Precision calculation in $b \to c$ decay process

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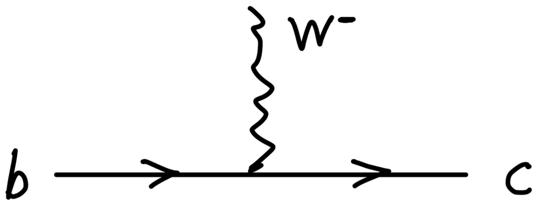
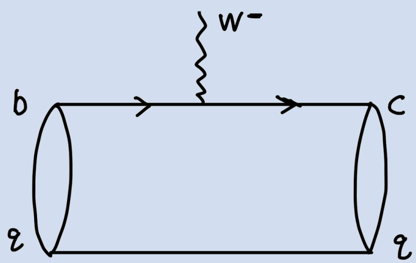
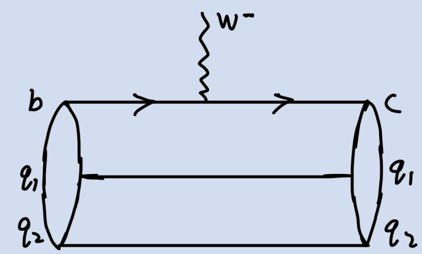
b→c Decay Process



- Flavor changing charge current
- Semi-leptonic weak decay process

B-meson decay process	b-baryon decay process
	
e.g. $B \to D$, $B \to D^*$ ✔ $B^* \to D$, $B^* \to D^*$ $B_s^{(*)} \to D_s^{(*)}$	e.g. $\Lambda_b \to \Lambda_c$ ✔ $\Xi_b \to \Xi_c$ $\cdots$
$\|V_{cb}\|$, R(D), R(D^*)	$\|V_{cb}\|$, R(Λ_c)

$|V_{cb}|$ Puzzle

- Exclusive process

$$|V_{cb}| = (39.1 \pm 0.5) \times 10^{-3}$$

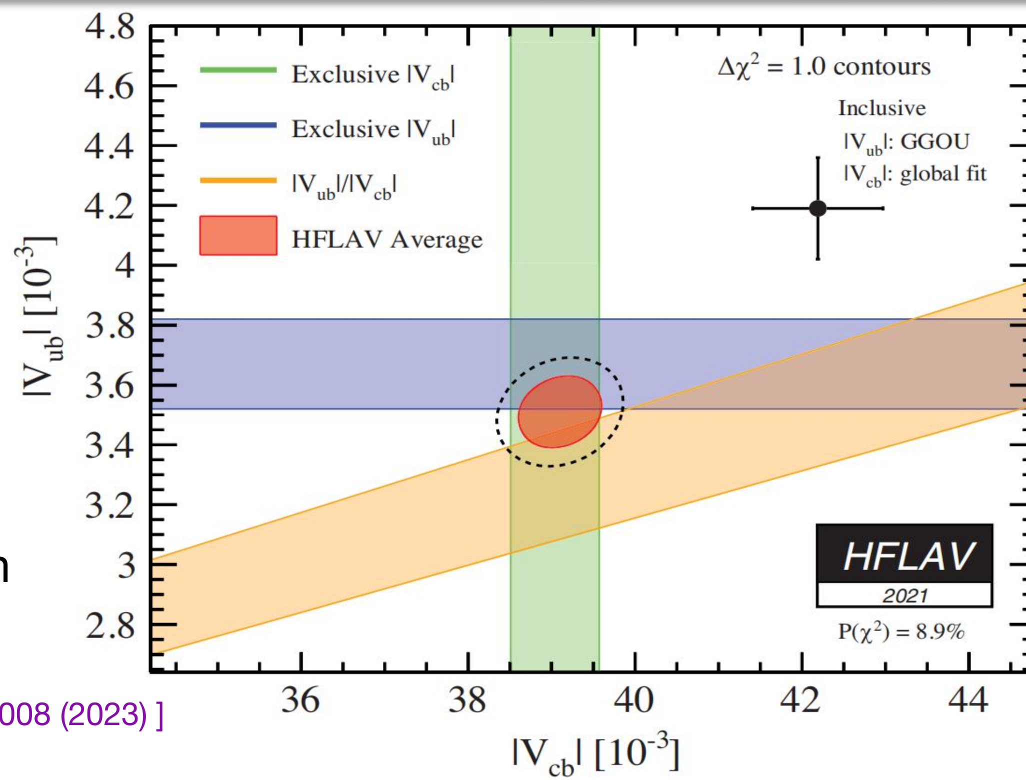
- Inclusive process

$$|V_{cb}| = (42.16 \pm 0.51) \times 10^{-3}$$

- Approximately 4σ deviation

[Belle II 2023, arXiv:2310.01170]

[HFLAV 2021, PHYSICAL REVIEW D 107, 052008 (2023)]



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Can the Differences in the Determinations of V_{ub} and V_{cb} Be Explained by New Physics?

Andreas Crivellin and Stefan Pokorski

Phys. Rev. Lett. **114**, 011802 – Published 7 January 2015

R(D) Anomaly

$$R(D) \equiv \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\mu\nu_\mu)}$$

Experimental result:

