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Hypernuclei from the NCSM

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- Motivation
- J-NCSM and SRG evolution of (hyper-)nuclear interactions
- Uncertainty of Λ separation energies and size of chiral 3BF contributions
- Determination of CSB contact interactions and Λn scattering length
- Application to $A = 7$ and 8 hypernuclei
- Light $\Lambda\Lambda$ hypernuclei and Ξ hypernuclei
- Conclusions & Outlook

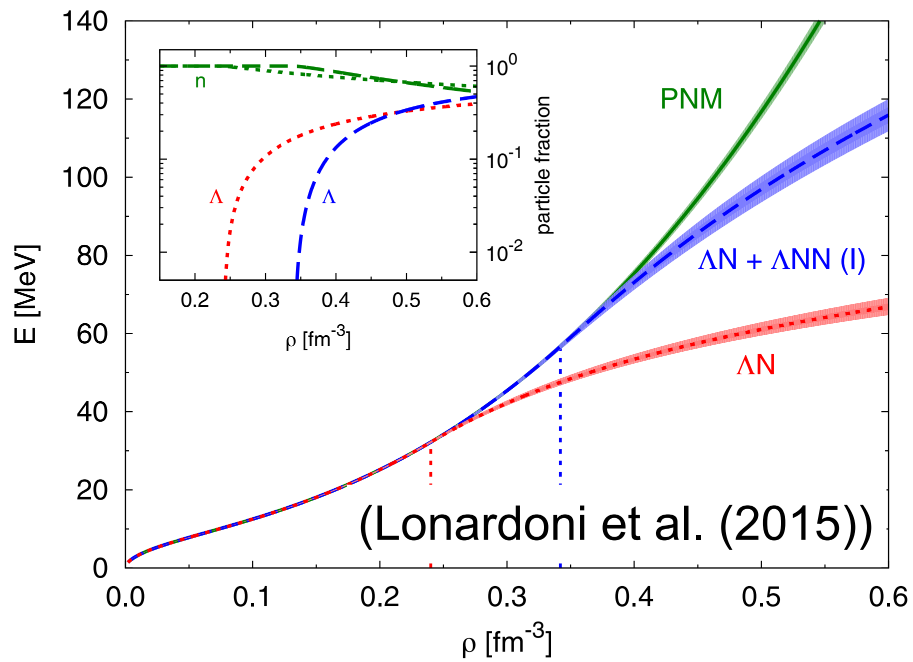
in collaboration with Johann Haidenbauer, **Hoai Le**, Ulf Meißner

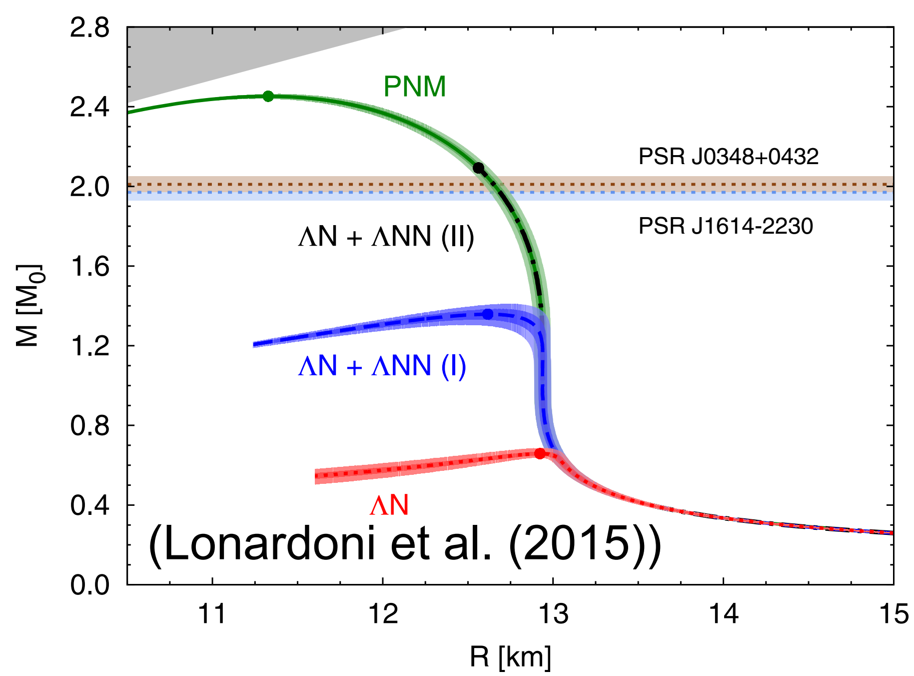
November 21st, 2023

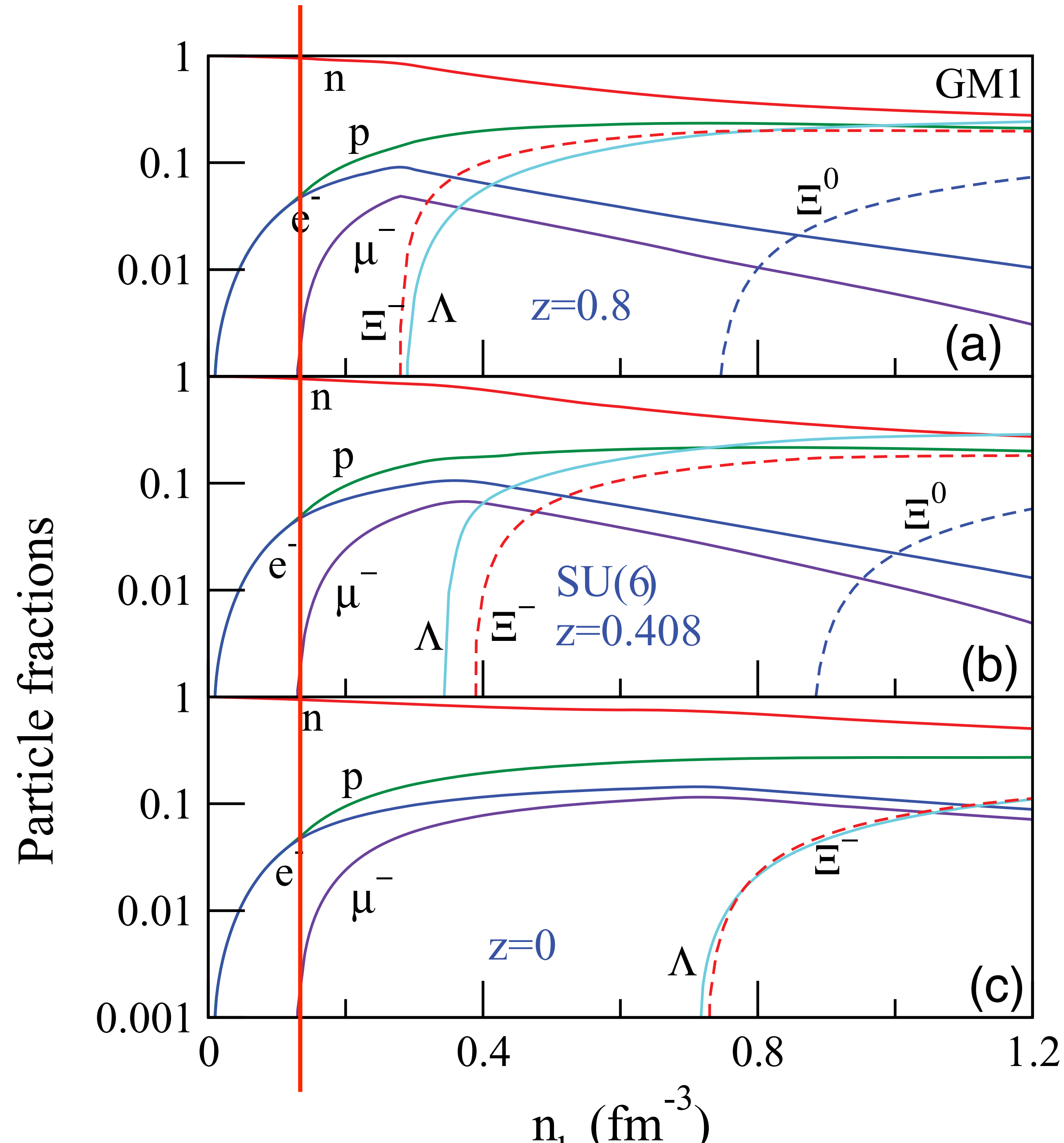
Motivation

Why is understanding hypernuclear interactions interesting?

- „phenomenologically“
  - *hyperon contribution to the EOS, neutron stars, supernovae*
  - *Λ as probe to nuclear structure*







(Weissenborn et al. 2012)

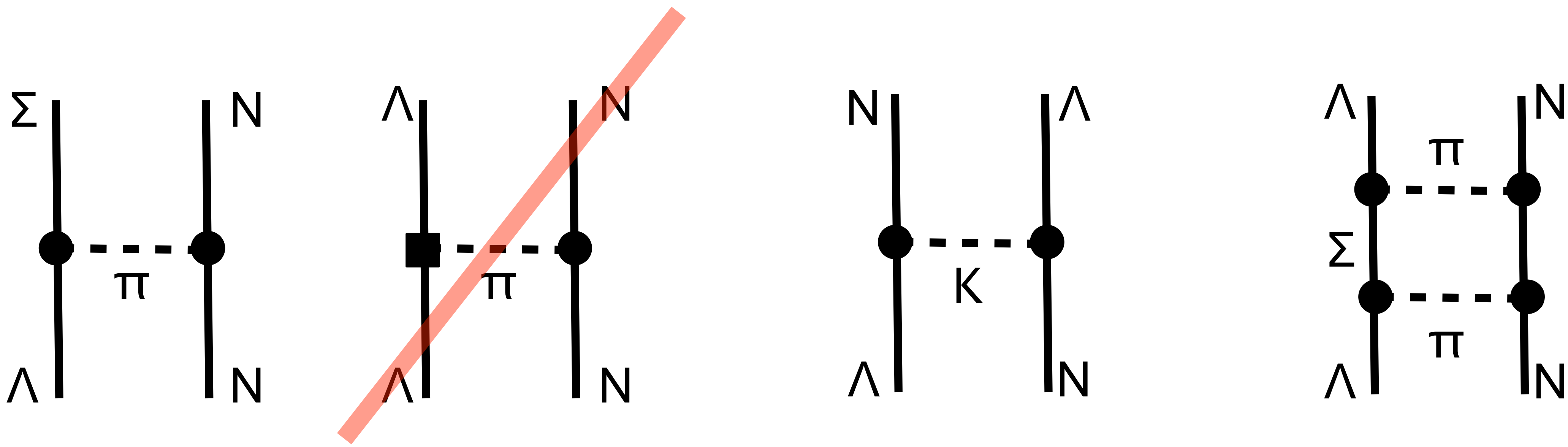
Testing hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

- Hypernuclear interactions have interesting properties

For example

- *Particle conversion process is sometimes long-range part of the interaction*
- *experimental access to explicit chiral symmetry breaking*



$m_K \approx 500$ MeV

suppressed by
isospin symmetry (CSB!)

Hypernuclei

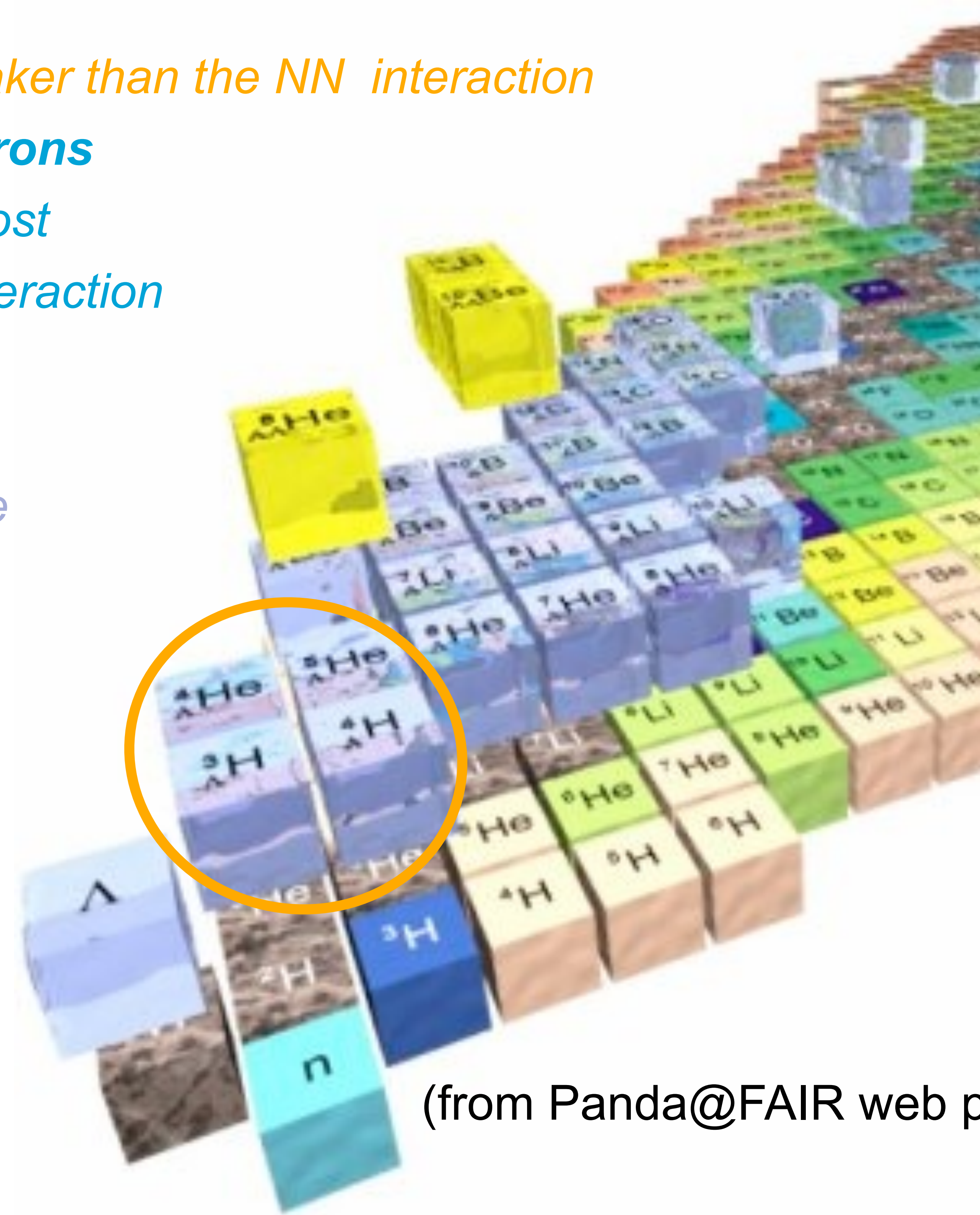
Hyperons can bind to nuclei. The binding energies are known experimentally.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies“ are almost independent from NN(+3N) interaction*

- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***

- ***non-trivial constraints*** *on the YN interaction even from lightest ones*

- *size of YNN interactions? need to include Λ-Σ conversion!*



(from Panda@FAIR web page)

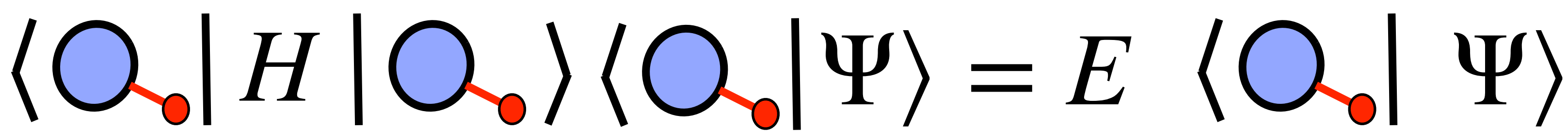
Jacobi-NCSM

Solve the Schrödinger equation using HO states

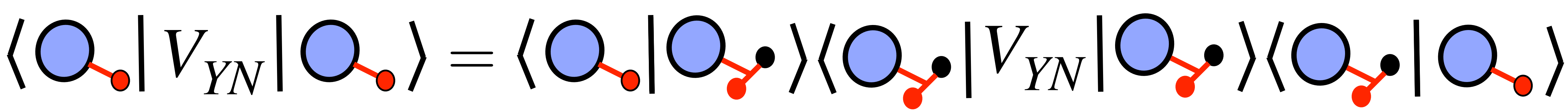
Two ingredients are necessary:

- **cfp** — antisymmetrized states for nucleons
- **transition coefficients** to separate off NN, YN, 3N and YNN

Schrödinger equation



e.g. for YN interaction



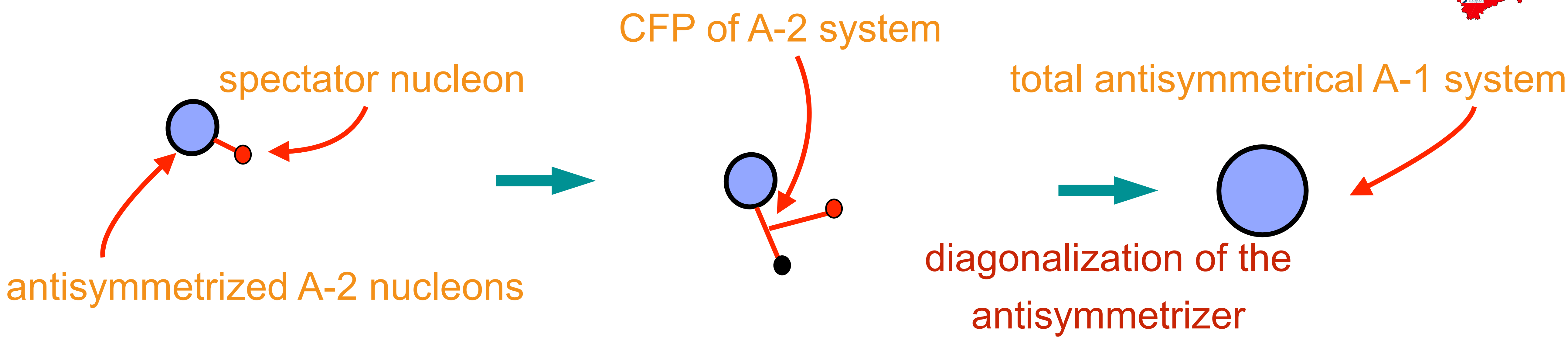
Application of to NN, YN, 3N and YNN interactions require the representation of particle transitions.

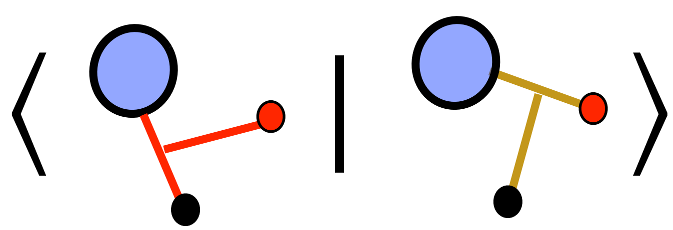
(see Liebig, Meißner, AN (2016),
Le, Haidenbauer, Meißner, AN (2020))

For **combinatorical factors** see Le, Haidenbauer, Meißner, AN (2021).

Jacobi-NCSM — CFP

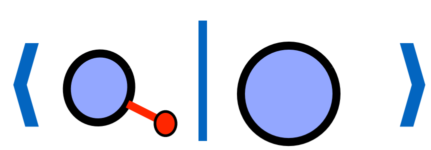
First, generate **antisymmetrized states** for the A-1 nucleon system





antisymmetrizer is equivalent to coordinate trafo
expression in terms of Talmi-Moshinsky brackets

(Navrátil, Kamuntavičius, Barrett (2000))

The CFP coefficients  are obtained by diagonalization of the antisymmetrizer.

HO states guarantee:

- complete separation of antisymmetrized and other states
- **independence** of HO **length/frequency**

These coefficients will be openly accessible as **HDF5** data files

(download server is in preparation *(please contact me when interested!)*)

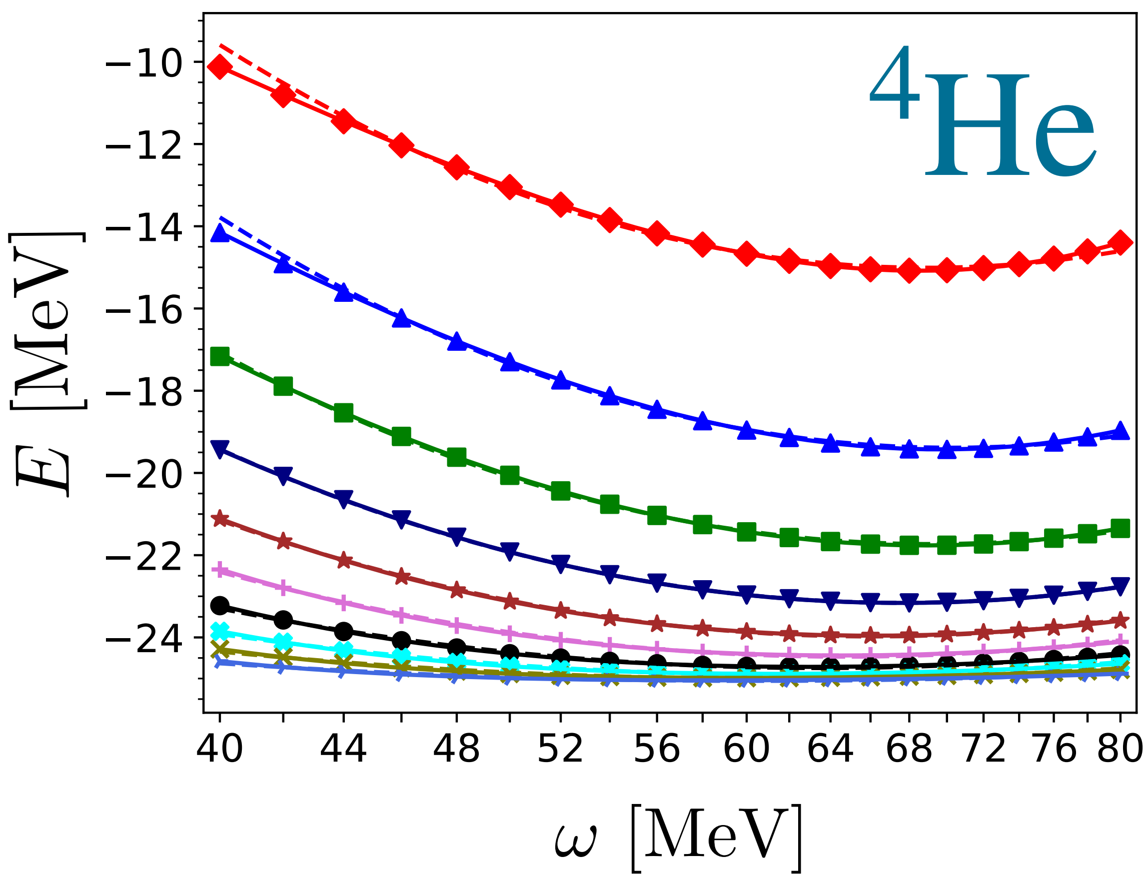
(Liebig, Meißner, AN (2016))

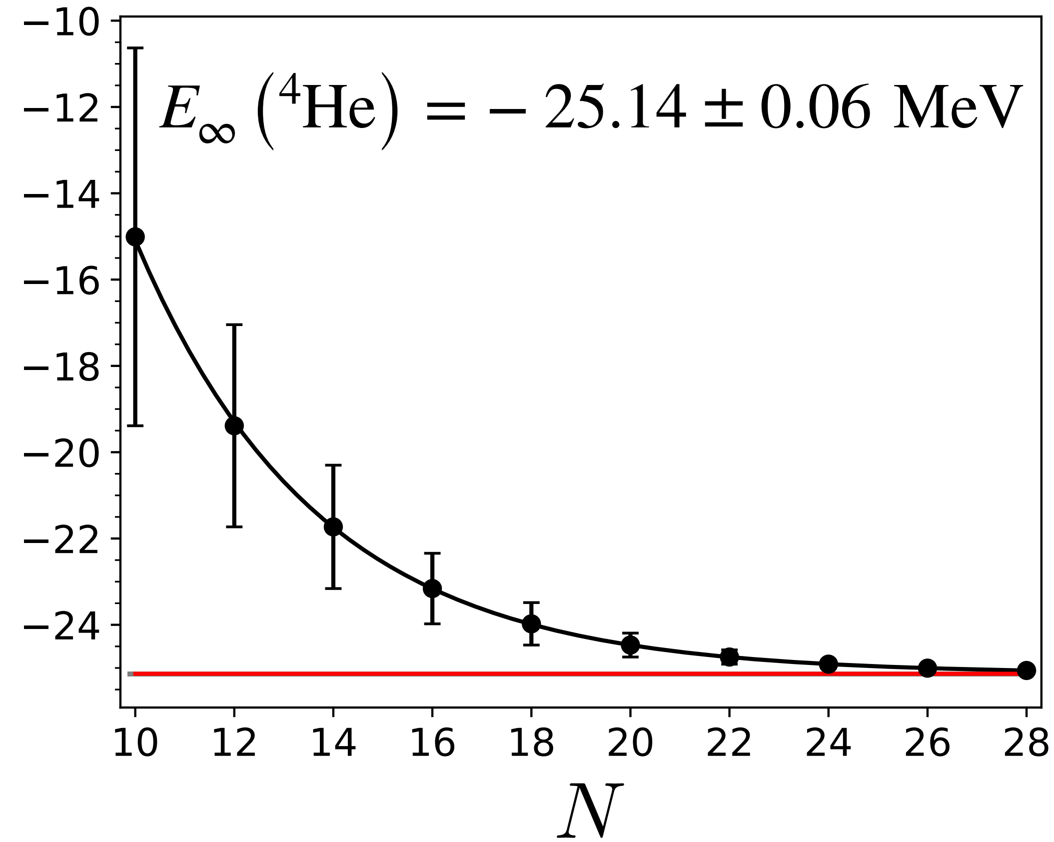
Convergence for Jacobi-NCSM

Simple example: ^{4}He with SMS N^{2}LO(550)

observed dependence on ω and N

$$E(\omega) = E_N + \kappa \left(\log(\omega) - \log(\omega_{opt})\right)^2 \longrightarrow E_N = E_\infty + A\, e^{-bN}$$





Conservative uncertainty estimate: difference of $E_{N_{\max}}$ and E_∞

Numerical uncertainties for light nuclei are small.

