Numerical computation of loops & x-sections

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On the basis of work done in coll. with:

Z. Capatti, D. Kermanschah, A. Pelloni, B. Rujil

Currently in coll. with:

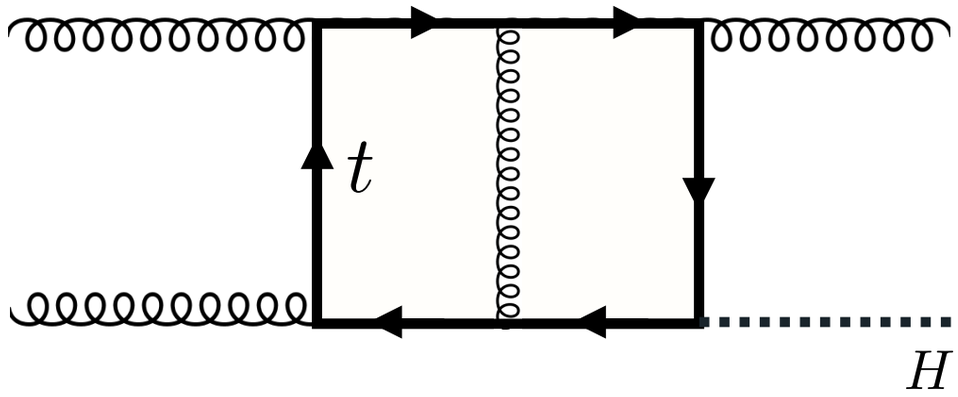
Z. Capatti, B. Ruijl

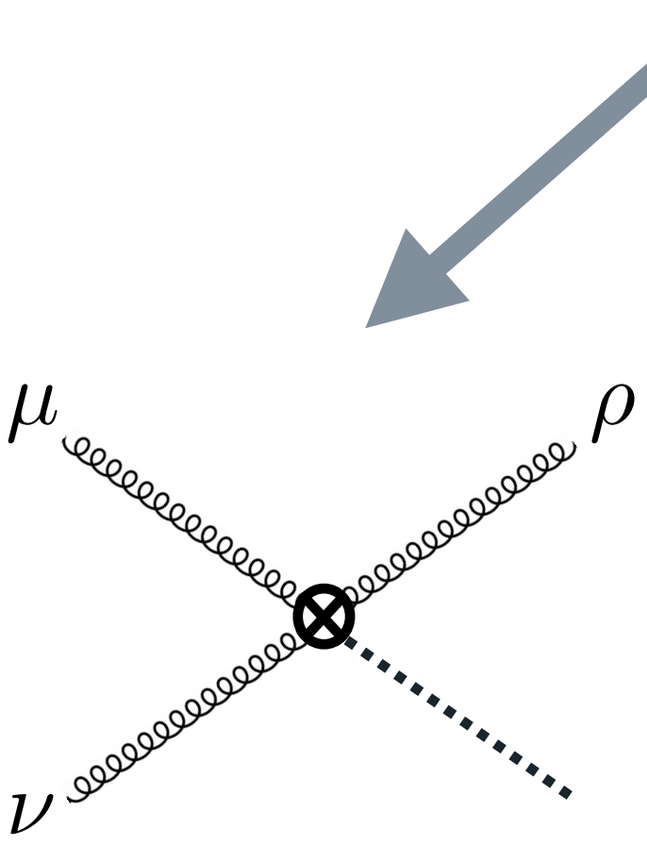
Bonn MadGraph Workshop

November, 17th, 2021

Multi-loop computations in madgraph

A) Evaluation within MG5





$$= \sum_{i=1}^{4} \mathrm{C}_i(s, t, M_H^2, M_t^2, \mu_r^2, \epsilon) \;\; T_i^{\mu\nu\rho}$$

A) Evaluation within MG5

vertices.py

```
GGHG2LQCD = Vertex(name = 'GGHG2LQCD',
                   particles = [ P.G, P.G, P.G, P.H ],
                   color = [ 'f(1,2,3)' ],
                   lorentz = [ L.gggHTensStruc1QCD2L, L.gggHTensStruc2QCD2L,
                               L.gggHTensStruc3QCD2L, L.gggHTensStruc4QCD2L
                             ],
                   couplings = {(0,0):C.GGGH2LQCD_C1, (0,1):C.GGGH2LQCD_C2,
                                (0,2):C.GGGH2LQCD_C3, (0,3):C.GGGH2LQCD_C4
                                }
                   )
```

lorentz.py

```
gggHTensStruc1QCD2L = Lorentz(
    name='gggHTensStruc1QCD2L',
    spins=[3, 3, 3, 1],
    structure='( ( P(-10,1)*P(-10,2)*Metric(1,2) - P(1,2)*P(2,1) ) * ( P(-11,2)*P(-11,3)*P(3,1) - P(-12,1)*P(-12,3)*P(3,2) ) ) / ( P(-13,1)*P(-13,3) ) '
)
```

couplings.py

```
GGGH2LQCD_C1 = Coupling(name = 'GGGH2LQCD_C1',
                        value = '(AllGGGH2LQCDCoup)*( GGGH2LQCD_ForFac1_RE + complex(0,1)*GGGH2LQCD_ForFac1_IM )',
                        order = {'PPHJ':1,'GGHG2LQCD':1,"QCD":5,"QED":1})
```

parameters.py

```
GGGH2LQCD_ForFac1_RE = Parameter(name='GGGH2LQCD_ForFac1_RE',
                                 nature='external',
                                 type='complex',
                                 value=66.025,
                                 texname='\\text{GGGH2LQCD_ForFac1_RE}',
                                 lhablock='GGGH2LQCD',
                                 lhacode=[10401])
```

A) Evaluation within MG5

generate_card.mg5

```
import model ./PLUGIN/linlo/UFO_model_gggH
add process g g  > h g GGHG1LQCD<=1 GGHG2LQCD<=1 PPHJ^2==2 GGHG2LQCD^2==1 GGHG1LQCD^2==1 @10
add process d d~ > h g GGHG1LQCD<=1 GGHG2LQCD<=1 PPHJ^2==2 GGHG2LQCD^2==1 GGHG1LQCD^2==1 @20
[...]
output standalone_ggHg My2LME
launch -f
```

./bin/mg5_aMC --mode=linlo generate_card.mg5

My2LME/Source/DHELAS/gggHTensStruc1QCD1L_0.f

```
SUBROUTINE GGGHTENSSTRUC1QCD1L_0(V1, V2, V3, S4, COUP,VERTEX)
IMPLICIT NONE
COMPLEX*16 CI
PARAMETER (CI=(0D0,1D0))
COMPLEX*16 V2(*)

[...]
pg(0,1) = P1(0)
[...]

call PROC_10_set_pphj_gghg_qcd_1l_coeffs(pg)
[...]

END
```

A) Evaluation within MG5

```
cd My2LME/SubProcesses/P10_gg_hg; make check; ./check
```

```
-----------------------------
n   E   px   py   pz   m
1   0.5000000E+03  0.0000000E+00  0.0000000E+00  0.5000000E+03  0.0000000E+00
2   0.5000000E+03  0.0000000E+00  0.0000000E+00 -0.5000000E+03  0.0000000E+00
3   0.5078125E+03  0.1091911E+03  0.4378803E+03 -0.1964349E+03  0.1250000E+03
4   0.4921875E+03 -0.1091911E+03 -0.4378803E+03  0.1964349E+03  0.9725608E-05
-----------------------------
1) Matrix element for (GGHG1LQCD=0 GGHG2LQCD=2 PPHJ=2) =    11491.131651979882
2) Matrix element for (GGHG1LQCD=1 GGHG2LQCD=1 PPHJ=2) =   -6.850619668029394 5
3) Matrix element for (GGHG1LQCD=2 GGHG2LQCD=0 PPHJ=2) =    1.175927094170577 2E-003
Selected squared coupling orders combination for the sum below:

Total Matrix element =   -6.850619668029394 5      GeV^          0
-----------------------------
```

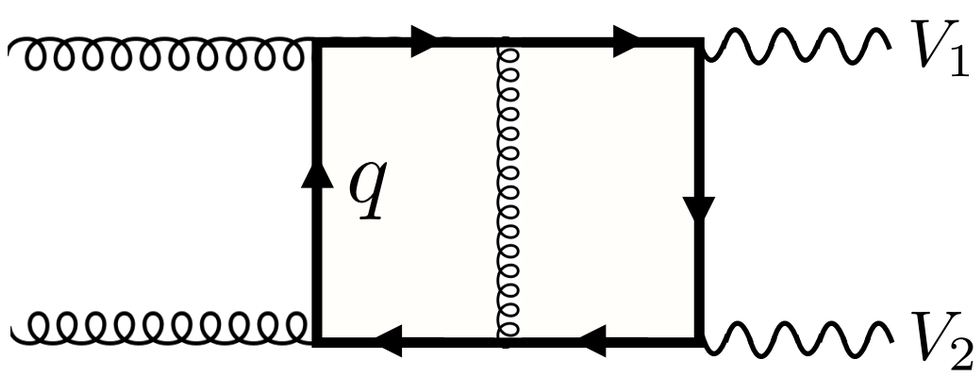
Ok but what is

```
call PROC_10_set_pphj_gghg_qcd_1l_coeffs(pg)
```

really doing ?

A) Evaluation within MG5

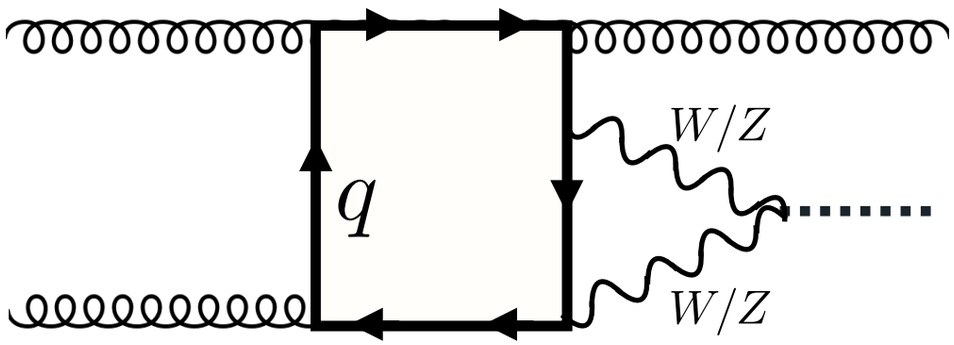
It depends !



Calling a giant C-library for the computation of the 20 FFs.

[VVAMP : Manteuffel, Tancredi arxiv:1503.08835]

`lp:~maddevelopers/mg5amcnlo/GGVV_plugin`

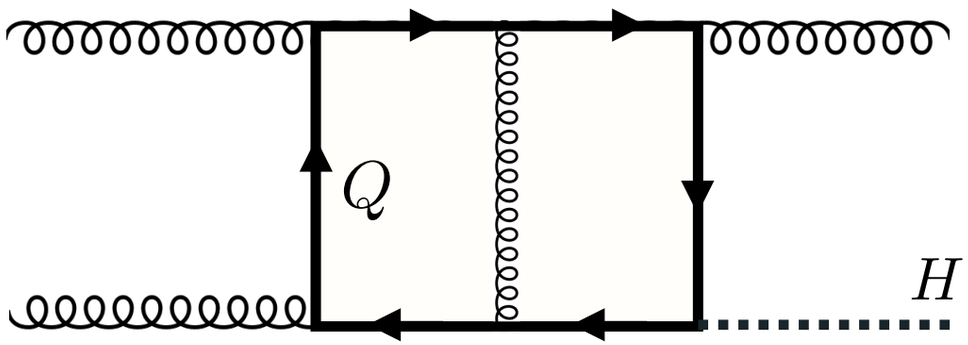


Calling Mathematica on-the-fly to run ancestor of DiffExp.

Computed exact NLO mixed QCD-EW for Higgs inclusive:

[Becchetti, Bonciani,Del Duca, VH, Moriello arxiv : 2010.09451]

`bitbucket.org/aschweitzer/mg5_higgs_ew_plugin`



DiffExp on Mathematica on-the-fly **or** interpolation grid.

[TO APPEAR arxiv : ??]

`bitbucket.org/vahirschi/linlo_pp_hj` (still private)

B) MG for MultiLoop People

bitbucket.org/vahirschi/alphaloop

```
./bin/mg5_aMC --mode=alphaloop generate_card.mg5
```

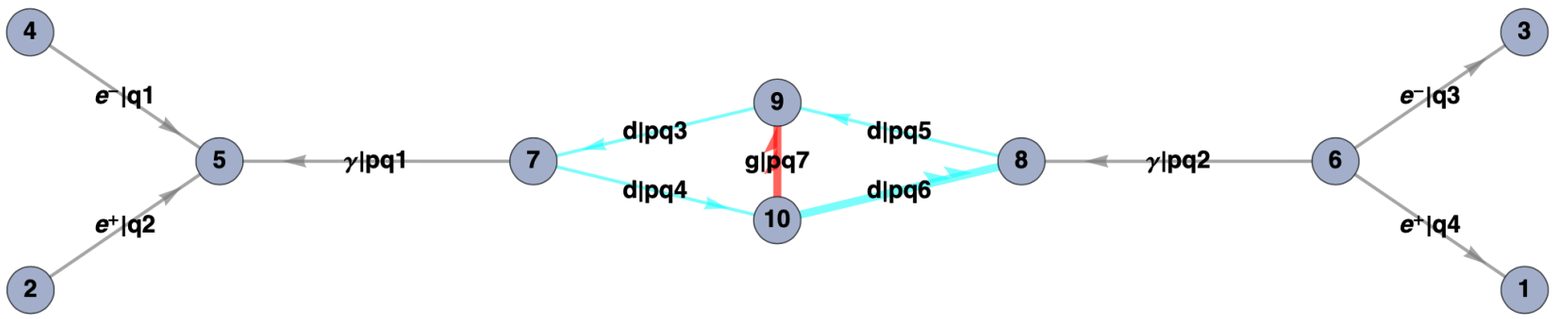
```
import model aL_sm-no_widths
set_alphaLoop_option FORM_processing_output_format raw
set_alphaLoop_option apply_graph_isomorphisms True
set_alphaLoop_option qgraf_template_model epem
qgraf_generate e+ e- > a > d d~ / t u s c b QCD^2==1 QED^2==2 [QCD]
output qgraf MyAmplitude
```

```
cd MyAmplitude; ls
```

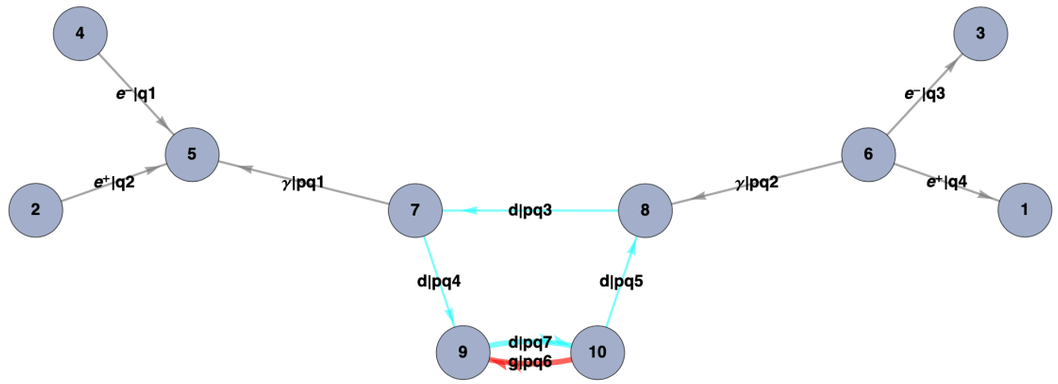
```
Cards/    Drawings/    FORM/    Rust_inputs/ Source/    checkpoints/ lib/    qgraf/
```

```
cd MyAmplitude/Drawings/all_graphs
```

Graph 1



Graph 2



B) MG for MultiLoop People

QGRAF

MyAmplitude/qgraph/model.dat

```
[...]

[higgs, higgs, +; pdg=('25')]
[gluon,gluon,+; pdg=('21')]

* Psi PsiBar
[ghost,ghostbar,-; pdg=('82','-82')]
[photon,photon,+;pdg=('22')]
[eminus, eplus, - , external; pdg= ('+11', '-11')]
[d, dbar, - ; pdg=('1','-1')]
[u, ubar, - ; pdg=('2','-2')]
[c, cbar, - ; pdg=('3','-3')]
[s, sbar, - ; pdg=('4','-4')]
[b, bbar, - ; pdg=('5','-5')]
[t, tbar, - ; pdg=('6','-6')]


*****************
* vertices
*****************

* Total QED Coupling supported: 0-99
* Total QCE Coupling supported: 0-..

* EW
[eplus,photon,eminus; QCD_QED='1']
[dbar,photon,d; QCD_QED='1']
[ubar,photon,u; QCD_QED='1']
[cbar,photon,c; QCD_QED='1']
[sbar,photon,s; QCD_QED='1']
[bbar,photon,b; QCD_QED='1']
[tbar,photon,t; QCD_QED='1']

[...]
```

MyAmplitude/qgraph/qgraf.dat

```
[...]
in = eplus[p1], eminus[p2];
out = eplus[p1], eminus[p2];

loops = 2;
loop_momentum = k;

options = onshell, notadpole, nosnail;

* final states
true=iprop[d,1,10];
true=iprop[dbar,1,10];

* coupling order
true=vsum[QCD_QED,204,204];

* exactly 2 photons
true=iprop[photon,2,2];
[...]
```

Then simply running **`./qgraf`** witha customised style file generates '**`output.py`**' which can directly be loaded by **Python**.

B) MG for MultiLoop People

FORM

MyAmplitude/FORM/workspace/input_0.h

```
[...]
L F = (1)*(-1)/4*
 vx(-11,22,11,p1,-p1-p2,p2,4,6,2)*
 vx(-11,22,11,-p2,p1+p2,-p1,1,8,3)*
 vx(-1,22,1,-k1-k2,p1+p2,k1+k2-p1-p2,13,7,10)*
 vx(-1,22,1,-k1+p1+p2,-p1-p2,k1,15,9,16)*
 vx(-1,21,1,-k1-k2+p1+p2,k2,k1-p1-p2,11,18,14)*
 vx(-1,21,1,-k1,-k2,k1+k2,17,19,12)*
 prop(-11,in,p1,4)*
 prop(11,in,p2,2)*
 prop(-11,out,p1,3)*
 prop(11,out,p2,1)*
 prop(22,virtual,-p1-p2,7,6)*
 prop(22,virtual,-p1-p2,8,9)*
 prop(1,virtual,k1+k2-p1-p2,11,10)*
 prop(1,virtual,k1+k2,13,12)*
 prop(1,virtual,k1-p1-p2,15,14)*
 prop(1,virtual,k1,17,16)*
 prop(21,virtual,k2,19,18)
;
```

MyAmplitude/FORM/workspace/raw_numerator.frm

```
#-
On statistics;
CF dotp, myv;
#include numerator.frm # setup
* Load the diagrams
#include- input_`SGID'.h
.sort
* Do algebra
#include numerator.frm # feynman-rules
#call FeynmanRules()
B+ x;
.sort
#write<out_raw_numerator_`SGID'.txt> "%E" F
.sort
.end;
```

FORM will take care of doing all the Lorentz/color algebra (incl. projectors if need be) **efficiently** !

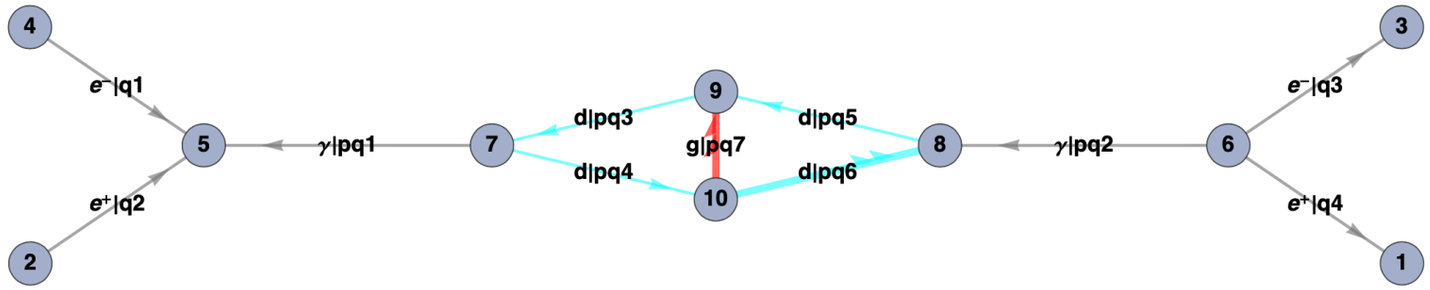
B) MG for MultiLoop People

MyAmplitude/FORM/workspace/numerator.frm

```
[...]
* vertices
id vx(x1?{`QBAR'}, `GLU', x2?{`Q'}, p1?, p2?, p3?, idx1?, idx2?, idx3?) = gamma(dirac[idx1], lorentz[idx2], dirac[idx3]);
id vx(`GHOBAR', `GLU', `GHO', p1?, p2?, p3?, idx1?, idx2?, idx3?) = (p3(lorentz[idx2])-p1(lorentz[idx2]));
id vx(x1?{`QBAR'}, `PHO', x2?{`Q'}, p1?, p2?, p3?, idx1?, idx2?, idx3?) = gamma(dirac[idx1], lorentz[idx2], dirac[idx3]);
id vx(x1?{`LBAR'}, `PHO', x2?{`L'}, p1?, p2?, p3?, idx1?, idx2?, idx3?) = gamma(dirac[idx1], lorentz[idx2], dirac[idx3]);
id vx(x1?{`QBAR'}, `H', x2?{`Q'}, p1?, p2?, p3?, idx1?, idx2?, idx3?) = d_(dirac[idx1], dirac[idx3]);
id vx(x1?{`LBAR'}, `H', x2?{`L'}, p1?, p2?, p3?, idx1?, idx2?, idx3?) = d_(dirac[idx1], dirac[idx3]);
id vx(`H', `H', `H', p1?, p2?, p3?, idx1?, idx2?, idx3?) = 1;
id vx(`H', `GLU', `GLU', p1?, p2?, p3?, idx1?, idx2?, idx3?) = p3(lorentz[idx2])*p2(lorentz[idx3])
                                                              - p2.p3 * d_(lorentz[idx2], lorentz[idx3]);
id vx(`H', `GLU', `GLU', `GLU', p4?, p1?, p2?, p3?, idx4?, idx1?, idx2?, idx3?) =
    - d_(lorentz[idx1], lorentz[idx3]) * p1(lorentz[idx2])
    + d_(lorentz[idx1], lorentz[idx2]) * p1(lorentz[idx3])
    + d_(lorentz[idx2], lorentz[idx3]) * p2(lorentz[idx1])
    - d_(lorentz[idx1], lorentz[idx2]) * p2(lorentz[idx3])
    - d_(lorentz[idx2], lorentz[idx3]) * p3(lorentz[idx1])
    + d_(lorentz[idx1], lorentz[idx3]) * p3(lorentz[idx2])
;
[...]
```

B) MG for MultiLoop People

Final result in:




```
cd MyAmplitude/FORM/raw_SG_expressions; ls;
```

```
SG_QG0.yaml    SG_QG2.yaml    numerator_0.txt  numerator_1.txt
```

MyAmplitude/FORM/raw_SG_expressions/SG_QG0.yaml

```
denominators_string: (1/(-q1-q2)^2*1/(-q1-q2)^2*1/(+k0+k1-q1-q2)^2*1/(+k0+k1)^2*1/(+k0-q1-q2)^2*1/k0^2*1/k1^2)
edges:
[...]
- - pq3
  - 9
  - 7
  - 1
  - - - 1
      - 1
    - - -1
      - -1
      - 0
      - 0
[...]
loop_momenta:
- pq6
- pq7
numerator: numerator_0.txt
```

EDGE NAME

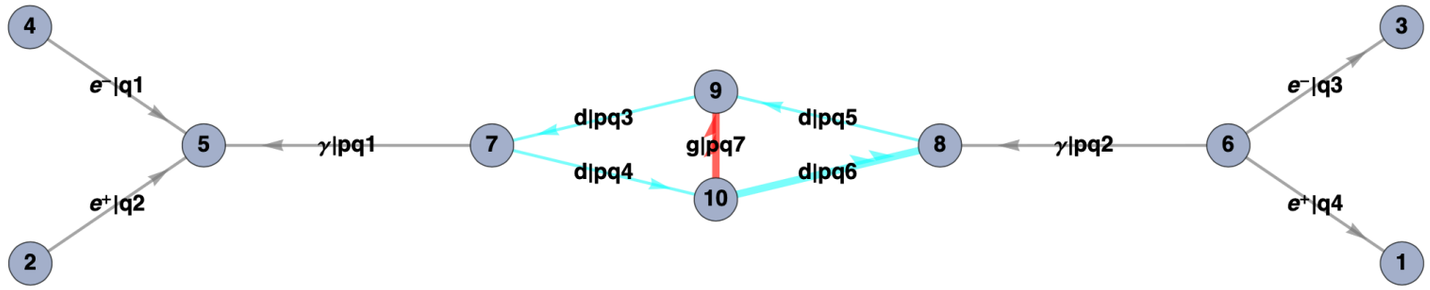
EDGE START & END NODES

PROPAGATOR POWER NAME

LOOP MOMENTUM "SIGNATURE" : $p_{q_3} = (+1) \times k + (+1) \times l + (-1) \times p_1 + (-1) \times p_2 + (0) \times p_3 + (0) \times p_4$

B) MG for MultiLoop People

Final result in:



