# Halo EFT: News and Views

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# Outline

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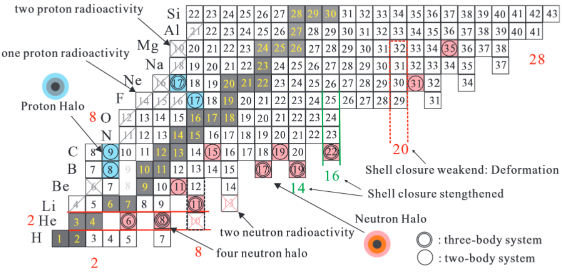
- Halo nuclei and Halo EFT
- Efimov physics in halo nuclei
  Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023)
- Nuclear reactions with neutrons
  HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021)
- Summary and Outlook

# Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT




C.-B. Moon, Wikimedia Commons

- EFT for halo nuclei

  (Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

# Halo Effective Field Theory

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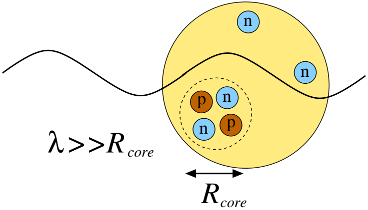



- Separation of scales:

  $1/k = \lambda \gg R_{core}$

- Limited resolution at low energy:

  $\longrightarrow$ expand in powers of $kR_{core}$

  $\longrightarrow$ contact interactions

- Short-distance physics not resolved

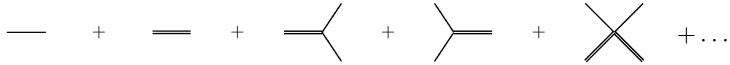
  $\longrightarrow$ capture in low-energy constants using renormalization

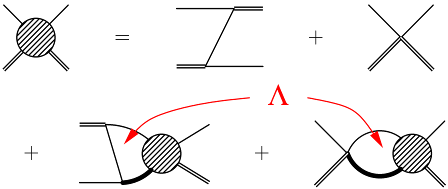
  $\longrightarrow$ include long-range physics explicitly if present

- Systematic, model independent $\Longrightarrow$ universal properties
- Nucleon degrees of freedom: $\Longrightarrow$ pionless EFT
- Exploit cluster substructures $\Longrightarrow$ Halo EFT

# EFT Framework

- Exploit scale separation in EFT framework
- Here: $S$-wave case, higher $L$ states can also be treated
- Effective Lagrangian

$\mathcal{L}_{eff} =$ 

- 2-body amplitude: 
- 2-body coupling $g_2$ near fixed point $(1/a = 0)$ $\Longleftrightarrow$ unitary limit
- 3-body amplitude: 


$g_3(\Lambda)$ $\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.

# Limit Cycle

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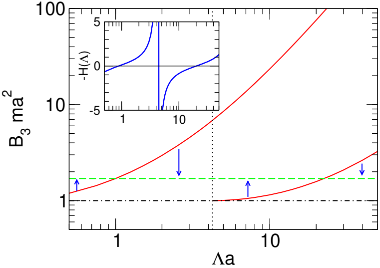
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$
  - $H(\Lambda)$ periodic: limit cycle

    $$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

    (cf. Wilson, 1971)
  - Anomaly: scale invariance broken to discrete subgroup



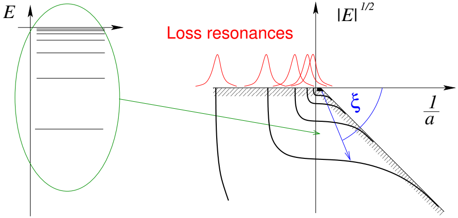

$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance

# Limit Cycle: Efimov Effect

- Universal spectrum of three-body states (Efimov, 1970)




  - Discrete scale invariance for fixed angle $\xi$
  - Geometrical spectrum for $1/a \to 0$

