Symmetry Preserving Regularization of Nuclear Potentials

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Outline

- Path-integral approach for derivation of nuclear forces
- Symmetry preserving regularization
- Status report on construction of 3N interactions

Path-Integral Framework for Derivation of Nuclear Forces

HK, Epelbaum, arXiv:2311.10893

Why a new Framework?

Difficulties in formulation of regularized chiral EFT

- Regularization should preserve chiral and gauge symmetries

- Regularization should not affect long-range pion physics

Pion-propagator in Euclidean space: $q^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$

$$\frac{1}{q^2 + M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2} = \frac{1}{q^2 + M_\pi^2} - \frac{1}{\Lambda^2} + \frac{q^2 + M_\pi^2}{2\Lambda^4} + \dots$$

all $1/\Lambda$-corrections are short-range interactions

q_0 - dependence in exponential requires second and higher order time-derivatives in pion field in the chiral Lagrangian

→ Canonical quantization of the regularized theory becomes difficult (Ostrogradski - approach, Constrains, …)

Canonical vs Path-Integral Quantization

Canonical Quantization of QFT

Hamiltonian & Hilbert space

Creation/annihilation operators

Time-ordered perturbation theory

↔

Path-Integral Quantization of QFT

Lagrangian & action

Summation over all classical paths

Loop expansion & Feynman rules

- Path-Integral approach is a natural choice in pionic and single-nucleon sector

  Gasser, Leutwyler, Annals Phys. 158 (1984) 142;
  Bernard, Kaiser, Kambor, Meißner, Nucl. Phys. B 388 (1992) 315

- In two - and more - nucleon sector Weinberg used canonical quantization language

  Weinberg Nucl. Phys. B 362 (1991) 3

> In using old-fashioned perturbation theory we must work with the Hamiltonian rather than the Lagrangian. The application of the usual rules of canonical quantization to the leading terms in (1) and (9) yields the total

Can we choose a formulation where we can work with the Lagrangian?

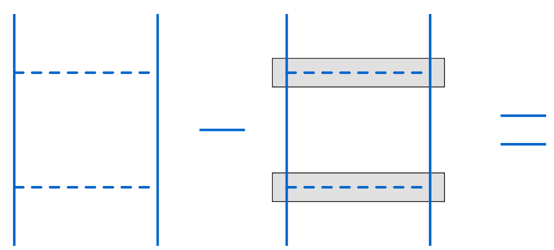
Lagrangian Formulation of Chiral EFT

Lagrangian formulation of chiral EFT so far

- Lagrangian formulation with subtractions: diagrammatic approach

  Kaiser, Brockmann, Weise, Nucl. Phys. A 625 (1997) 758

  → Less transparent in quantification of off-shell ambiguities



Irreducible part of the box diagram

- Lagrangian formulation with instant subtractions: T - matrix approach

  Gasparyan, Epelbaum, Phys. Rev. C 105 (2022) 2, 024001

- Nucleon-field transformation in a derivation of isospin violating nuclear forces

  Friar, van Kolck, Rentmeester, Timmermans, Phys. Rev. C 70 (2004) 044001

- Path-integral formulation of chiral EFT with instant interactions on the lattice

  Borasoy, Epelbaum, HK, Lee, Meißner, EPJA 31 (2007)105

  - Instant interactions generate only iterative part of the NN amplitude

Path-Integral over Nucleons and Pions

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x \big(\mathscr{L} + \eta^\dagger(x) N(x) + N^\dagger(x)\eta(x) \big) \right)$$

Yukawa toy-model:

$$\mathscr{L} = N^\dagger \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} + \frac{g}{2F} \vec{\sigma} \cdot \vec{\nabla} \pi \cdot \tau \right) N + \frac{1}{2} \left(\partial_\mu \pi \cdot \partial^\mu \pi - M^2 \pi^2 \right)$$

- Perform a Gaussian path-integral over the pion fields

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN] \exp\left(i S_N + i \int d^4x \big(\eta^\dagger(x) N(x) + N^\dagger(x)\eta(x) \big) \right)$$

$$S_N = \int d^4x \, N^\dagger(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} \right) N(x) \; - \; V_{NN}$$

