Hypernuclei from the Lattice

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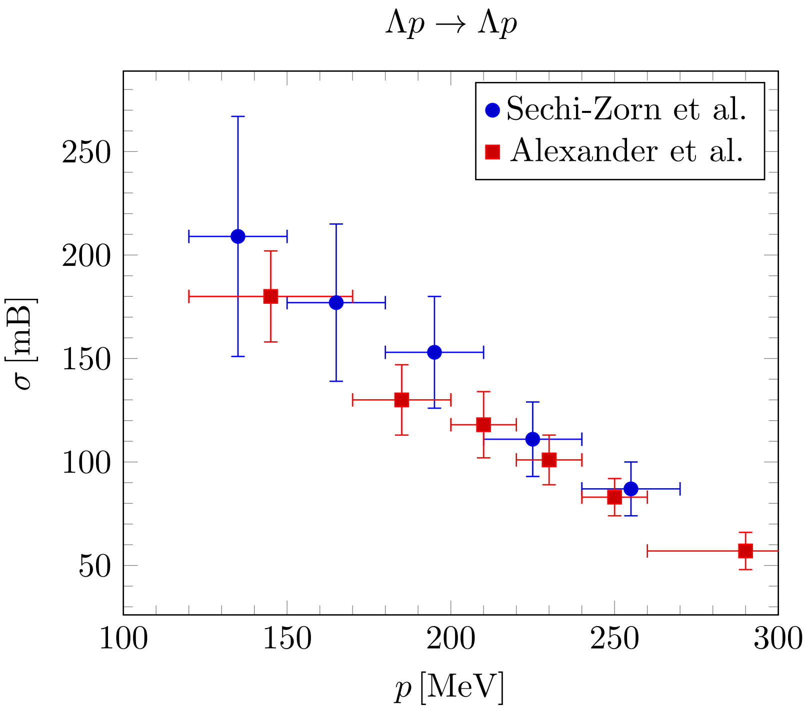
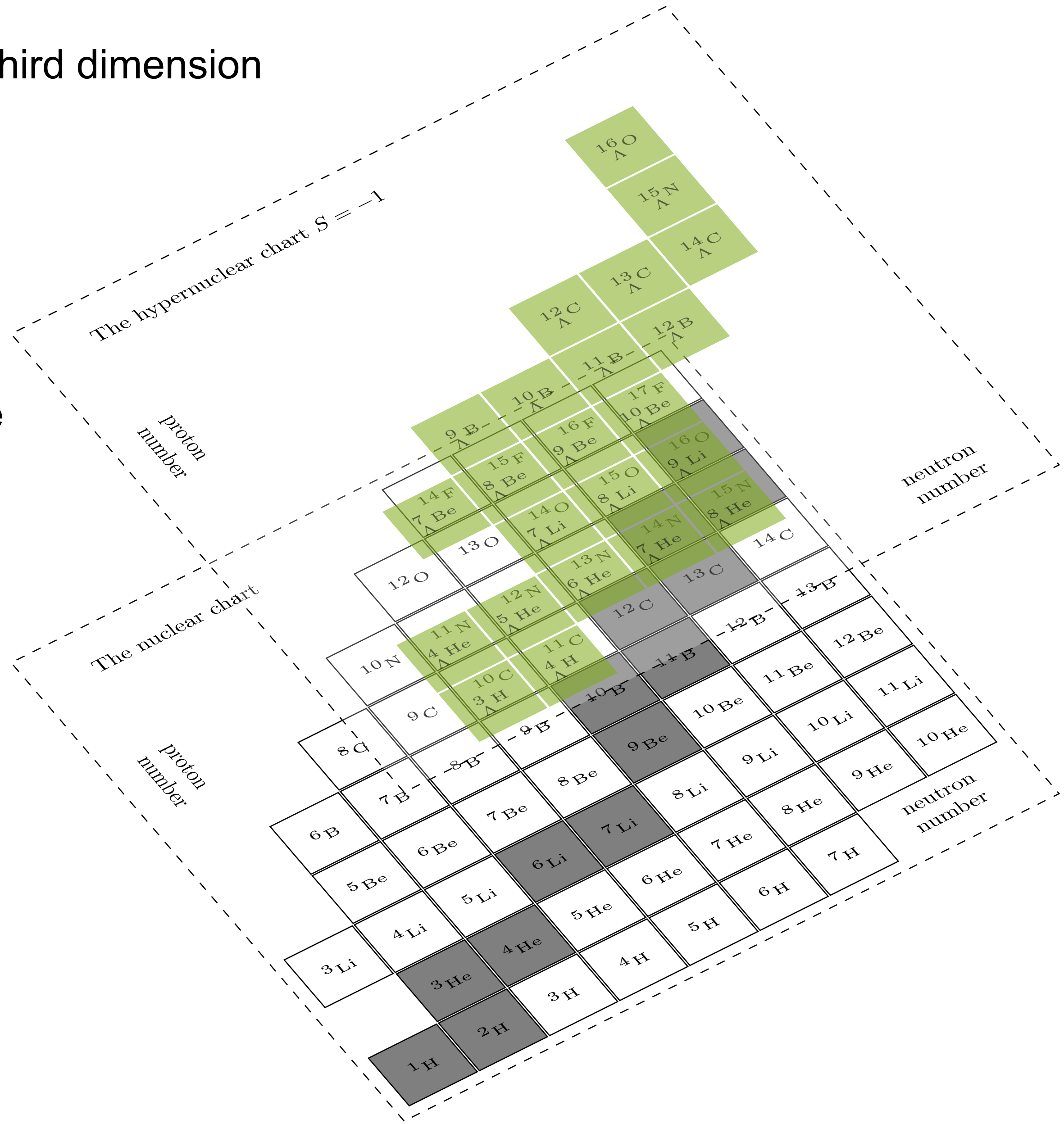
Outline