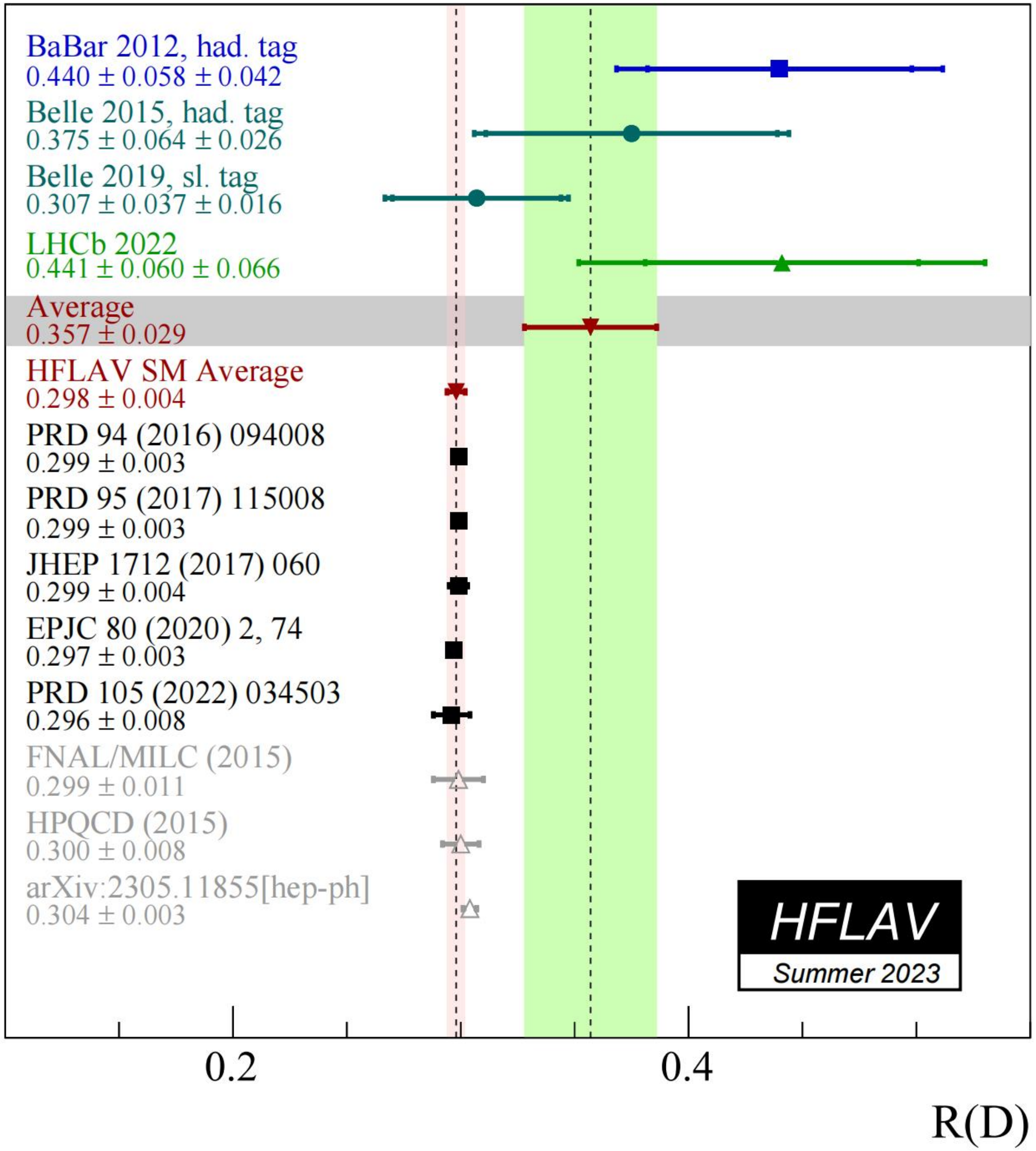
$R(D) = 0.357 \pm 0.029$

SM prediction:

$R(D) = 0.298 \pm 0.004$

Approximately 2σ deviation

[HFLAV 2023]

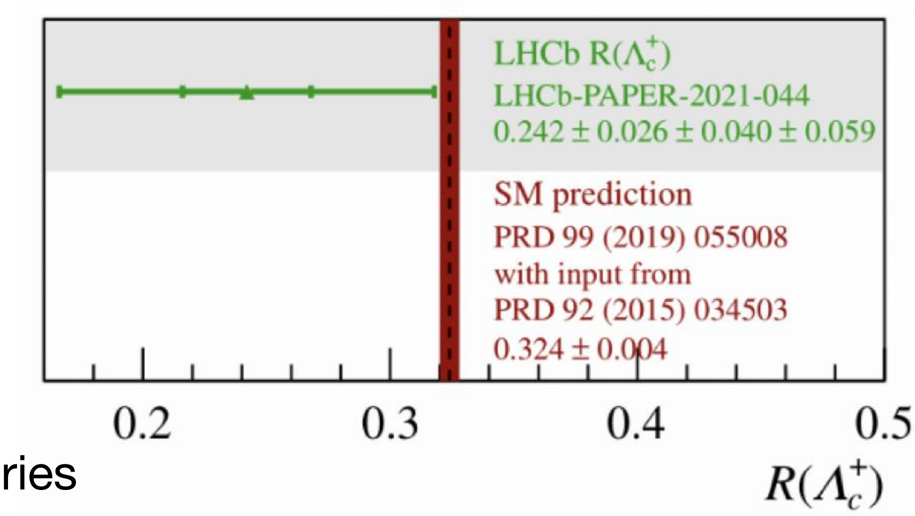


$R(\Lambda_c)$ Anomaly

$$R(\Lambda_c) \equiv \frac{B(\Lambda_b \to \Lambda_c \tau \nu_\tau)}{B(\Lambda_b \to \Lambda_c \mu \nu_\mu)}$$

Approximately 1.4σ deviation

[PRL 128, 191803 (2022)]



$R(\Lambda_c)$ calculated in different theories

Method	$R(\Lambda_c)$	Referece
LQCD	0.333 ± 0.013	[PRD 92, 034503 (2015)]
LCSRs	$0.274^{+0.009}_{-0.005}$ *or* $0.239^{+0.070}_{-0.021}$ 0.268 ± 0.015	[EPJC 82, 10, 951 (2022)] [Chin.Phys.C 46, 11, 113107 (2022)]
QCDSRs	0.31 ± 0.11	[Phys.Rev.D 97, 7, 074007 (2018)]
HQET	0.324 ± 0.004	[Phys.Rev.Lett. 121 (2018) 20, 202001]
LFQM	*0.28*	[EPJC 79, 6, 540 (2019)]

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Definitions and Conventions

Form factors :

$$\langle \Lambda_c(p',s')|\bar{c}\,\gamma_\mu\, b|\Lambda_b(p,s)\rangle = \bar{u}_{\Lambda_c}(p',s')\bigg[f_0(q^2)\,\frac{m_{\Lambda_b}-m_{\Lambda_c}}{q^2}\,q_\mu + f_+(q^2)\,\frac{m_{\Lambda_b}+m_{\Lambda_c}}{s_+}\left((p+p')_\mu - \frac{m^2_{\Lambda_b}-m^2_{\Lambda_c}}{q^2}\,q_\mu\right)$$

$$+ f_\perp(q^2)\left(\gamma_\mu - \frac{2\,m_{\Lambda_c}}{s_+}p_\mu - \frac{2\,m_{\Lambda_b}}{s_+}p'_\mu\right)\bigg]u_{\Lambda_b}(p,s)$$

$$\langle \Lambda_c(p',s')|\bar{c}\,\gamma_\mu\gamma_5\, b|\Lambda_b(p,s)\rangle = -\bar{u}_{\Lambda_c}(p',s')\gamma_5\bigg[g_0(q^2)\,\frac{m_{\Lambda_b}+m_{\Lambda_c}}{q^2}\,q + g_+(q^2)\,\frac{m_{\Lambda_b}-m_{\Lambda_c}}{s_-}\left((p+p')_\mu - \frac{m^2_{\Lambda_b}-m^2_{\Lambda_c}}{q^2}\,q_\mu\right)$$

$$+ g_\perp(q^2)\left(\gamma_\mu + \frac{2\,m_{\Lambda_c}}{s_-}p_\mu - \frac{2\,m_{\Lambda_b}}{s_-}p'_\mu\right)\bigg]u_{\Lambda_b}(p,s)$$