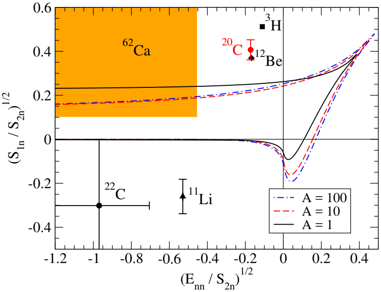
$$B_3^{(n)}/B_3^{(n+1)} \stackrel{1/a\to 0}{\longrightarrow} \left(e^{\pi/s_0}\right)^2 = 515.035...$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

# Efimov Physics in Halo Nuclei

- Efimov effect in halo nuclei? (Fedorov, Jensen, Riisager, 1994)

  $\Longrightarrow$ excited states obeying scaling relations
- Correlation plot: $E_{nn} \leftrightarrow S_{1n}$ (Amorin, Frederico, Tomio, 1997)




HWH, Ji, Phillips, J. Phys. G **44**, 103002 (2017)

- Alternative ways to observe Efimov physics in halo nuclei?

# Neutron Scattering

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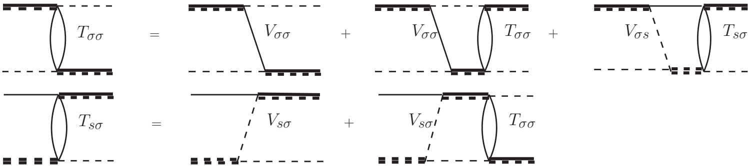
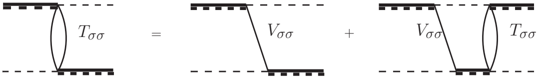
- Neutron scattering off $J^P = 1/2^+$ one-neutron halos ($^{11}$Be, $^{15}$C, $^{19}$C)
- $J = 0$ channel (three-body force suppressed)




- $J = 1$ channel (no three-body force)




Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023)

# $J = 1$ channel

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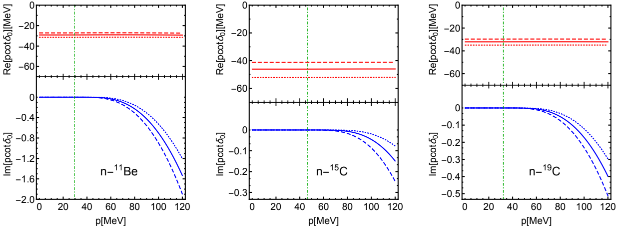
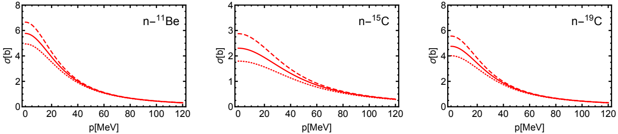
- *S*-wave scattering amplitude (Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023))

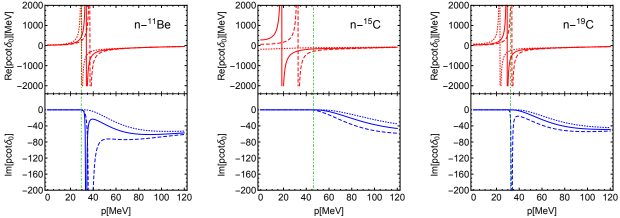



- Total *S*-wave cross section

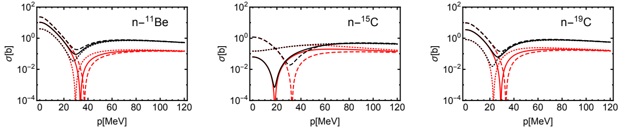


# $J = 0$ channel

- *S*-wave scattering amplitude (Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023))

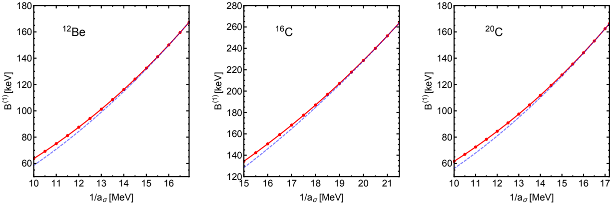



- Total cross section



# Efimov physics in halo nuclei?

- Alternative ways to see Efimov physics in halo nuclei?
  - Pole in $p \cot \delta_0$ due to virtual Efimov state close to threshold
  - Real excited state appears for $1/a_\sigma$ smaller than some critical value




