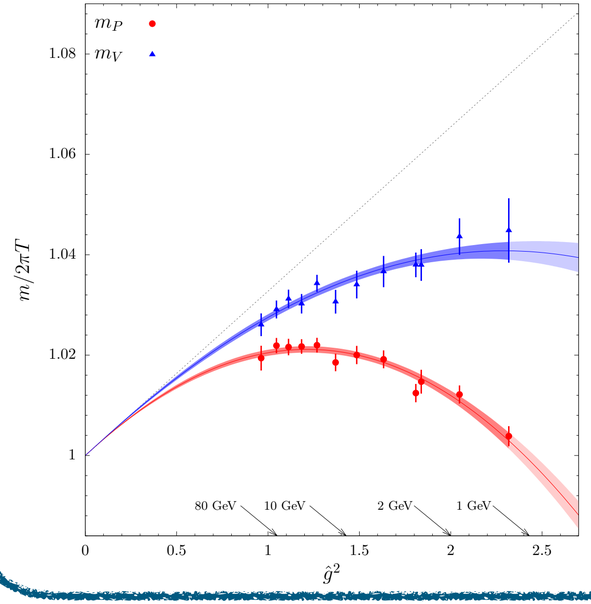
Dalla Brida et al. '22 (JHEP)

QCD non-perturbatively on the lattice even at very high temperatures

- Large range of temperatures from $T \sim 1$ GeV up to $T \sim 160$ GeV, thanks to scale-setting strategy that
  - Exploits non-perturbative, finite-volume, definition of the strong coupling constant to renormalize the theory
  - Employs step scaling techniques

Enables extended comparison with perturbation theory!

Flavor non-singlet meson screening masses (I)...

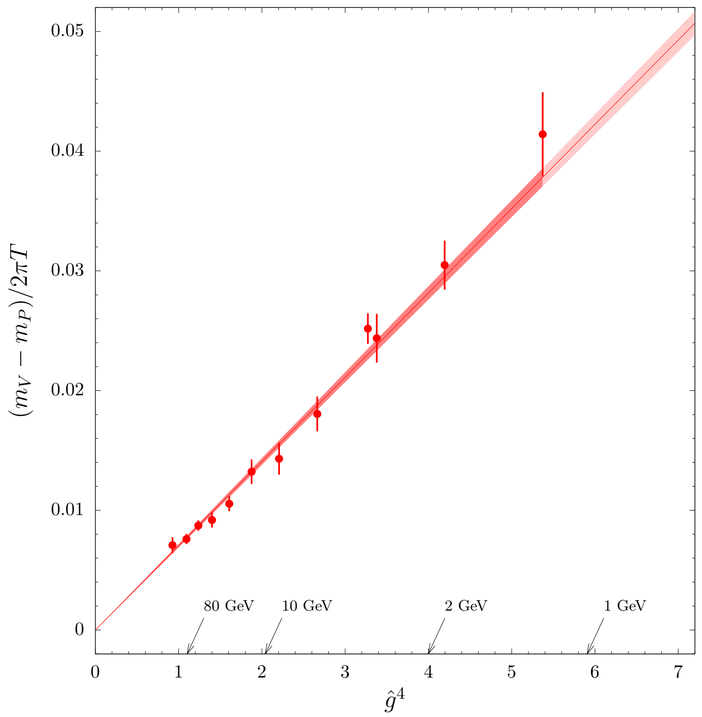


...in QCD with $N_f = 3$ massless quarks, for $T \in [1 - 160]$ GeV

- T-dependence of PS- and V-screening masses (normalized to their free value) compared to NLO-PT
  - Large relative corrections to the perturbative result even at the electroweak scale

Flavor non-singlet meson screening masses (II)...

...in QCD with $N_f = 3$ flavors of massless quarks, for $T \in [1 - 160]$ GeV



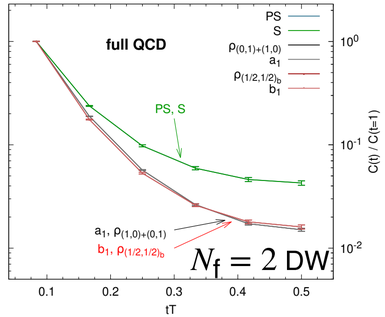
- Splitting between V and PS screening masses
  - Non-negligible up to EW scale, due to higher order effects not explained by NLO-PT
- Chiral symmetry restoration, instead, manifests itself through the degeneracy of the PS and the S channels and of the V and the A ones.

Davide Laudicina, Wed 17:10

From the hadronic to the QGP phase: correlators, screening masses, spectral functions

Emergent chiral spin symmetry and the QCD phase diagram

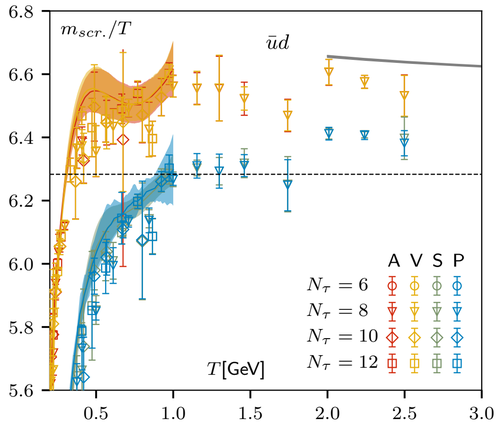
- $T < T_c$, with spontaneously broken chiral symmetry
- $T_c \lesssim T \lesssim 3T_c$, with larger approx chiral spin and SU($2N_f$) symmetries



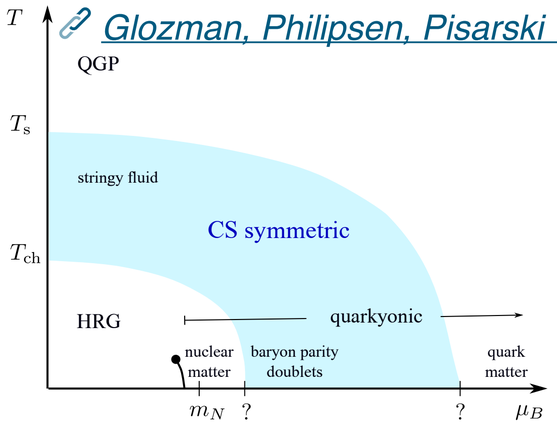
- multiplet patterns, of SU(2)$_{CS}$ and SU($2N_f$) groups, of meson correlation functions
- Dominance of color-electric q-g interactions (dof chirally symmetric quarks bound to color singlet objects by color electric field) *Rohrhofer, Aoki, Glozman '20 (PLB)*

- $T \gtrsim 3T_c$, with screening & usual χ-symmetry recovered via crossover

Additional evidence based on $T \in [0.5 - 0.7]$ GeV meson screening masses: Change of dynamics at the "knee"… *Bazavov et al. '19 (PRD)*



Glozman, Philipsen, Pisarski '22



Pion spectral properties above T_c

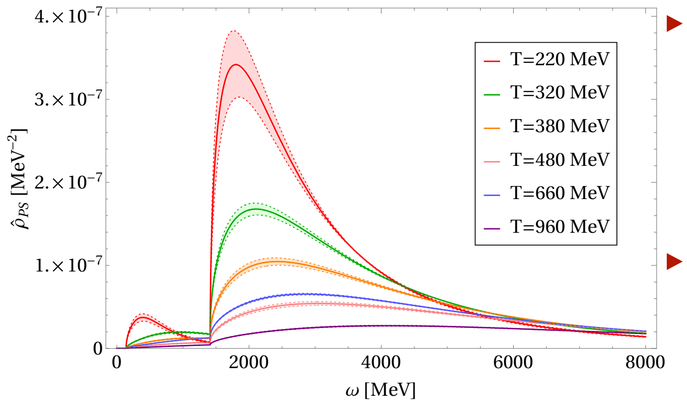
Lowdon, Philipsen '22

Spectral functions: Key quantities to access the transport coefficients parametrizing the in-medium response of QGP to any perturbations

- Related by an integral transform to the conserved current Euclidean correlators measured in lattice QCD
  - Inversion required to extract spectral functions is an ill-posed problem!