[Phys.Rev.D 86, 079901 (2012)]

where $q^2 = (p-p')^2$, $s_\pm = (m_{\Lambda_b} \pm m_{\Lambda_c})^2 - q^2$

Power counting :

$$p'_\mu = (n\cdot p'/2)\bar{n}_\mu + (\bar{n}\cdot p'/2)n_\mu$$

$\mathcal{O}(1)$ $\mathcal{O}(\lambda)$

$$p_b = (m_b/2)\bar{n}_\mu + (m_b/2)n_\mu$$

$$m_c \sim \mathcal{O}(\sqrt{\lambda}), \quad \lambda = \Lambda_{QCD}/m_b$$

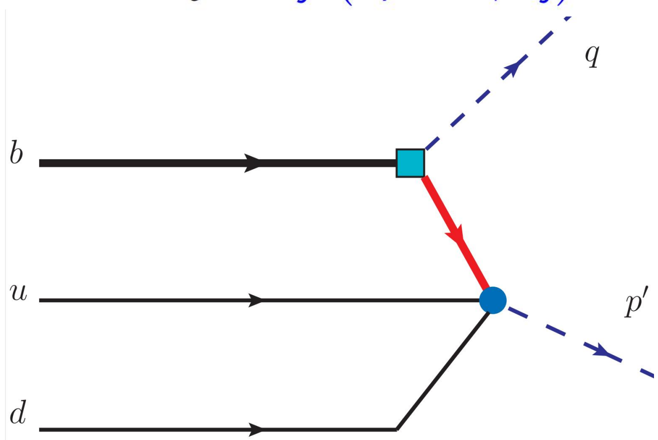
Correlation Function

- Correlation function in quark level [JHEP 02 (2016) 179]

$$\Pi_{\mu,a}(p,q) = i\int d^4x\, e^{iq\cdot x}\langle 0|T\{j_{\Lambda_c}(x), j_{\mu,a}(0)\}|\Lambda_b(p)\rangle\,,$$

where $j_{\mu,a} = \bar{c}\Gamma_{\mu,a}b, \quad \Gamma_{\mu,V(A)} = \gamma_\mu(\gamma_\mu\gamma_5)$ Heavy to light weak current

$$j_{\Lambda_c} \equiv \epsilon_{ijk}\left(u_i^{\mathrm{T}} C\gamma_5 \not{n} d_j\right) c_k$$ Leading-power Λ_c interpolating current



In Euclidean space, $p'^2 < 0$

In large recoil region, $q^2 \to 0$

Perform OPE

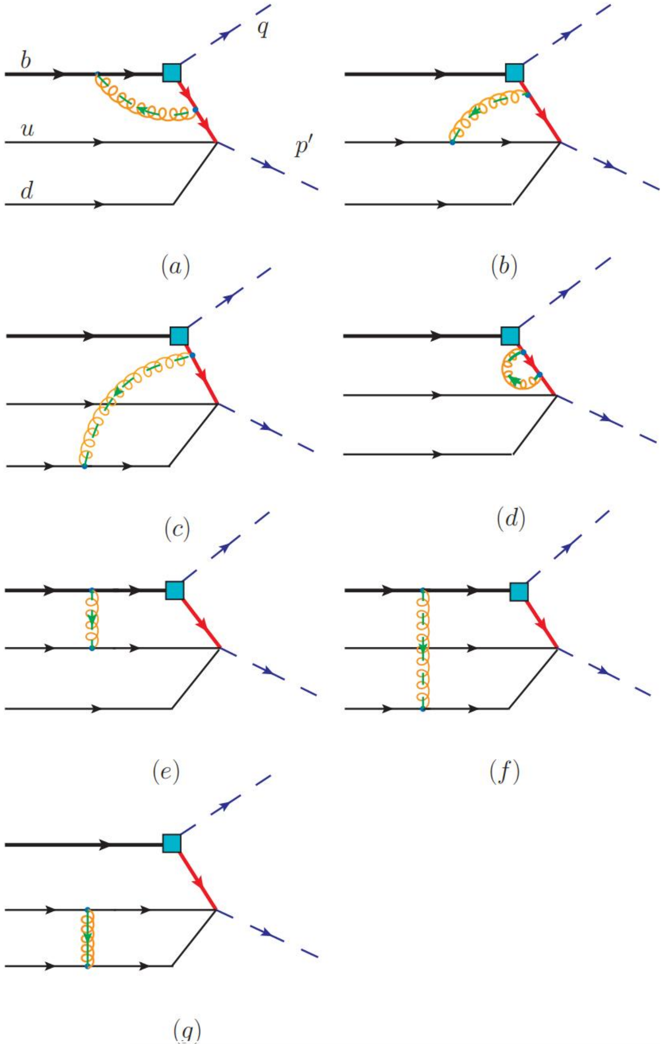
- Tree level

$$\Pi_\mu^{(0)} = f_{\Lambda_b}^{(2)}(\mu)\int_0^\infty d\omega_1\int_0^\infty d\omega_2\frac{\psi_4(\omega_1,\omega_2)}{\omega_1+\omega_2+\omega_c-\bar{n}\cdot p'+i\epsilon}(1,\gamma_5)\frac{\not{\bar{n}}}{2}(\gamma_{\perp\mu}+\bar{n}_\mu)\Lambda_b(v)$$

where $\omega_1 = \bar{n}\cdot k_1, \quad \omega_2 = \bar{n}\cdot k_2, \quad \omega_c = m_c^2/n\cdot p'$

Correlation Function

- One- loop diagrams



- Method of regions

- Hard function → gluon is hard

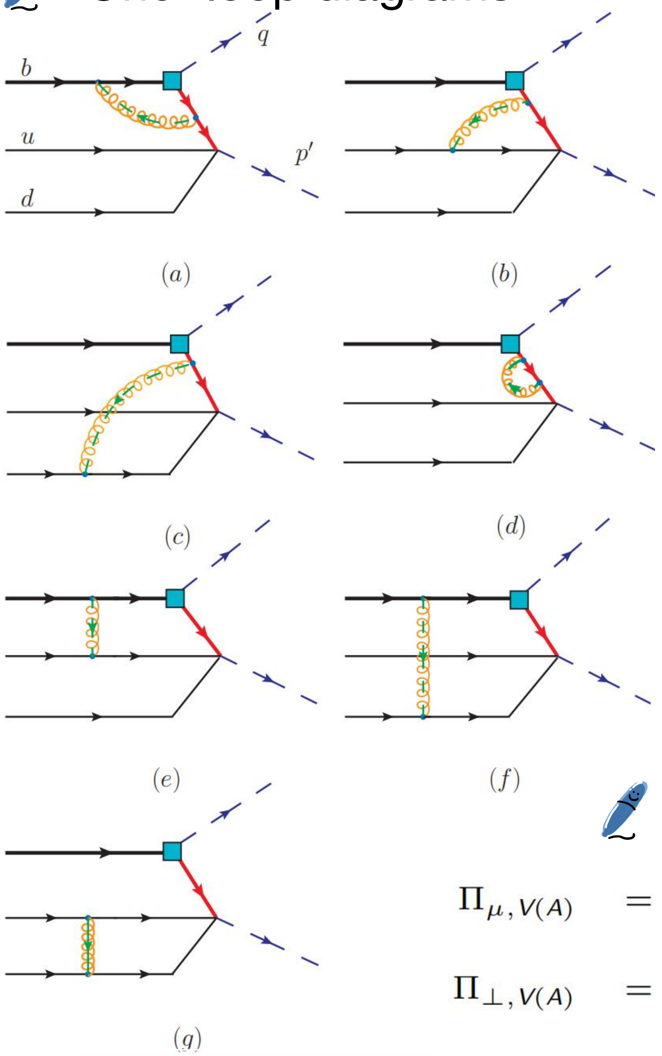
$$C_{\perp,V(A)}(n\cdot p',\mu) = 1 - \frac{\alpha_s(\mu)\,C_F}{4\,\pi}\left[2\ln^2\frac{\mu}{n\cdot p'} + 5\ln\frac{\mu}{m_b} - 2\mathrm{Li}_2\left(1-\frac{1}{r}\right) - \ln^2 r + \frac{3r-2}{1-r}\ln r + \frac{\pi^2}{12} + 6\right]$$

