NoTORIous Neutrinos

Mu-Chun Chen

University of California at Irvine

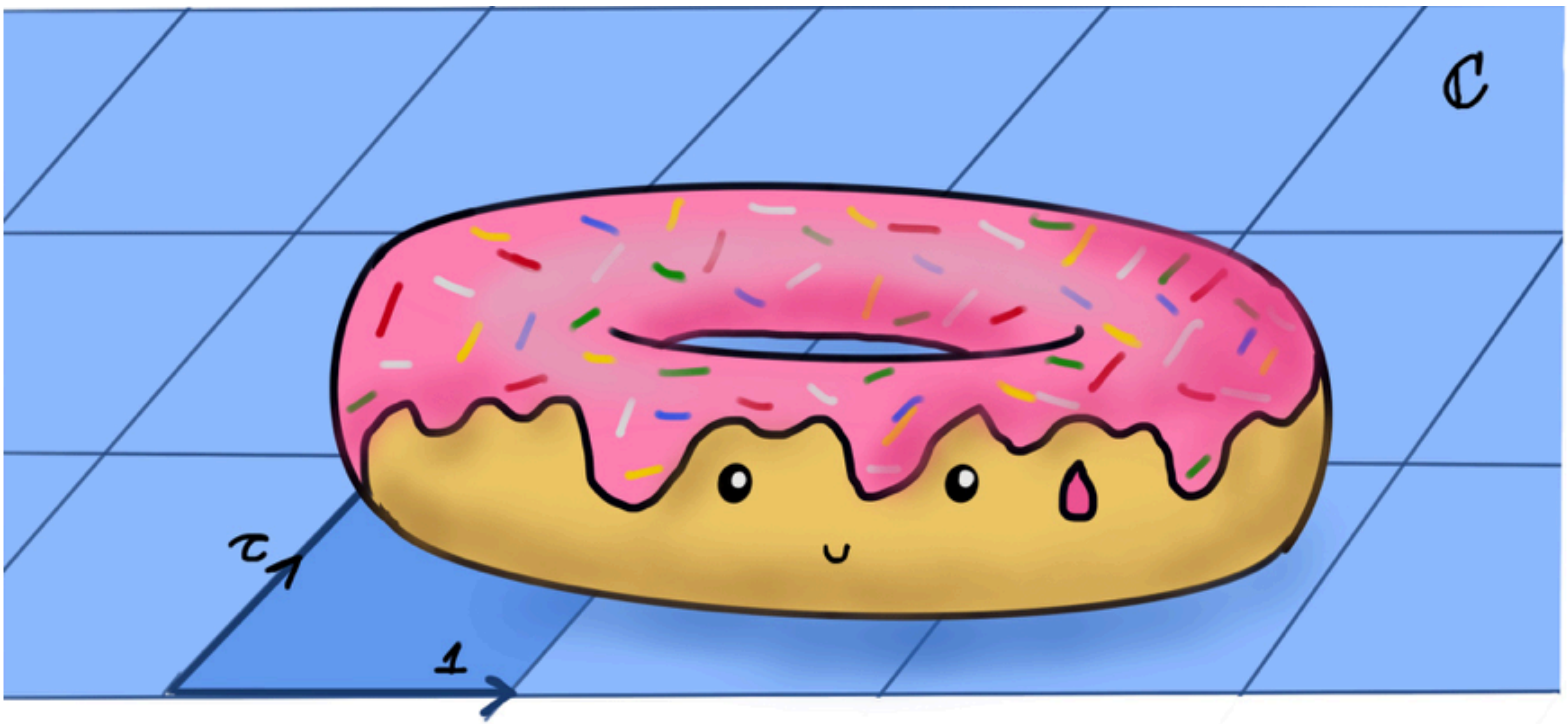
Bethe Colloquium, Universität Bonn, May 5, 2022