For p-shell, numerical uncertainty is more sizable due to smaller $N_{\max}$.

Hypernuclei convergence is slower since separation energies are smaller

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

$$\frac{dH_s}{ds} = \Big[\underbrace{[T\,,\,H(s)]}_{\equiv\eta(s)}\,,\,H(s) \Big] \qquad H(s) = T + V(s)$$

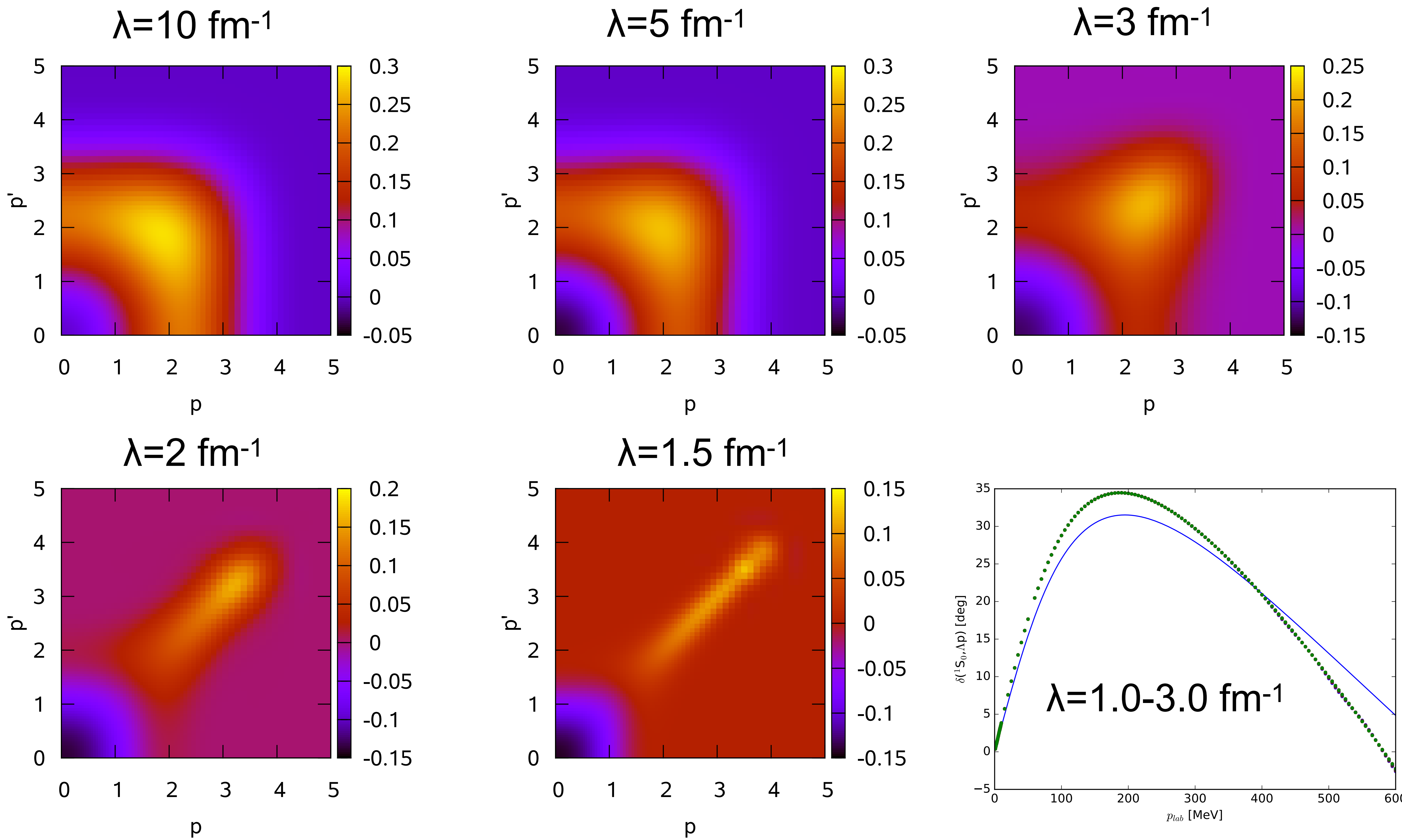
this choice of generator drives *V(s)* into a diagonal form in momentum space

- *V(s)* will be **phase equivalent** to original interaction
- short range *V(s)* will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, … body level** (approximation)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- **dependence** of results on λ or s is a measure for **missing terms**

(Bogner et al., 2007)

SRG interactions (YN)

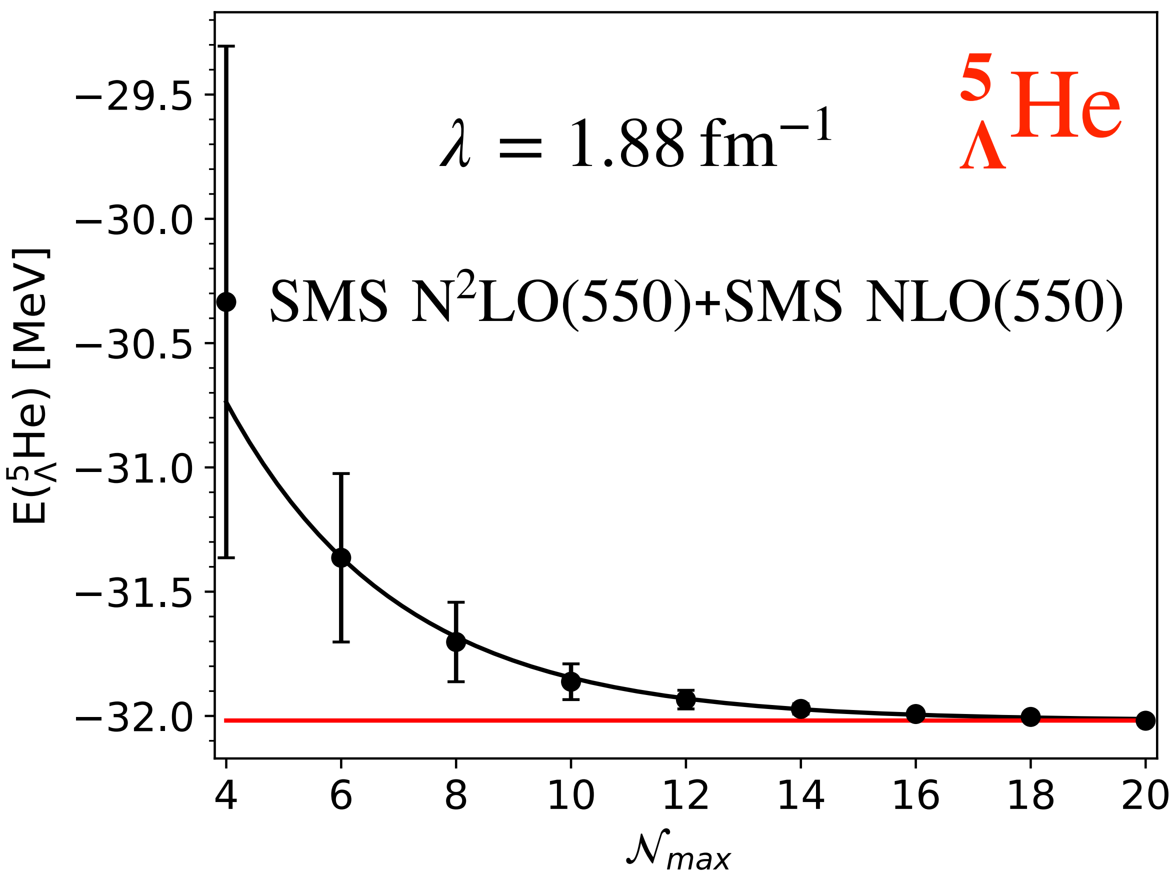
Λp-Λp matrix element for the $^1S_0$ depending on incoming and outgoing momenta