IDEA: Exploit constraints imposed by field locality at finite T:

- Non perturbative representation of the spectral function generalizing for T > 0 the Källén-Lehmann representation and featuring so-called "damping factors" (controlling the thermal broadening of peaks)



- Fit lattice spatial PS meson correlator for $T \in [220 - 960]$ MeV
  → damping factors
  → analytic spectral function
- Test: spectral function reproduces temporal correlators @$T = 220$ MeV

- Distinct pion state above T_c + contributions from its first excitation + gradual melting as T increases

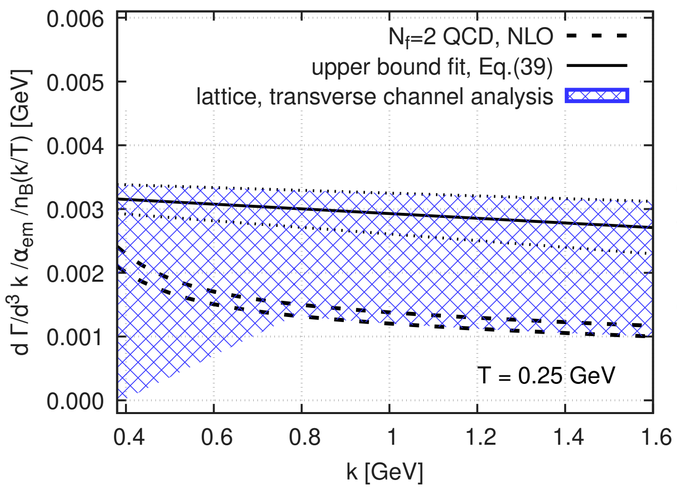
Owe Philipsen, Tue 14:00

Transport properties - Thermal photons from the QGP

How brightly does the quark-gluon plasma glow?

Photon emissivity of the QGP important input to predict photon yield in heavy-ion collisions

- Kin. factors × spectral function associated with 2-point correlator of EM current at light-like kinematics
- Probes in medium interactions: Differs between weakly & strongly coupled plasma in soft-γ regime



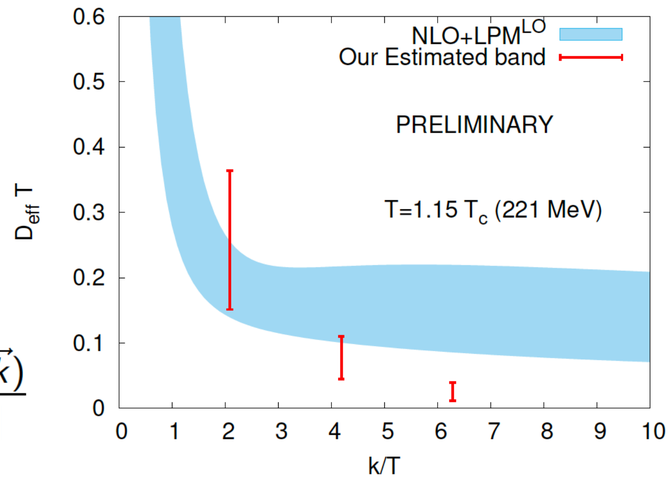
Photon emissivity (and lepton-pair production rate) based on continuum-extrapolated (Euclidean) time-dependent correlators with $N_f = 2$ $\mathcal{O}(a)$-impr. Wilson at $T \sim 1.2T_c$ and $m_\pi \sim 270$ MeV

Cè et al. '22

Harvey B. Meyer, Fri 15:10

$N_f = 2+1$ flavor HISQ configurations with $m_l = m_s/5$ corresponding to $m_\pi \sim 310$ MeV, at $T \sim 1.15T_c$

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)\chi_q}{\pi^2} D_{eff}(k) \qquad D_{eff}(k) = \frac{\rho_H(|\vec{k}|,\vec{k})}{2\chi_q|\vec{k}|}$$



Dibyendu Bala, Fri 15:30

Correlators at imaginary spatial momentum

Cè et al. '22 (PoS LAT21)

New perspective circumventing the inverse problem:

- Dispersion relations at fixed spacelike virtuality, rather than at fixed spatial momentum, to access moments of the spectrum of emitted photons
- Directly computing the analytic continuation of the retarded correlator at fixed, vanishing virtuality of the photon via calculation of the appropriate Euclidean correlator at imaginary spatial momentum

Test in $N_f = 2$ $\mathcal{O}(a)$-impr. Wilson fermions at $T \sim 1.2T_c$ with $m_\pi \sim 270$ MeV

Csaba Török, Tue 17:30

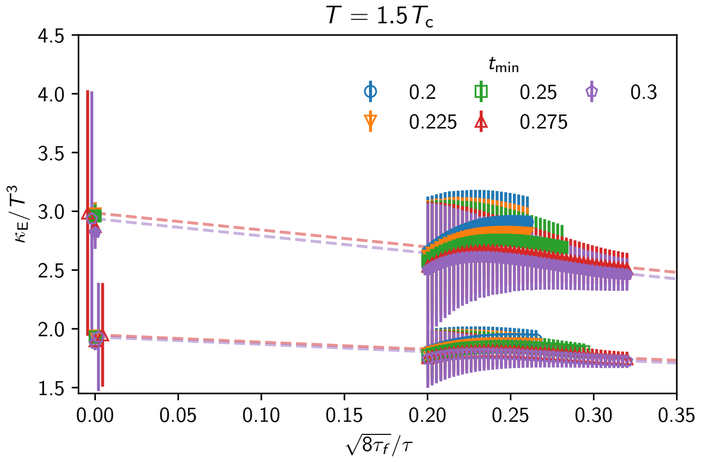
Transport properties - Heavy quarks equilibration time in the QGP

How fast do heavy quarks thermalize in a hot medium?

- Equilibration time ↔ heavy quark diffusion (Brownian motion) described by Langevin equation depending on three related transport coefficients
  - Heavy quark diffusion coefficient κ related, in thermal equilibrium, to the heavy quark diffusion coefficient D via $D = 2T^2/\kappa$
- κ ↔ Euclidean correlators of field strength tensor components in the heavy quark limit $M \gg \pi T$ (HQET)
  - Correlator of two chromo-electric fields E ↔ Leading contribution in the T/M expansion κ_E
  - Correlator of two chromo-magnetic fields B ↔ T/M correction κ_B

Heavy quark diffusion coefficient in SU(3) pure gauge

Brambilla et al. '22

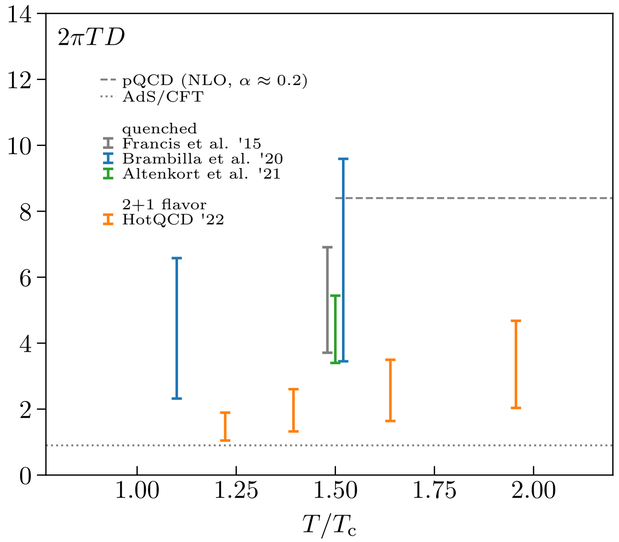


- Chromo- electric & -magnetic correlators
  - $1.70 \leq \kappa_E/T^3 \leq 3.12$
  - $1.23 \leq \kappa_B/T^3 \leq 2.74$
- Mass suppressed effects in the heavy quark diffusion coefficient are 20% for bottom and 34% for charm quark at $T = 1.5T_c$