Bethe Forum

Modular Flavor Symmetries

May 2-6, 2022, Bonn Germany

Donuts = TORI



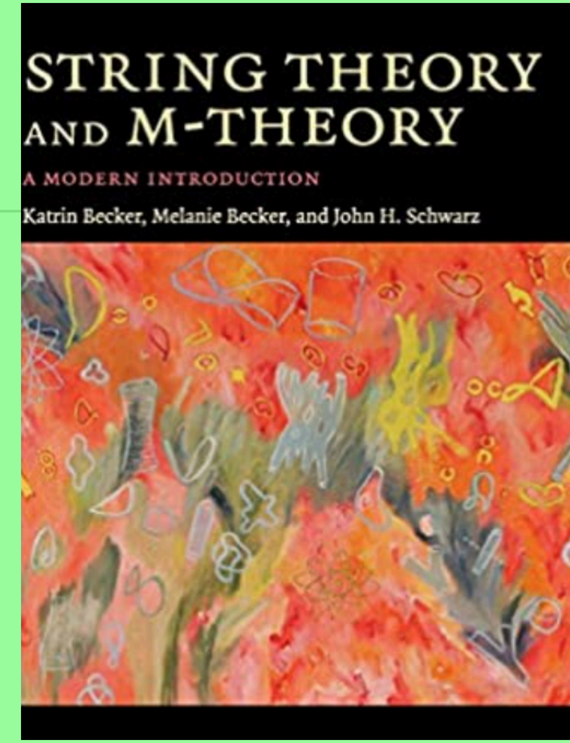
Artist rendition by Shreya Shukla

Equivalent TORI related by Modular Symmetries

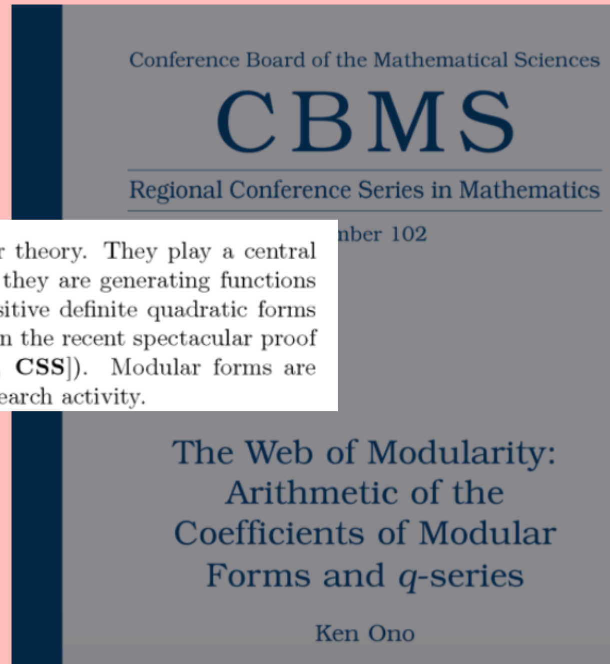
Artist rendition by Shreya Shukla

Modular Symmetries

String Theory



Number Theory



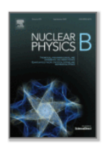
Modular forms appear in many ways in number theory. They play a central role in the theory of quadratic forms; in particular, they are generating functions for the number of representations of integers by positive definite quadratic forms (for example, see [**Gro**]). They are also key players in the recent spectacular proof of Fermat's Last Theorem (see for example, [**Bos, CSS**]). Modular forms are presently at the center of an immense amount of research activity.

Condensed Matter Physics

ELSEVIER

Nuclear Physics B

Volume 474, Issue 3, 2 September 1996, Pages 543-574



Modular invariance, self-duality and the phase transition between quantum Hall plateaus

Eduardo Fradkin [a], Steven Kivelson [b]

Neutrino Physics

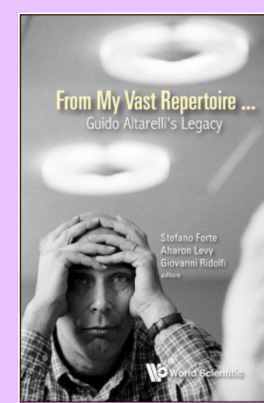
arXiv.org > hep-ph > arXiv:1706.08749

High Energy Physics – Phenomenology

[Submitted on 27 Jun 2017 (v1), last revised 29 Sep 2017 (this version, v2)]

Are neutrino masses modular forms?

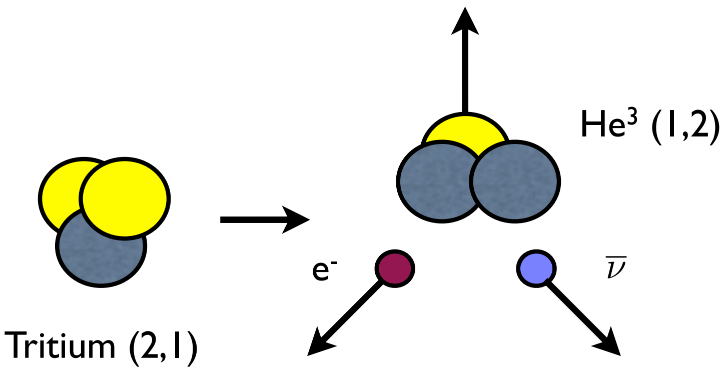
Ferruccio Feruglio

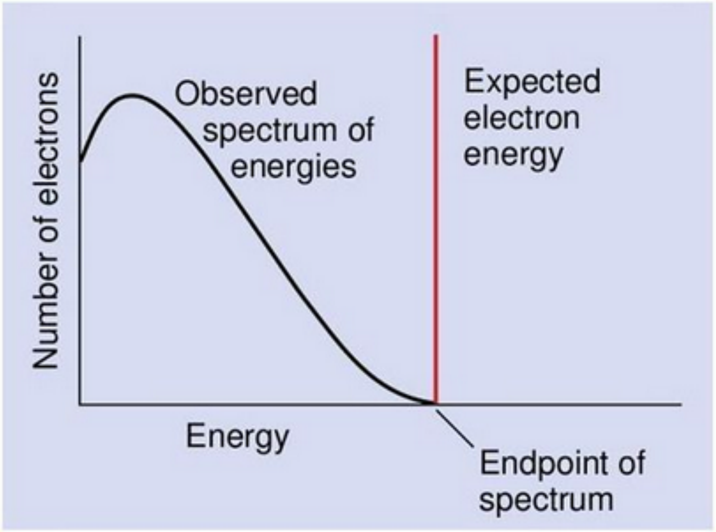


Neutrino: Solution to the "Energy Crisis" !