MyAmplitude/FORM/raw_SG_expressions/numerator_0.txt

```
+ p1.p1*p1.p2*k1.k1*i_*ge^4*gs^2*rat( - 64*ep^2 + 64,9) + p1.p1*p1.p2*k1.k2*
        i_*ge^4*gs^2*rat( - 64*ep^2 + 64,9) + p1.p1*p2.p2*k1.k1*i_*ge^4*gs^2*
        rat(128,9) + p1.p1*p2.p2*k1.k2*i_*ge^4*gs^2*rat(128,9) + p1.p1*p2.k1*
        p2.k2*i_*ge^4*gs^2*rat(-128,9) + p1.p1*p2.k1*k2.k2*i_*ge^4*gs^2*rat(
         - 64*ep + 64,9) + p1.p1*p2.k1^2*i_*ge^4*gs^2*rat(-128,9) + p1.p1*
        p2.k2*k1.k2*i_*ge^4*gs^2*rat(64*ep - 64,9) + p1.p2*p1.k1*p1.k2*i_*
        ge^4*gs^2*rat(128*ep - 128,9) + p1.p2*p1.k1*p2.k1*i_*ge^4*gs^2*rat(
        256*ep,9) + p1.p2*p1.k1*p2.k2*i_*ge^4*gs^2*rat(128*ep,9) + p1.p2*
        p1.k1*k1.k1*i_*ge^4*gs^2*rat(128*ep^2 - 256*ep + 128,9) + p1.p2*p1.k1
        *k1.k2*i_*ge^4*gs^2*rat(128*ep^2 - 256*ep + 128,9) + p1.p2*p1.k1*
        k2.k2*i_*ge^4*gs^2*rat(64*ep^2 - 64*ep,9) + p1.p2*p1.k1^2*i_*ge^4*
        gs^2*rat(128*ep - 128,9) + p1.p2*p1.k2*p2.k1*i_*ge^4*gs^2*rat(128*ep,
        9) + p1.p2*p1.k2*k1.k1*i_*ge^4*gs^2*rat(64*ep^2 - 128*ep + 64,9) +
        p1.p2*p1.k2*k1.k2*i_*ge^4*gs^2*rat( - 64*ep + 64,9) + p1.p2*p2.p2*
        k1.k1*i_*ge^4*gs^2*rat( - 64*ep^2 + 64,9) + p1.p2*p2.p2*k1.k2*i_*ge^4
        *gs^2*rat( - 64*ep^2 + 64,9) + p1.p2*p2.k1*p2.k2*i_*ge^4*gs^2*rat(128
        *ep - 128,9) + p1.p2*p2.k1*k1.k1*i_*ge^4*gs^2*rat(128*ep^2 - 256*ep
         + 128,9) + p1.p2*p2.k1*k1.k2*i_*ge^4*gs^2*rat(128*ep^2 - 256*ep +
        128,9) + p1.p2*p2.k1*k2.k2*i_*ge^4*gs^2*rat(64*ep^2 - 64*ep,9) +
        p1.p2*p2.k1^2*i_*ge^4*gs^2*rat(128*ep - 128,9) + p1.p2*p2.k2*k1.k1*i_
        *ge^4*gs^2*rat(64*ep^2 - 128*ep + 64,9) + p1.p2*p2.k2*k1.k2*i_*ge^4*
        gs^2*rat( - 64*ep + 64,9) + p1.p2*k1.k1*k1.k2*i_*ge^4*gs^2*rat( - 128
        *ep^2 + 256*ep - 128,9) + p1.p2*k1.k1*k2.k2*i_*ge^4*gs^2*rat( - 64*
        ep^2 + 64,9) + p1.p2*k1.k1^2*i_*ge^4*gs^2*rat( - 64*ep^2 + 128*ep -
        64,9) + p1.p2*k1.k2^2*i_*ge^4*gs^2*rat(128*ep - 128,9) + p1.p2^2*
        k1.k1*i_*ge^4*gs^2*rat( - 128*ep^2,9) + p1.p2^2*k1.k2*i_*ge^4*gs^2*
        rat( - 128*ep^2,9) + p1.k1*p1.k2*p2.p2*i_*ge^4*gs^2*rat(-128,9) +
        p1.k1*p1.k2*p2.k2*i_*ge^4*gs^2*rat( - 64*ep + 64,9) + p1.k1*p2.p2*
        k2.k2*i_*ge^4*gs^2*rat( - 64*ep + 64,9) + p1.k1*p2.k1*k2.k2*i_*ge^4*
        gs^2*rat(128*ep - 128,9) + p1.k1*p2.k2*k1.k2*i_*ge^4*gs^2*rat( - 128*
        ep + 128,9) + p1.k1*p2.k2^2*i_*ge^4*gs^2*rat(64*ep - 64,9) + p1.k1^2*
        p2.p2*i_*ge^4*gs^2*rat(-128,9) + p1.k2*p2.p2*k1.k2*i_*ge^4*gs^2*rat(
        64*ep - 64,9) + p1.k2*p2.k1*p2.k2*i_*ge^4*gs^2*rat( - 64*ep + 64,9)
         + p1.k2*p2.k1*k1.k2*i_*ge^4*gs^2*rat( - 128*ep + 128,9) + p1.k2*
        p2.k2*k1.k1*i_*ge^4*gs^2*rat(128*ep - 128,9) + p1.k2^2*p2.k1*i_*ge^4*
        gs^2*rat(64*ep - 64,9)
```

In this case, all in terms of **dot-products**, **couplings**, evaluated in **d-dimensions.**

NOTHING original here, every loop person has his own version of something like this, but that's actually the problem! Nowhere is it done **in general** and **efficiently**…

MG5 + FeynRules3
c(sh)ould solve this !

B) MG for MultiLoop People

And there are motivated young people for this !

e.g. Johann Usovitch (Kira) + Martin Higgins (DiffExp)

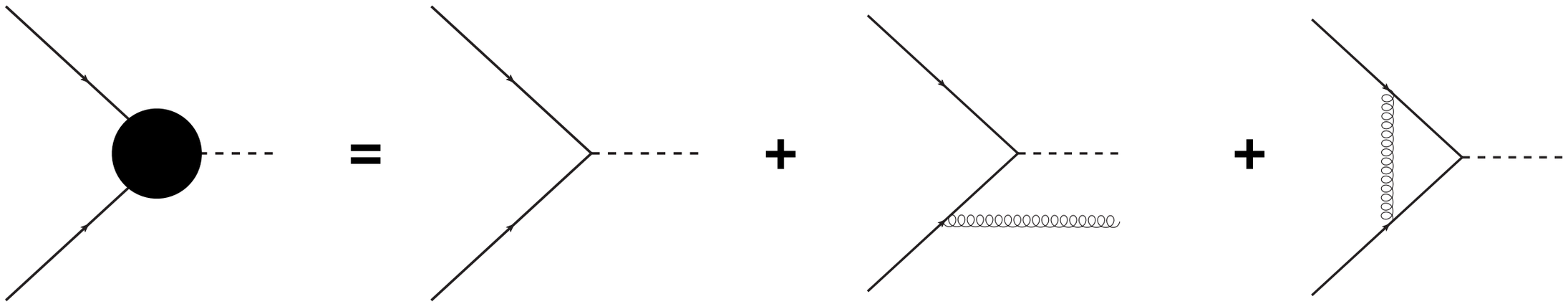
[see: https://indico.cern.ch/event/958085/contributions/4329104/attachments/2246417/3809850/RADCORLoopfestSlides.pdf]

I believe that the MadGraph5 collaboration would benefit from making sure to offer support and encourage such endeavour.

A new way of computing perturbative expansions:

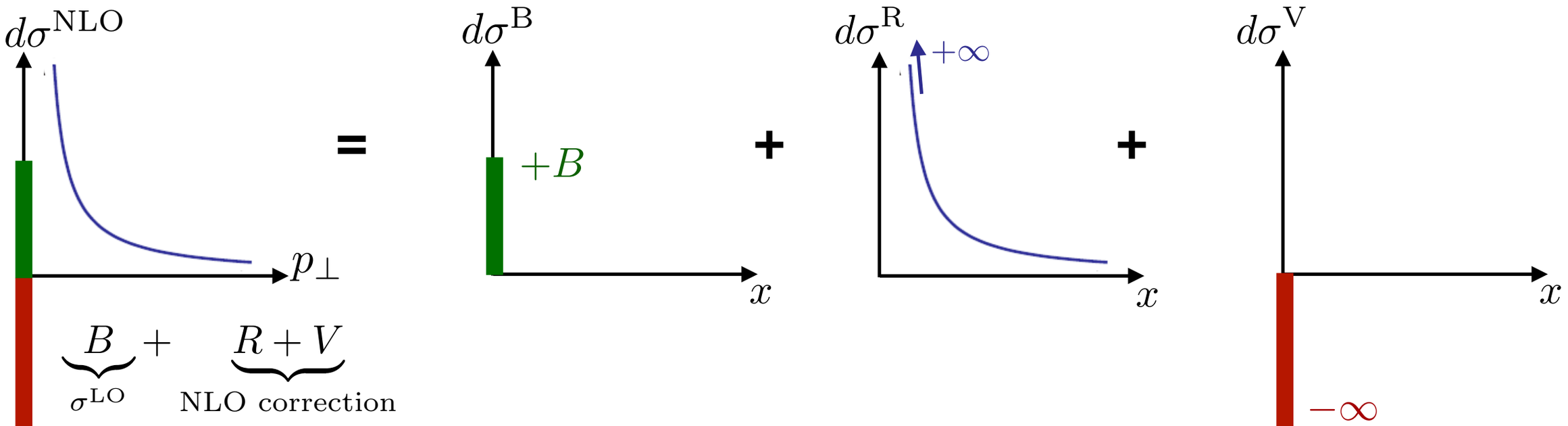
LOCAL UNITARITY

NLO Anatomy



$$\sigma^{\mathrm{NLO}} = 1 + \int_0^1 dx \left[\frac{\cos(x)}{x}\right] + \int_0^{10} dy \left[\frac{-e^{-y}}{y}\right]$$

NLO inclusive — Born (B) — Real-emission (R) — Virtual (V)



One-dimensional toy example

- Toy expression with $\mathcal{J}$ a measurement function, over $x \in [0, 10]$

$$\sigma^{(R+V)}(\mathcal{J}) = \int_0^{10} dx \left[\frac{\cos(x)}{x} \mathcal{J}(x) \right] + \left[\int_0^1 dy \frac{-e^{-y}}{y} \right] \mathcal{J}(0)$$

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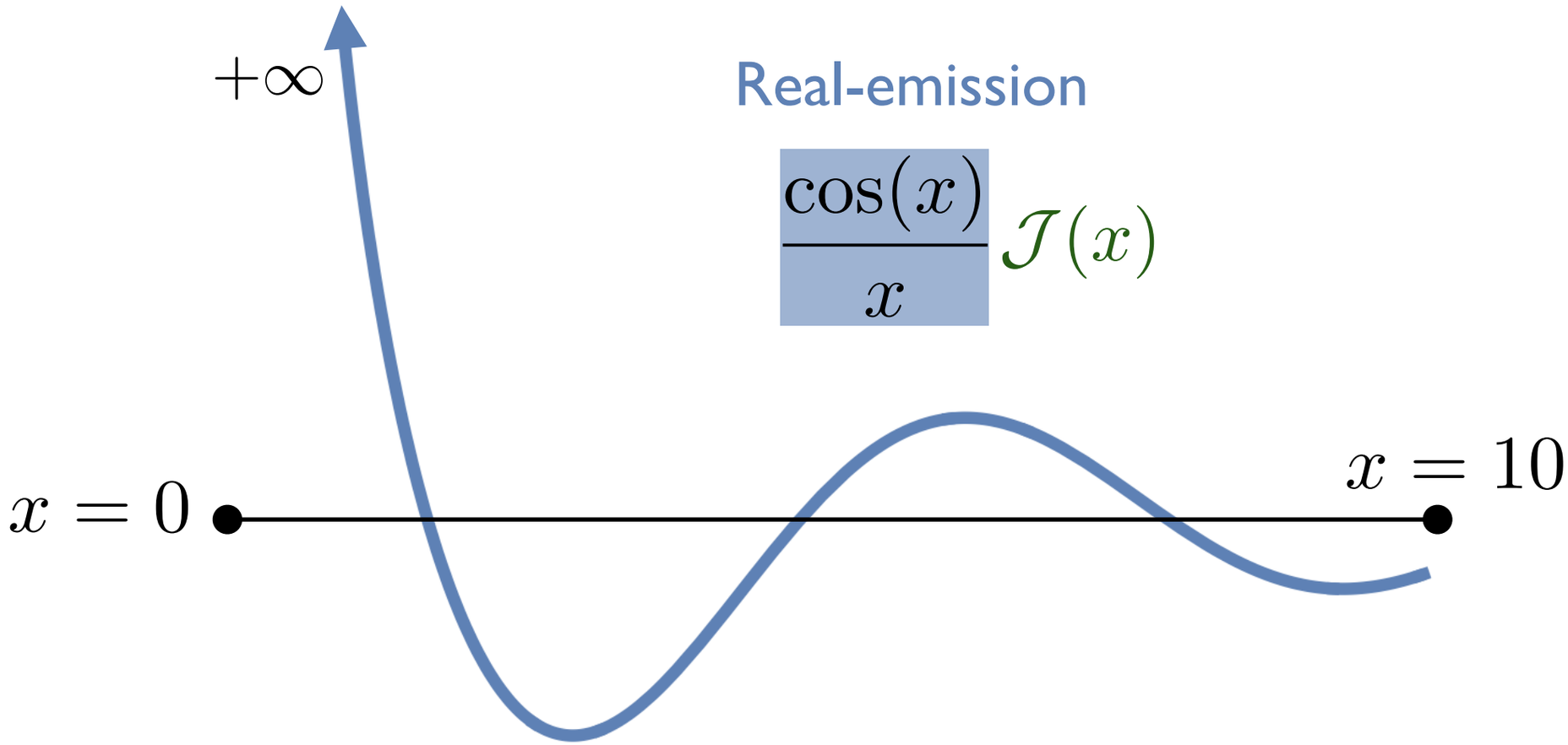

Real-emission

$$\frac{\cos(x)}{x} \mathcal{J}(x)$$

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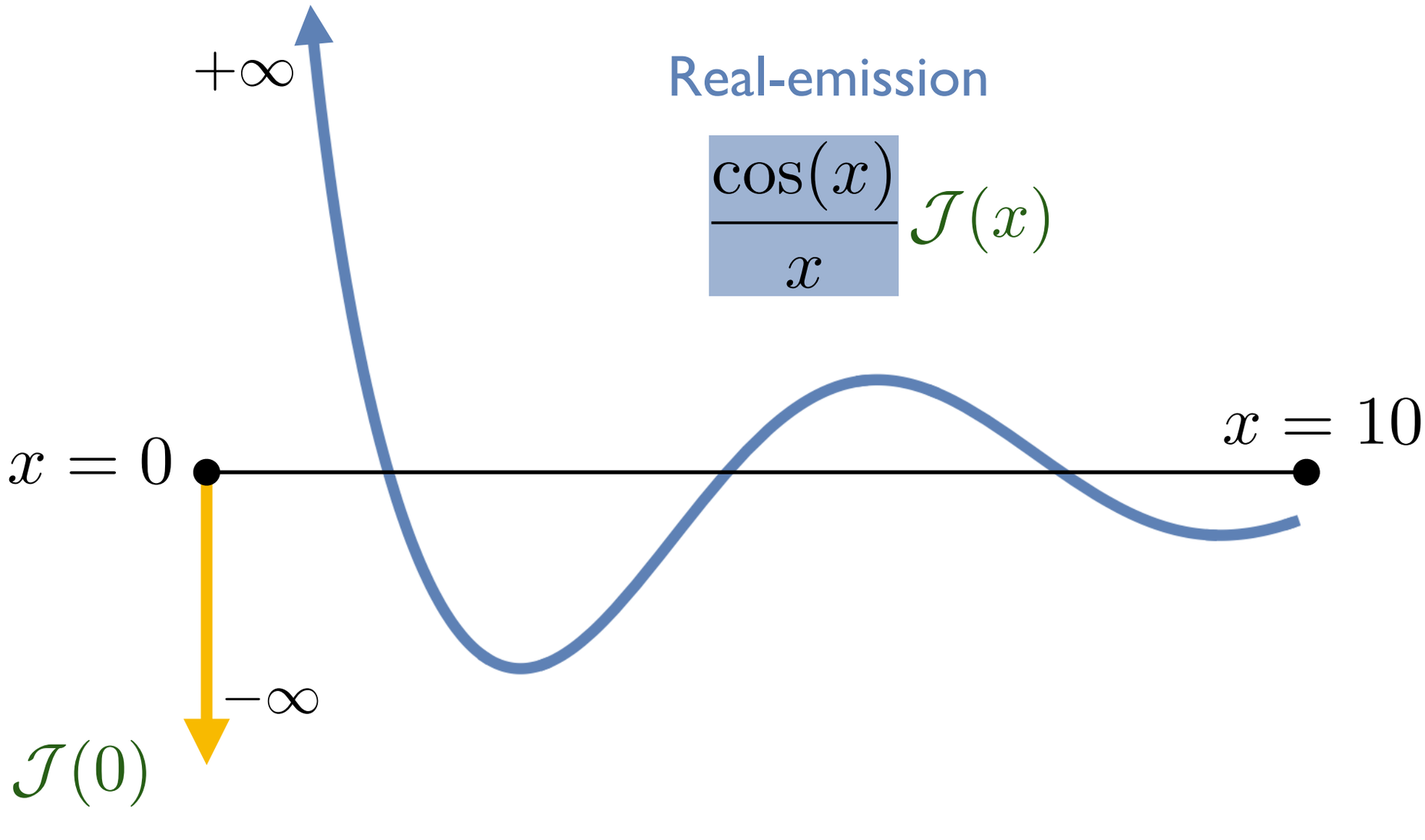

Real-emission

$$\frac{\cos(x)}{x} \mathcal{J}(x)$$

Virtual

$$\left[\int_0^1 dy \frac{-e^{-y}}{y} \right] \mathcal{J}(0)$$

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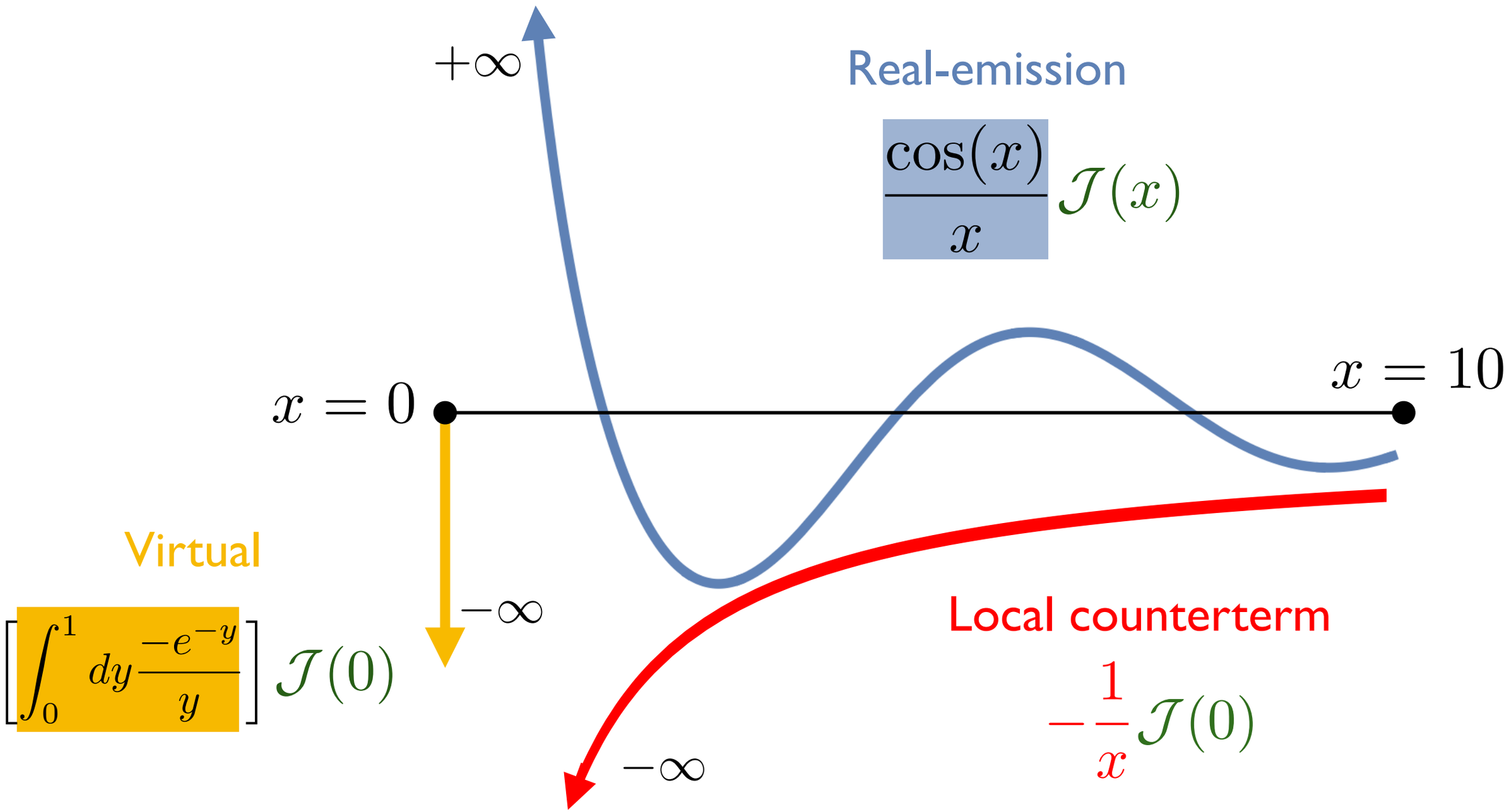

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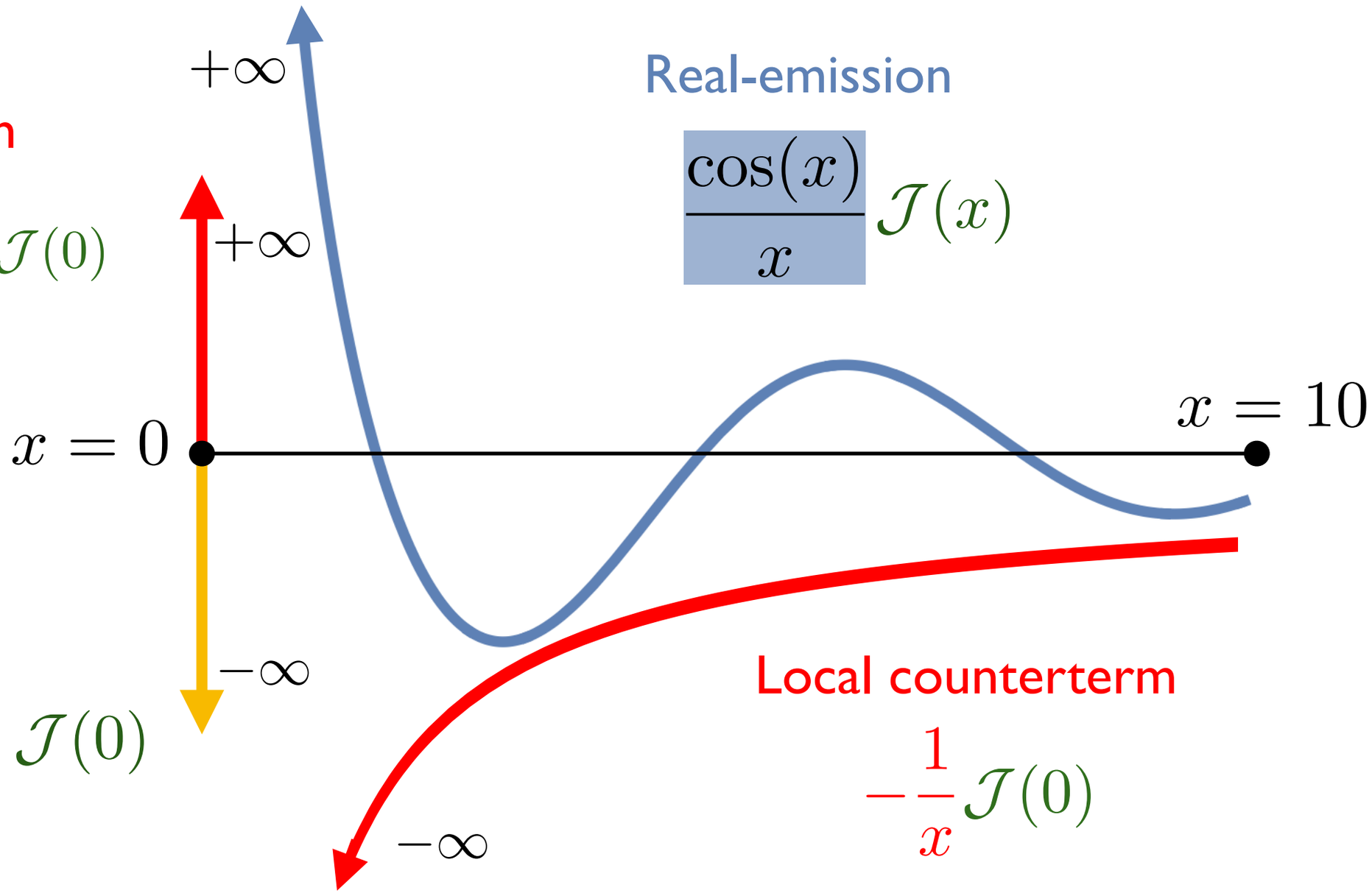
Integrated counterterm

$$+ \left[\int_0^{10} dx \frac{1}{x} \right] \mathcal{J}(0)$$



Real-emission

$$\frac{\cos(x)}{x} \mathcal{J}(x)$$

Virtual

$$\left[\int_0^1 dy \frac{-e^{-y}}{y} \right] \mathcal{J}(0)$$

Local counterterm

$$-\frac{1}{x} \mathcal{J}(0)$$

Subtraction vs Local Unitarity

- One can introduce the following local (in x) **counterterm**:

$$\sigma^{(R+V)}(\mathcal{J}) = \int_0^{10} dx \left[\frac{\cos(x)}{x}\mathcal{J}(x) - \frac{1}{x}\mathcal{J}(0)\right] + \left(\left[\int_0^1 dy \frac{-e^{-y}}{y}\right] + \left[\int_0^{10} dx \frac{1}{x}\right]\right)\mathcal{J}(0)$$

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NUMERICAL

Subtraction vs Local Unitarity

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- My research introduces a new paradigm, **Local Unitarity**, whereby :

$$\sigma^{(R+V)}(\mathcal{J}) = \int_0^{10} dx \left[\frac{\cos(x)}{x}\mathcal{J}(x) + \frac{-e^{-x}}{x}\mathcal{J}(0)\Theta(1-x)\right]$$

OBSTACLES: INFRARED LIMITS

$$\sigma \sim \boxed{\int d^4p_1 \dots d^4p_N} \left|\mathcal{M}^{(L)}(\{p_i\})\right|^2 \delta p_1^2 \dots \delta p_N^2 \delta^4\left(p_1 + \dots p_N\right)$$

Numerical

LO	NLO	NNLO	N3LO	…
$d\bar{d} \to Z$	$d\bar{d} \to Zg$	$d\bar{d} \to Zgg$	$d\bar{d} \to Zggg$	

{}	`| L0 : ((S(4),),)`
	`| L1 : ((C(1,4),),)`
	`| L2 : ((C(2,4),),)`
	`| L3 : ((C(S(4),1),),)`
	`| L4 : ((C(S(4),2),),)`
		`| L0 : ((S(4),),)`
		`| L1 : ((S(5),),)`
		`| L2 : ((C(4,5),),)`
		`| L3 : ((C(1,4),),)`
		`| L4 : ((C(2,4),),)`
		`[ … 71 limits … ]`
		`| L69 : ((C(C(S(4),2),S(5),),),)`
		`| L70 : ((C(S(C(S(4),5),),2),),)`
		`| L71 : ((C(S(C(S(5),4),),2),),)`
			`| L0 : ((S(4),),)`
			`| L1 : ((S(5),),)`
			`| L2 : ((S(6),),)`
			`| L3 : ((C(4,5),),)`
			`| L4 : ((C(4,6),),)`
			`[ … 1396 limits … ]`
			`| L1394 : ((C(S(C(S(C(S(6),4),),5),),2),),)`
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			`| L1396 : ((C(S(C(S(C(S(6),5),),4),),2),),)`	

On top of :

- R^kV^{k-n} proliferation of contributions
- Convolution with PDF counterterms
- Growth of number of partonic channels

} Very challenging at and beyond NNLO…

OBSTACLES: MULTI-LOOPS MEs

$$\sigma \sim \boxed{\int d^4p_1 \dots d^4p_N}\ |\mathcal{M}^{(L)}(\{p_i\})|^2\, \delta p_1^2 \dots \delta p_N^2 \delta^4\left(p_1 + \dots p_N\right)$$

Numerical

$$|\mathcal{M}^{(L)}(\{p_i\})|^2 \sim \boxed{\int d^dk_1 \dots \int d^dk_L}\ I(\{p_i\},\{k_i\})|$$

Analytical ?