Viljami Leino, Wed 15:00

Heavy quark diffusion coefficient, with $N_f = 2+1$ HISQ

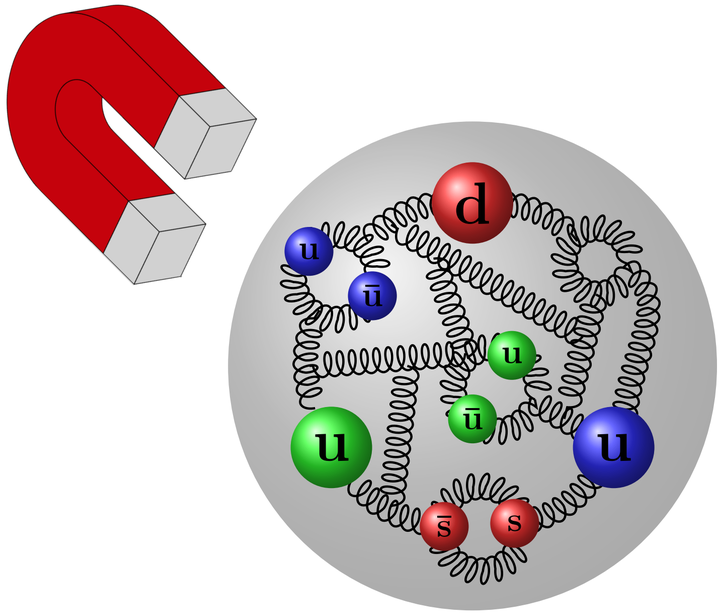
Altenkort et al. '22 (PRD)

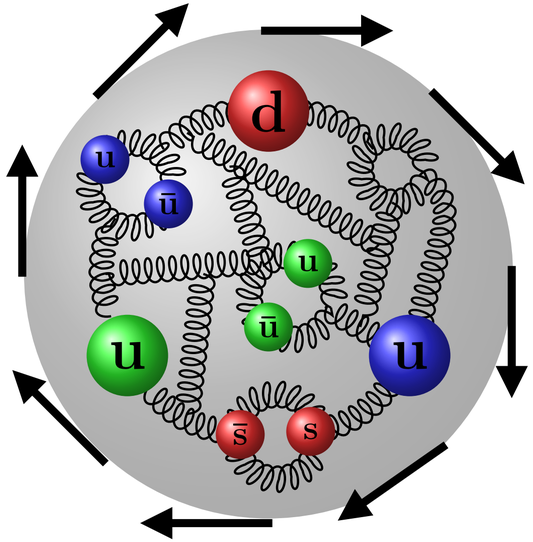


- First study on heavy quark momentum diffusion with dynamic quarks
- Significant decreasing of $2\pi TD$ with dynamical quarks as compared to the quenched case

Luis Altenkort, Wed 15:20

Nonzero EM fields or vorticity



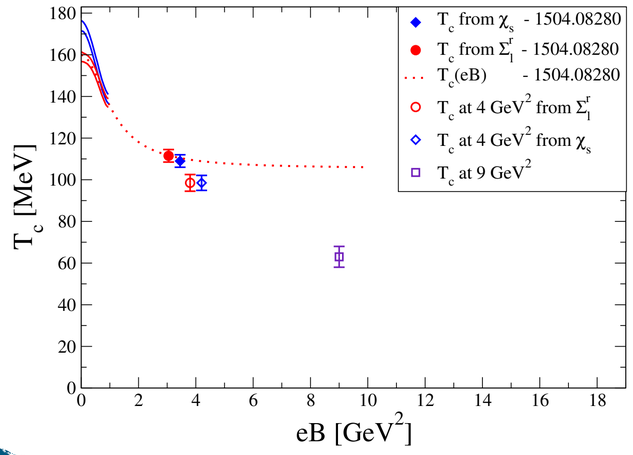


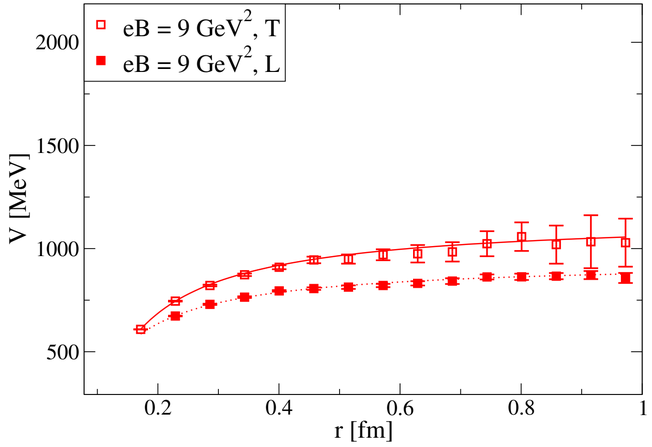
- Nonzero EM fields
  - Magnetic phase diagram
  - Topological aspects
  - Anomalous transport
  - Inhomogeneous B fields
- Vorticity

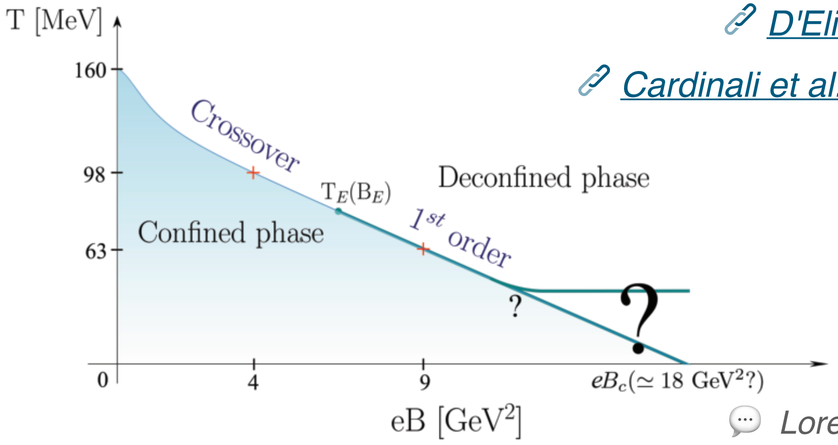
Phase diagram in a magnetic/vortical background & inhomogeneous B fields

QCD phase diagram in a magnetic background, with $N_f = 2+1$ stout-improved staggered quarks at physical quark masses

- Crossover at $eB = 4\ \mathrm{GeV}^2$ (with $T_c = (98 \pm 3)\ \mathrm{MeV}$) and strong 1st order phase transition at $eB = 9\ \mathrm{GeV}^2$ (with $T_c = (63 \pm 5)\ \mathrm{MeV}$)
- Transition from strongly anisotropic confined phase to completely deconfined phase: $\sigma = 0$ in all directions
- Steady decrease of $T_c \to$ Will $T_c(B)$ continue its drop until $T_c(B_c) = 0$?





