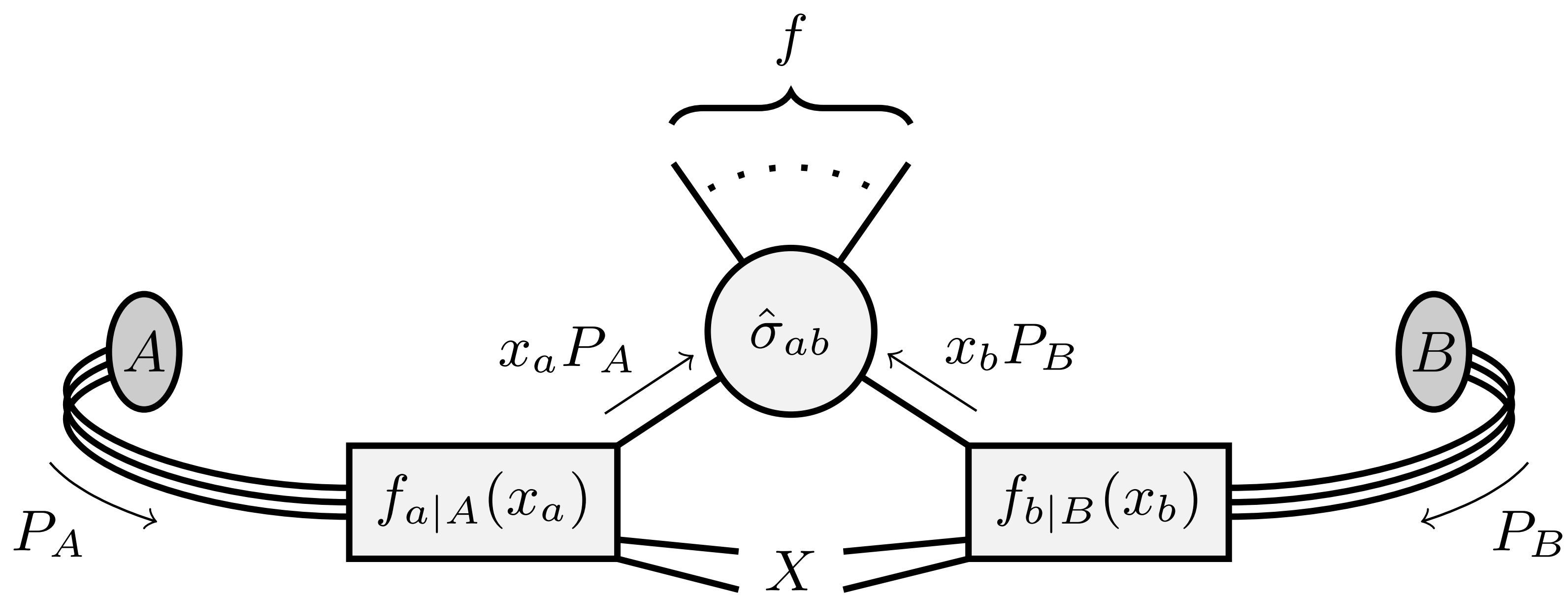
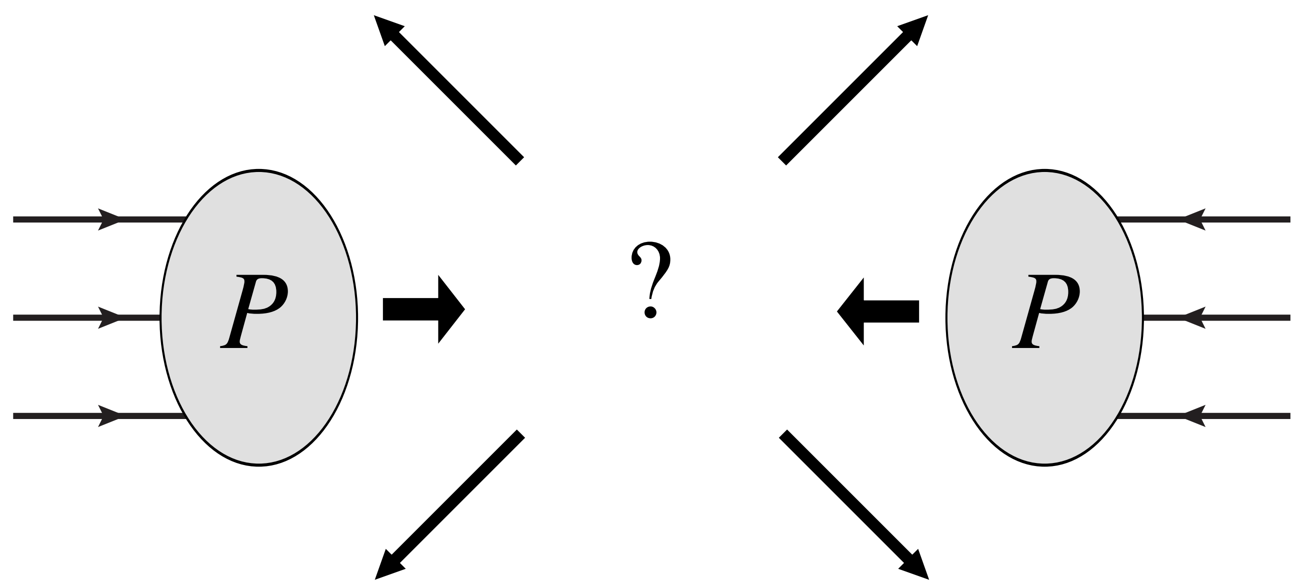


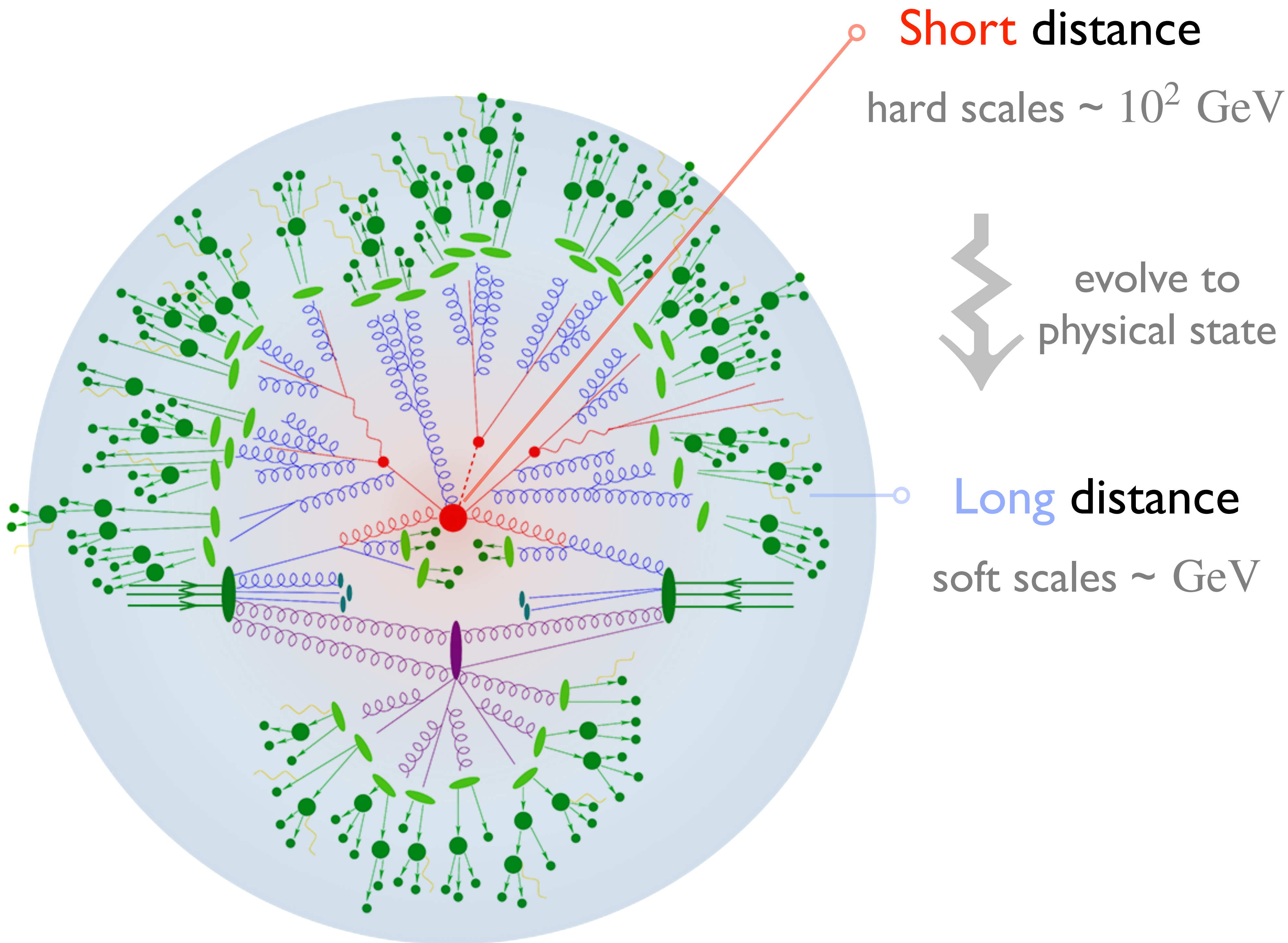
Recent advances in NNLO computations

Rhorry Gauld

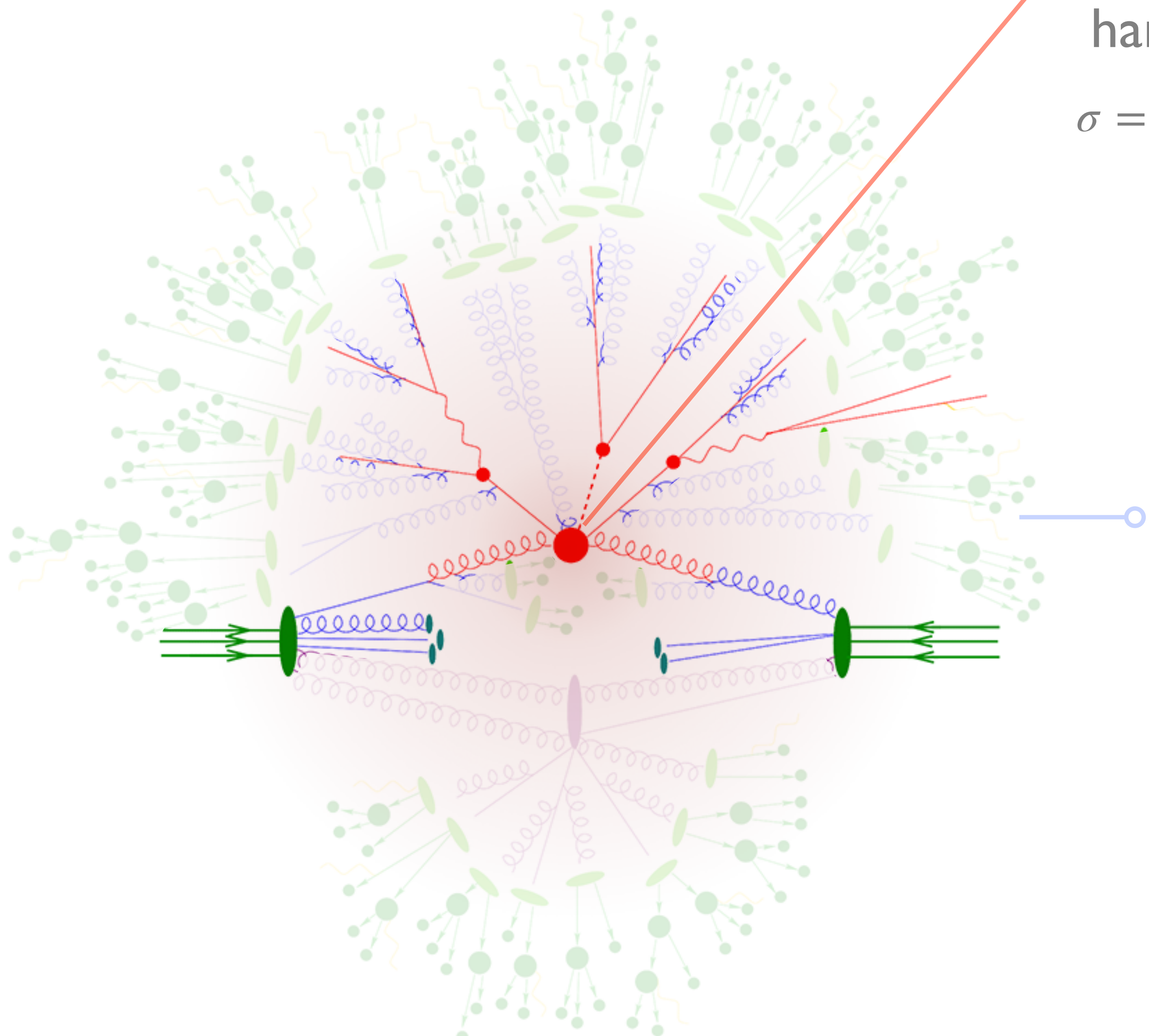
MadGraph / FeynRules meeting [17/11/]2021



$$PP \to f + X$$


Short distance
hard scales ~ 10^2 GeV
evolve to
physical state
Long distance
soft scales ~ GeV

$$PP \to f + X$$



Short distance

hard scales ~ 10^2 GeV

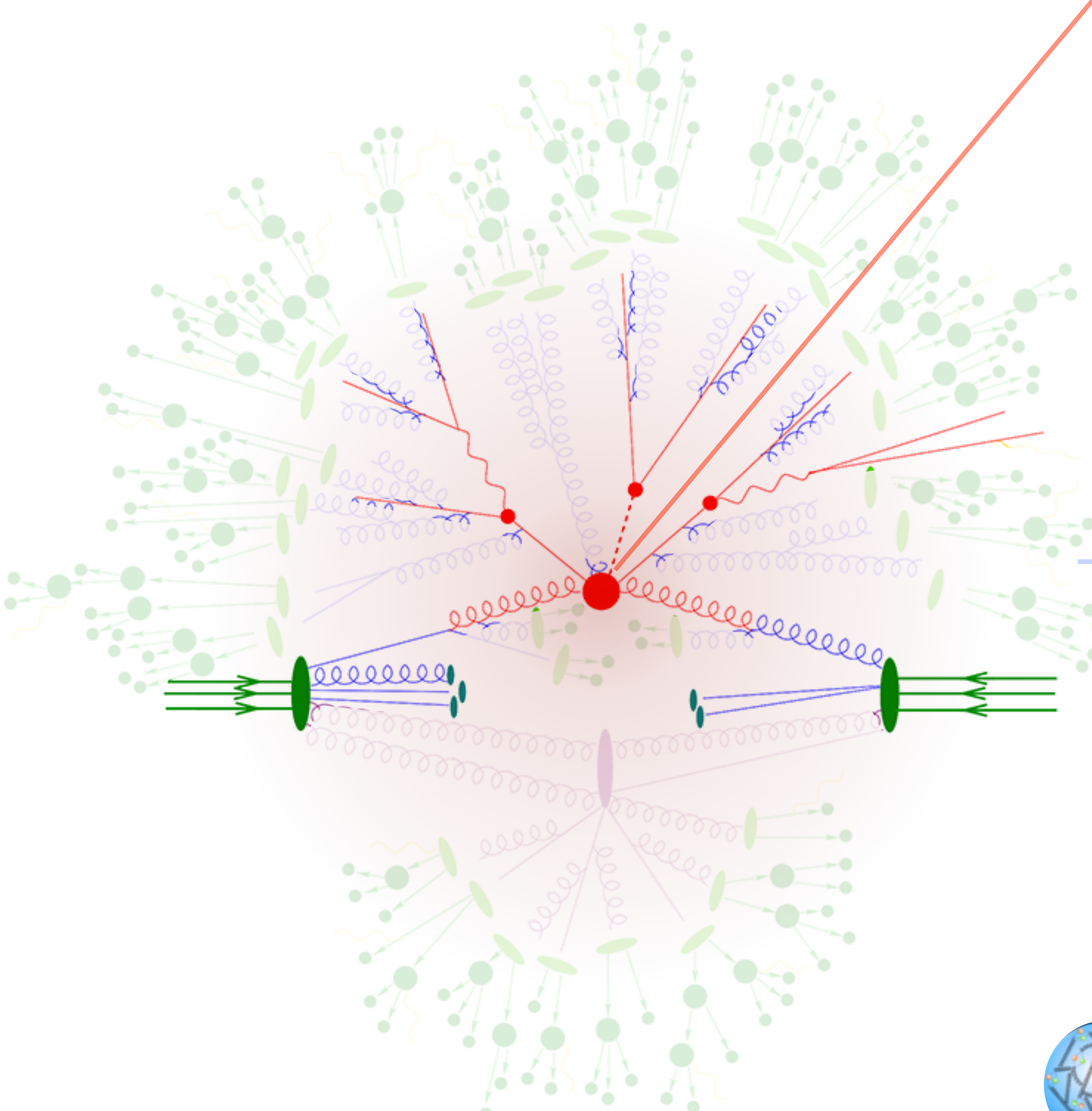
$\sigma = \sigma_0 \left(1 + \alpha_s c_1 + \alpha_s^2 c_2 + \ldots\right)$