Timings	Tree-level MG5aMC [arXiv:1405.0301]		One-loop MadLoop [arXiv:1103.0621]		Two-loop VVamp [arXiv:1503.08835]
$d\bar{d} \to ZZ$	7 µs	x 10^2 →	0.6 ms	x 10^3 →	$\mathcal{O}$(~1s)
$d\bar{d} \to ZZg$	35 µs	x 10^3 →	38 ms	? →	N/A
$d\bar{d} \to ZZgg$	220 µs	x 10^4 →	1200 ms		N/A

A claim :

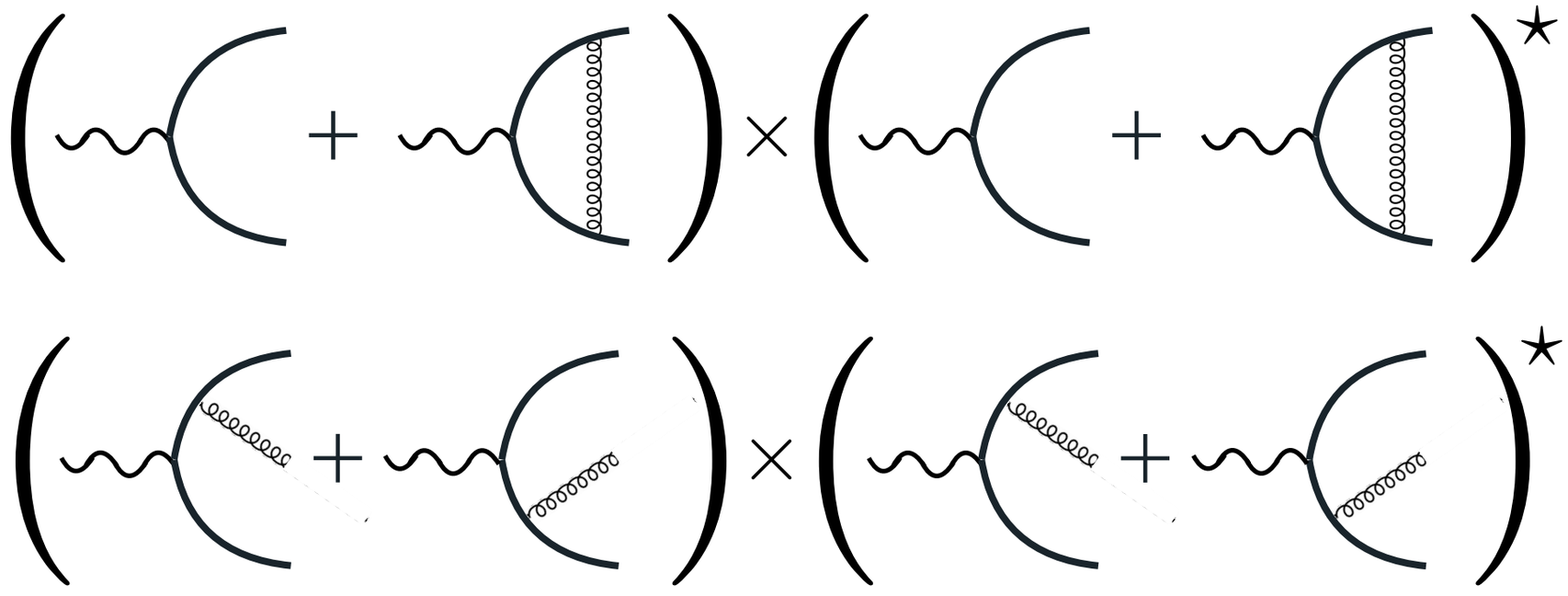
The more N's you add …

… the less natural it is to look at amplitudes.

Alternative : Local Unitarity

[Z. Capatti, VH, A. Pelloni, B. Ruijl, arXiv : 2010.01068] [Also more details in my QCD Seminar talk]

$$\sigma_{\gamma^\star \to d\bar{d}} =$$



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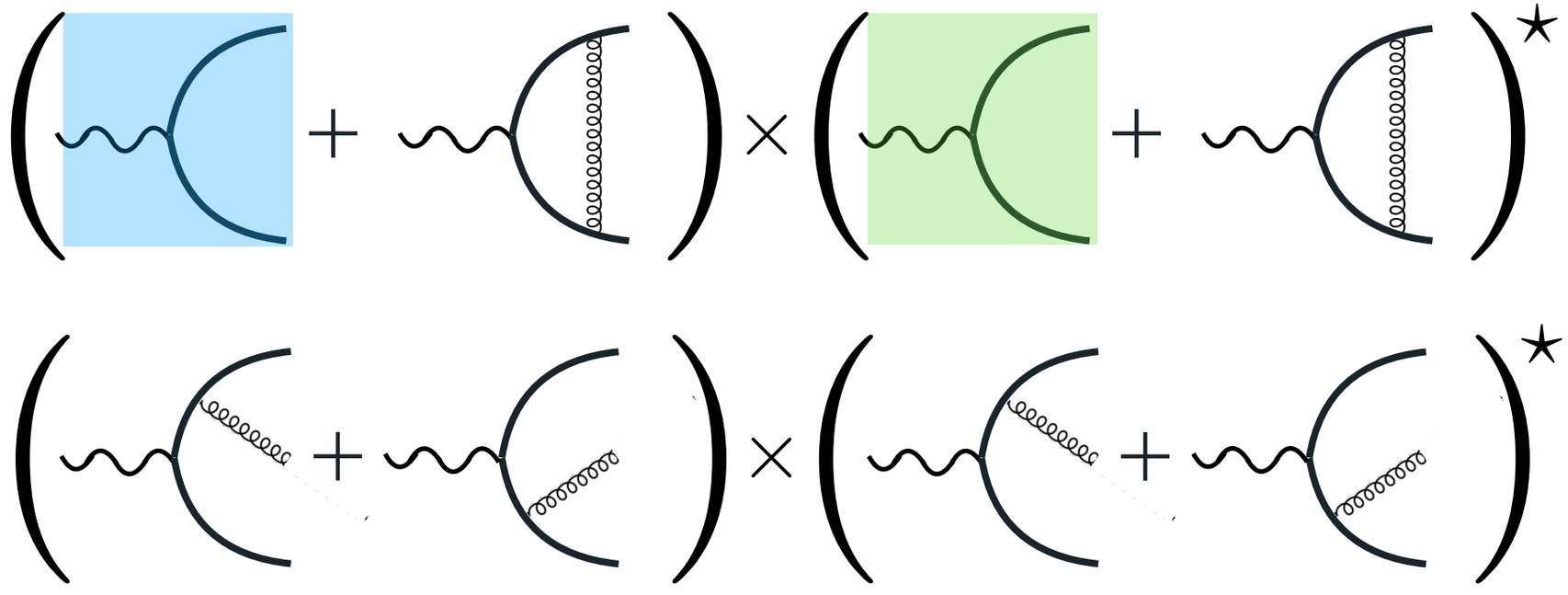

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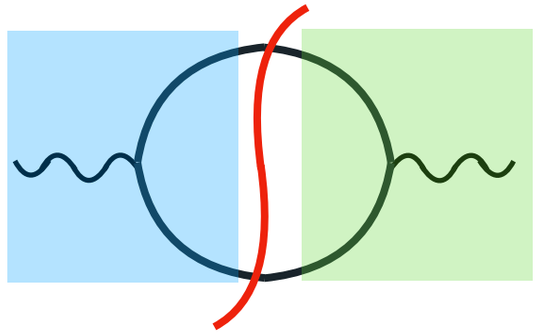

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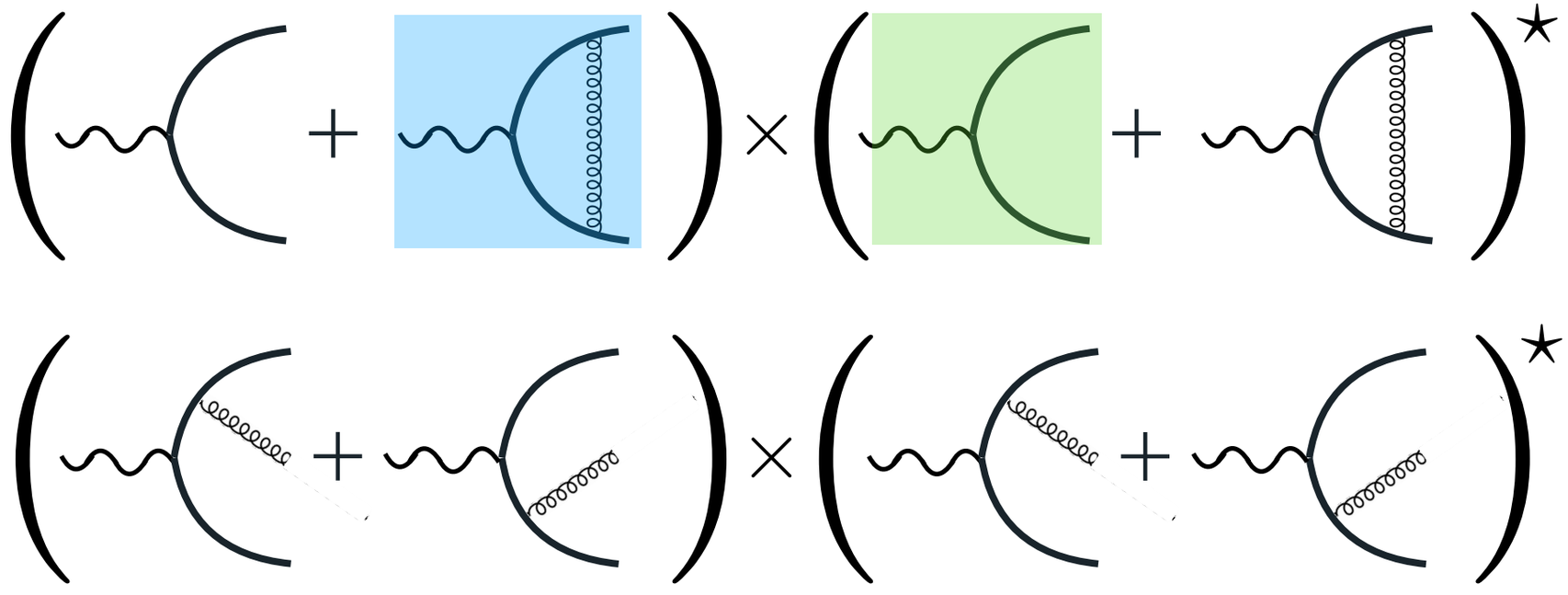

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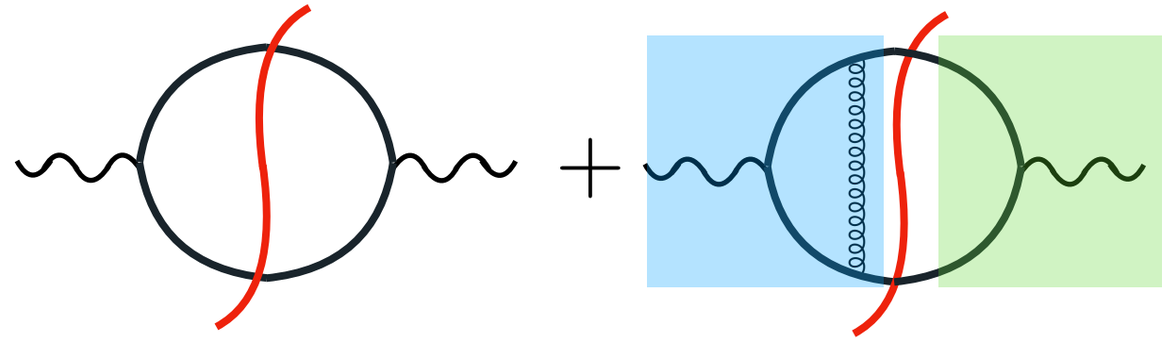

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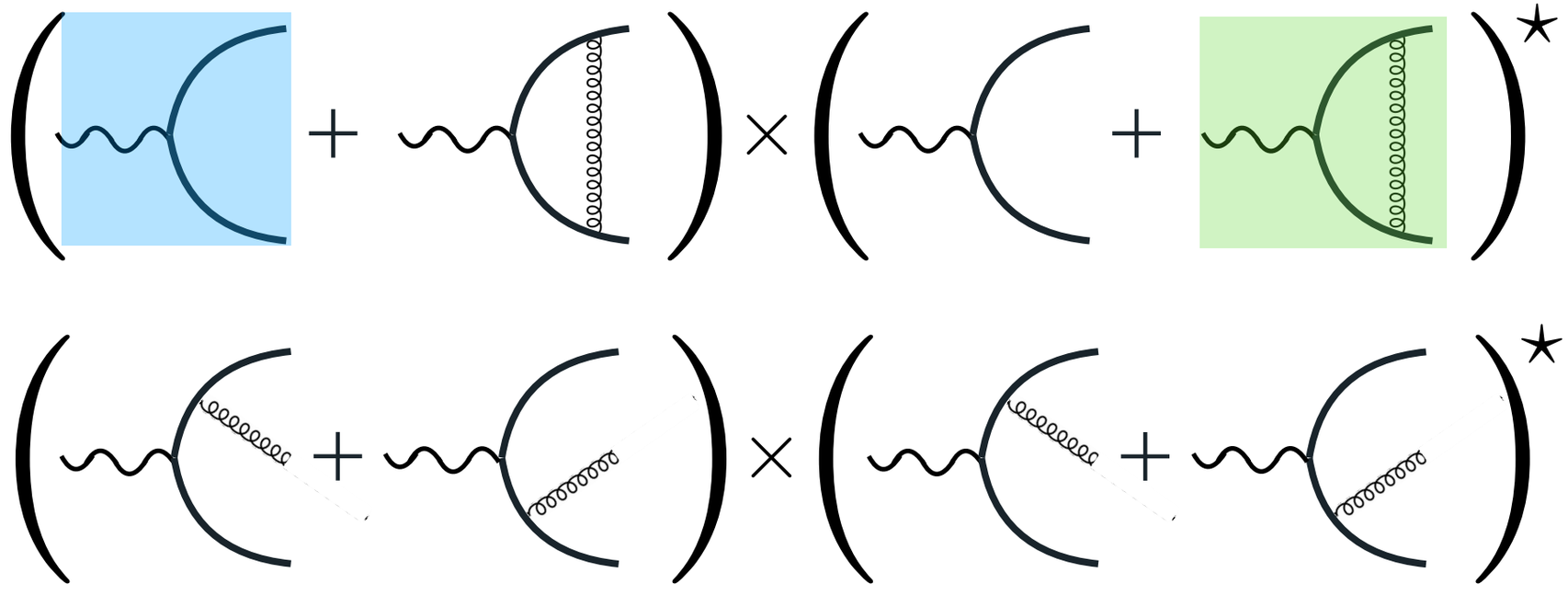

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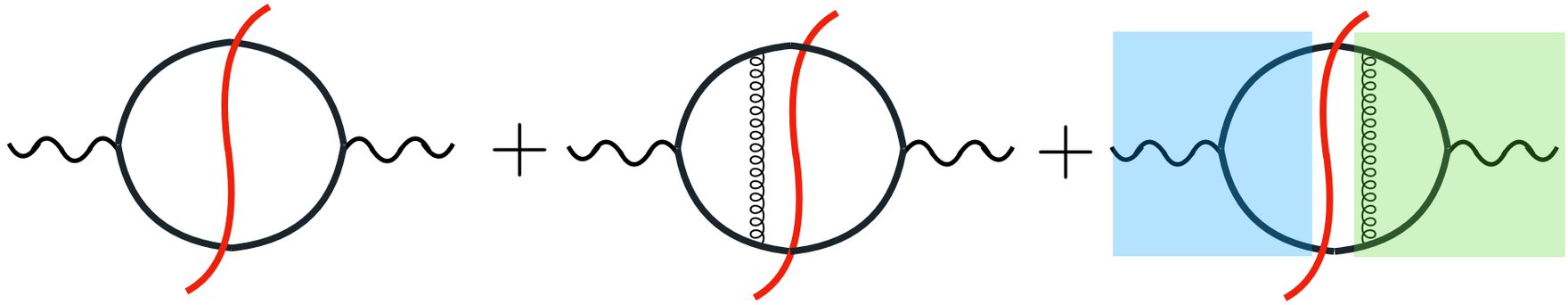

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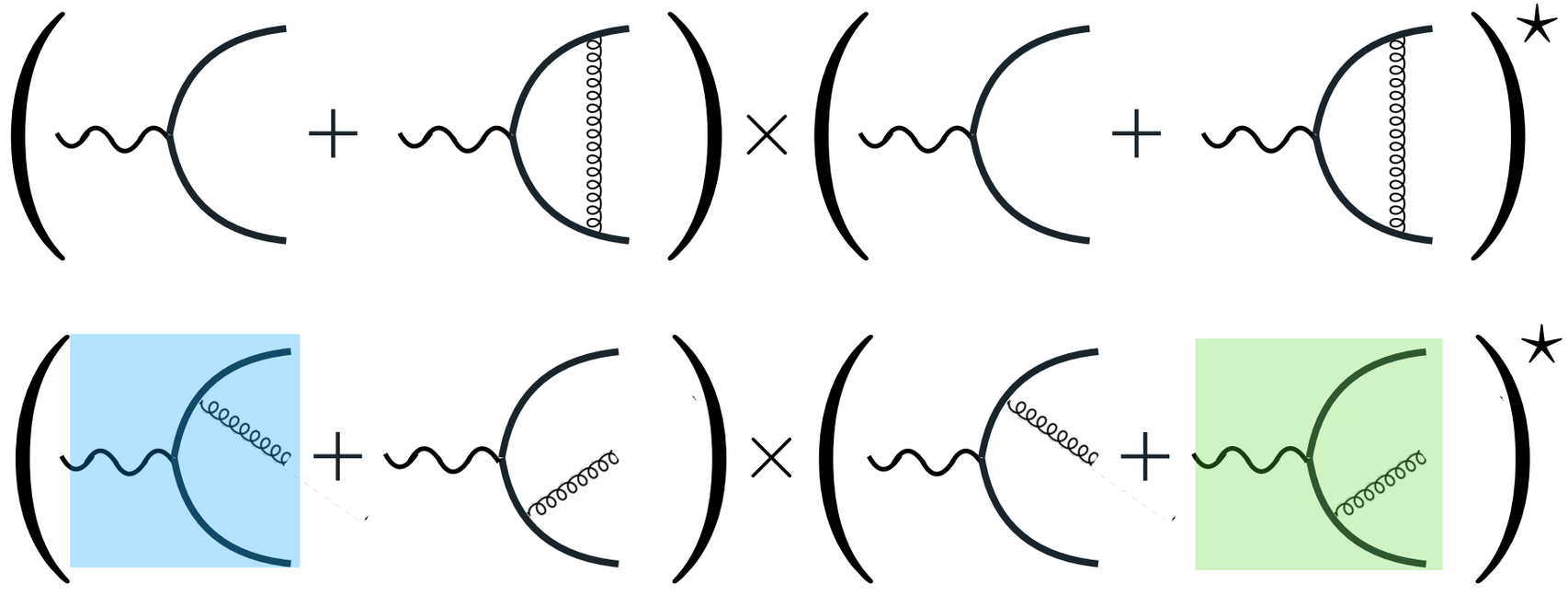

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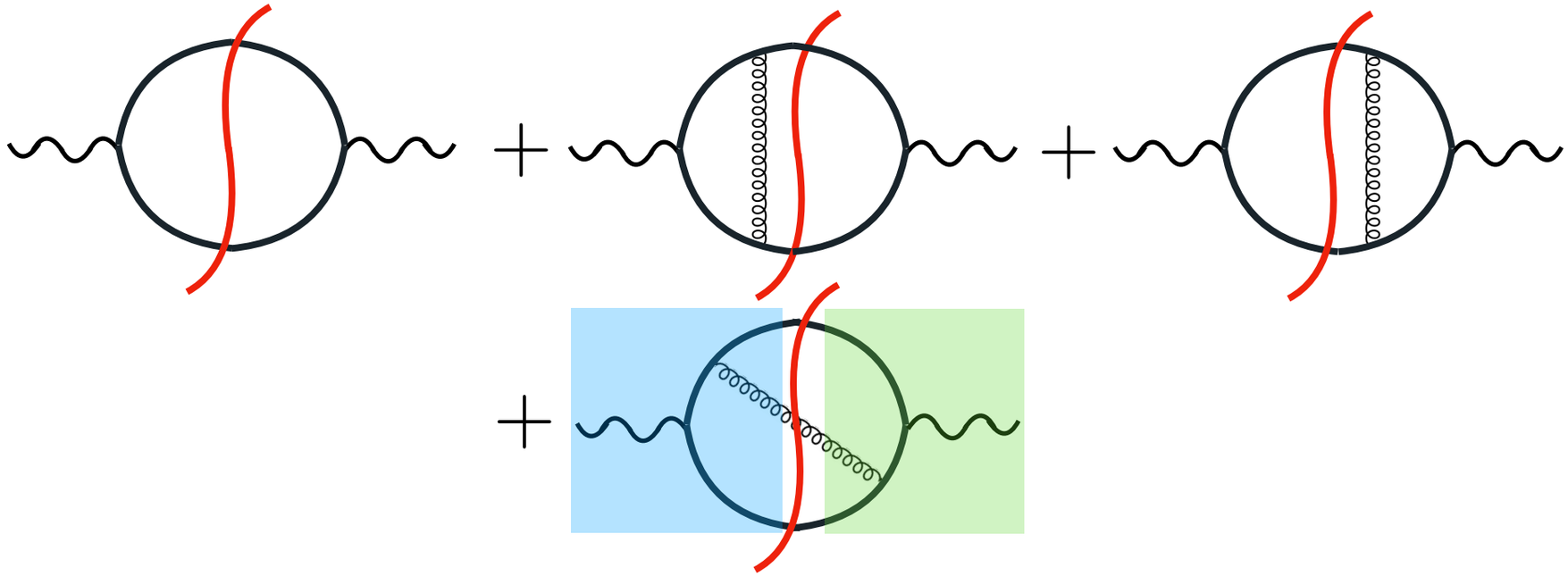

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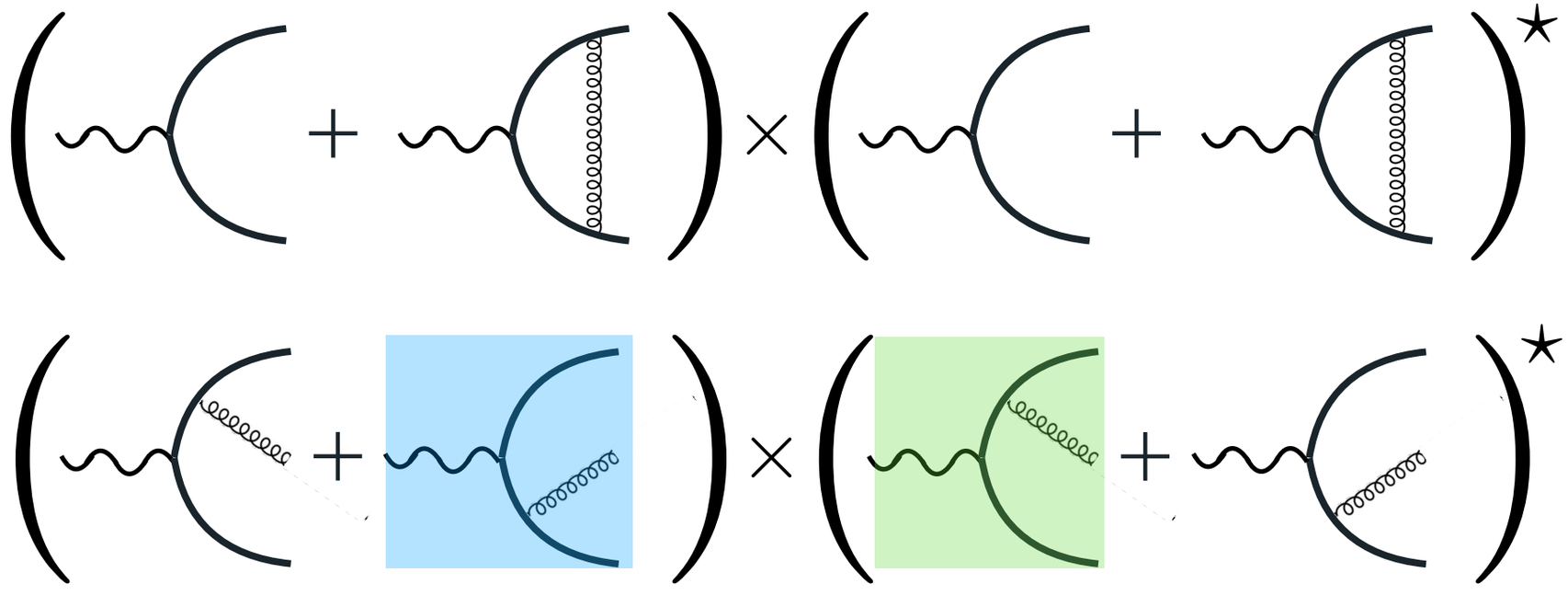

