Primordial nucleosynthesis with varying $\alpha_{\rm EM}$

ERC EXOTIC Workshop – Frontiers in Nuclear Physics

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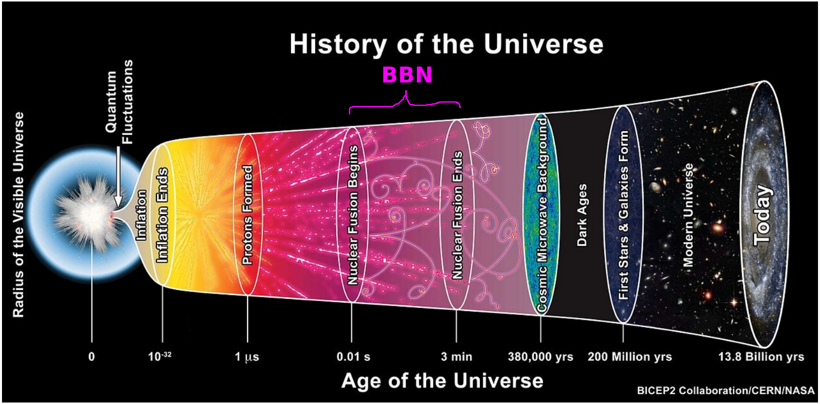
Introduction



BBN probe of physics[1]: are fundamental constants really constant?[2]

[1] reviews: Olive, Steigman, and Walker, 2000; Iocco et al., 2009; Cyburt et al., 2016; Pitrou et al., 2018a

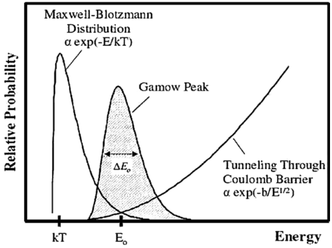
[2] Dirac, 1973 and many others

Introduction

Want to study variation of electromagnetic coupling constant [Meißner, Metsch, and Meyer, 2023; Bergström, Iguri, and Rubinstein, 1999; Nollett and Lopez, 2002; Dent, Stern, and Wetterich, 2007; Coc et al., 2007]

$$\alpha_0 = 7.297\,352\,569\,3(11) \times 10^{-3}$$ [PDG]

Goal: find a bound on $\delta\alpha = \Delta\alpha/\alpha_0$ through comparing calculations with experimental values for light element abundances



: from Trache, 2010

Where does α appear in BBN?

- Nuclear Rates: Coulomb barrier $\rightarrow$ Gamow-factor [Gamow, 1928]
- Weak rates: final state Coulomb interactions in $n \leftrightarrow p$ rates and β-decays
- Indirectly: Neutron-Proton mass difference $Q_n = m_n - m_p$, nuclear binding energies ($\rightarrow$ reaction "Q-values")

Timescales

Fusion of d, ^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li and ^{7}Be

3 min

1 min:
"deuterium bottleneck":
$n + p \to d + \gamma$ possible

1 s: $n \leftrightarrow p$
freeze-out

Weak interactions $n \leftrightarrow p$ with $\frac{n_n}{n_p} = e^{-Q_n/T}$, $Q_n = m_n - m_p = 1.293\,\text{MeV}$
[PDG]

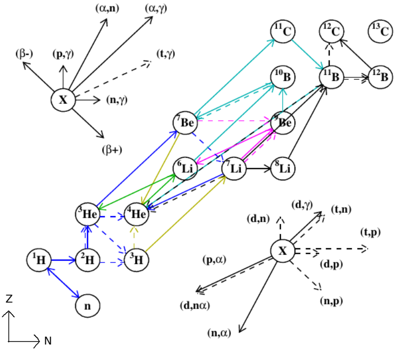
freeze out at $T_f = 1\,\text{MeV} \to$ free neutron decay: $\frac{n_n}{n_p} = e^{-Q_n/T_f} e^{-(t-t_f)/\tau_n}$

Evolution of Abundances

Define abundance $Y_i = n_i / n_b$, with n_i density of species i and n_b total baryon density. Evolution depends on

- Cosmoligical model: Hubble expansion
- Particle reactions (rate $\Gamma_{ij \to kl} \equiv n_b \langle \sigma v \rangle_{ij \to kl}$) and decays (rate $\Gamma_{i \to \dots}$)