Long distance

soft scales ~ GeV

(non-perturbative)

$$PP \to f + X$$



Short distance

hard scales ~ 10^2 GeV

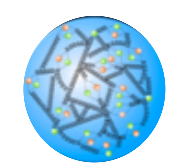
$$\sigma = \sigma_0 \left(1 + \alpha_s c_1 + \alpha_s^2 c_2 + \ldots\right)$$

Long distance

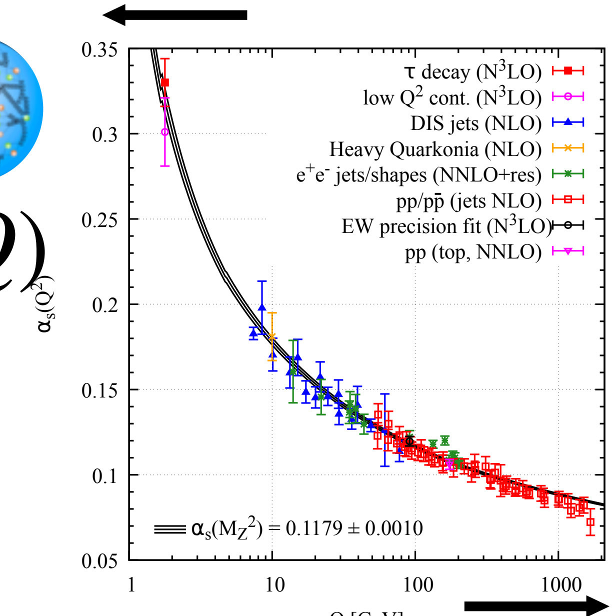
soft scales ~ GeV

(non-perturbative)

Confinement



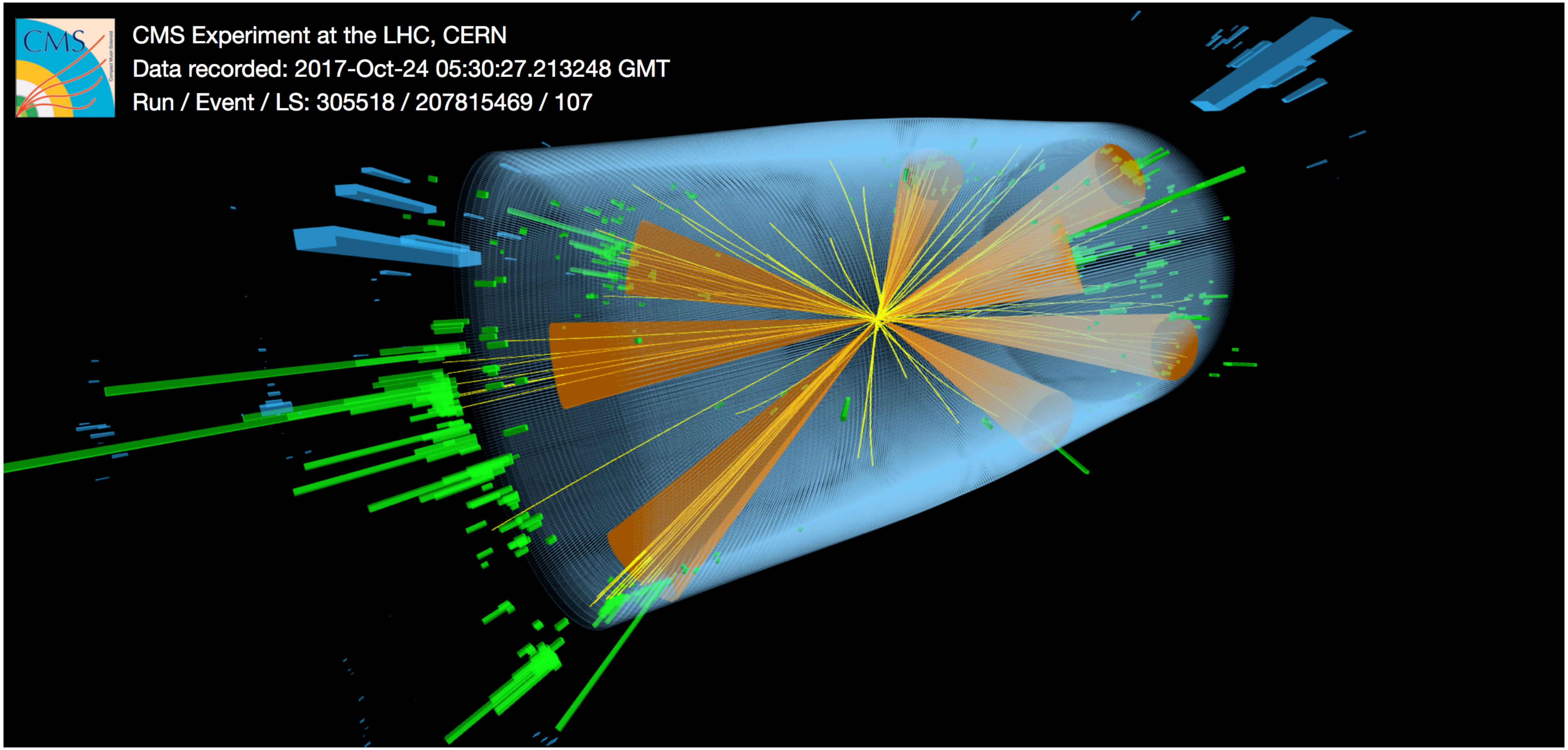
$\alpha_s(Q)$



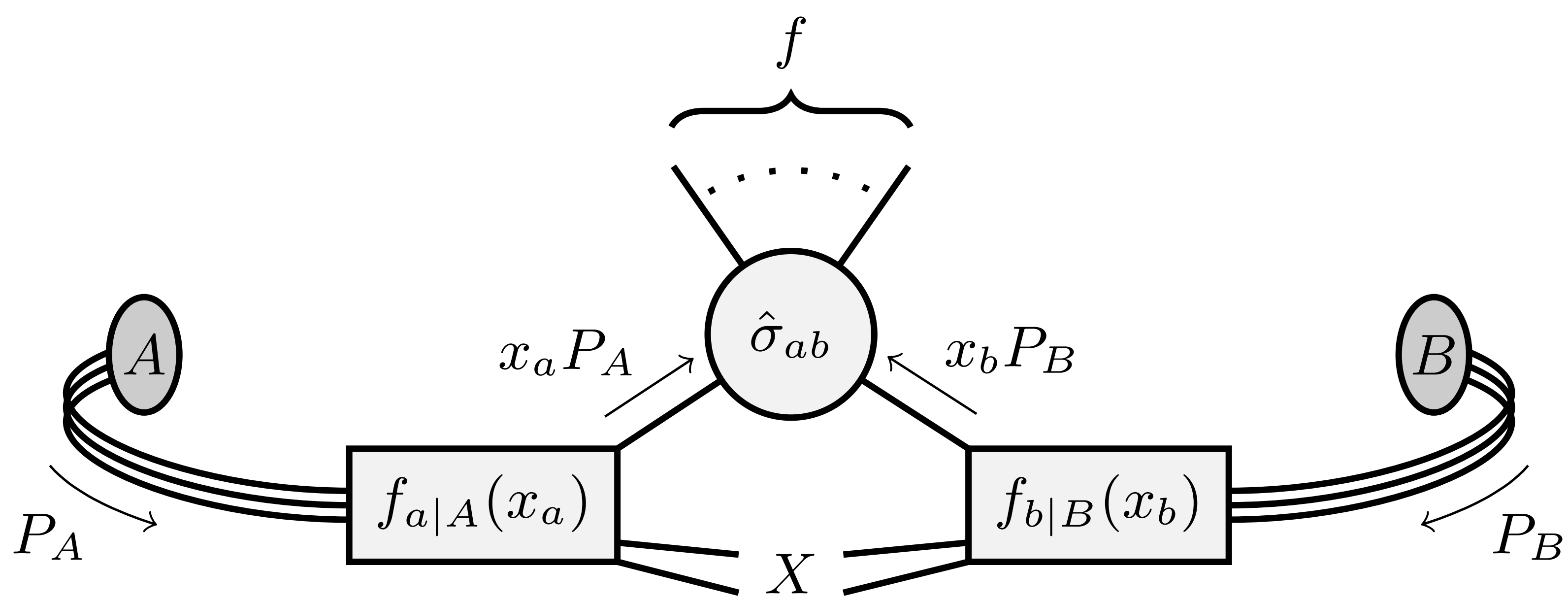
Freedom

$$PP \to f + X$$

Event display for a $t\bar{t}H$ candidate



$$PP \to f + X$$



$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; \underline{f_{a|A}(x_a)\; f_{b|B}(x_b)}\; \underline{\hat{\sigma}_{ab}(x_a, x_b)}\; \big(1+\underline{\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)}\big)$$

parton distribution functions (PDFs)

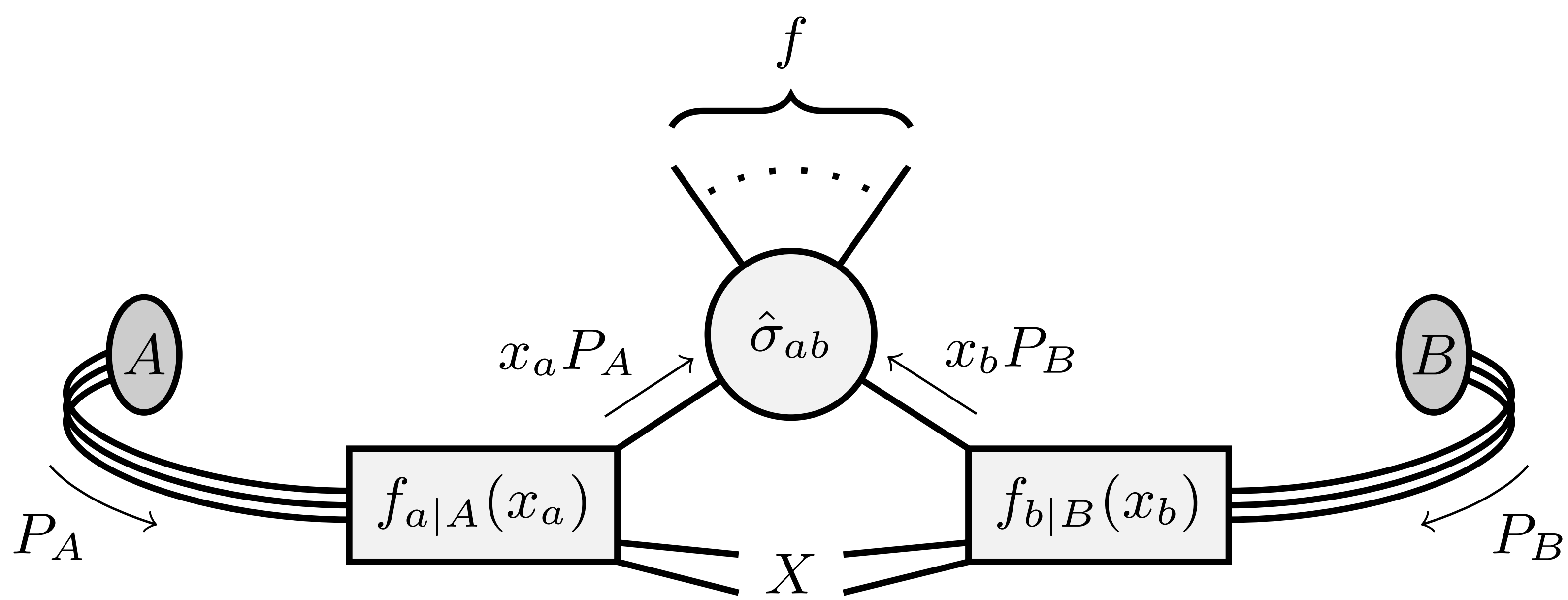
non-perturbative, data-driven

hard scattering

perturbation theory

hadronisation corrections, …

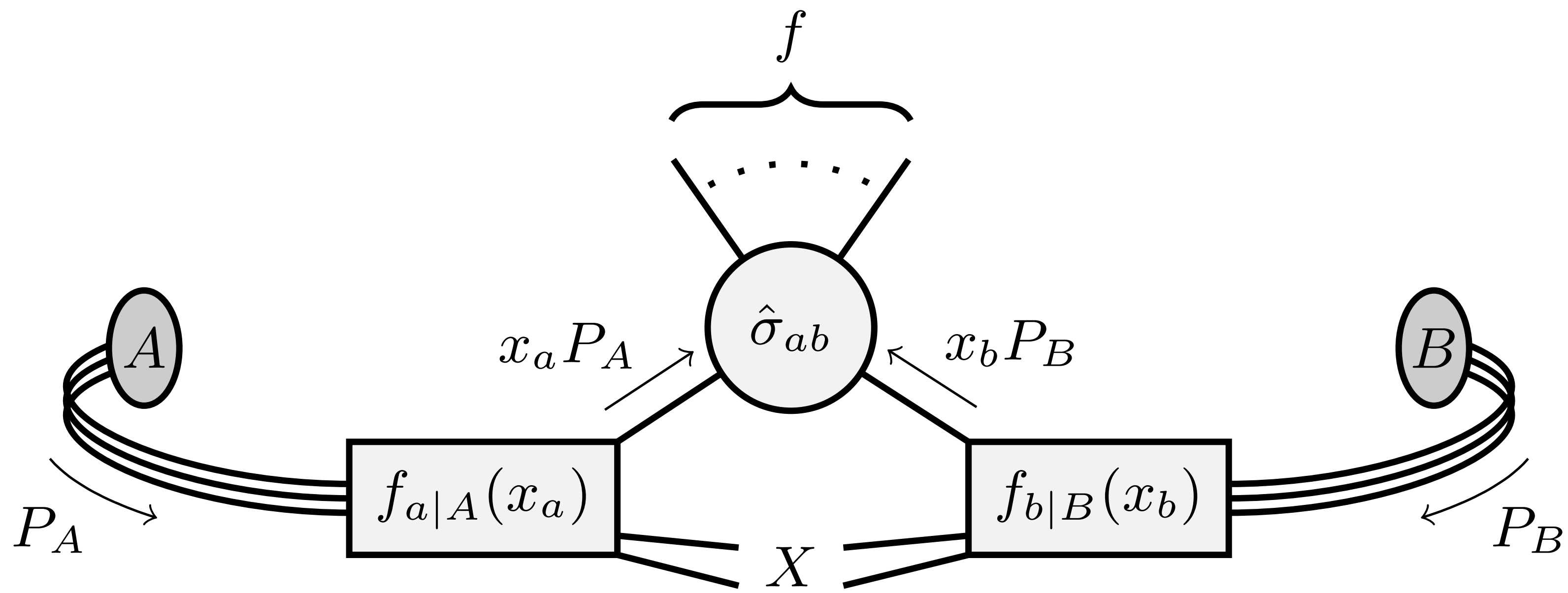
non-perturbative effects



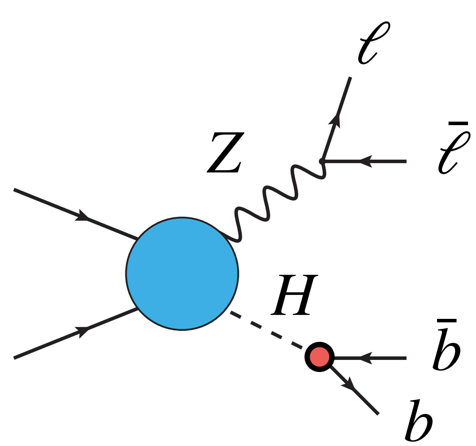
$$\mathrm{d}\sigma^{\text{Measured}}_{pp\to f+X} \quad \textbf{vs.} \quad \mathrm{d}\sigma^{\text{Theory(SM)}}_{pp\to f+X}$$

Theoretically: improve modelling of both background and signal

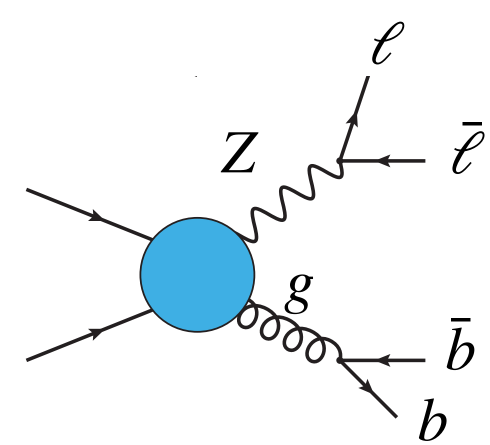
Enables direct and indirect searches for Beyond-the-SM physics



