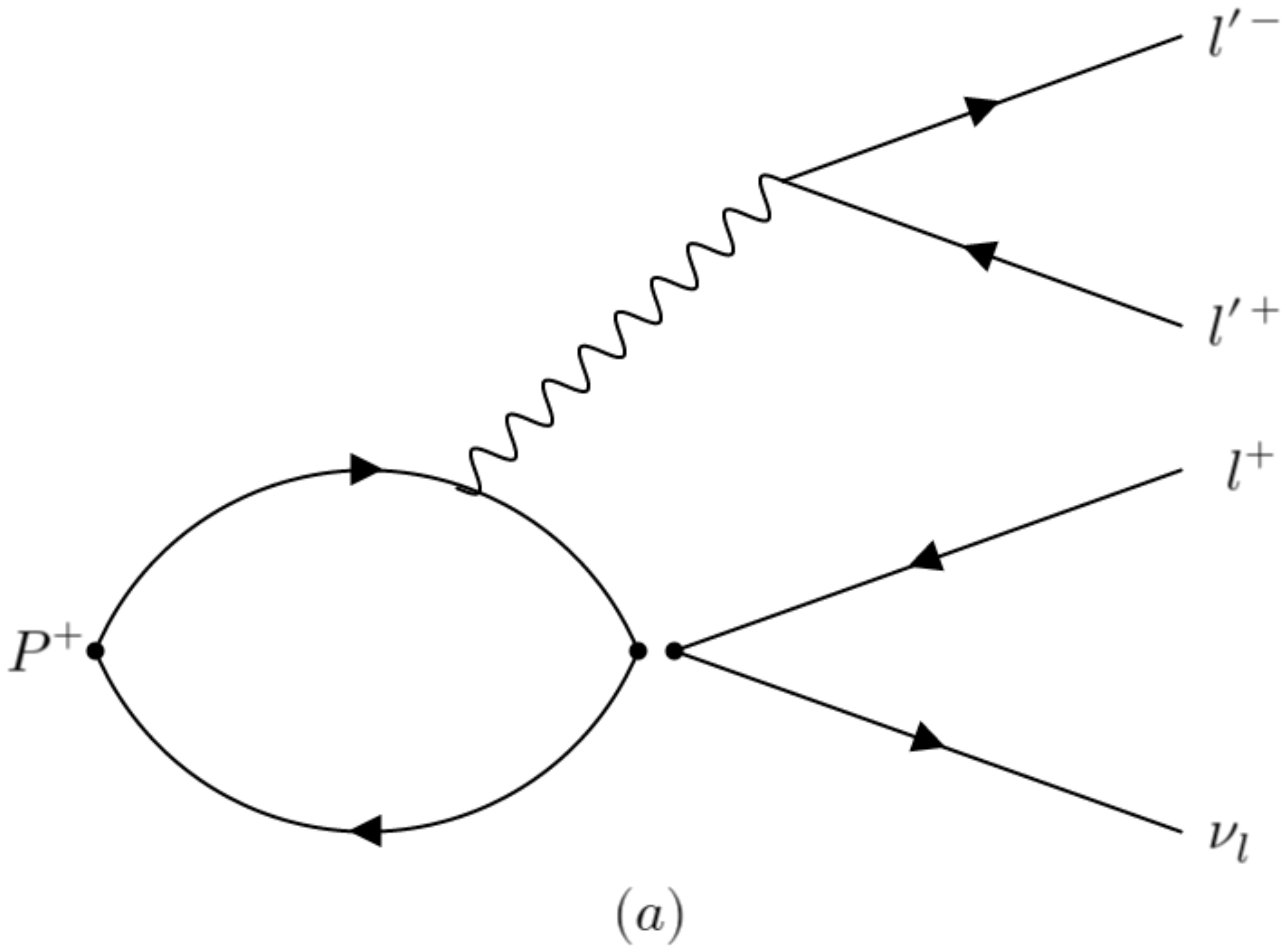
Lattice Results for the $K^+ \to \ell^+ \nu_\ell \ell'^+ \ell'^-$ Form Factors and Branching Ratios



G. Gagliardi, F. Sanfilippo, S. Simula, V. Lubicz, F. Mazzetti, G. Martinelli, C. T. Sachrajda, and N. Tantalo,
Phys. Rev. D 105, 114507 (2022).

Filippo Mazzetti

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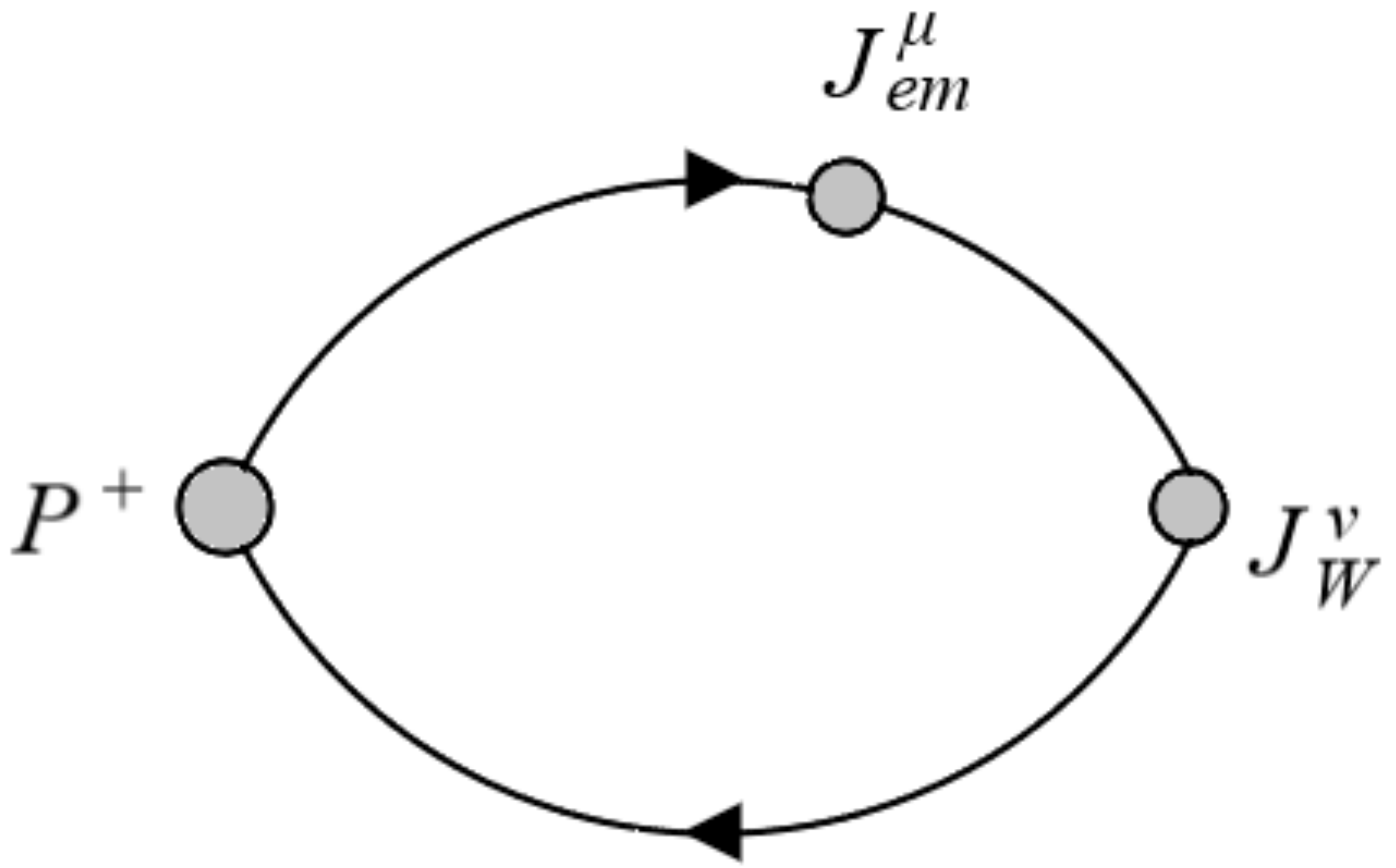