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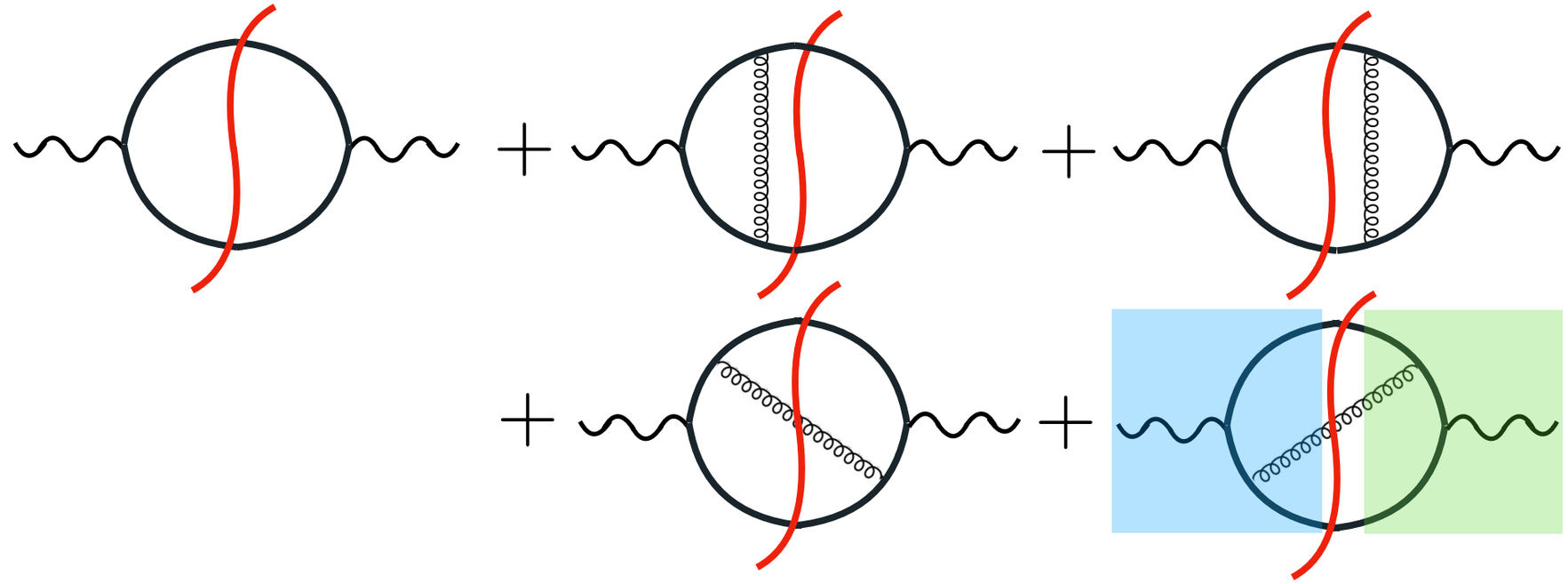

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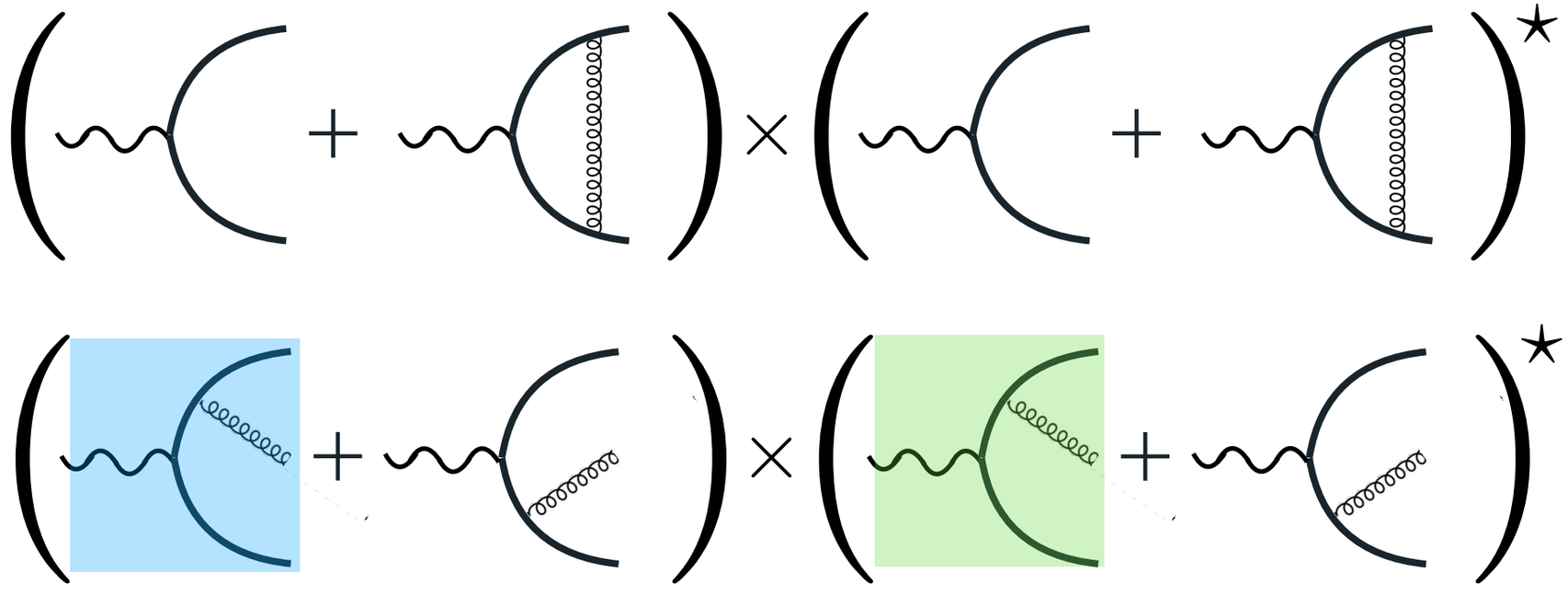

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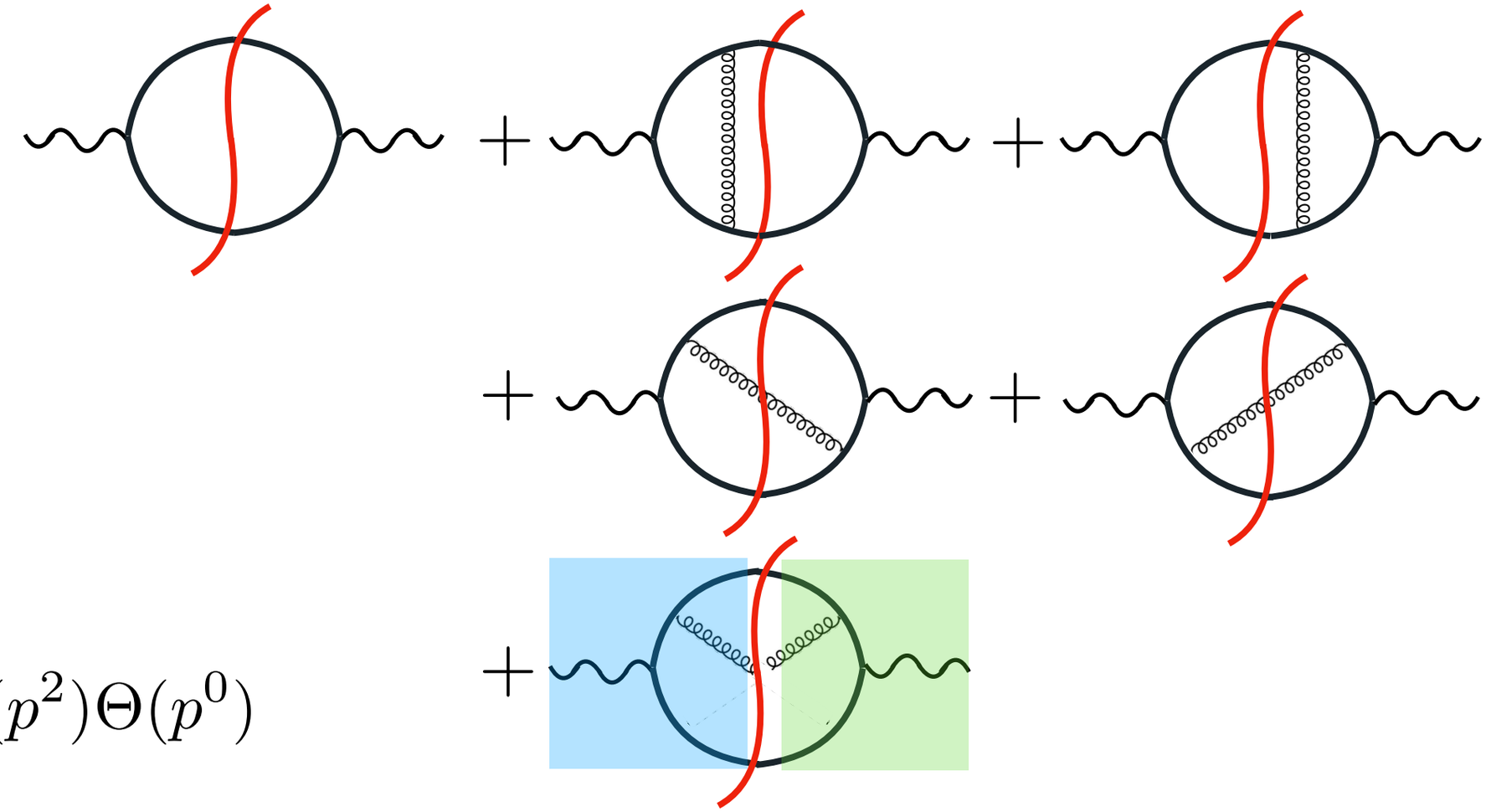

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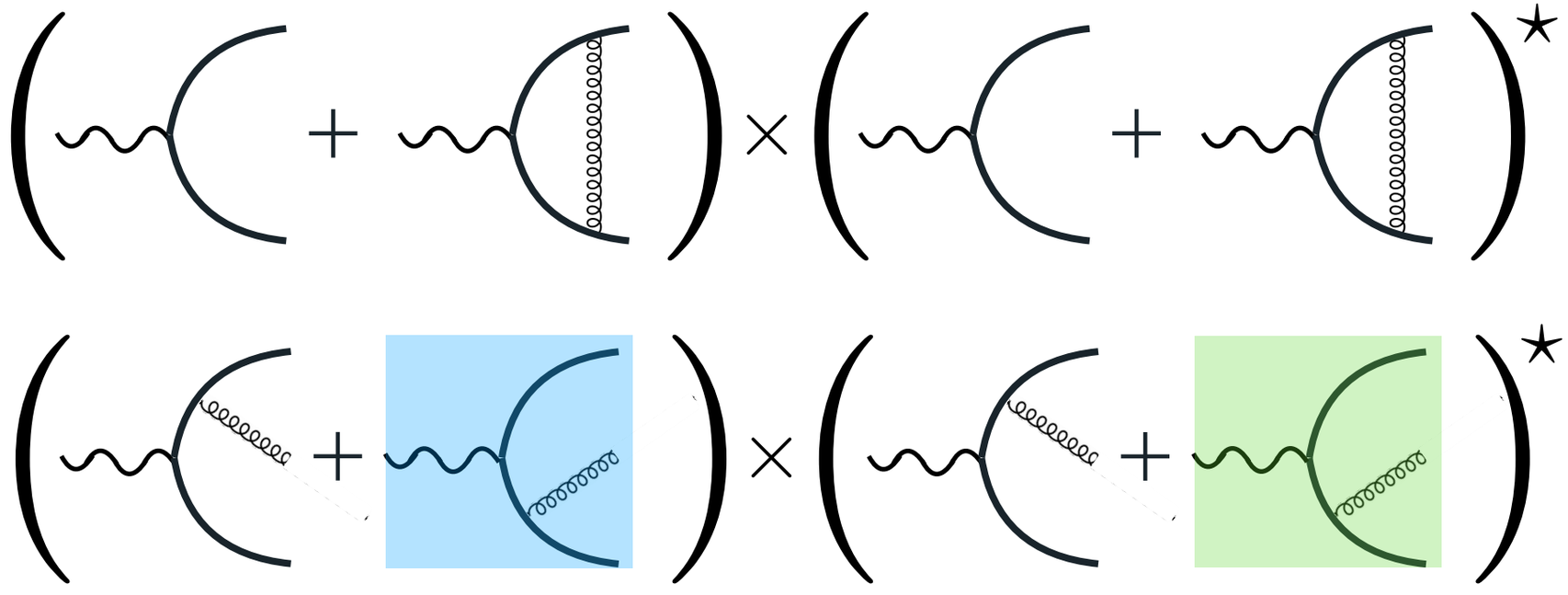

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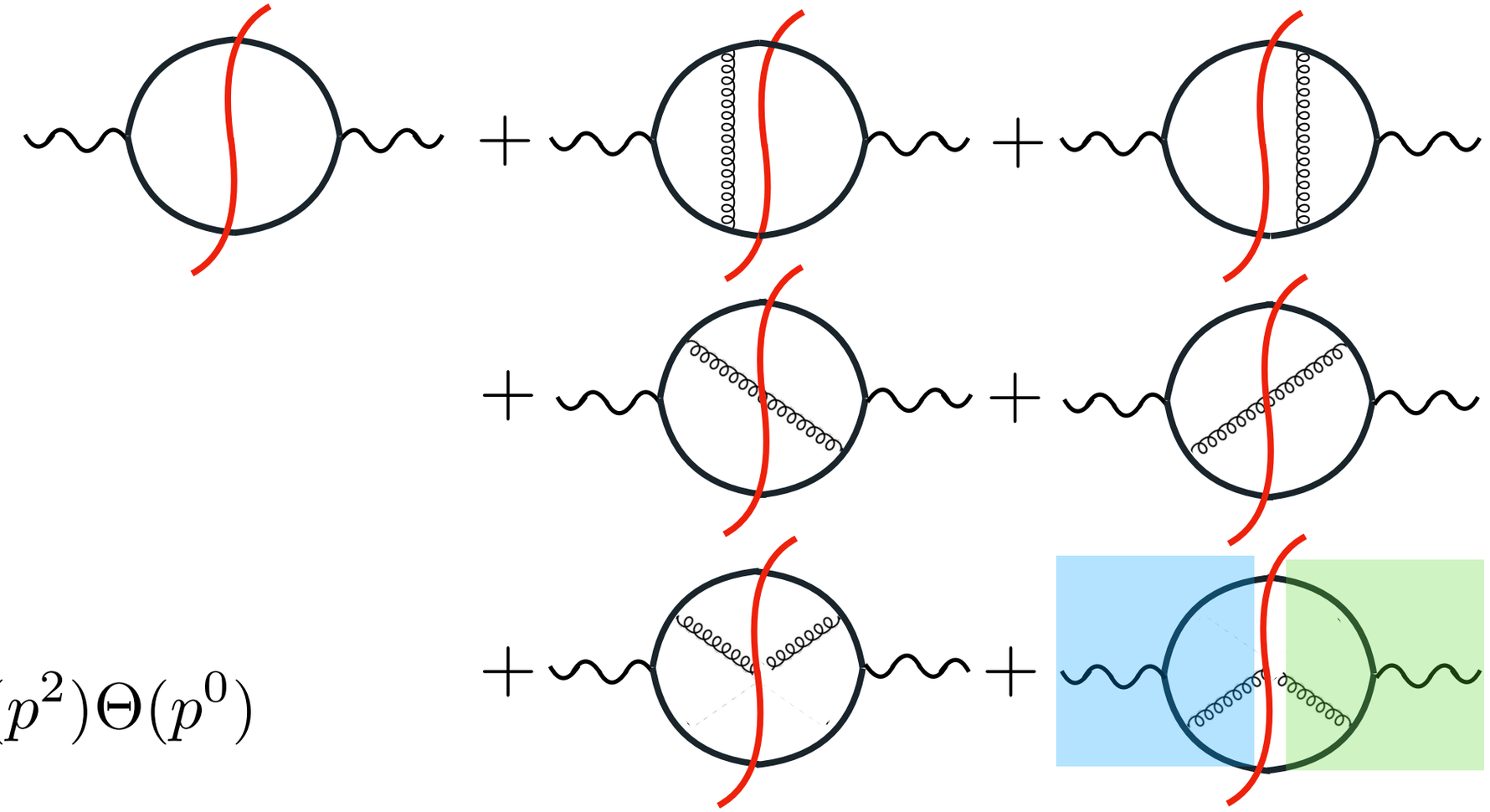

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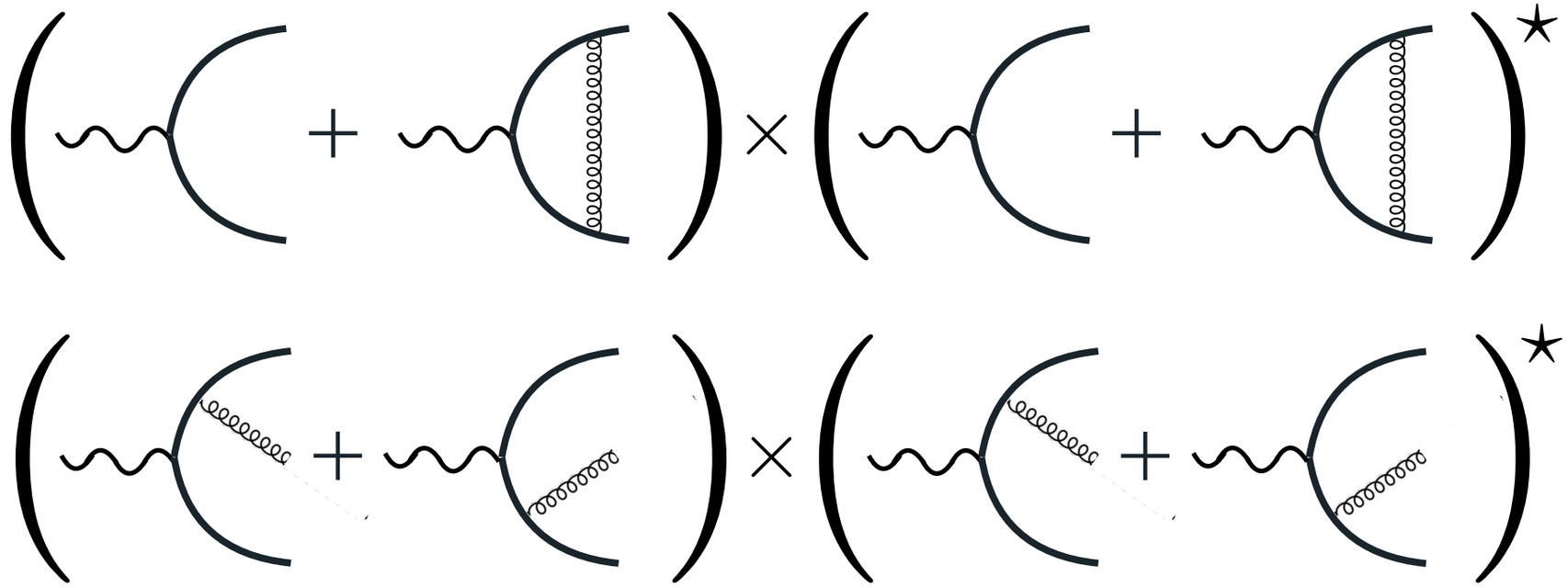

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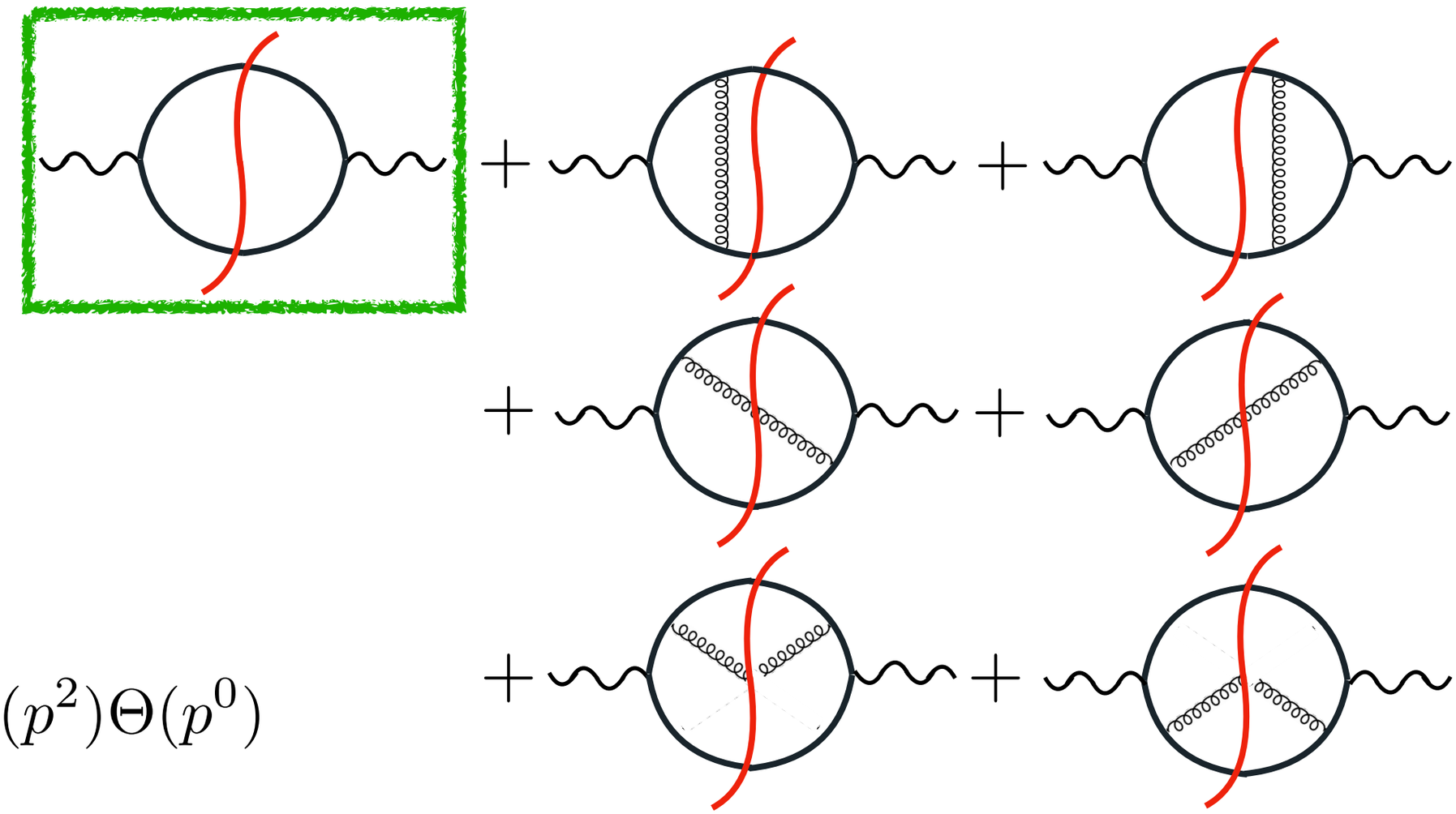

LO

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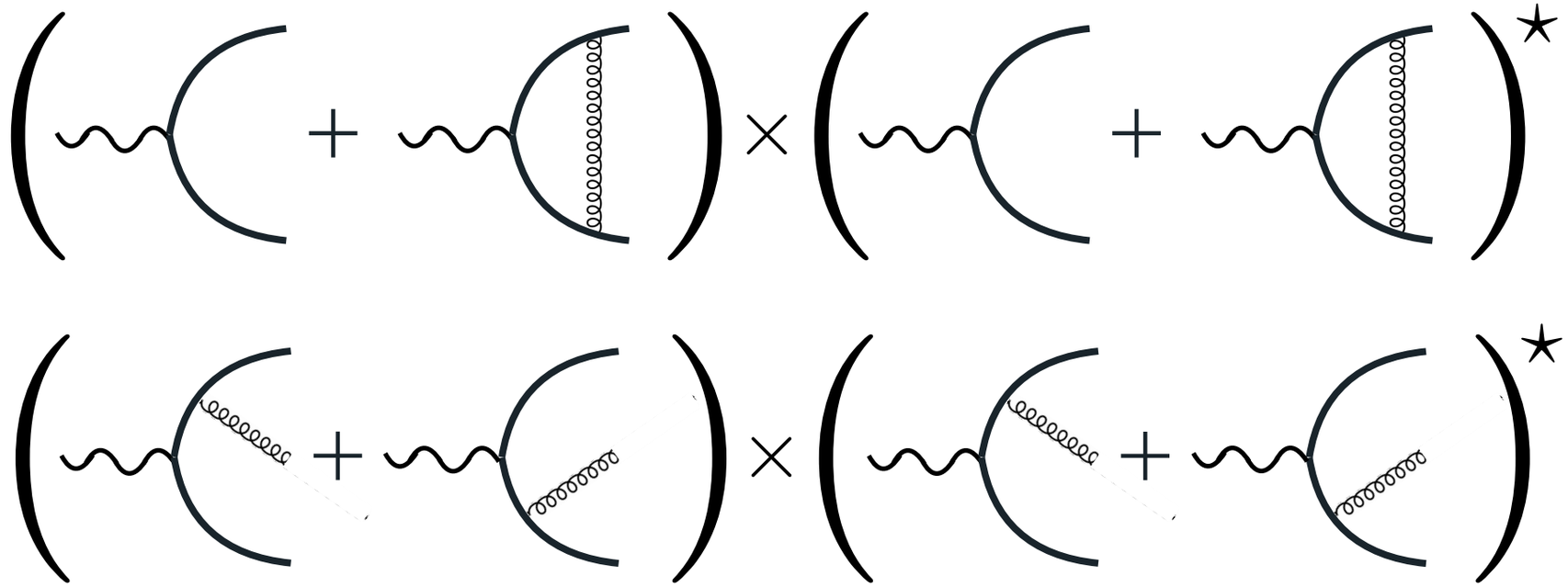
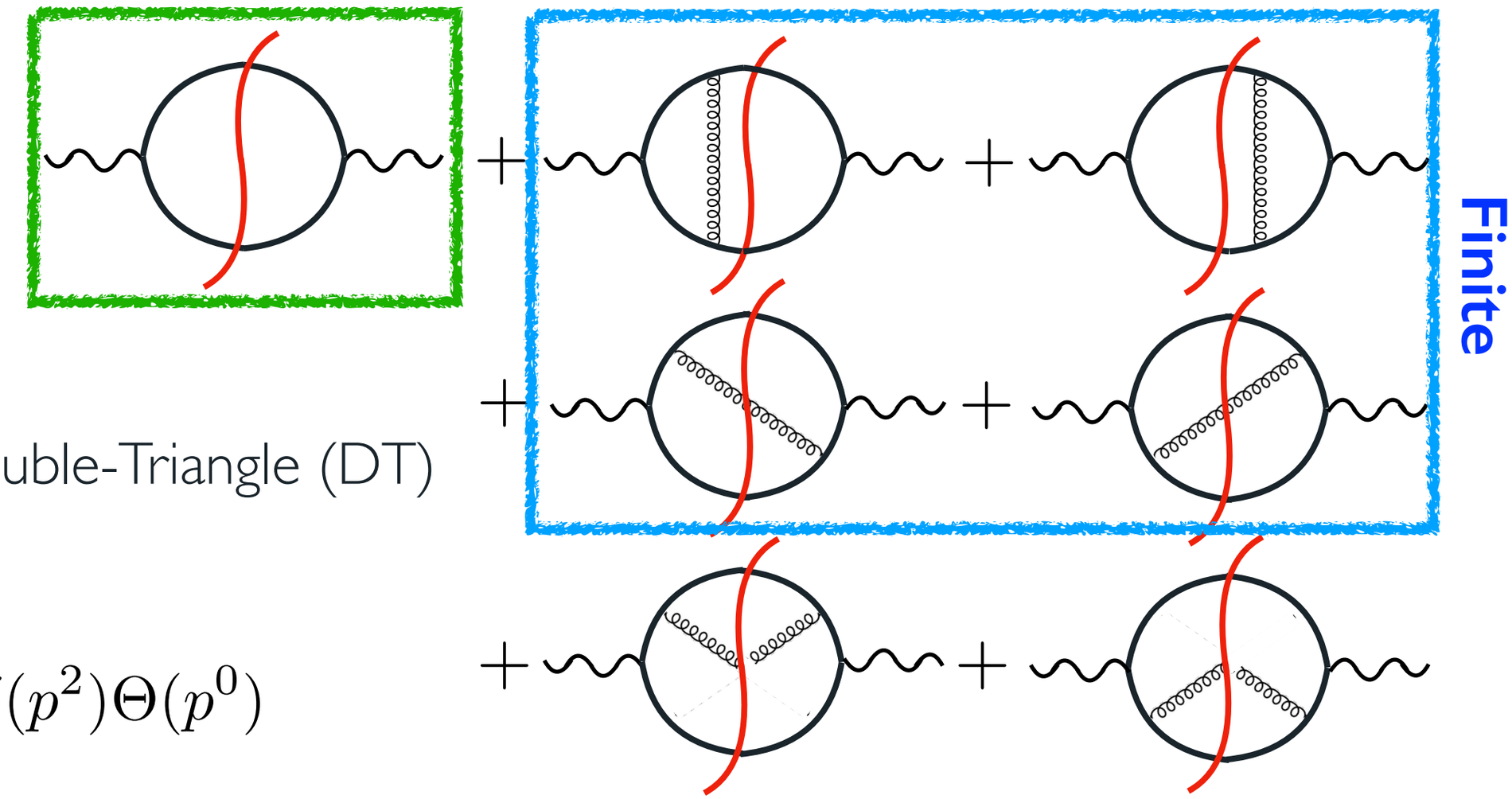


$$\sigma^{(\mathrm{LU})}_{\gamma^\star \to d\bar{d}} =$$



LO

NLO, Double-Triangle (DT)