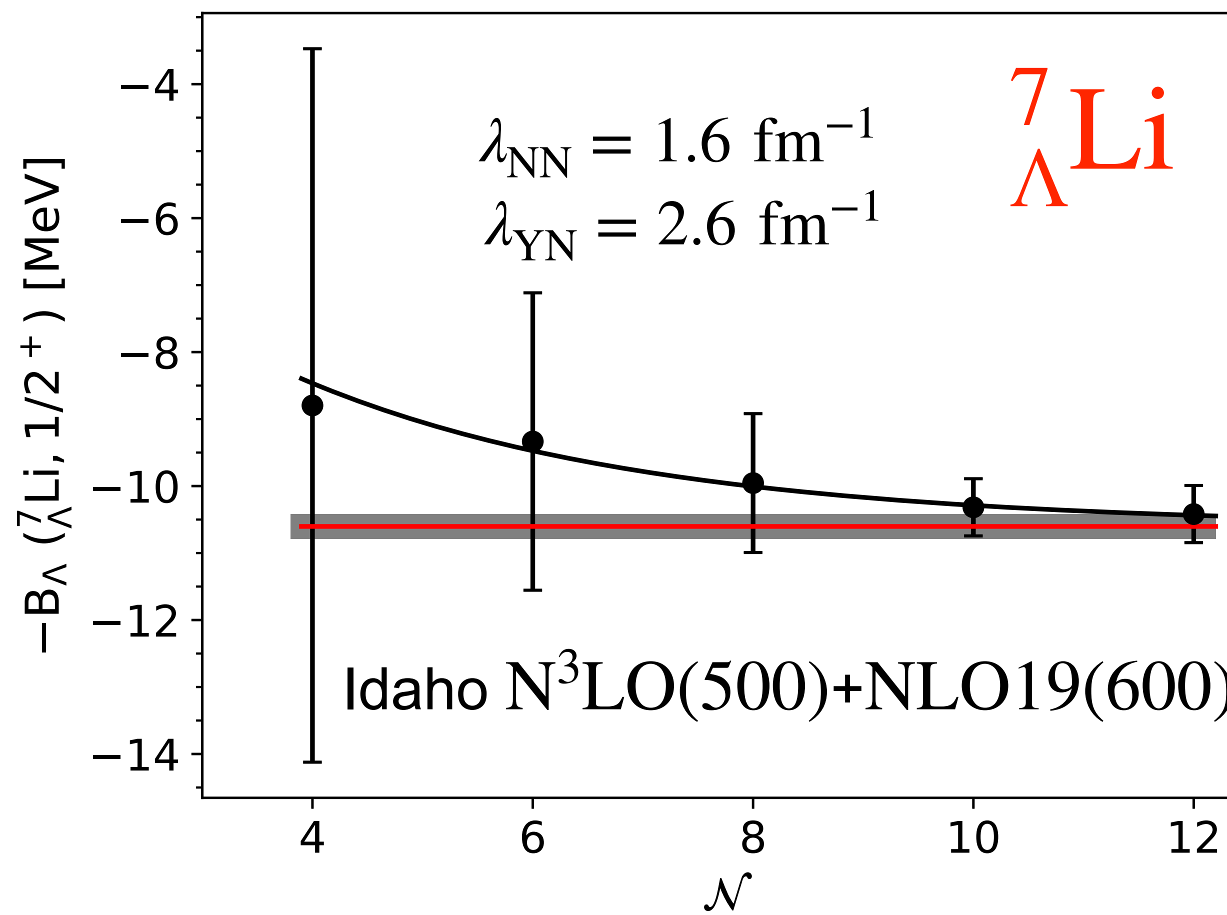


SC97f compared to SRG of EFT-NLO-600

J-NCSM convergence

SRG evolution improves convergence



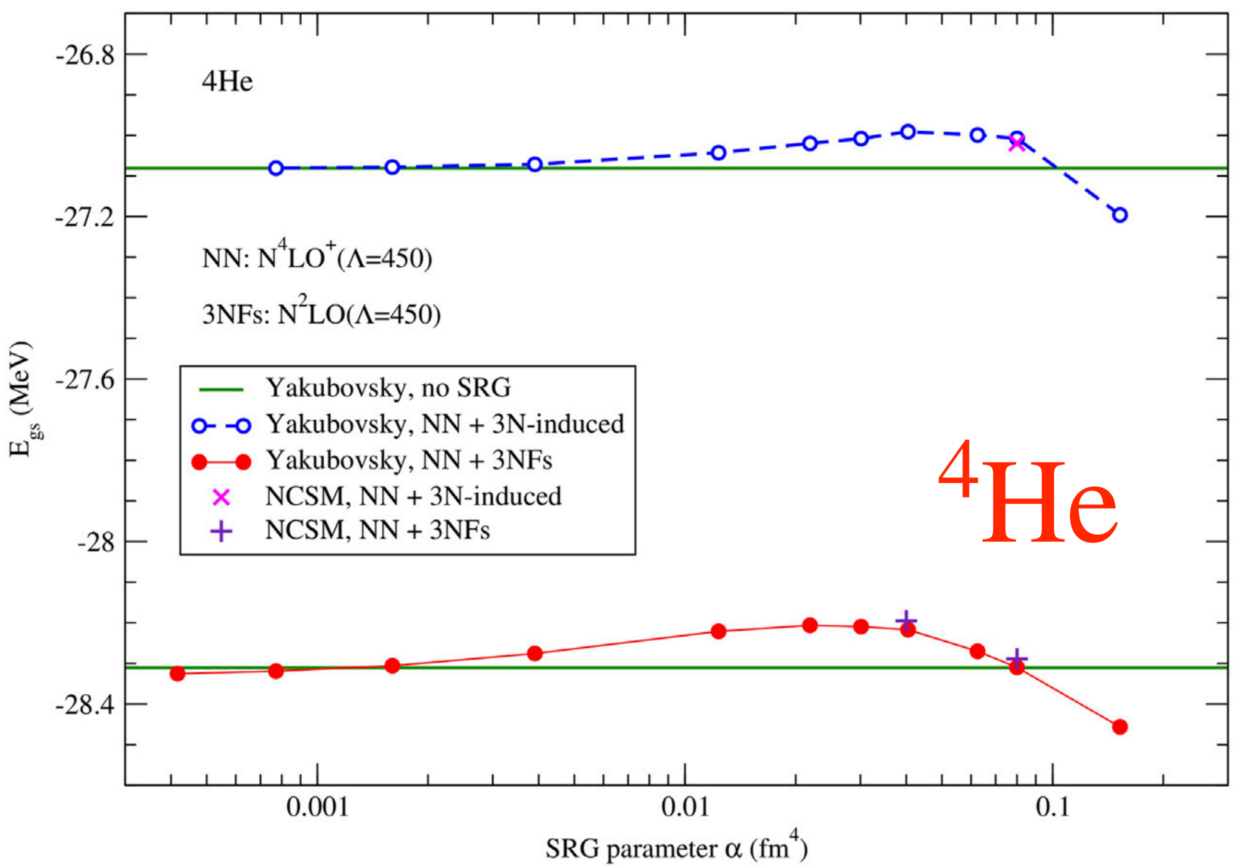


$$E\left({}^{5}_{\Lambda}\mathrm{He}\right) = -32.018 \pm 0.001 \ \mathrm{MeV} \qquad E_{\Lambda}\left({}^{7}_{\Lambda}\mathrm{Li}\right) = 10.6 \pm 0.2 \ \mathrm{MeV}$$

- for light nuclei and hypernuclei, the numerical uncertainty is negligible.
- for p-shell nuclei/hypernuclei, the uncertainty is visible
- extrapolation of separation energy can reduce uncertainty of this quantity

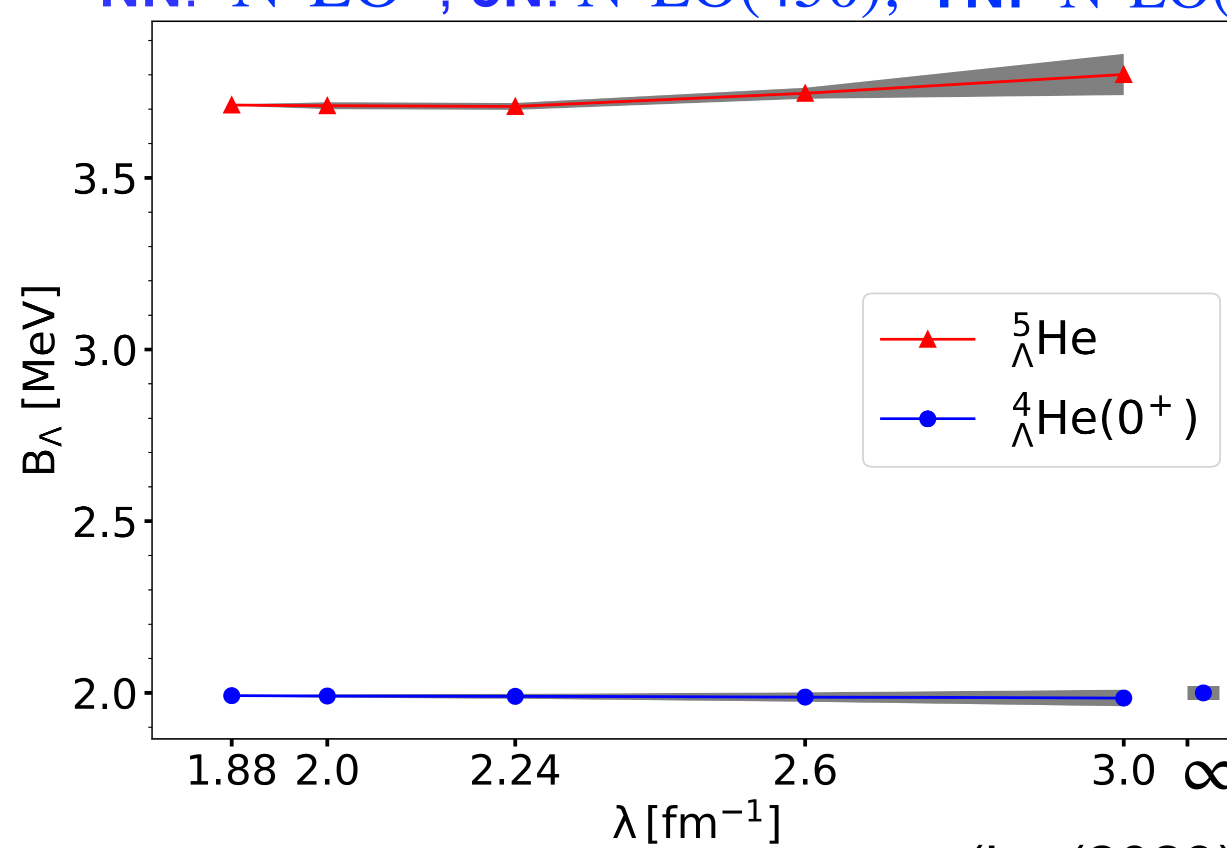
SRG dependence of results

- SRG-induced 3N and YNN interactions
- ^{4}He binding energies varies by $\approx 100 - 200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris, Le, Nogga, Roth, Vary (2023))

NN: N^{4}LO$^+$, **3N**: N^{2}LO(450); **YN**: N^{2}LO(550)



(Le (2023))

For hypernuclei, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

Study uncertainty due to chiral expansion of NN and YN interactions

Uncertainty analysis to $A = 3$ to 5

Order N²LO requires combination of chiral NN, YN, 3N and YNN interaction

Need calculation of separation energies (use Faddeev, Yakubovsky eq. or J-NCSM) and use **different orders** for uncertainty estimate.

Assuming a negligible numerical uncertainty and the following ansatz for the order by order convergence

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, } \ldots)$$

a **Bayesian analysis** of the uncertainty is possible (see Melendez et al. 2017,2019)

Extracting c_k for $k \leq K$ **from calculations** and assuming identical probability distributions for c_k for $k > K$ the uncertainty is given by the distribution of

$$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$$

Uncertainty analysis to $A = 3$ to 5

How to obtain the distribution for c_k ?

EFT expectation: c_k **are natural-sized**, i.e. of order 1.

→ defines prior distribution (usually normal distribution with width $\bar{c}$)

$\bar{c}$ is distributed using an inverse-χ^2 distribution (parameters ν_0, τ_0)

For this choice, the posterior then follows the same distribution (conjugate prior) with shifted parameters given the data:

$$\nu = \nu_0 + n_c \qquad \nu\tau^2 = \nu_0\tau_0^2 + \vec{c}_k^{\,2} \qquad (\vec{c}_k^{\,2} = \sum c_k^2 \text{ for } n_c \text{ values extracted})$$

→ uncertainty follows so-called student t distribution (analytically known)

allows to extract degree of believe intervals (DoB)

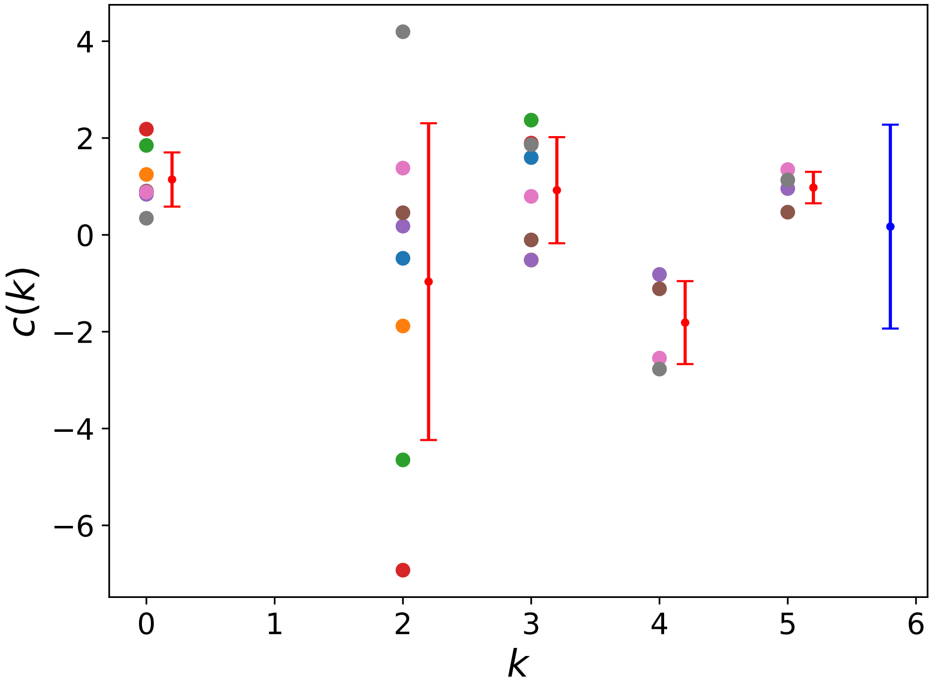
dependence on choice of prior will be less for large n_c !

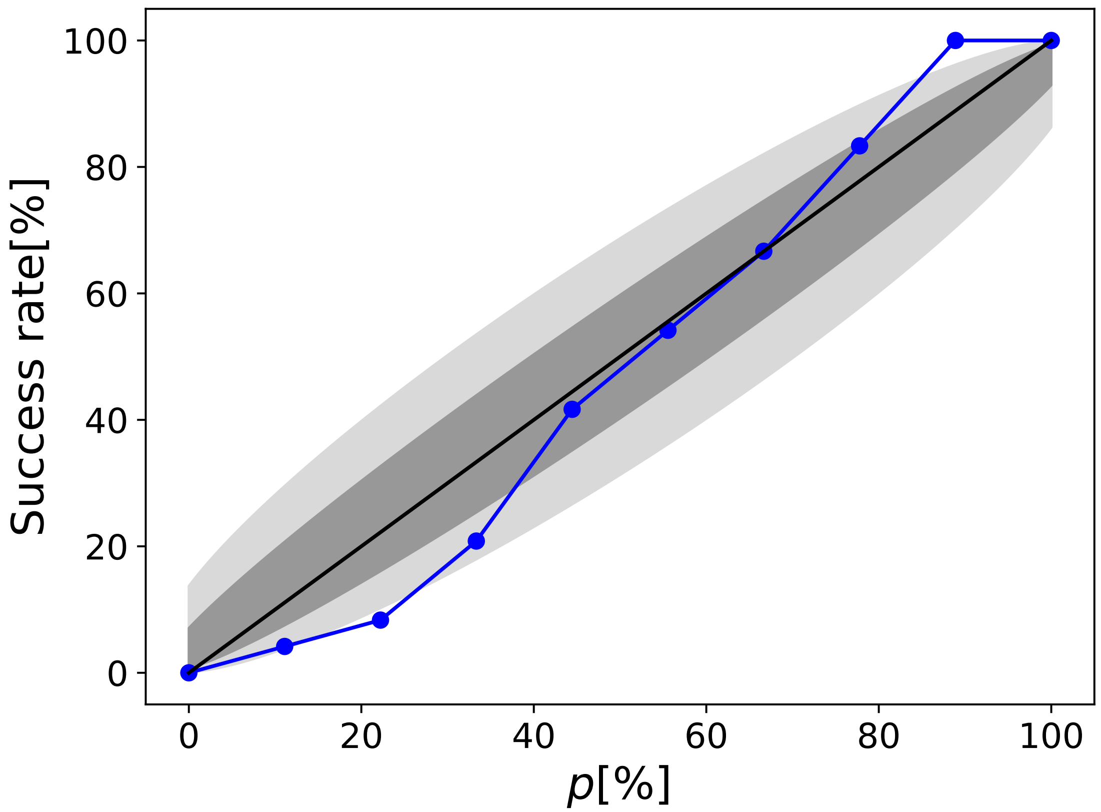
Uncertainty analysis to $A = 3$ to 5

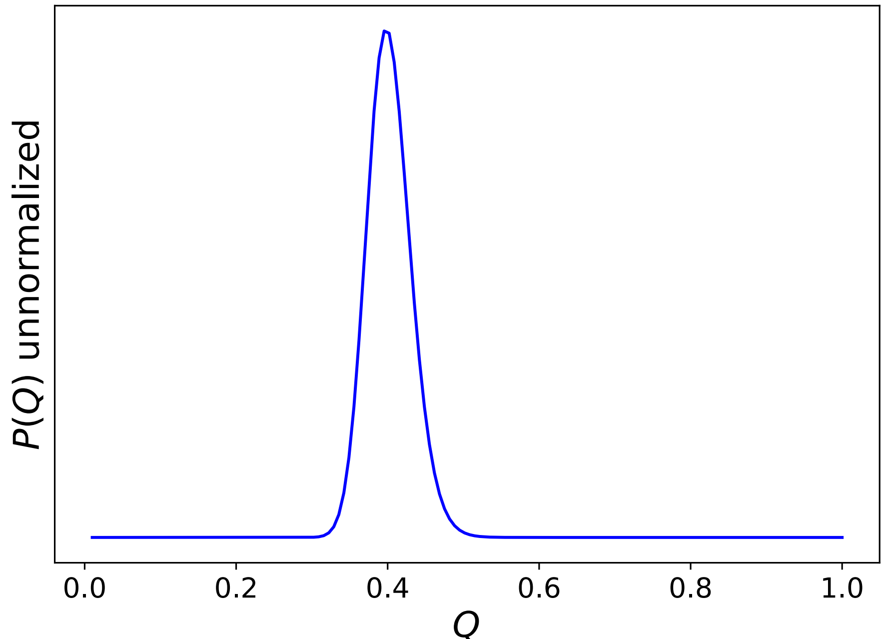
- expansion parameter Q should be consistent with assumption of k independent distribution of c_k
- distribution of of prior should be consistent with observed pattern for c_k
- few orders used cannot entirely remove prior dependence

$$Q = 0.4$$
$$\tau_0^2 = 2.25$$
$$\nu_0 = 1.5$$

(see also Maris et al. 2022)







Hoai Le et al. arXiv:2308.01756.