To Sum Up

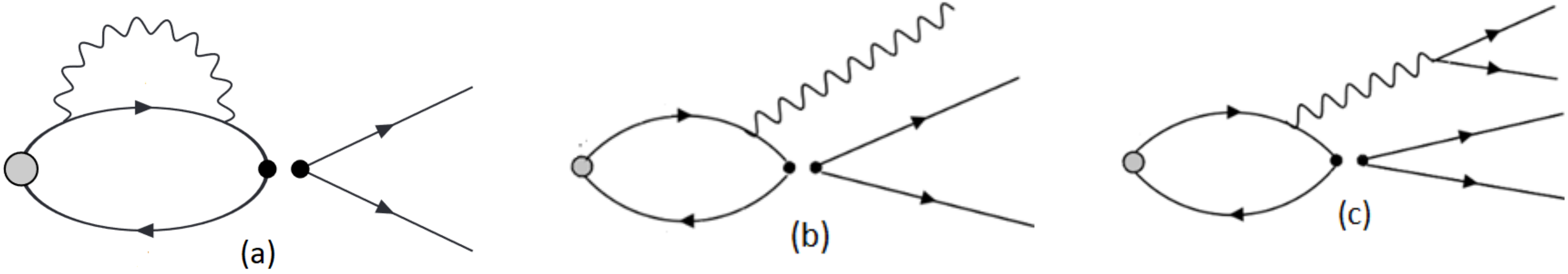
$O(\alpha_{em})$ corrections

NP constraints

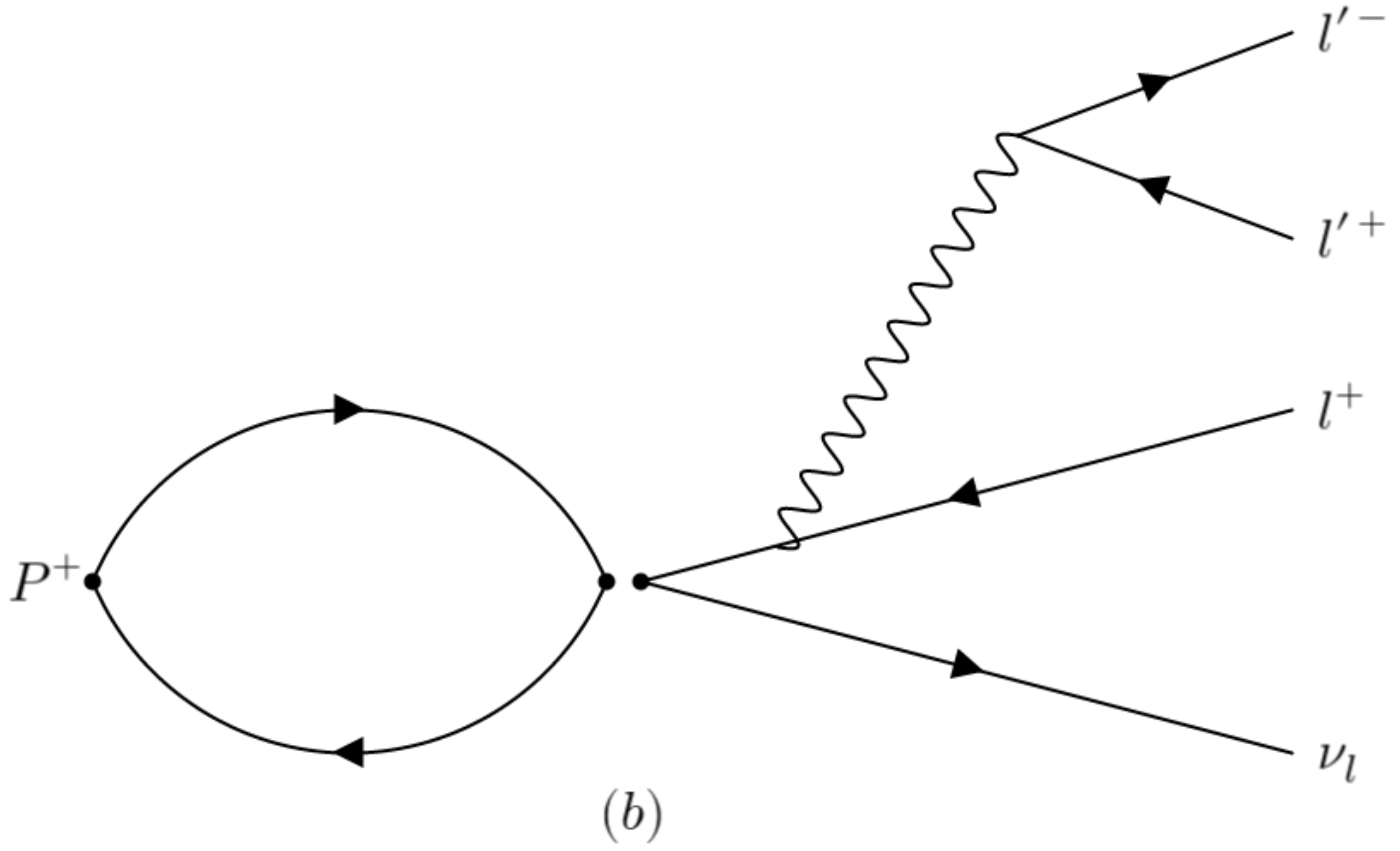
- Virtual corrections (a)
- Real photon emission (b)
- Virtual photon emission (c)

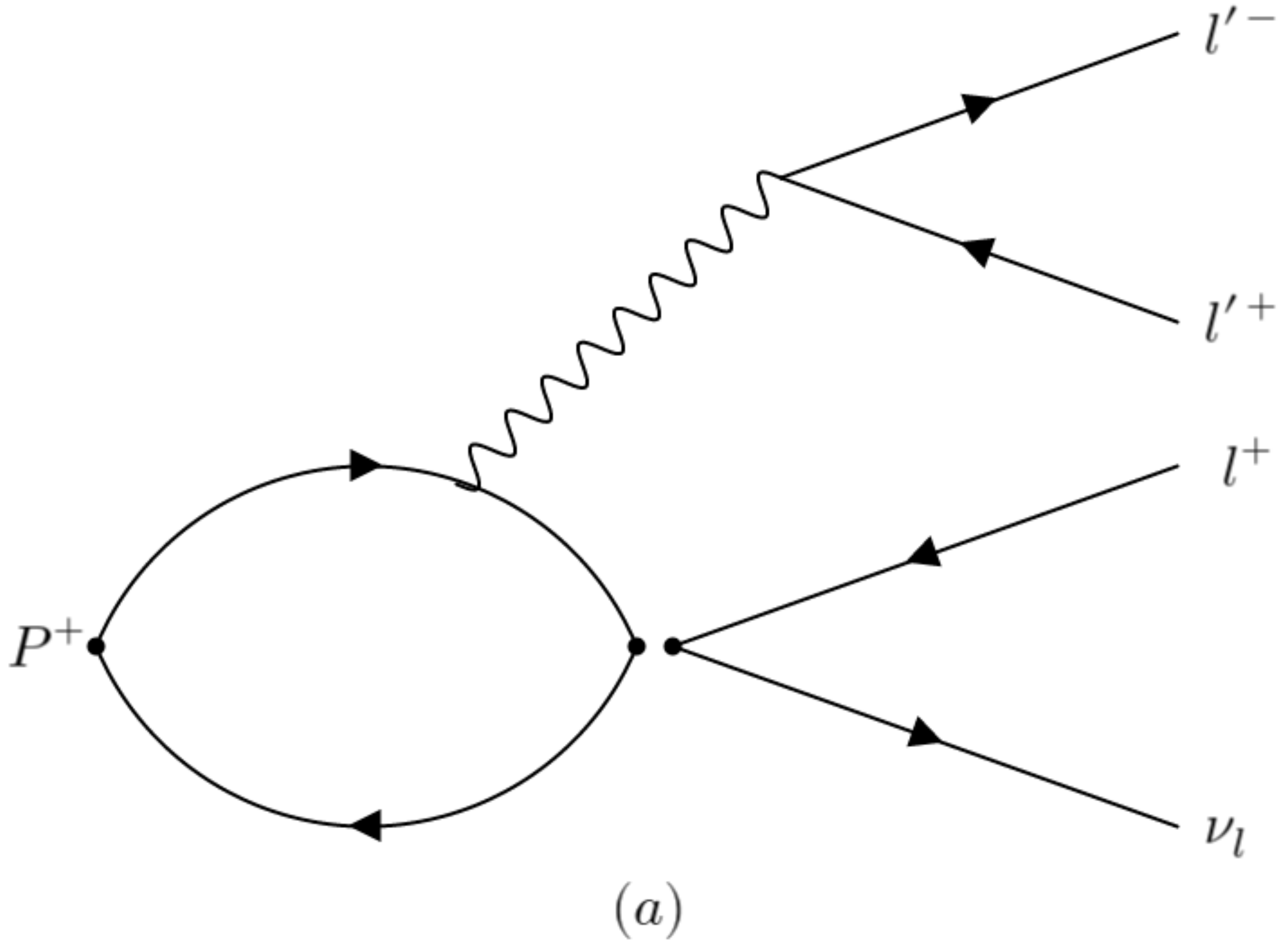
dynamical enhancement despite α_{em} suppression