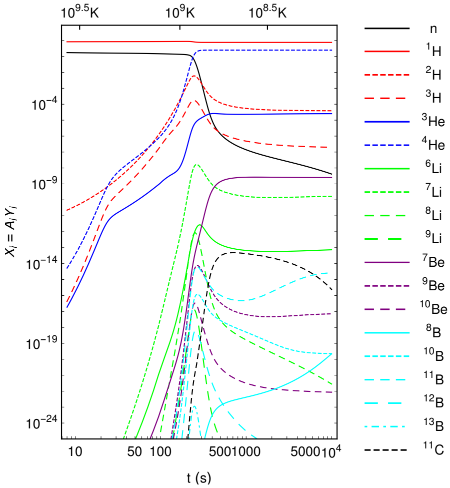
Need to solve system of rate equations



: Pitrou et al., 2018a

$$\dot{Y}_i \supset -Y_i \Gamma_{i \to \dots} + Y_j \Gamma_{j \to i + \dots} + Y_k Y_l \Gamma_{kl \to ij} - Y_i Y_j \Gamma_{ij \to kl}$$

Evolution of Abundances – Codes



: PRIMAT

Codes used for solving network of rate equations [Wagoner, Fowler, and Hoyle, 1967]:

- PRIMAT [Pitrou et al., 2018b]
- AlterBBN [Arbey et al., 2020]
- PArthENoPE [Gariazzo et al., 2022]
- NUC123 [Kawano, 1992]
- New: PRyMordial [Burns, Tait, and Valli, 2023]

Nuclear Reaction Rates – Coulomb Barrier

$$\Gamma_{ab\to cd}(T) = N_A \langle \sigma v \rangle \propto \int_0^\infty \mathrm{d}E\, \sigma_{ab\to cd}(E) \cdot E \cdot e^{-\frac{E}{k_B T}}, \quad E = \frac{1}{2}\mu_{ab} v^2$$

(1) Coulomb Barrier

Cross section is proportional to penetration factor [Blatt and Weisskopf, 1979]

$$\sigma \propto v_0 = \frac{2\pi\eta}{e^{2\pi\eta} - 1},$$

with Sommerfeld parameter

$$\eta = \frac{Z_a Z_b \alpha c}{\hbar v} = \frac{1}{2\pi}\sqrt{E_G/E},$$

and Gamow-energy

$$E_G = 2\mu_{ab} c^2 \pi^2 Z_a^2 Z_b^2 \alpha^2, \quad \mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$

Nuclear Reaction Rates – Radiative Capture

(2) Radiative capture reactions

- Coupling $\propto e \Rightarrow$ Cross section $\sigma \propto \alpha \propto e^2$
- External capture processes [Christy and Duck, 1961]: parameterized in $f(\delta\alpha)$ [Nollett and Lopez, 2002]
- Assume dipole dominance
- For some reactions: Halo EFT cross sections $\Rightarrow$ work in progress

α-dependence of cross section ($q_\gamma = 1$ for radiative capture, zero else)

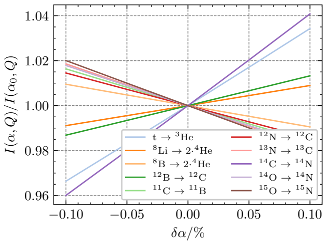
$$\sigma(\alpha, E) \propto \left(\frac{\sqrt{E_G^{\text{in}}/E}}{e^{\sqrt{E_G^{\text{in}}/E}} - 1} \right) \cdot \left(\frac{\sqrt{E_G^{\text{out}}/(E+Q)}}{e^{\sqrt{E_G^{\text{out}}/(E+Q)}} - 1} \right) \cdot (\alpha f(\delta\alpha))^{q_\gamma}$$

$$Q = m_a + m_b - m_c - m_d$$

Weak Rates – Fermi Function

β-decay rate (assume $|M_{fi}|^2$ to be p-independent) [Segrè, 1964]:

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 c^3 \hbar^7} \underbrace{\int_0^{p_{e,\max}} \left(W - \sqrt{m_e^2 c^4 + p_e^2 c^2} \right)^2 F(Z, \alpha, p_e) p_e^2 \, \mathrm{d}p_e}_{= I(\alpha, Q)},$$



$$p_{e,\max} = \frac{1}{c}\sqrt{W^2 - m_e^2 c^4}, W \approx M_a - M_b = Q$$

Fermi function (for $Z\alpha \ll 1$):

$$F(\pm Z, \alpha, \epsilon_e) \approx \frac{\pm 2\pi\nu}{1 - \exp(\mp 2\pi\nu)}, \quad \nu \equiv \frac{Z\alpha\epsilon_e}{\sqrt{\epsilon_e^2 - 1}}$$