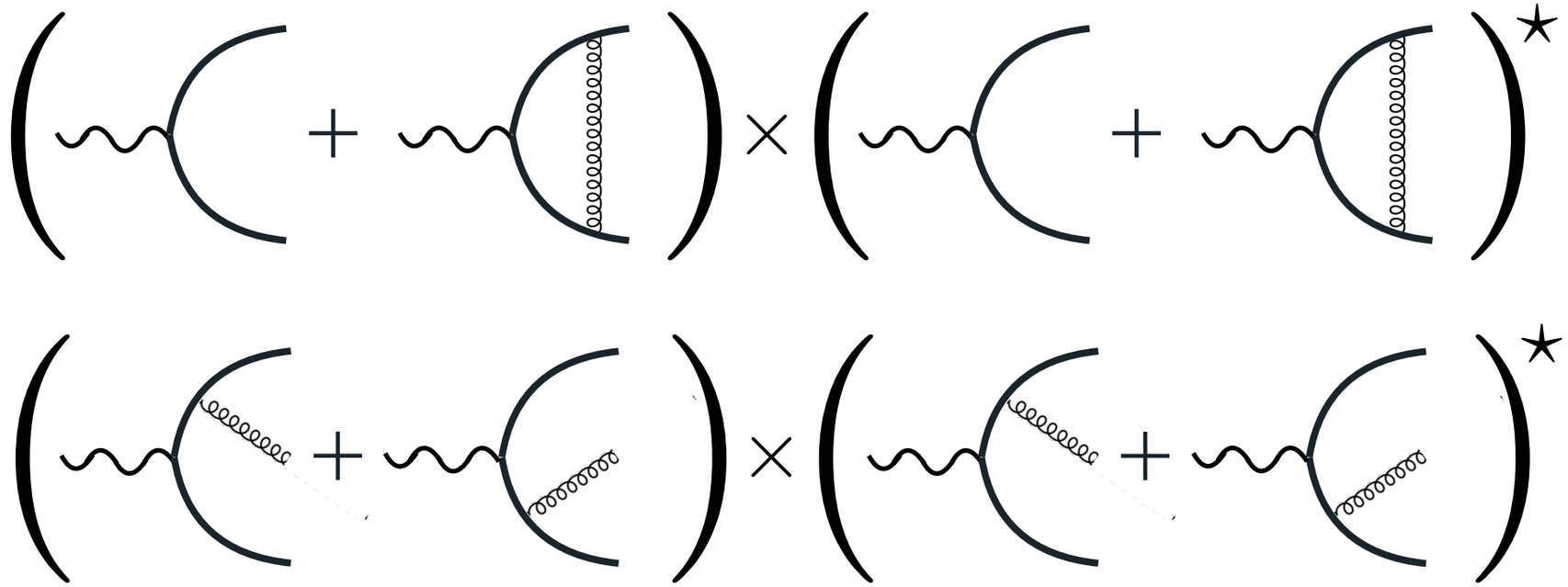
Finite

$$\equiv \frac{p^2}{2p^0}\delta(p^2)\Theta(p^0)$$

Alternative : Local Unitarity

[Z. Capatti, VH, A. Pelloni, B. Ruijl, arXiv : 2010.01068] [Also more details in my QCD Seminar talk]

$$\sigma_{\gamma^\star \to d\bar{d}} =$$



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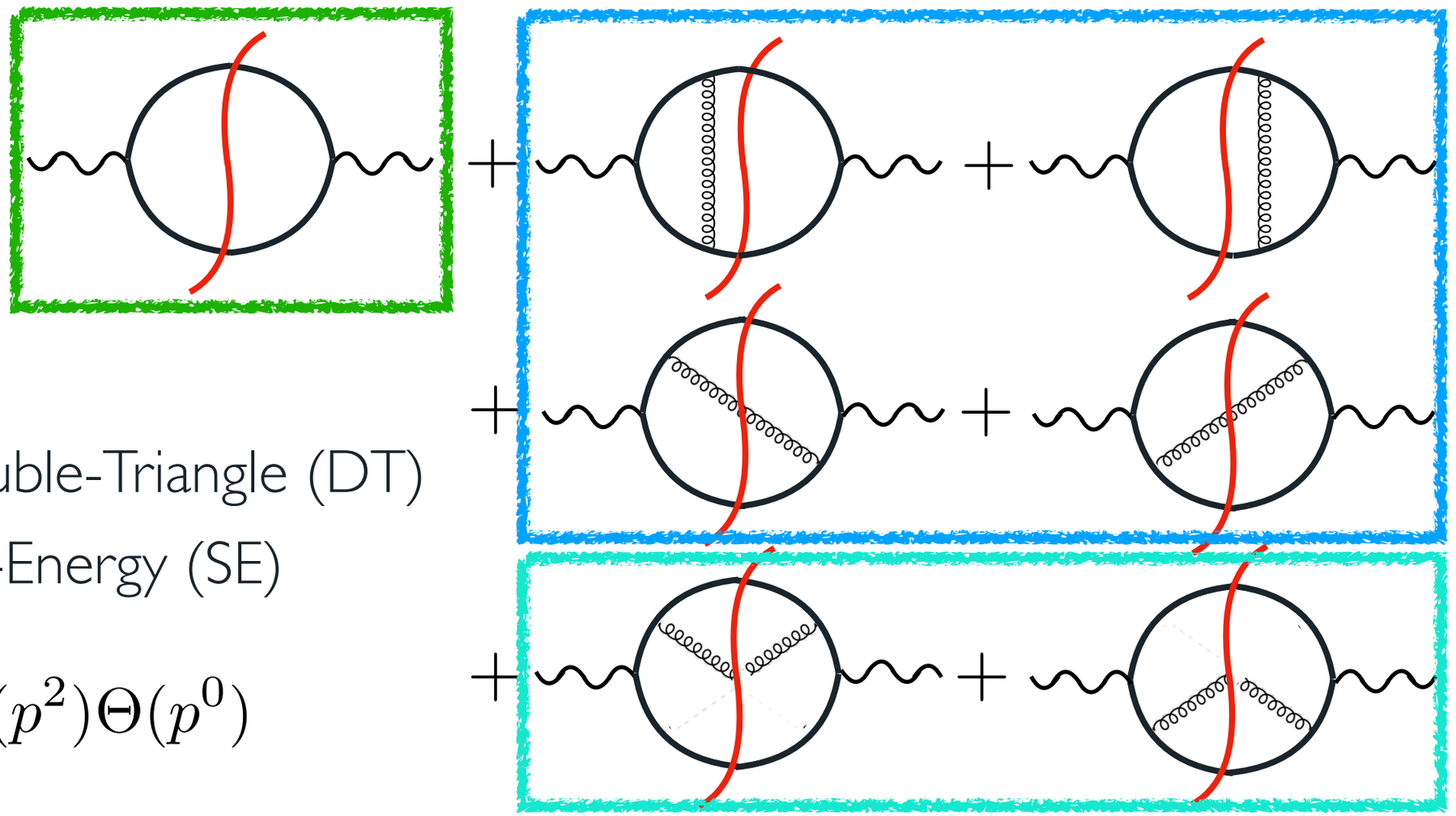
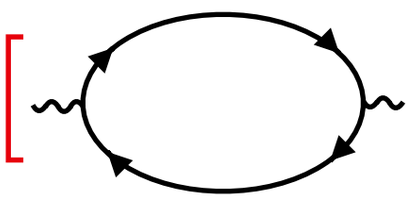
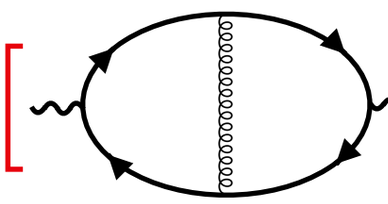
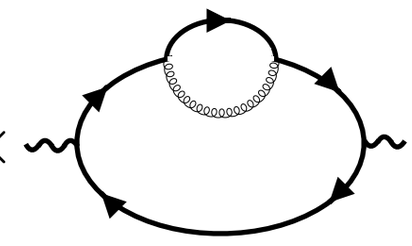


- **LO**
- **NLO**, Double-Triangle (DT)
- **NLO**, Self-Energy (SE)

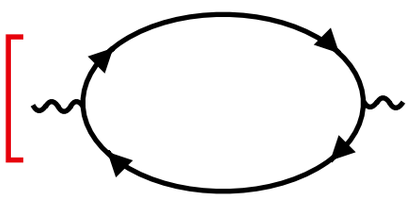
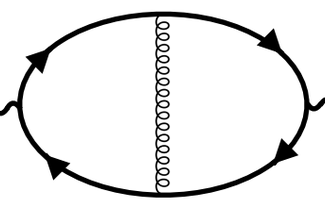
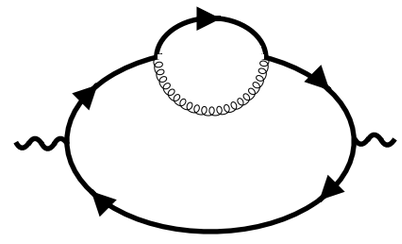
$$\equiv \frac{p^2}{2p^0}\delta(p^2)\Theta(p^0)$$

So what? It's just the optical theorem no?

$$\sigma^{(\mathrm{LU})}_{\gamma^\star \to d\bar{d}} = \mathrm{LU}\Big[$$  $$\Big] + \mathrm{LU}\Big[$$  $$\Big] + \mathrm{LU}\Big[2\times$$  $$\Big]$$

At the **integral** level, the **Optical Theorem** simply gives: $\mathrm{LU}[\cdot] \propto \mathrm{Im}[\cdot]$

So what? It's just the optical theorem no?

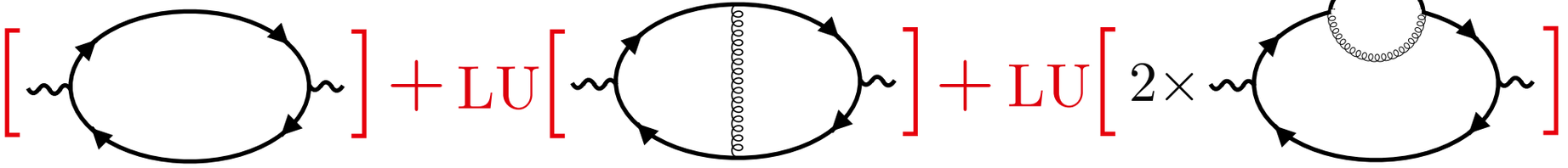
$$\sigma^{(\mathrm{LU})}_{\gamma^\star \to d\bar{d}} = \mathrm{LU}\Big[$$  $$\Big] + \mathrm{LU}\Big[$$  $$\Big] + \mathrm{LU}\Big[2\times$$  $$\Big]$$

At the **integral** level, the **Optical Theorem** simply gives: $\mathrm{LU}[\cdot] \propto \mathrm{Im}[\cdot]$

The **differential** version of LU is our new paradigm: **Local Unitarity**

So what? It's just the optical theorem no?

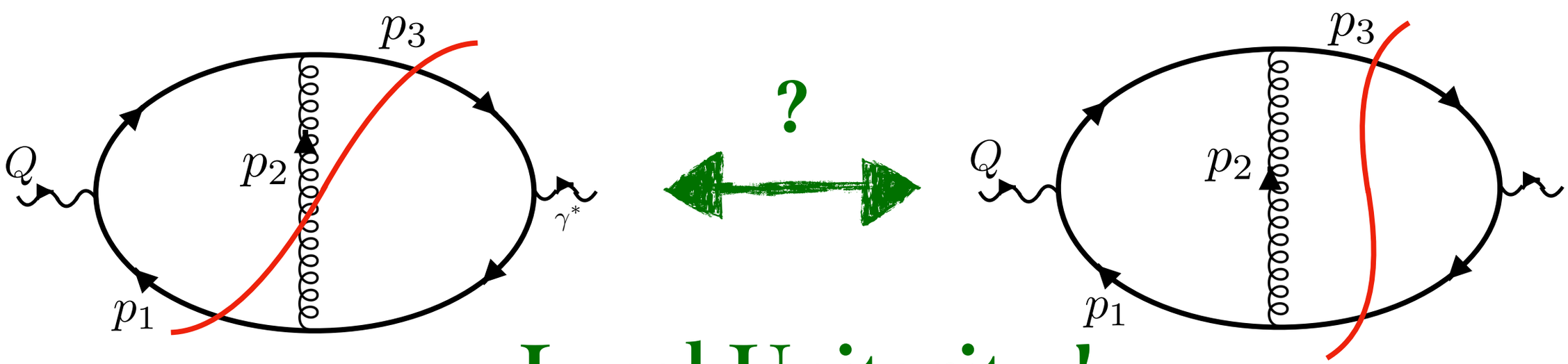
$$\sigma^{(\mathrm{LU})}_{\gamma^\star \to d\bar{d}} = \mathrm{LU}\Big[\Big] + \mathrm{LU}\Big[\Big] + \mathrm{LU}\Big[\, 2\times \Big]$$



At the **integral** level, the **Optical Theorem** simply gives: $\mathrm{LU}[\cdot] \propto \mathrm{Im}[\cdot]$

The **differential** version of LU is our new paradigm: **Local Unitarity**

Each supergraph is **individually locally finite**, how you ask?



Local Unitarity !

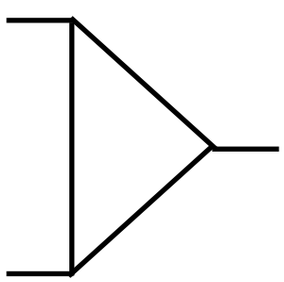
$$\propto \delta^{(+)}(p_3^2)\delta^{(-)}(p_1^2)\delta^{(-)}(p_2^2) \quad ? \quad \propto \delta^{(+)}(p_3^2)\delta^{(-)}((p_1+p_2)^2)$$

Loop-Tree Duality

$$\frac{1}{k^2 - M^2 + i\delta} = \frac{1}{(k^0)^2 - |\vec{k}|^2 - M^2 + i\delta} = \frac{1}{\left(k^0 - \sqrt{|\vec{k}|^2 + M^2 - i\delta}\right)\left(k^0 + \sqrt{|\vec{k}|^2 + M^2 - i\delta}\right)}$$

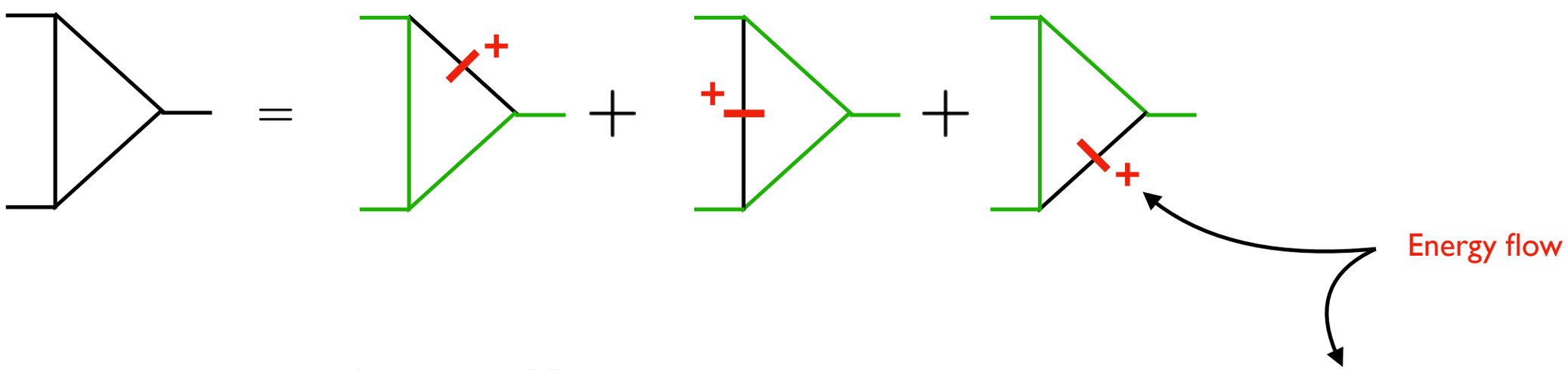
Pole selected for each propagator

Then integrate the energy component using residue theorem



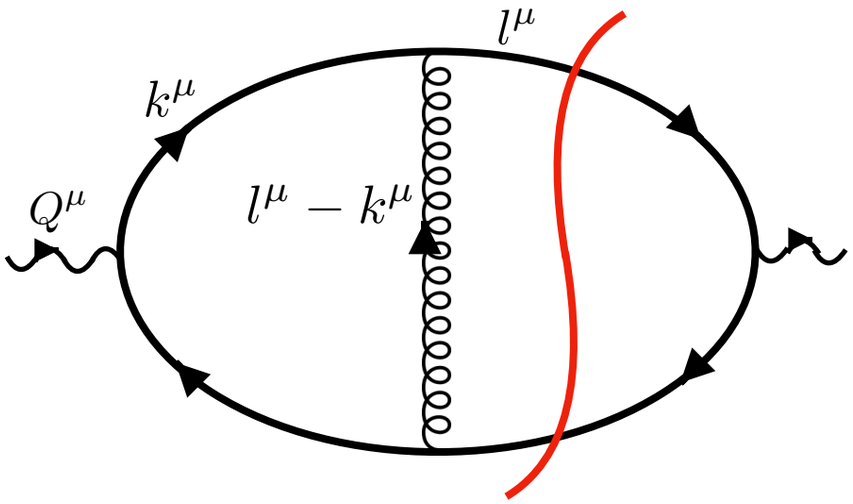
$$= \int \mathrm{d}^3\vec{k} \left[\mathrm{Res}_1 \left[\frac{N}{D_1 D_2 D_3} \right] + \mathrm{Res}_2 \left[\frac{N}{D_1 D_2 D_3} \right] + \mathrm{Res}_3 \left[\frac{N}{D_1 D_2 D_3} \right] \right]$$

Residues can be represented as cuts:



$$= \int \mathrm{d}^4 k \frac{N}{D_1 D_2 D_3} (D_1 \delta^{(+)}(D_1) + D_2 \delta^{(+)}(D_2) + D_3 \delta^{(+)}(D_3))$$

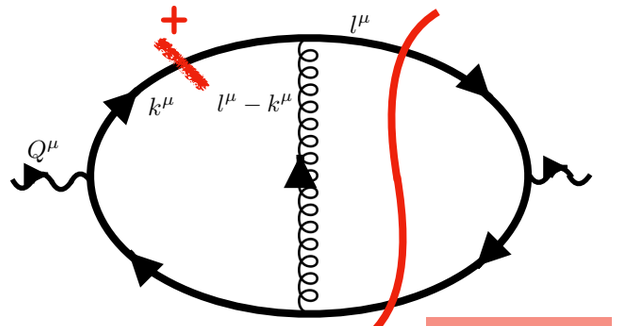
Loop-Tree Duality



$$\propto \delta^+(l^2)\ \delta^-((l-Q)^2)$$

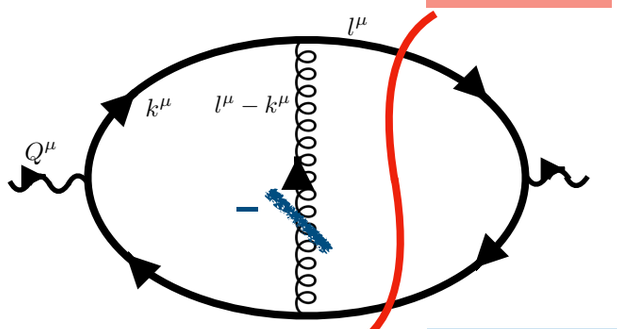
LTD

$=$



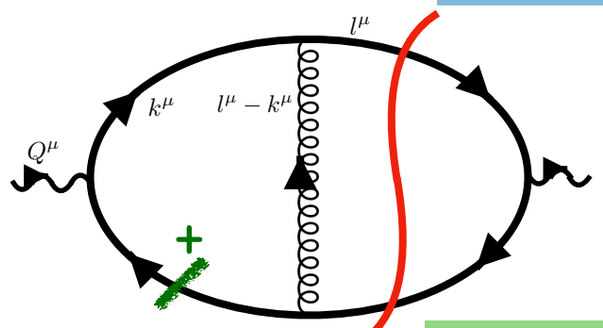
$$\propto \delta^+(l^2)\ \delta^-((l-Q)^2)\ \delta^+(k^2)$$

$+$



$$\propto \delta^+(l^2)\ \delta^-((l-Q)^2)\ \delta^+((l-k)^2)$$

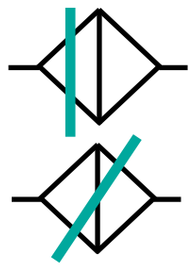
$+$



$$\propto \delta^+(l^2)\ \delta^-((l-Q)^2)\ \delta^+((k-Q)^2)$$

Causal flow

The measure now differs only in the **delta enforcing on shell energy conservation**



$$\sim \delta(E_1 + E_2 - Q_0)$$

$$\sim \delta(E_1 + E_3 + E_5 - Q_0)$$

How to solve all these deltas enforcing on shell energy conservation **with a single variable** ?

The idea:

$$\delta(|\vec{k}| - Q_0) \underset{\vec{k} \to t\vec{k}}{\longrightarrow} \delta(t|\vec{k}| - Q_0) \longrightarrow t = \frac{Q_0}{|\vec{k}|}$$

Soper,
arXiv: 9804454 (1998)

Soper,
arXiv: 0102031 (2001 @ RADCOR)

ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

General FSR cancellations
For N to M NkLO processes

A toy example:

$$\int \mathrm{d}^3\vec{k}\, \delta(|\vec{k}| - Q_0) f(\vec{k})$$

Causal flow

Causal flow

$$= \int \mathrm{d}^3\vec{k} \int \mathrm{d}t\, h(t)\, \delta(|\vec{k}| - Q_0) f(\vec{k}) \qquad \text{using} \qquad 1 = \int \mathrm{d}t\, h(t)$$

Causal flow

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$$= \int \mathrm{d}^3\vec{k}\, \frac{Q_0^3}{|\vec{k}|^4}\, h(Q_0/|\vec{k}|)\, f(Q_0\vec{k}/|\vec{k}|) \qquad \text{with} \qquad t^\star = Q_0/|\vec{k}|$$

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Solve all deltas in the common scaling variable. This completes the alignment of the measure!

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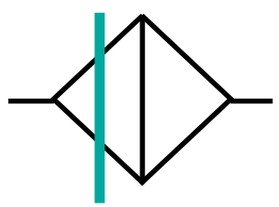
When applying this construction to LU we get:



$$= \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\,\delta(E_1 + E_2 - Q_0) f_{\text{virt}}$$

Causal flow

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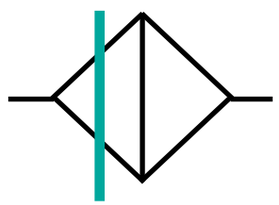

$$= \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\delta(E_1 + E_2 - Q_0) f_{\text{virt}} = \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\; g_{\text{v}}(t_{\text{v}}^\star)$$

where $t_{\text{v}}^\star = t_{\text{v}}^\star(\vec{k}, \vec{p}) = \dfrac{Q_0}{E_1 + E_2}$

Causal flow

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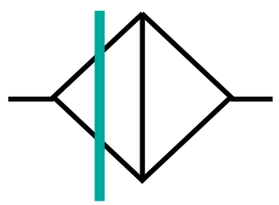

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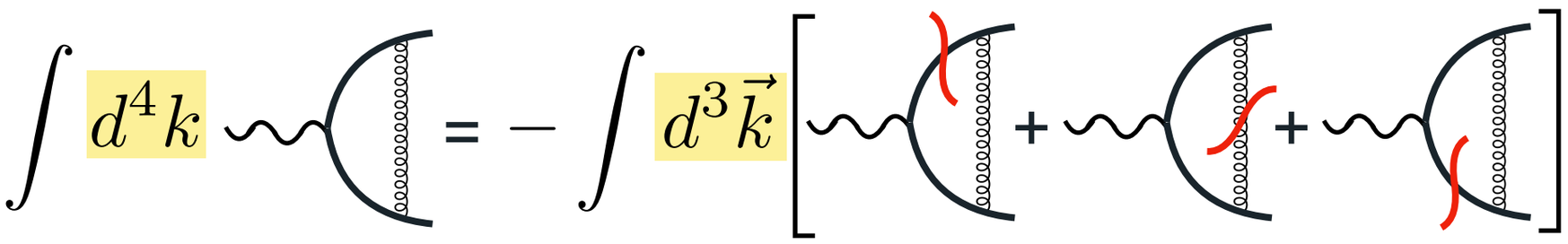
$$(\vec{p}, \vec{k}) \to \vec{\phi}(t, (\vec{p}, \vec{k})) \qquad \begin{cases} \partial_t \vec{\phi} = \vec{\kappa} \circ \vec{\phi} \\ \vec{\phi}(0, (\vec{k}, \vec{l})) = (\vec{k}, \vec{l}) \end{cases}$$

"Causal flow" is called like this because of the generalisation of the simple scaling from Soper.