$\propto \alpha_{em}^2$



$P^+ \to l^+ \nu_l l'^+ l'^-$ decays



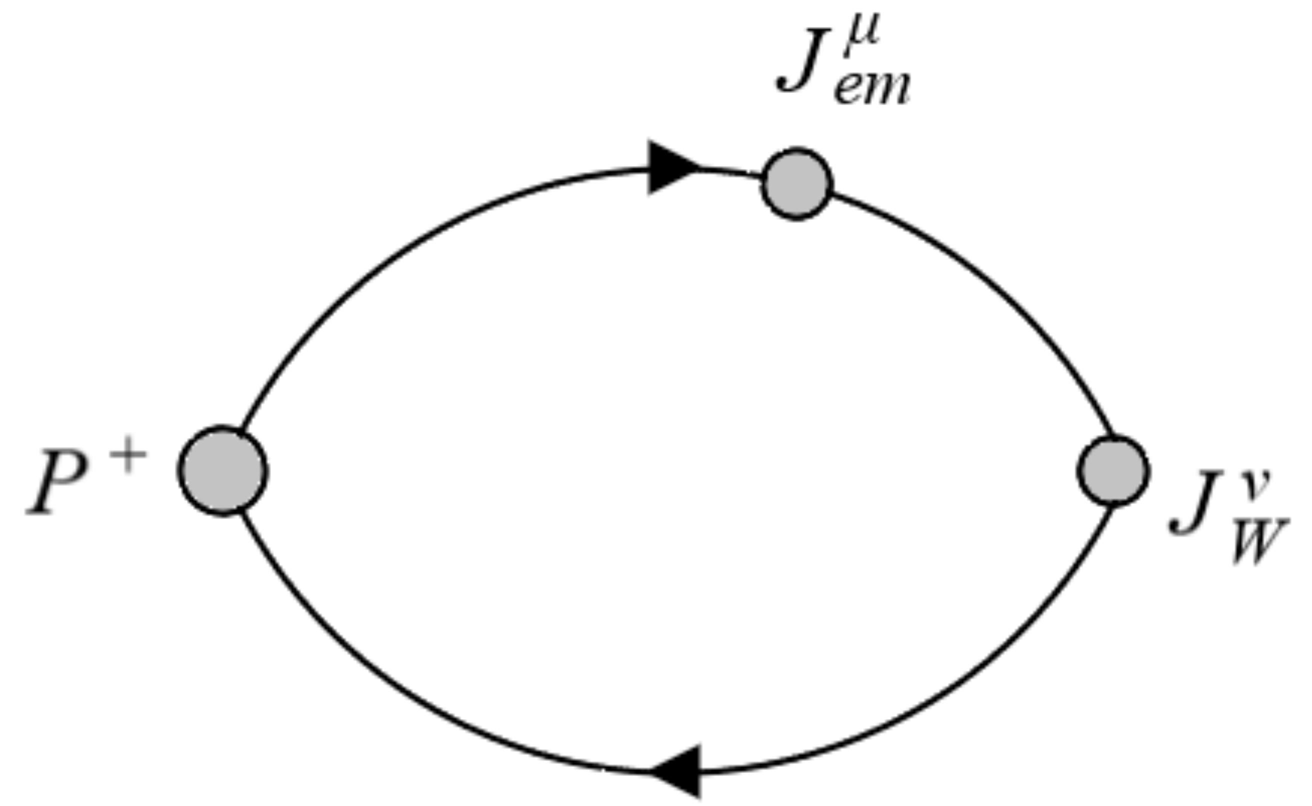


- Can be computed in perturbation theory, by simply knowing f_P

- Virtual photon interacts with the internal hadronic structure of P
- Non perturbative strong dynamics encoded in the hadronic tensor

Hadronic Tensor and Form Factors

$$H^{\mu\nu}(k,p) = \int d^4x e^{ik\cdot x} \langle 0|T[J^\mu_{em}(x) J^\nu_W(0)]|P(p)\rangle$$



$$H^{\mu\nu} = H^{\mu\nu}_{pt} + H^{\mu\nu}_{SD}\,,$$

$$H^{\mu\nu}_{pt} = f_P\left[g^{\mu\nu} - \frac{(2p-k)^\mu (p-k)^\nu}{(p-k)^2 - m_P^2}\right]\,,$$

Point-like, IR, contribution

$$H^{\mu\nu}_{SD} = \frac{H_1}{m_P}(k^2 g^{\mu\nu} - k^\mu k^\nu) + \frac{H_2}{m_P}\frac{[(k\cdot p - k^2)k^\mu - k^2(p-k)^\mu]}{(p-k)^2 - m_P^2}(p-k)^\nu$$
$$+ \frac{F_A}{m_P}[(k\cdot p - k^2)g^{\mu\nu} - (p-k)^\mu k^\nu] - i\frac{F_V}{m_P}\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta\,.$$

SD form factors

Non perturbative functions of k^2 and $(p-k)^2$

Goal of the Work

How do we include the photon in the lattice simulation?

- **Extraction of the SD form factors from suitable lattice Euclidean correlators**

Is the analytic continuation to Euclidean time “legit”?

How to account for their momentum dependence?

How to separate the point-like contribution?

- **Reconstruction of the Branching Ratios for different final states**

Our Lattice Analysis

- SD form factors evaluation
- Reconstruction of differential decay rate in momentum space
- SD contribution to the decay width is individually computed

Tuo et All Analysis*

- Only the whole matrix element is computed
- Numerical integration on the whole phase space directly
- No separation of the trivial point-like contribution to the decay width

Both studies are based on one gauge ensemble, without a proper study of systematics

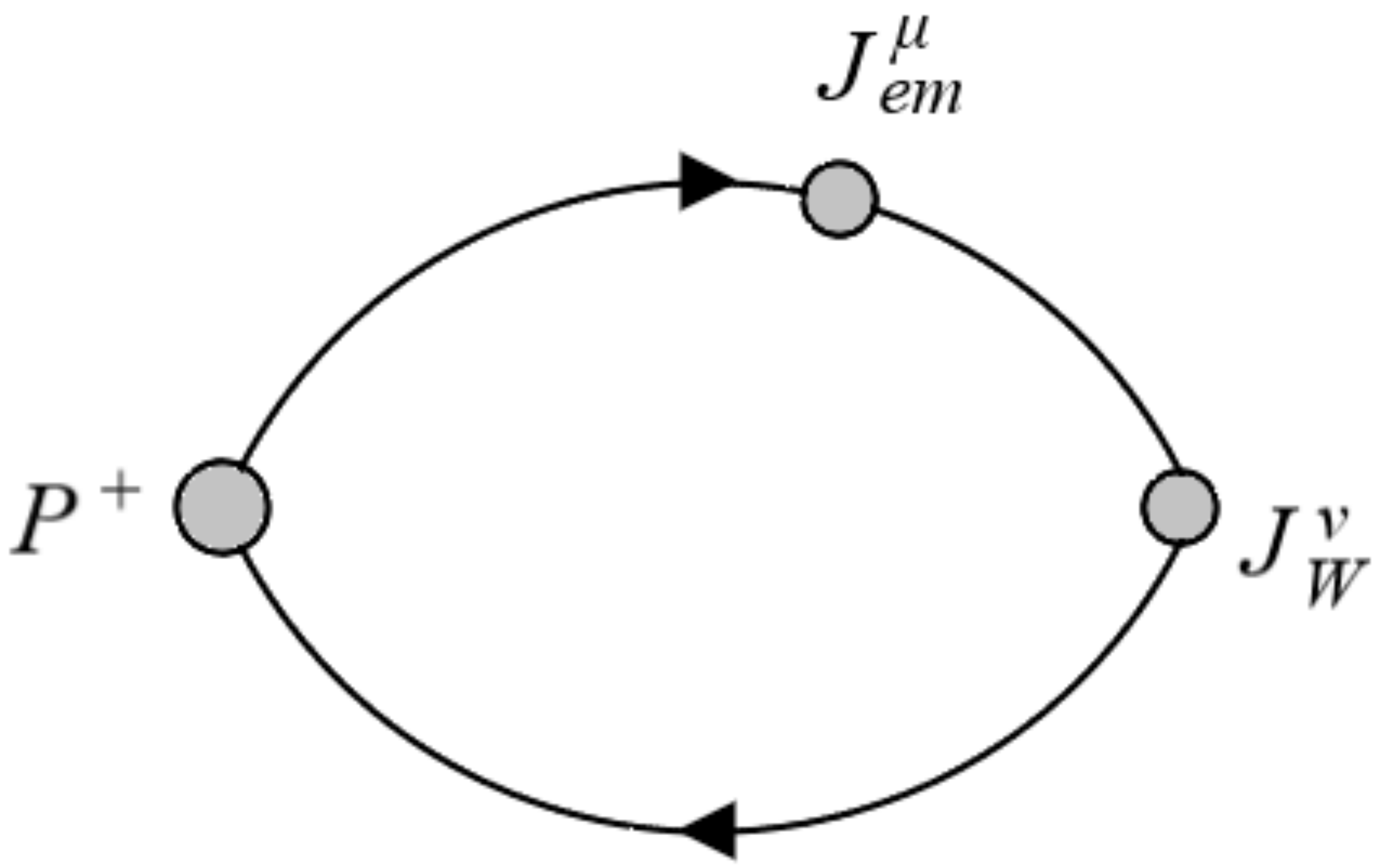
*X.-Y. Tuo, X. Feng, L.-C. Jin, and T. Wang, Phys. Rev. D 105, 054518 (2022).