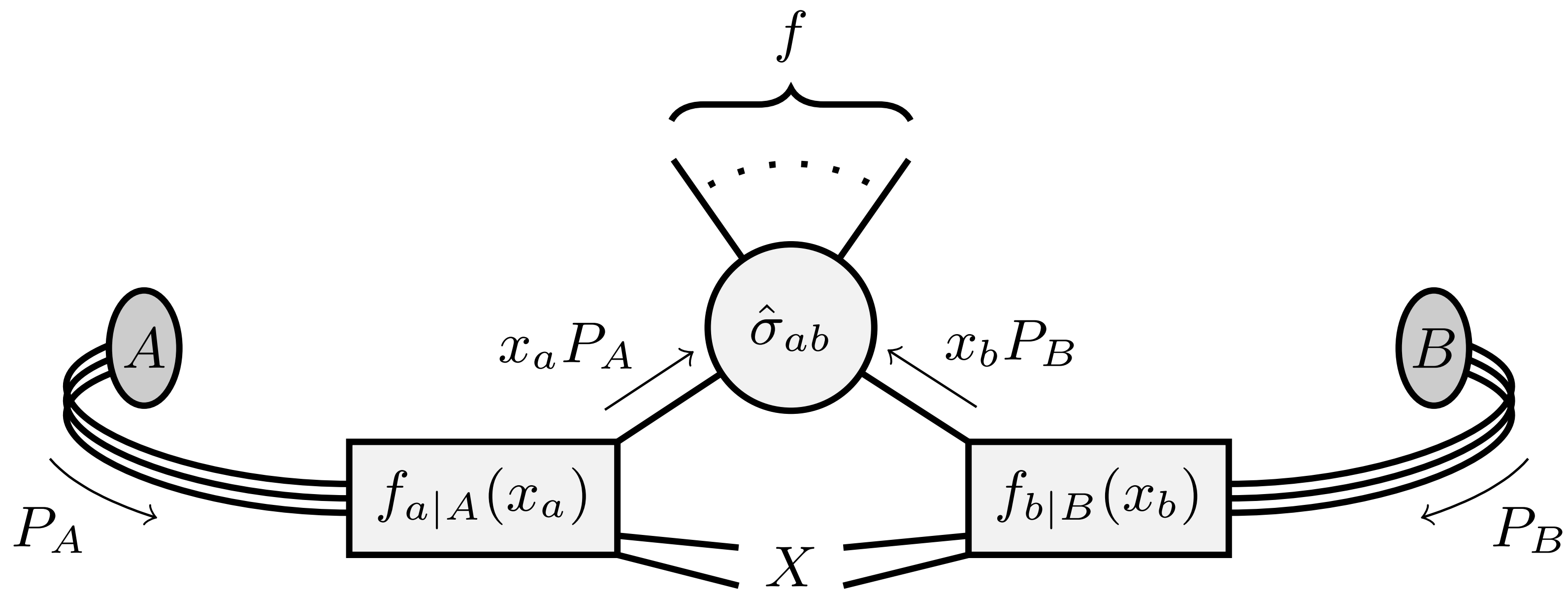
E.g. Associated production of a Higgs boson (VH)



Signal



Background



particle nature of dark matter

mass hierarchies of fermions

origin of neutrino masses

matter anti-matter asymmetry

…

improved understanding of strong interactions

Overview of talk

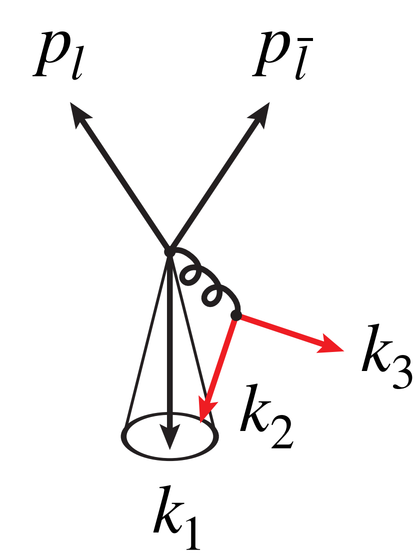
NNLO corrections for the LHC

- Introduction
- State-of-the-art

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}_{ab} + \ldots$$

Physics highlights

- 2 to 3, fragmentation
- Processes with flavoured jets



Outlook

Perturbative corrections and "NNLO"

$$d\sigma_{pp\to f+X} = \sum_{i,j} \int dx_1 dx_2\, f_a(x_1)\, f_b(x_2)\, \boxed{d\hat{\sigma}_{ab\to f+\hat{X}}(\hat{s},\ldots)} \Big(1 + \mathcal{O}(\Lambda_{QCD}/Q)\Big)$$

$$\boxed{d\hat{\sigma}_{ab\to f+..} = d\hat{\sigma}^{\mathrm{LO}}_{ab\to f+..} + \alpha_s\, d\hat{\sigma}^{\mathrm{NLO}}_{ab\to f+..} + \alpha_s^2\, d\hat{\sigma}^{\mathrm{NNLO}}_{ab\to f+..} + \ldots}$$

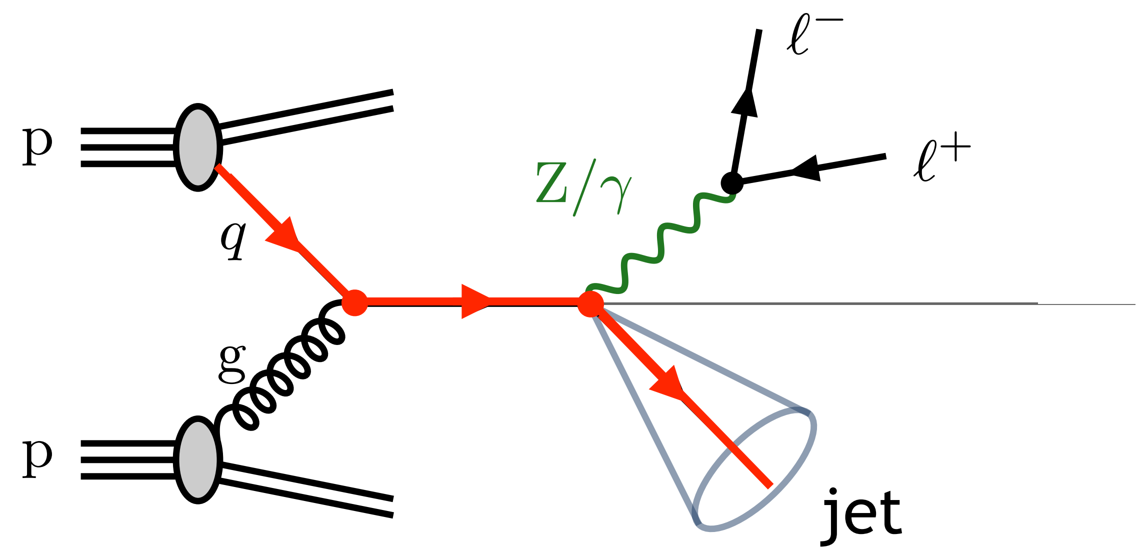
precision $\approx 20\,\%$ $\approx 10\,\%$ $\approx 5\,\%$

Perturbative corrections and “NNLO”

$$d\sigma_{pp\to f+X} = \sum_{i,j} \int dx_1 dx_2\, f_a(x_1)\, f_b(x_2)\, \boxed{d\hat\sigma_{ab\to f+\hat X}(\hat s, \ldots)} \Big(1 + \mathcal{O}(\Lambda_{QCD}/Q)\Big)$$

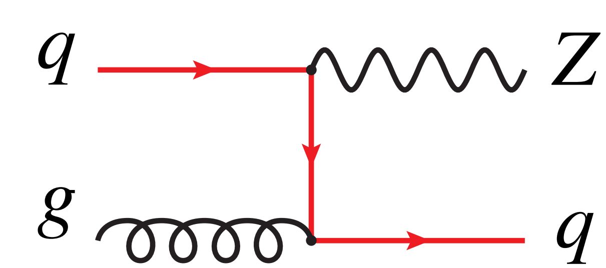
$$\boxed{d\hat\sigma_{ab\to f+..} = d\hat\sigma^{\rm LO}_{ab\to f+..} + \alpha_s\, d\hat\sigma^{\rm NLO}_{ab\to f+..} + \alpha_s^2\, d\hat\sigma^{\rm NNLO}_{ab\to f+..} + \ldots}$$

e.g. f = Z+jet



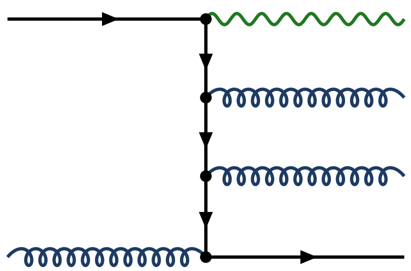
At LO: $qg \to Zq$

$$d\hat\sigma^{\rm LO}_{ab\to f} \sim \int d\phi_{Zq}\, |M_{ab\to Zq}|^2$$

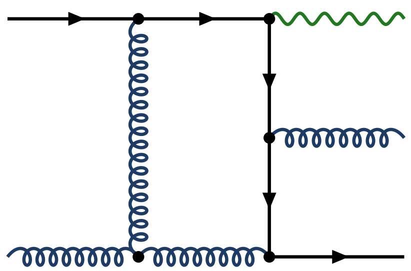


Ingredients of an NNLO computation

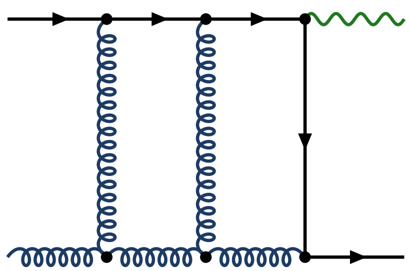
Double Real (RR)



Real-Virtual (RV)



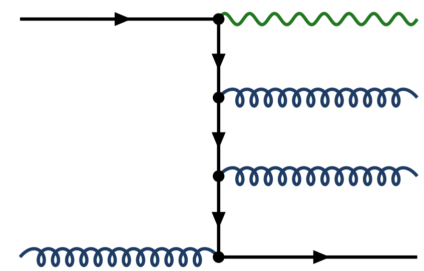
Double Virtual (VV)



$$\mathrm{d}\hat{\sigma}_{ab\to n+i} \sim \int \mathrm{d}\phi_{n+i} \, |M_{ab\to n+i}|^2$$

Ingredients of an NNLO computation

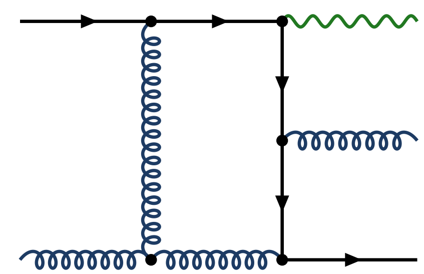
Double Real (RR)



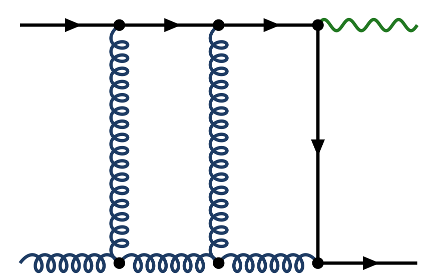
Solved

Madgraph/loop
[Hirschi et al., 11]
OpenLoops
[Cascioli et al., 11]

Real-Virtual (RV)



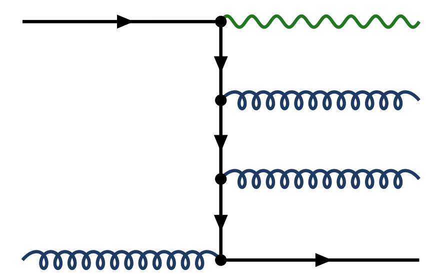
Double Virtual (VV)



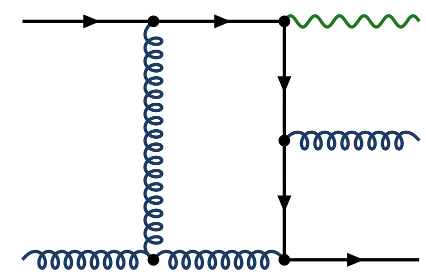
$$\mathrm{d}\hat{\sigma}_{ab\to n+i} \sim \int \mathrm{d}\phi_{n+i}\, |M_{ab\to n+i}|^2$$

Ingredients of an NNLO computation

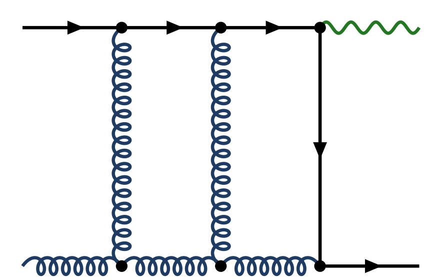
Double Real (RR)



Real-Virtual (RV)



Double Virtual (VV)

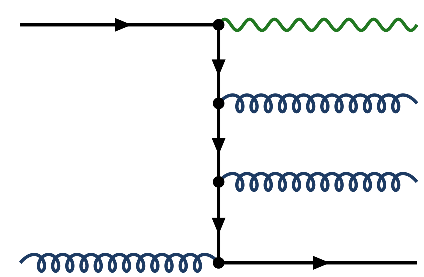


Available:
4-point (masses)
5-point (massless)

$$\mathrm{d}\hat{\sigma}_{ab\to n+i} \sim \int \mathrm{d}\phi_{n+i}\,|M_{ab\to n+i}|^2$$

Ingredients of an NNLO computation

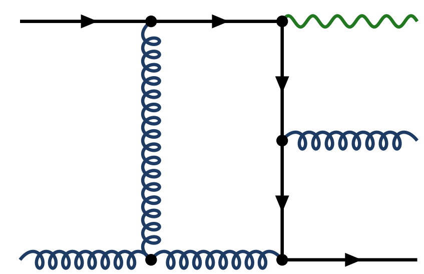
$$\sigma_{\mathrm{NNLO}} = \int_{\phi_{n+2}} \mathrm{d}\sigma^{RR}_{\mathrm{NNLO}}$$



Single unresolved

Double unresolved

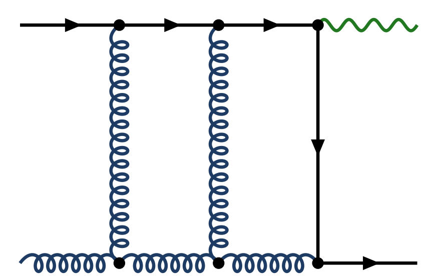
$$+ \int_{\phi_{n+1}} \mathrm{d}\sigma^{RV}_{\mathrm{NNLO}}$$



Single unresolved

$1/\epsilon,\ 1/\epsilon^2$

$$+ \int_{\phi_{n+0}} \mathrm{d}\sigma^{VV}_{\mathrm{NNLO}}$$



$1/\epsilon,\ 1/\epsilon^2,\ 1/\epsilon^3,\ 1/\epsilon^4$

$$\sum = \mathrm{Finite}$$

Non-trivial cancellation of IR divergences

Ingredients of an NNLO computation

$$
\begin{aligned}
\sigma_{\mathrm{NNLO}} = & \int_{\phi_{n+2}} \left(\mathrm{d}\sigma^{RR}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{S}_{\mathrm{NNLO}}\right) \\
& + \int_{\phi_{n+1}} \left(\mathrm{d}\sigma^{RV}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{T}_{\mathrm{NNLO}}\right) \\
& + \int_{\phi_{n+0}} \left(\mathrm{d}\sigma^{VV}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{U}_{\mathrm{NNLO}}\right)
\end{aligned}
$$

mimic unresolved

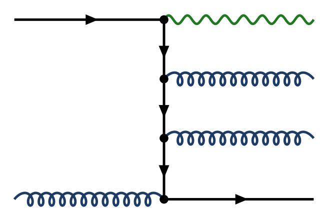
explicit pole cancellation

$$\sum = \mathrm{Finite} - 0$$

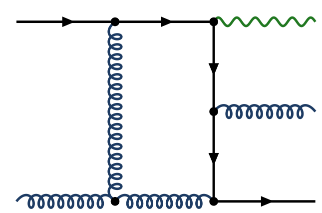
Each line individually finite, can be integrated in 4-d

NNLO methods

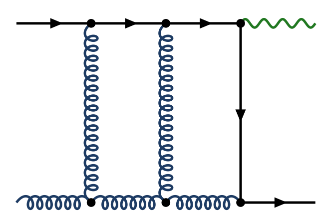
$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} d\sigma^{RR}_{\text{NNLO}}$$



$$+ \int_{\Phi_{Z+2}} d\sigma^{RV}_{\text{NNLO}}$$



$$+ \int_{\Phi_{Z+1}} d\sigma^{VV}_{\text{NNLO}}$$



$\sum$ finite

◉ Apply measurement function $\mathscr{F}$ to each line:
jets, fiducial cross-section, differential observables

➡ These methods operate on each line to render intermediate expression finite (i.e. suitable for numerical integration)

Antenna
[Gehrmann-De Ridder, Gehrmann, Glover '05]

CoLorFul
[Del Duca, Somogyi, Trocsanyi '05]

Geometric
[Herzog '18]

Local analytical sectors
[Magnea et al. '18]

Local unitarity
[Capatti et al. '20]

Nested soft-collinear
[Caola, Melnikov, Rontsch '17]

N-jettiness
[Gaunt, Stahlhofen, Tackmann, Walsh '15]
MCFM [Boughezal, Focke, Liu, Petriello '15]

Projection-to-Born
[Cacciari et al. '15]

qT-subtraction
MATRIX [Catani, Grazzini '07]