- Motivation
  - From NLEFT to (Hyper) NLEFT
    - Lattice Interaction
    - First Results for light nuclei
- Impurity Worldline Monte-Carlo

Hypernuclear physics in a nutshell

- Strangeness extents the nuclear chart to a third dimension
- Unique opportunity to study the strong force
  Without the Pauli principle
- Typical approach from nuclear physics
  does not work since two-body data is sparse





- Gateway : Three-Body Systems

Starting point for (Hyper) Nuclear Lattice EFT

Very Successful Nuclear Program:

Using AFMC and shuttle algorithm

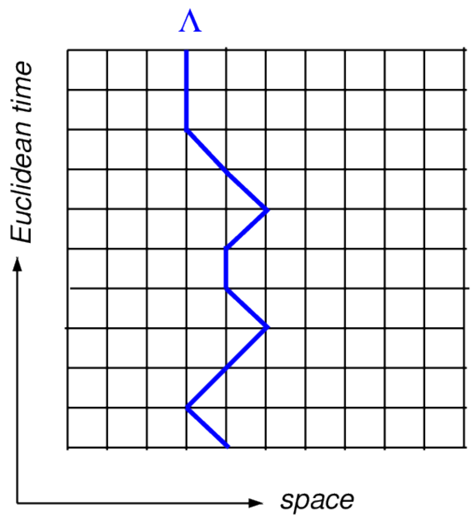
WFM to obtain precise results for Nuclei and charge radii

AFMC does not converge as good as in a pure nuclear matter simulation

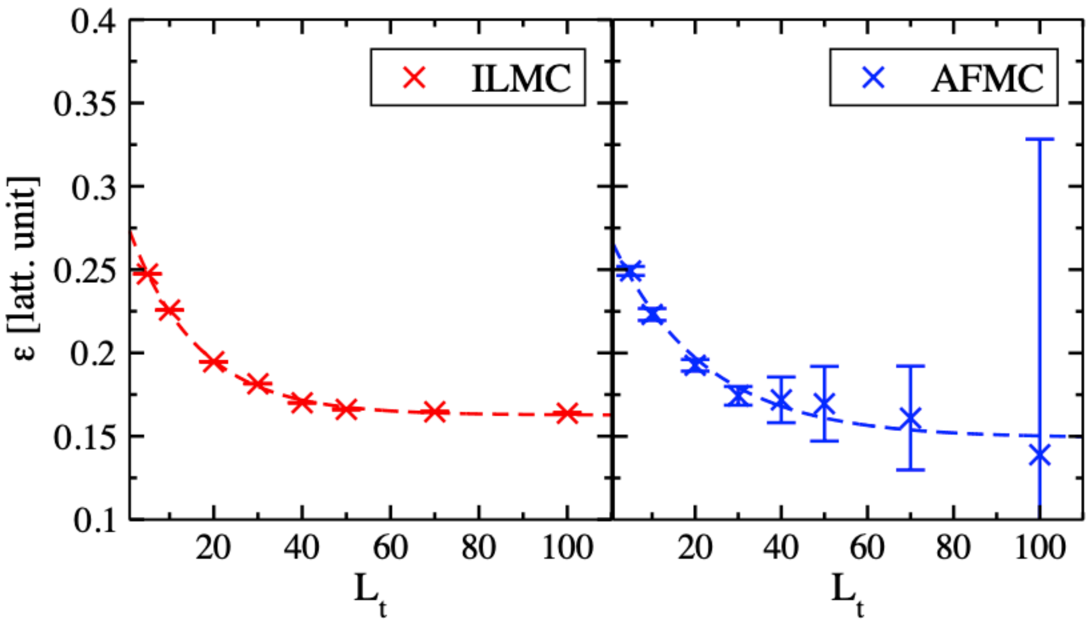
Need to develop a method that threats this impurities more efficient

Treat Impurity as worldline:

(S.Bour, D.Lee, H.-W. Hammer, U.-G. Meißner)



(D. Frame, T. A. Lähde, D. Lee, U.-G. Meißner)



Starting point for (Hyper) Nuclear Lattice EFT

- Challenge with IFMC, need to collect millions of Worldlines

→ Can we still do Hypernuclear calculations with AFMC ?

→ Important for possible applications with many Hyperons

- Taylor interaction to work non-perturbative with our best NN interaction

→ Evolve together with NN counterparts

Constraints smearing parameters to the NN ones

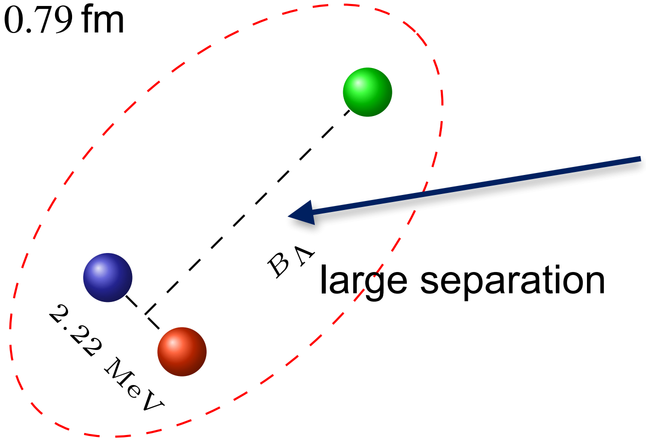
	Phase
$A = 3$	0.97
$A = 4$	0.89
$A = 5$	1 ← α – core
$A = 7$	0.92