Euclidean Correlator

we do not consider the photon on the lattice, only the e.m. current that carries momentum k

finite volume effects are exponentially suppressed (the lighter state is the massive pion)

$$H_E^{\mu\nu}(k,p) = \int d^4y\, d^3x\, e^{t_y E_\gamma - i\mathbf{k}\cdot\mathbf{y} + i\mathbf{p}\cdot\mathbf{x}}\, \mathrm{T}\langle 0| j_W^{\alpha}(t) j_{em}^{\mu}(y) P(0,\mathbf{x}) |0\rangle$$



Euclidean extraction of the photon momentum k

source of the pseudoscalar meson with momentum p

Conditions on the Intermediate States

Convergence of time-integrals is related to the safe analytical continuation to Euclidean time, namely by the absence of propagating states between the operators with energies smaller than the external state ones

conditions

$$E_\gamma + E_n - E > 0$$

$$E_n - E_\gamma > 0$$

$$\Downarrow$$

$$k^2 < 4m_\pi^2$$

$$-i \sum_{n:\vec{p}_n=\vec{k}} \frac{\langle 0|J^\mu_{em}(0)|n\rangle \; \langle n|J^\nu_W(0)|P\rangle}{2E_n} \int_0^{+\infty} dt_x e^{-t_x(E_n-E_\gamma)}$$

divergent if

$E_n < E_\gamma$

For now we restrict our numerical analysis to kaon decays within ensembles such that $m_K^{latt} < 2m_\pi^{latt}$, as in (*)

*X.-Y. Tuo, X. Feng, L.-C. Jin, and T. Wang, Phys. Rev. D 105, 054518 (2022).

Form Factor Extraction

Meson Rest Frame and $\overrightarrow{k} \propto \hat{z}$

VECTOR FORM FACTOR

Signal proportional to F_V

$$\frac{H_V^{12} - H_V^{21}}{i\, k_z} \xrightarrow{0 \ll t \ll T/2} F_V\,,$$

AXIAL FORM FACTORS

$$\tilde{H}_A^{33}(k_z, k^2) = H_A^{33}(k_z, k^2) - H_A^{33}(0,0)\frac{E_\gamma\left(2m_P - E_\gamma\right)}{2m_P E_\gamma - k^2}$$

By inverting the coefficient matrix of three independent expressions of the SD form factors, in which the f_P term has been subtracted

$$\tilde{H}_A^{11}(k_z, k^2) = H_A^{11}(k_z, k^2) - H_A^{11}(0,0)$$

$$H_A^{30+03}(k_z, k^2) = H_A^{30}(k_z, k^2) - H_A^{03}(k_z, k^2) \cdot \left(\frac{m_P - E_\gamma}{2m_P - E_\gamma}\right)$$

Numerical Results

(based on one gauge ensemble, without a proper study of systematics)