Then:

$$\lambda(\alpha) = \lambda(\alpha_0) \frac{I(\alpha, Q)}{I(\alpha_0, Q)}$$

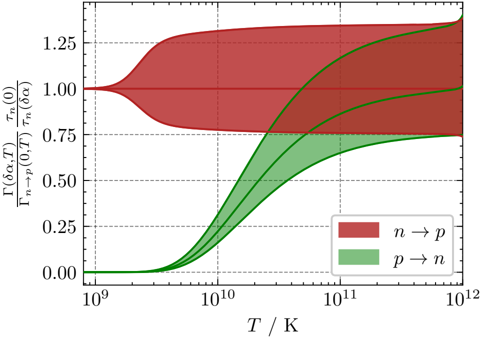
$n \leftrightarrow p$ Rates

Free neutron decay: lifetime

$$\tau_n(\alpha) = \tau_n(\alpha_0)\frac{I(\alpha_0, Q)}{I(\alpha, Q)}$$

But: Ignored Fermi-Dirac distribution of neutrino and electron

$\Rightarrow$ temperature dependence in α-variation for high temperatures



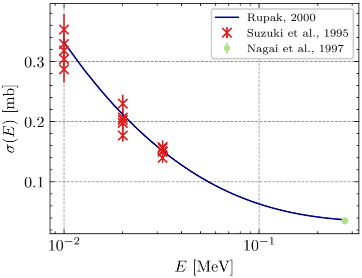
Nuclear Reaction Rates – $n + p \to d + \gamma$

Some corrections due to α variation are energy-dependent
$\Rightarrow$ need reaction cross section!

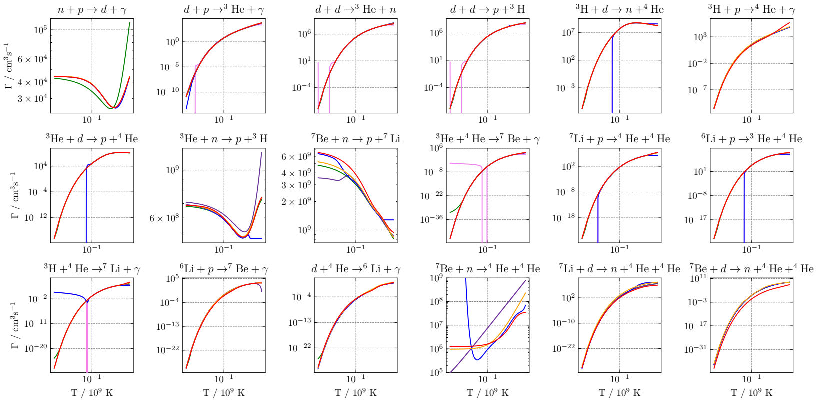
For $n + p \to d + \gamma$:

- Pionless EFT (N^4LO) approach by Rupak, 2000
- $\sigma(n + p \to d + \gamma)$ depends linearly on α

Other reaction cross section need to be parameterized by fitting to data EXFOR database



Nuclear Reaction Rates – Leading Reactions

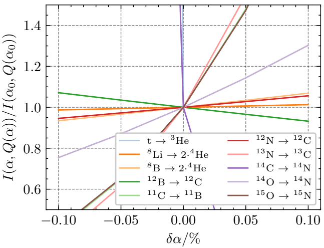


This work ; PRIMAT ; AlterBBN ; PArthENoPE; NUC123 ; NACRE II ;
(PRyMordial uses the PRIMAT rates)

Indirect Effects – Binding energies [Meißner and Metsch, 2022]

Coulomb interaction between protons in nucleus
$\Rightarrow$ Electromagnetic contribution to binding energy [Elhatisari et al., 2022a]
Change in Q-value:

$$\Delta Q = \delta\alpha \left(-\sum_i B_C^i + \sum_j B_C^j \right)$$



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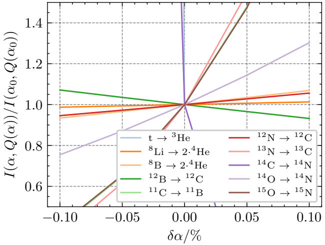