$$C_{\bar{n},V(A)}(n\cdot p',\mu_h) = 1 - \frac{\alpha_s(\mu)\,C_F}{4\pi}\left[2\ln^2\frac{\mu}{n\cdot p'} + 5\ln\frac{\mu}{m_b} - 2\,\mathrm{Li}_2\left(1-\frac{1}{r}\right) - \ln^2 r + \frac{2-r}{r-1}\ln r + \frac{\pi^2}{12} + 5\right],$$

$$C_{n,V(A)}(n\cdot p',\mu) = -\frac{\alpha_s(\mu)\,C_F}{4\pi}\left[\frac{1}{r-1}\left(1+\frac{r}{1-r}\ln r\right)\right],$$

where $r = n\cdot p'/m_b$

One- loop diagrams



Method of region

Jet function → gluon is hard-collinear

$$\begin{aligned}
J\left(\frac{\mu^2}{\bar n\cdot p',\omega_i},\frac{\omega_i}{\bar n\cdot p'}\right) &= 1+\frac{\alpha_s C_F}{4\pi}\Big\{2\Big[\mathrm{Li}_2(\frac{1}{r_2+1}) - 2\mathrm{Li}_2(\frac{r_3+1}{r_2+r_3+1})\Big] \\
&- \frac{r_2\ln r_2}{2(r_3+1)^2(r_2+r_3+1)}\{r_2^2[r_3(r_3+2)+3]+r_2(r_3+1)[r_3(r_3+6)+11] \\
&+ 4(r_3-1)(r_3+1)^2\} + (r_2+1)\ln(r_2+1)(\frac{r_2+3}{2}-\frac{2}{r_3}) \\
&+ [\ln R+\ln(r_3+1)]\Big[-\frac{6r_2}{r_2+r_3+1}-\frac{1}{2}-4\ln(r_2+r_3+1)\Big] \\
&+ \frac{\ln(r_2+r_3+1)}{(r_3+1)^2}\Big[\frac{2(r_2+1)}{r_3}+(r_2^2+4r_2r_3+6r_2-r_3^2+3)\Big] \\
&+ 2\ln^2(r_2+r_3+1)-\ln^2(r_2+1)+2\ln(r_2+1)[\ln R+\ln(r_3+1)] \\
&+ [\ln R+\ln(r_3+1)]^2-\frac{r_2}{r_3+1}-\frac{8r_2}{r_2+r_3+1}+\frac{\pi^2}{6}-\frac{r_2+1}{2}\Big\}.
\end{aligned}$$

where $r_2=\omega_c/(\omega_2-\bar n\cdot p'), r_3=\omega_1/(\omega_2-\bar n\cdot p'), R=\mu^2/n\cdot p'(\omega_2-\bar n\cdot p')$

Factorization result

$$\Pi_{\mu,V(A)} = (I,\gamma_5)\,\frac{\not{\bar n}}{2}\,\left[\Pi_{\perp,V(A)}\gamma_{\perp\mu}+\Pi_{\bar n,V(A)}\,\bar n_\mu+\Pi_{n,V(A)}\,n_\mu\right]u_{\Lambda_b}(v),$$

$$\Pi_{\perp,V(A)} = f^{(2)}_{\Lambda_b}(\mu)C_{\perp,V(A)}(n\cdot p',\mu)\int_0^\infty d\omega_1\int_0^\infty d\omega_2\,\frac{1}{\omega_1+\omega_2+\omega_c-\bar n\cdot p'-i\epsilon}\times J\left(\frac{\mu^2}{\bar n\cdot p'\omega_i},\frac{\omega_i}{\bar n\cdot p'}\right)\psi_4(\omega_1,\omega_2,\mu)$$

Light-Cone Sum Rules

- Correlation function in hadronic level

$$\Pi_{\mu,V}(p,q) = \frac{f_{\Lambda_c}(\mu)(n\cdot p')}{m_{\Lambda_c}^2/n\cdot p' - \bar{n}\cdot p'} \frac{\not{\bar{n}}}{2}\left[f_\perp(q^2)\gamma_{\perp\mu} + \frac{f_0(q^2)-f_+(q^2)}{2(1-n\cdot p'/m_{\Lambda_b})}n_\mu + \frac{f_0(q^2)+f_+(q^2)}{2}\bar{n}_\mu\right]u_{\Lambda_b}(p)$$

$$+\int_{\omega_s}^{+\infty} d\omega \frac{1}{\omega+\omega_c-\bar{n}\cdot p'} \times \frac{\not{\bar{n}}}{2}\left[\rho_{V,\perp}^h(\omega, n\cdot p')\gamma_{\perp\mu} + \rho_{V,n}^h(\omega', n\cdot p')n_\mu + \rho_{V,\bar{n}}^h(\omega, n\cdot p')\bar{n}_\mu\right]u_{\Lambda_b}(p).$$

- Dispersion relation $\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s-q^2-i\epsilon}$

- Quark-hadron duality ansatz

$$\int_{s_0}^{\infty} ds \frac{\rho^h(s)}{s-p^2} \simeq \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{Im\Pi^{(pert)}(s)}{s-p^2}$$

effctive threshold s_0

- Borel transformation

$$\mathcal{B}_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{\exp(-m^2/M^2)}{M^{2(k-1)}}$$

Borel mass M^2

[hep-ph/0010175]