← Non-instant one-pion-exchange interaction

$$V_{NN} = -\frac{g^2}{8F^2} \int d^4x \, d^4y \, \vec{\nabla}_x \cdot \left[N^\dagger(x) \vec{\sigma} \tau \right] N(x) \, \Delta_F(x-y) \, \vec{\nabla}_y \cdot \left[N^\dagger(y) \vec{\sigma} \tau \right] N(y)$$

with non-instant pion propagator: $\Delta_F(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-i q \cdot x}}{q^2 - M^2 + i\epsilon}$

Instant Interactions from Path-Integral

To transform V_{NN} into an instant form we rewrite a pion propagator

$$\frac{1}{q_0^2-\omega_q^2} = -\frac{1}{\omega_q^2} + \frac{1}{q_0^2-\omega_q^2} + \frac{1}{\omega_q^2} = -\frac{1}{\omega_q^2} + q_0^2 \frac{1}{\omega_q^2}\frac{1}{q_0^2-\omega_q^2}, \quad \omega_q = \sqrt{\vec{q}^2+M^2}$$

In coordinate space this corresponds to $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x)$ with

$$\Delta_S(x) = -\int \frac{d^4q}{(2\pi)^4}\frac{e^{-i\,q\cdot x}}{\omega_q^2} = -\,\delta(x_0)\int \frac{d^3q}{(2\pi)^3}\frac{e^{i\,\vec{q}\cdot\vec{x}}}{\omega_q^2}, \quad \Delta_{FS}(x) = \int \frac{d^4q}{(2\pi)^4}\frac{e^{-i\,q\cdot x}}{\omega_q^2(q_0^2-\omega_q^2)}$$

- The decomposition $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x)$ can be generalized

  $G(x) = \int \frac{d^4q}{(2\pi)^4} e^{-i\,q\cdot x}\tilde{G}(q_0^2, q^2)$ and $\tilde{G}(q_0^2, q^2)$ is differentiable at $q_0 = 0$

  Defining $G_S(x) = \int \frac{d^4q}{(2\pi)^4} e^{-i\,q\cdot x}\tilde{G}(0,q^2)$ and $G_{FS}(x) = \int \frac{d^4q}{(2\pi)^4} e^{-i\,q\cdot x}\frac{\tilde{G}(q_0^2, q^2)-\tilde{G}(0,q^2)}{q_0^2}$

  $$\rightarrow \quad G(x) = G_S(x) - \frac{\partial^2}{\partial x_0^2}G_{FS}(x)$$

Instant Interactions from Path-Integral

Perform an instant decomposition of the pion propagator $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x)$

$$V_{NN} = -\frac{g^2}{8F^2}\int d^4x\, d^4y \vec{\nabla}_x \cdot \left[N^\dagger(x)\vec{\sigma}\tau\right]N(x)\, \Delta_F(x-y)\, \vec{\nabla}_y \cdot \left[N^\dagger(y)\vec{\sigma}\tau\right]N(y)$$

$$\rightarrow \quad V_{NN} = V_{OPE} + V_{FS}$$

$$V_{OPE} = -\frac{g^2}{8F^2}\int d^4x\, d^4y \vec{\nabla}_x \cdot \left[N^\dagger(x)\vec{\sigma}\tau\right]N(x)\, \Delta_S(x-y)\, \vec{\nabla}_y \cdot \left[N^\dagger(y)\vec{\sigma}\tau\right]N(y)$$ is instant

$$V_{FS} = \frac{g^2}{8F^2}\int d^4x\, d^4y \vec{\nabla}_x \cdot \left[N^\dagger(x)\vec{\sigma}\tau\right]N(x)\, \frac{\partial^2}{\partial x_0^2}\Delta_{FS}(x-y)\, \vec{\nabla}_y \cdot \left[N^\dagger(y)\vec{\sigma}\tau\right]N(y)$$ is non-instant

V_{FS} is time-derivative dependent and thus can be eliminated by a non-polynomial field redefinition

$$N(x) \rightarrow N'(x) = N(x) + i\frac{g^2}{8F^2}\int d^4y\, [\vec{\sigma}\tau N(x)] \cdot [\, \vec{\nabla}_x \frac{\partial}{\partial x_0}\Delta_{FS}(x-y)]\, \vec{\nabla}_y \cdot [N^\dagger(y)\vec{\sigma}\tau N(y)]$$

$$N^\dagger(x) \rightarrow N'^\dagger(x) = N^\dagger(x) - i\frac{g^2}{8F^2}\int d^4y \vec{\nabla}_y \cdot [N^\dagger(y)\vec{\sigma}\tau N(y)][\, \vec{\nabla}_y \frac{\partial}{\partial y_0}\Delta_{FS}(y-x)] \cdot [N^\dagger(x)\vec{\sigma}\tau]$$

Instant Interactions from Path-Integral

Non-local field transformations remove time-derivative dependent two-nucleon interactions but generate time-derivative dependent three-nucleon interactions.