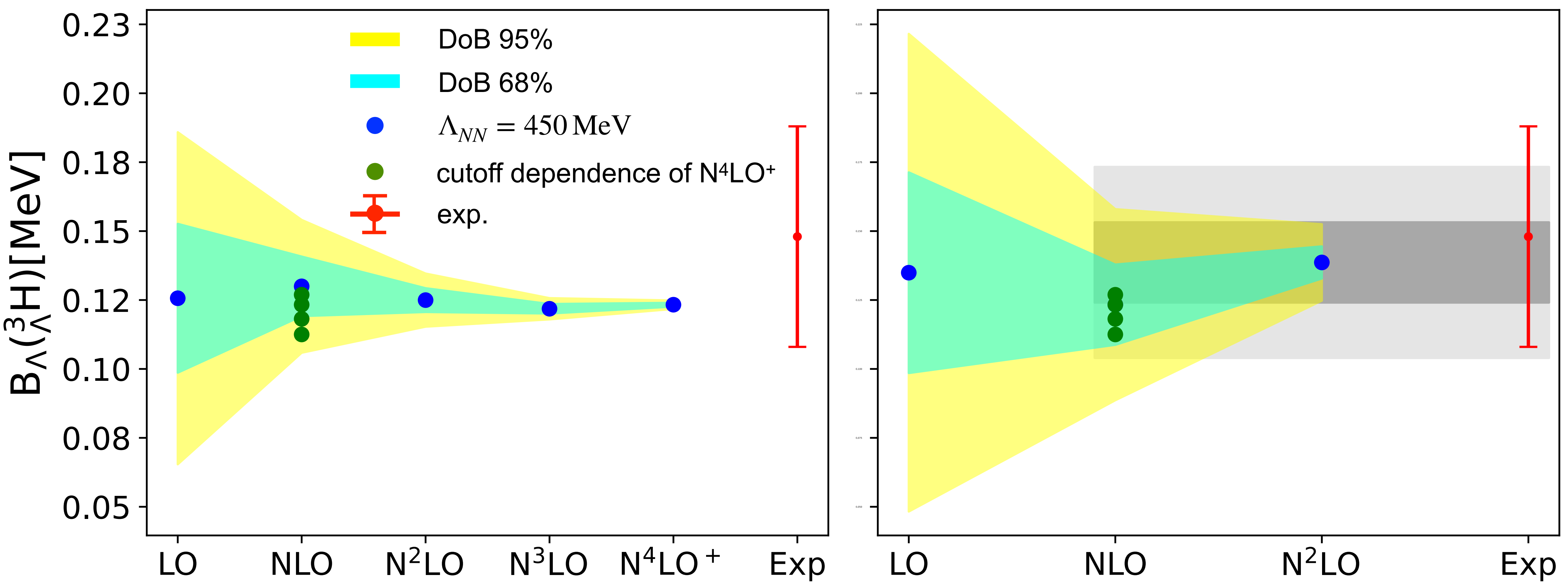
Application to ${}^{3}_{\Lambda}\mathrm{H}$

- Q, ν_0 and τ_0 are chosen using all available data (NN and YN convergence)
- uncertainties are extracted using c_k for NN or YN convergence
- use c_k of individual hypernuclei

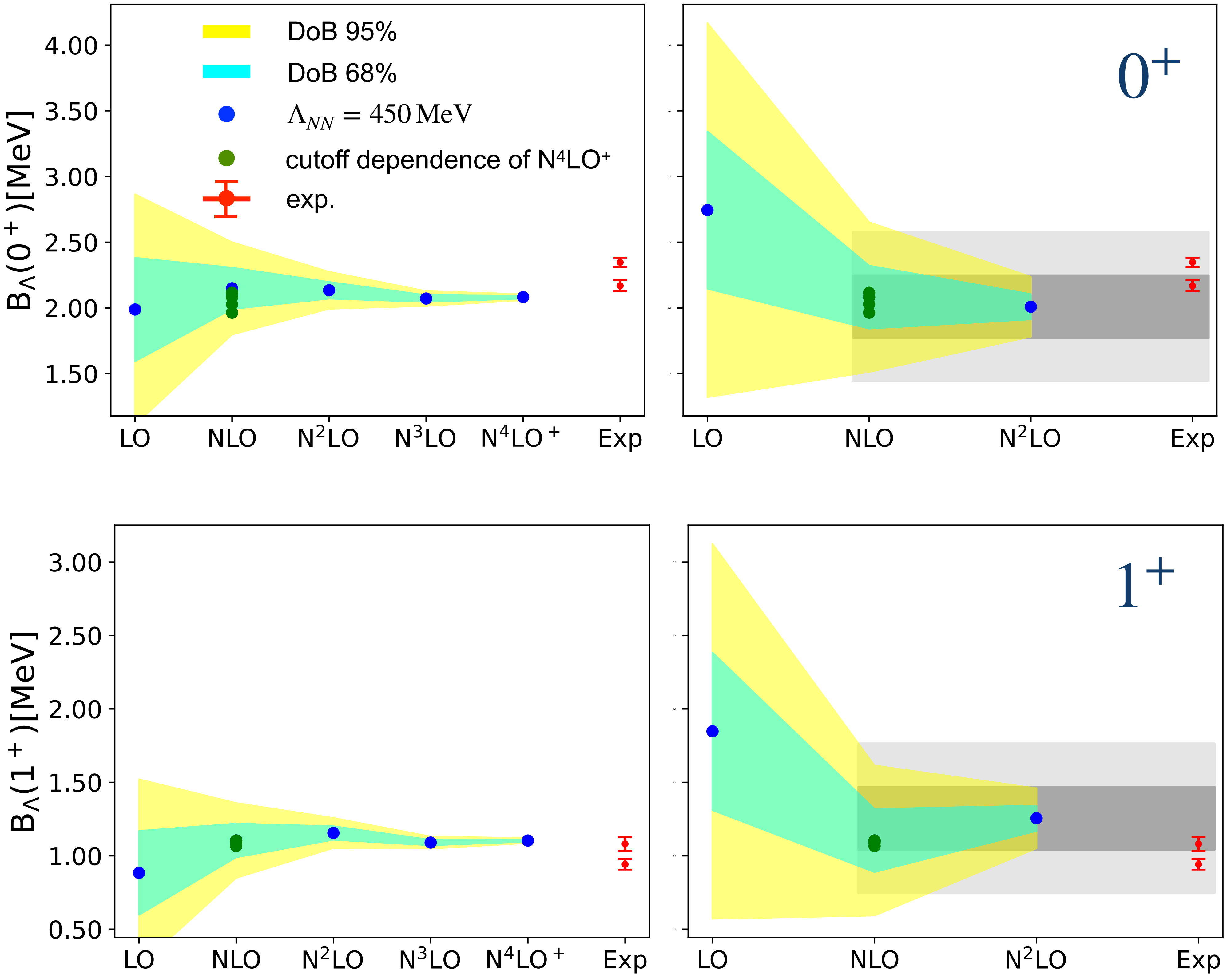
→ individual uncertainties for NN and YN convergence for each separation energy

consistent with experimental data

cutoff dependence always at least NLO (YNN missing!)



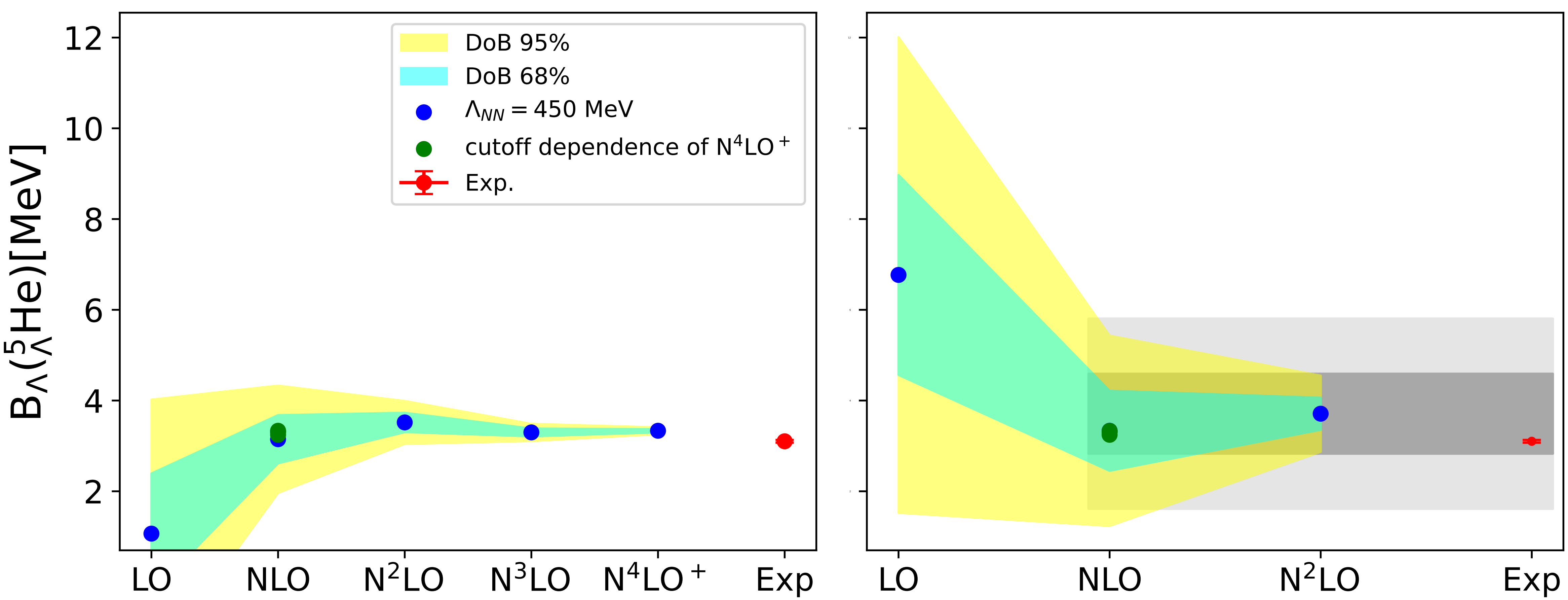
Application to $^{4}_{\Lambda}\text{He}$



Application to ${}^5_\Lambda\mathrm{He}$ and summary

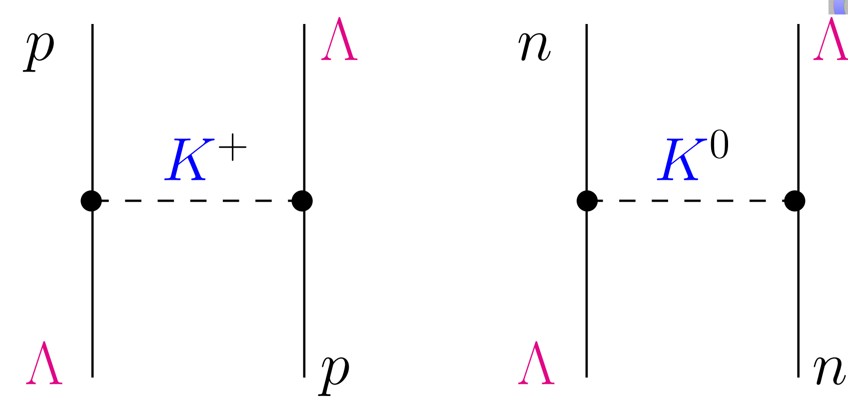
- without YNN: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^3_\Lambda\mathrm{H}$	0.011	0.015
${}^4_\Lambda\mathrm{He}\,(0^+)$	0.157	0.239
${}^4_\Lambda\mathrm{He}\,(1^+)$	0.114	0.214
${}^5_\Lambda\mathrm{He}$	0.529	0.881