Nuclear reaction cross sections ($p_\gamma = 3, q_\gamma = 1$ for radiative capture, $p_\gamma = 1/2, q_\gamma = 0$ else)

$$\sigma(E,\alpha) \propto \underbrace{(E + Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\rm in}(\alpha)/E}}{\exp\left(\sqrt{E_G^{\rm in}(\alpha)/E}\right) - 1} \frac{\sqrt{E_G^{\rm out}(\alpha)/(E + Q(\alpha))}}{\exp\left(\sqrt{E_G^{\rm out}(\alpha)/(E+Q(\alpha))}\right) - 1}$$

Indirect Effects – Neutron-proton mass difference

$Q_n = m_n - m_p$ has QED contribution [Gasser, Leutwyler, and Rusetsky, 2021]:

$$\Rightarrow \Delta Q_n = Q_n^{\mathrm{QED}} \cdot \delta\alpha = -0.58(16)\,\mathrm{MeV} \cdot \delta\alpha$$

Affects

- weak $n \leftrightarrow p$ rates
- Q-values of β-decays
- $m_N = (m_n + m_p)/2$ appearing in $n + p \to d + \gamma$ cross section? $\to$ neglect α-dependence!

Results

Baryon-to-photon ratio $\eta = 6.14 \times 10^{-10}$; neutron lifetime $\tau_n(\alpha_0) = 879.4\,\mathrm{s}$ [PDG]

Parameter fit

$$\frac{Y(\alpha) - Y(\alpha_0)}{Y(\alpha_0)} = a \cdot \frac{\Delta\alpha}{\alpha_0} + b \cdot \left(\frac{\Delta\alpha}{\alpha_0}\right)^2$$

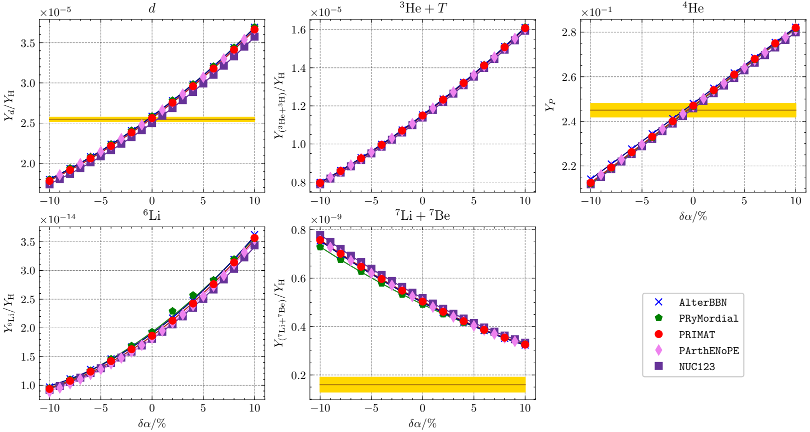
Main results see Meißner, Metsch, and Meyer, 2023:

- For most elements: change in nuclear reaction rates biggest effect.
- $^4\mathrm{He}$ indeed very sensitive to ΔQ_n.
- Lithium Problem

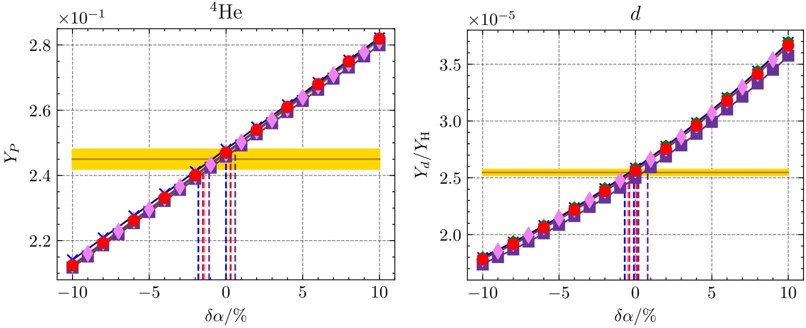
Differences to existing literature:

- Updated experimental values for masses, physical constants etc., more recent calculation of Q_n^{QED}
- Different reaction rates due to parameterization of cross section.
- Calculating the corrections exactly or using temperature-dependent approximations.