These contributions can be eliminated by similar field transformations

$$Z[\eta^\dagger,\eta] = \int [DN'^\dagger][DN'] \det\left(\frac{\delta(N'^\dagger,N')}{\delta(N^\dagger,N)}\right)\exp\left(i\,S_{N(N'^\dagger,N')} + i\int d^4x\left(\eta^\dagger(x)N(N'^\dagger,N')(x) + N(N'^\dagger,N')^\dagger(x)\eta(x)\right)\right)$$

$$\simeq \int [DN'^\dagger][DN'] \det\left(\frac{\delta(N'^\dagger,N')}{\delta(N^\dagger,N)}\right)\exp\left(i\,S_{N(N'^\dagger,N')} + i\int d^4x\left(\eta^\dagger(x)N'(x) + N'^\dagger(x)\eta(x)\right)\right)$$

Equivalence theorem: nucleon pole-structure is unaffected by the field-transf.

$$S_{N(N'^\dagger,N')} = \int d^4x\, N'^\dagger(x)\left(i\frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m}\right)N'(x) \;-\; V_{OPE} + \mathcal{O}(g^4)$$

$$V_{OPE} = -\frac{g^2}{8F^2}\int d^4x\, d^4y\, \vec{\nabla}_x\cdot\left[N'^\dagger(x)\vec{\sigma}\tau\right]N'(x)\,\Delta_S(x-y)\,\vec{\nabla}_y\cdot\left[N'^\dagger(y)\vec{\sigma}\tau\right]N'(y)$$

Instant one-pion-exchange interaction

One-Loop Corrections to Interaction

One loop corrections to NN & NNN interaction come from functional determinant

$$\det\left(\frac{\delta(N'^{\dagger},N')}{\delta(N^{\dagger},N)}\right) = \exp\left(\mathrm{Tr}\log\frac{\delta(N'^{\dagger},N')}{\delta(N^{\dagger},N)}\right)$$

Due to non-local structure of field transformations $\det\left(\frac{\delta(N'^{\dagger},N')}{\delta(N^{\dagger},N)}\right) \neq 1$

$$S_{N(N'^{\dagger},N')} = \int d^4x\, N'^{\dagger}(x)\left(i\frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} + \frac{3g^2M^3}{32\pi F^2}\right)N'(x) \;-\; V_{OPE} + \mathcal{O}(g^4)$$

Nucleon mass-shift **Gasser, Zepeda, NPB 174 (1980) 445**
is reproduced from functional determinant

Note: The Z-factor of the nucleon is equal to one. This is due to the replacement

$\eta^{\dagger}N + N^{\dagger}\eta \rightarrow \eta^{\dagger}N' + N'^{\dagger}\eta$ in the generating functional $Z[\eta^{\dagger},\eta]$

The original Z-factor of the nucleon is reproduced if we remove this replacement

$$Z = 1 - \frac{9M^2g^2}{2F^2}\left(\bar{\lambda} + \frac{1}{16\pi^2}\left(\log\frac{M}{\mu} + \frac{1}{3} - \frac{\pi}{2}\frac{M}{\mu})\right)\right)$$

Generalization to Chiral EFT

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x \left(\mathscr{L}_\pi + \mathscr{L}_{\pi N} + \mathscr{L}_{NN} + \mathscr{L}_{NNN} + \eta^\dagger(x) N(x) + N^\dagger(x) \eta(x) \right) \right)$$

- Integrate over pion fields via loop-expansion of the action

  → expansion of the action around the classical pion solution

- Perform instant decomposition of the remaining interactions between nucleons

- Perform nucleon-field redefinitions to eliminate non-instant part of the interaction

- Calculate functional determinant to get one-loop corrections to few-nucleon forces

Connection to Unitary Transformations

Previous derivation of nuclear forces was based on unitary transformation technique