Dec. 1930: invented by Pauli to explain missing energy spectrum in beta decay

$$(N, Z) \rightarrow (N-1, Z+1) + e^- + \overline{\nu}$$





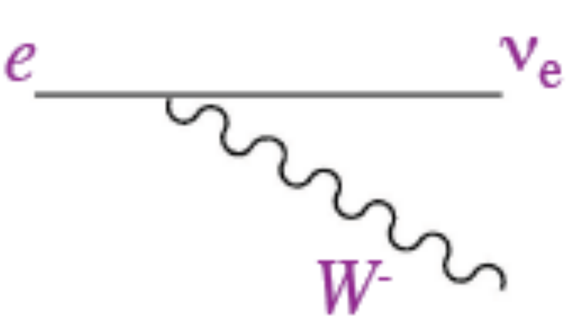
Three Neutrino “Flavors”

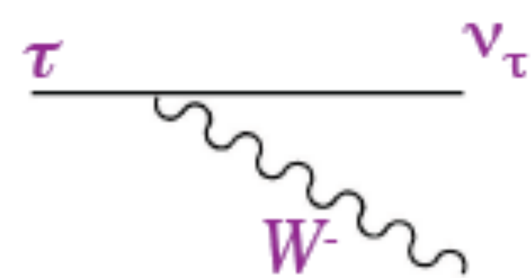
Illustration: © Johan Jarnestad/The Royal Swedish Academy of Sciences

electron
neutrino

muon
neutrino

tau
neutrino

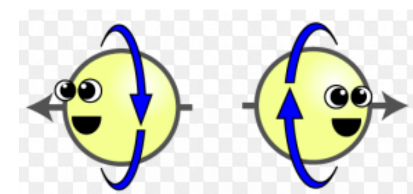




Standard Model of Particle Physics

[Picture credit: http://particlezoo.net]

- 3 generations of quarks and leptons
- LH & RH partners for all particles except for neutrinos



Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar

Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

A

electr

We h

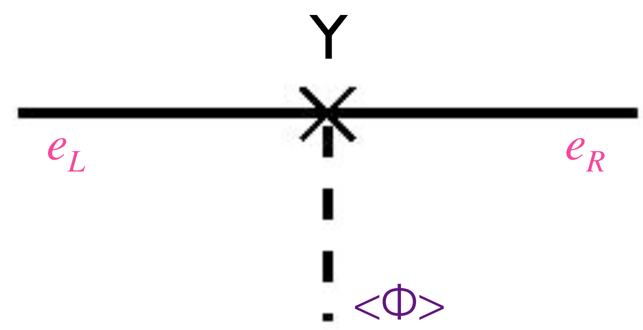
which

the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\sigma_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

only LH neutrinos have been observed

Fermion Mass Generation

- Higgs Mechanism
- Yukawa Interactions
- In Standard Model: no RH neutrinos
  - LH neutrinos cannot interact with Higgs BEC
  - Neutrinos stay massless