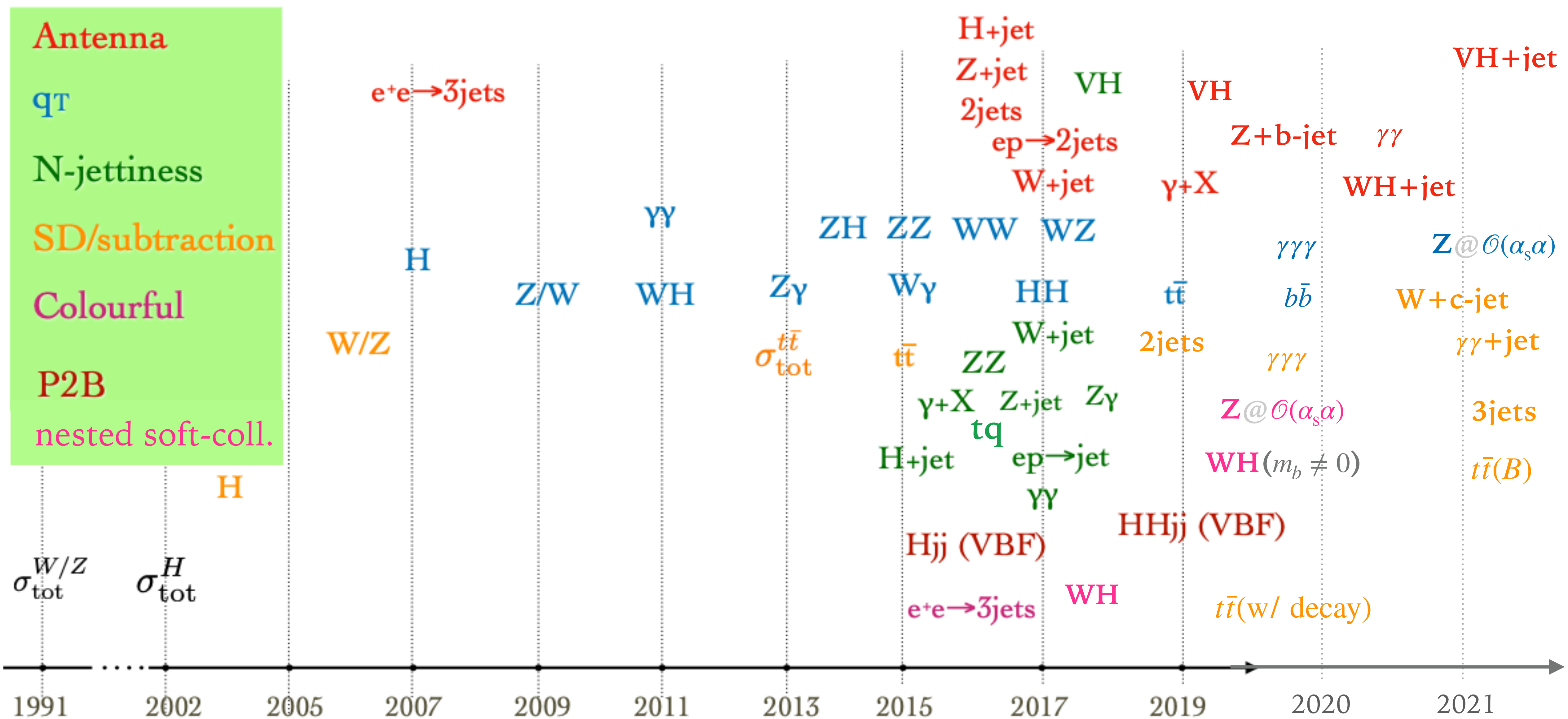
STRIPPER
[Cazkon '10]

subtraction & slicing
public implementation

NNLO timeline

Explosion in progress since ~2015 (development of methods)

$2 \to 1$ | $2 \to 2$ (with masses) | $2 \to 3$ (massless)

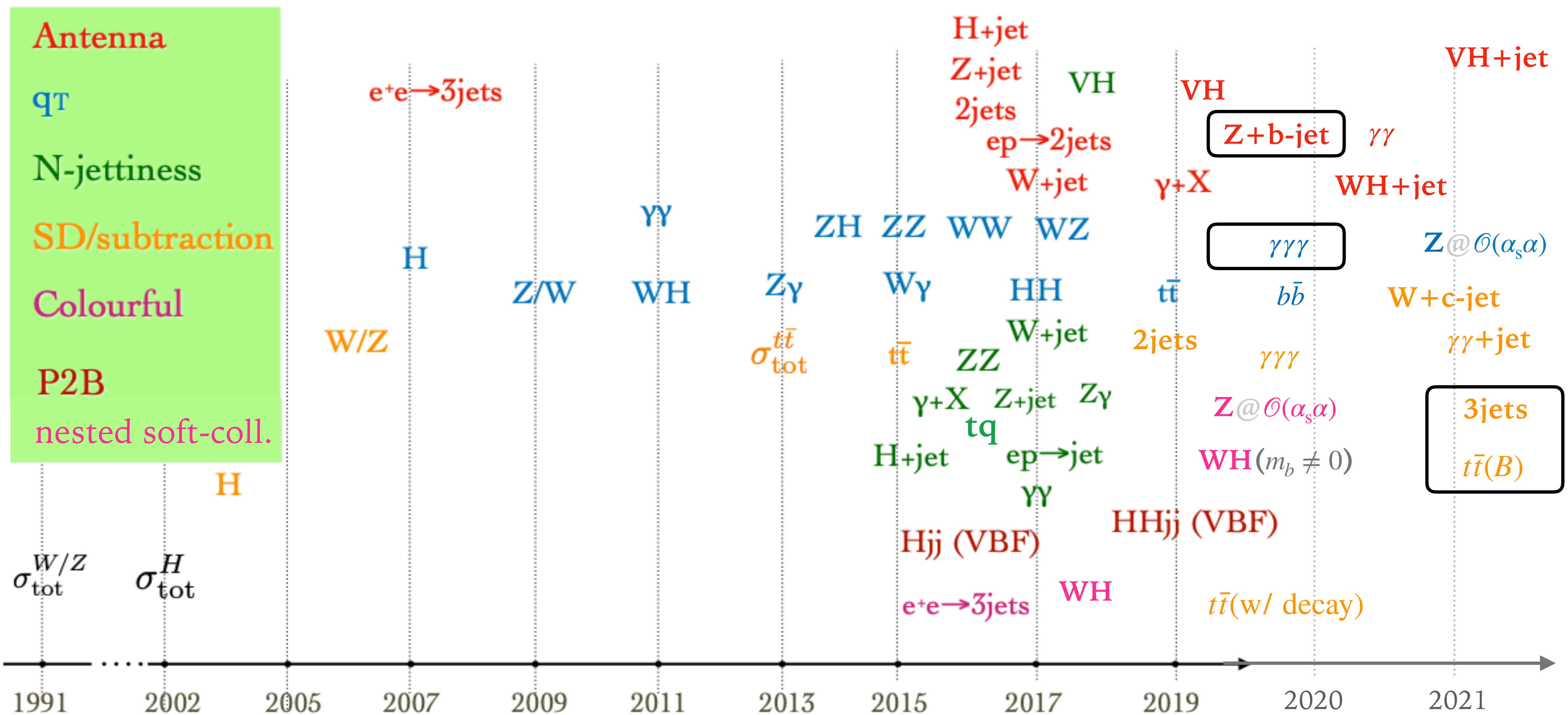


[based on slide by M. Grazzini; QCD@LHC 2019]

NNLO timeline

Explosion in progress since ~2015 (development of methods)

$2 \rightarrow 1$ $\quad$ $2 \rightarrow 2$ (with masses) $\quad$ $2 \rightarrow 3$ (massless)



[based on slide by M. Grazzini; QCD@LHC 2019]

I will focus on these cases

Foreword

These calculations are complex:

They rely on independent methods or implementations
Require careful validation and benchmarking
(historically, has led to `bug fixes' etc.)

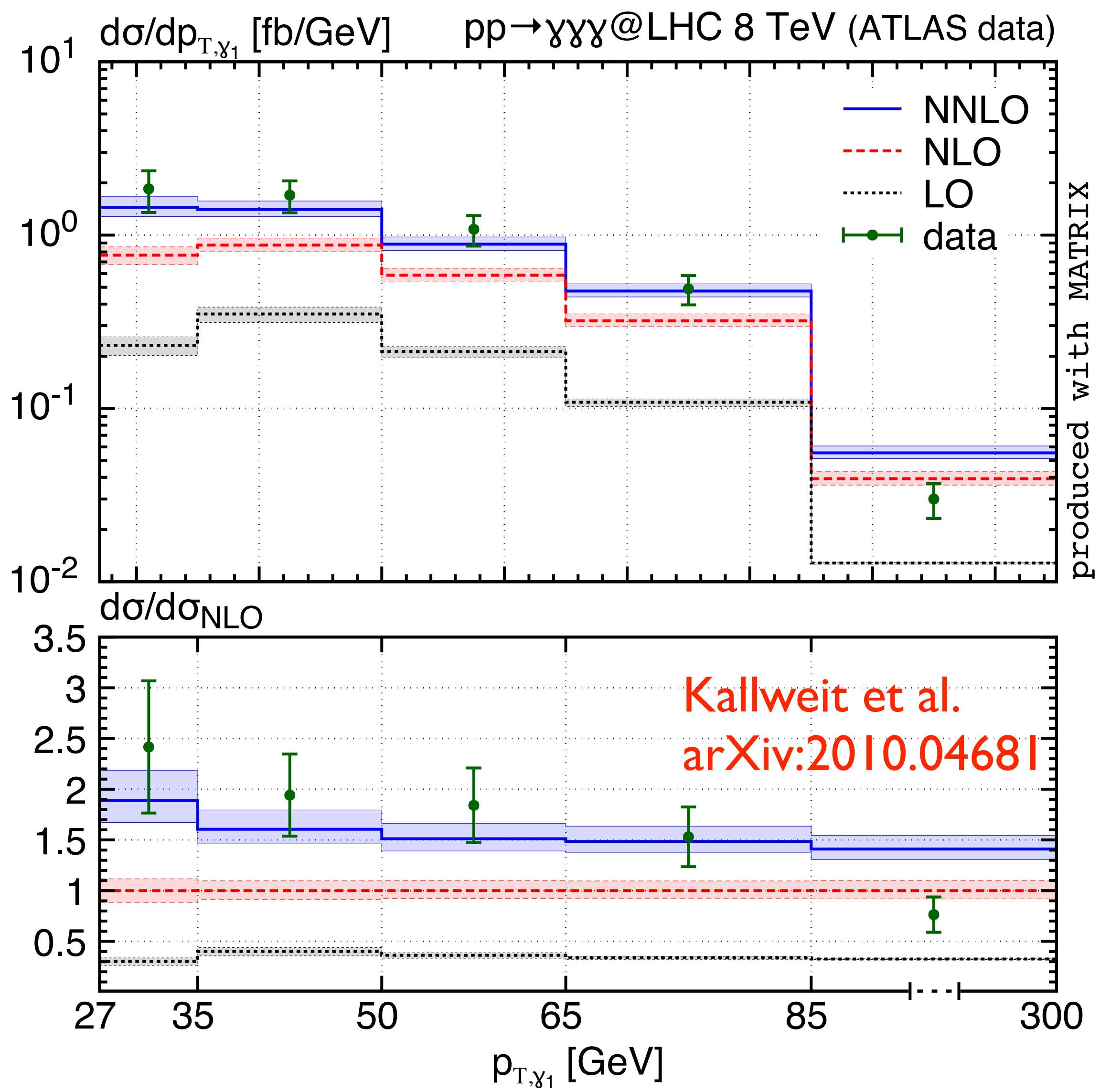
CPU intensive:

Typically an NNLO distribution ~ O(100k) CPU hours
For special cases (e.g. Angular coefficients, RG et al, arXiv:1708.00008 O(1m) CPU hours)
Not suitable for PDF fits etc. (grid tables a must)

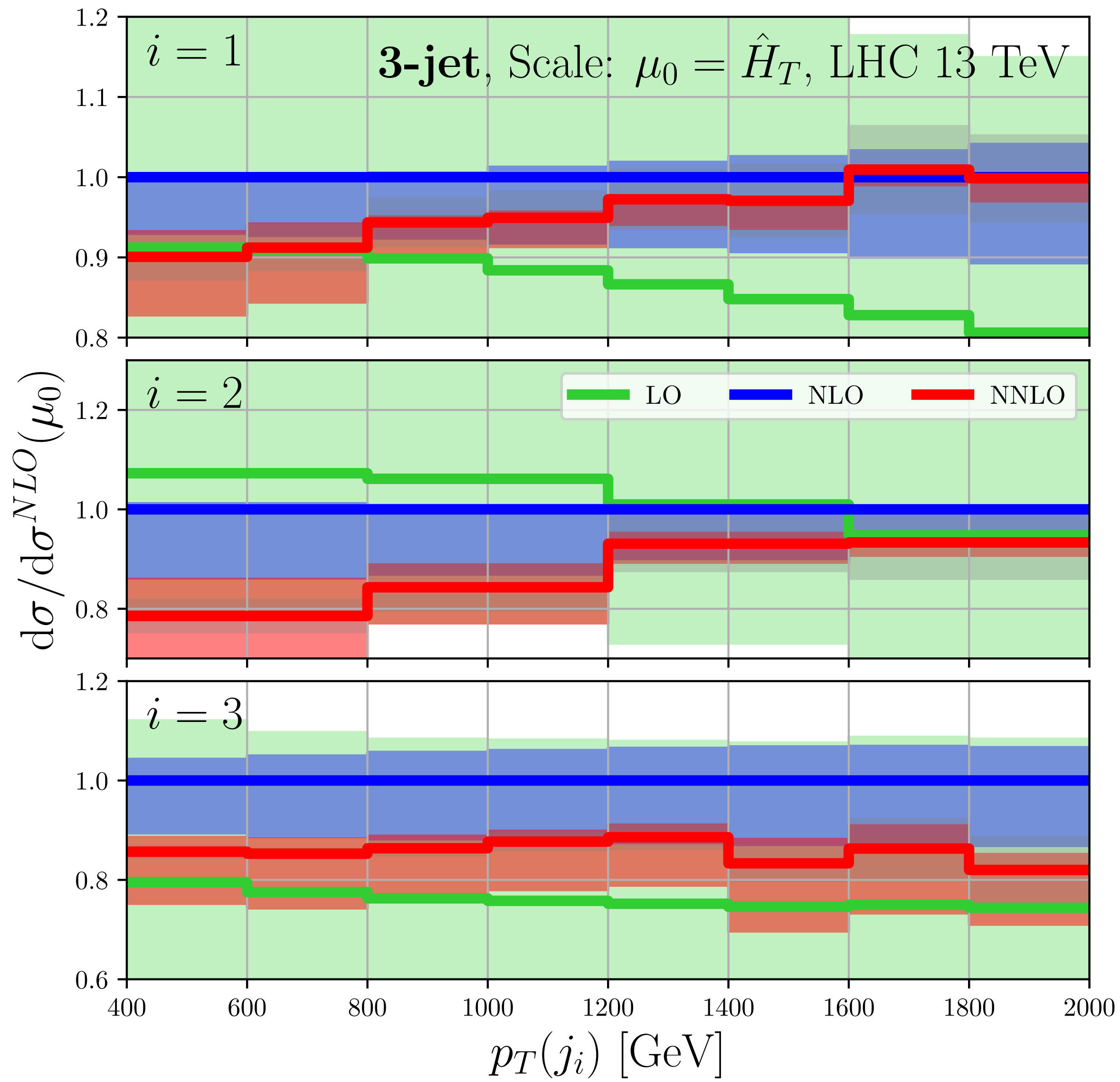
Many/most of the codes are private:

Running the computation and processing of data is involved
Ironically, available public codes currently based on slicing methods

$2 \to 3$ results



$pp \to \gamma\gamma\gamma$



$pp \to jjj$

See also, Chawdry et al., arXiv:1911.00479

Czakon et al., arXiv:2106.05331

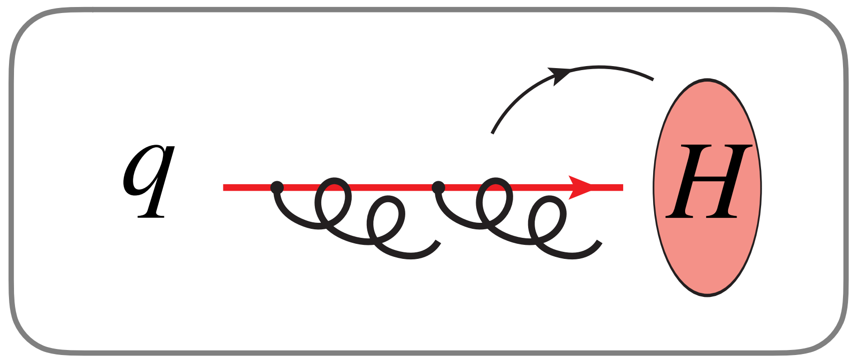
*VV at leading colour

$t\bar{t}$ with fragmentation in decay

$$d\sigma_{pp\to H+X} = \sum_{i,j} \int dx_1 dx_2\, f_a(x_1)\, f_b(x_2) \sum_q \int dz\, d\hat{\sigma}_{ab\to q+\hat{X}}(\hat{s},\ldots) \otimes D_{q\to H}(z,\ldots)$$