Total result

- Resummation for hard function [Eur.Phys.J.C 71 (2011) 1818]

$$C(n\cdot p',\mu) = U(n\cdot p',\mu_h,\mu)C(n\cdot p',\mu_h)$$ solve RGEs

- Resummation improved form factors

$$\begin{aligned}
\{f_\perp(q^2), g_\perp(q^2)\} &= \frac{f^{(2)}_{\Lambda_b}(\mu)}{f_{\Lambda_c}(\nu)\, n\cdot p'}\,\mathrm{Exp}\left[\frac{m^2_{\Lambda_c}}{n\cdot p'\,\omega_M}\right] \\
&\times \left[U_1(n\cdot p',\mu_h,\mu)\, C_{\perp,V(A)}(n\cdot p',\mu_h)\right]\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\;\psi_{4,\mathrm{eff}}(\omega',\mu,\nu)\,, \\
\{f_0(q^2), g_0(q^2)\} &= \frac{f^{(2)}_{\Lambda_b}(\mu)}{f_{\Lambda_c}(\nu)\, n\cdot p'}\,\mathrm{Exp}\left[\frac{m^2_{\Lambda_c}}{n\cdot p'\,\omega_M}\right] \\
&\times\Bigg\{U_1(n\cdot p',\mu_h,\mu)\, C_{\bar{n},V(A)}(n\cdot p',\mu_h)\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\;\psi_{4,\mathrm{eff}}(\omega',\mu,\nu)\,, \\
&+\left(1-\frac{n\cdot p'}{m_{\Lambda_b}}\right)U_1(n\cdot p',\mu_h,\mu)C_{n,V(A)}(n\cdot p',\mu_h)\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\psi^{(0)}_{4,\mathrm{eff}}(\omega',\mu)\Bigg\} \\
\{f_+(q^2), g_+(q^2)\} &= \frac{f^{(2)}_{\Lambda_b}(\mu)}{f_{\Lambda_c}(\nu)\, n\cdot p'}\,\mathrm{Exp}\left[\frac{m^2_{\Lambda_c}}{n\cdot p'\,\omega_M}\right] \\
&\times\Bigg\{U_1(n\cdot p',\mu_h,\mu)\, C_{\bar{n},V(A)}(n\cdot p',\mu_h)\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\;\psi_{4,\mathrm{eff}}(\omega',\mu,\nu)\,, \\
&-\left(1-\frac{n\cdot p'}{m_{\Lambda_b}}\right)U_1(n\cdot p',\mu_h,\mu)C_{n,V(A)}(n\cdot p',\mu_h)\int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\psi^{(0)}_{4,\mathrm{eff}}(\omega',\mu)\Bigg\}
\end{aligned}$$

Contents

Input parameters

Parameter	value/interval	unit	prior	source/comments
quark masses				
$\overline{m_b}(\overline{m_b})$	4.193 ± 0.035	GeV	Gaussian @ 68%	[Beneke,2014]
$\overline{m_c}(\overline{m_c})$	1.288 ± 0.02	GeV	Gaussian @ 68%	[Dehnadi,2015]
m_b^{Pole}	4.8 ± 0.1	GeV	Gaussian @ 68%	
hadron masses				
m_{Λ_b}	5619.60	MeV	—	[PDG,2020]
m_{Λ_c}	2286.46	MeV	—	
vacuum condensate densities				
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$0.012^{+0.006}_{-0.012}$	GeV^4	Uniform @ 100%	[Duplancic,2008]
parameters of the Λ_b DAs				
$f^{(2)}_{\Lambda_b}(1\mathrm{GeV})$	$(3.0 \pm 0.5) \times 10^{-2}$	GeV^3	Gaussian @ 68%	[Groote,1997]
sum rule parameters and scales				
μ	[1.0,2.0]	GeV	Uniform @ 100%	[Wang,2015]
ν	[1.0,2.0]	GeV	Uniform @ 100%	
μ_h	$[\overline{m_b}/2, 2\overline{m_b}]$	GeV	Uniform @ 100%	
M^2_{2pt}	2.5 ± 0.5	GeV^2	Uniform@ 100%	[Khodjamirian,2011]
☑ M^2_{SR}	5.0 ± 1.0	GeV^2	Uniform@ 100%	
☑ s_0	[7.0,8.0]	GeV^2	Uniform@ 100%	[Duplancic,2008]

Λ_b LCDAs

Three-parameter model [JHEP 07 (2018) 154]

$$\phi_4(\omega;\mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\beta(\beta+1)}{\alpha(\alpha+1)} \frac{\mathcal{N}}{\omega_0^2} e^{-\omega/\omega_0}\, U(\beta-\alpha,\ 2-\alpha,\ \omega/\omega_0)$$

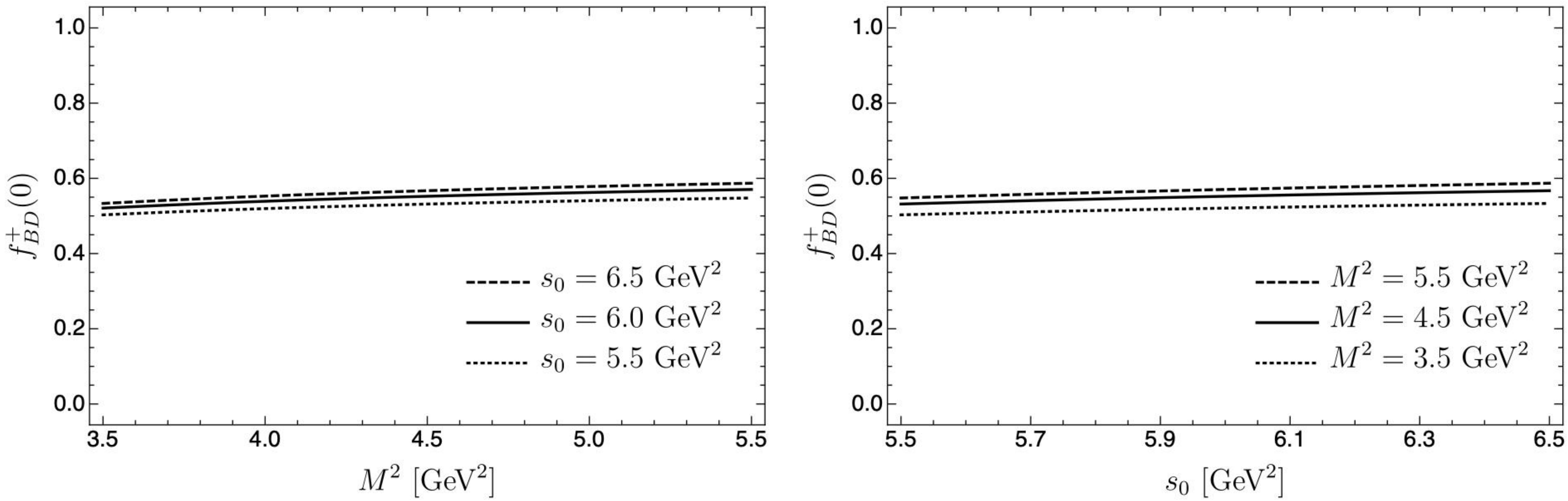
Three parameters at $\mu_0 = 1Gev$ satisfy