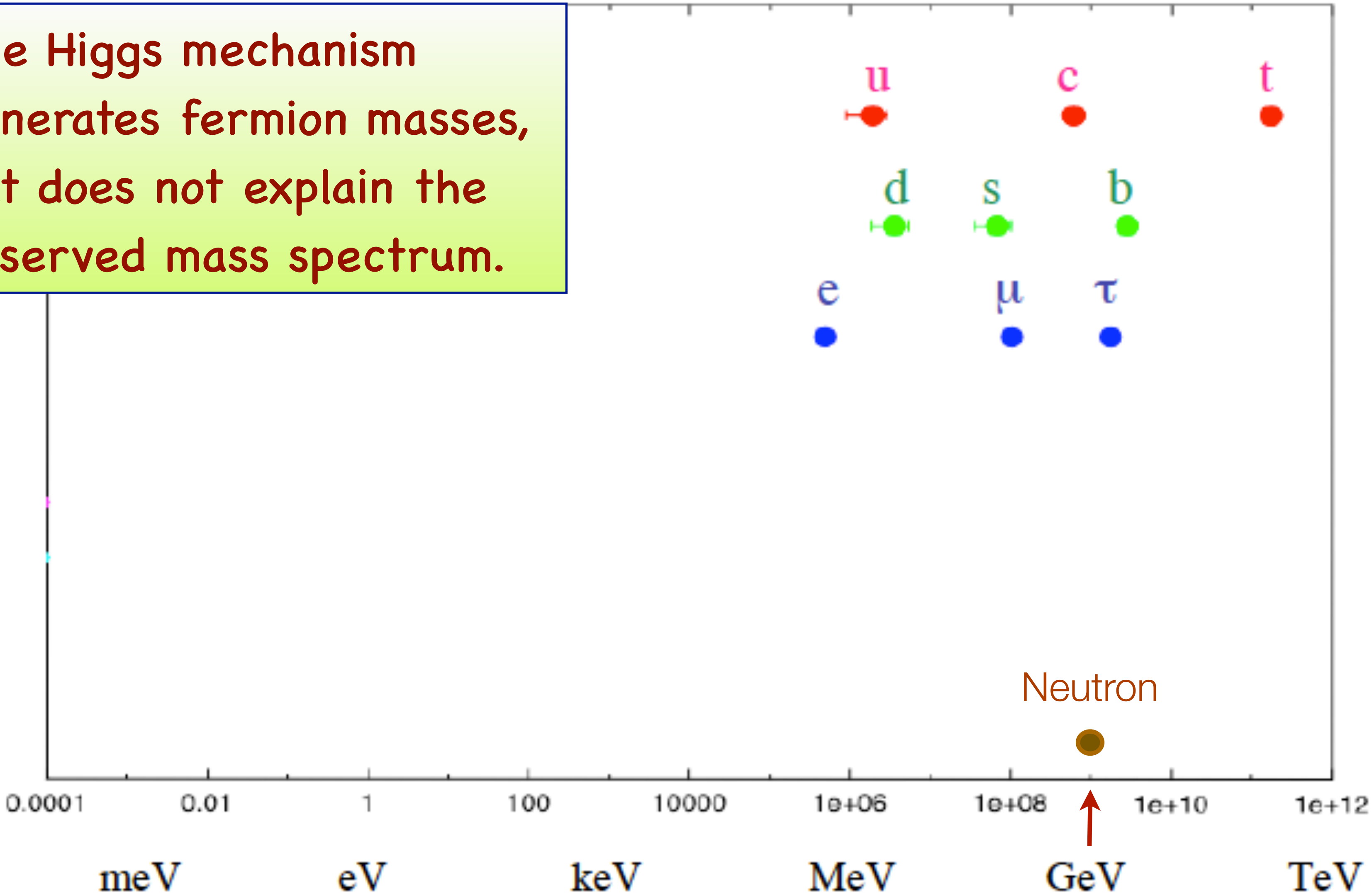


Y: Yukawa coupling constant

Standard Model predicts massless neutrinos

Mysteries of Masses in SM

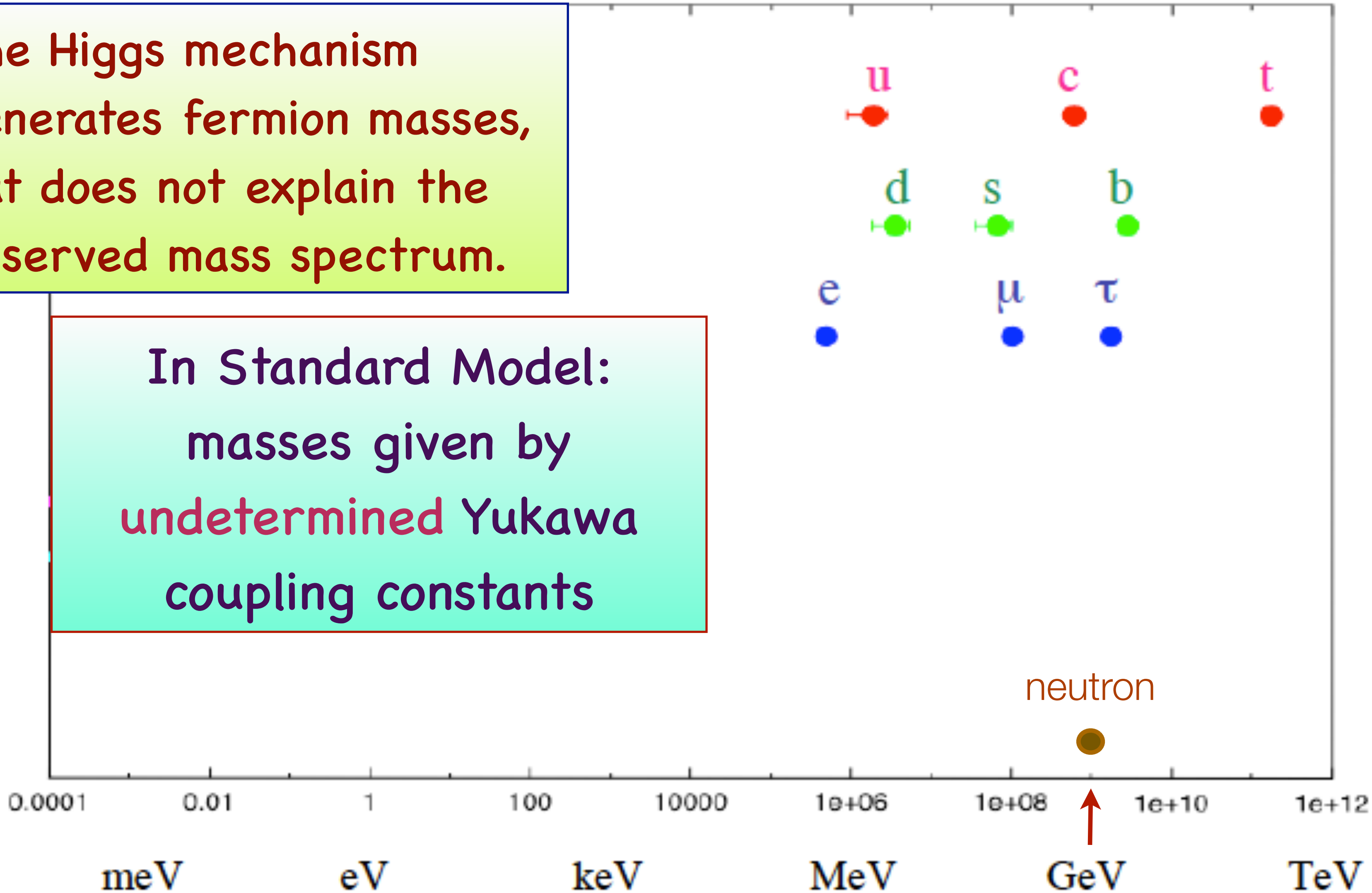
The Higgs mechanism generates fermion masses, but does not explain the observed mass spectrum.



Mysteries of Masses in SM

The Higgs mechanism generates fermion masses, but does not explain the observed mass spectrum.