Inclusion of (resummed) perturbative fragmentation functions at NNLO QCD

Czakon et al., arXiv:2102.08267

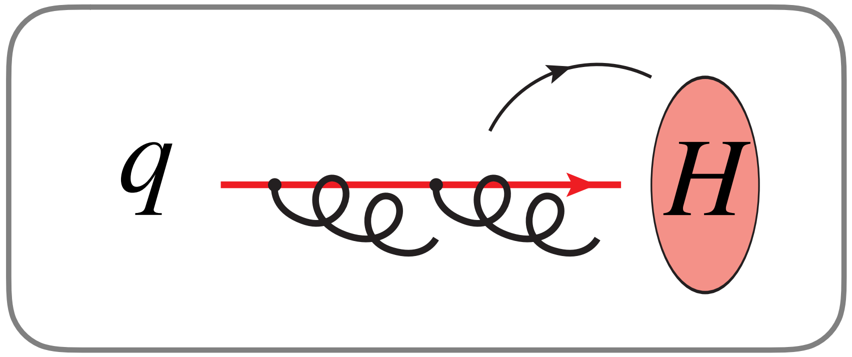


$t\bar{t}$ with fragmentation in decay

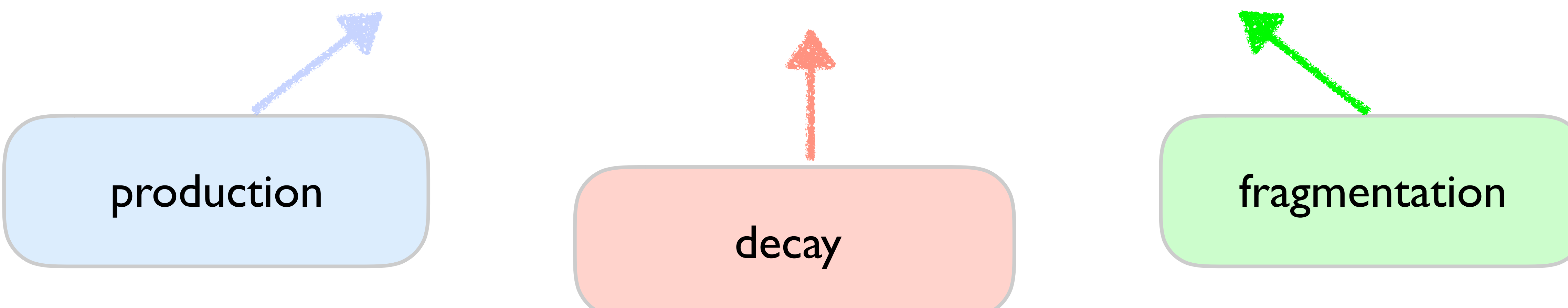
$$\mathrm{d}\sigma_{pp\to H+X} = \sum_{i,j} \int \mathrm{dx}_1 \mathrm{dx}_2\, f_a(x_1)\, f_b(x_2) \sum_q \int \mathrm{d}z\, \mathrm{d}\hat{\sigma}_{ab\to q+\hat{X}}(\hat{s},\ldots) \otimes D_{q\to H}(z,\ldots)$$

Inclusion of (resummed) perturbative fragmentation functions at NNLO QCD

Czakon et al., arXiv:2102.08267



$$\mathrm{d}\sigma^{(n)}_{pp\to t\bar{t}+i} \otimes \mathrm{d}\Gamma^{(n)}_{t(\bar{t})\to Wbj} \otimes D_{i(j)\to B}$$



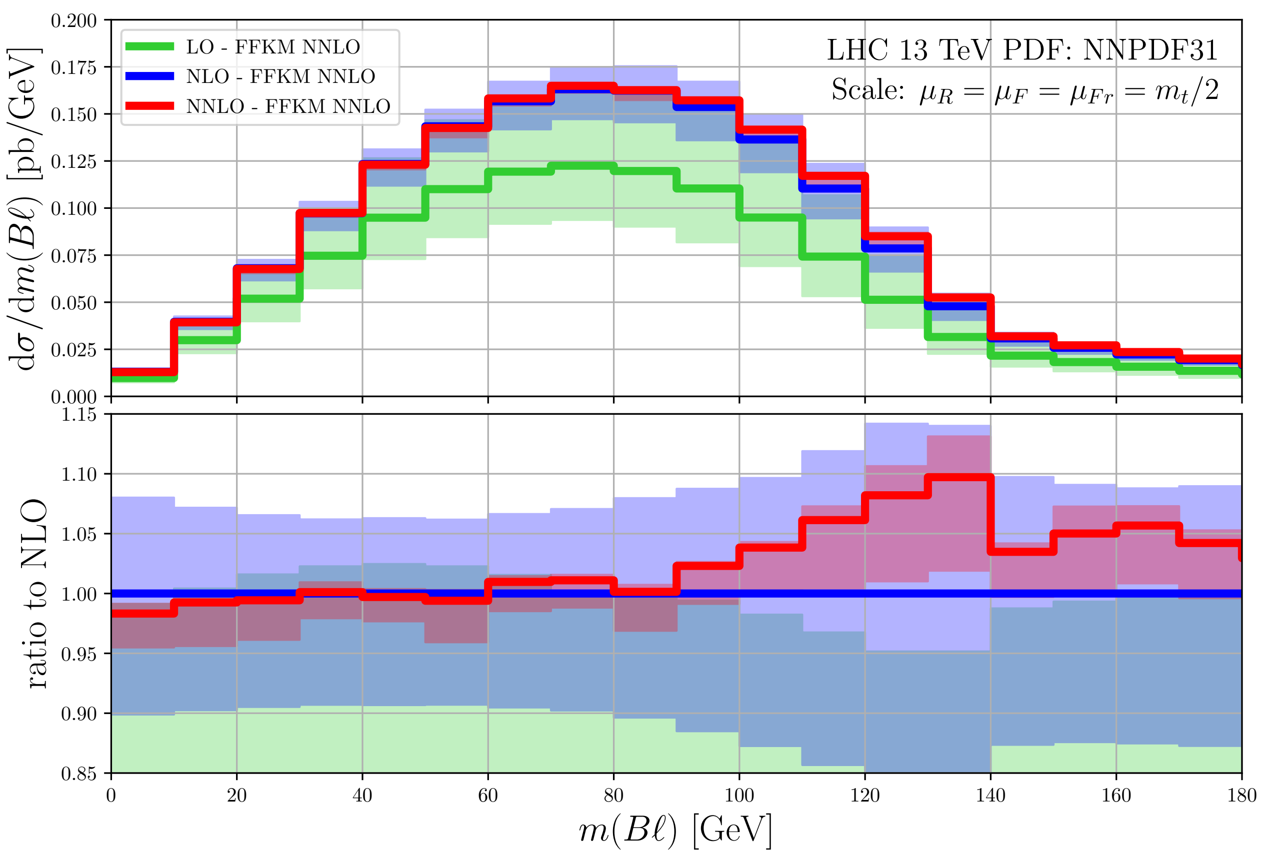
1. More direct access to experimental observables
2. Precision framework to extract the universal fragmentation functions

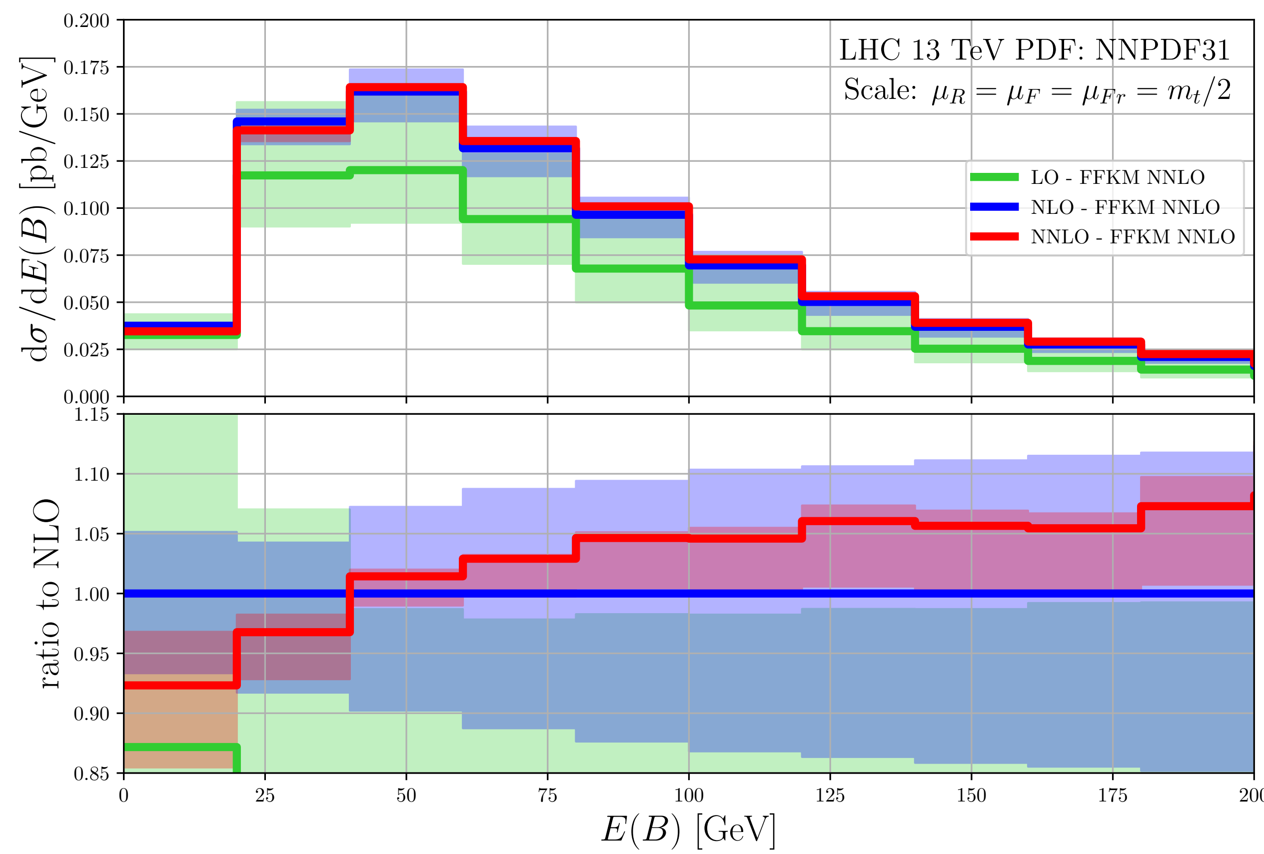
$t\bar{t}$ with fragmentation in decay

$$d\sigma_{pp\to H+X} = \sum_{i,j} \int dx_1 dx_2\, f_a(x_1)\, f_b(x_2) \sum_q \int dz\, d\hat{\sigma}_{ab\to q+\hat{X}}(\hat{s},\ldots) \otimes D_{q\to H}(z,\ldots)$$

Inclusion of (resummed) perturbative fragmentation functions at NNLO QCD

Czakon et al., arXiv:2102.08267





Future applications: inclusive B/D hadron production

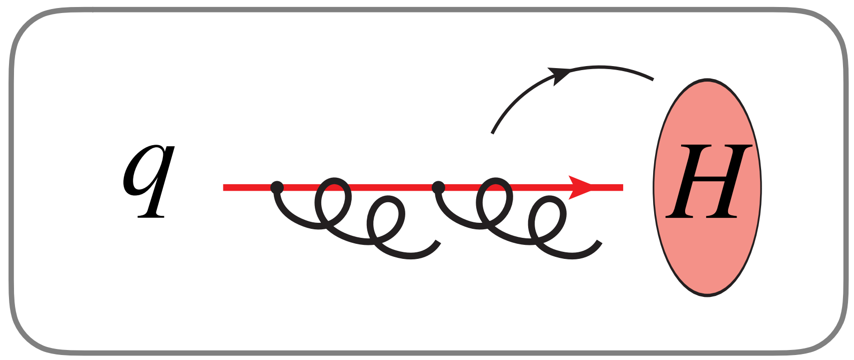
Relevant for extracting fragmentation functions (and also nPDFs)

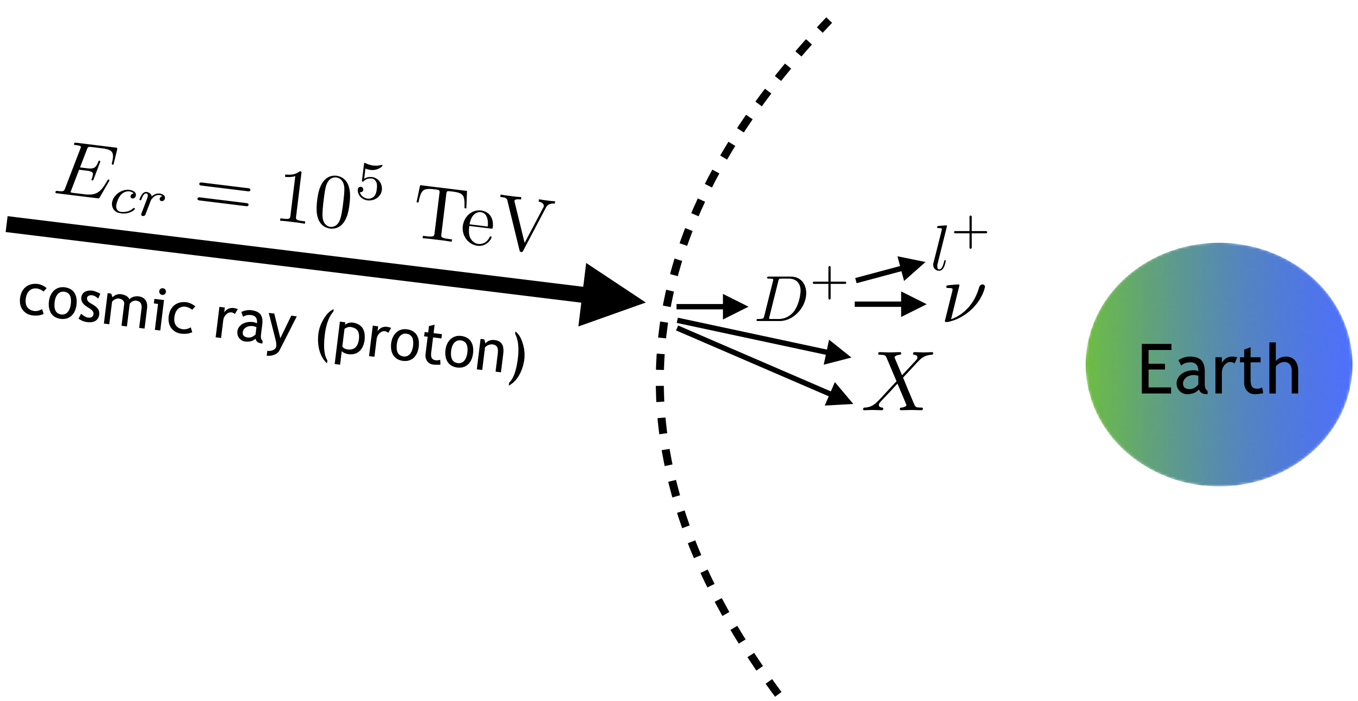
$t\bar{t}$ with fragmentation in decay

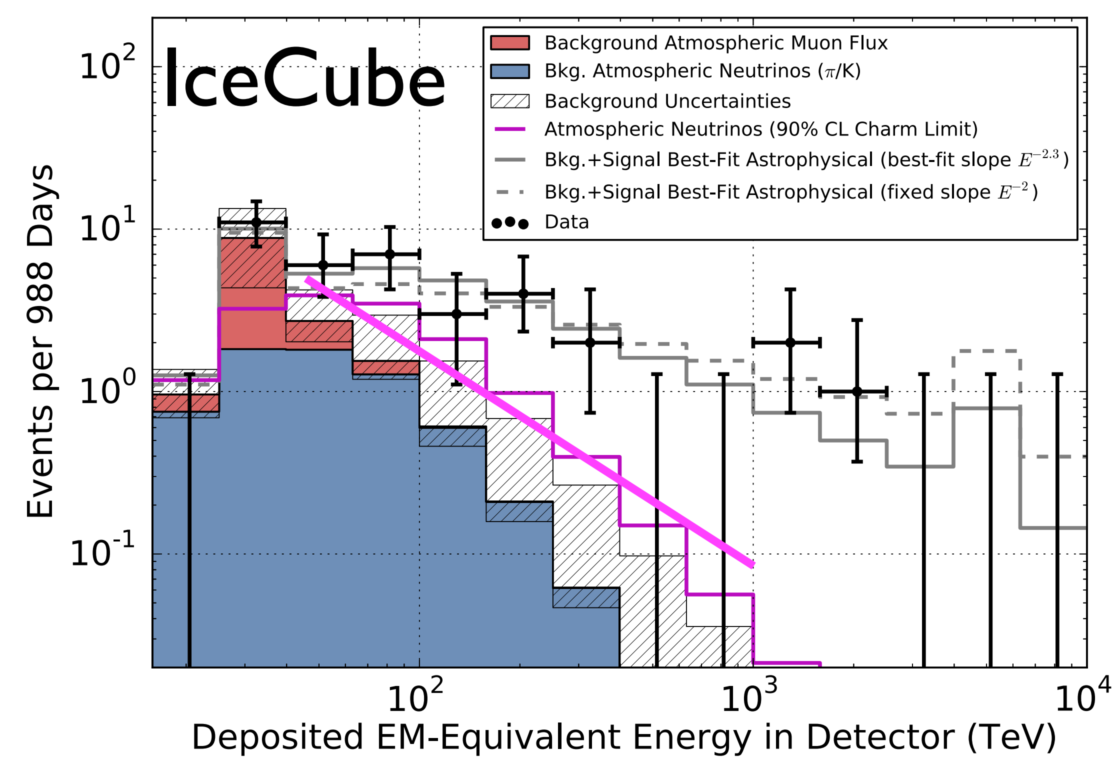
$$\mathrm{d}\sigma_{pp\to H+X} = \sum_{i,j} \int \mathrm{d}x_1 \mathrm{d}x_2\, f_a(x_1)\, f_b(x_2) \sum_q \int \mathrm{d}z\, \mathrm{d}\hat{\sigma}_{ab\to q+\hat{X}}(\hat{s},\ldots) \otimes D_{q\to H}(z,\ldots)$$