Results


d
$\times 10^{-5}$
Y_d/Y_H
^{3}He + T
$Y_{(^3He+^3H)}/Y_H$
^{4}He
$\times 10^{-1}$
Y_P
^{6}Li
$\times 10^{-14}$
$Y_{^6Li}/Y_H$
^{7}Li + ^{7}Be
$\times 10^{-9}$
$Y_{(^7Li+^7Be)}/Y_H$
$\delta\alpha/\%$
AlterBBN
PRyMordial
PRIMAT
PArthENoPE
NUC123

Experimental constraints



1σ-bounds on α-variation: $\Rightarrow$ From ^{4}He: $|\delta\alpha| < 1.8\%$

Conclusion: what we discussed so far

- Goal: Study α-dependence of primordial abundances of d, ^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li and ^{7}Be in BBN
- Considered α in
  - nuclear reaction rates (Coulomb penetration factor)
  - final state Coulomb interactions in weak $n \leftrightarrow p$ and β-decays rates (Fermi function) $\rightarrow$ neutron lifetime τ_n
  - the Coulomb contribution to (binding energies) $\rightarrow$ reaction Q-values
  - the neutron-proton mass difference Q_n
- Parameterized 18 relevant reaction cross sections.
- Computed primordial abundances for different α.
- Constrain α-variation to $|\delta\alpha| < 1.8\%$

Outlook: what we are working on right now

How does a variation of the quark masses (u, d) influence BBN?

- Interesting: combined study of α- and m_q-variation $\Rightarrow$ constraints?
- Include Halo EFT results for radiative capture cross sections

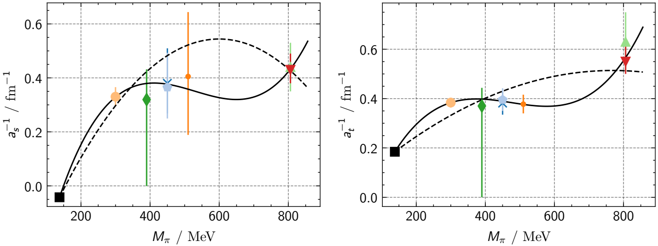
Where do quark masses come in?

- Nuclear binding energies depend (mainly) on average quark mass $\hat{m} = \frac{m_u + m_d}{2}$
- This in turn may affect reaction parameters $(a,\ r_{\mathrm{eff}}) \Rightarrow$ Halo EFT rates
- How does quark mass difference change? $\Rightarrow$ changes Q_n

Quark mass dependence of binding energies

- Gell-Mann-Oakes-Renner relation: $M_\pi^2 \propto \hat{m}$
- From (pionless) EFT [Bedaque, Luu, and Platter, 2011] or using Nuclear Lattice EFT [Lähde, Meißner, and Epelbaum, 2020; Elhatisari et al., 2022b]

  $\Rightarrow$ binding energies depend on $\frac{\partial a_s^{-1}}{\partial M_\pi}$, $\frac{\partial a_t^{-1}}{\partial M_\pi}$ (N-N-scattering)

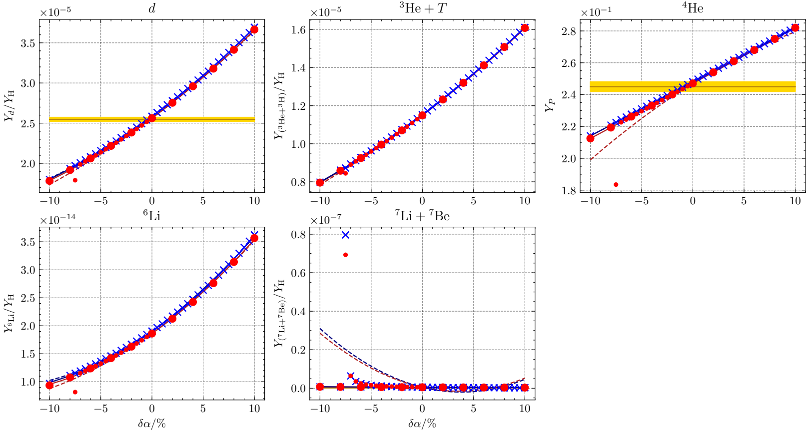


Thank you for your attention!