Local Unitarity: ingredients

[Z. Capatti, VH, A. Pelloni, B. Ruijl, arXiv : 2010.01068] [Summary in proceedings, arXiv : 2110.15662]



Loop-Tree Duality

[arXiv: 1906.06138]

[arXiv: 1912.09291]

[arXiv: 2009.05509]

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$$\int d^4k \; \text{[diagram]} = -\int d^3\vec{k} \left[\text{[diagram]} + \text{[diagram]} + \text{[diagram]} \right]$$

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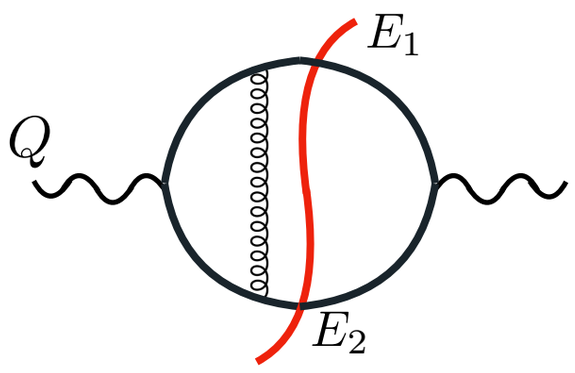
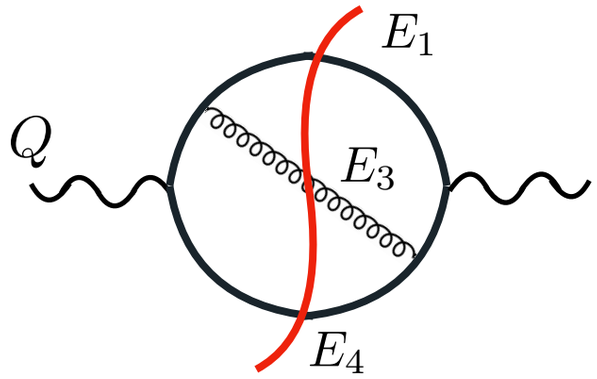


$$\propto \delta(E_1 + E_2 - Q)$$

?



$$\propto \delta(E_1 + E_3 + E_4 - Q)$$

Causal Flow

[arXiv : 2010.01068]
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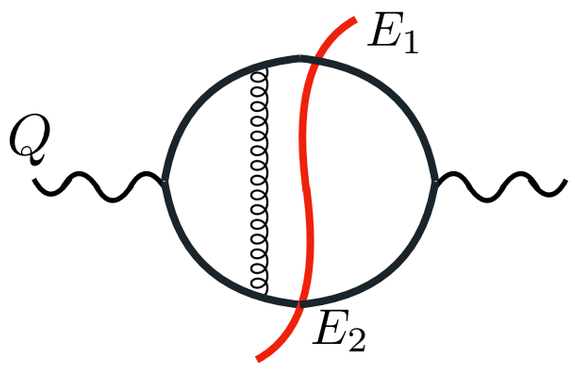

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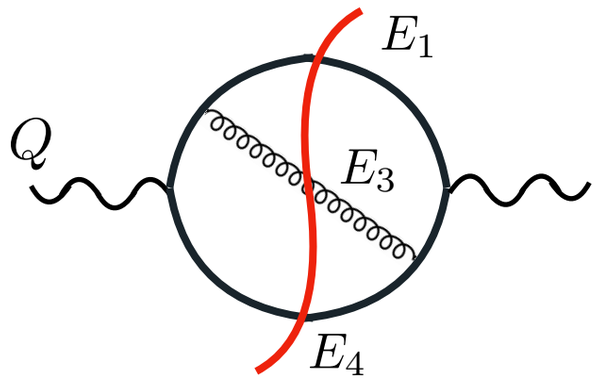


$$\propto \delta(E_1 + E_3 + E_4 - Q)$$

Causal Flow

[arXiv : 2010.01068]
[arXiv : 2110.15662]
[arXiv : 9910292]

$$\text{UV}\left[\text{[diagram: } k, l, m \text{]} \right] = \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]}$$

$$t_{klm} + t_{km} + t_{ml} + t_{kl}$$

$$- t_{klm}(t_{km}) - t_{klm}(t_{ml}) - t_{klm}(t_{kl})$$

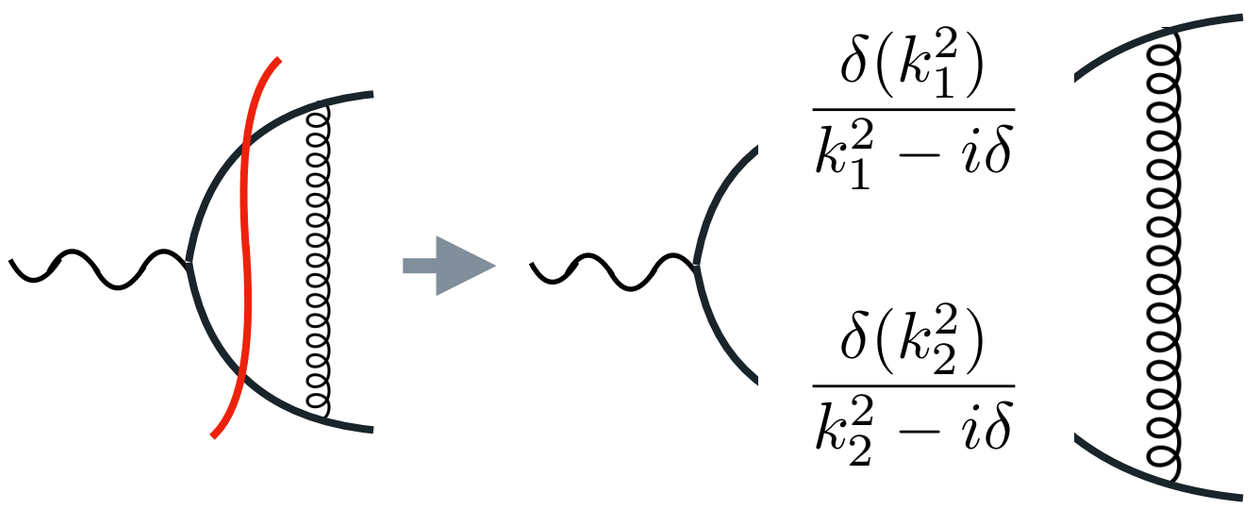
Local UV subtr.

[BPHZ refs.]

Local Unitarity: ingredients

[Z. Capatti, VH, A. Pelloni, B. Ruijl, arXiv : 2010.01068] [Summary in proceedings, arXiv : 2110.15662]

Regularising (non-pinched) threshold singularities



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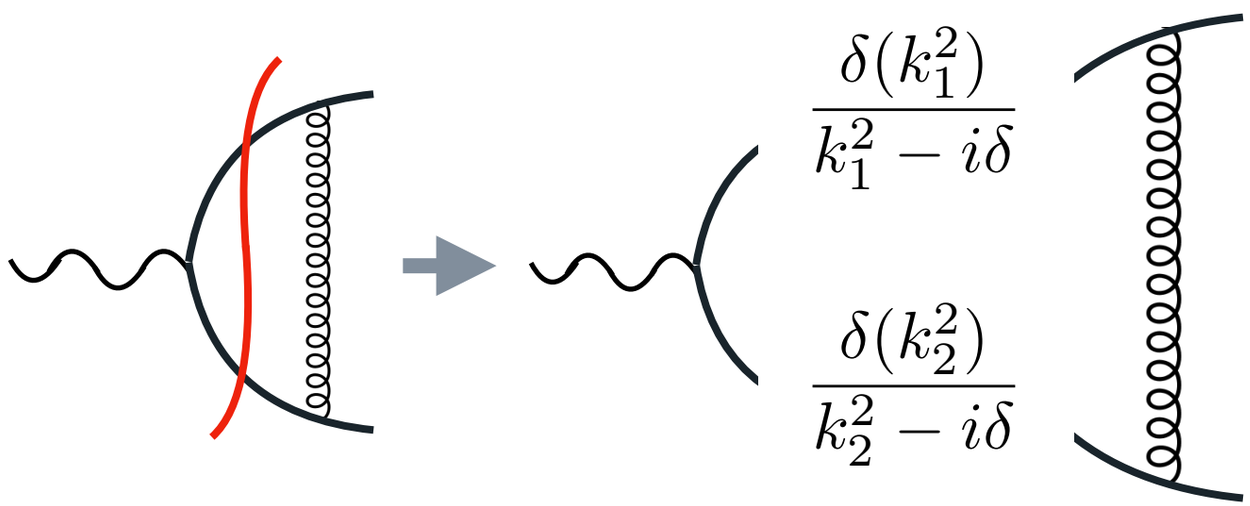
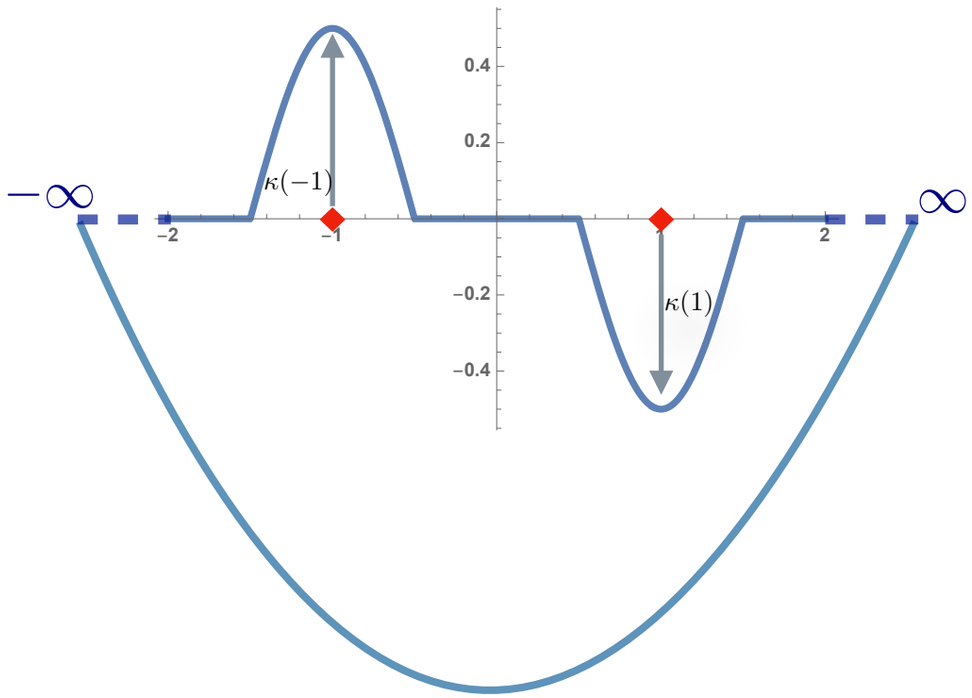


$$\int_{-\infty}^{\infty} dx I[x]$$

$$\bar{x} = x + i\kappa(x)$$

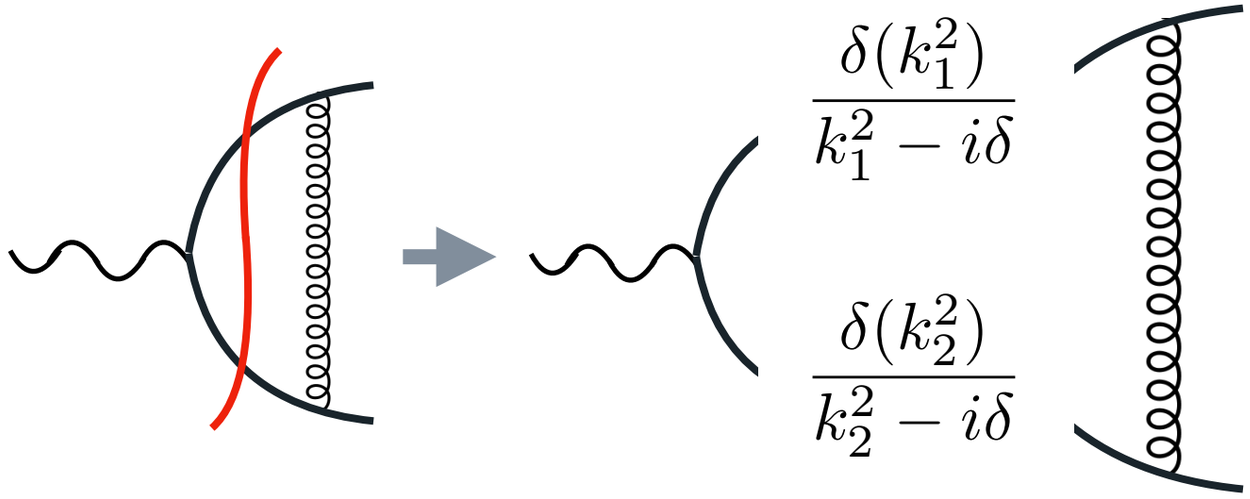
$$\int_{-\infty}^{\infty} dx \frac{d\bar{x}}{dx} I[\bar{x}(x)]$$



Local Unitarity: ingredients

[Z. Capatti, VH, A. Pelloni, B. Ruijl, arXiv : 2010.01068] [Summary in proceedings, arXiv : 2110.15662]

Regularising (non-pinched) threshold singularities



$$\frac{\delta(k_1^2)}{k_1^2 - i\delta}$$

$$\frac{\delta(k_2^2)}{k_2^2 - i\delta}$$

$$\int_{-\infty}^{\infty} dx I[x]$$

$$\bar{x} = x + i\kappa(x)$$

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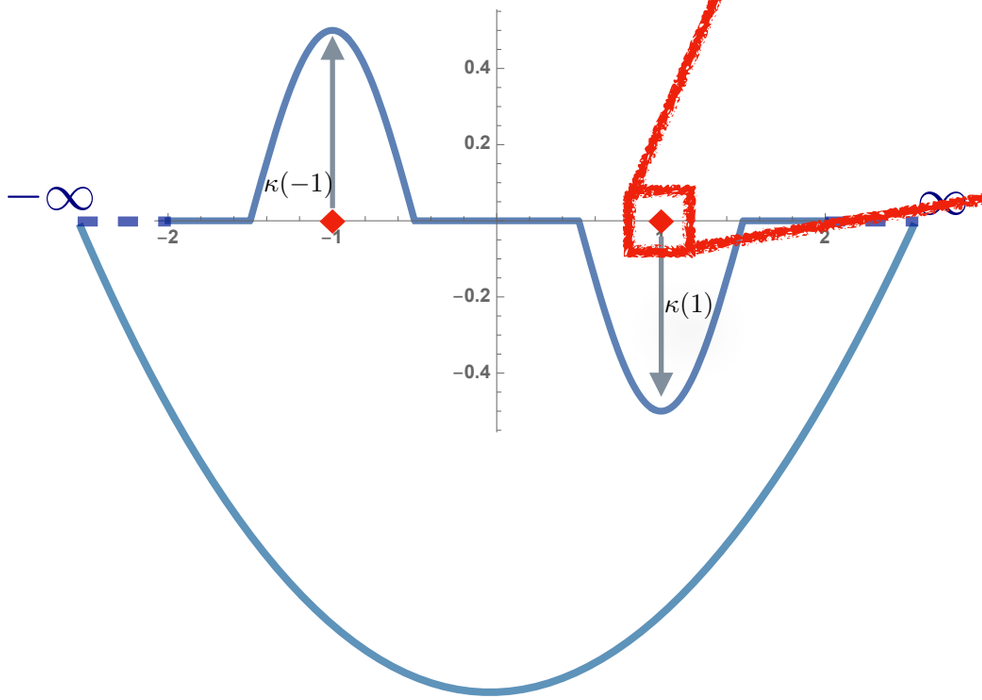
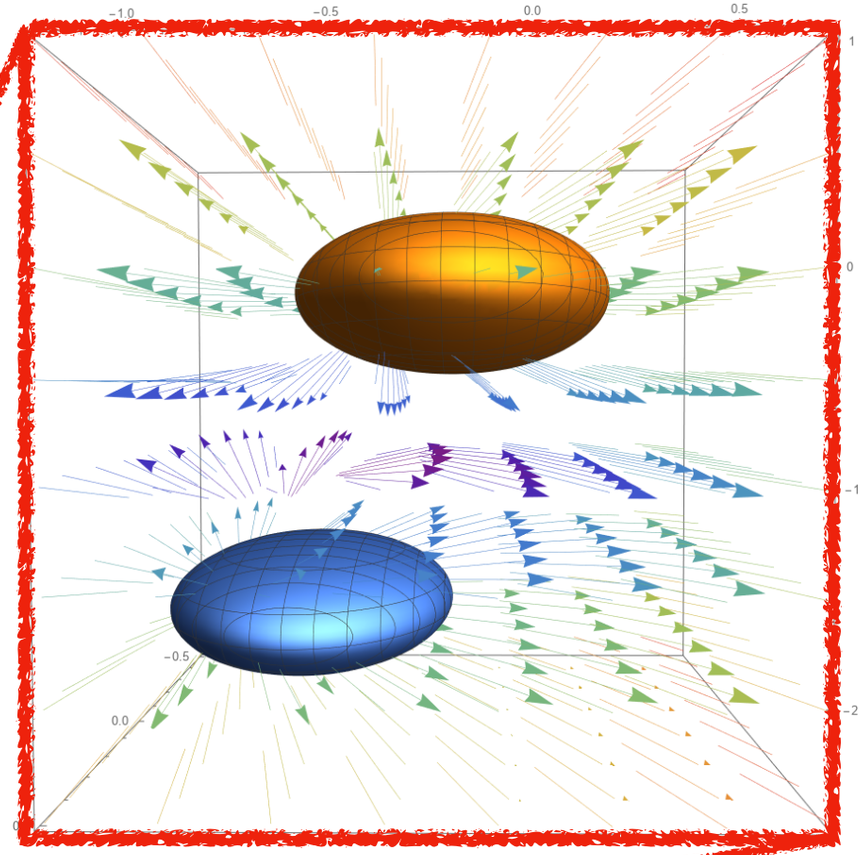


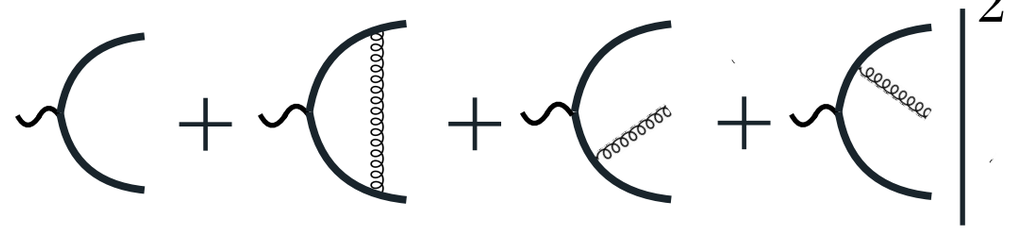


Contour deformation

[arXiv : 1912.09291]

Local Unitarity: A conceptual shift

$$\sigma^{(\text{normal})}_{\gamma^\star \to d\bar{d}} = \int \Pi^{(\text{phase-space})} \left| \; + \; + \; + \; \right|^2$$



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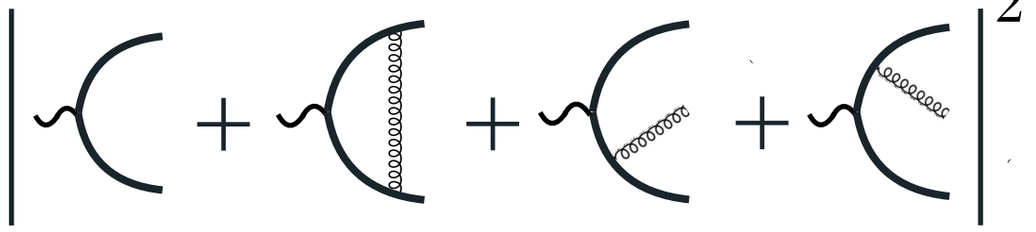
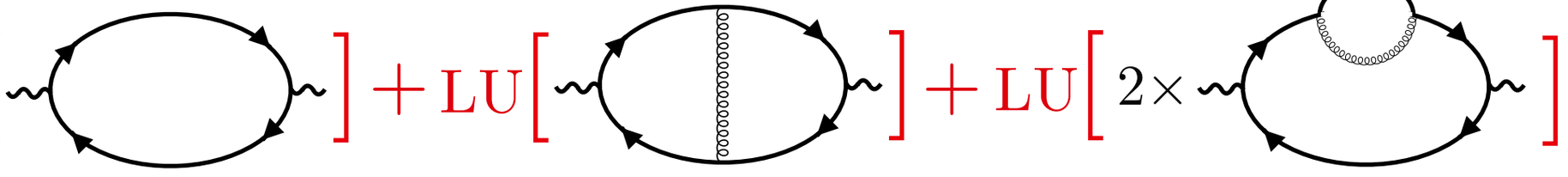


$$\sigma^{(\text{LU})}_{\gamma^\star \to d\bar{d}} = \text{LU}\Big[\quad\Big] + \text{LU}\Big[\quad\Big] + \text{LU}\Big[\,2\times\quad\Big]$$



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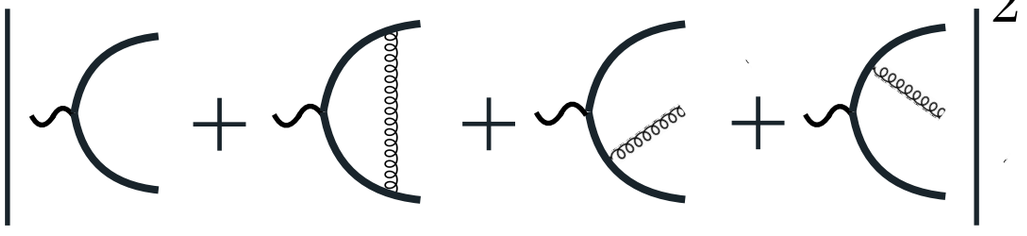

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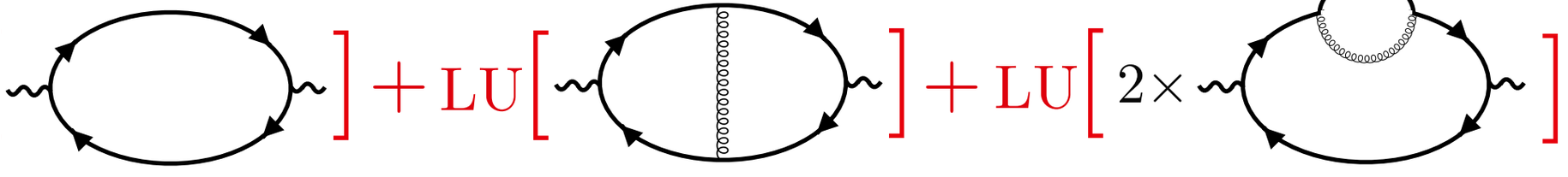

$$\sum_{c\in\{RRR,RRV,RVV,\dots\}} \int \Pi_c^{(\text{phase-space})} \left| \sum_{i_c=1}^{n_{\text{amplitudes}}(c)} \int \Pi_{i_c}^{(\text{loop})} \mathcal{A}_{i_c} \right|^2_{\text{truncated}}$$

IR-subtraction numerical $d = 4$ analytic $d = 4 - 2\epsilon$

Local Unitarity: A conceptual shift

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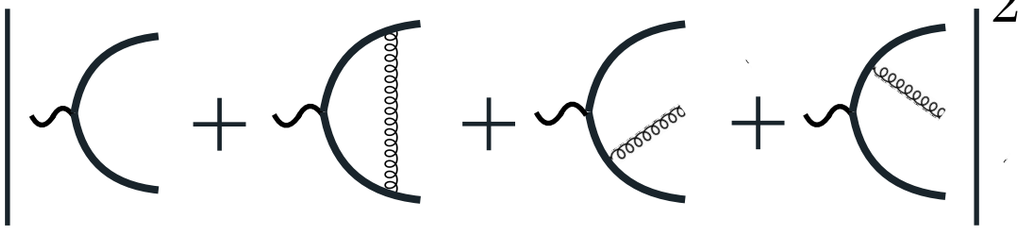

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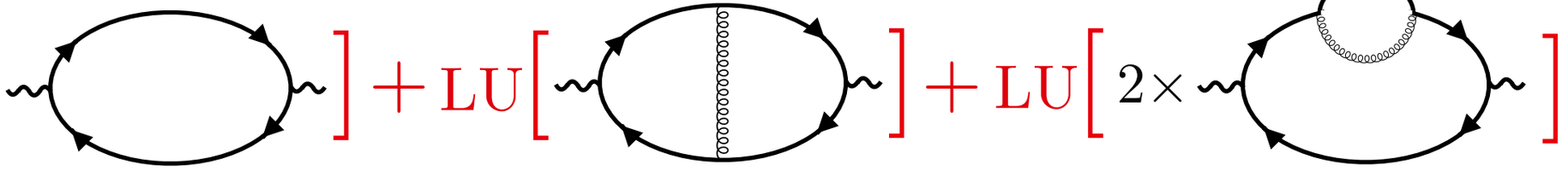