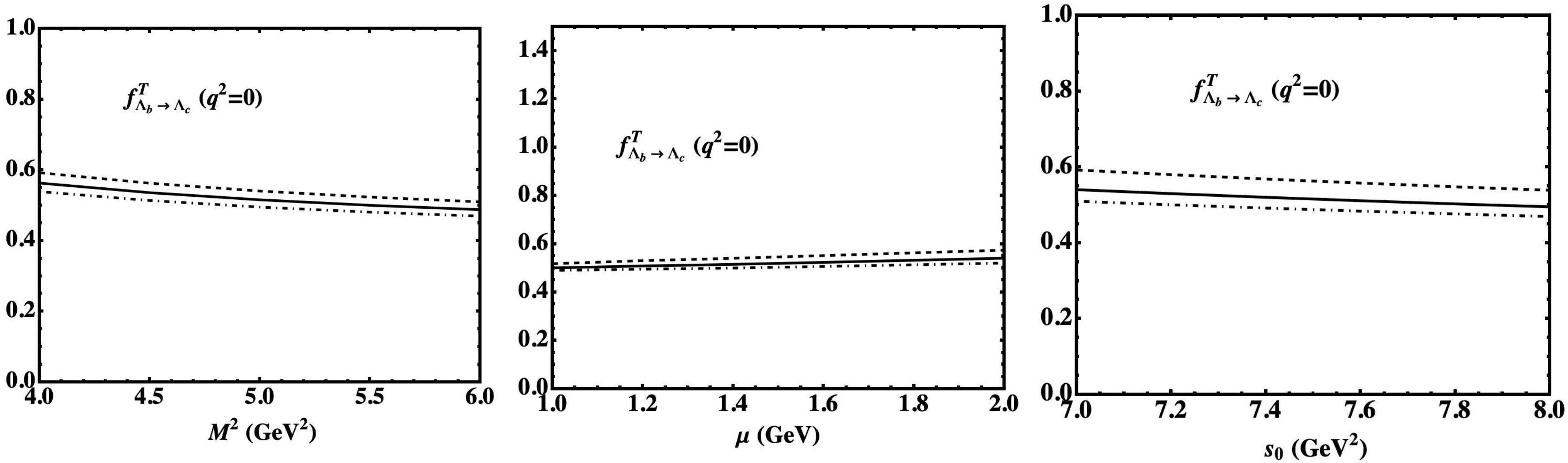
$$\lambda_B(\mu) = \frac{\alpha-1}{\beta-1}\,\omega_0\,,\ \widehat{\sigma}_1(\mu) = \psi(\beta-1) - \psi(\alpha-1) + \ln\frac{\alpha-1}{\beta-1}\,,$$

$$\widehat{\sigma}_2(\mu) = \widehat{\sigma}_1^2(\mu) + \psi'(\alpha-1) - \psi'(\beta-1) + \frac{\pi^2}{6}$$

	ω_0	α	β	λ_B	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\mathcal{N}$
ϕ_4	0.28(10)GeV	1.2583	1.2583	0.28(10)GeV	0	$\pi^2/6$	1
	0.329(118)GeV	1.29771	1.35034	0.28(10)GeV	0.40	5.00	1
	0.264(94)GeV	1.1361	1.12827	0.28(10)GeV	−0.40	−5.00	1