$L = 12 \quad Lt = 500$

This is very promising, for larger hypernuclei

Construction of a first Lattice ΛN interaction

$\sqrt{\langle r^2_{\Lambda d} \rangle} \approx 10.79$ fm



Emulsion:

$B_\Lambda = 0.130 \pm 0.050$ MeV Juric 1973

Heavy Ion:

$B_\Lambda = 0.406 \pm 0.120$ MeV Star 2020

$B_\Lambda = 0.102 \pm 0.063$ MeV Alice 2023

World Average:

$B_\Lambda = 0.164 \pm 0.043$ MeV Mainz 2023

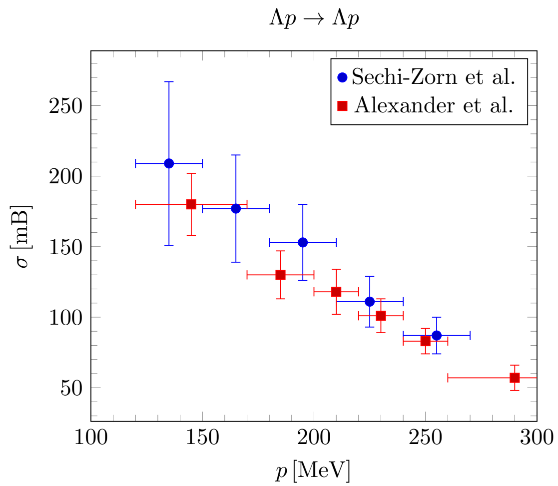
Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

Distinguishable

$$I = 0 \Rightarrow \frac{1}{\sqrt{2}}(pn - np)\Lambda$$

Combine 2-Body data with hypertriton in exact calculation



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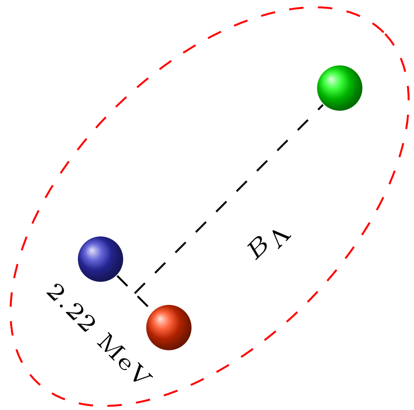
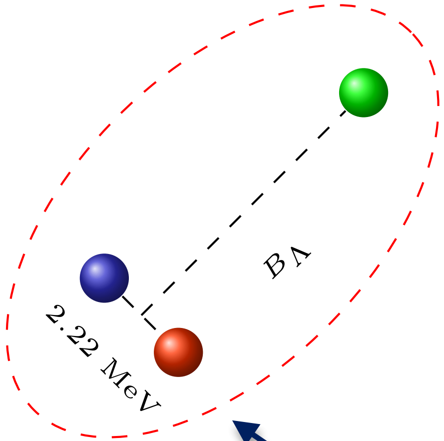


Construction of a first Lattice ΛN interaction



large separation → Large box sizes needed

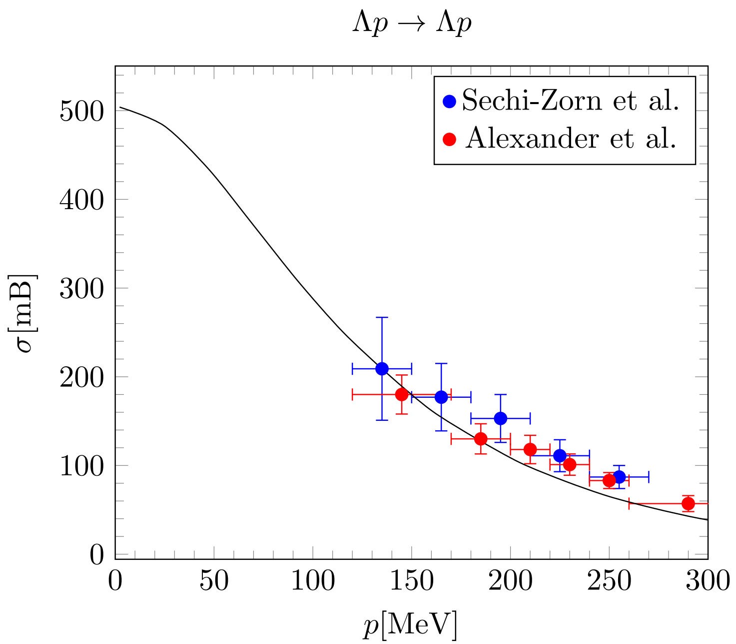
GPU Lanczos Code to fix 2-Body forces → Allows L=18