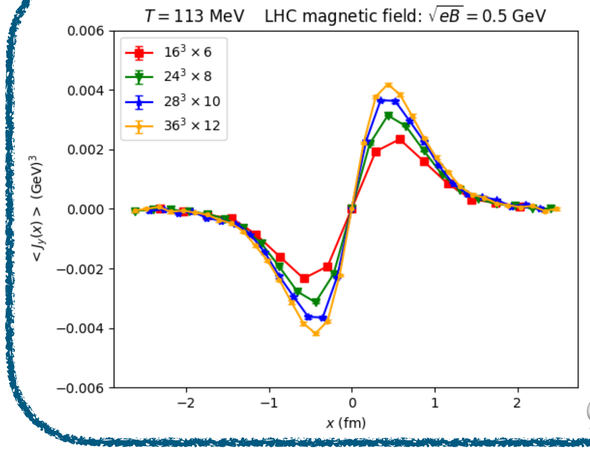


D'Elia et al. '22 (PRD)

Cardinali et al. '22 (PoS LAT21)

Lorenzo Maio, Fri 14:30

Inhomogeneous $\mathbf{B}$, with $N_f = 2+1$ stout-improved staggered



Brandt et al. '22 (PoS LAT21)

$$\mathbf{B}(x_1) = B\left(\cosh\frac{x_1 - L_1/2}{\epsilon}\right)^{-2}\hat{\mathbf{z}}$$

- Prominent steady electric currents at LHC-like B fields

Adeilton Dean Marques Valois, Fri 14:10

Rotating QCD with $N_f = 2$ dynamical quarks

Braguta et al. '21 (PRD)

- Phenomenological effective models predict decrease in T_c in rotating QCD
- T_c in gluodynamics found to increase due to rotation, with $C_2 > 0$ in

$$T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2$$

- Rotation of gluons and fermions found to have opposite effect on T_c…
  - …with some dependence on the pion mass

Artem Roenko, Fri 17:20

Non-dissipative transport effects and topological aspects

Anomalous transport phenomena: Chiral Separation Effect (CSE)

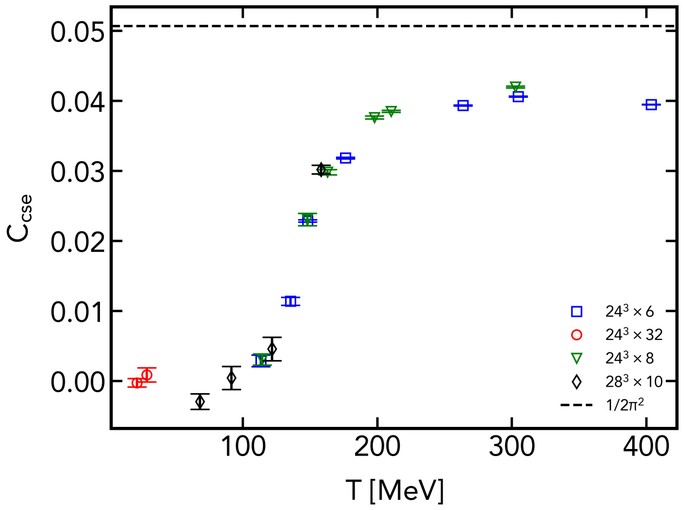
Quantum anomalies + EM fields → non-dissipative transport, e.g. CSE

- At nonzero μ and (strong) B-field → CP-odd axial current appears

$$j_i^A = C_{\text{cse}} \mu B_i$$

- On the lattice one can measure, from $\mu = 0$ simulations, at nonzero B_z,

$$\left.\frac{\mathrm{d}j_z^A}{\mathrm{d}\mu}\right|_{\mu=0} = C_{\text{cse}} B_z$$



- Free case: confirmed analytic prediction $C_{\text{cse}} = 1/(2\pi^2)$
- Interacting case ($N_f = 2+1$ stout-impr. staggered at physical masses):
  - suppression at low T, approach free case value at high T
  - m-dependence to be checked, continuum limit to be taken

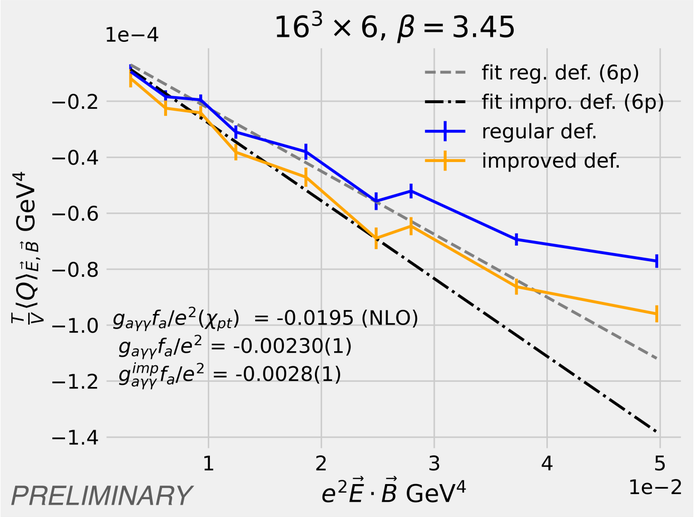
Eduardo Garnacho Velasco, Wed 17:50

Topology in electromagnetic fields and the axion-photon coupling

- Direct axion-photon coupling $g_{a\gamma\gamma}^{\text{QCD}}$ by linear response of Q_{top} to weak background EM fields (f_a, energy scale suppressing axion dynamics)

$$\frac{T}{V}\left\langle Q_{\text{top}} \right\rangle_{E,B} \approx \frac{g_{a\gamma\gamma}^{\text{QCD}} \cdot f_a}{e^2} e^2 \vec{E} \cdot \vec{B}$$

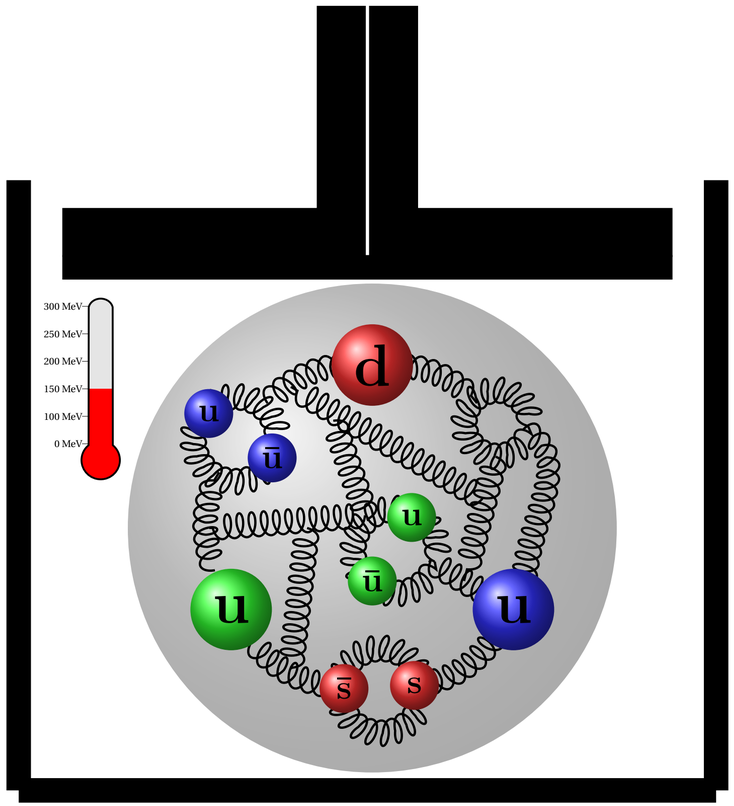
- $N_f = 2+1$ stout-impr. staggered at physical masses with B and $\imath E$
- Weight of topological sectors with non-zero topological charge enhanced



- Two different lattice discretizations for Q_{top}
- Fit in the linear region
- Comparison with chiral perturbation theory prediction

José Javier Hernández Hernández, Fri 14:50

Nonzero density



- Phase transitions
- Fluctuations of conserved charges
- Equation of State
- Speed of sound
- Properties of dense QGP
- The cold and dense regime

Phase transitions in QCD at nonzero density

Multi-point Padè for the study of phase transitions

Singh et al. '22 (PoS LAT21)

Method to investigate the QCD phase diagram based on

- Computation of Taylor series coefficients at both $\mu = 0$ and $\mu = \imath\mu_B$
- Poles of multi-point Padè approximants for locating Lee Yang edge singularities