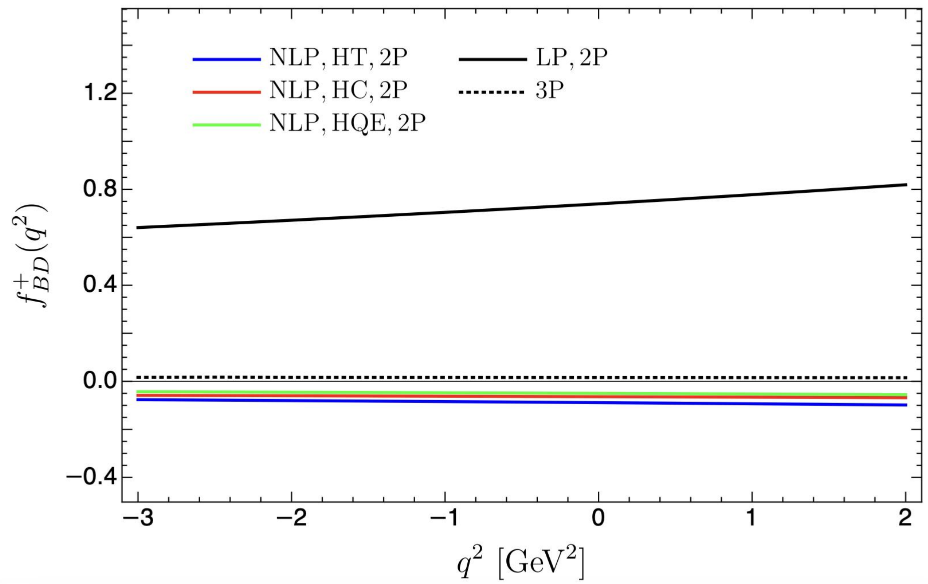
Dependence of parameters



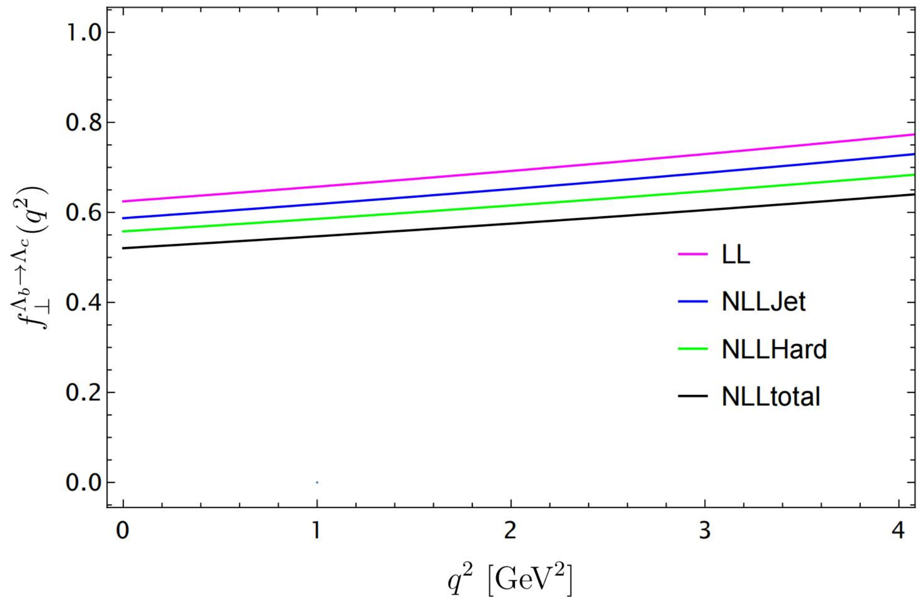


Power correction and Radiative correction

Power correction in $B \to D$



Radiative correction in $\Lambda_b \to \Lambda_c$



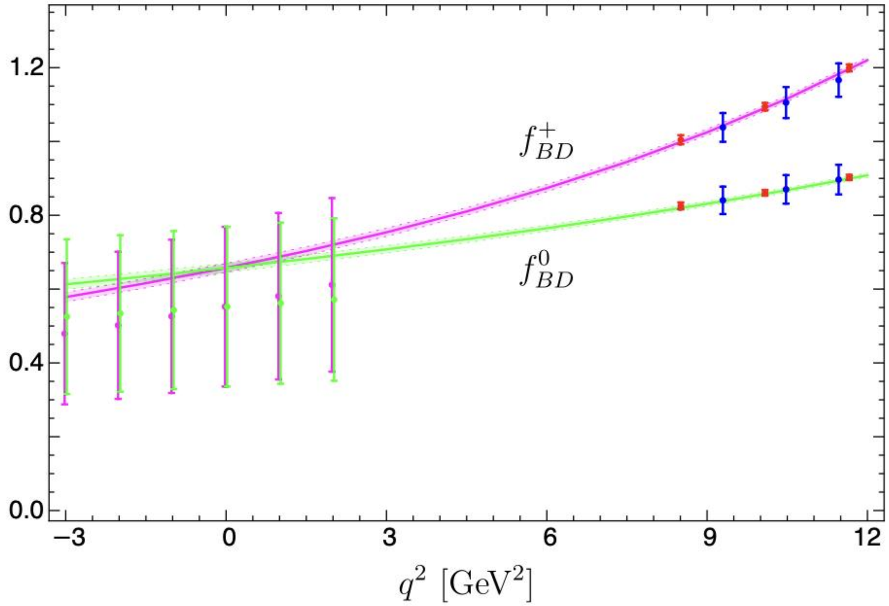
- 3-particle contribution is too small compared to others.
- 2-particle high-twist contribution gives the largest contribution.
- Total power correction is 20%.
- NLL hard function correction is larger than NLL jet function correction.
- NLL correction is 17%

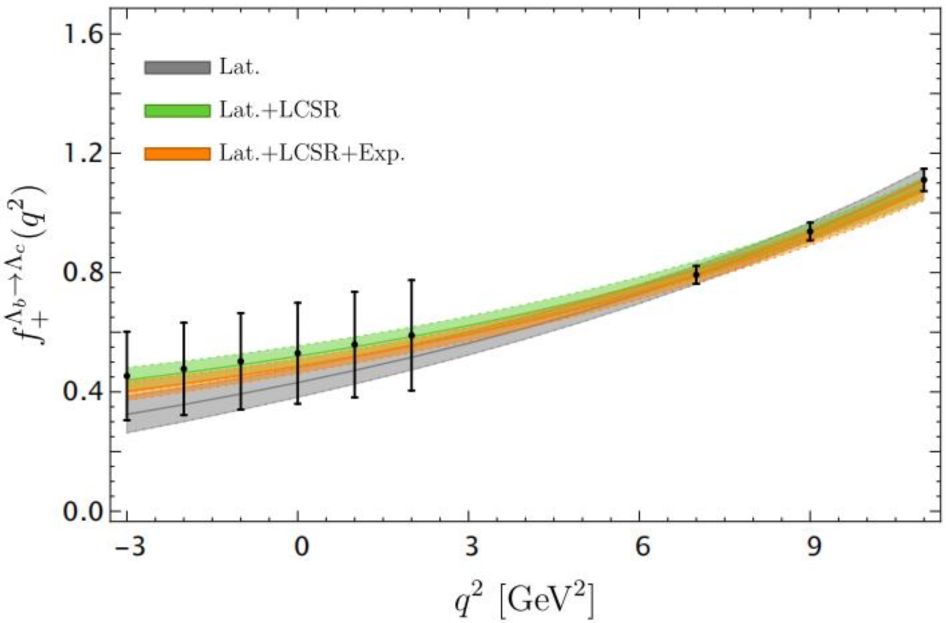
Form factors in entire momentum region

- BGL (BCL) parameterization

$$f_i(t) = \frac{1}{B_i(t)\phi_i(t;\, t_0)} \sum_{n=0}^{2} a_n^i z(t;\, t_0)^n \qquad \sum_{i;n=0}^{2} (a_n^i)^2 \le 1$$

[Phys.Rev.D 92 (2015) 3, 034503]





- include $B_q^{(*)} \to D_q^{(*)}$ form factors ☞ the strong unitarity bound
- LCSR reduce the uncertainty of form factors significantly

$|V_{cb}|$

Differential decay width

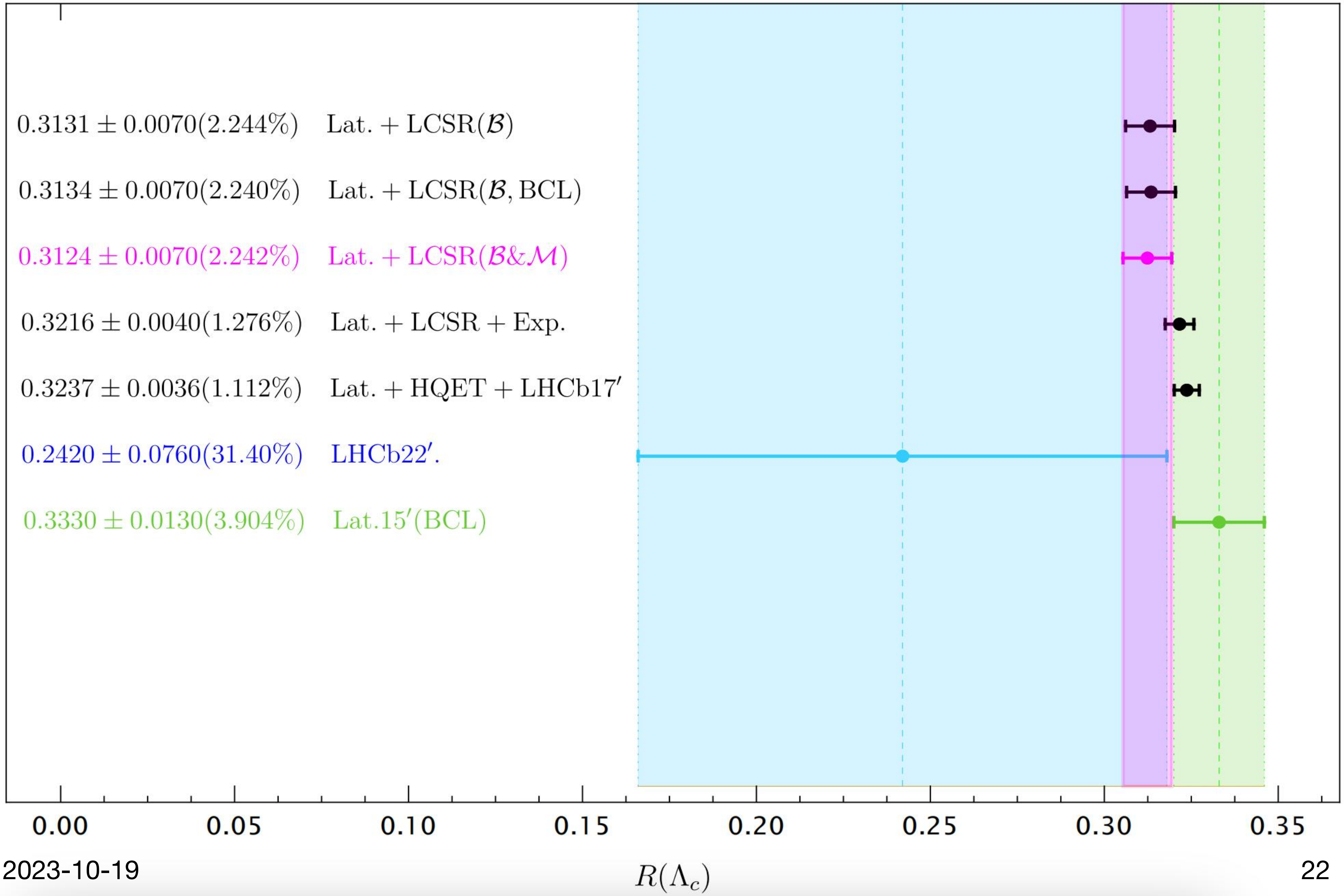
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{qb}^L|^2 \sqrt{s_+ s_-}}{768\pi^3 m_{\Lambda_b}^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \Bigg\{ 4\left(m_\ell^2 + 2q^2\right) \left(s_+ \left[(1-\epsilon_q^R) g_\perp\right]^2 + s_- \left[(1+\epsilon_q^R) f_\perp\right]^2\right)$$
$$+2\frac{m_\ell^2 + 2q^2}{q^2} \left(s_+ \left[(m_{\Lambda_b} - m_X)(1-\epsilon_q^R) g_+\right]^2 + s_- \left[(m_{\Lambda_b} + m_X)(1+\epsilon_q^R) f_+\right]^2\right)$$
$$+\frac{6m_\ell^2}{q^2} \left(s_+ \left[(m_{\Lambda_b} - m_X)(1+\epsilon_q^R) f_0\right]^2 + s_- \left[(m_{\Lambda_b} + m_X)(1-\epsilon_q^R) g_0\right]^2\right) \Bigg\},$$