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IR-subtraction numerical $d = 4$ analytic $d = 4 - 2\epsilon$

$$\sum_{j=1}^{n_{\text{supergraphs}}} \int \Pi \; g_j^{(\text{LU})}$$

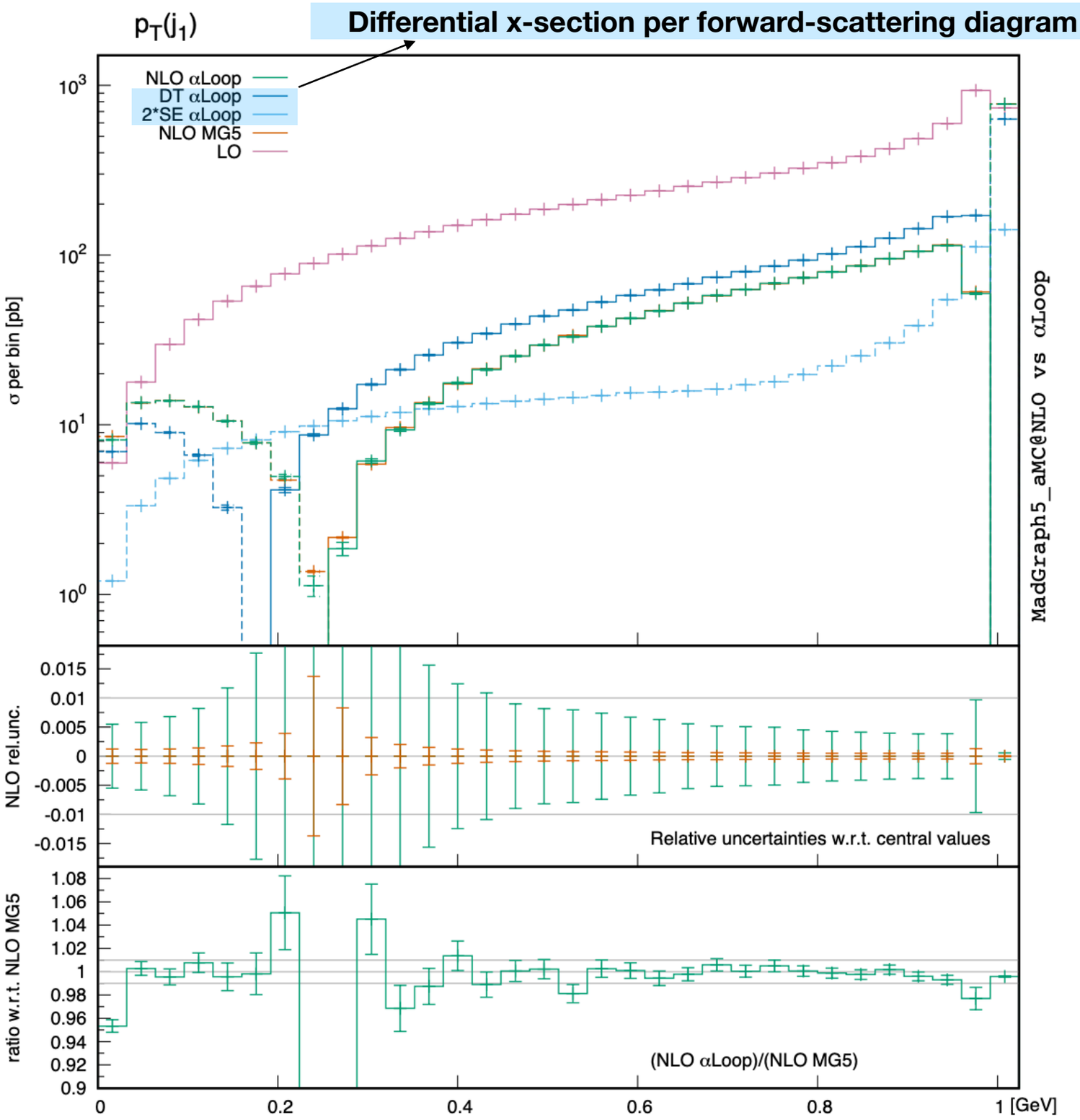
numerical $d = 4$

NO IR-subtraction

LOCAL UNITARITY

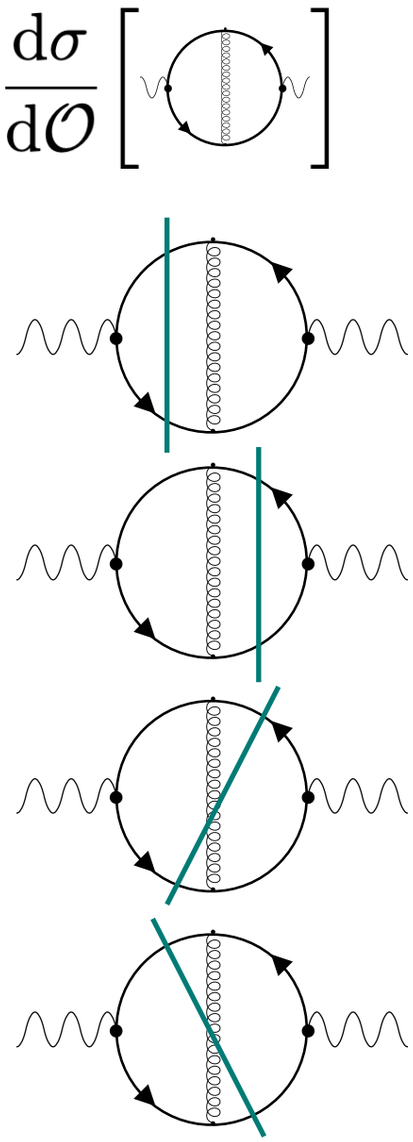
First results:

Numerical results from αLoop: Physical processes

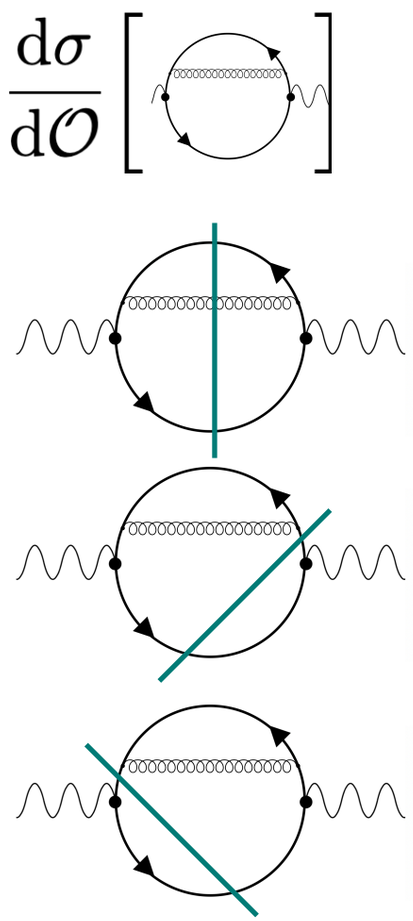


Numerical results from αLoop: Physical processes

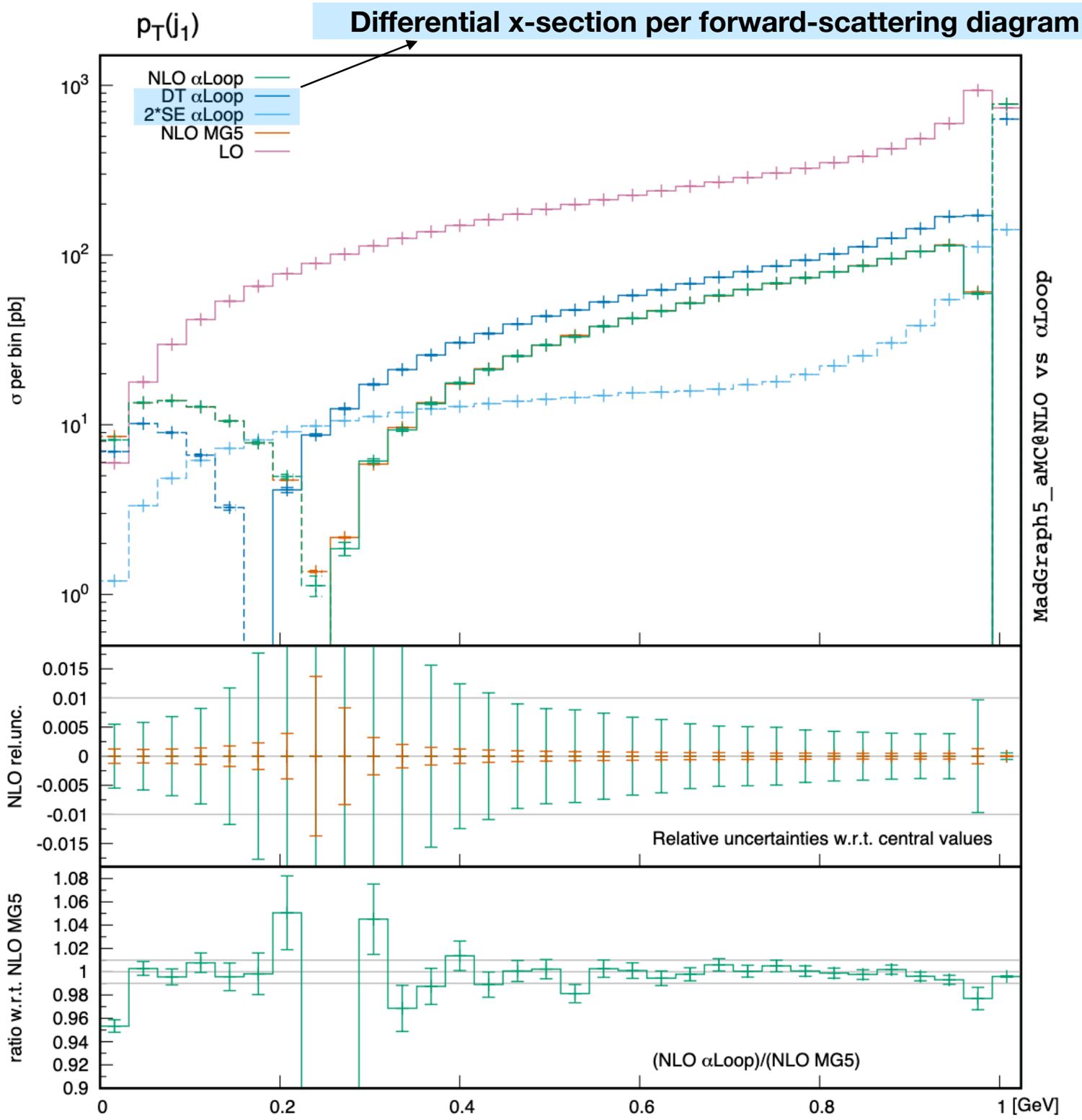
DT



SE



Differential x-section per forward-scattering diagram



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	name	multiplicity	neval	real	real_err	eval_time
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2	SG_QG6	1.000000e+00	2080000	-8.346356e-05	9.293410e-08	0 days 00:00:53.087342000
3	SG_QG7	1.000000e+00	2310000	-8.339337e-05	8.996056e-08	0 days 00:01:03.678127000
4	SG_QG10	2.000000e+00	2550000	-8.387958e-05	9.418465e-08	0 days 00:01:57.402755000
5	SG_QG11	2.000000e+00	35250000	1.042949e-04	9.357861e-08	0 days 02:05:01.854560000
6	SG_QG14	2.000000e+00	2080000	1.358842e-05	3.100517e-08	0 days 00:01:07.722351000
7	SG_QG15	2.000000e+00	5850000	-6.903777e-05	9.267193e-08	0 days 00:01:30.960251000
8	SG_QG18	2.000000e+00	2080000	1.358842e-05	3.100517e-08	0 days 00:01:06.815650000
9	SG_QG19	2.000000e+00	5850000	-6.903777e-05	9.267193e-08	0 days 00:01:31.330903000
10	SG_QG38	2.000000e+00	2550000	-5.948191e-05	9.390897e-08	0 days 00:01:10.004909000
11	SG_QG39	2.000000e+00	2800000	-5.930572e-05	9.044551e-08	0 days 00:01:13.862147000
12	SG_QG42	2.000000e+00	2080000	-8.904994e-06	7.817902e-08	0 days 00:00:25.930139000
13	SG_QG43	2.000000e+00	2310000	-8.718230e-06	9.172632e-08	0 days 00:00:29.550519000
14	SG_QG46	2.000000e+00	2080000	3.534058e-05	3.903003e-08	0 days 00:00:59.076110000
15	SG_QG47	2.000000e+00	2080000	-1.618672e-06	1.686635e-09	0 days 00:00:09.248204000
16	renorm_SG_QG2DS2	2.000000e+00	2080000	-6.505343e-06	5.163914e-09	0 days 00:00:03.773708000
17	renorm_SG_QG6DS3	1.000000e+00	2080000	8.328622e-06	3.274328e-09	0 days 00:00:03.594100000
18	renorm_SG_QG7DS2	1.000000e+00	2080000	8.327790e-06	3.432804e-09	0 days 00:00:03.593342000
19	renorm_SG_QG10DS2	2.000000e+00	2080000	3.574834e-05	1.473165e-08	0 days 00:00:03.439091000
20	renorm_SG_QG14DS2	2.000000e+00	2080000	-5.381537e-07	1.960019e-09	0 days 00:00:06.044348000
21	renorm_SG_QG15DS2	2.000000e+00	2080000	6.505343e-06	5.163914e-09	0 days 00:00:02.293894000
22	renorm_SG_QG18DS2	2.000000e+00	2080000	-5.381537e-07	1.960019e-09	0 days 00:00:05.866005000
23	renorm_SG_QG19DS2	2.000000e+00	2080000	6.505343e-06	5.163914e-09	0 days 00:00:02.299732000
24	renorm_SG_QG38DS2	2.000000e+00	2080000	2.787320e-05	1.661677e-08	0 days 00:00:02.324085000
25	renorm_SG_QG39DS2	2.000000e+00	2080000	2.788824e-05	2.280913e-08	0 days 00:00:02.305769000
26	renorm_SG_QG42DS1	2.000000e+00	2080000	9.137823e-06	9.422893e-09	0 days 00:00:02.835274000
27	renorm_SG_QG43DS1	2.000000e+00	2080000	9.150816e-06	1.231864e-08	0 days 00:00:02.819873000
28	renorm_SG_QG46DS2	2.000000e+00	2080000	-6.501564e-06	3.892669e-09	0 days 00:00:04.103701000

```
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MG res:        -1.38400e-04 +/- 1.4e-07
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```

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6	SG_QG14					
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27	renorm_SG_QG43DS1	2.000000e+00	2080000	9.150816e-06	1.231864e-08	0 days 00:00:02.819873000
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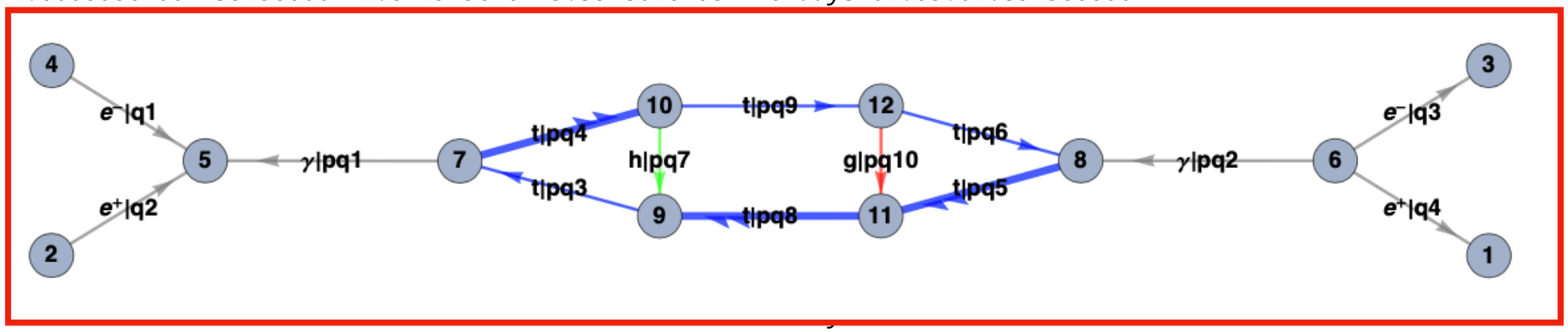

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15	SG_QG47					
16	renorm_SG_QG2DS2					
17	renorm_SG_QG6DS3					
18	renorm_SG_QG7DS2					
19	renorm_SG_QG10DS2					
20	renorm_SG_QG14DS2					
21	renorm_SG_QG15DS2					
22	renorm_SG_QG18DS2					
23	renorm_SG_QG19DS2					
24	renorm_SG_QG38DS2					
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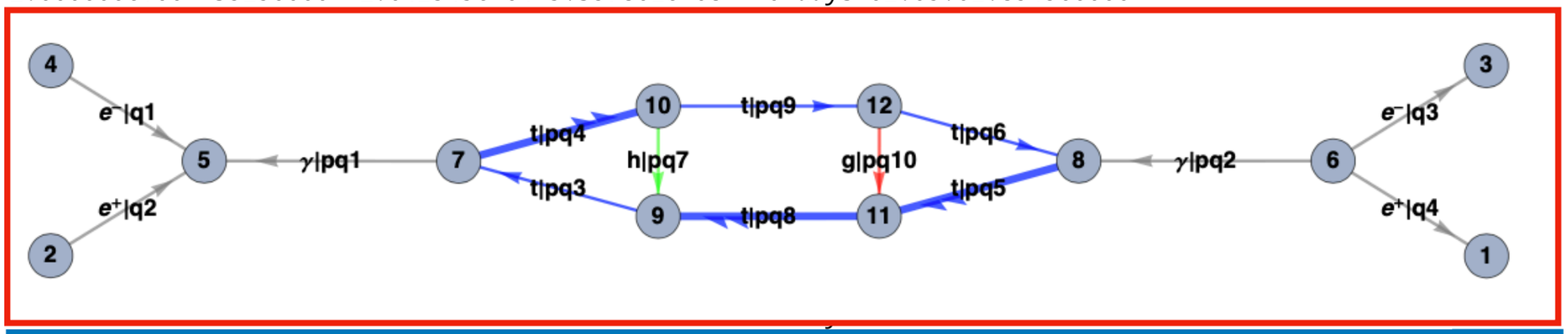

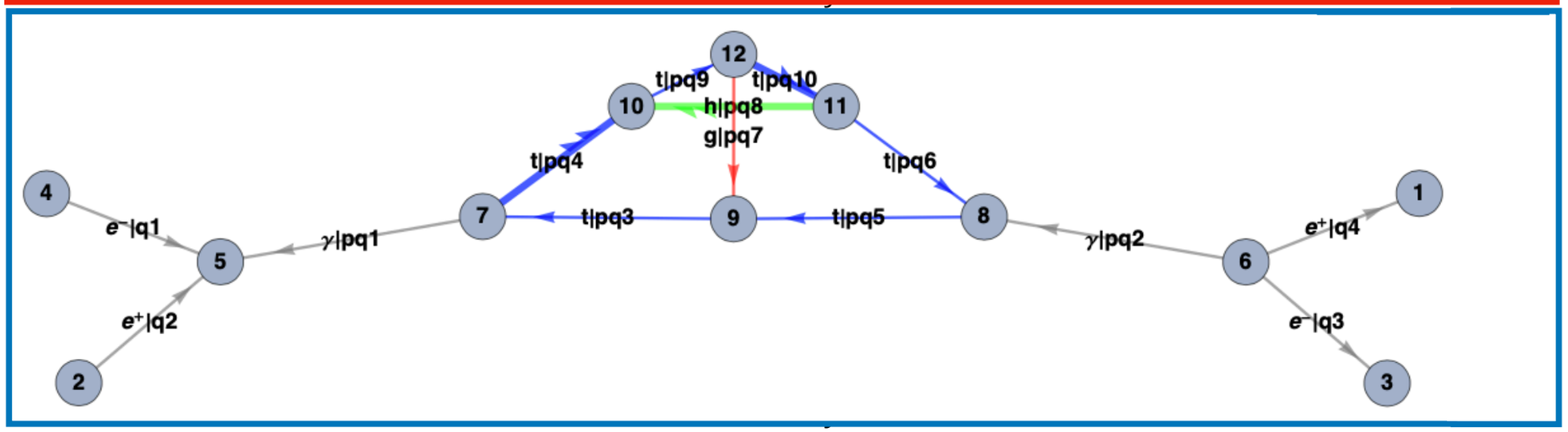


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11	SG_QG39					
12	SG_QG42					
13	SG_QG43					
14	SG_QG46					
15	SG_QG47					
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17	renorm_SG_QG6DS3					
18	renorm_SG_QG7DS2					
19	renorm_SG_QG10DS2					
20	renorm_SG_QG14DS2					
21	renorm_SG_QG15DS2					
22	renorm_SG_QG18DS2					
23	renorm_SG_QG19DS2					
24	renorm_SG_QG38DS2					
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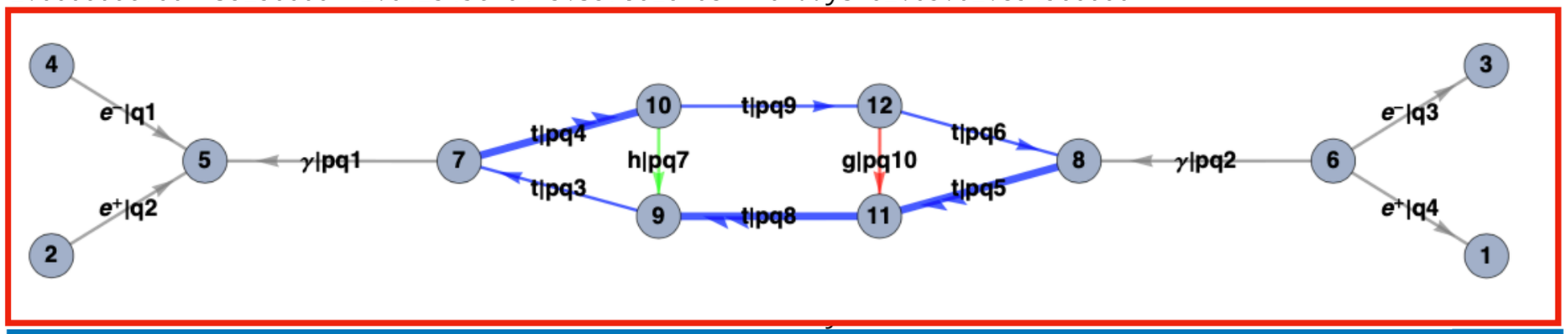
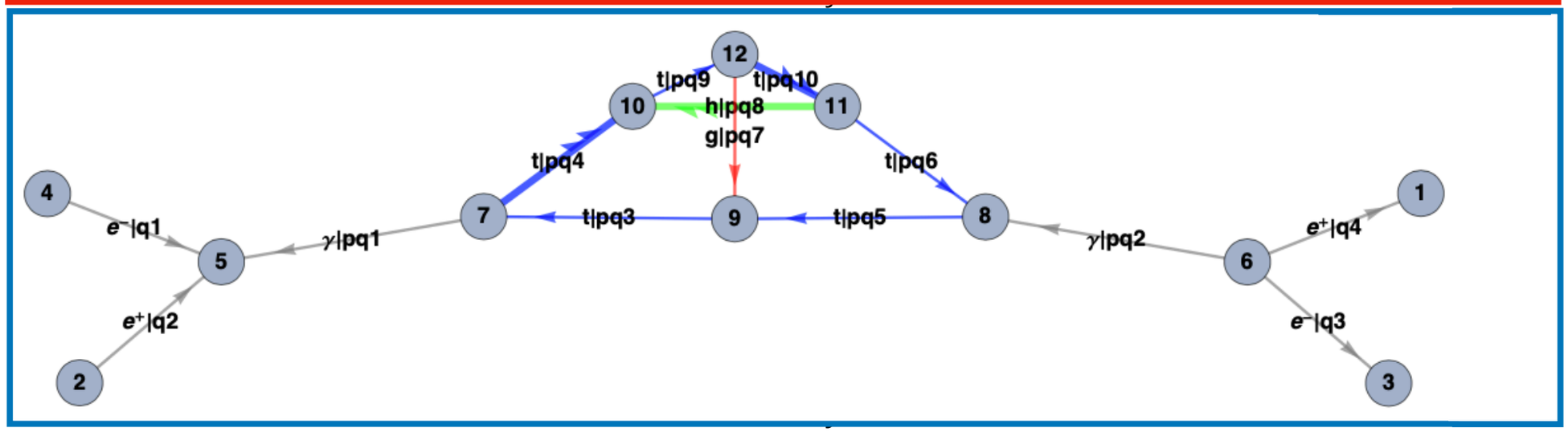
26	renorm_SG_QG42DS1					
27	renorm_SG_QG43DS1					
28	renorm_SG_QG46DS2					



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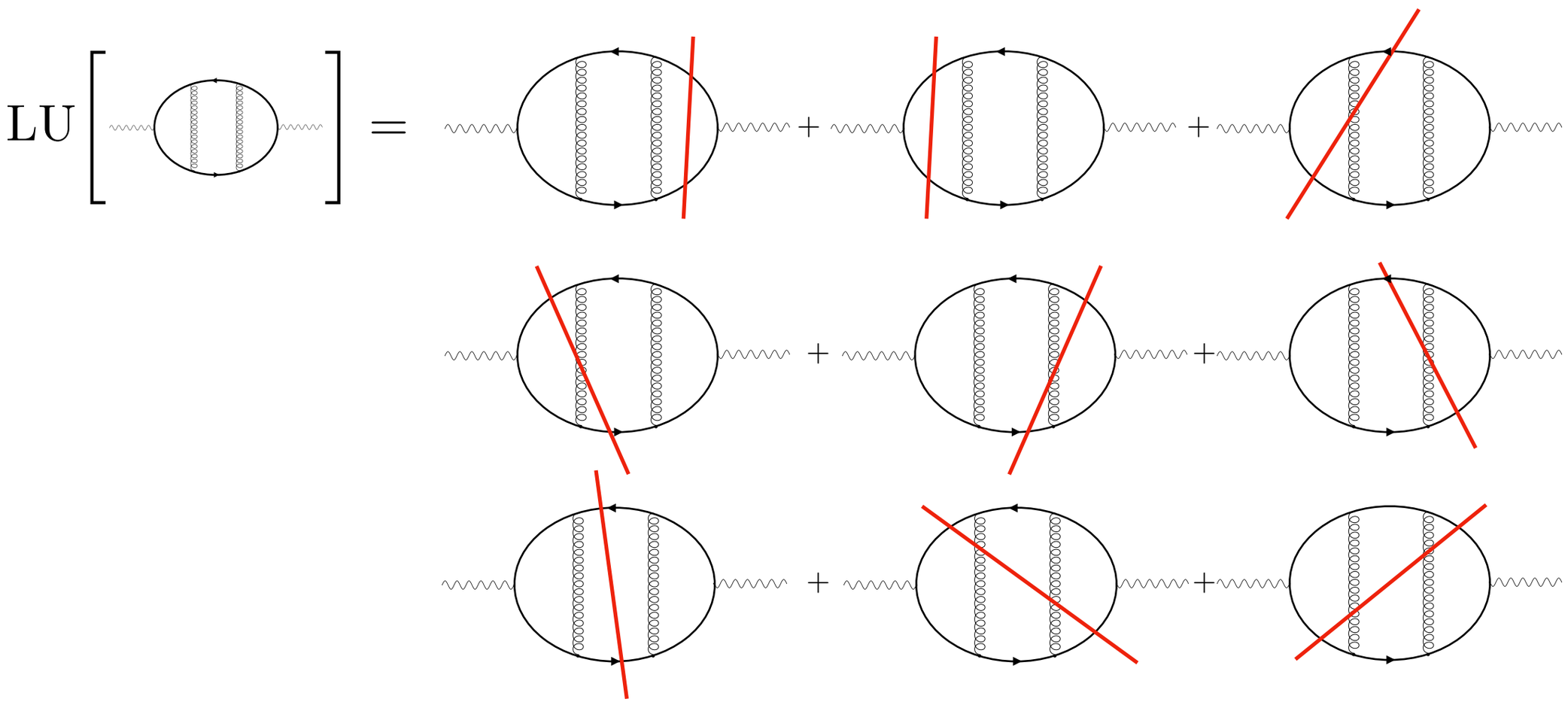
```

Two vastly different approaches collapsing to the same number

Numerical results from αLoop : ongoing

$$e^+e^- \to \gamma^\star \to t\bar{t} \text{ @ NNLO}$$

Example of one super-graph:



And there are 19 supergraphs in total, all automatically generated and renormalised in MSbar