In Standard Model: masses given by undetermined Yukawa coupling constants

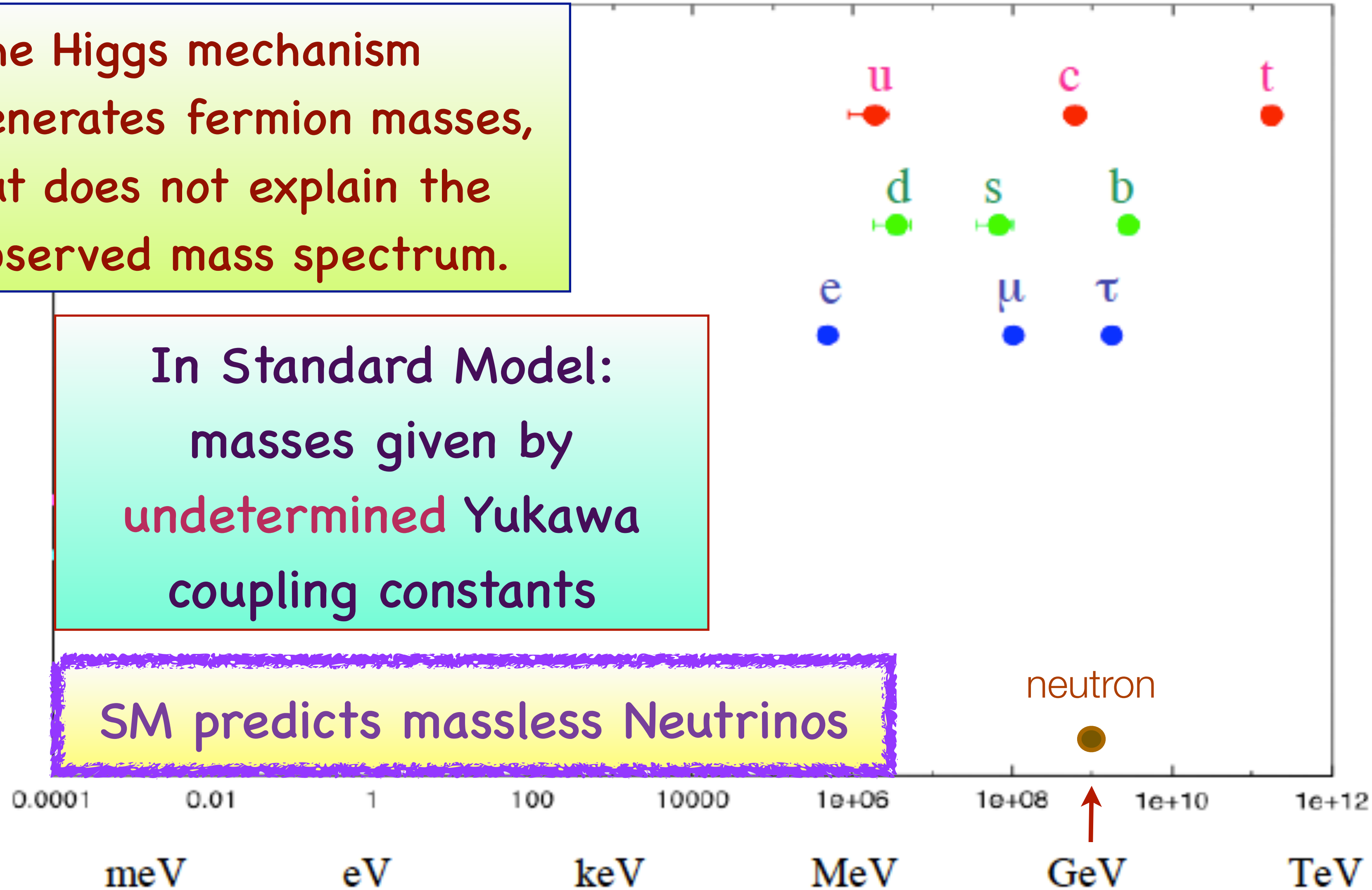


Mysteries of Masses in SM

The Higgs mechanism generates fermion masses, but does not explain the observed mass spectrum.

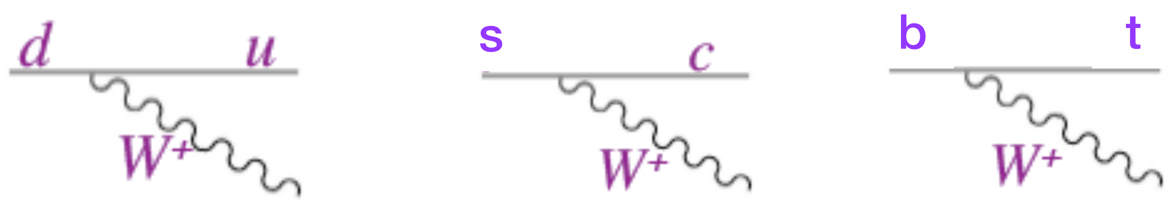
In Standard Model: masses given by undetermined Yukawa coupling constants

SM predicts massless Neutrinos



Mysteries of Masses and Flavor Mixing in SM

- Charged current weak interaction mediated by $W^{\pm}$ gauge boson:

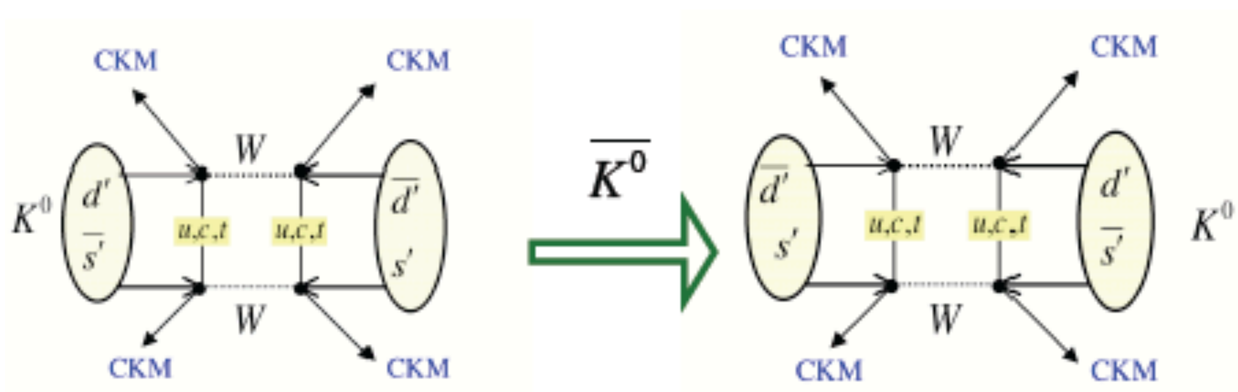


weak eigenstates = mixture of mass eigenstates

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{mass eigenstates}} = \underbrace{\begin{pmatrix} V_{ud} & V_{cd} & V_{td} \\ V_{us} & V_{cs} & V_{ts} \\ V_{ub} & V_{cb} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{weak eigenstates}}$$