(Zhang, Fu, Guo, HWH, Phys. Rev. C **108**, 044304 (2023))

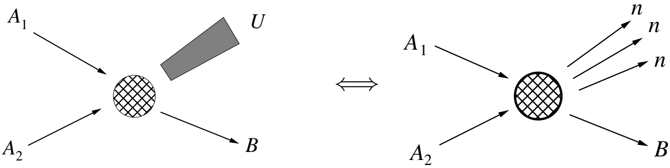
- Pole position directly correlated with virtual state energy

$\Longrightarrow$ pole position determined by Efimov physics

- Also present in deuteron-halo scattering?

# Reactions with neutrons

- High-energy nuclear reaction with multi-neutron final state
  (HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))

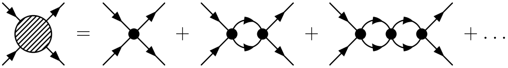



$$E_{\text{kin}} = (M_{A_1} + M_{A_2} - M_B - M_{\mathcal{U}})c^2 + \frac{p_{A_1}^2}{2M_{A_1}} + \frac{p_{A_2}^2}{2M_{A_2}} = E_B + E_{\mathcal{U}}$$

- Assumption: energy scale of primary reaction $\gg E_{\mathcal{U}} - \frac{\boldsymbol{p}^2}{2M_{\mathcal{U}}} = E_n^{cms}$
- Factorization: $\dfrac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \, \mathrm{Im}\, G_{\mathcal{U}}(E_{\mathcal{U}}, \boldsymbol{p})$
- Reproduces Watson-Migdal treatment of FSI for $2n$
  (Watson, Phys. Rev. **88**, 1163 (1952); Migdal, Sov. Phys. JETP **1**, 2 (1955))

# Unitary limit

- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e$)



- Renormalization group equation: $$\Lambda \frac{d}{d\Lambda}\tilde{g}_2 = \tilde{g}_2(1+\tilde{g}_2)$$
- Two fixed points:

  $-\ \tilde{g}_2 = 0 \ \Leftrightarrow \ a = 0 \quad \Rightarrow$ no interaction, free particles

  $-\ \tilde{g}_2 = -1 \ \Leftrightarrow \ 1/a = 0 \quad \Rightarrow$ unitary limit

  $\Rightarrow$ conformal/Schrödinger symmetry

  (Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...)

- Neutrons: $a \approx -18.6$ fm, $\quad r_e \approx 2.8$ fm

  $\Rightarrow$ neutrons are close to the unitary limit

# Conformal field theory

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- Two-point function of primary field operator $\mathcal{U}$ ("unnucleus")

$$G_{\mathcal{U}}(t,\boldsymbol{x}) = -i\langle T\mathcal{U}(t,\boldsymbol{x})\mathcal{U}^{\dagger}(0,\boldsymbol{0})\rangle = C\,\frac{\theta(t)}{(it)^{\Delta}}\exp\left(\frac{iM\boldsymbol{x}^2}{2t}\right)$$

- Determined by symmetry up to overall constant $C$
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega,\boldsymbol{p}) = -C\left(\frac{2\pi}{M}\right)^{3/2}\Gamma\left(\frac{5}{2}-\Delta\right)\left(\frac{\boldsymbol{p}^2}{2M}-\omega-i\epsilon\right)^{\Delta-\frac{5}{2}}$$

  - pole only for $\Delta = 3/2$ (free field)
  - branch cut for $\Delta > 3/2$
- General unnucleus/unparticle does not behave like a particle

$\Rightarrow$ continuous energy spectrum

# Reactions with neutrons

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- Two ways to do experiments
  - (a) detect recoil particle $B$

$$\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta - 5/2}, \qquad E_0 = (1 + M_B/M_{\mathcal{U}})^{-1} E_{\text{kin}}$$