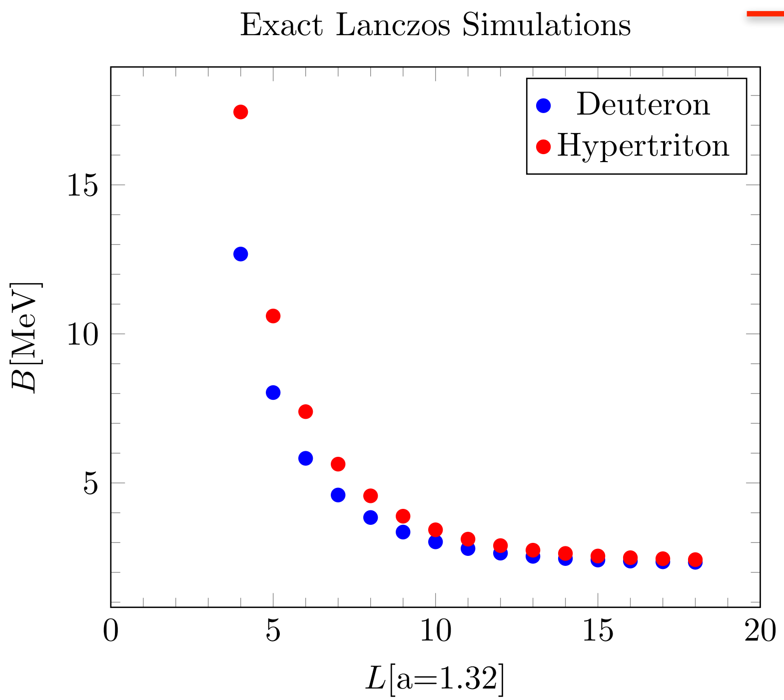
Tightly bound compared to Λ separation

Use simpler nuclear Interaction

Non-pertubative LO interaction → $^3S_1$, $^1S_0$ interaction

Construction of a first Lattice ΛN interaction





Best SMS N^2LO interaction

(Haidenbauer et al.)

$a_s = -2.80 \qquad r_s \sim 2.89$

$a_t = -1.58 \qquad r_t \sim 3.09$

This interaction

$a_s = -2.89 \qquad r_s \sim 3.28$

$a_t = -1.60 \qquad r_t \sim 3.94$

Phase shift similar to $p \sim 60$ MeV

$$E(L) = E_{L\to\infty} + \frac{A}{L} e^{\frac{L}{L_0}}$$

$\approx$ Emulsion

$B^{\Lambda}_{L\to\infty} = (90 + 40)$ keV $= 130$ keV

2-Body

GIR corrections

Results: Two Body interaction (L=12 l.u.)

During Evolution:

Spin-averaged Interaction:

$$C = \frac{3\,{}^3S_1 + {}^1S_0}{4}$$

Perturbative part:

Spin-dependent Interaction:

$$C_S = \frac{{}^3S_1 - {}^1S_0}{4}$$

Nuclear Interaction:

N^{3}LO interaction, same as in WFM results

Results: Two Body	Experiment
$B_\Lambda({}^3\text{H}_\Lambda) = 0.31 \pm 0.19$ MeV	0.164 ± 0.48 MeV

→ Box effect, consistent with exact L=12 result

Results: Two Body	Experiment
$B_\Lambda({}^4\text{H}_\Lambda^{0^+}) = 1.71 \pm 0.42$ MeV	2.169 ± 0.005 MeV
$B_\Lambda({}^4\text{H}_\Lambda^{1^+}) = 0.62 \pm 0.41$ MeV	1.081 ± 0.005 MeV

→ Splitting good, missing 0.4 MeV Binding

Results: Two Body	Experiment
$B_\Lambda({}^5\text{He}_\Lambda) = 3.57 \pm 0.62$ MeV	3.102 ± 0.003 MeV

→ Smaller overbinding compared to other LO Calculations

Results: Two Body	Experiment
$B_\Lambda({}^7\text{Li}_\Lambda) = 5.19 \pm 0.71$ MeV	5.619 ± 0.06 MeV

→ Typically overbound by ~1 MeV in LO calculations

Results: Two Body interaction, further analysis

Missing 0.4 MeV in A=4 as well as A=7 systems

A=5 system only slightly overbound

More Attraction → Modify C

→ Fixes four body systems

→ Strong overbinding of $^{5}\text{He}_{\Lambda}$

Strong overbinding of $^{7}\text{Li}_{\Lambda}$

$B_{\Lambda}(^{3}\text{H}_{\Lambda}) \sim 0.2 - 0.3$

→ Can three-body forces help us here?

Structure of contact three-body forces

(Petschauer et al.)

$$V_{ct}^{\Lambda NN} = C_1(1 - \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)(3 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3) \quad \leftarrow I = 1$$

$$+C_2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$

$$+C_3(3 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$

$I = 0$ → Can Influence Hypertriton

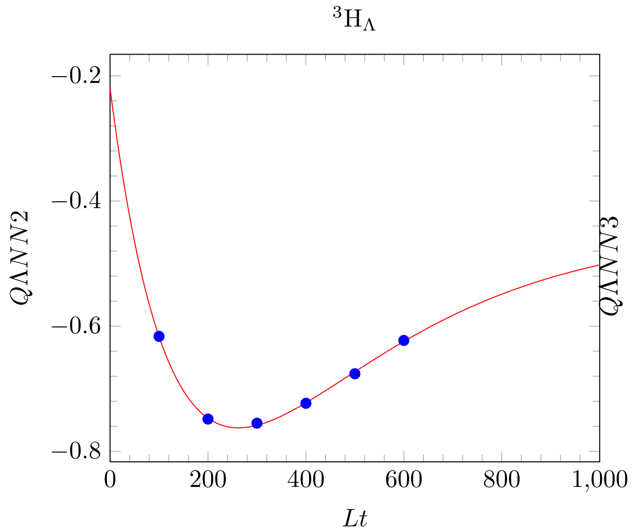
C_2 only interaction that depends on Λ spin → ${}^4\mathrm{H}_\Lambda^{0^+} - {}^4\mathrm{H}_\Lambda^{1^+}$ Splitting