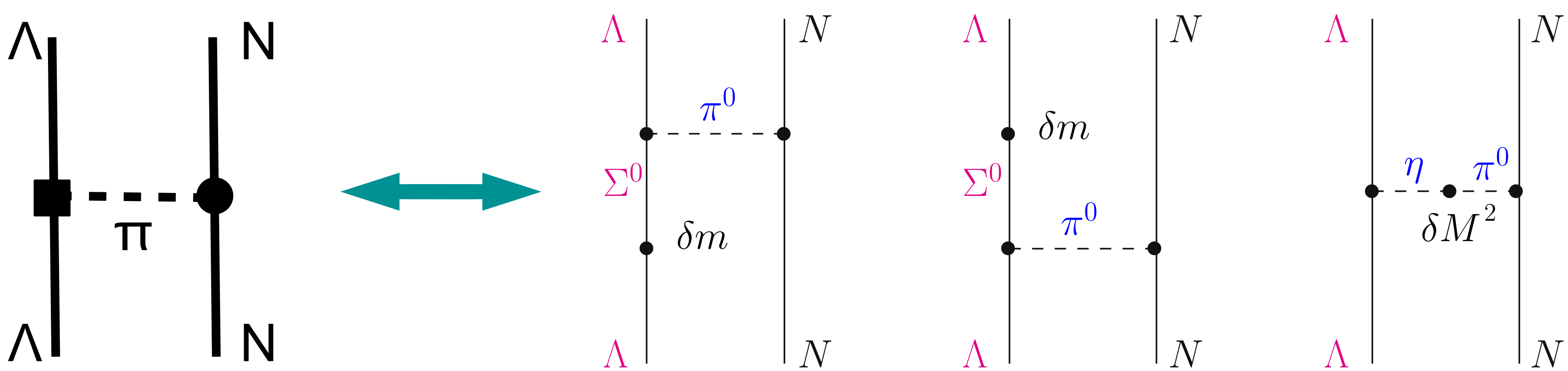
CSB contributions to YN interactions

- **formally leading** contributions:
  Goldstone boson mass difference
  - — very small due to the small relative difference of kaon masses

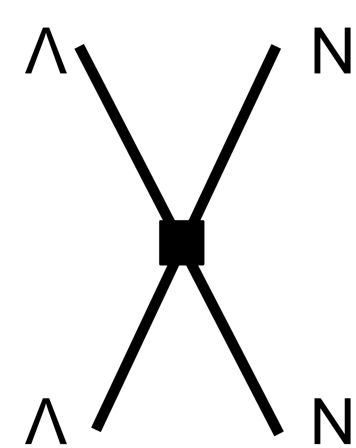


- **subleading but most important**
  — effective CSB ΛΛπ coupling constant (Dalitz, van Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0|\delta m|\Lambda\rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0|\delta M^2|\eta\rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$



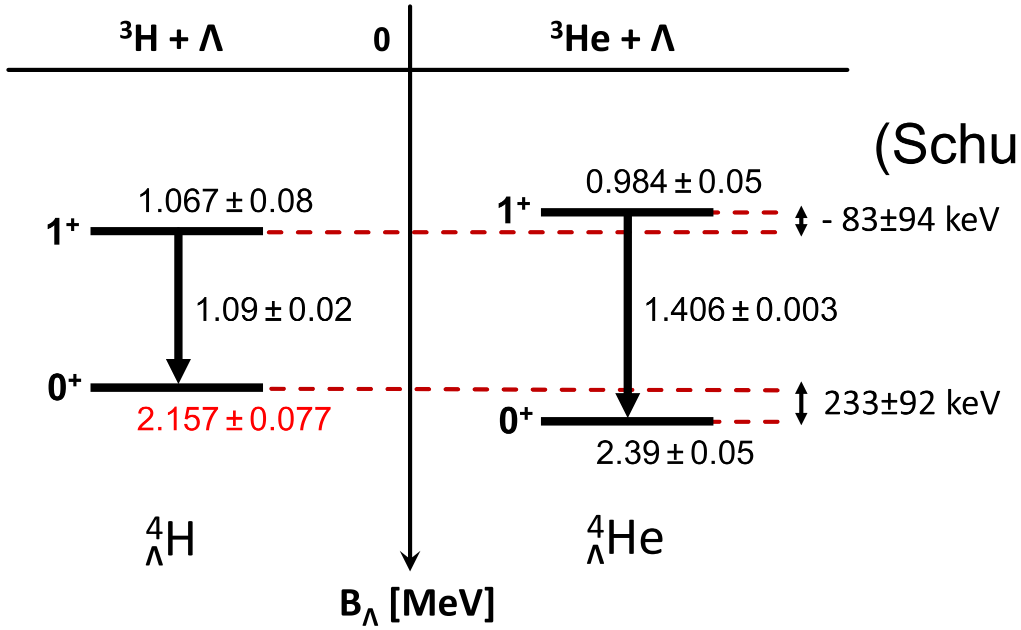
- **so far less considered, but equally important**
  — CSB contact interactions (for singlet and triplet)



Aim: use A=4 hypernuclei to determine the two unknown CSB LECs and predict Λn scattering

(so far: NLO13 and NLO19)

Fit of contact interactions



(Schulz et al.,2016; Yamamoto, 2015)

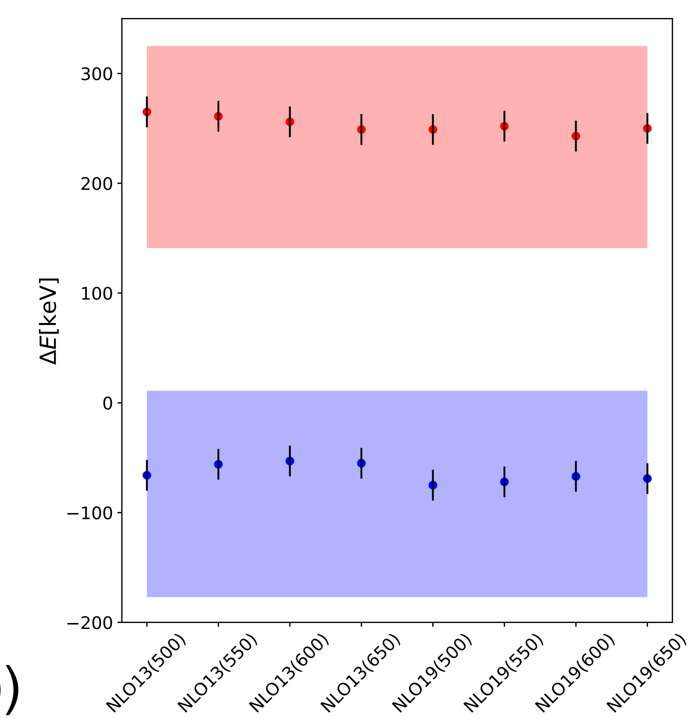
- Adjust the two CSB contact interactions to one main scenario (**CSB1**)

Λ	NLO13		NLO19	
	C_s^{CSB}	C_t^{CSB}	C_s^{CSB}	C_t^{CSB}
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

The values of the LECs are in 10^4 GeV^{-2}

- Size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d}\left(\frac{M_\pi}{\Lambda}\right)^2 \quad C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4\,\text{GeV} \propto 6 \cdot 10^{-3} \cdot 10^4\,\text{GeV}$$

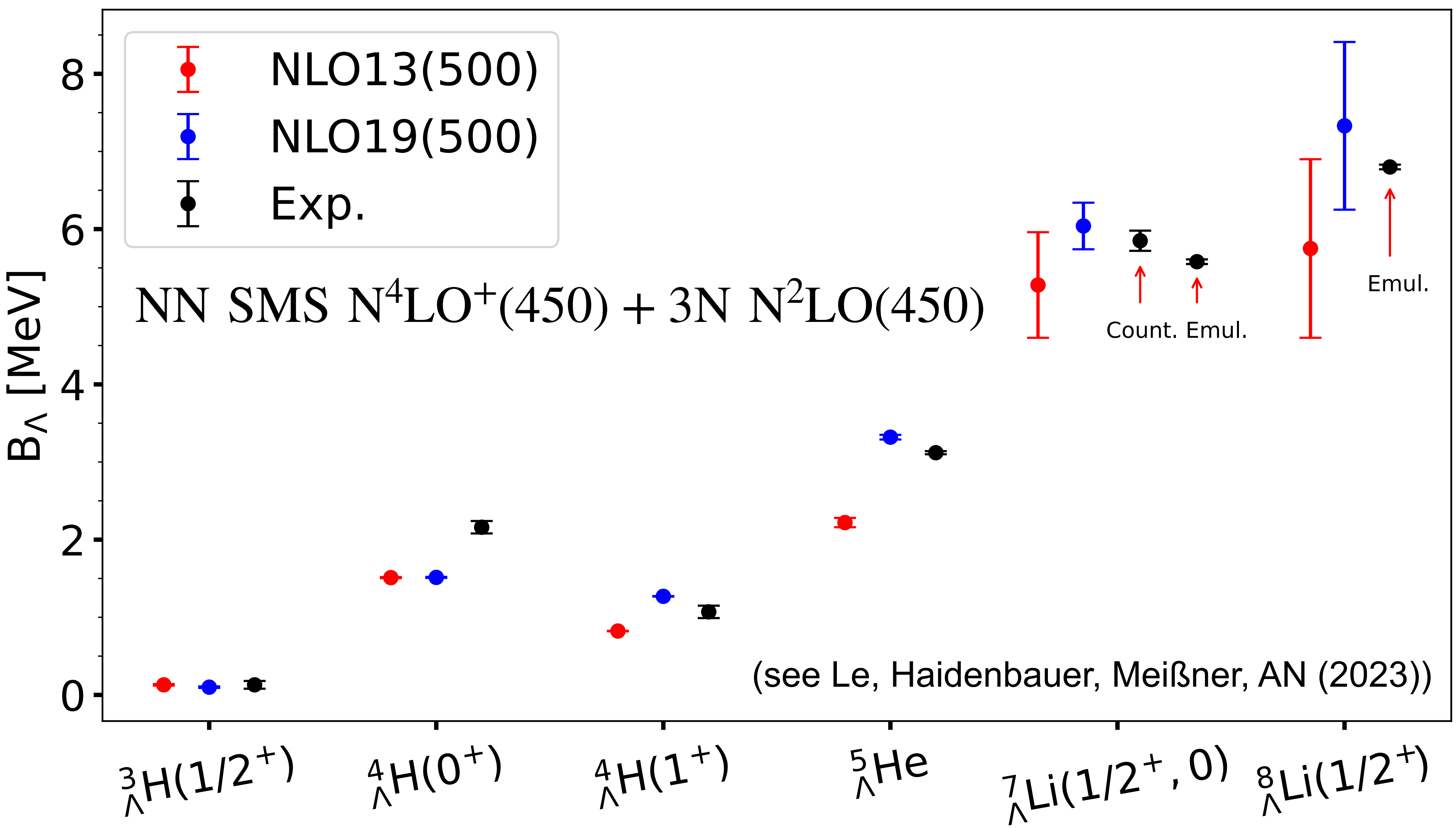


- Problem: large experimental uncertainty of experiment
- here only **fit to central values** to test theoretical uncertainties

(see Haidenbauer, Meißner, AN (2021))