Field transformations (FT) within path-integral approach	? ⟷	Unitary transformations (UT) within canonical quantization approach

- Interactions generated by FT have always a form of heavy-baryon like tree-level or 4-dim loop-integrals

- Interactions generated by UT can be matched by 4-dim loop-integrals, *only* if some unitary phases are fixed

  → UT technique is more flexible

In practical calculation we do not want to explore the flexibility of UT in constructing non-renormalizable nuclear forces

- FT which don't generate interactions with time-derivatives describe off-shell ambiguities

- Allows to study unitary ambiguities of e.g. relativistic corrections

UT & FT path-integral approach lead to the same chiral EFT nuclear forces up to N^4LO

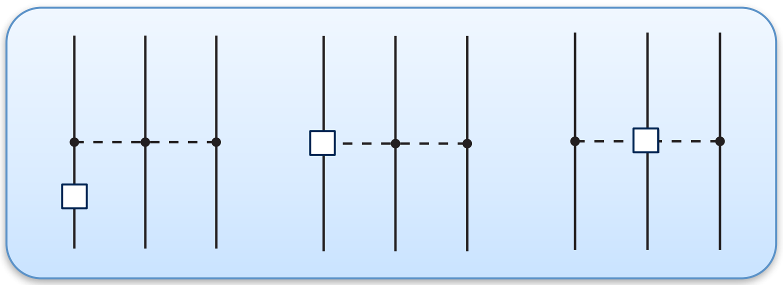
Fazit: Path-integral formulation of nuclear forces is as powerful as UT technique, however it allows consideration of a wider class of theories

Symmetry Preserving Regulator

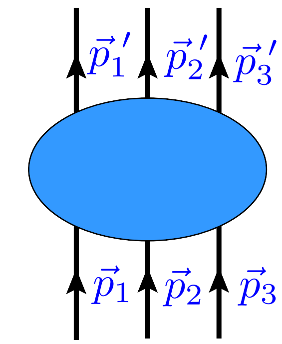
Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.

Epelbaum, HK, Reinert, Front. in Phys. 8 (2020) 98



1/m - corrections to TPE 3NF $\sim g_A^2$



$$V_{2\pi,1/m}^{g_A^2} = i\frac{g_A^2}{32mF_\pi^4}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}\tau_1\cdot(\tau_2\times\tau_3)(2\vec{k}_1\cdot\vec{q}_3+4\vec{k}_3\cdot\vec{q}_3+i\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}(\vec{p}_i{}' + \vec{p}_i)$$

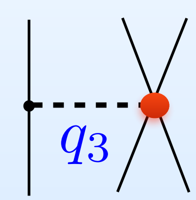
Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)\exp\left(-\frac{q_3^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

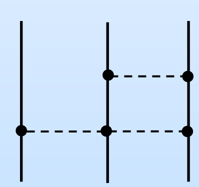
First iteration with OPE NN potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{1\pi}^{Q^0,\Lambda} + V_{1\pi}^{Q^0,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda\frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)\frac{\vec{q}_2\cdot\vec{\sigma}_2\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2}+\ldots$$

No such D-like term in chiral Lagrangian



The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi}$ =  + … is finite

Higher Derivative Lagrangian

To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i\, u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u\, l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi\, u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + i\,p), \quad u = \sqrt{U}, \quad \mathrm{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

$$\mathcal{L}_{\pi,\Lambda}^{(2)} = \mathcal{L}_\pi^{(2)} + \frac{F^2}{4} \mathrm{Tr} \left[\mathrm{EOM} \frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2} \right)}{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+} \mathrm{EOM} \right]$$

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right] \qquad \mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