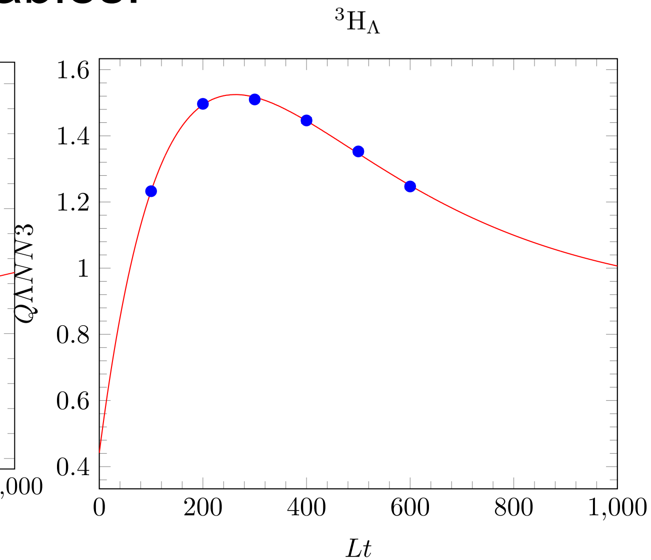
C_1, C_3 are iso/spin interchanged to each other → Might not split for small Hypernuclei

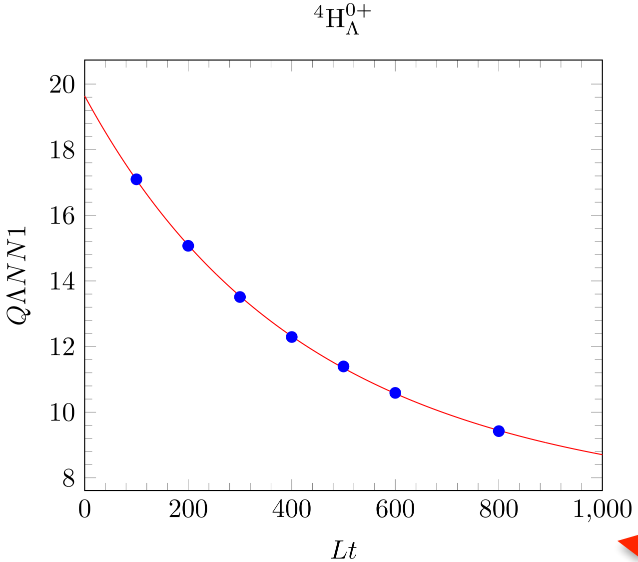
Structure of contact three-body forces

Extract TBF Terms in a contained fit with all other observables:

$$E(Lt) = E_\infty + Ae^{-(\Delta E)Lt} + Be^{-\frac{(\Delta E)Lt}{2}}$$

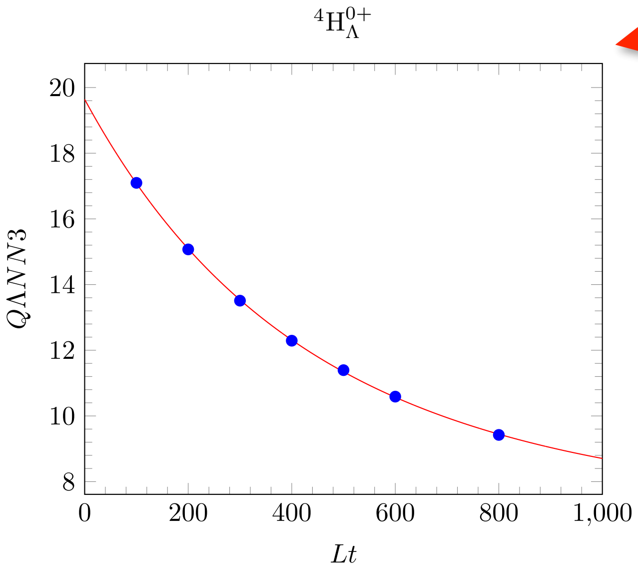


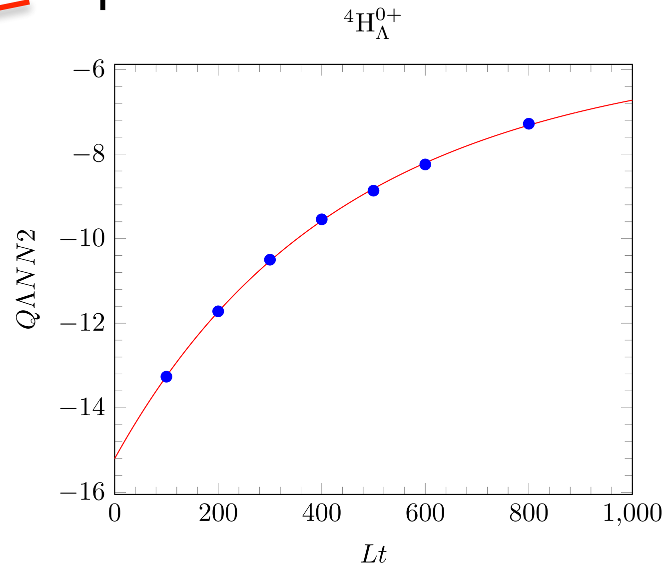


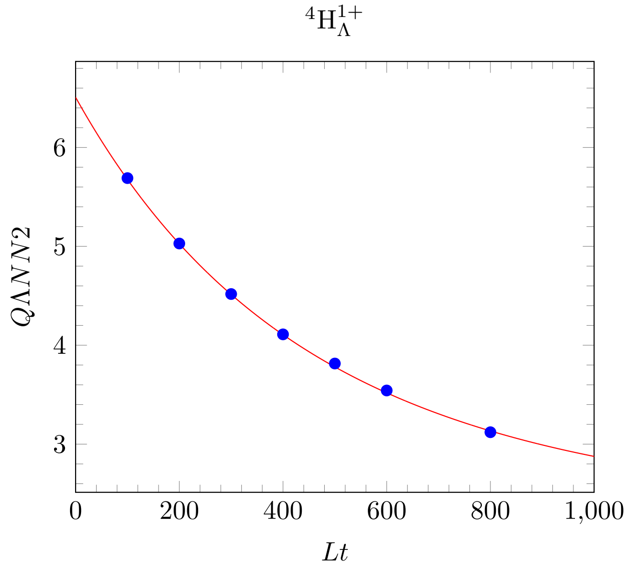


We must keep under control

Split for A=7







Three-Body Results

$$V_{ct}^{\Lambda NN} = C_1(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(3+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$ ← $I=1$