ETMC A40.32 ensemble

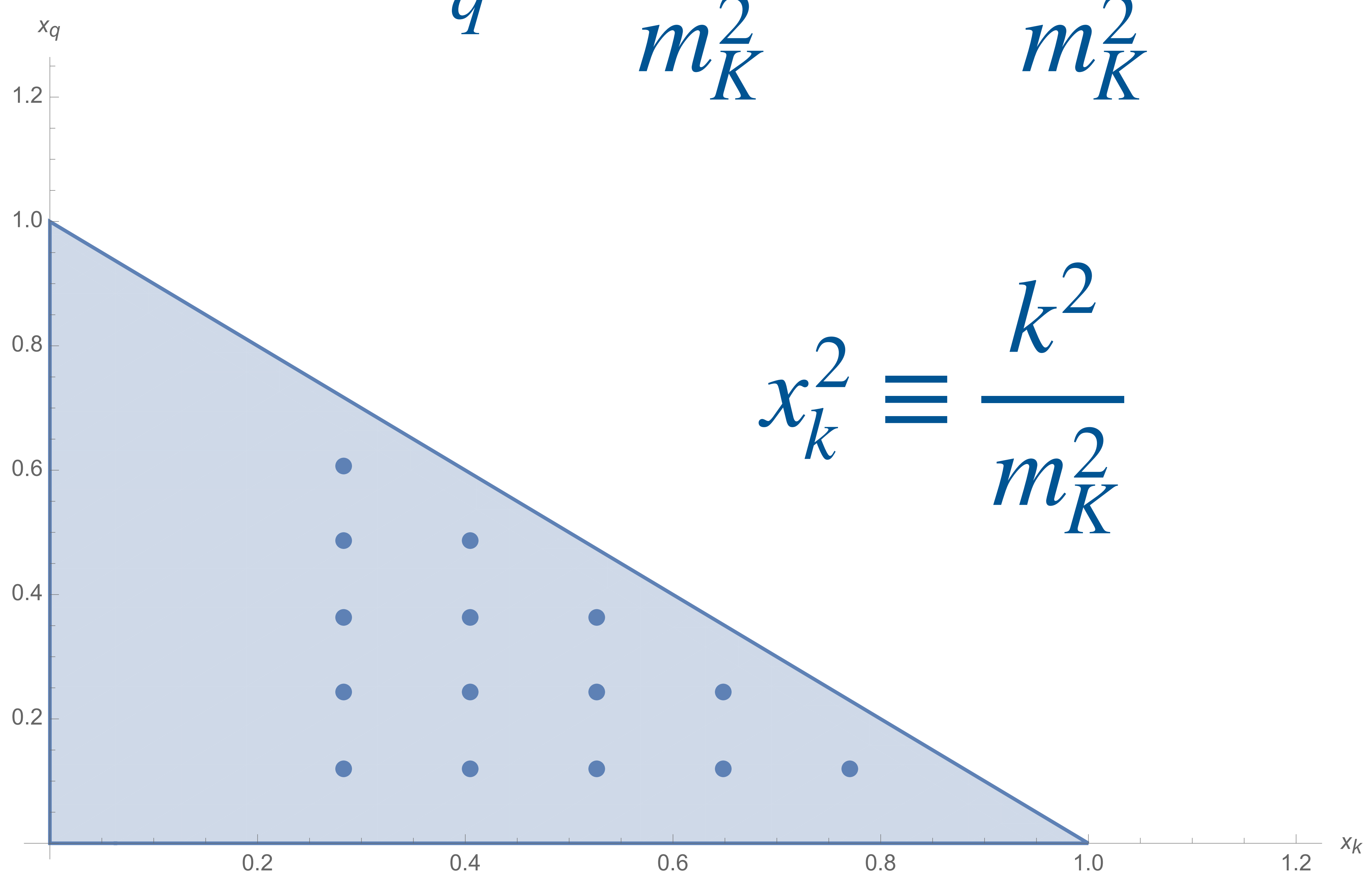
Extended Twisted Mass Collaboration

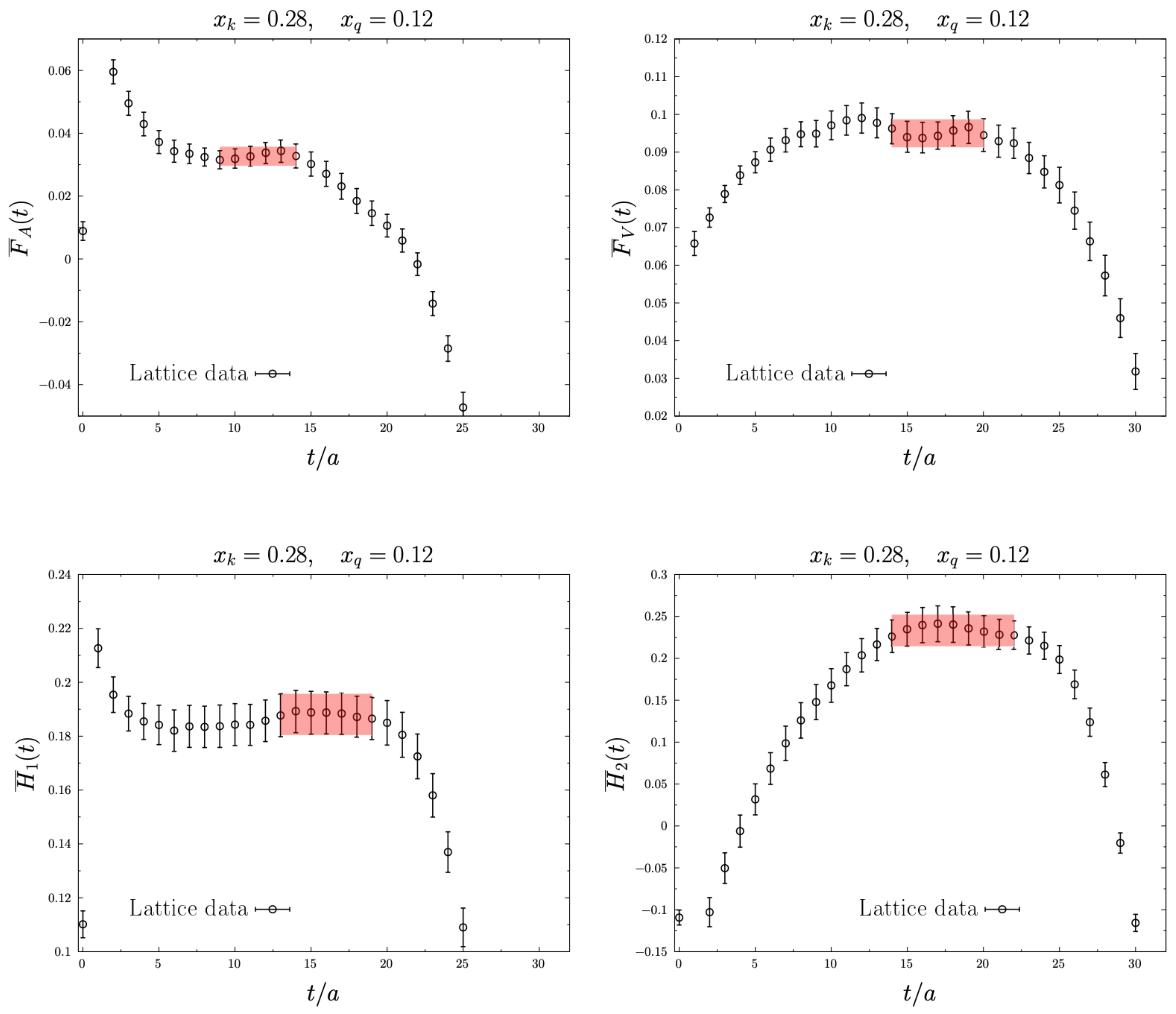
- 2+1+1 twisted mass fermions
- Electro-quenched approximation
- 21 momentum configurations via twisted boundary conditions
- $a = 0.0885(36)$ fm, $m_\pi^{latt} = 315$ MeV, $m_K^{latt} = 530$ MeV

Numerical Results

$$x_q^2 \equiv \frac{q^2}{m_K^2} = \frac{(p-k)^2}{m_K^2}$$

$$x_k^2 \equiv \frac{k^2}{m_K^2}$$





Numerical Results

$$F_{polyn}(x_k, x_q) = a_0 + a_k x_k^2 + a_q x_q^2 + a_{kq} x_k^2 x_q^2$$

$$F_{pole}(x_k, x_q) = \frac{A}{\left[\left(1 - R_k x_k^2\right)\left(1 - R_q x_q^2\right)\right]}$$

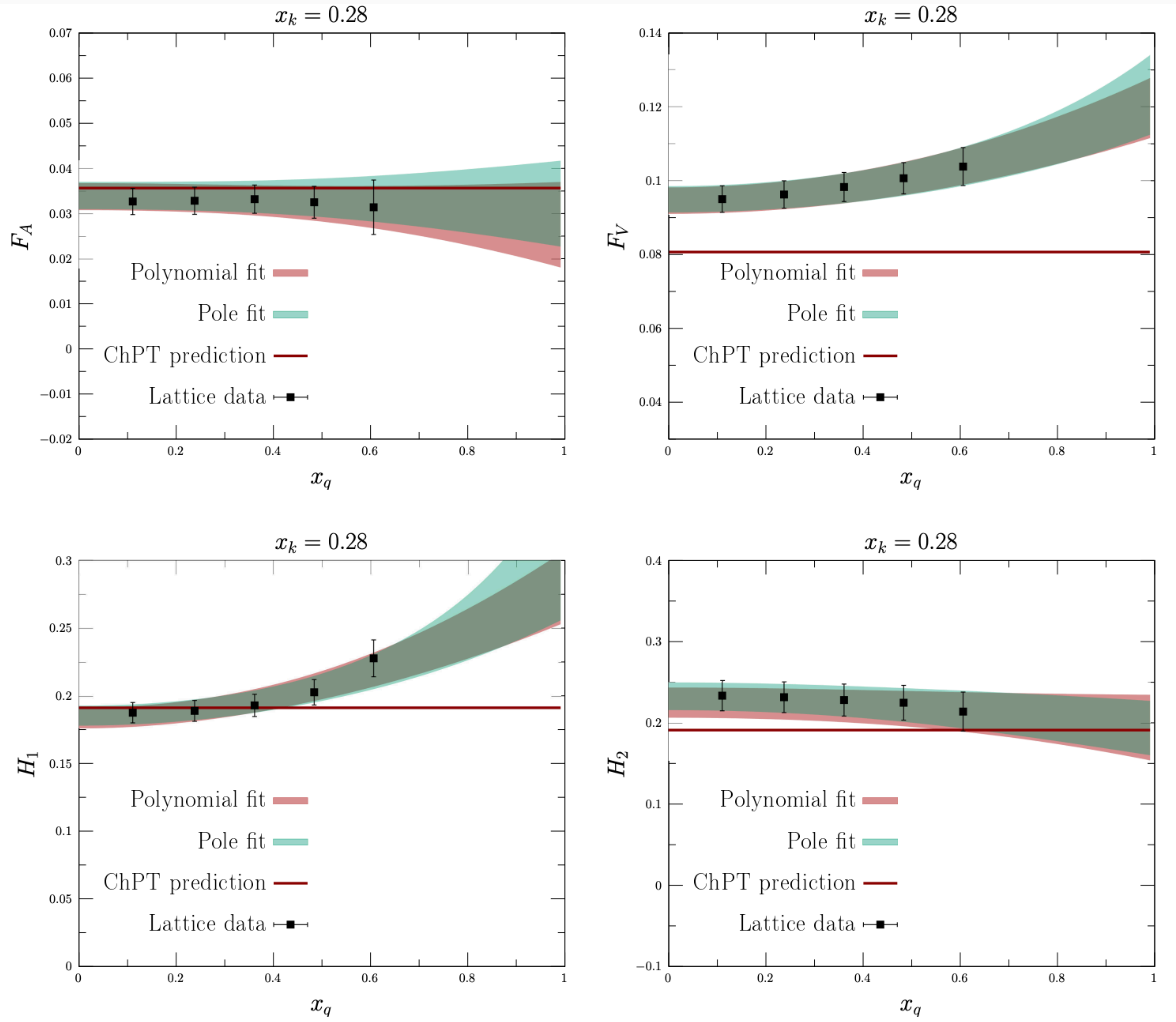
	a_0	a_k	a_q	a_{kq}	A	R_k	R_q
H_1	0.175(9)	0.122(28)	0.121(30)	-0.30(12)	0.179(8)	0.45(9)	0.40(10)
H_2	0.198(21)	0.35(9)	-0.02(4)	-0.1(3)	0.217(17)	0.87(12)	-0.2(2)
F_A	0.032(5)	0.02(4)	-0.037(13)	0.42(20)	0.0320(30)	0.7(5)	0.0(3)
F_V	0.091(4)	0.045(18)	0.028(6)	-0.035(32)	0.092(4)	0.38(13)	0.23(5)

Numerical Results

ChPT predictions at NLO

$$F_V = \frac{m_K}{4\pi^2 f_K},$$
$$F_A = \frac{8m_K}{f_K}(L_9^r + L_{10}^r).$$

$$H_1(k^2) = 2f_K m_K \frac{(F_V^K(k^2) - 1)}{k^2},$$
$$H_2(k^2) = 2f_K m_K \frac{(F_V^K(k^2) - 1)}{k^2},$$