✓ Leads to regularized nuclear forces up to N[4]LO

✖ Leads to unregularized nuclear currents starting from N[3]LO

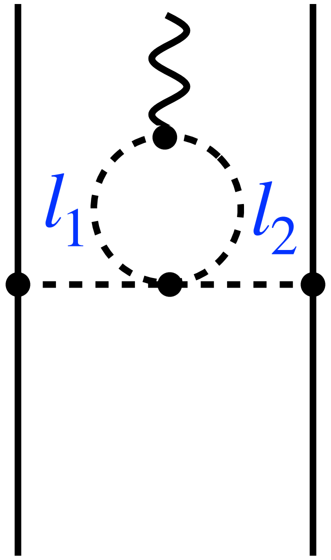
→ We need a better formalism

Problems with Currents

$$\mathscr{L}^{E(2)}_{\pi\gamma,\Lambda} = \frac{e}{2}\epsilon_{3ij}A^E_\mu\Bigg(\pi_j e^{\frac{-\partial^2+M^2}{\Lambda^2}}\partial_\mu\pi_i - (\partial_\mu\pi_i)e^{\frac{-\partial^2+M^2}{\Lambda^2}}\pi_i + \frac{1}{\Lambda^2}\int_0^1 ds\Bigg\{\Bigg[e^{s\frac{-\partial^2+M^2}{\Lambda^2}}(-\partial^2+M^2)\partial_\mu\pi_i\Bigg]e^{(1-s)\frac{-\partial^2+M^2}{\Lambda^2}}\pi_j$$

$$-\Bigg[e^{s\frac{-\partial^2+M^2}{\Lambda^2}}(-\partial^2+M^2)\pi_i\Bigg]e^{(1-s)\frac{-\partial^2+M^2}{\Lambda^2}}\partial_\mu\pi_j\Bigg\}\Bigg) + \mathcal{O}(\pi^4)$$

Additionally there is exponential increase in momenta in four-pion vertex



$$l_1 = l, \quad l_2 = l + k$$

Coming from pion-propagators

$$\sim \int d^4l \left[e^{\frac{l_1^2+M^2}{\Lambda^2}} e^{\frac{l_1^2+M^2}{\Lambda^2}} e^{-\frac{l_1^2+M^2}{\Lambda^2}} e^{-\frac{l_2^2+M^2}{\Lambda^2}} + \ldots = e^{\frac{l_1^2+M^2}{\Lambda^2}} e^{-\frac{l_2^2+M^2}{\Lambda^2}} + \ldots = e^{-\frac{2k\cdot l+k^2}{\Lambda^2}} + \ldots\right]$$

Coming from $\gamma 2\pi$ - vertex   Coming from 4π - vertex

- ✖ Leads to unregularized nuclear currents starting from N³LO

→ We need a better formalism

Gradient-Flow Equation (GFE)

Yang-Mills gradient flow in QCD: **Lüscher, JHEP 04 (2013) 123**

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu} \quad \text{with} \quad B_\mu|_{\tau=0} = A_\mu \quad \& \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

B_μ is a regularized gluon field

- Apply this idea to ChPT **(Proposed in various talks by D. Kaplan for nuclear forces)**

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying GFE

$$\partial_\tau W = i\, w\, \mathrm{EOM}(\tau)\, w \quad \text{with } w = \sqrt{W} \text{ and } \mathrm{EOM}(\tau) = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i\, r_\mu)w - w(\partial_\mu - i\, l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Note: The shape of regularization is dictated by the choice of the right-hand side of GFE

- Our choice is motivated by a Gaussian regularization of one-pion-exchange in NN

Properties under Chiral Transformation

Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

$$\mathscr{L}_{\pi N}^{(1)} = N^\dagger \Big(D^0 + g\, u \cdot S \Big) N \to N^\dagger \Big(D_w^0 + g\, w \cdot S \Big) N$$

Chiral transformation: by induction, one can show

$$U \to R U L^\dagger \longrightarrow W \to R W L^\dagger$$

- Regularized pion fields transform under τ - independent transformations

- Nucleon fields transform in τ - dependent way

$$N \to K N, \quad K = \sqrt{L U^\dagger R^\dagger} R \sqrt{U} \longrightarrow N \to K_\tau N, \quad K_\tau = \sqrt{L W^\dagger R^\dagger} R \sqrt{W}$$

Gradient-Flow Equation

Analytic solution is possible of $1/F$ - expanded gradient flow equation:

$$W = 1 + i\tau \cdot \phi(1 - \alpha\phi^2) - \frac{\phi^2}{2}\left[1 + \left(\frac{1}{4} - 2\alpha\right)\phi^2\right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty} \frac{1}{F^n}\phi_b^{(n)}$$

In the absence of external sources we have

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi_b^{(1)}(x,\tau) = 0, \quad \phi_b^{(1)}(x,0) = \pi_b(x)$$