$$+C_2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$

$$+C_3(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$ ← $I=0$

→ Behave like expected

Clean data

$^4\mathrm{H}_\Lambda^{0^+} - {}^4\mathrm{H}_\Lambda^{1^+}$ Splitting was already good

Best Fit

$B_\Lambda(^3\mathrm{H}_\Lambda) = 0.01$ MeV

$B_\Lambda(^4\mathrm{H}_\Lambda^{0^+}) = 2.12$ MeV

$B_\Lambda(^4\mathrm{H}_\Lambda^{1^+}) = 0.56$ MeV

$B_\Lambda(^5\mathrm{He}_\Lambda) = 3.18$ MeV

$B_\Lambda(^7\mathrm{Li}_\Lambda) = 5.62$ MeV

→ We could use QNN1 and QNN3
To get 0.4 MeV of Binding

→ Will overbind A=5 and A=7
Systems

→ Or use QNN2 to fix A=5
And A=7, but will destroy
Splitting

Possible Paths to improvement

- Go to higher orders in the two-body interaction → Typical LO problems go away in other methods
- Include two-Pion exchange/Pion exchange 3B Forces → Long-Range behaviour of the interaction
- fit two-body forces with better nuclear interaction → Removes any dependence of the NN Force on the YN Force
- Improve statistics in the NN part of the hypernuclei → Main uncertainty from sampling of the NN part of the nucleus

$$\hat{H}_0 = \frac{1}{2m} \sum_{s=\uparrow_a,\uparrow_b,\downarrow} \int \mathrm{d}^3\boldsymbol{r} \nabla a_s^\dagger(\boldsymbol{r}) \nabla a_s(\boldsymbol{r})$$

← Kinetic Energy Term

$$\hat{H}_I = C_{II} \int \mathrm{d}^3\boldsymbol{r} \hat{\rho}_{\uparrow_b}(\boldsymbol{r}) \hat{\rho}_{\uparrow_a}(\boldsymbol{r}) + C_{IB} \int \mathrm{d}^3\boldsymbol{r} \left[\hat{\rho}_{\uparrow_a}(\boldsymbol{r}) \hat{\rho}_{\downarrow}(\boldsymbol{r}) + \hat{\rho}_{\uparrow_b}(\boldsymbol{r}) \hat{\rho}_{\downarrow}(\boldsymbol{r}) \right]$$

← Contact Interactions

Worldline - Worldline Interaction

Worldline - Background Interaction

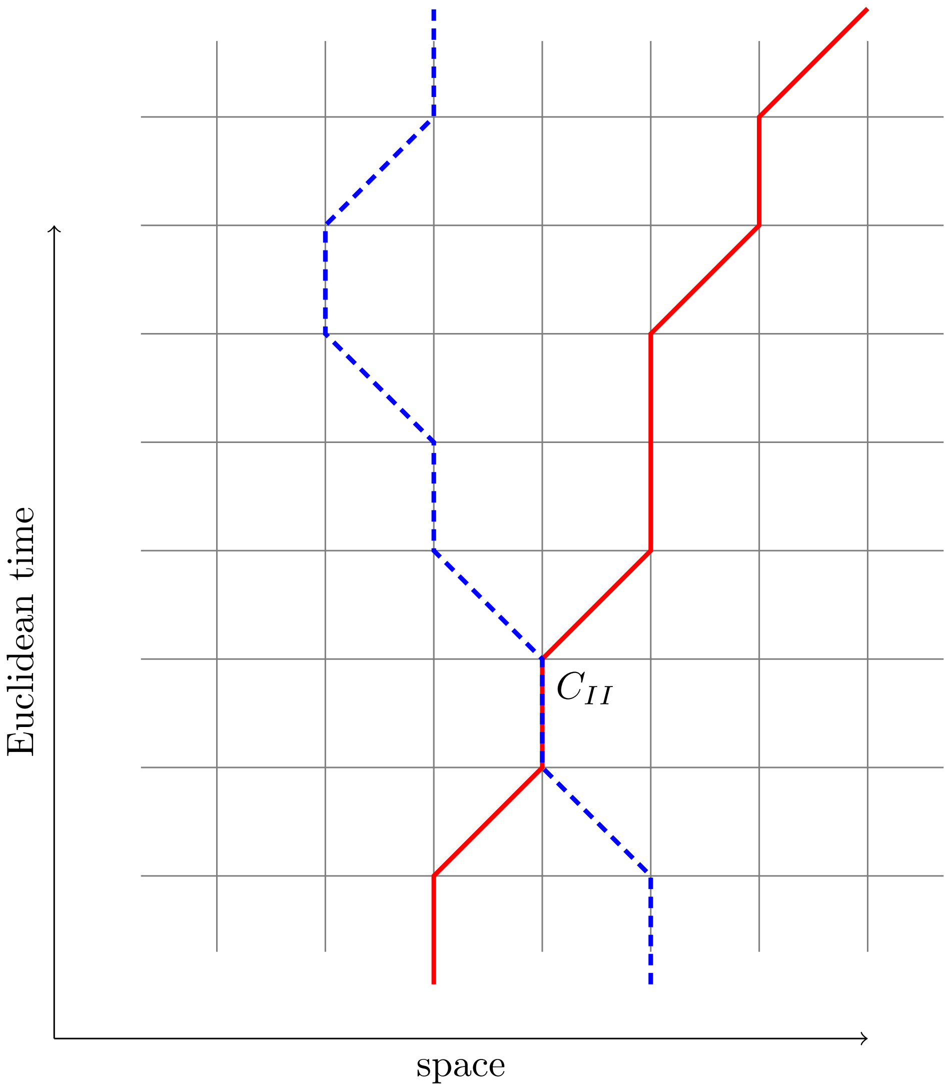
Idea: Integrate out the impurities from the lattice action :