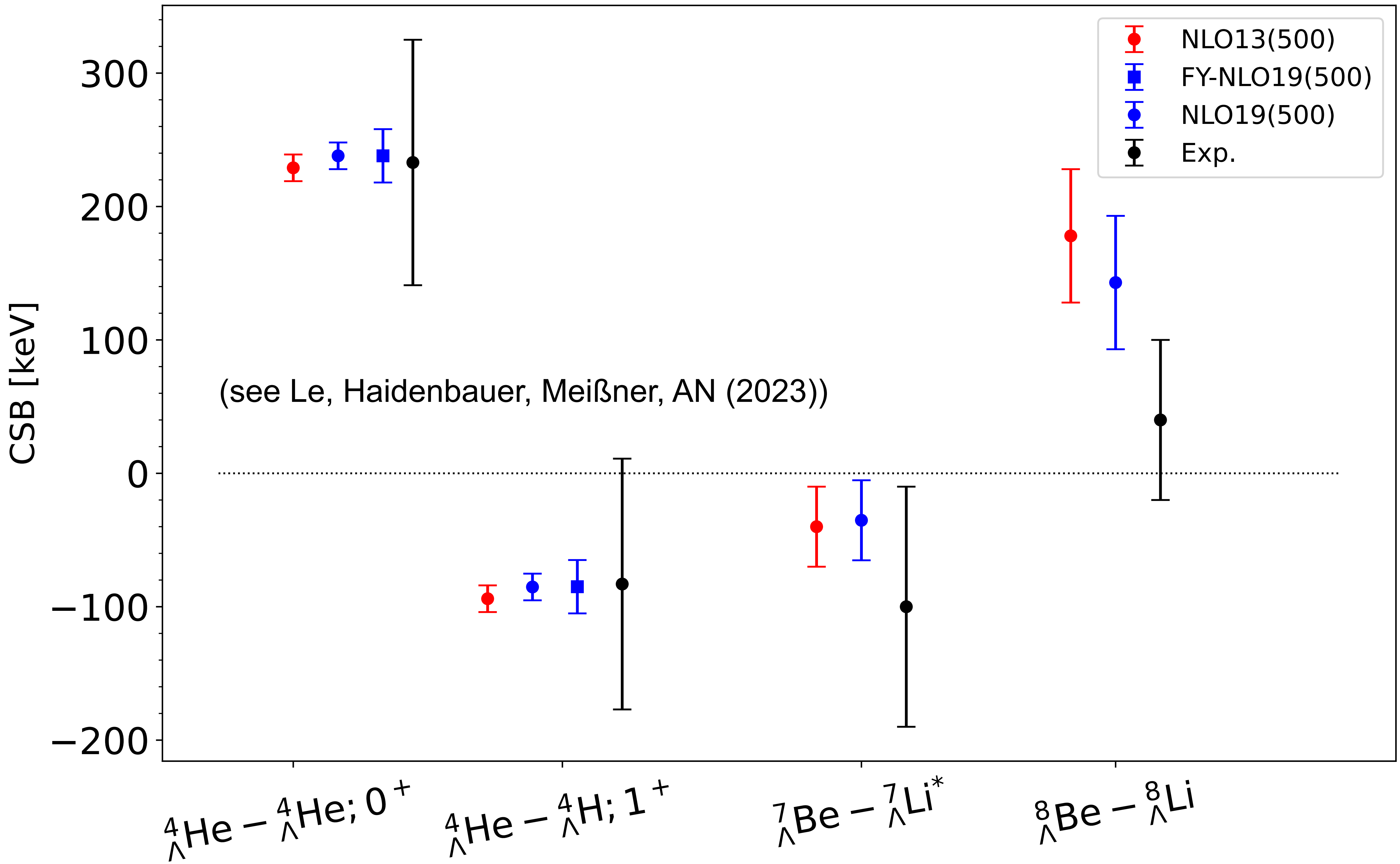
Application to $A = 7$ and 8

- YN interaction adjusted to the hypertriton — YNN is small
- based only on YN interactions: splitting for ${}^4_\Lambda\mathrm{H}$ is not well reproduced — YNN(?)
- NLO19 gives better results for ${}^5_\Lambda\mathrm{He}$ and heavier hypernuclei
  — accidentally small YNN interaction?
- uncertainties are numerical — no estimate of chiral uncertainties yet



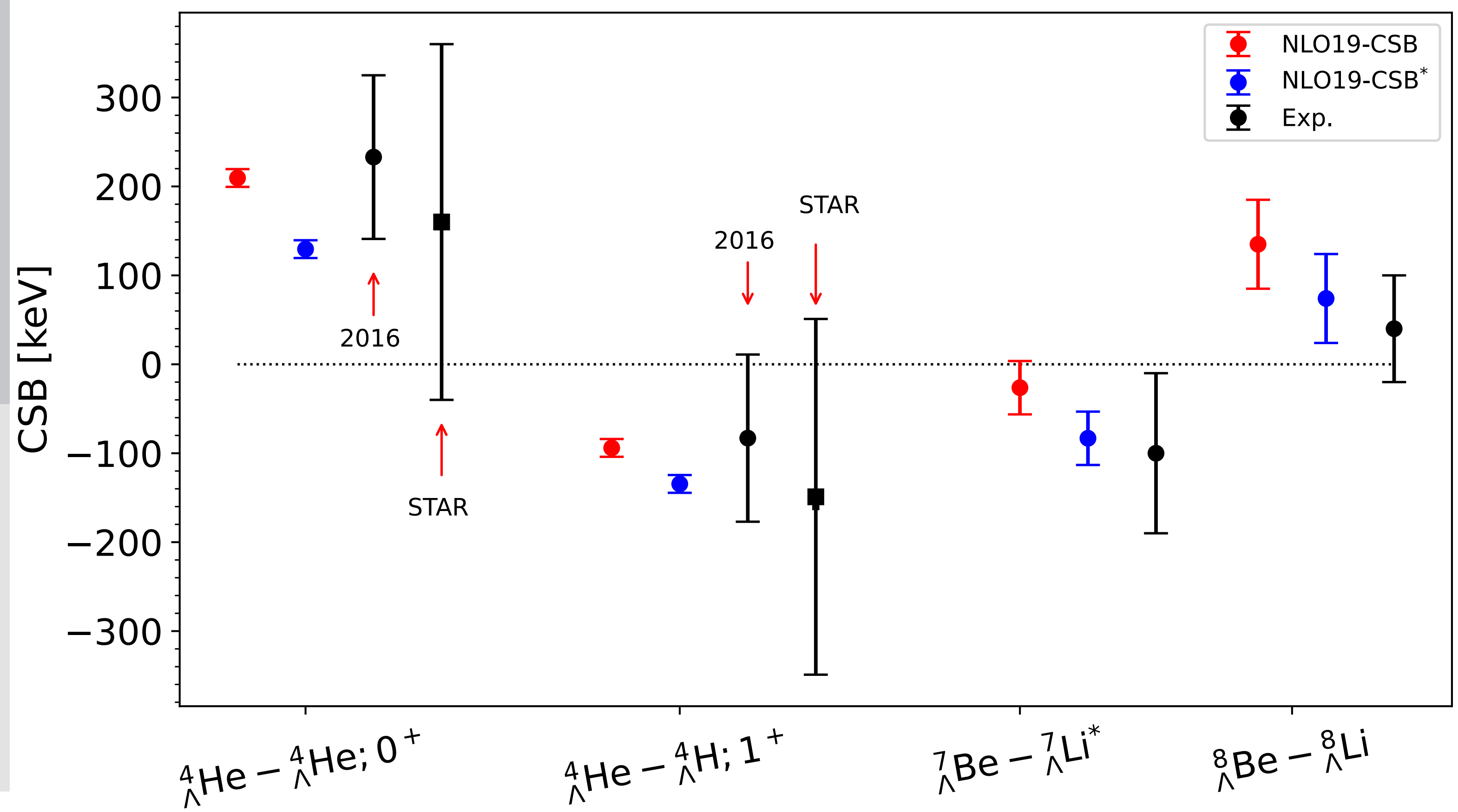
Application to $A = 7$ and 8

- CSB of singlet and triplet states interferes differently
- CSB still not fixed — experimental uncertainty is large
- scenario studied here is only marginally consistent with CSB in $A = 8$



New STAR data for $A = 4$ CSB

- fit to STAR data only
- only slight adjustment required
- improves description to p-shell CSB
- higher experimental accuracy is desirable
- good example of using hypernuclei to determine YN interactions



	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

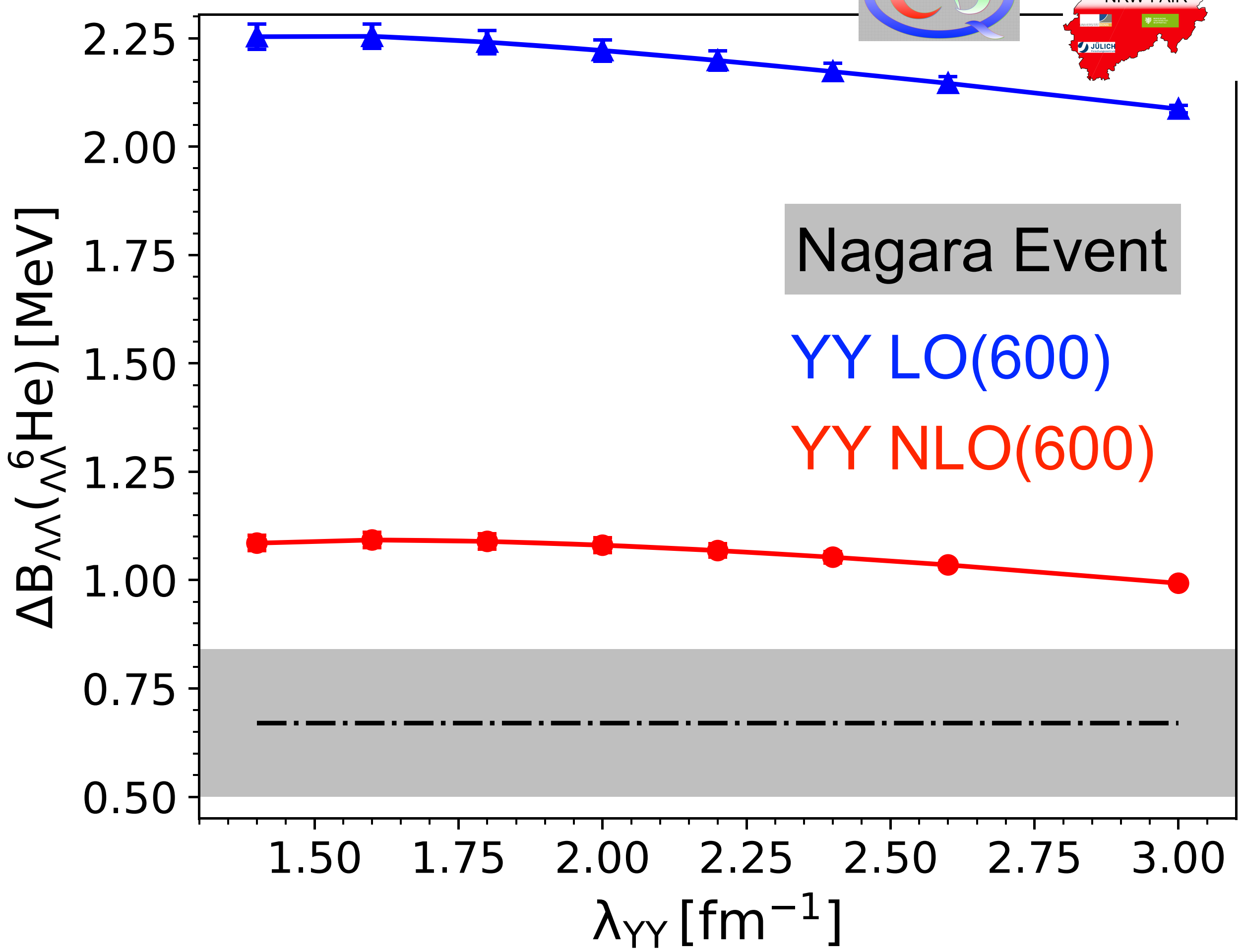
(see Le, Haidenbauer, Meißner, AN (2023))

$S = -2$ hypernuclei — $^{6}_{\Lambda\Lambda}\mathrm{He}$

- $\Lambda\Lambda$ excess binding energy

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$$
$$= 2E\left({}^{A-1}_{\ \Lambda}X\right) - E\left({}^{\ A}_{\Lambda\Lambda}X\right) - E\left({}^{A-2}X\right)$$

- NN, YN and YY interactions contribute
- use NN and YN that describe nuclei and single Λ hypernuclei
- small λ_{YY} dependence (no induced YYN forces used!)
- LO overbinds YY
- **NLO predicts binding fairly well**