Inclusion of (resummed) perturbative fragmentation functions at NNLO QCD

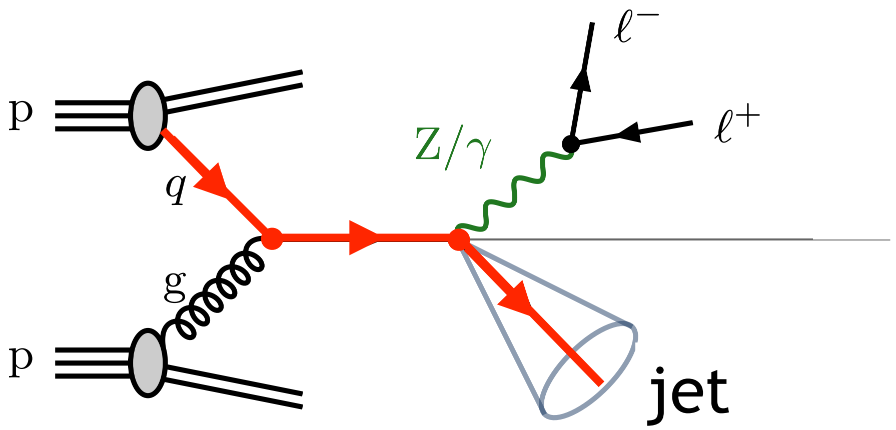
Czakon et al., arXiv:2102.08267







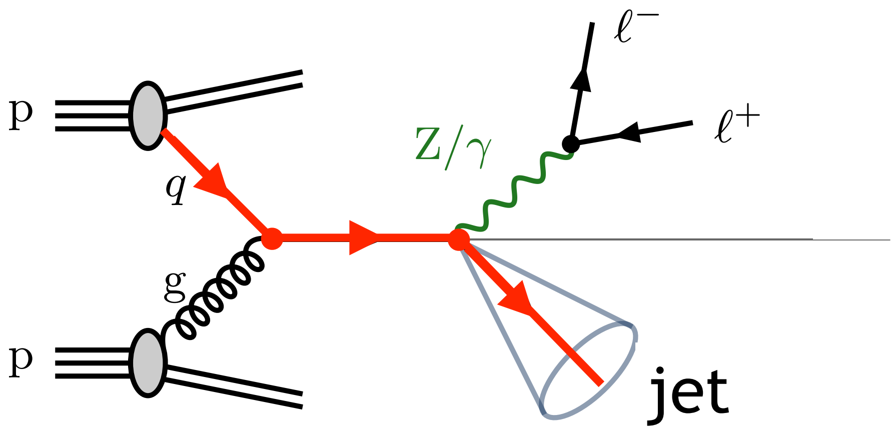
Validation and flavour



- **Z+jet:**
  - Antenna: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan '15]
  - N-jettiness: [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15]
- **W+jet:**
  - N-jettiness: [Boughezal, Liu, Petriello '15]
  - Antenna: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Walker '17]
- **γ+jet:**
  - N-jettiness: [Campbell, Ellis, Williams '16]
  - Antenna: [Chen, Gehrmann, Glover, Höfer, Huss '19]

NNLO QCD for $2 \to 2$ well established… now to identify jet flavours

Validation and flavour



- Z+jet:
  - Antenna: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan '15]
  - N-jettiness: [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15]
- W+jet:
  - N-jettiness: [Boughezal, Liu, Petriello '15]
  - Antenna: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Walker '17]
- γ+jet:
  - N-jettiness: [Campbell, Ellis, Williams '16]
  - Antenna: [Chen, Gehrmann, Glover, Höfer, Huss '19]

NNLO QCD for $2 \to 2$ well established… now to identify jet flavours

- Z+b-jet
  [RG, Gehrmann-De Ridder, Glover, Huss, Majer '20]
- W+c-jet
  [Czakon, Mitov, Pellen, Poncelet '20]

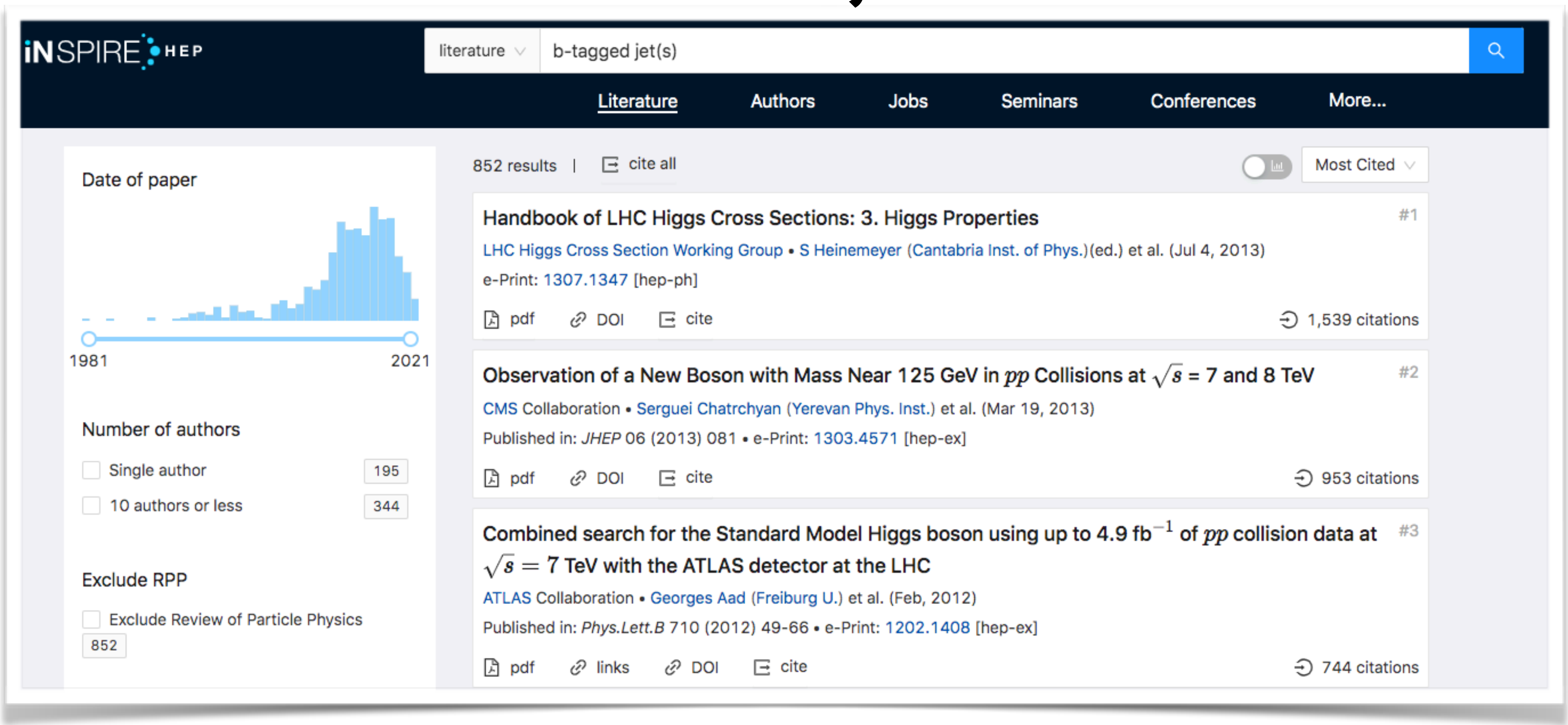
Core idea:

Identify ("tag") QCD radiation as that originating from specific quark flavours

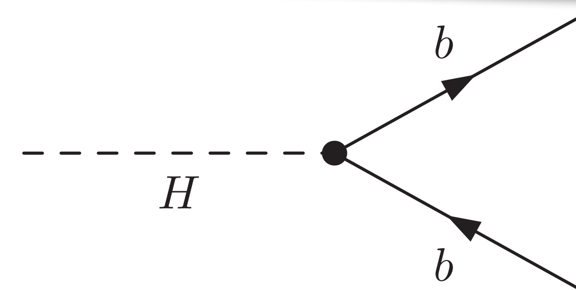
Issue:

Experimental tag is IRC unsafe

Relevance of jet flavour



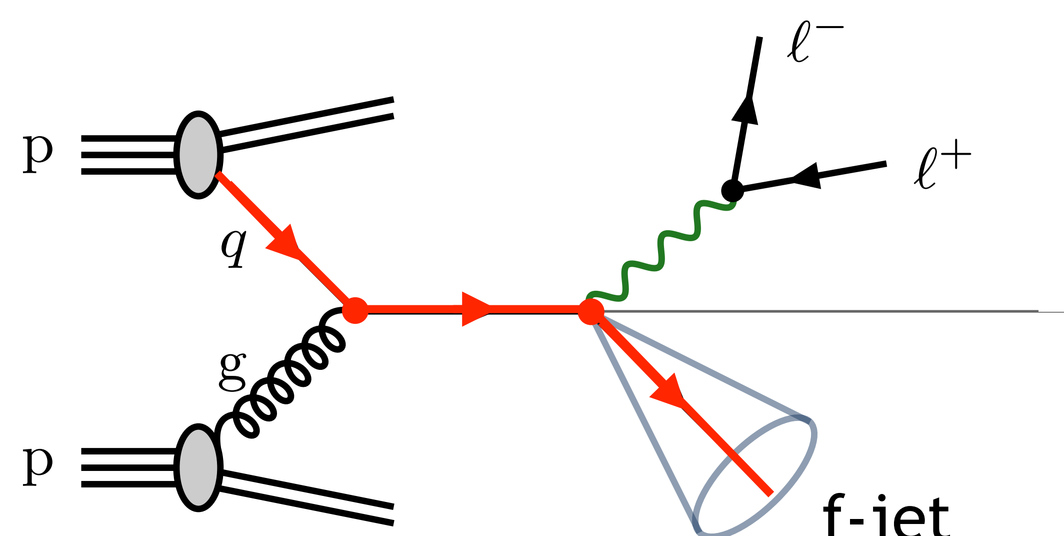
(i) Higgs physics (hadronic decays)



(ii) Top-quark physics ($|V_{tb}| \sim 1$)

(iii) New physics searches (f-jet $+E_T^{\mathrm{miss}}$)

(iv) Gauge-boson + heavy-flavour



Jet flavour

Experimental approach:

(1) Construct flavour-blind anti-k_T jets

(2) Assign flavour of jet based on event properties (B-hadron signature)

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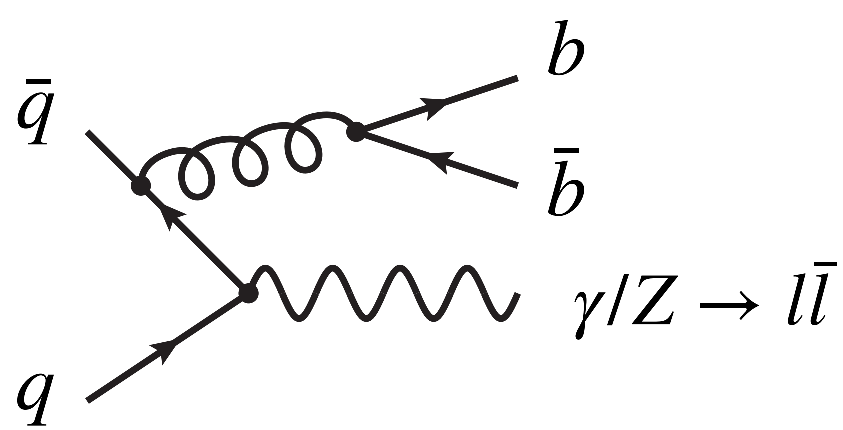
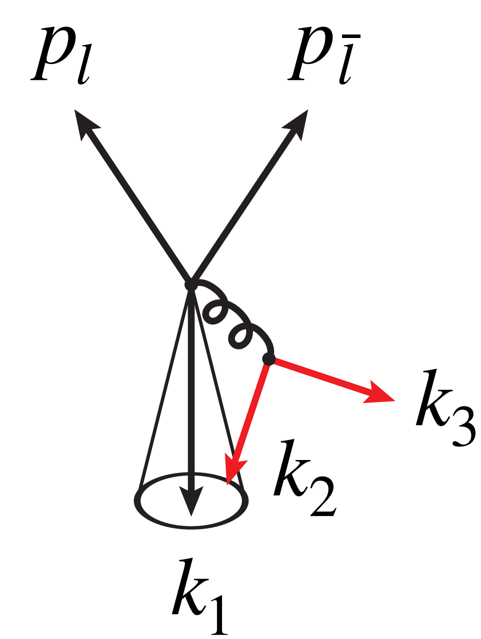
Theoretically, what happens if experimental jet-tagging definition applied?



Collinear safety



Soft safety

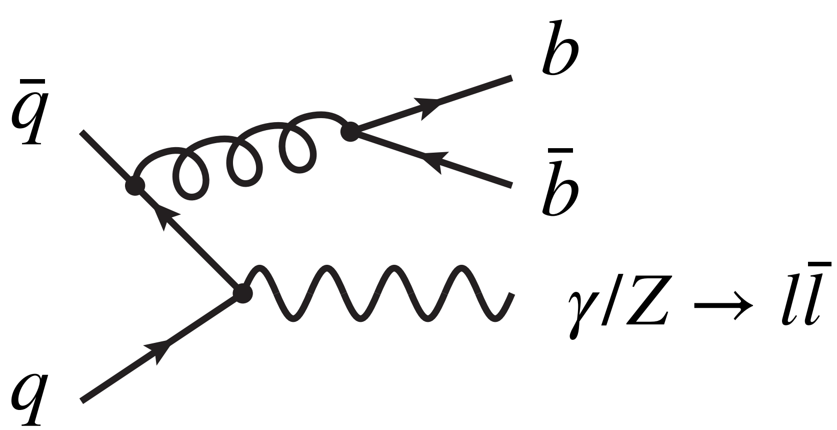
Jet flavour

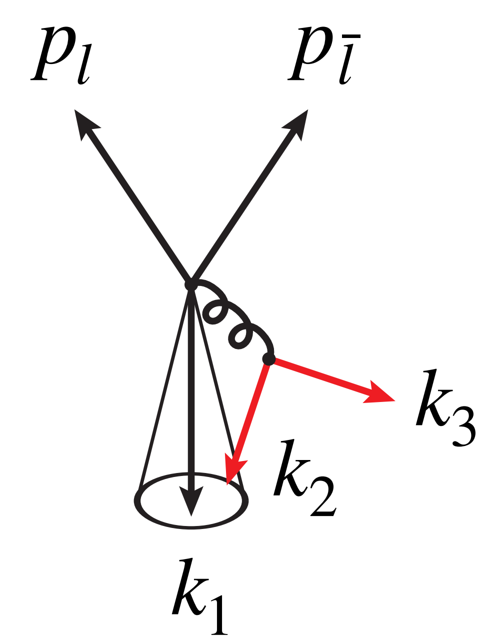
Experimental approach:

(1) Construct flavour-blind anti-k_T jets

(2) Assign flavour of jet based on event properties (B-hadron signature)

Theoretically, what happens if experimental jet-tagging definition applied?





Collinear safety

$C \propto \alpha_s \ln[Q^2/m_b^2]$

Soft safety

$S \propto \alpha_s^2 \ln^2[Q^2/m_b^2]$

Solution: apply a flavour-dependent jet algorithm Banfi et al., *Eur. Phys .J.C* 47 (2006) 113-124

Jet flavour

Target: resummed heavy-flavour PDFs and avoid large "C, S" corrections

Theoretical Physics | Published: 19 May 2006

Infrared-safe definition of jet flavour

A. Banfi, G.P. Salam & G. Zanderighi

The European Physical Journal C - Particles and Fields **47**, 113–124(2006) | Cite this article

109 Accesses | **71** Citations | Metrics

(1) Quantum flavour assignment:

$$b = +1, \ \bar{b} = -1$$

(2) Flavour specific clustering

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}, k_{tj})^\alpha & \text{softer of } i, j \text{ is unflavoured} \end{cases}$$

Solution: apply a flavour-dependent jet algorithm Banfi et al., *Eur. Phys .J.C* 47 (2006) 113-124

NNLO with jet flavour

Currently, no way to directly compare data with precise (NNLO) theory

$V + (H \to b\bar{b})$	Ferrera et al. (1705.10304), Caola et al. (1712.06974), Gauld et al. (1907.05836)
$Z + \mathrm{b-jet}$	Gauld et al. (2005.03016)
$W^{\pm} + \mathrm{c-jet}$	Czakon et al. (2011.01011)

flavoured-jet algorithm applied

$t\bar{t}$ with decay	Behring et al. (1901.05407), Czakon et al. (2008.11133)
$t, \bar{t}$ (t-chan with decay)	Berger et al. (1606.08463,1708.09405), Campbell et al. (2012.01574)
$V + (H \to b\bar{b})$ [4fs]	Behring et al. (2003.08321)

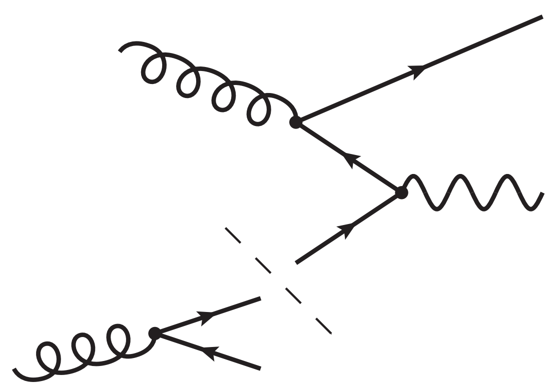
anti-k_T algorithm applied (regulated by m_b , a tech. cut, or ‘prescription’)

Why not just compute with massive quarks everywhere?