$$\langle \chi^{\downarrow}_{n_{t+1}}, \chi^{\uparrow_a}_{n_{t+1}}, \chi^{\uparrow_b}_{n_{t+1}} | \hat{M} | \chi^{\downarrow}_{n_t}, \chi^{\uparrow_a}_{n_t}, \chi^{\uparrow_b}_{n_t} \rangle \Rightarrow \langle \chi^{\downarrow}_{n_{t+1}} | \hat{\tilde{M}} | \chi^{\downarrow}_{n_t} \rangle$$

With any state in occupation number basis is given by:

$$|\chi^{\downarrow}_{n_t}, \chi^{\uparrow_a}_{n_t}, \chi^{\uparrow_b}_{n_t}\rangle = \prod_{\boldsymbol{n}} \left[a_{\downarrow}^\dagger(\boldsymbol{n}) \right]^{\chi^{\downarrow}_{n_t}(\boldsymbol{n})} \left[a_{\uparrow_a}^\dagger(\boldsymbol{n}) \right]^{\chi^{\uparrow_a}_{n_t}(\boldsymbol{n})} \left[a_{\uparrow_b}^\dagger(\boldsymbol{n}) \right]^{\chi^{\uparrow_b}_{n_t}(\boldsymbol{n})} |0\rangle$$

What can happen?



- both worldline hop

$$\overline{M}^{\boldsymbol{n}\pm\hat{l},\boldsymbol{n}}_{\boldsymbol{n}'\pm\hat{l}',\boldsymbol{n}'} = W_h^2 : e^{-\alpha H_0^{\downarrow}} :$$