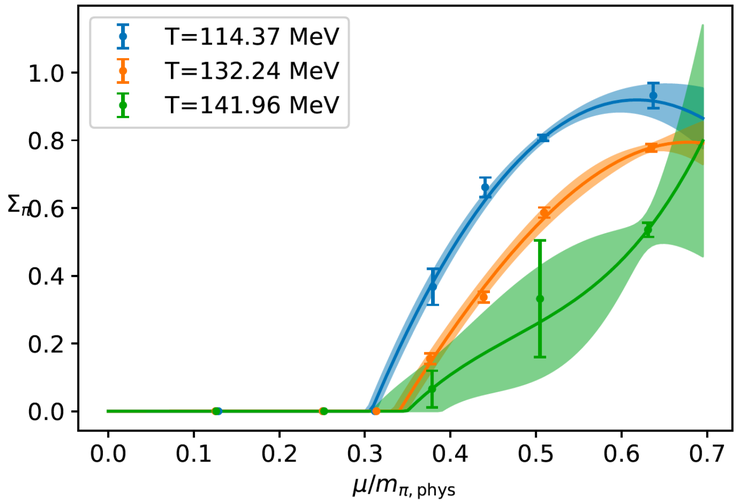
Method applied to:

- Two-dimensional Ising model — *Francesco Di Renzo, Mon 14:00*
- $N_f = 2 + 1$ QCD at physical masses, with HISQ — *Kevin Zambello, Mon 14:20*
  - identifying singularities of the net-baryon number density in the complex μ_B plane using conserved charges
  - close to the Roberge-Weiss (RW) transition, the location of the LYE singularities scales according to the 3-d $\mathbb{Z}2$ universality class
  - preliminary results for singularities close to the chiral phase transition

Pion condensation at lower than physical quark masses

Isospin imbalance in QCD drives transition to Bose-Einstein Condensation (BEC) phase with condensation of charged pions.

- Phase diagram affected by the flavor content
  - At the physical point → BEC boundary known in large μ_I and T ranges
  - In the chiral limit → $T = 0$ condensation at infinitesimally small μ_I
- $\mu = 0$ chiral transition might be affected, depending on the shape of the BEC boundary, by its proximity!



- First step towards the chiral limit: simulations of $N_f = 2 + 1$ stout-impr. staggered fermions at half the physical quark masses
- BEC boundary mapped and compared with the results at physical masses

Volodymyr Chelnokov, Mon 16:30

Fluctuations of conserved charges

Charm fluctuations - $N_f = 2+1$ QCD @high T & physical masses

Measuring dimensionless generalized susceptibilities of conserved charges

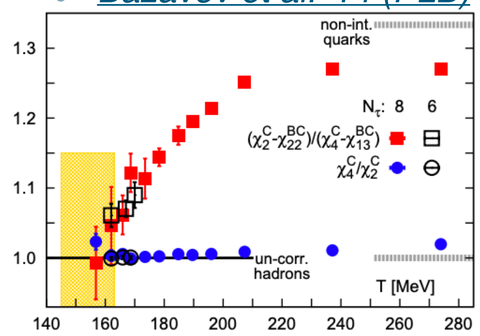
$$\chi^{\mathrm{BQSC}}_{klmn} = \left.\frac{\partial^{(k+l+m+n)}\left[p(\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S,\hat{\mu}_C)/T^4\right]}{\partial\hat{\mu}_B^k\partial\hat{\mu}_Q^l\partial\hat{\mu}_S^m\partial\hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

for comparison with:

- Confined phase phenomenology by hadron resonance gas model (HRG)
- Ratios of identified particles yields in heavy-ion collisions

Validity range of quark model extended HRG description of open charm sector and dissociation temperature of charmed hadrons by extraction of

Bazavov et al. '14 (PLB)



- 2nd and 4th order cumulants of charm fluctuations
- Correlations of charm with lighter conserved flavor quantum numbers
- Appropriate combinations of the above

Sipaz Sharma, Tue 16:30

Electric charge fluctuations using 4HEX quarks

- Most severe lattice artefacts among fluctuations

P. Parotto, Tue 16:50

  - Data with a novel discretization suppressing artefacts

→ continuum extrapolated results in T region of chemical freeze-out

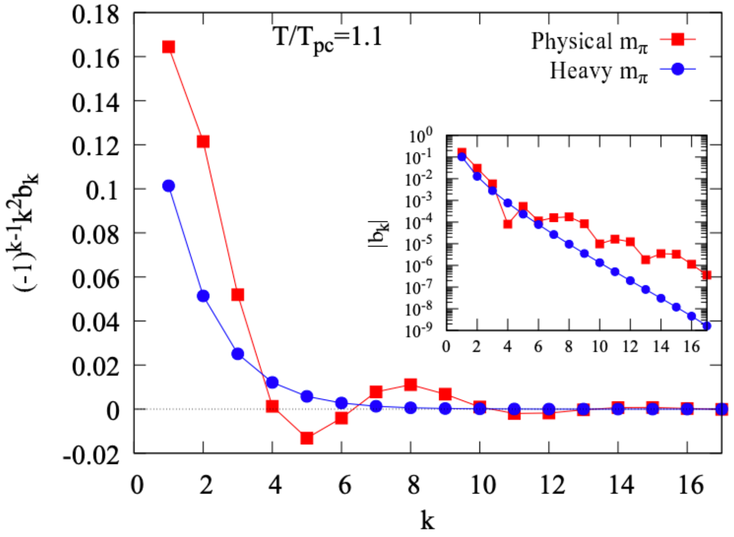
Fourier coefficients of net-baryon number as function of $\mu = \imath\mu_\imath$

Coefficients' asymptotics at large k governed by singularity structure of $\mathcal{Z}_{\mathrm{QCD}}$ in the complex chemical potential plane

→ They encode information on phase transitions

→ characteristic behavior reflected also in the baryon number fluctuations

- Calculation via asymptotic numerical quadrature designed for highly oscillatory integrals
- Estimate the position of the nearest singularities in the complex μ plane
- Compare:



- Data from simulations of QCD with $N_f = 2+1$ HISQ at physical quark masses at imaginary chemical potential on $N_\tau \in \{4,6,8\}$ lattices
- Predictions from phenomenological models

Almasi et al. '19 (PRD)

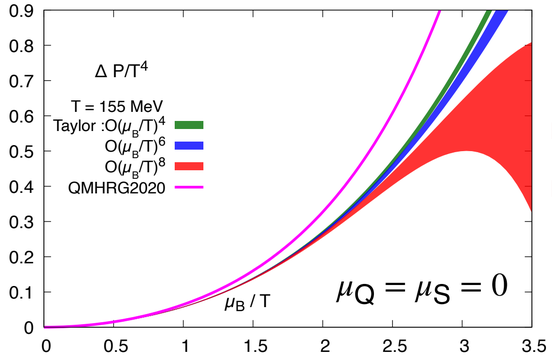
Christian Schmidt, Mon 14:40

Equation of State (EoS)

Isentropic equation of state in $N_f = 2+1$ QCD

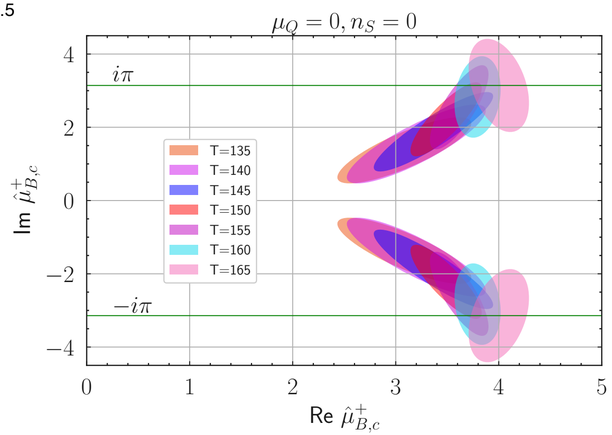
Bollweg et al. '22 (PRD)