Fitting results

Scheme	$V_{cb}(10^{-3})$	Relative uncertainty
$B_q^{(*)} \to D_q^{(*)}$ [Cui,Huang,Wang,Zhao,2023]	39.63 ± 0.63	1.59%
$\Lambda_b \to \Lambda_c$	40.10 ± 3.50	8.73%
$\Lambda_b \to \Lambda_c \otimes B_q^{(*)} \to D_q^{(*)}$	39.79 ± 0.62	1.56%
$\Lambda_b \to \Lambda_c$ [(Exp. Err)/2]	40.07 ± 1.91	4.77%
$\Lambda_b \to \Lambda_c \otimes B_q^{(*)} \to D_q^{(*)}$ [(Exp. Err)/2]	39.80 ± 0.60	1.51%

R(D) and R(Λ_c) in SM

- $$R(D) \equiv \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\mu\nu_\mu)} = 0.302 \pm 0.003$$



Contents

Summary and Prospect

Conclusions

- $B \to D$ NLP correction is 20% and $\Lambda_b \to \Lambda_c$ NLL correction is 17%

- Joint fitting LCSR results in large recoil and LQCD in small recoil,
  we get the most reliable and accurate form factors in SM

- Improve the accuracy of $|V_{cb}|$,R(D),R(Λ_c), A^l_{FB} signifiacntly

Outlook

- Precision calculation of Λ_b LCDAs

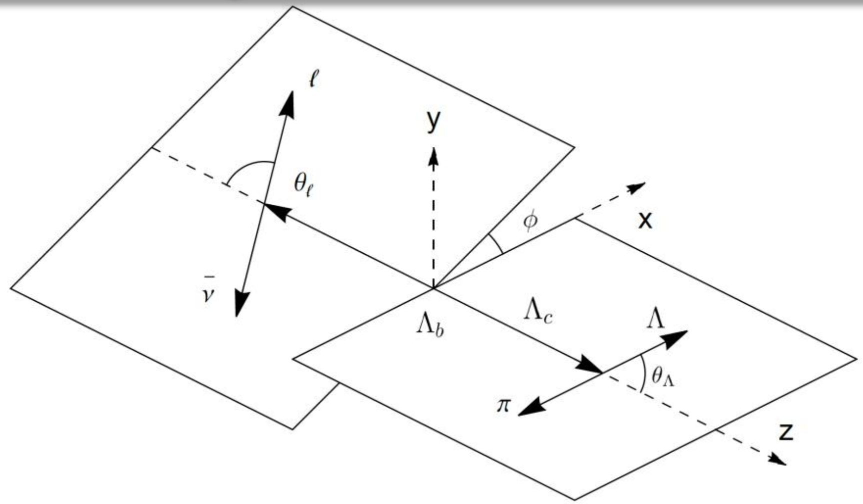
- Appropriate fittting methods

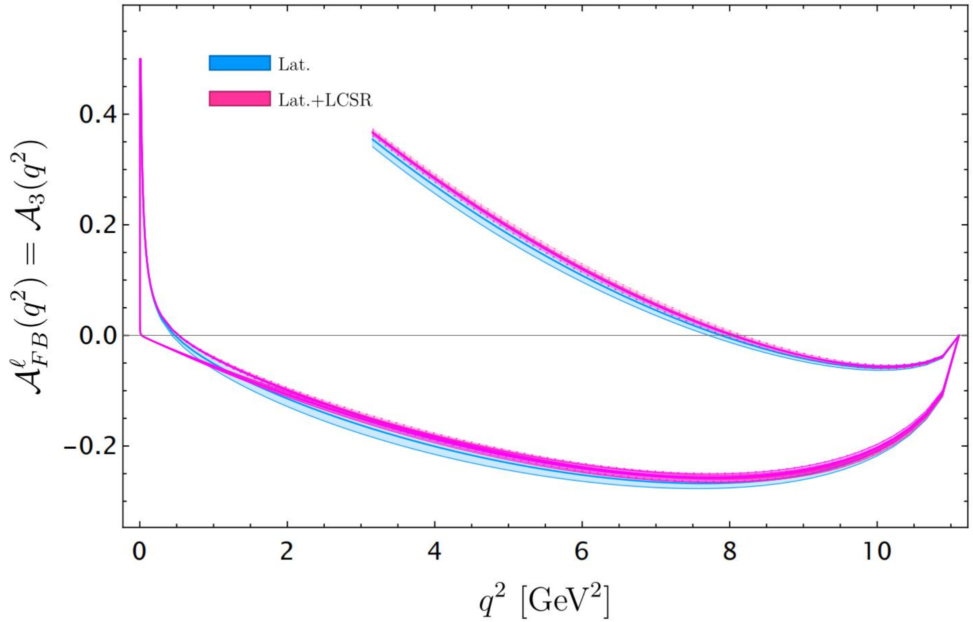
Thanks for your attention!

Forward-backward asymmetry

$$\mathcal{A}_{FB}^{\ell} = \frac{\left[\int_0^1 - \int_{-1}^0\right] \frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell} d\cos\theta_\ell}{d\Gamma/dq^2}$$

[JHEP08(2017)131]





$$\langle A_{FB}^{\tau}\rangle = +(0.0725 \pm 0.0062)$$
$$\langle A_{FB}^{\mu}\rangle = -(0.1912 \pm 0.0056)$$
$$\langle A_{FB}^{e}\rangle = -(0.1970 \pm 0.0056)$$
$$\Delta A_{FB} = \langle A_{FB}^{\mu}\rangle - \langle A_{FB}^{e}\rangle = (5.86 \pm 0.02) \times 10^{-3}$$

[BELLE 2301.07529]