Numerical Results

Branching Ratios

$e^+ e^-$ final state has an invariant mass IR cut of 140 MeV

Channels	our Lattice	Tuo et al.*	ChPT**	experiments
$\mathrm{Br}[K \to \mu\nu_\mu e^+ e^-]$	$8.26(13) \times 10^{-8}$	$10.59(33) \times 10^{-8}$	$9.8 - 8.2 \times 10^{-8}$	$7.93(33) \times 10^{-8}$***
$\mathrm{Br}[K \to e\nu_e \mu^+ \mu^-]$	$0.762(49) \times 10^{-8}$	$0.72(5) \times 10^{-8}$	$1.1 - 0.6 \times 10^{-8}$	$1.72(45) \times 10^{-8}$****
$\mathrm{Br}[K \to e\nu_e e^+ e^-]$	$1.95(11) \times 10^{-8}$	$1.77(16) \times 10^{-8}$	$3.4 - 1.7 \times 10^{-8}$	$2.91(23) \times 10^{-8}$***
$\mathrm{Br}[K \to \mu\nu_\mu \mu^+ \mu^-]$	$1.178(35) \times 10^{-8}$	$1.45(6) \times 10^{-8}$	$1.5 - 1.1 \times 10^{-8}$	——

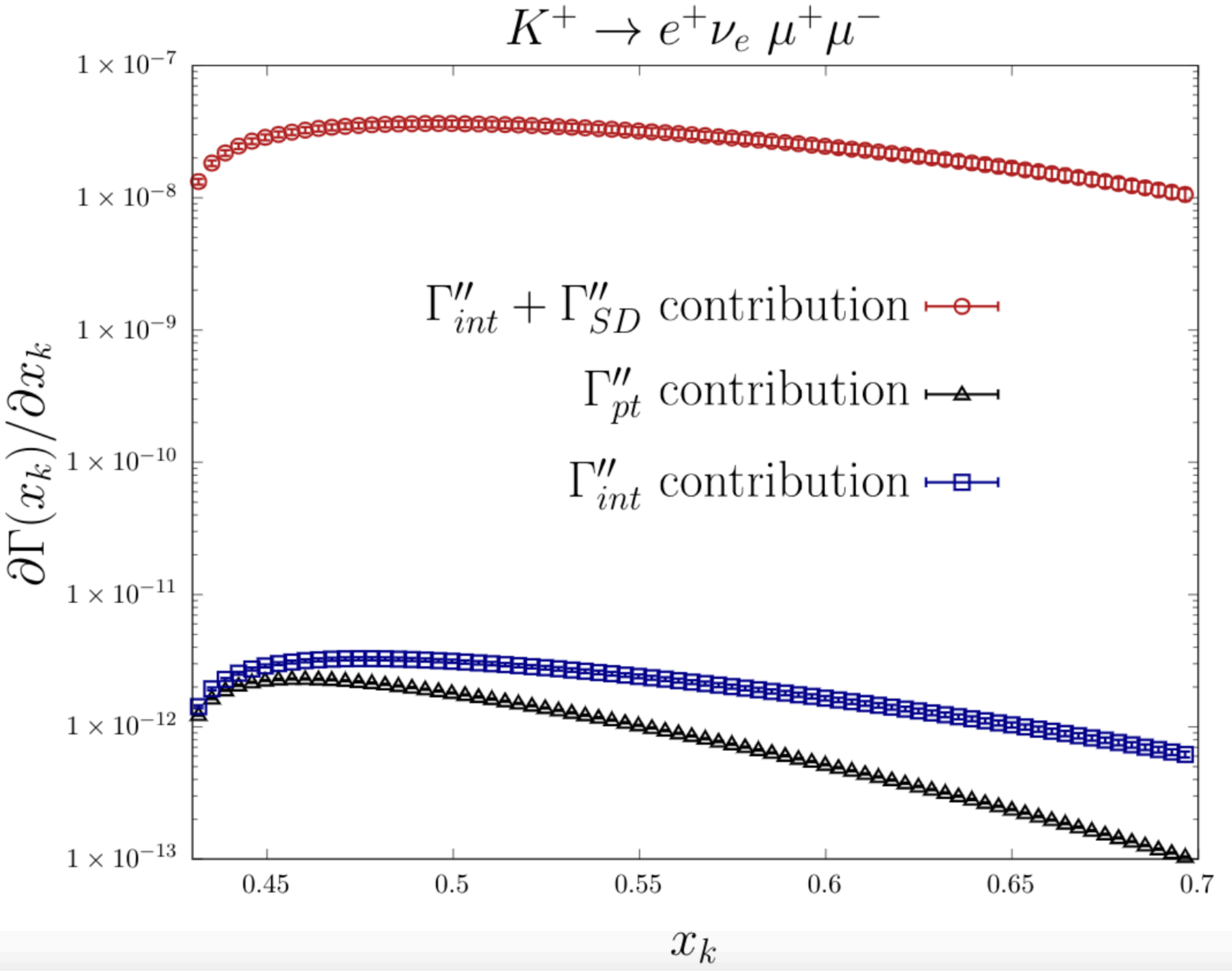
**X.-Y. Tuo, X. Feng, L.-C. Jin, and T. Wang, Phys. Rev. D 105, 054518 (2022).

**J. Bijnens, G. Ecker and J. Gasser, Nucl. Phys. B 396 (1993)

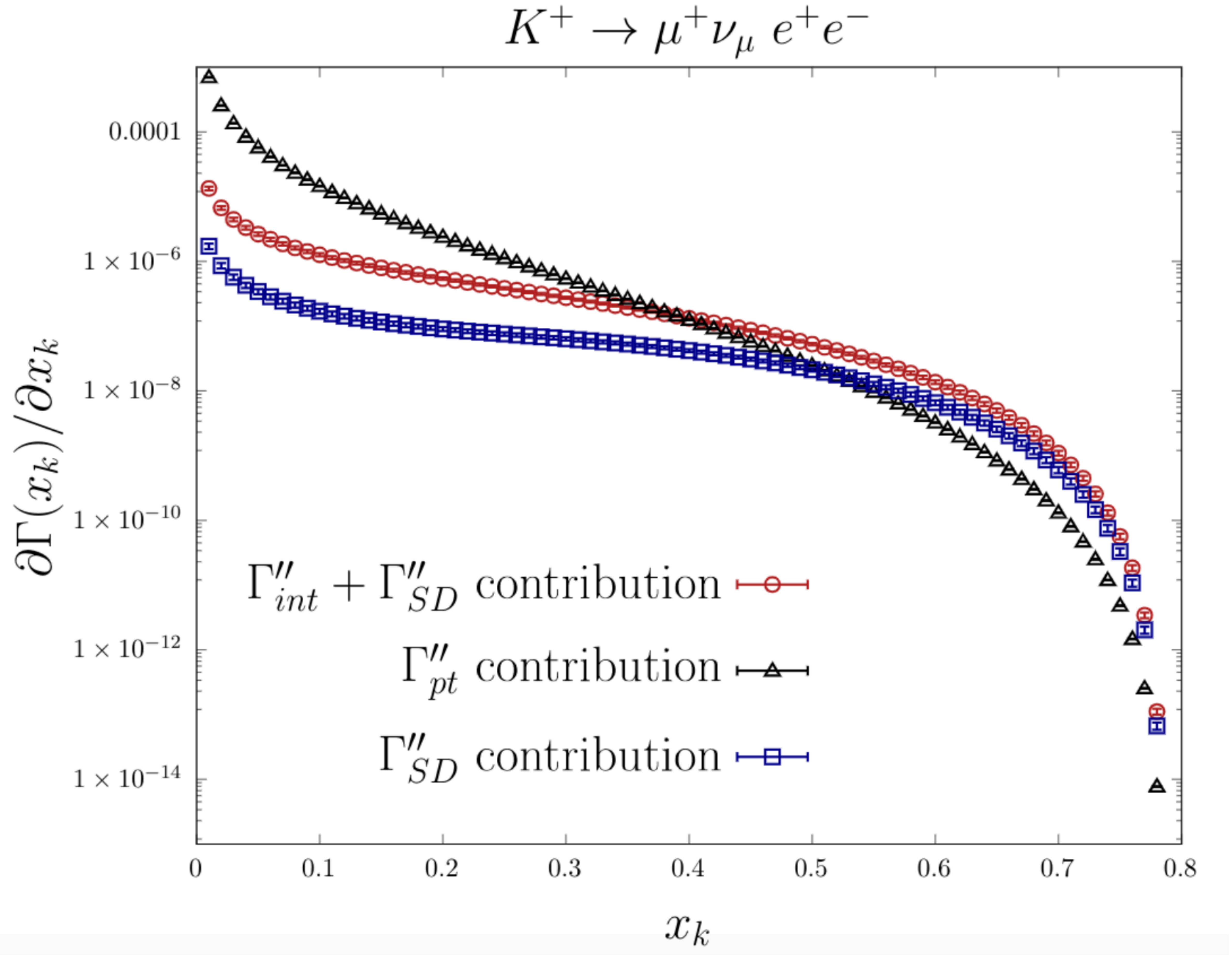
**** H. Ma *et al.* Phys. Rev. D 73 (2006) , 037101