Based on high precision Taylor expansion and Padé-resummed expansion



- Energy density and pressure along lines of constant entropy per net baryon-number density
- 8th order Taylor series for p ($N_\tau = 8$)
- Straightforward Taylor series expansion for $\Delta P/T^4$
  → well controlled description up to $\mu_B/T \leq 2.5$

- Location of singularities in the complex μ_B-plane (influencing convergence of Taylor series)



- Studied isospin symmetric case $\mu_Q = 0$ at $\mu_S = 0$, and at $n_S = 0$
- Results compared at low-T with HRG model calculations based on QMHRG2020 hadron list including resonances in relativistic quark models

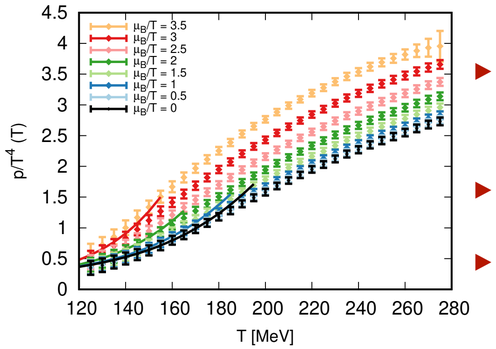
Jishnu Goswami, Thu 09:40

Resummed equation of state at nonzero baryon density

Borsanyi et al. '22 (PRD)

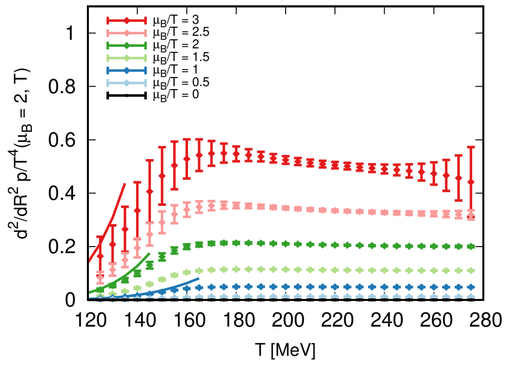
- Continuum extrapolated results with extrapolations from imaginary μ using 4stout-impr. staggered fermions with $N_\tau \in \{8,10,12,16\}$
- Physically motivated extrapolation scheme based on observation, that the main effect of an imaginary μ is parametrized by a shift (F observable of interest, of sigmoid shape in T) Borsanyi et al. '21 (PRL)

$$\frac{F(T,\hat{\mu}_B)}{F^{SB}(\hat{\mu}_B)} = \frac{F(T'_F,0)}{F^{SB}(0)}, \quad \text{with } T'_F = T\left(1 + \kappa_2^F(T)\hat{\mu}_B^2 + \kappa_4^F(T)\hat{\mu}_B^4 + \dots\right)$$



- Scheme generalized to the case of non-zero μ_S (focusing on $n_S = 0$)
- Larger T dividing by Stefan-Boltzmann limit
- Up to $\mu_B/T \leq 3.5$, $T \in [130-280]$ MeV

- Relax $n_S = 0$: Extrapolate to small $R = n_S/n_B$
  - In HIC, only global strangeness neutrality guaranteed with large local charge fluctuations



Jana N. Guenther, Thu 10:40

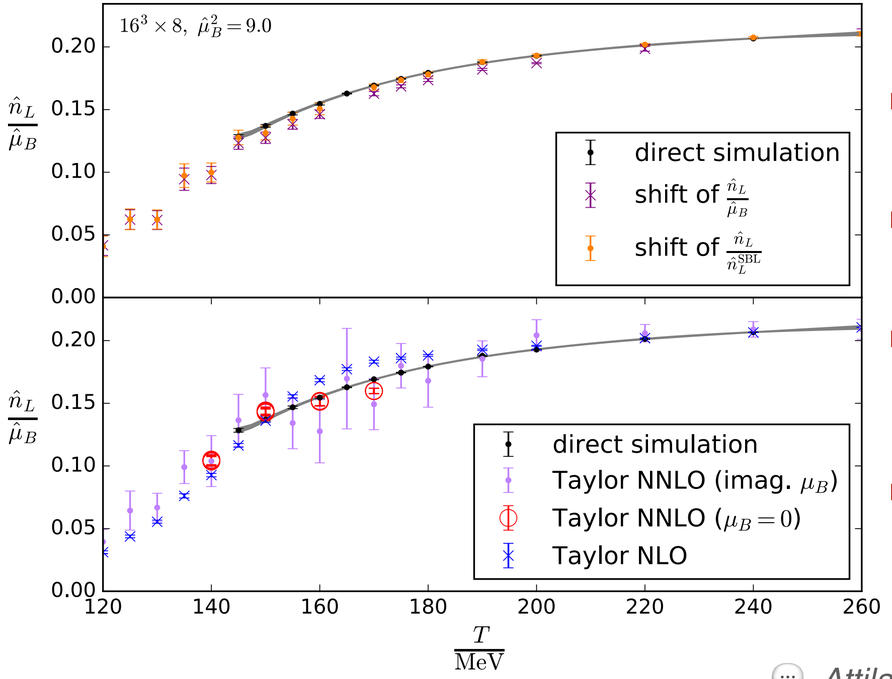
Equation of State (EoS)

Hot & dense QGP EoS: direct results *vs.* extrapolations

Direct reweighting Giordano et al. '20 (JHEP) Borsanyi et al. '22 (PRD)

2-stout-impr. staggered fermions, with $N_\tau = 8$ and at phyical quark masses

- Results up to $\mu_B/T = 3$ in the QGP phase
- Comparison with results from different extrapolation procedures:
  - Taylor expansion around zero chemical potential,
  - some of its resummations Borsanyi et al. '21 (PRL)



- Tension with Taylor up to χ_4^B
- Large errors with Taylor up to χ_6^B
- Newer schemes for extrapolation perform better
- Superiority of direct method

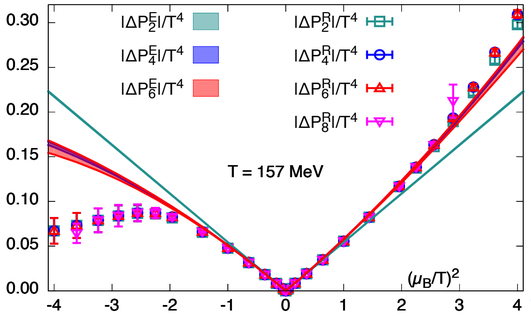
Attila Pasztor, Thu 10:20

Yet another way of extending the QCD EoS to larger μ_B/T

Resummation scheme for the Taylor expansion of the EoS in μ_B based on n-point correlation functions of the conserved (baryon) current D_n

$$\frac{\Delta P_N^E}{T^4} = \sum_{n=1}^{N} \frac{\chi_n^B}{n!}\left(\frac{\mu_B}{T}\right)^n \longleftrightarrow \frac{\Delta P_N^R}{T^4} = \frac{1}{T^3 V}\ln\left\langle \exp\left[\sum_{n=1}^{N}\frac{D_n}{n!}\left(\frac{\mu_B}{T}\right)^n\right]\right\rangle$$

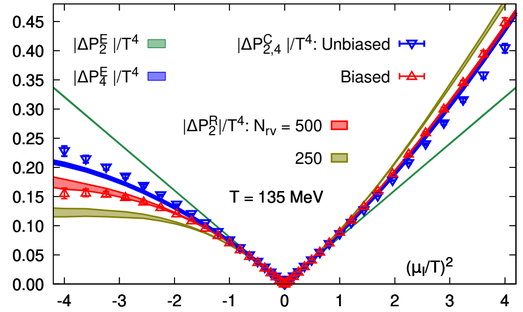
resums contributions of the first N correlation function $D_1; \ldots; D_N$ to the Taylor expansion of $\mathcal{Z}_{\rm QCD}$ to all orders in μ_B