Z + b-jet production

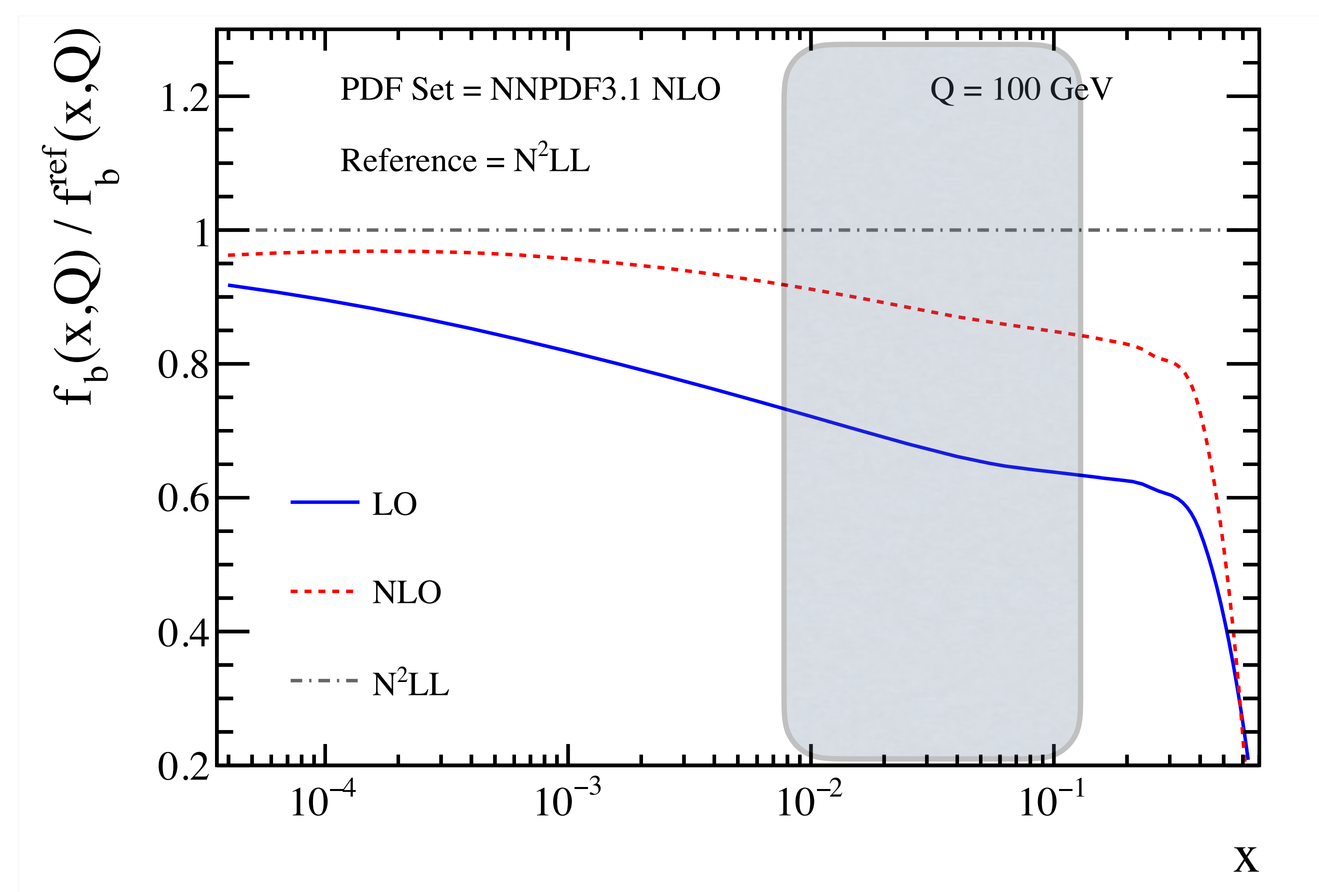
$$d\sigma^{\mathrm{M-VFNS}} = d\sigma^{5fs} + \left(d\sigma^{4fs} - d\sigma^{4fs}_{m_b \to 0} \right)$$

We construct a 'massive' variable flavour number scheme



Logarithmic part dominates

(the b-quark PDF)

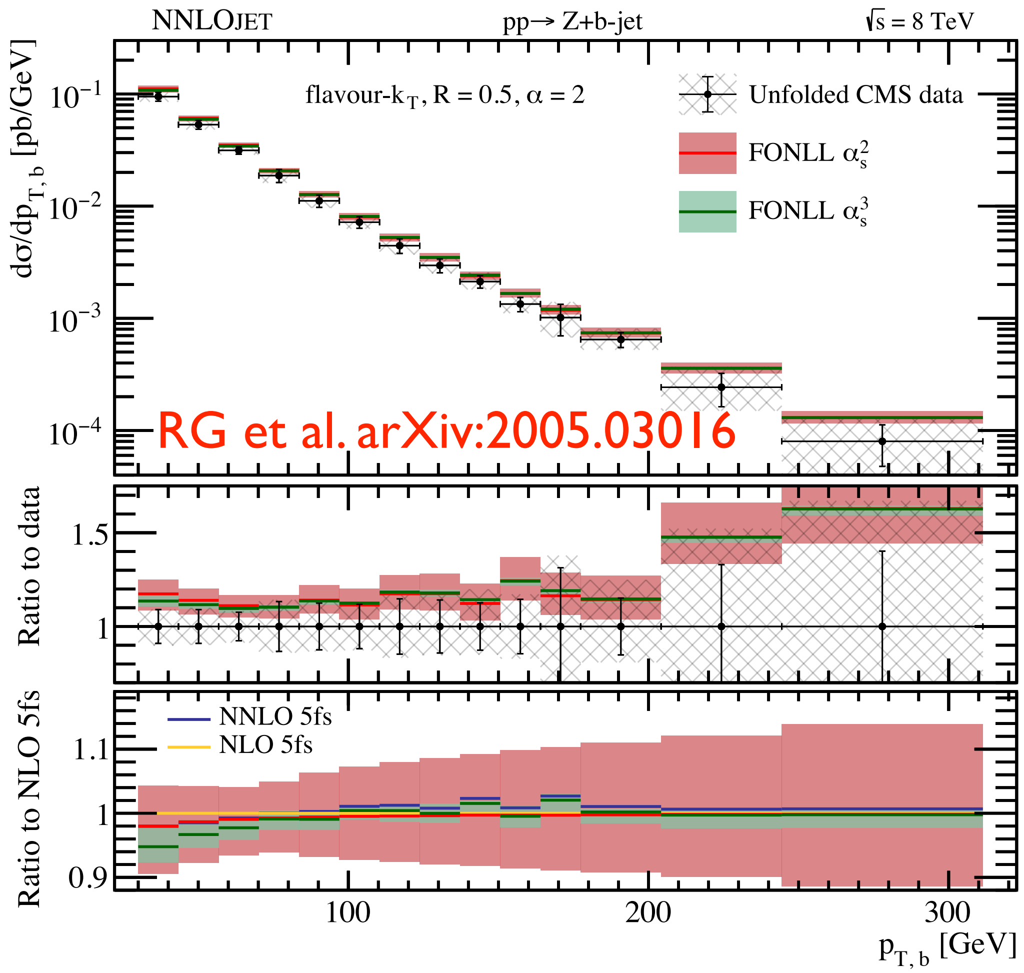


$$d\sigma^{4fs} = d\sigma^{m_b=0} \quad + \quad d\sigma^{\ln[m_b]} \quad + \quad d\sigma^{m_b}$$

100% = -32% +135% -3%

Z + b-jet production

A temporary solution: unfold the experimental data (anti-k_T to flav-k_T)



$p_{T,b}$

Theory precision: $\left\{ \begin{array}{l} \text{removal of FSR logs} \\ \text{resummed ISR logs} \end{array} \right.$

⟵ (2-3)% unc.

Exact m_b effects included

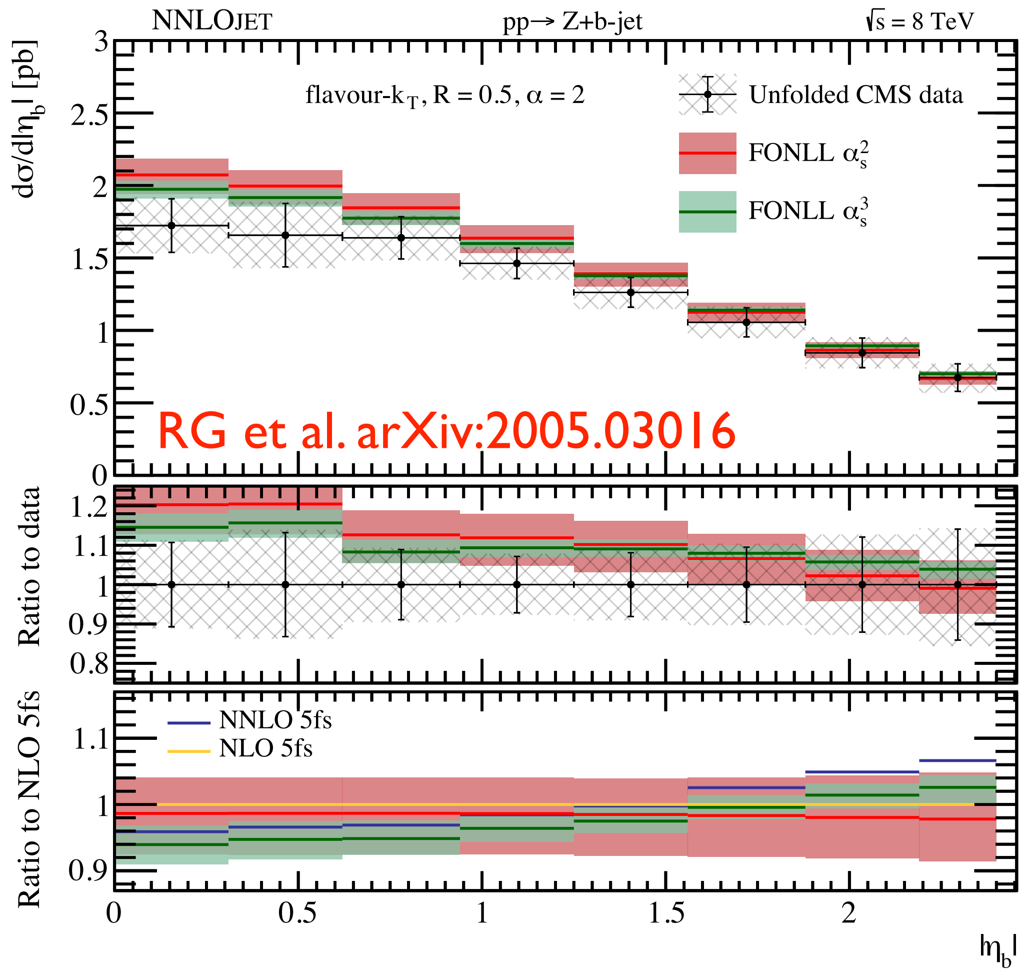
$$\chi^2/N_{\mathrm{dat}} \qquad 1.9 \xrightarrow{m_b} 1.7 \xrightarrow{\mathcal{O}(\alpha_s^3)} 1.5$$

NLO (5fs)

$$\chi^2 = \sum_{\mathrm{bins}} \frac{(\sigma_{\mathrm{theory}} - \sigma_{\mathrm{data}})^2}{\delta\sigma^2_{\mathrm{data}}}$$

Z + b-jet production

A temporary solution: unfold the experimental data (anti-k$_T$ to flav-k$_T$)



η_b

Theory precision: { removal of FSR logs / resummed ISR logs

← (2-3)% unc.

Exact m_b effects included

$$m_b \qquad \mathcal{O}(\alpha_s^3)$$

$\chi^2/N_{\rm dat}$ 2.0 → 1.6 → 1.0

NLO (5fs)

$$\chi^2 = \sum_{\rm bins} \frac{(\sigma_{\rm theory} - \sigma_{\rm data})^2}{\delta\sigma^2_{\rm data}}$$

The High Stats Frontier

will collect a factor of 20 more data

The High Stats Frontier

will collect a factor of 20 more data

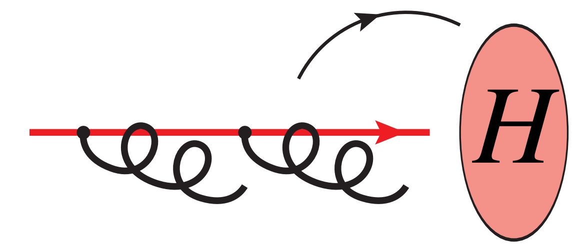
Improved precision from stats. (factor of 5)

Improved systematics (e.g. working at different 'tagging' point)

Theory modelling of signal and background will be a

Conclusions / Outlook

NNLO:



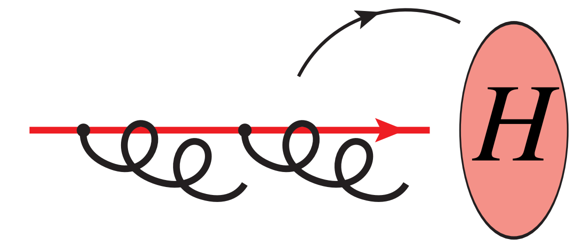
Huge progress in last ~5 years

Attention turning to: flavour, fragmentation, and $2 \to 3$

Important steps toward automation [Talks by Paolo, Valentin, Tiziano]

Conclusions / Outlook

NNLO:



Huge progress in last ~5 years

Attention turning to: flavour, fragmentation, and $2 \rightarrow 3$

Important steps toward automation [Talks by Paolo, Valentin, Tiziano]

Other relevant topics:

N3LO

N2LO+PS matching

Resummation

Mixed QCD-EW

Heavy-quark mass effects

A massive variable flavour number scheme for the Drell-Yan process

R. Gauld[1,*]

[1] *Nikhef, Science Park 105, NL-1098 XG Amsterdam, The Netherlands*

(Dated: 6th July 2021)

RG arXiv:2107.01226

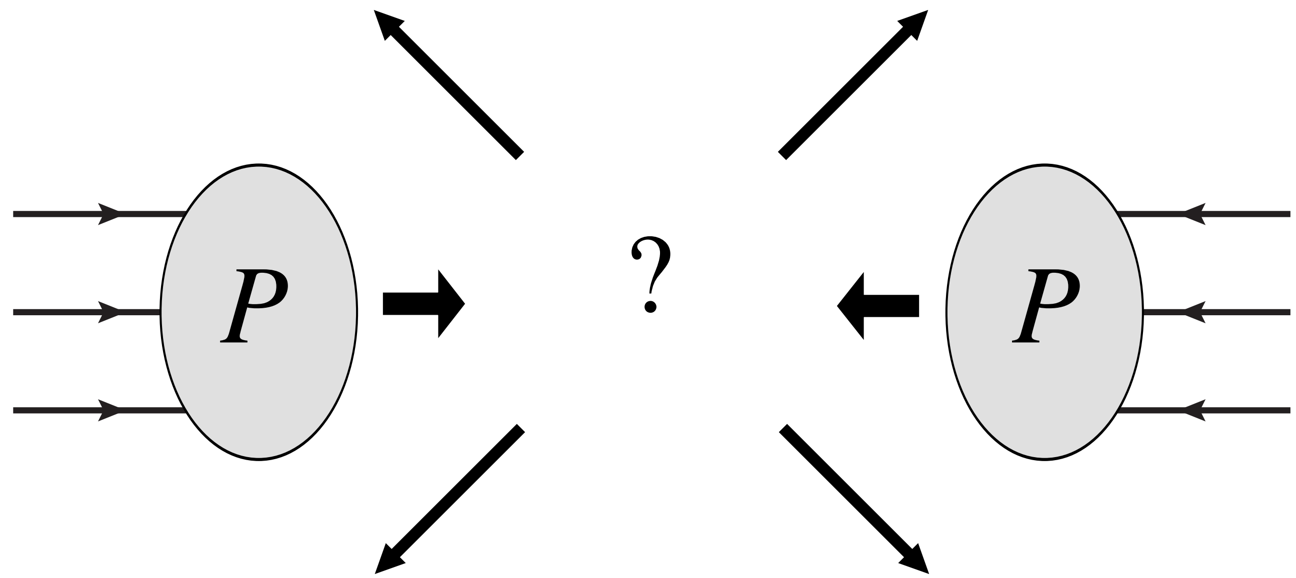
$$\mathrm{d}\sigma^{M-VFNS} = \mathrm{d}\sigma^{ZM} + \sum_{i=c,b,..}^{n_f^{\max}} \mathrm{d}\sigma_i^{\mathrm{pc}}$$

differential extraction of massive power corrections

Whiteboard

Whiteboard

General treatment of flavour in pp



Theoretical basis is a factorisation theorem

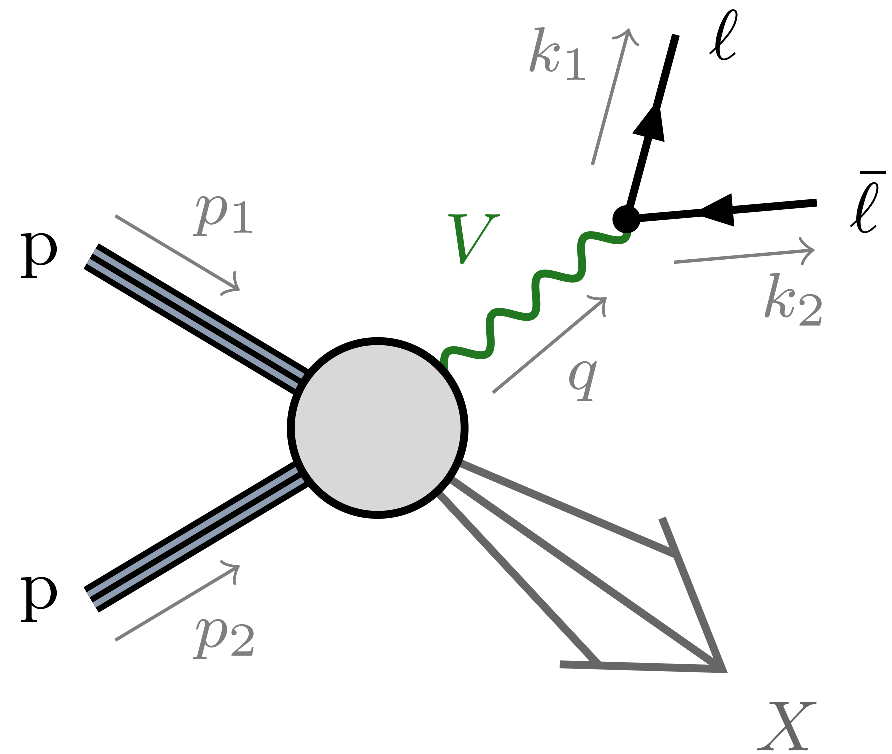
$$d\sigma_{pp\to X} = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1)\, f_j(x_2)\, d\hat{\sigma}_{ij\to\hat{X}}(\hat{s}, \ldots)\, T(\hat{X} \to X)$$