  - (b) detect all final state particles including neutrons

$$\frac{d\sigma}{dE} \sim (E_n^{cms})^{\Delta - 5/2}$$

(HWH, D.T. Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))

- Consistent with previous experiments for ${}^3\mathrm{H}(\pi^-, \gamma)3n$

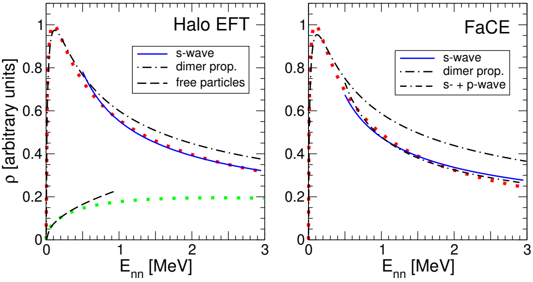
(Miller et al., Nucl. Phys. A **343**, 347 (1980))

- Two few events in recent tetraneutron experiment: ${}^4\mathrm{He}({}^8\mathrm{He}, {}^8\,\mathrm{Be})4n$

(Kisamori et al., Phys. Rev. Lett. **116**, 052501 (2016))

## Reaction calculations

- Two-neutron spectrum for $^6\mathrm{He}(p, p\alpha)2n$ (Göbel et al., Phys. Rev. C **104**, 024001 (2021))




- Can be understood from dimer propagator ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

# Reaction calculations

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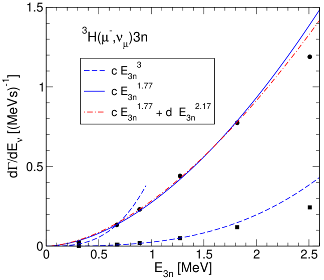
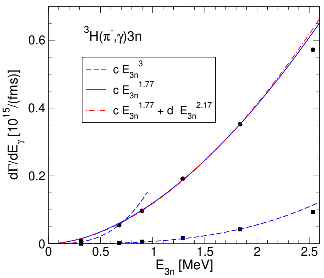
- Radiative muon/pion capture on the triton (AV18 + UIX)




Golak et al., PRC **98**, 054001 (2018)




Golak et al., PRC **94**, 054001 (2016)

- Unnucleus behavior prediction

$$\frac{d\Gamma}{dE} \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}, \qquad 0.1\,\text{MeV} \ll E_{3n} \ll 5\,\text{MeV}$$

# New experiments

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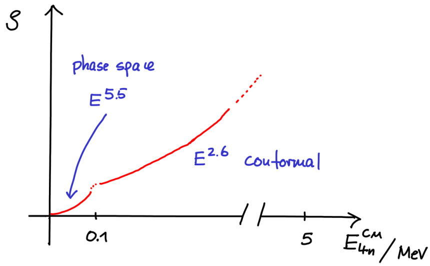
- New experiments in complete kinematics at RIBF/RIKEN
- Measurement of $a_{nn}$ in $^6\mathrm{He}(p, p\alpha)2n$
  (T. Aumann et al., NP2012-SAMURAI55R1 (2020))
- Search for tetraneutron resonances in $^8\mathrm{He}(p, p\alpha)4n$
  (S. Paschalis et al., NP1406-SAMURAI19R1 (2014))

  - unnucleus prediction for point source:

    $\rho \sim (E_{4n})^{5.07-5/2} \sim (E_{4n})^{2.57}$

    $0.1\,\mathrm{MeV} \ll E_{4n} \ll 5\,\mathrm{MeV}$



# New experiments

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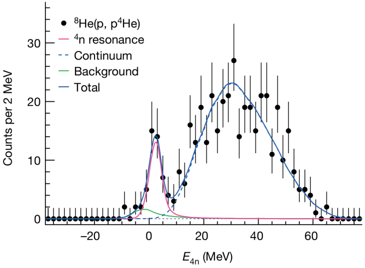
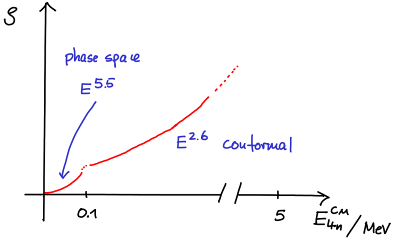
- Search for tetraneutron resonances in $^8\mathrm{He}(p, p\alpha)4n$