- Resummed $\mathcal{Z}$, approximation to the reweighted $\mathcal{Z}$ at $\mu_B \neq 0$
- Improved convergence, breakdown reflects singularities of $\mathcal{Z}$ & severity of sign problem

Mondal, Mukherjee, Hegde '22 (PRL)

- D_n calculated stochastically → bias in exp factor (esp. @large N, μ_B)
  - Resum based on cumulant expansion Mitra, Hegde, Schmidt '22



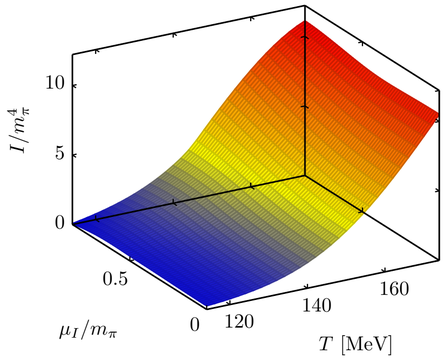
- Each term evaluated using unbiased powers of the operators
- Truncating at max order M, only finite unbiased products of D_n

Sabarnya Mitra, Thu 10:00

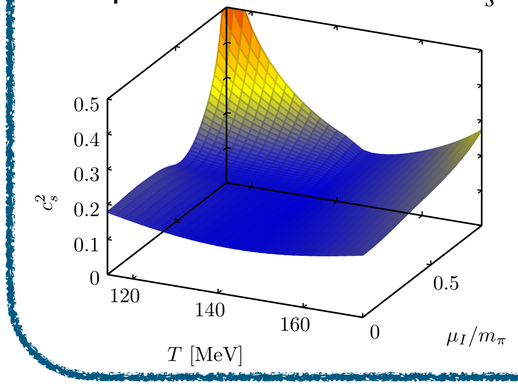
Equation of State (EoS) and speed of sound

Equation of state and Taylor expansions at $\mu_I \neq 0$

Brandt et al. '22 (PoS LAT21)



- QCD EoS at $\mu_I \neq 0$ and zero and non-zero T from simulations, with $N_f = 2+1$ improved staggered fermions at physical quark masses and $N_\tau \in \{8,10,12\}$
- First steps toward extending the EoS to small $\mu_B \neq 0$ via Taylor exp.
- Speed of sound. $c_s^2 = \frac{\partial p}{\partial \epsilon}\Big|_{@\left[\frac{s}{n_I} = \text{const};\ \frac{n_I}{n_L} = \text{const};\ \ldots\right]}$



- Violation of holographic bound:

$$T \ll T_c \;\&\; \mu_I \gg m_\pi/2 \longrightarrow c_s^2 > 1/3$$

implications for EoS modelling based on neutron star radii & masses

Bastian Brandt, Thu 09:00

Isothermal & isentropic c_s^2 in $N_f = 2+1$ QCD at nonzero μ_B

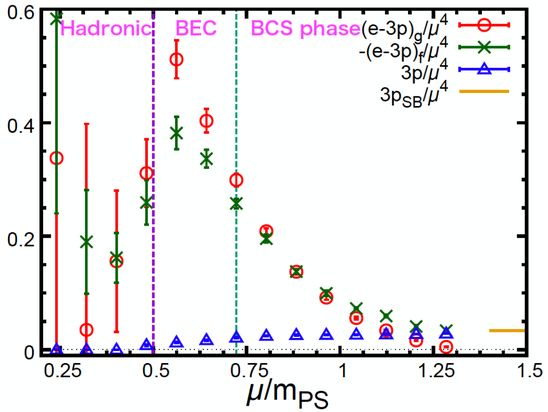
- c_T^2 and c_S^2 & comparison with HRG at low T & vanishing μ_S or n_S
- Imprint of chiral transition on c_T^2 and c_S^2 (sensitive a peak in specific heat, characteristic for critical behavior of 3d, O(N) spin models) at low m_π

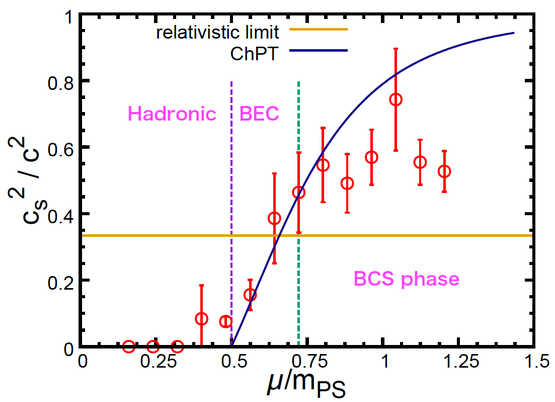
David Anthony Clarke, Thu 09:20

Dense 2-color QCD: Bump of sound velocity & hadron masses

Iida, Itou '22

- $p(\epsilon)$ and $\frac{c_s^2}{c^2} = \frac{\partial p}{\partial \epsilon}$ for 2-color QCD at low T and nonzero μ





Chiral Perturbation Theory (ChPT), $c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$

In BEC phase, our result is consistent with ChPT.
In high-density, it seems to peak around $\mu \approx m_{PS}$.

in the superfluid phase $c_s^2/c^2 > 1/3$

- Result consistent with results from effective models which have shown the bump of sound velocity.

Etsuko Itou, Thu 12:10

Kotaro Murakami, Thu 12:30

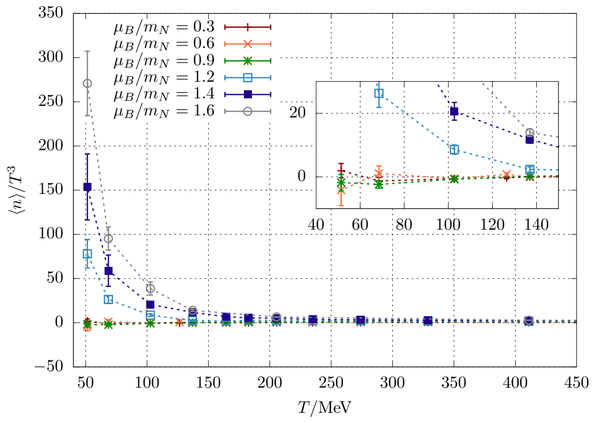
- Flipping of spectral ordering near the transition between the hadronic and superfluid phases of π and ρ meson masses masses ($N_f = 2$)
- 2-point functions for $I = 0$, $J^P = 0^\pm$ consistent with expected mixing (lin. σ model) of meson, diquark, antidiquark states due to $U(1)_B$ breaking in superfluid phase

Equation of State (EoS), EM conductivity and screening masses

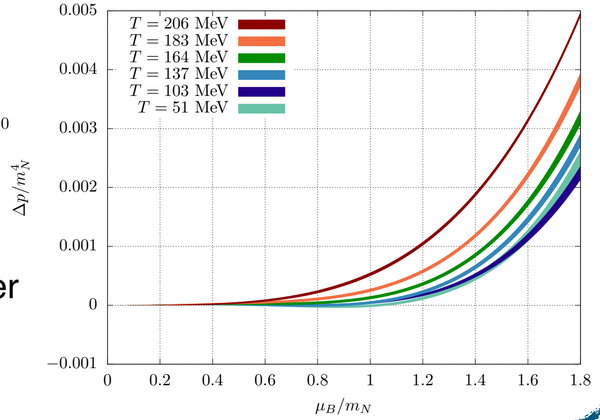
$N_f = 2$ QCD EoS via the complex Langevin method

Attanasio, Jäger, Ziegler '22

First step towards low T and physical pion mass, with $m_\pi \approx 480$ MeV



- Silver Blaze phenomenon: vanishing $\langle n \rangle / T^3$, at T = 0 for $\mu < m_N/3$