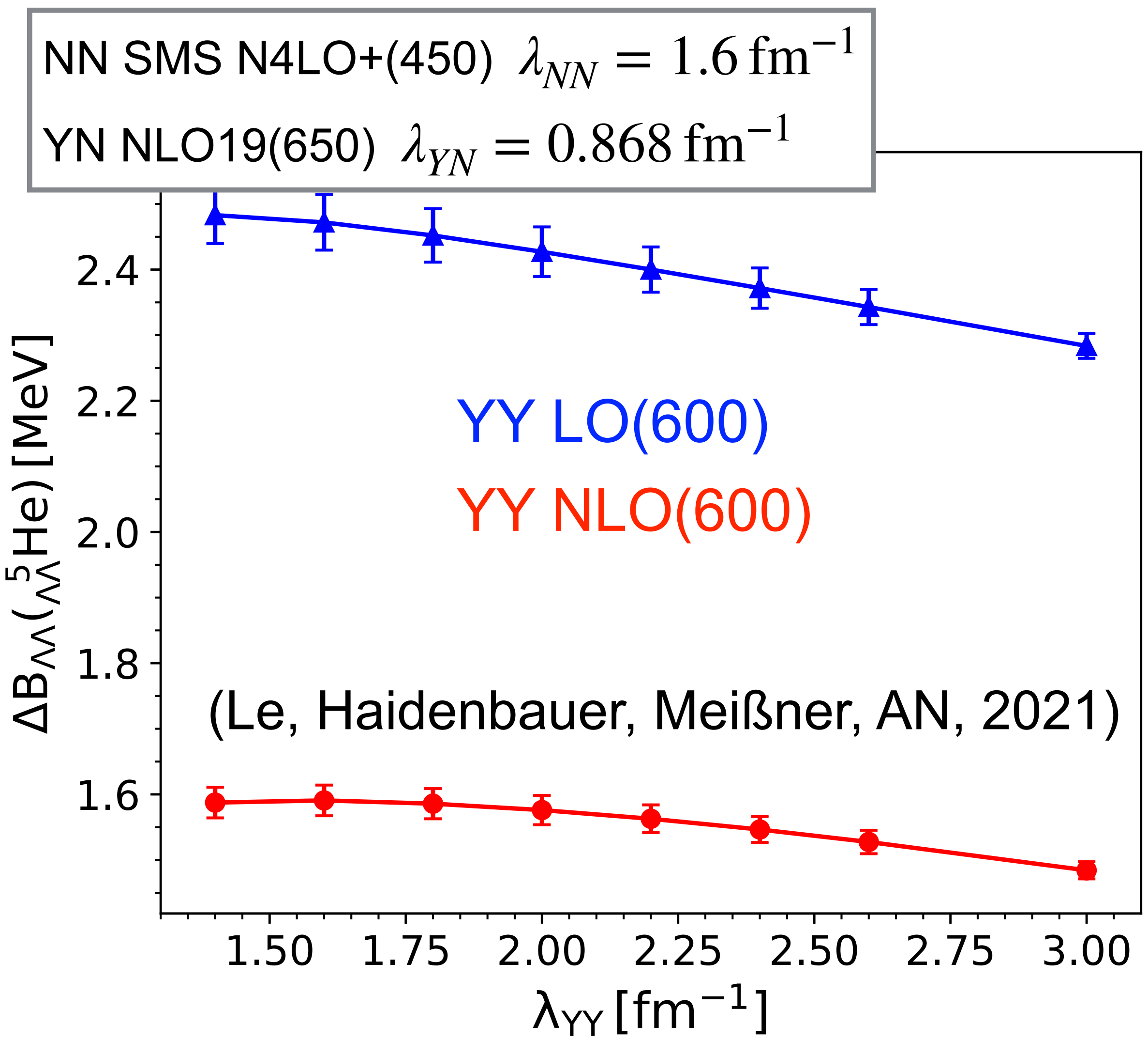


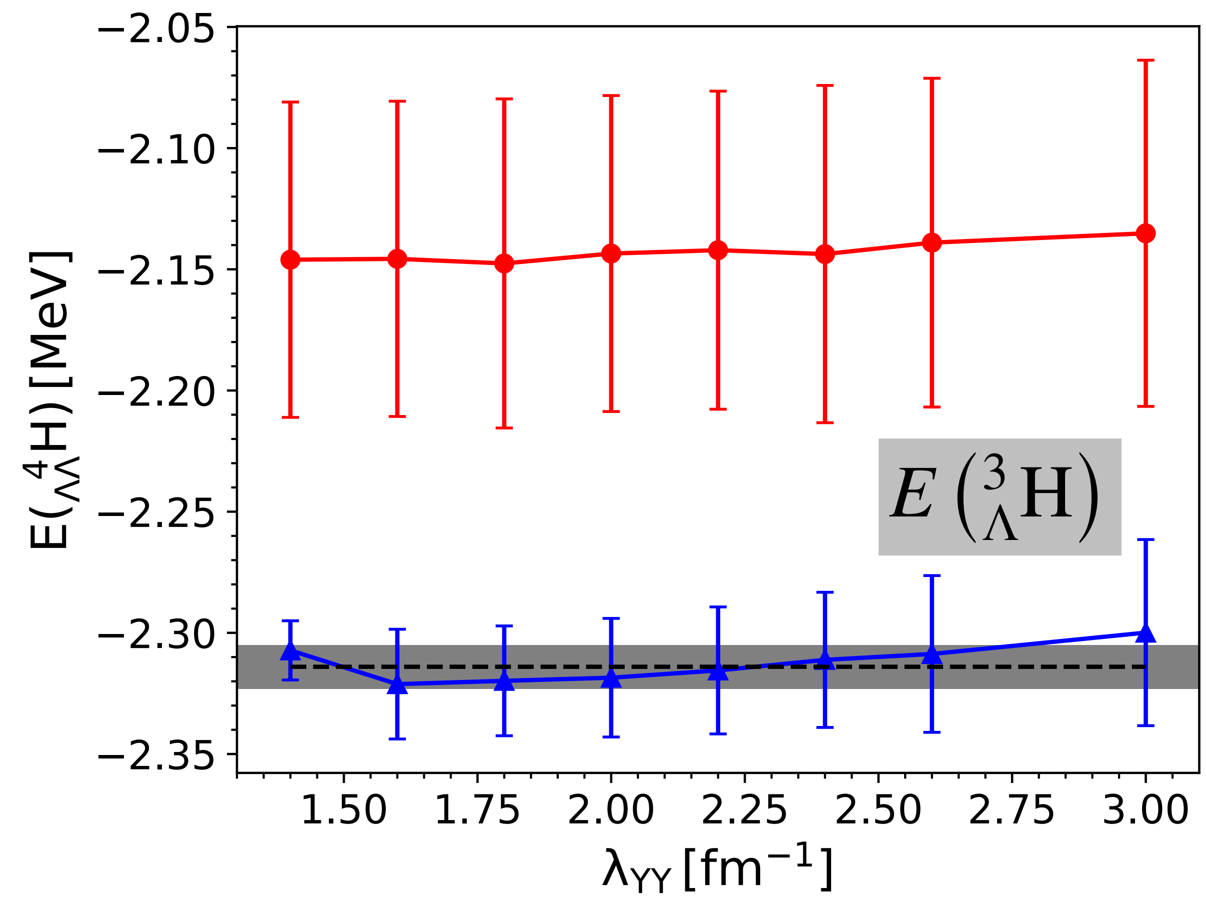
NN SMS N4LO+(450) $\lambda_{NN} = 1.6\ \mathrm{fm}^{-1}$
YN NLO19(650) $\lambda_{YN} = 0.868\ \mathrm{fm}^{-1}$

Can an $S = -2$ bound state for $A = 4{,}5$ be expected?

(Le, Haidenbauer, Meißner, AN, 2021)

$S = -2$ hypernuclei — ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ & ${}^{4}_{\Lambda\Lambda}\mathrm{H}$





- $A = 5$: $\Lambda\Lambda$ excess binding energy & $A = 4$: binding energy
- $A = 5$: LO & NLO predicts bound state
- $A = 4$: NLO unbound, LO at threshold to binding (see also Contessi et al., 2019)
- excess energy larger for $A = 5$ than for $A = 6$ (in contrast to Filikhin et al., 2002!)

$S = -2$ bound state for $A = 5$ can be expected,

for $A = 4$ less likely but not ruled out!

Ξ hypernuclei

- experimentally accessible: Ξ^- capture process (experimental data for ${}^{15}_{\Xi}\mathrm{C}$ and ${}^{12}_{\Xi}\mathrm{Be}$)
- $\Xi N - \Lambda\Lambda$ conversion channel open: possibly short life times/difficult calculations
- HAL QCD & chiral YY interactions indicate suppression $\Xi N - \Lambda\Lambda$ transition
- ΞN interaction relevant: Ξ is often the second hyperon to appear in neutron matter

Identify possibly interesting states:

calculations based on chiral interactions neglecting $\Xi N - \Lambda\Lambda$ transitions

(keeping $\Xi N - \Lambda\Sigma, \Sigma\Sigma$) → states are bound states

finetuning of ${}^{11}S_0$ interaction to correct for missing $\Lambda\Lambda$ channel

neglect YN interaction to avoid transitions to $\Lambda\Lambda$

perturbative width estimates indicate small widths ✔

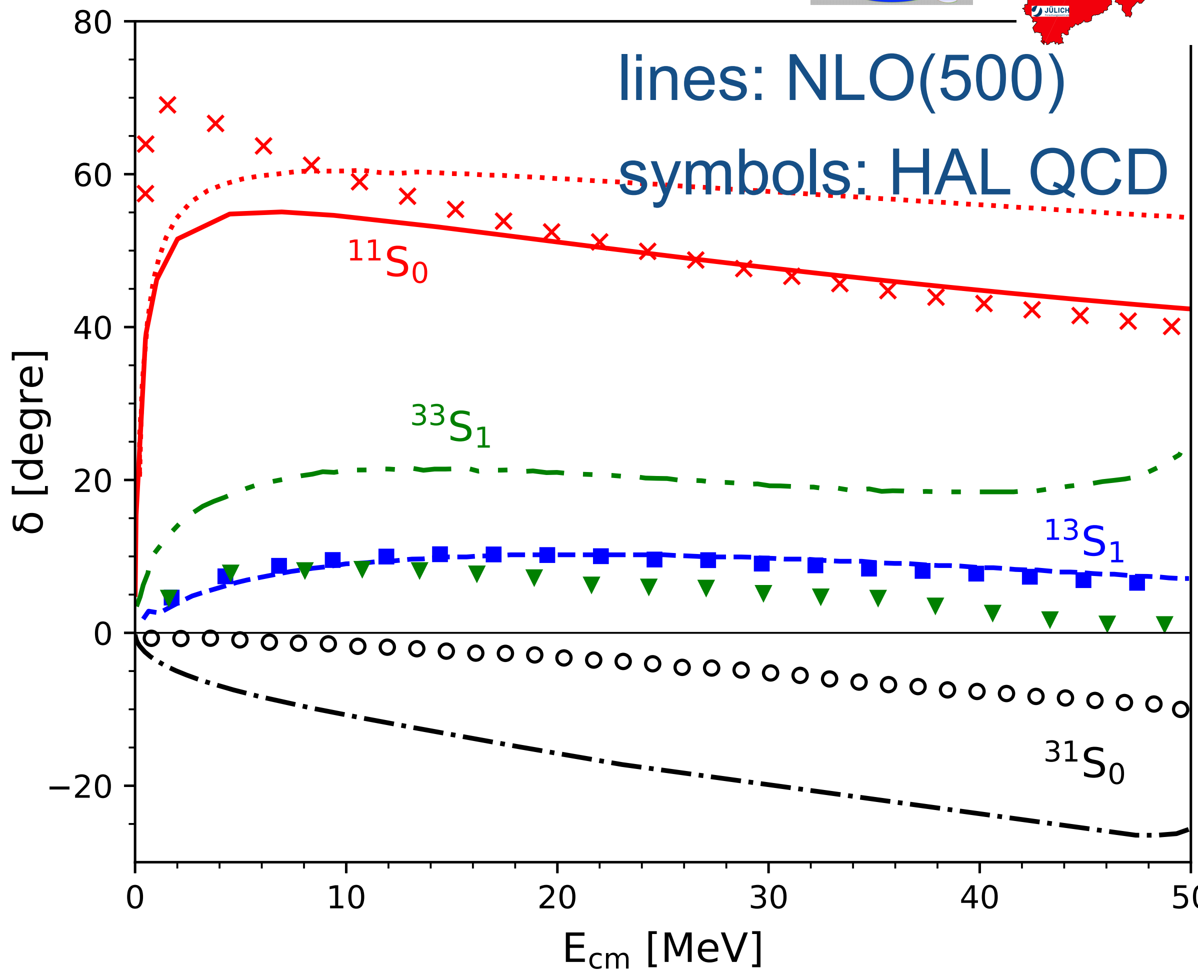
Here: look at ${}^{7}_{\Xi}\mathrm{H}$ (exp. expected), ${}^{5}_{\Xi}\mathrm{H}$, ${}^{4}_{\Xi}\mathrm{H}$ and ${}^{4}_{\Xi}\mathrm{n}$

explore possible bound states and their widths

Ξ hypernuclei

Ξ separation energies (NLO(500) and SMS $N^4LO^+(450)$)

	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}H(1^+, 0)$	0.48 ± 0.01	0.74
${}^{4}_{\Xi}n(0^+, 1)$	0.71 ± 0.08	0.2
${}^{4}_{\Xi}n(1^+, 1)$	0.64 ± 0.11	0.01
${}^{4}_{\Xi}H(0^+, 0)$	–	–
${}^{5}_{\Xi}H(\frac{1}{2}^+, \frac{1}{2})$	2.16 ± 0.10	0.19
${}^{7}_{\Xi}H(\frac{1}{2}^+, \frac{3}{2})$	3.50 ± 0.39	0.2



	$V^{S=-2}$ $^{11}S_0$	$^{31}S_0$	$^{13}S_1$	$^{33}S_1$
${}^{4}_{\Xi}H(1^+, 0)$	− 1.95	0.02	− 0.7	− 2.31
${}^{4}_{\Xi}n(0^+, 1)$	− 0.6	0.25	− 0.004	− 0.74
${}^{4}_{\Xi}n(1^+, 1)$	− 0.02	0.16	− 0.13	− 1.14
${}^{4}_{\Xi}H(0^+, 0)$	− 0.002	0.08	− 0.01	− 0.006
${}^{5}_{\Xi}H(1/2^+, 1/2)$	− 0.96	0.94	− 0.58	− 3.63
${}^{7}_{\Xi}H(1/2^+, 3/2)$	− 1.23	1.79	− 0.79	− 6.74

(Le, Haidenbauer, Meißner, AN, 2021)

Conclusions & Outlook

- **Hypernuclei provide important constraints on YN and YY interactions**
  - $^1S_0$ ΛN *scattering length &* ${}^{3}_{\Lambda}\mathrm{H}$
  - $^1S_0$ $\Lambda\Lambda$ *scattering length &* ${}^{\;\;6}_{\Lambda\Lambda}\mathrm{He}$ *& predictions for A=4,5*
  - *Light* Ξ*-hypernuclei exist and provide information on the* Ξ*N interaction*
  - *CSB of* ΛN *scattering &* ${}^{4}_{\Lambda}\mathrm{He}$ $/$ ${}^{4}_{\Lambda}\mathrm{H}$

- ***J-NCSM***
  - *reliable predictions are possible for ranges of interactions for* $S=-1$ *and* -2

- ***next steps***
  - *estimates of **chiral 3BFs** are needed (implementing Petschauer et al., (2016))*
  - *study CSB of p-shell hypernuclei*
  - *study dependence of* $S=-2$ *results on chiral orders and regulators.*