M. Duer et al., Nature **606** (2022) 678

$$E_R = 2.37 \pm 0.38(stat) \pm 0.44(sys)\ \text{MeV}, \quad \Gamma_R = 1.75 \pm 0.22(stat) \pm 0.30(sys)\ \text{MeV}$$

- Genuine resonance or dineutron correlations/initial state effect?

R. Lazauskas et al., Phys. Rev. Lett. **130**, 102501 (2023) $\Longrightarrow$ dineutron correlations

## Summary and Outlook

- Efimov physics in halo nuclei
- High-energy nuclear reactions with final state neutrons

  $\Rightarrow$ (approximate) conformal symmetry

  $\Rightarrow$ power law behavior of observables determined by $\Delta$

- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles

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- Efimov physics in halo nuclei
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  $\Rightarrow$ (approximate) conformal symmetry
  $\Rightarrow$ power law behavior of observables determined by $\Delta$

- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles
- Other applications & extensions
  - Two-component Fermions in ultracold atom physics
  - Neutral charm mesons
    (Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022), Phys. Rev. D **107**, 034017 (2023))
  - Systems with the Efimov effect?

    $\Rightarrow$ bosonic atoms, nucleons, $\alpha$ particles
    $\Rightarrow$ complex scaling dimensions
    $\Rightarrow$ scale symmetry broken

# Additional Slides

# Conformal field theory

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- Imaginary part of propagator

$$\operatorname{Im} G_{\mathcal{U}}(\omega, \boldsymbol{p}) \sim \begin{cases} \delta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta = \frac{3}{2}\,, \\ \left(\omega - \frac{\boldsymbol{p}^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

- Examples of unnuclei
  - free field: $\mathcal{U} = \psi, \quad M = m_\psi, \quad \Delta = 3/2$
  - $N$ free fields: $\mathcal{U} = \psi_1 \ldots \psi_N, \quad M = N m_\psi, \quad \Delta = 3N/2$
  - $N$ interacting fields: $\mathcal{U} = \psi_1 \ldots \psi_N, \quad M = N m_\psi, \quad \Delta > 3/2$

- In our case: unnucleus is strongly interacting multi-neutron state with

$$1/(ma^2) \sim 0.1\text{ MeV} \ll E_n^{cms} \ll 1/(mr_e^2) \sim 5\text{ MeV}$$

- Corrections from finite $a$ and $r_0$ (S. Dutta, R. Mishra, D.T. Son, arXiv:2309.15177)

$$\operatorname{Im} G_{\mathcal{U}}(\omega, 0) \sim \omega^{\Delta - \frac{5}{2}} \theta(\omega) \left(1 + \frac{c_1}{a\sqrt{m\omega}} + c_2 r_0 \sqrt{m\omega}\right), \quad c_2 = 0$$

# Scaling dimension

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- How to calculate scaling dimension $\Delta$?

  (1) $\Delta$ can be obtained from field theory calculation

  (2) $\Delta$ can be obtained from operator state correspondence

$\Delta$ of primary operator = (Energy of state in HO)/$\hbar\omega$

(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

| N | S | L | $\mathcal{O}$ | $\Delta$ |
|---|---|---|---|---|
| 2 | 0 | 0 | $\psi_1\psi_2$ | 2 |
| 3 | 1/2 | 1 | $\psi_1\psi_2\nabla_j\psi_2$ | 4.27272 |
| 3 | 1/2 | 0 | $\psi_1\nabla_j\psi_2\nabla_j\psi_2$ | 4.66622 |
| 4 | 0 | 0 | $\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$ | 5.07(1) |
| 5 | 1/2 | 1 | $\ldots$ | 7.6(1) |

$\Rightarrow$ connection between $\Delta$ and energy of particles in a trap