Cabibbo, 1963;
Kobayashi, Maskawa, 1973

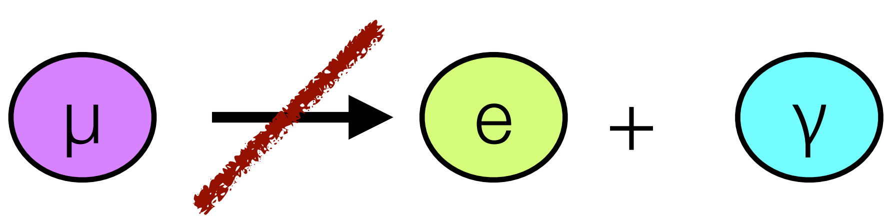
3 mixing angles
1 phase



Nobel prize to KM in 2008

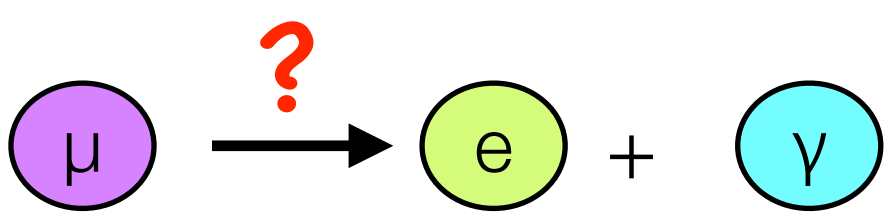
Mysteries of Masses and Flavor Mixing in SM

- Neutrino Masses are degenerate (all zero)
  - mass eigenstates = weak eigenstates
- Accidental symmetries in SM
  - lepton flavor numbers: L_e, L_μ, L_τ
  - no processes cross family lines in lepton sector
- As a result
  - no neutrino oscillation
  - lepton flavor violation decays forbidden



Mysteries of Masses and Flavor Mixing in SM

- Neutrino Masses are degenerate (all zero)
  - mass eigenstates = weak eigenstates
- Accidental symmetries in SM
  - lepton flavor numbers: L_e, L_μ, L_τ
  - no processes cross family lines in lepton sector
- As a result
  - no neutrino oscillation
  - lepton flavor violation decays forbidden



Active experimental program searching for these rare processes, MEG, Mu2E, ...

Neutrino: a particle w/ 4 Nobel Prizes under Belt

1988 - Lederman, Schwartz, Steinberger, detection of muon neutrino in **1962**

1995 - Reines (UCI): detection of electron antineutrino in **1958**

2002 - Davis and Koshiba: solar and supernova neutrino detections, **1968**, **1987**

2015 - Kajita and McDonald: detection of neutrino oscillations, **1998**, **2002**

The Nobel Prize in Physics 2015

Photo © Takaaki Kajita

Takaaki Kajita

Super-Kamiokande Collaboration

Photo: K. MacFarlane. Queen's University /SNOLAB

Arthur B. McDonald

Sudbury Neutrino Observatory Collaboration

Neutrino Oscillations

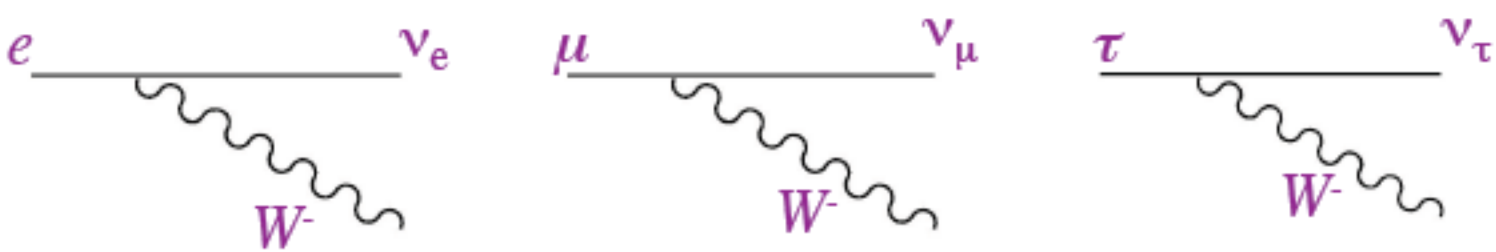
⇩

Neutrinos have Mass

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

What if Neutrinos Have Mass?