*** A. A. Poblaguev *et al.* Phys. Rev. Lett. 89 (2002), 061803

Point-like Vs Structure-Dependent

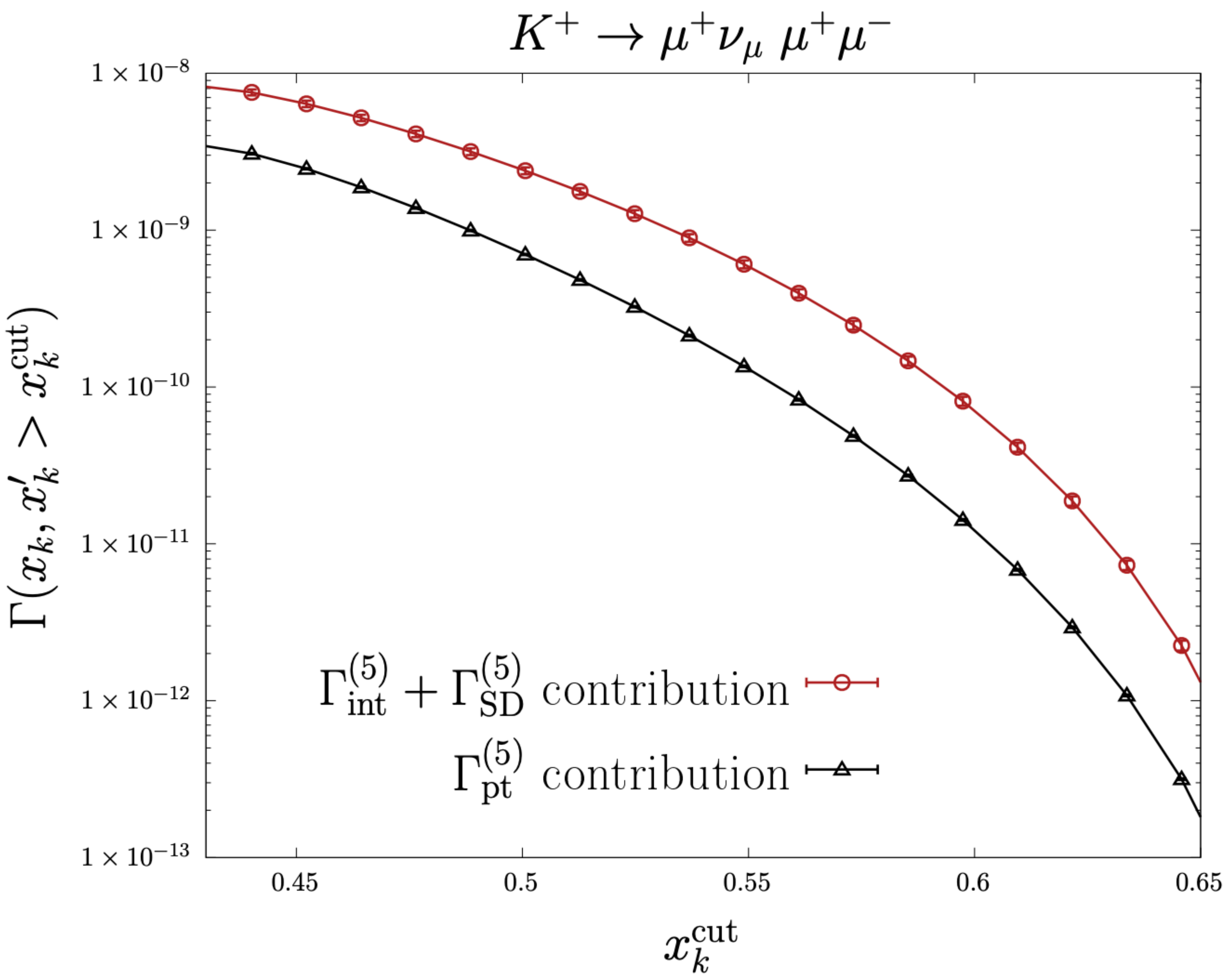


$x_k > 0.4$

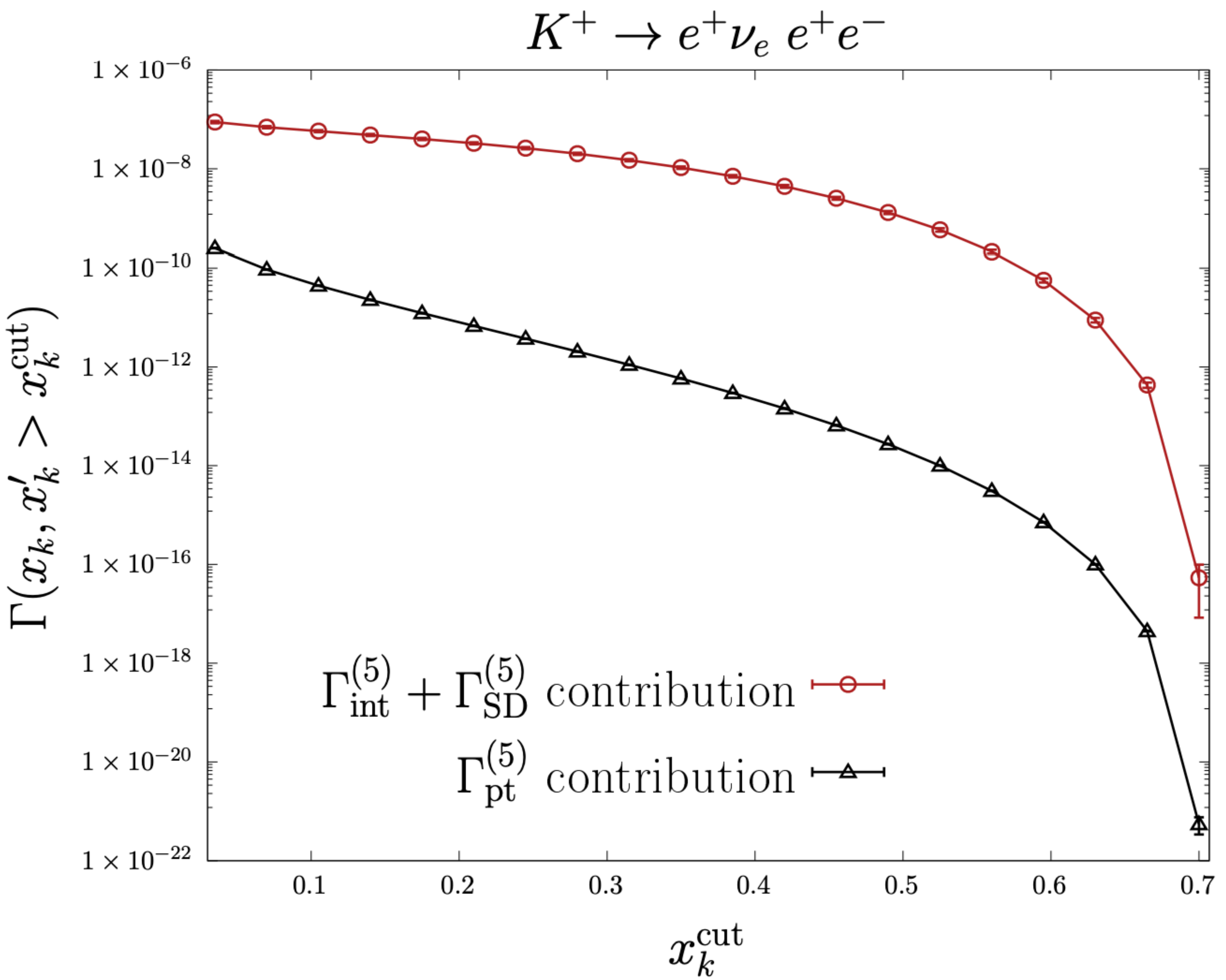


$x_k > 0.3$

Point-like Vs Structure-Dependent



$x_k > 0.4$



$x_k > 0.3$

Conclusion

Lattice computation for $P^+ \to l^+ \nu_l l'^+ l'^-$ processes **is possible**

- We developed a strategy to extract all the relevant form factors for the **virtual photon emission** in leptonic decays of pseudoscalar mesons from suitable **lattice Euclidean Correlators**
- We studied the **issue of internal lighter states**, related to the analytic continuation to Euclidean time
- We obtained **lattice results** for the **form factors** and **branching ratios** in case of **Kaon decays,** comparing them to ChPT predictions and experiments.