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi_b^{(3)}(x,\tau) = (1-2\alpha)\partial_\mu\phi^{(1)} \cdot \partial_\mu\phi^{(1)}\phi_b^{(1)} - 4\alpha\partial_\mu\phi^{(1)} \cdot \phi^{(1)}\partial_\mu\phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha)\phi^{(1)} \cdot \phi^{(1)}\phi_b^{(1)}, \quad \phi_b^{(3)}(x,0) = 0$$

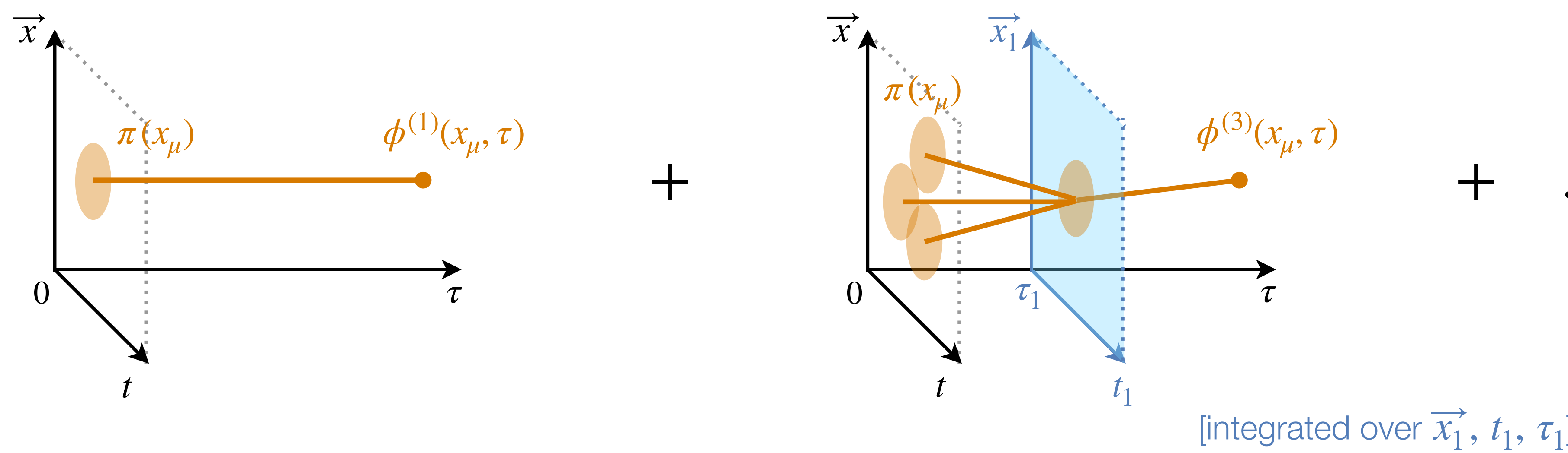
Iterative solution in momentum space: $\tilde{\phi}^{(n)}(q,\tau) = \int d^4x\, e^{iq\cdot x}\phi_b^{(n)}(x,\tau)$

$$\tilde{\phi}_b^{(1)}(q) = e^{-\tau(q^2+M^2)}\tilde{\pi}_b(q)$$

$$\tilde{\phi}_b^{(3)}(q) = \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}(2\pi)^4\delta(q - q_1 - q_2 - q_3)\int_0^\tau ds\, e^{-(\tau-s)(q^2+M^2)}e^{-s\sum_{j=1}^3(q_j^2+M^2)} \times \left[4\alpha\, q_1 \cdot q_3 - (1-2\alpha)q_1 \cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right]\tilde{\pi}(q_1) \cdot \tilde{\pi}(q_2)\tilde{\pi}_b(q_3)$$

Integration over momenta of pion fields with Gaussian prefactor introduces smearing

Iterative solution in Coordinate Space

$\phi(x_\mu, \tau) \quad = $  $+ \quad \ldots$

Light-shaded area visualizes smearing in Euclidean space of size $\sim \sqrt{2\tau}$