- Similar to the quark sector, there can be a mismatch between mass eigenstates and weak eigenstates
- weak interactions eigenstates: ν_e, ν_μ, ν_τ



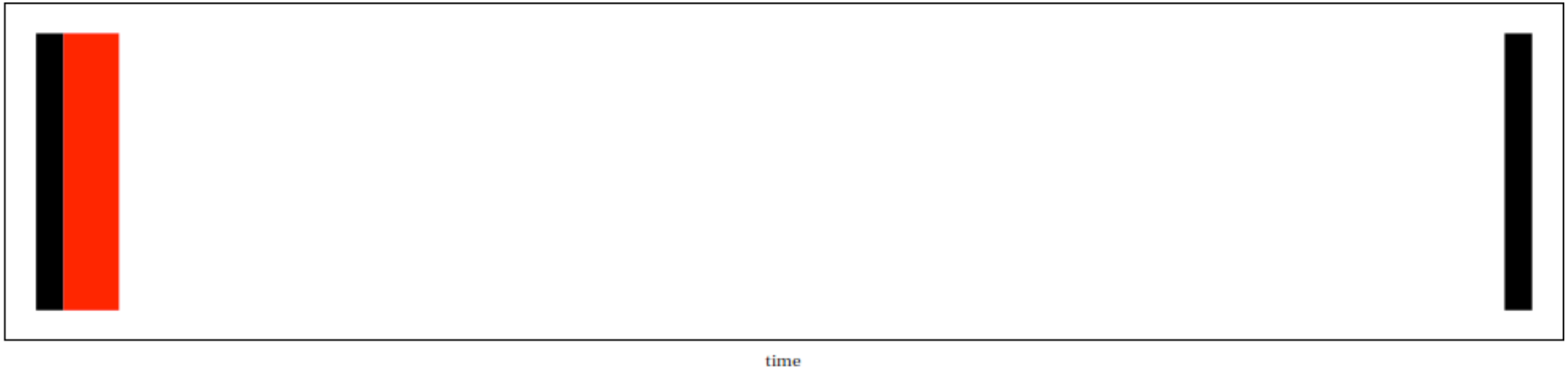
- mass eigenstates: ν_1, ν_2, ν_3
- Pontecorvo-Maki-Nakagawa-Sakata (PMNS) Matrix

Maki, Nakagawa, Sakata, 1962 ; Pontecorvo, 1967

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino Oscillation: Macroscopic Quantum Mechanics

- **production**: neutrinos of a definite flavor produced by weak interaction
- **propagation**: neutrinos evolve according to their masses
- **detection**: neutrinos of a different flavor composition detected