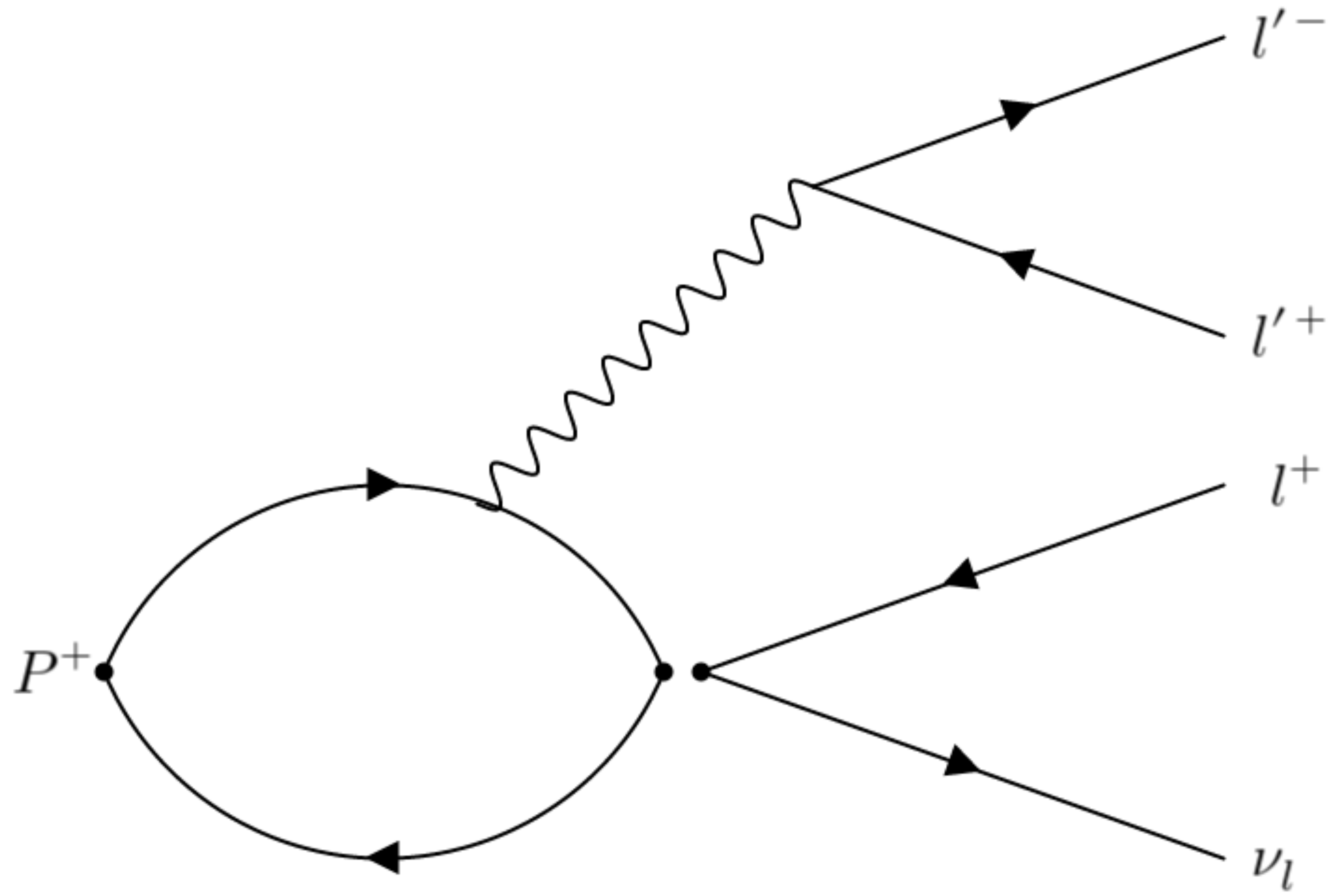
Outlook

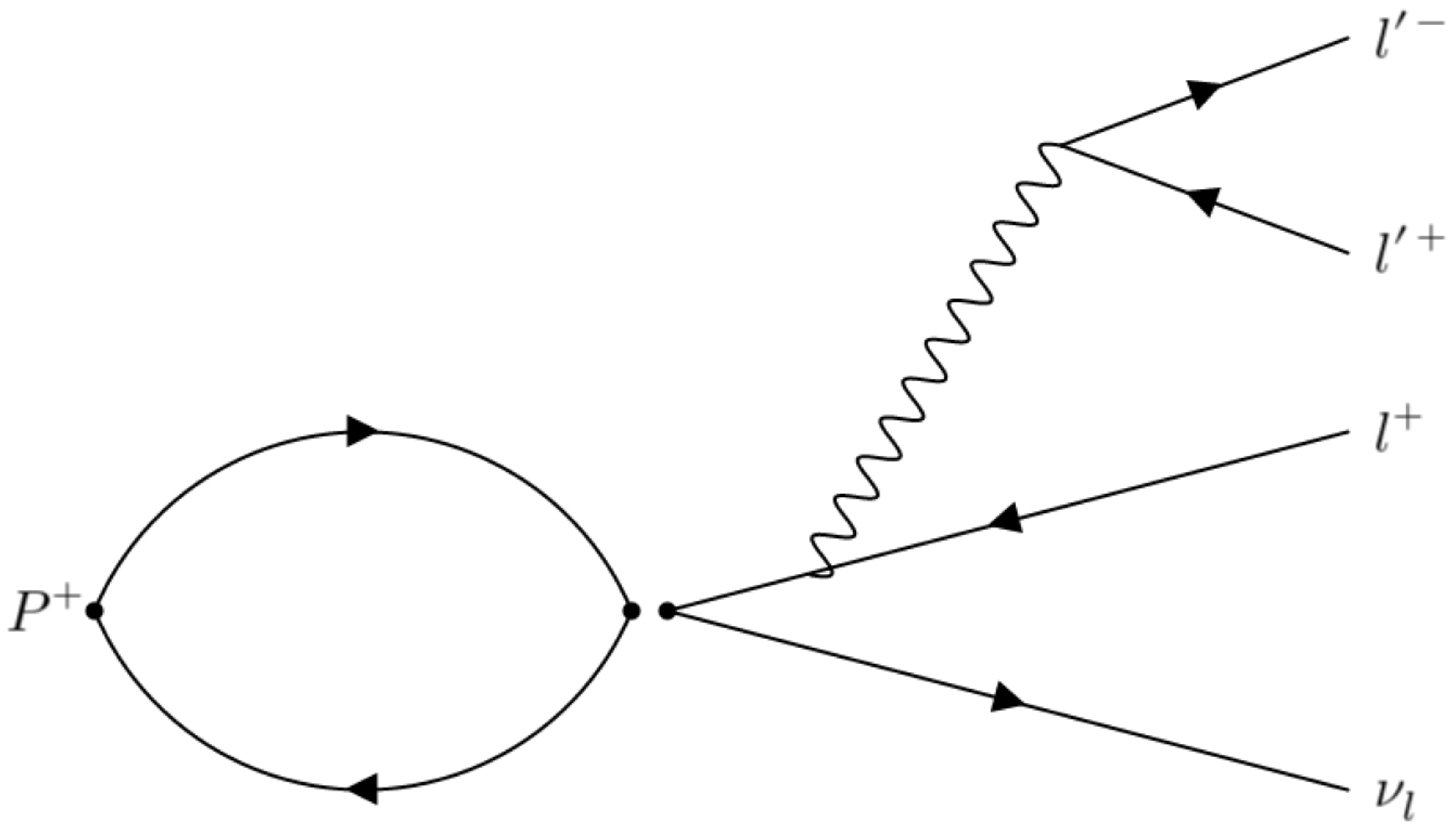
- A proper **study of all the systematics** (unphysical quark masses, finite volume, finite lattice spacing)

- Accounting for the internal lighter states so to **extend the method to heavier mesons**

This will led to a **non-perturbative, model independent, theoretical prediction** for all the $P^+ \rightarrow l^+ \nu_l l'^+ l'^-$ processes

Such predictions will be very useful **to test SM** and **for the search of New Physics**





Thanks for your attention!