Solid line stands for Green-function:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right] G(x-y, \tau - s) = \delta(x-y)\delta(\tau - s)$$

$$G(x, \tau) = \theta(\tau) \int \frac{d^4 q}{(2\pi)^4} e^{-\tau(q^2+M^2)} e^{-i\, q\cdot x}$$

$$\phi^{(1)}_b(x, \tau) = \int d^4 y\, G(x-y, \tau) \pi_b(y)$$

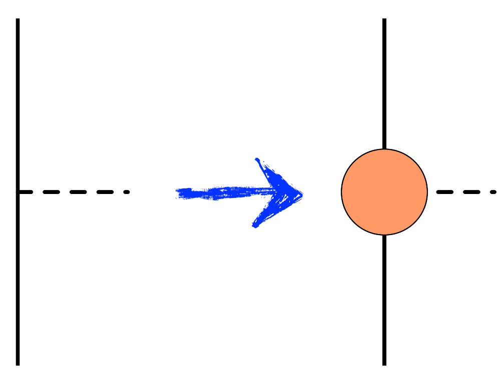
$$\phi^{(3)}_b(x, \tau) = \int_0^\tau ds \int d^4 y\, G(x-y, \tau - s) \Big[(1-2\alpha)\partial_\mu \phi^{(1)}(y,s)\cdot \partial_\mu \phi^{(1)}(y,s) \phi^{(1)}_b(y,s)$$
$$- 4\alpha\, \partial_\mu \phi^{(1)}(y,s) \cdot \phi^{(1)}(y,s) \partial_\mu \phi^{(1)}_b(y,s) + \frac{M^2}{2} \phi^{(1)}(y,s)\cdot \phi^{(1)}(y,s) \phi^{(1)}_b(y,s)\Big]$$

Regularization for Nuclear Forces

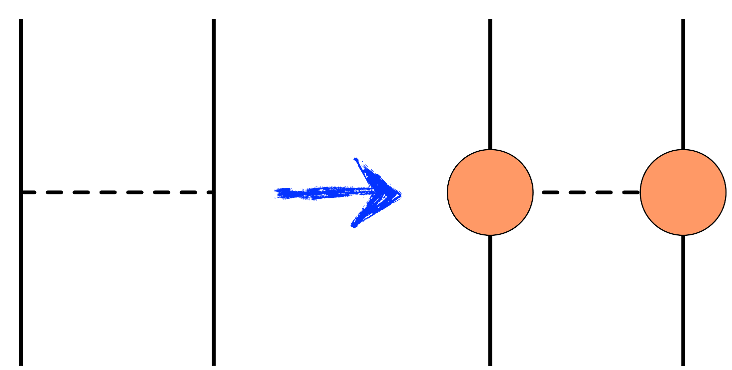
To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathcal{L}_{\pi}^{(2)}$ & $\mathcal{L}_{\pi}^{(4)}$ unregularized (essential)
- Replace all pion fields in pion-nucleon Lagrangians $\mathcal{L}_{\pi N}^{(1)}, \ldots, \mathcal{L}_{\pi N}^{(4)}$: $U \to W$

$$\mathcal{L}_{\pi N}^{(1)} = N^{\dagger}\Big(D^0 + g\, u \cdot S\Big)N \to N^{\dagger}\Big(D_w^0 + g\, w \cdot S\Big)N$$