Any Questions?

Halo EFT – Abundances

Halo EFT cross sections for $^3\mathrm{H} + {}^4\mathrm{He} \to {}^7\mathrm{Li} + \gamma$ and $^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma$



Measurement of Primordial Abundances

Deuterium *d*:

- Almost completely destroyed in stars
- Observe high red-shift, low-metallicity systems

Helium-4 $^4\mathrm{He}$:

- Recombination lines of He and H in metal-poor extra-galactic HII regions
- Metal Production in stars positively correlated to stellar $^4\mathrm{He}$ contribution
  $\rightarrow$ Primordial abundance found by extrapolation to zero metallicity

Lithium-7 $^7\mathrm{Li}$:

- Observe stars in the galactic halo with very low metallicities
- $^7\mathrm{Li}$ dominant over $^6\mathrm{Li}$
- Lithium problem[3]: theoretical prediction three times higher

[3] Fields, 2011

Temperature-Dependent Approximation

Charged particle reactions

- Define $S(E) = \sigma(E) E e^{\sqrt{E_G^{\text{in}}/E}}$ and assume $S \approx \text{const.}$
- Reaction rate

$$\Gamma = \int \mathrm{d}E \, \frac{S(E)}{E} e^{-\sqrt{E_G^{\text{in}}/E}} E e^{E/(k_B T)}$$

- E at maximum of integrand

$$E \to \bar{E}_c = \left(\frac{k_B T}{2}\right)^{\frac{2}{3}} (E_G^{\text{in}})^{\frac{1}{3}}.$$

Neutron induced reactions

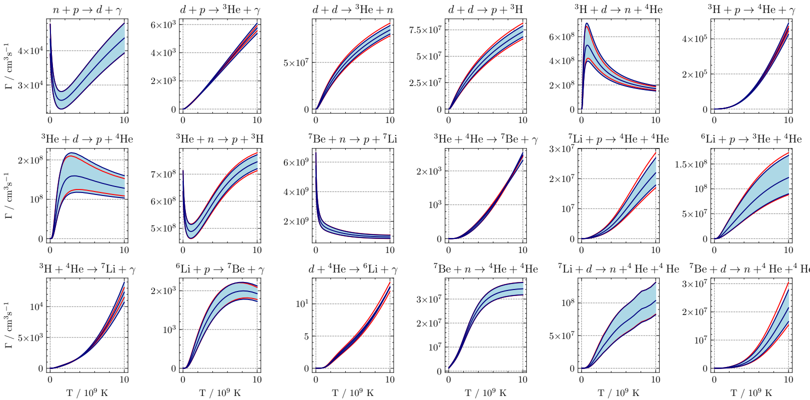
- Define $R(E) = \sigma(E)\sqrt{E}$ and assume $R \approx \text{const.}$
- Reaction rate

$$\Gamma = \int \mathrm{d}E \, \frac{R(E)}{\sqrt{E}} E e^{E/(k_B T)}$$

- E at maximum of integrand

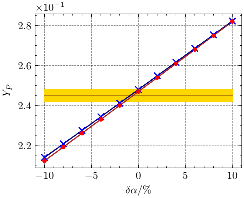
$$E \to \bar{E}_\gamma = \frac{1}{2} k_B T$$

Reaction Rates for Approximation

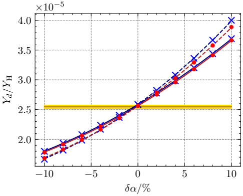


Reaction rates for $\delta\alpha = 0, \pm 10\%$ calculated exactly (blue) and with temperature-dependent approximation (red)

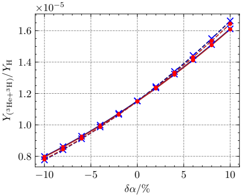
Abundances with Approximation



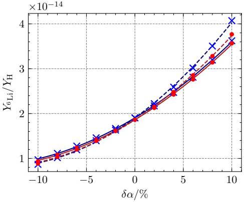
(a) Y_P (^{4}He mass fraction)



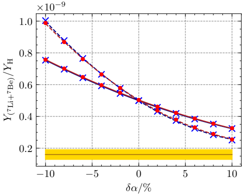
(b) Deuterium d



(c) ^{3}He + ^{3}H



(d) ^{6}Li



(e) ^{7}Li + ^{7}Be