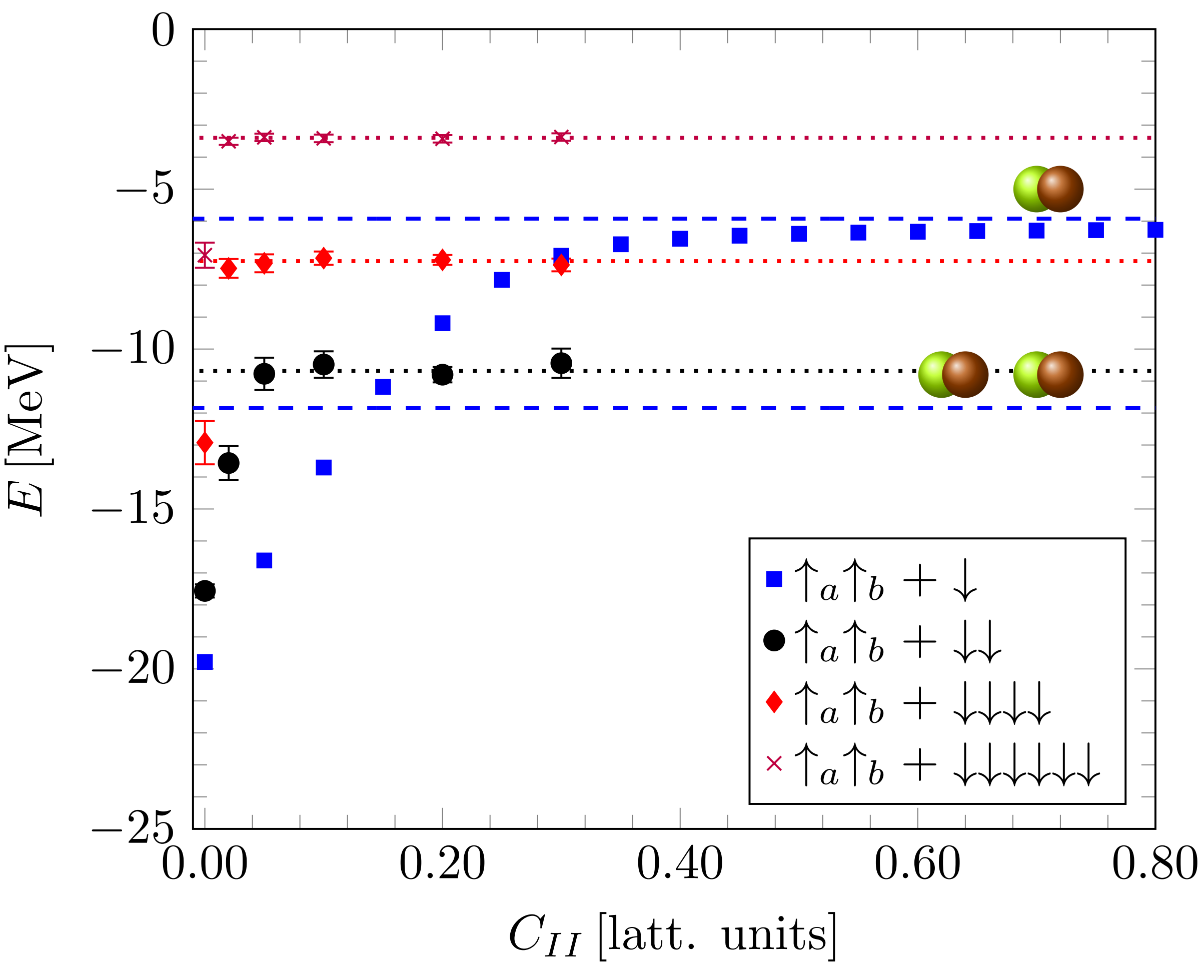
- one worldline hops, one stays

$$\overline{M}^{\boldsymbol{n}\pm\hat{l},\boldsymbol{n}}_{\boldsymbol{n}',\boldsymbol{n}'} = W_h W_s : e^{-\alpha H_0^{\downarrow} - \frac{\alpha C_{IB}\,\rho_{\downarrow}(\boldsymbol{n}')}{W_s}} :$$

- both worldlines stay

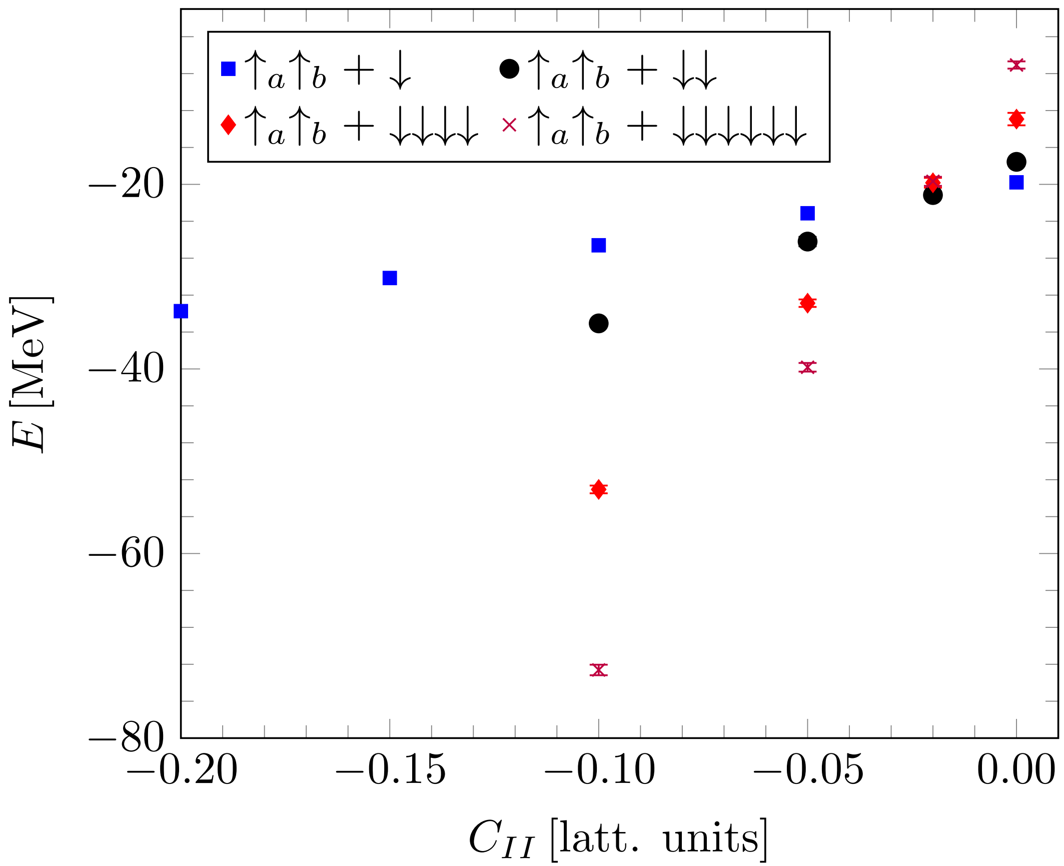
$$\overline{M}^{\boldsymbol{n},\boldsymbol{n}}_{\boldsymbol{n}',\boldsymbol{n}'} = W_s^2 : e^{-\alpha H_0^{\downarrow}} \exp\left[\frac{-\delta_{\boldsymbol{n},\boldsymbol{n}'}\alpha C_{II}}{W_s^2} - \frac{\alpha C_{IB}\rho_{\downarrow}(\boldsymbol{n})}{W_s} - \frac{\alpha C_{IB}\,\rho_{\downarrow}(\boldsymbol{n}')}{W_s} + \mathcal{O}(\alpha^2)\right] :$$

Results: Attractive Impurity-Background Interaction
Repulsive Impurity-Impurity interaction



- Impurity-Background interaction chosen to be attractive $a \sim 3$ fm
- Trimer stays bound even for very repulsive C_{II}
- The four particle bound state however consists out of two dimers
- Further particles fill up the fermi sea of the box and do not contribute to the binding

Results: Attractive Impurity-Background Interaction
Attractive Impurity-Impurity interaction



- Around $C_{II} \sim -0.02$ the four particle system is deeper bound than the 3-body system
- Higher-particle systems show a similar behaviour at the same point
- Indication of a rich phase structure

Summary and Outlook

Promising Results for light hypernuclei nuclei A=3-7 with $N^3LO(NN)$ and $LO(YN)$ interaction

Briefly introduced 2WL Method, which treat hyperons or other impurities in a sea of nuclei.

Many possible path ways to improve the results

Calculate the hypernuclear chart

Many excited states in A=7/9 hypernuclei