Numerical results from αLoop : ongoing

`./bin/mg5_aMC --mode=alphaloop epem_a_ttx_NNLO.aL`

```
import model aL_sm-no_widths
set_alphaLoop_option apply_graph_isomorphisms True
set_alphaLoop_option qgraf_template_model epem
set_FORM_option cores 10
set_alphaLoop_option qgraf_model SM_no_aqq
set_FORM_option generate_integrated_UV_CTs True
set_FORM_option generate_renormalisation_graphs False
set_alphaLoop_option FORM_compile_optimization 0
set_alphaLoop_option FORM_integrand_type PF
#
qgraf_define j = d d~ g gh gh~
qgraf_generate e+ e- > a > t t~ / u s c b QCD^2==2 QED^2==2 [QCD QCD]
output qgraf EPEM_A_TTBAR_NNLO
#
launch -i EPEM_A_TTBAR_NNLO
set_hyperparameter Selectors.active_selectors []
set_hyperparameter CrossSection.incoming_momenta [[200.0,0.,0.,200.0],[200.0,0.,0.,-200.0]]
set_hyperparameter CrossSection.picobarns True
set_hyperparameter General.deformation_strategy "none"
[...]
integrate all \
-i havana -bs 20000 --n_start 10000000 --n_max 1000000000000 --n_increase 0 \
-nw 40 -c 1 --cluster_type local \
--run_id 1000 --use_redis --max_iteration_time 108000.0 --redis_max_job_time 18000 --havana_integrand_phase real \
--bulk_redis_enqueuing 200 \
--run_description epem_a_ttx_nnlo_exploration_MC_with_DS_FIXED_16_11_2021 \
--show_selected_phase_only \
--no_use_optimal_channels
```

Numerical results from αLoop : ongoing

```
./bin/mg5_aMC --mode=alphaloop epem_a_ttx_NNLO.aL
```

```
import model aL_sm-no_widths
set_alphaLoop_option apply_graph_isomorphisms True
set_alphaLoop_option qgraf_template_model epem
set_FORM_option cores 10
set_alphaLoop_option qgraf_model SM_no_aqq
set_FORM_option generate_integrated_UV_CTs True
set_FORM_option generate_renormalisation_graphs False
set_alphaLoop_option FORM_compile_optimization 0
set_alphaLoop_option FORM_integrand_type PF
#
qgraf_generate e+ e- > a > t t~ / u s c b QCD^2==2 QED^2==2 [QCD QCD]
#
launch -i EPEM_A_TTBAR_NNLO
set_hyperparameter Selectors.active_selectors []
set_hyperparameter CrossSection.incoming_momenta [[200.0,0.,0.,200.0],[200.0,0.,0.,-200.0]]
set_hyperparameter CrossSection.picobarns True
set_hyperparameter General.deformation_strategy "none"
[...]
integrate all
-t havana -ds 20000 --n_start 10000000 --n_max 1000000000000 --n_increase 0 \
-nw 40 -c 1 --cluster_type local \
--run_id 1000 --use_redis --max_iteration_time 108000.0 --redis_max_job_time 18000 --havana_integrand_phase real \
--bulk_redis_enqueuing 200 \
--run_description epem_a_ttx_nnlo_exploration_MC_with_DS_FIXED_16_11_2021 \
--show_selected_phase_only \
--no_use_optimal_channels
```

Numerical results from αLoop : ongoing

First NNLO result in pure MSbar scheme "from MadGraph": VERY PRELIMINARY

Live integration monitoring dashboard: $e^+e^- \to \gamma^\star \to t\bar{t}$ @ NNLO

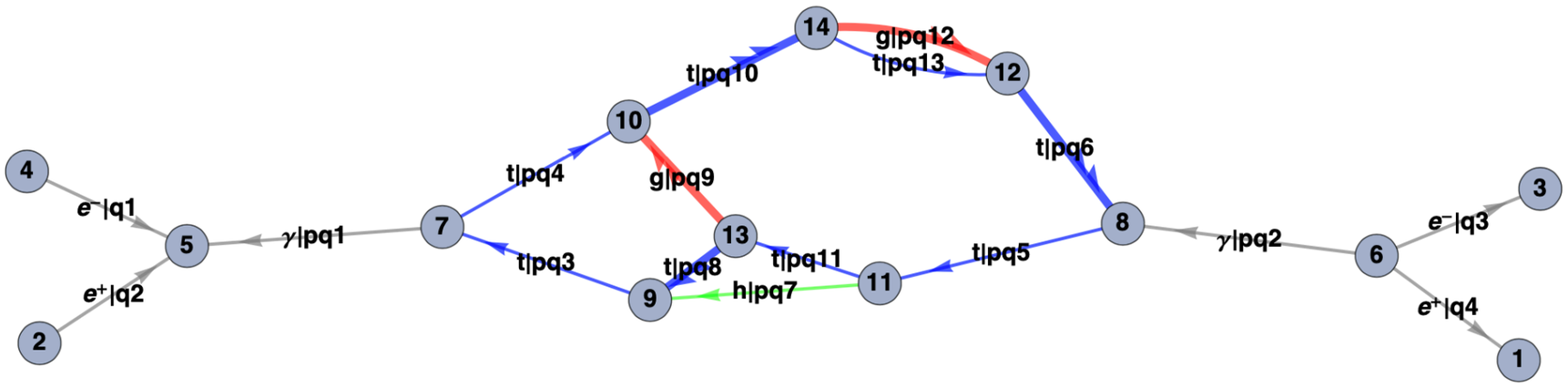
```
madgraph.havana 2021-11-17,00:16:17.401398 Now running iteration #6
| Status of 40 workers: 0 pending, 4 available, 36 running.
| Status of 36 jobs: 0 scheduled, 36 submitted for this iteration, 36 started, 0 finished and not processed
| Jobs completed: overall = 1164 (avg 35.6 min/job), and for this iteration = 164/200/200 on 40 cpus.
| Total n_pts: 11.6M ( 227 ms / pt on one core,  0.2K pts / s overall ). n_pts for this iteration #6: 1.64M/2.00M (82.0%).
| Wall time: 18.370 h, IO: 0.00%, processing: 0.00%, jobs: 691.328 CPU-hours, efficiency: 94.09%

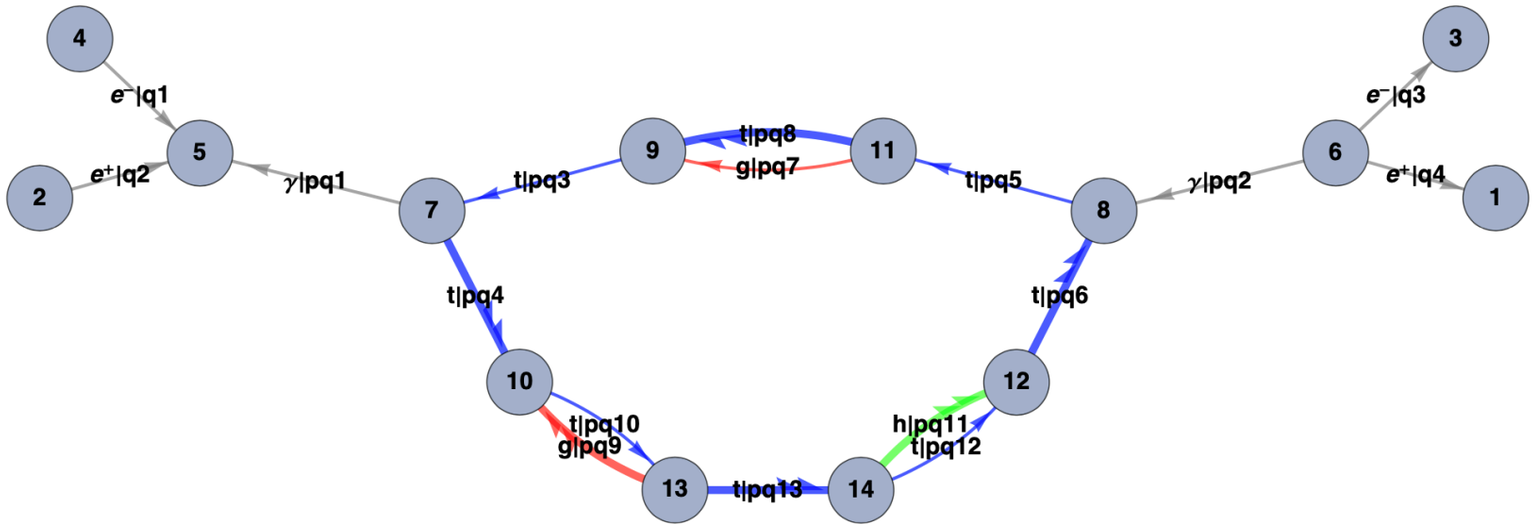
Run description: epem_a_ttx_nnlo_exploration_MC_no_DS_FIXED_16_11_2021 (run id: 2000)
Result after 11.6M evaluations and 5 iterations: 0.00690671 +/- 0.0001409 (2%), chi2=0.237, max_wgt_infl=0.162, zero_evals=0.0152%
  -> Re[I0(epem_a_ttx_NNLO_mUVxOne_MC )] = 0.0069067 +/- 0.00014086 (2%), chi2=1.19, max_wgt_infl=0.162, zero_evals=0.0152%
     Im[I0(epem_a_ttx_NNLO_mUVxOne_MC )] = 0 +/- 6.0834e-09 (0%), chi2=0, max_wgt_infl=0, zero_evals=100%
     Re[I1(epem_a_ttx_NNLO_mUVxHalf_MC)] = 0.0068684 +/- 0.00011846 (1.7%), chi2=0.976, max_wgt_infl=0.149, zero_evals=0.0154%
     Im[I1(epem_a_ttx_NNLO_mUVxHalf_MC)] = 0 +/- 6.0834e-09 (0%), chi2=0, max_wgt_infl=0, zero_evals=100%
     Re[I2(epem_a_ttx_NNLO_mUVxTwo_MC )] = 0.0069117 +/- 0.0002138 (3.1%), chi2=4.03, max_wgt_infl=0.111, zero_evals=0.0151%
     Im[I2(epem_a_ttx_NNLO_mUVxTwo_MC )] = 0 +/- 6.0834e-09 (0%), chi2=0, max_wgt_infl=0, zero_evals=100%
+---------+-----------------------------------------------------------------------------+-----------------------------------+-----------------------------------+
| Result  | Re[I0(epem_a_ttx_NNLO_mUVxOne_MC)]                                          | Re[I1(epem_a_ttx_NNLO_mUVxHalf_MC)] | Re[I2(epem_a_ttx_NNLO_mUVxTwo_MC)] |
+---------+-----------------------------------------------------------------------------+-----------------------------------+-----------------------------------+
| Total   | +0.00690671  +- 0.000141 (2%)    chi2=0.237|mwi=0.162|n=11.6M(1e+02%)|n0=0.0M(0.015%)  | +0.00686844  +- 0.000118 (1.7%)   | +0.00691174  +- 0.000214 (3.1%)   |
|         |                                                                             | +0.00683482  +- 0.000128 (1.9%)   | +0.0068886   +- 0.000233 (3.4%)   |
| SG_QG36 | -0.000281572 +- 6.49e-05 (23%)   | chi2=2.65|mwi=0.13|n=1.0M(8.5%)|n0=0.0M(0.008%)|p=11%  | -0.000258307 +- 5.02e-05 (19%)    | -0.00034488  +- 0.000106 (31%)    |
| SG_QG11 | +0.00478977  +- 5.18e-05 (1.1%)  | chi2=1.14|mwi=0.299|n=0.9M(7.8%)|n0=0.0M(0%)|p=9.1%    | +0.00478695  +- 4.23e-05 (0.88%)  | +0.00477191  +- 7.17e-05 (1.5%)   |
| SG_QG9  | +0.00471363  +- 5.17e-05 (1.1%)  | chi2=0.223|mwi=0.235|n=0.9M(7.8%)|n0=0.0M(0.00022%)|p=9.1% | +0.0047279   +- 4.22e-05 (0.89%)  | +0.00465241  +- 7.17e-05 (1.5%)   |
| SG_QG1  | +0.00267462  +- 4.92e-05 (1.8%)  | chi2=0.541|mwi=0.119|n=0.8M(7.1%)|n0=0.0M(0.00024%)|p=8.7% | +0.00264667  +- 3.09e-05 (1.2%)   | +0.00274466  +- 9.53e-05 (3.5%)   |
| SG_QG35 | -0.00111979  +- 4.66e-05 (4.2%)  | chi2=1.73|mwi=0.0853|n=0.7M(6.1%)|n0=0.0M(0.0053%)|p=8.2%  | -0.0011406   +- 4.46e-05 (3.9%)   | -0.00110935  +- 5.85e-05 (5.3%)   |
| SG_QG33 | +0.0040796   +- 4.55e-05 (1.1%)  | chi2=1.32|mwi=0.184|n=0.8M(6.6%)|n0=0.0M(0.1%)|p=8%      | +0.00405814  +- 3.33e-05 (0.82%)  | +0.00414805  +- 8.3e-05  (2%)     |
| SG_QG29 | +0.00331927  +- 4.19e-05 (1.3%)  | chi2=1.86|mwi=0.079|n=0.6M(5.6%)|n0=0.0M(0.0036%)|p=7.4%  | +0.00330099  +- 4.3e-05  (1.3%)   | +0.00327731  +- 6.34e-05 (1.9%)   |
| SG_QG6  | -0.0101491   +- 3.9e-05  (0.38%) | chi2=0.591|mwi=0.211|n=0.7M(5.9%)|n0=0.0M(0%)|p=6.9%     | -0.0101428   +- 3.33e-05 (0.33%)  | -0.0101444   +- 5.25e-05 (0.52%)  |
| SG_QG3  | -0.00306697  +- 3.73e-05 (1.2%)  | chi2=0.937|mwi=0.297|n=0.7M(6%)|n0=0.0M(0%)|p=6.6%      | -0.00306724  +- 3.74e-05 (1.2%)   | -0.00306564  +- 4.04e-05 (1.3%)   |
| SG_QG13 | -0.00124539  +- 3.12e-05 (2.5%)  | chi2=1.34|mwi=0.29|n=0.6M(4.8%)|n0=0.0M(0.018%)|p=5.5%  | -0.00123257  +- 2.76e-05 (2.2%)   | -0.00120716  +- 3.82e-05 (3.2%)   |
| SG_QG38 | +0.000175632 +- 2.04e-05 (12%)   | chi2=0.264|mwi=0.148|n=0.4M(3.1%)|n0=0.0M(0.0022%)|p=3.6% | +0.000180325 +- 1.48e-05 (8.2%)   | +0.000172404 +- 3.24e-05 (19%)    |
| SG_QG37 | +4.23273e-05 +- 1.85e-05 (44%)   | chi2=1.07|mwi=0.208|n=0.3M(2.9%)|n0=0.0M(0.0044%)|p=3.3%  | +4.76648e-05 +- 1.35e-05 (28%)    | +4.50705e-05 +- 2.84e-05 (63%)    |
| SG_QG39 | -0.00010226  +- 1.74e-05 (17%)   | chi2=0.19|mwi=0.137|n=0.3M(2.8%)|n0=0.0M(0.21%)|p=3.1%   | -0.000108084 +- 1.69e-05 (16%)    | -9.36485e-05 +- 2.71e-05 (29%)    |
| SG_QG30 | +0.000897469 +- 1.12e-05 (1.2%)  | chi2=1.06|mwi=0.319|n=0.2M(2%)|n0=0.0M(0.0039%)|p=2%     | +0.000901048 +- 1.09e-05 (1.2%)   | +0.000895013 +- 1.35e-05 (1.5%)   |
| SG_QG15 | +0.000284376 +- 1.11e-05 (3.9%)  | chi2=0.984|mwi=0.313|n=0.2M(2.1%)|n0=0.0M(0.0028%)|p=2%    | +0.000284718 +- 9.54e-06 (3.4%)   | +0.000286081 +- 1.34e-05 (4.7%)   |
| SG_QG5  | +0.00122644  +- 8.83e-06 (0.72%) | chi2=1.76|mwi=0.511|n=0.2M(1.8%)|n0=0.0M(0%)|p=1.6%     | +0.00122225  +- 8.86e-06 (0.73%)  | +0.0012331   +- 9.05e-06 (0.73%)  |
| SG_QG16 | +0.000931088 +- 8.09e-06 (0.87%) | chi2=1.01|mwi=0.325|n=0.2M(1.7%)|n0=0.0M(0.0025%)|p=1.4%  | +0.000931158 +- 8.15e-06 (0.88%)  | +0.000930464 +- 9.55e-06 (1%)     |
| SG_QG14 | +6.68456e-05 +- 7.96e-06 (12%)   | chi2=0.52|mwi=0.46|n=0.2M(1.8%)|n0=0.0M(0.0029%)|p=1.4%  | +6.69943e-05 +- 6.82e-06 (10%)    | +6.53915e-05 +- 9.58e-06 (15%)    |
| SG_QG0  | -0.000369437 +- 5.86e-06 (1.6%)  | chi2=1|mwi=0.398|n=0.2M(1.5%)|n0=0.0M(0%)|p=1%         | -0.000370376 +- 5.69e-06 (1.5%)   | -0.000368211 +- 6.07e-06 (1.6%)   |
+---------+-----------------------------------------------------------------------------+-----------------------------------+-----------------------------------+
```

No IR subtraction and no "manual" renormalisation !

Numerical results from αLoop : Prospects

In anticipation of: $e^+e^- \to \gamma^\star \to t\bar{t}H$ @ NNLO QCD

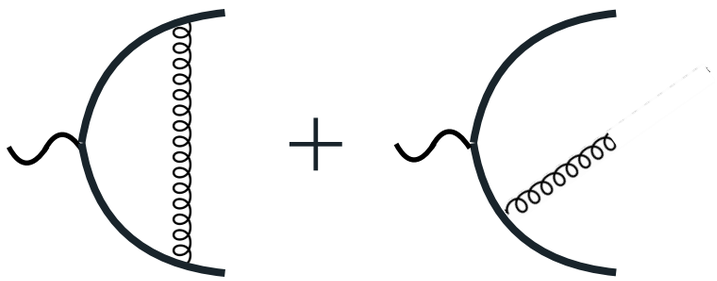




which are two examples from the $\mathcal{O}(200)$ contributing supergraphs

Local Unitarity: prospects ISR

Support for initial-state singularities:



$$\sum_{\text{final states}} \text{cuts} \quad \text{is finite !}$$

Local Unitarity: prospects ISR

Support for initial-state singularities:

$\sum_{\text{final states}}^{\text{cuts}}$ is finite ! …so logically… $\sum_{\text{initial states}}^{\text{cuts}}$ is finite too ?

YES ! But subtleties:

Local Unitarity: prospects ISR

Support for initial-state singularities:

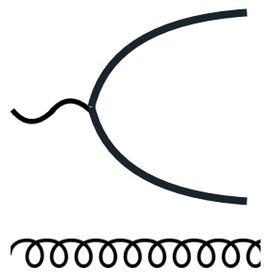
$\sum_{\text{final states}}^{\text{cuts}}$ is finite ! …so logically… $\sum_{\text{initial states}}^{\text{cuts}}$ is finite too ?

YES ! But subtleties:

> How to interpret $3 \to N$ scatterings ?

On the footsteps of

[arXiv : 1810.10022]

> What about disconnected graphs  appearing ?

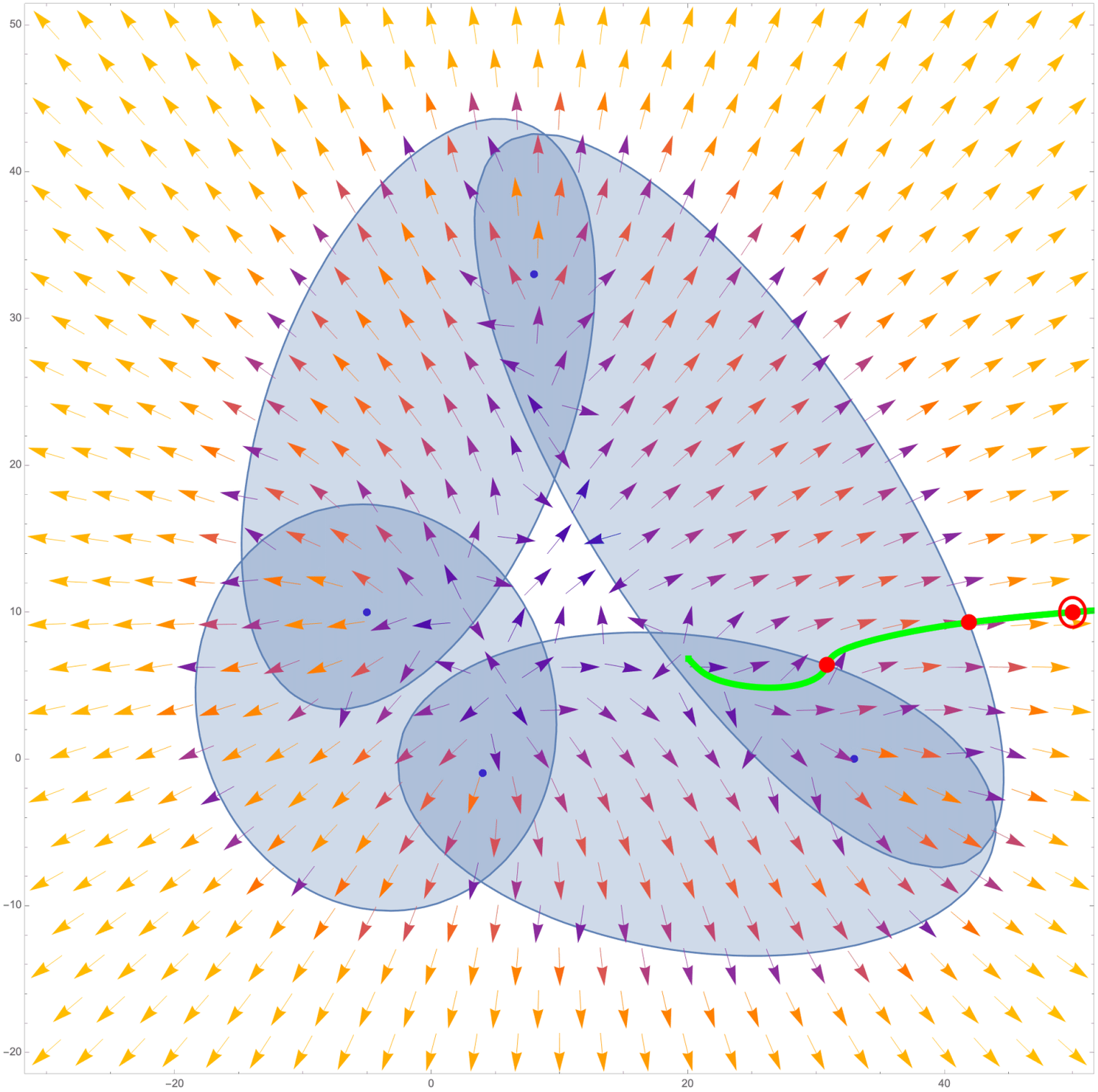
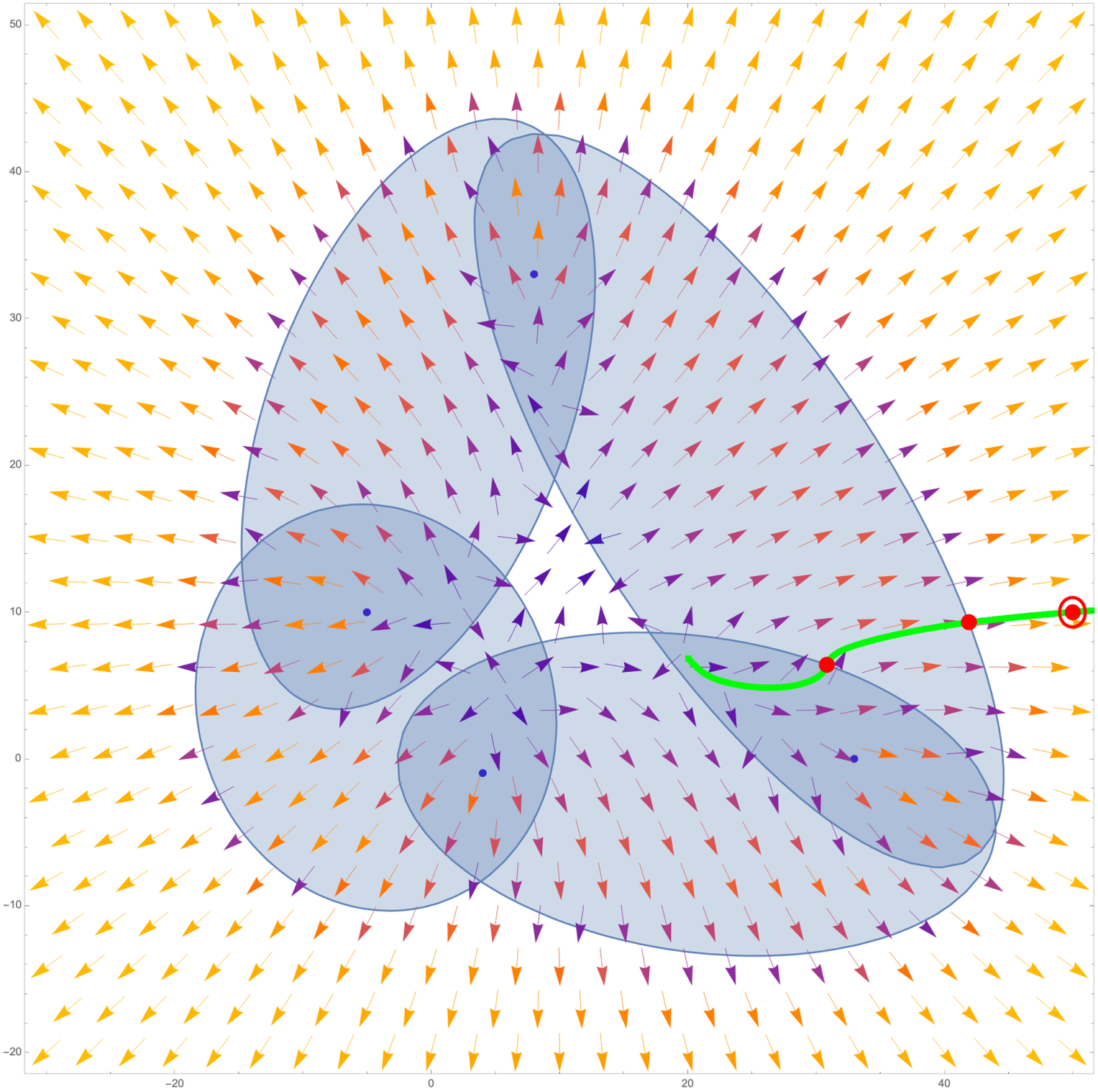
> How to connect to $\overline{\text{MS}}$ PDFs ? How to DGLAP resum?

Summary : my theory wishlist

" Build a numerical collider ! "

Wishes	Local Unitarity
Fully local / differential	✓
No restriction on number of scales	✓
No reduction to master integrals	✓
No special functions	✓
No dimensional regularisation	✓
No IR phase-space counterterms	✓
No IR loop counterterms	✓
Contour deformation only when needed	✓
Applicable to all processes and pert. orders	✓

I WELCOME YOUR QUESTIONS



I WELCOME YOUR QUESTIONS