How to treat the heavy-flavour quarks? Defines a flavour scheme

At the LHC, typically assumed that $m_b = m_c = 0$ (“5fs”)

(often theoretical benefits making this assumption)

General treatment of flavour in pp

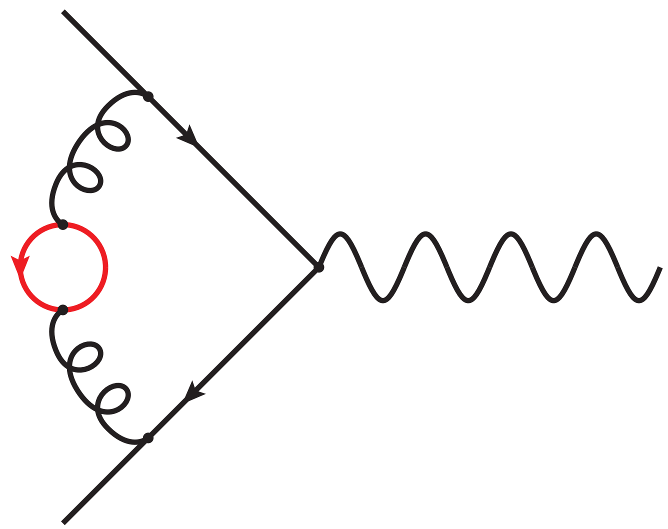


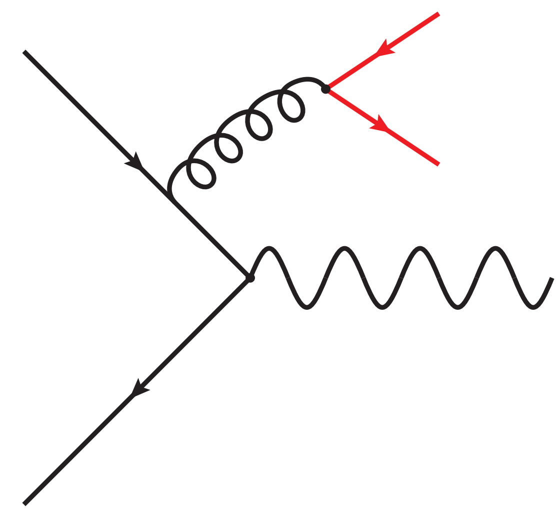
E.g. production of a Gauge boson (or Higgs)

How does the physical quark mass alter kinematic distributions?

"unknown" "negligibly small" "several %"

General treatment of flavour in pp





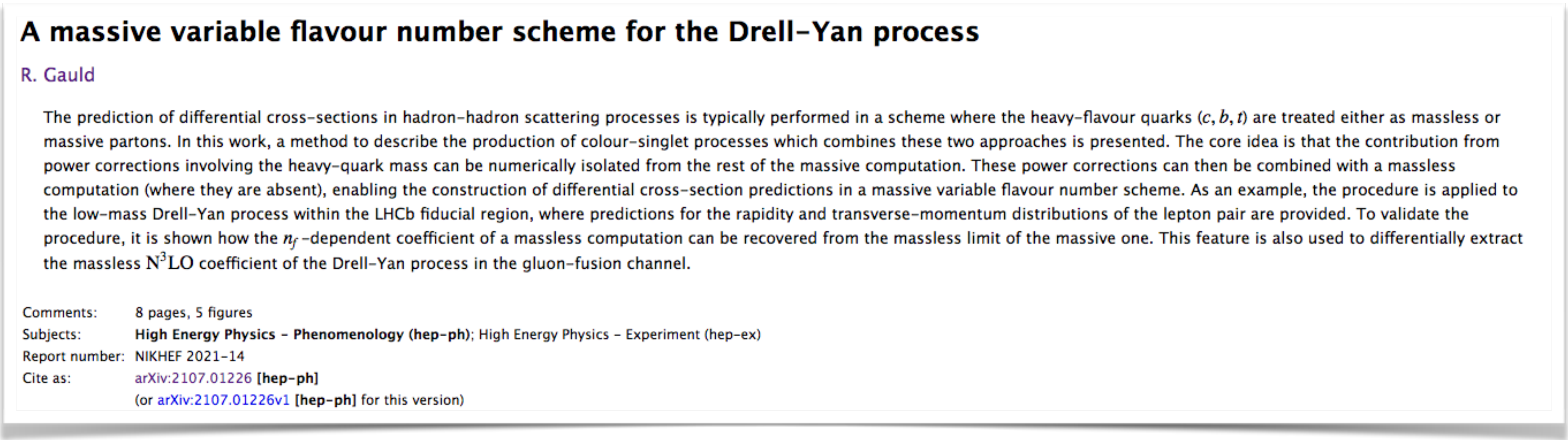
$$d\sigma^M = d\sigma^{n_f, m=0} + d\sigma^{\ln[m]} + d\sigma^{pc}$$

	$d\sigma^{n_f, m=0}$	$d\sigma^{\ln[m]}$	$d\sigma^{pc}$
Massive:	✓	✓	✓
Massless:	✓	✓ (resummed)	✗

How does the physical quark mass alter kinematic distributions?

"unknown" "negligibly small" "several %"

Issue largely solved, RG arXiv:2107.01226


A massive variable flavour number scheme for the Drell–Yan process

R. Gauld

The prediction of differential cross-sections in hadron–hadron scattering processes is typically performed in a scheme where the heavy-flavour quarks (c, b, t) are treated either as massless or massive partons. In this work, a method to describe the production of colour-singlet processes which combines these two approaches is presented. The core idea is that the contribution from power corrections involving the heavy-quark mass can be numerically isolated from the rest of the massive computation. These power corrections can then be combined with a massless computation (where they are absent), enabling the construction of differential cross-section predictions in a massive variable flavour number scheme. As an example, the procedure is applied to the low-mass Drell–Yan process within the LHCb fiducial region, where predictions for the rapidity and transverse-momentum distributions of the lepton pair are provided. To validate the procedure, it is shown how the n_f-dependent coefficient of a massless computation can be recovered from the massless limit of the massive one. This feature is also used to differentially extract the massless $\mathrm{N^3LO}$ coefficient of the Drell–Yan process in the gluon-fusion channel.

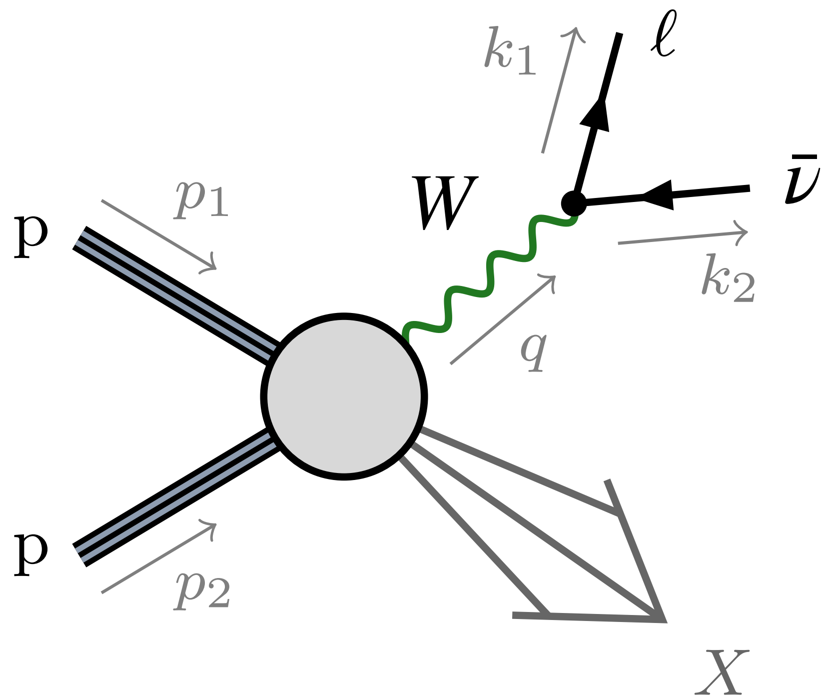
Comments: 8 pages, 5 figures
Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex)
Report number: NIKHEF 2021-14
Cite as: arXiv:2107.01226 **[hep-ph]**
(or arXiv:2107.01226v1 **[hep-ph]** for this version)

- A formalism to combine massive and massless approaches
- Differential and applicable to arbitrary* collider observables
- Includes calculation of initial-state mass effects (NLP)
- In passing, also computed N^3LO correction to DY (gg-channel)

*QCD inclusive and/or IRC safe

Issue largely solved, RG arXiv:2107.01226

Some future implications:



Exact description of recoil effects

m_W extraction (internal SM test)

Factorisation from formal p.o.v.

- A formalism to combine massive and massless approaches
- Differential and applicable to arbitrary* collider observables
- Includes calculation of initial-state mass effects (NLP)
- In passing, also computed N^3LO correction to DY (gg-channel)

*QCD inclusive and/or IRC safe

Perturbative computations

Primarily challenge: dealing with flavour

$$\mathrm{d}\hat{\sigma}_{ij,\mathrm{NLO}} = \int_{n+1} \left[\mathrm{d}\hat{\sigma}^{R}_{ij,\mathrm{NLO}} - \mathrm{d}\hat{\sigma}^{S}_{ij,\mathrm{NLO}}\right] + \int_{n} \left[\mathrm{d}\hat{\sigma}^{V}_{ij,\mathrm{NLO}} - \mathrm{d}\hat{\sigma}^{T}_{ij,\mathrm{NLO}}\right], \qquad (2.1)$$

Generic structure of higher-order terms (see arXiv: 1907.05836)

$$\mathrm{d}\hat{\sigma}^{R}_{ij,\mathrm{NLO}} = \mathcal{N}^{R}_{\mathrm{NLO}}\, \mathrm{d}\Phi_{n+1}\left(\{p_3,\ldots,p_{n+3}\};p_1,p_2\right)\, \frac{1}{S_{n+1}} \times \left[M^{0}_{n+3}\left(\{p_{n+3}\},\{f_{n+3}\}\right)\, J^{(n+1)}_{n}(\{p_{n+1}\},\{f_{n+1}\})\right]. \qquad (2.2)$$

Jet function acts on flavour and momenta of reduced MEs. In general (i, j, k) $\xrightarrow[\text{momentum}]{\text{flavour}}$ (I, K)

$$\mathrm{d}\hat{\sigma}^{S}_{ij,\mathrm{NLO}} = \mathcal{N}^{R}_{\mathrm{NLO}} \sum_{k} \mathrm{d}\Phi_{n+1}\left(\{p_3,\ldots,p_{n+3}\};p_1,p_2\right)\, \frac{1}{S_{n+1}} \times \left[X^{0}_{3}(\cdot,k,\cdot)\, M^{0}_{n+2}\left(\{\tilde{p}_{n+2}\},\{\tilde{f}_{n+2}\}\right)\, J^{(n)}_{n}\left(\{\tilde{p}_{n}\},\{\tilde{f}_{n}\}\right)\right], \qquad (2.3)$$

The ~ functions denoted mapped (in soft/collinear limits) momenta/flavour sets

Framework - antenna subtraction

- Exploits factorisation properties in IR limits
- Formalism operates on colour-ordered amplitudes

$$|\mathcal{M}^0_{m+1}(..,i,j,k,..)|^2 \xrightarrow{\text{j unresolved}} X^0_3(i,j,k)\,|\mathcal{M}^0_m(..,I,K,..)|^2$$

Partial amplitude

Antenna function

Reduced amplitude
$\{p_i,p_j,p_k\} \to \{p_I,p_K\}$

The antenna function captures multiple IR limits, e.g.

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{kj}}$	$p_i \to p_I\,,\ p_k \to p_K$
$p_j \parallel p_k$	$\dfrac{1}{s_{kj}}P_{kj}(z)$	$p_i \to p_I\,,\ (p_k + p_j) \to p_K$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}}P_{ij}(z)$	$(p_i + p_j) \to p_I\,,\ p_k \to p_K$

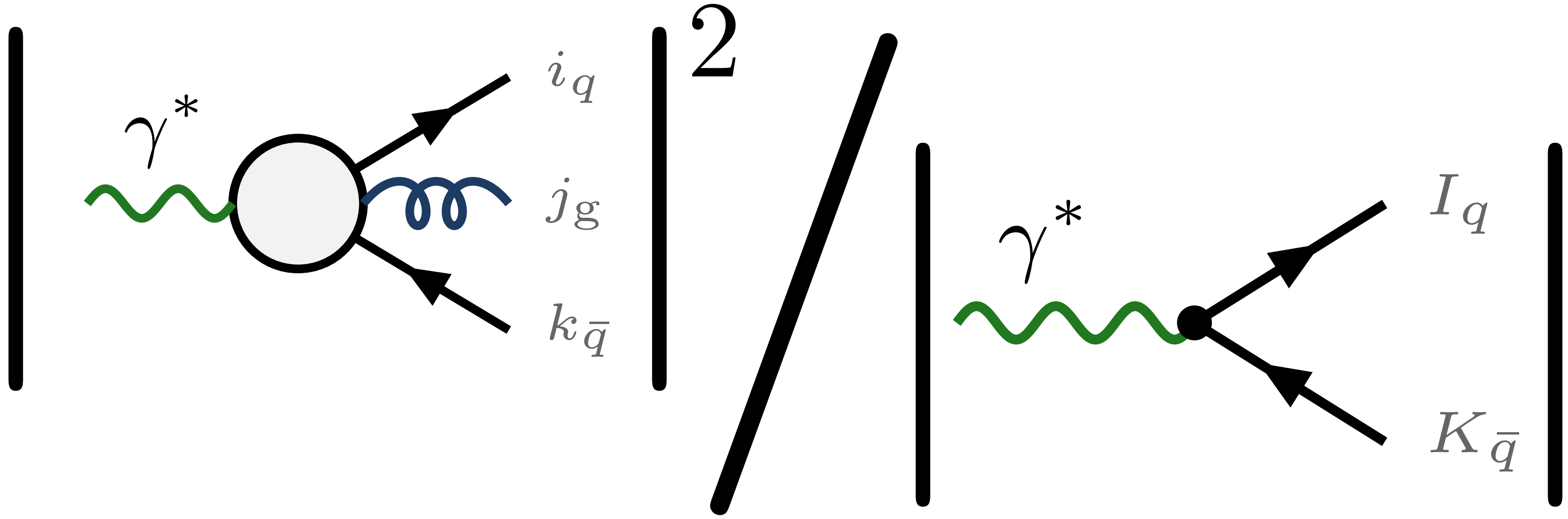
Framework - antenna subtraction

- Exploits factorisation properties in IR limits
- Formalism operates on colour-ordered amplitudes

$$|\mathcal{M}^0(\ldots,i,j,k,\ldots)|^2 \xrightarrow{\text{j unresolved}} X^0(i,j,k)\,|\mathcal{M}^0(\ldots,I,K,\ldots)|^2$$

Example: tree-level quark, anti-quark antenna

$$X_3^0(i_q,j_g,k_{\bar{q}}) \sim$$



$p_j \parallel p_k$	$\frac{1}{s_{kj}}P_{kj}(z)$	$p_i \to p_I,\ (p_k+p_j) \to p_K$
$p_j \parallel p_i$	$\frac{1}{s_{ij}}P_{ij}(z)$	$(p_i+p_j) \to p_I,\ p_k \to p_K$