- Predictions for EoS at $n \sim 15 n_0$
- EoS stiffer at low T: p grows slower
- Energy & entropy EoS measured

Felix Ziegler, Thu 11:30

QGP electromagnetic conductivity at $\mu_B \neq 0$

- $N_f = 2 + 1$ stout improved staggered at the physical point, at $\mu = \imath \mu_\imath$

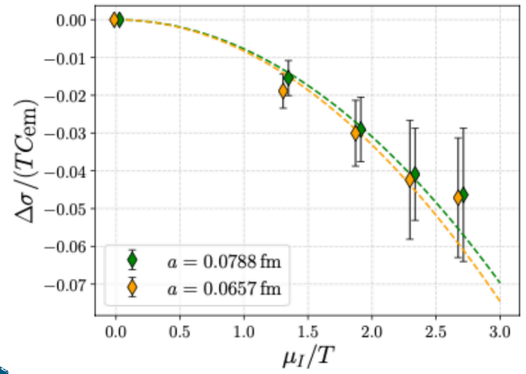
$$C_{ij}^{e,o}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh[\omega\beta/2]} \rho_{ij}^{e,o}(\omega), \quad \frac{\sigma_{ij}}{T} = \frac{1}{2T} \lim_{\omega \to 0} \frac{\rho_{ij}^e(\omega) + \rho_{ij}^o(\omega)}{\omega}$$

- Conductivity from $J - J$ correlators with modified Backus-Gilbert method

Hansen, Lupo, Tantalo '19 (PRD)

- $\frac{\Delta\sigma}{T C_{\text{em}}} = -c(T) \left(\frac{\mu_I}{T} \right)^2$, with $C_{\text{em}} = e^2 \sum_f q_f^2$, and $\Delta\sigma = \sigma_{\mu_I} - \sigma_{\mu_I = 0}$

well described by quadratic polynomial, analytically continued to real μ_B



- Electromagnetic conductivity of QGP grows rapidly with real μ_B

Manuel Naviglio, Mon 15:20

More on complex Langevin & contour deformations

- Boundary terms in Complex Langevin for full QCD: updates w.r.t. Sexty et al. '22 (PoS LAT21) — Michael W. Hansen, Thu 11:50
- Alleviating sign problem in (2+1)d XY model at $\mu \neq 0$ on deformed integration manifolds Giordano et al. '22 — Zoltan Tulipant, Mon 18:10

Psudoscalar meson screening mass at $T \neq 0$ and $\mu_B \neq 0$

Bazavov et al. '19 (PRD)

- By expanding the screening correlator in a Taylor series in μ_B
  - Results for the second derivative of the screening mass w.r.t. μ_B on $64^3 \times 8$ lattices generated using the $N_f = 2 + 1$ HISQ action

Rishabh Thakkar, Mon 15:00

The cold and dense regime

Chirally inhomogeneous phases in low-energy models for QCD

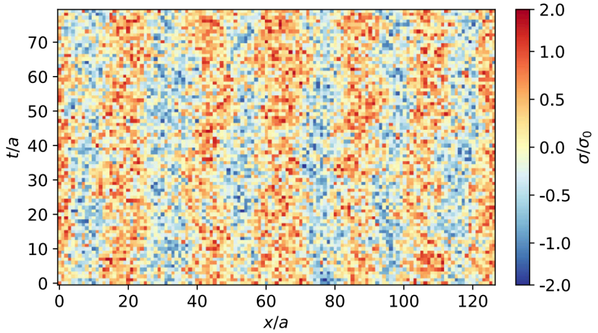
Four-Fermi theories modelling (some) chiral properties of QCD treated analytically by mean-field, or large-N expansions, numerically via MC

- From 1+3D Nambu-Jona-Lasinio model, prediction of chirally inhomogeneous phase (broken transl. symmetry) at low T & large μ_B

CAVEAT: Non-renormalizable→Possible regularization scheme dependence

Laurin Pannullo, Mon 17:50

- What about lower-d realisations of models with χ-symmetry (breaking)?



- **1+1D**→no spontaneous symmetry breaking, quasi-long-range order
  - Chiral crystals (consistent with BKT-like algebraically decaying correl. functions) at $N_f \in \{2,8,16\}$ *Lenz et al. '20 (PRD)*
- **1+2D** inhomogeneities shown to be pure cutoff effects in Gross-Neveu model in MF and in a variety of Four-fermion and Yukawa models

Marc Winstel, Tue 17:10

- Constant B-field brings inhomogeneities back?

Michael Mandl, Mon 15:40

- Or quantum-spin liquid phases in all considered cases?

The cold & dense regime from strong coupling

- Hamiltonian lattice QCD with staggered fermions from continuous time limit of strong coupling lattice QCD based on the dual representation that is obtained when integrating out the gauge links first
  - Formalism extended to $N_f = 2$ and, after a resummation, there is no sign problem both for $\mu_B \neq 0$ and for $\mu_I \neq 0$
  - progress on implementation of the Quantum Monte Carlo simulations
  - Computed baryon and isospin densities in the chiral limit
  - Looked at quark mass dependence of baryon mass & nuclear transition

Pratitee Pattanaik, Mon 16:50, Wolfgang Unger, Mon 17:10

The cold & dense regime from Polyakov loop effective theories

- For the strong coupling and heavy quark mass regime of QCD
  - New set of effective couplings whose T dependence worked out via a method to map correlators of Polyakov loops in the effective theory (high-order expressions using finite-cluster method) to those in full QCD

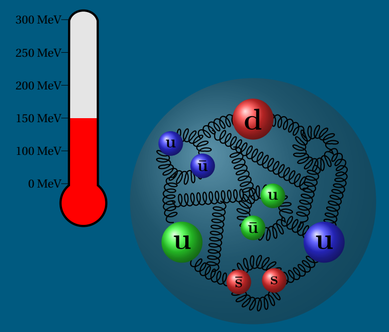
Christopher Winterowd, Fri 17:40

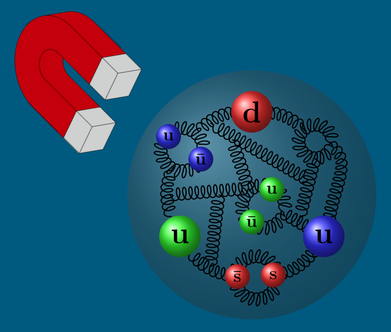
  - Mean-field approximations employed to determine the critical endpoint of the deconfinement transition and the phase diagram at non-zero baryon and iso-spin chemical potential

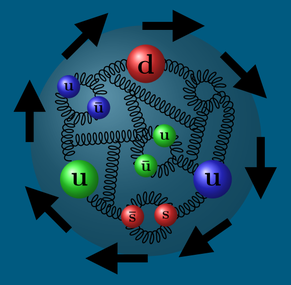
Amine Chabane, Mon 17:30 , Christoph Konrad, Fri 18:00

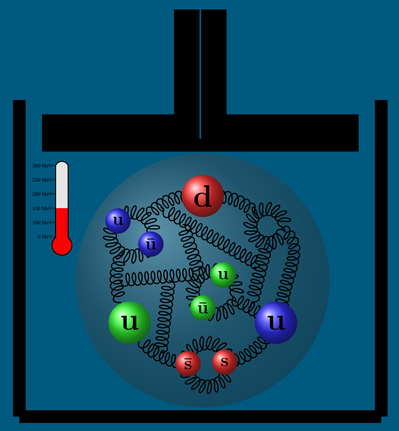
Thank you for your attention!

Please, drop by at the 55 talks (!!)
In the 'Non-zero Temperature' & 'Non-zero Density' parallel sessions!









A special thank you to those who wrote to me with their results!
[Sorba, Lenz, Laudicina, Günther, Jäger, Bietenholz, Itou, Aarts]