$$\sim e^{-\tau(q^2+M^2)}$$



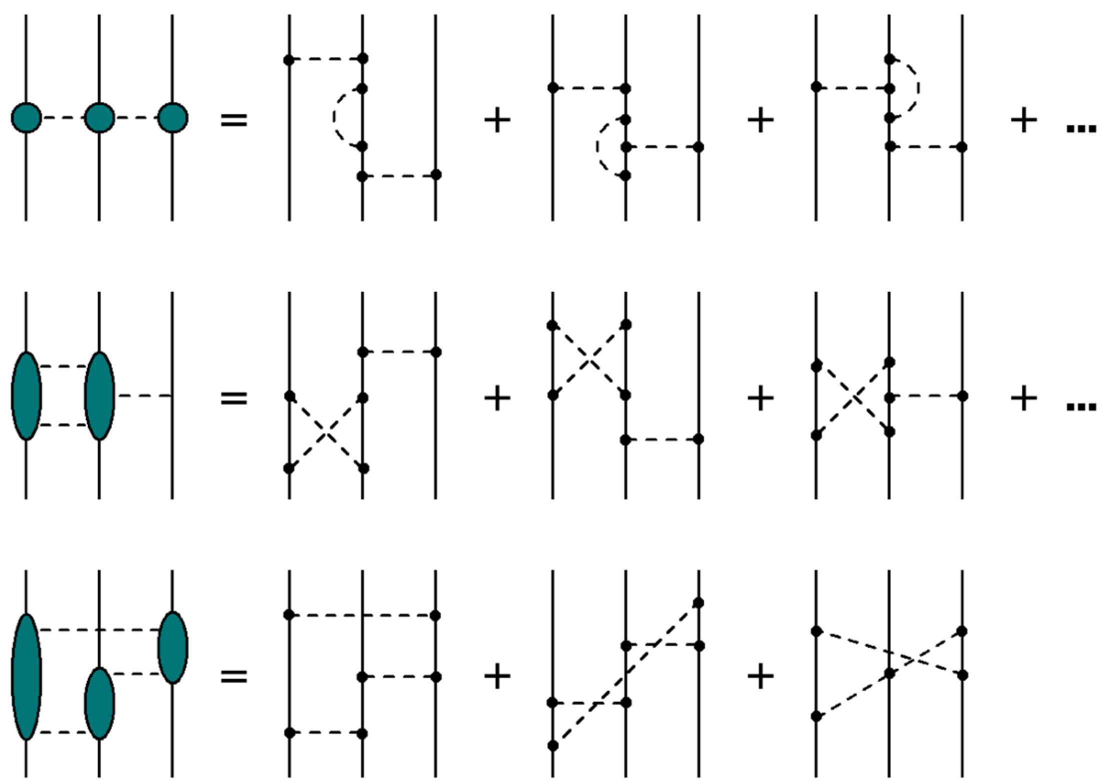
$$\sim e^{-2\tau(q^2+M^2)}\frac{1}{q^2+M^2}$$

For $\tau = \dfrac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

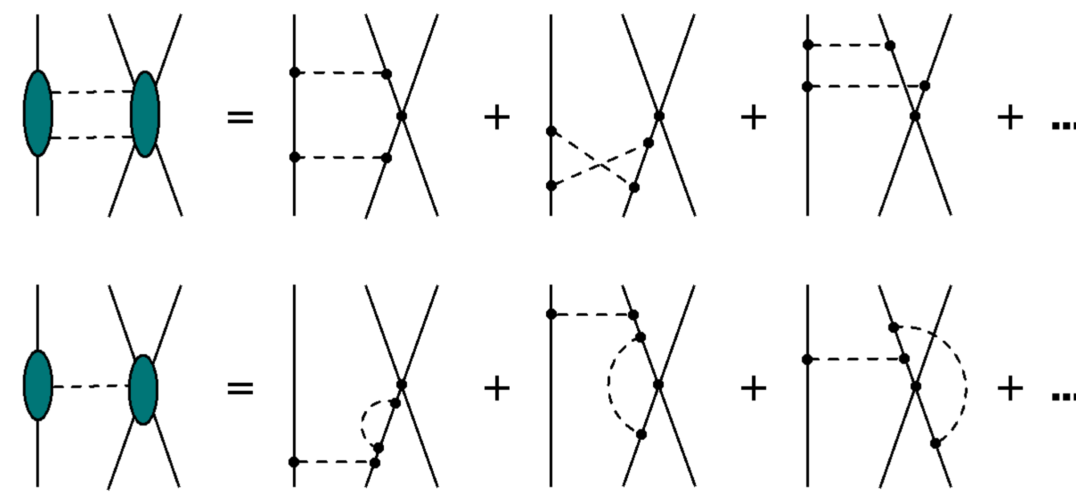
Status Report on 3NF

Status Report on 3NF

- Long-range part of 3NF is calculated up to N³LO ✓



- Short-range part of 3NF is being calculated up to N³LO



To get a finite 3NF in $\Lambda \to \infty$ limit we have to perform 5 additional field-transformations which include second power of the pion propagator

- In $\Lambda \to \infty$ limit we reproduce dim. reg. results: **Bernard, Epelbaum, HK, Meißner ´08, ´11**

- Partial-wave decomposition is needed for practical implementations

Kai Hebeler (Darmstadt), Andreas Nogga (Jülich), Kacper Topolnicki (Krakow)

Summary

- Path-integral approach for derivation of nuclear forces has been developed
  - Applicable for EFT's with interactions involving second or higher number of time-derivatives
  - All results from unitary transformation technique are reproduced within path-integral approach
- Symmetry preserving regularization
  - Pion fields which couple to nucleons are smoothed within a gradient-flow equation approach
- Status report on construction of 3N interactions