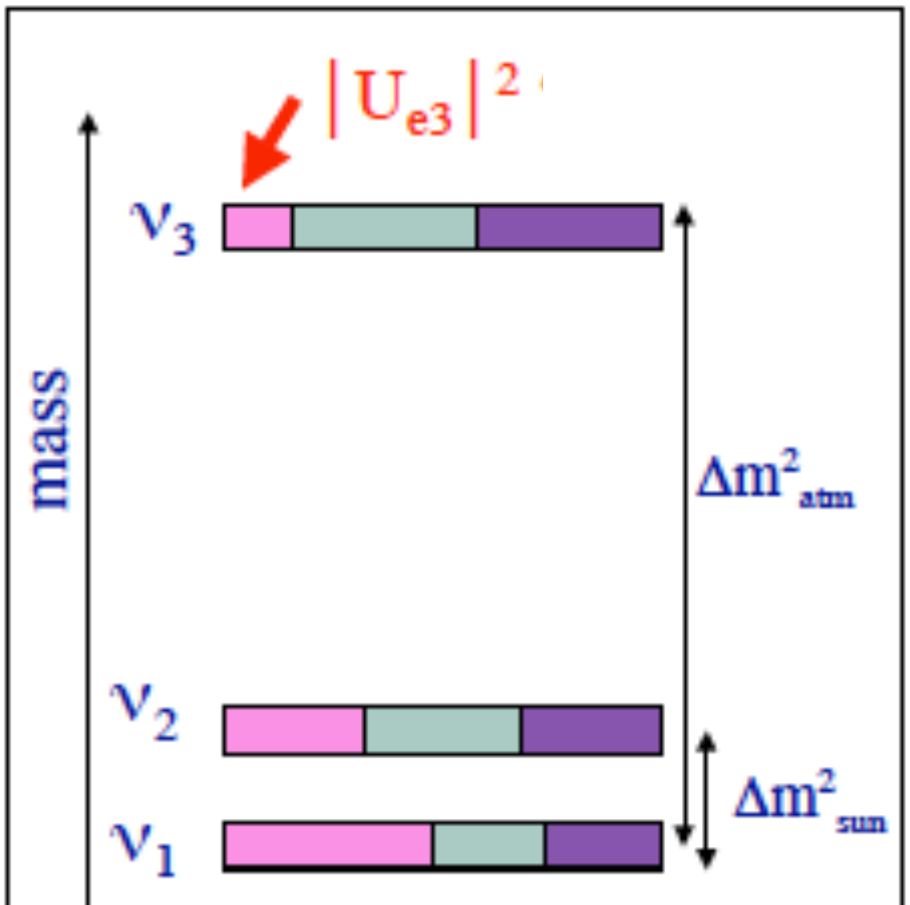
neutrino source

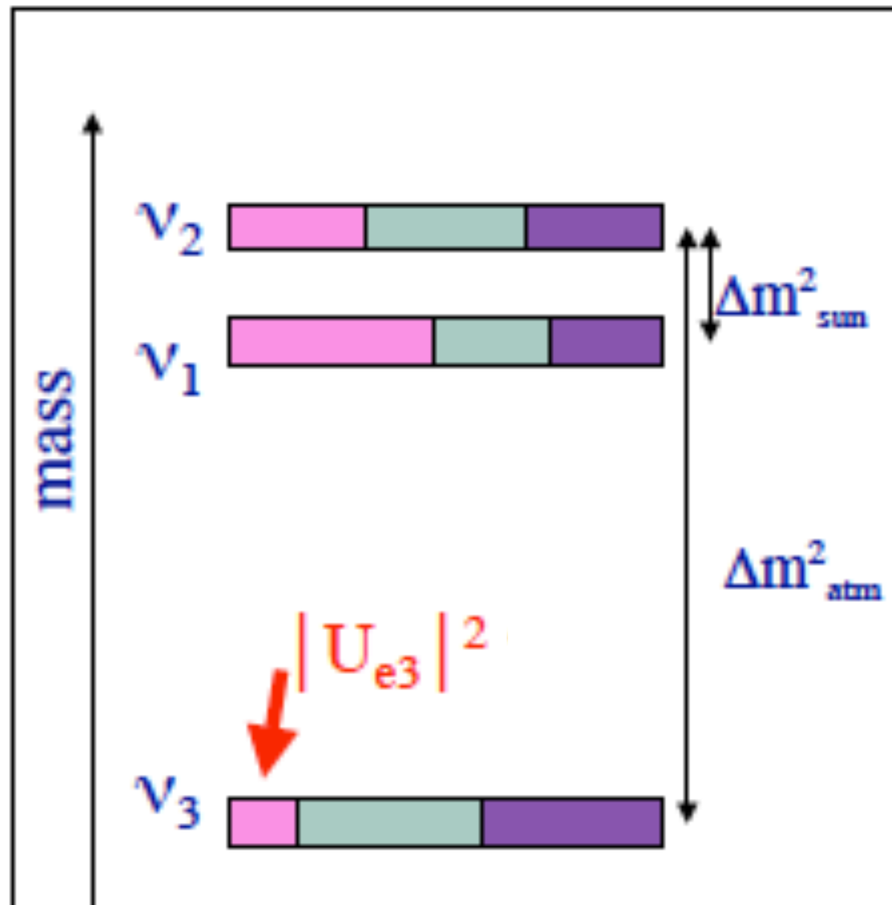
detector

Where Do We Stand?

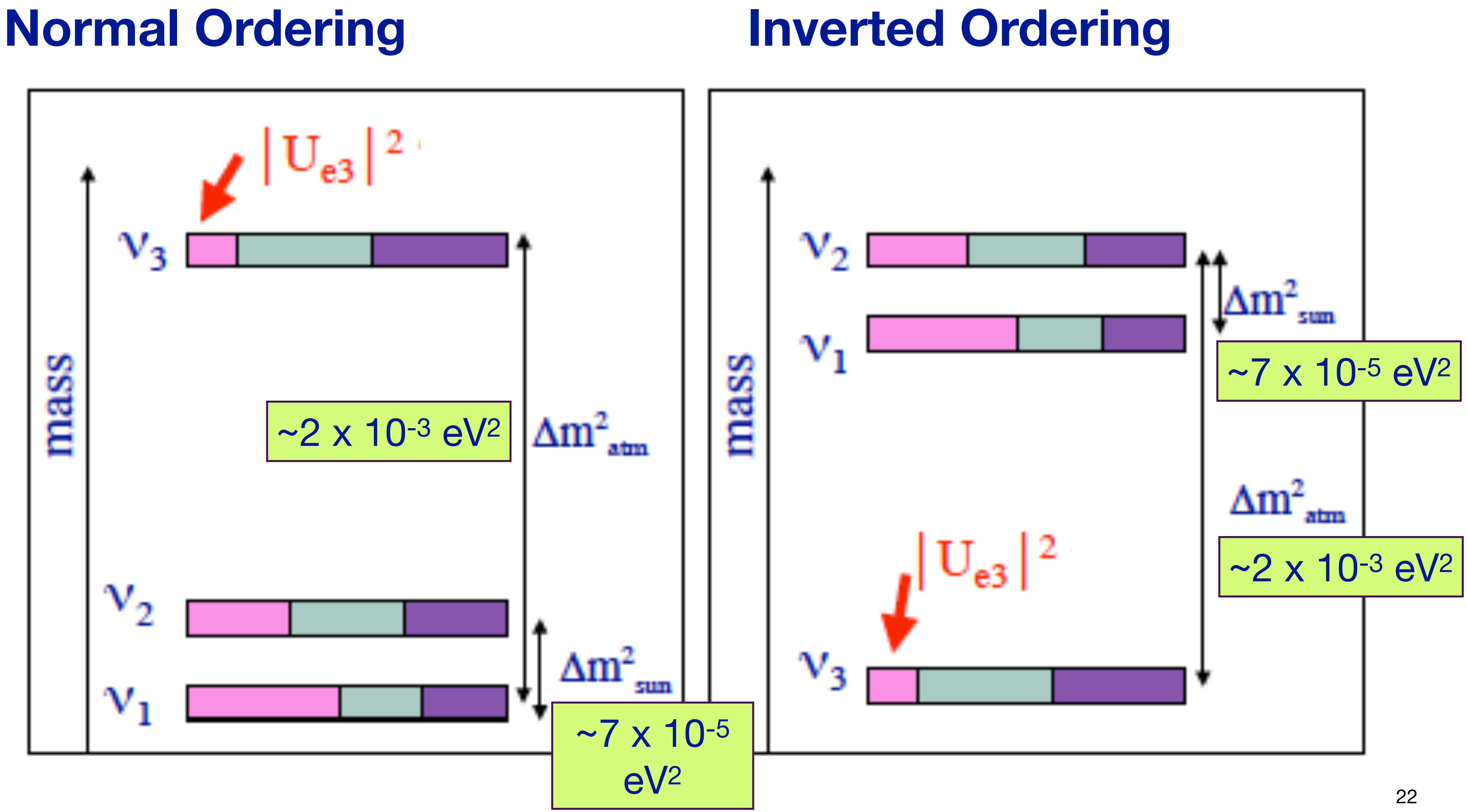
Normal Ordering

Inverted Ordering





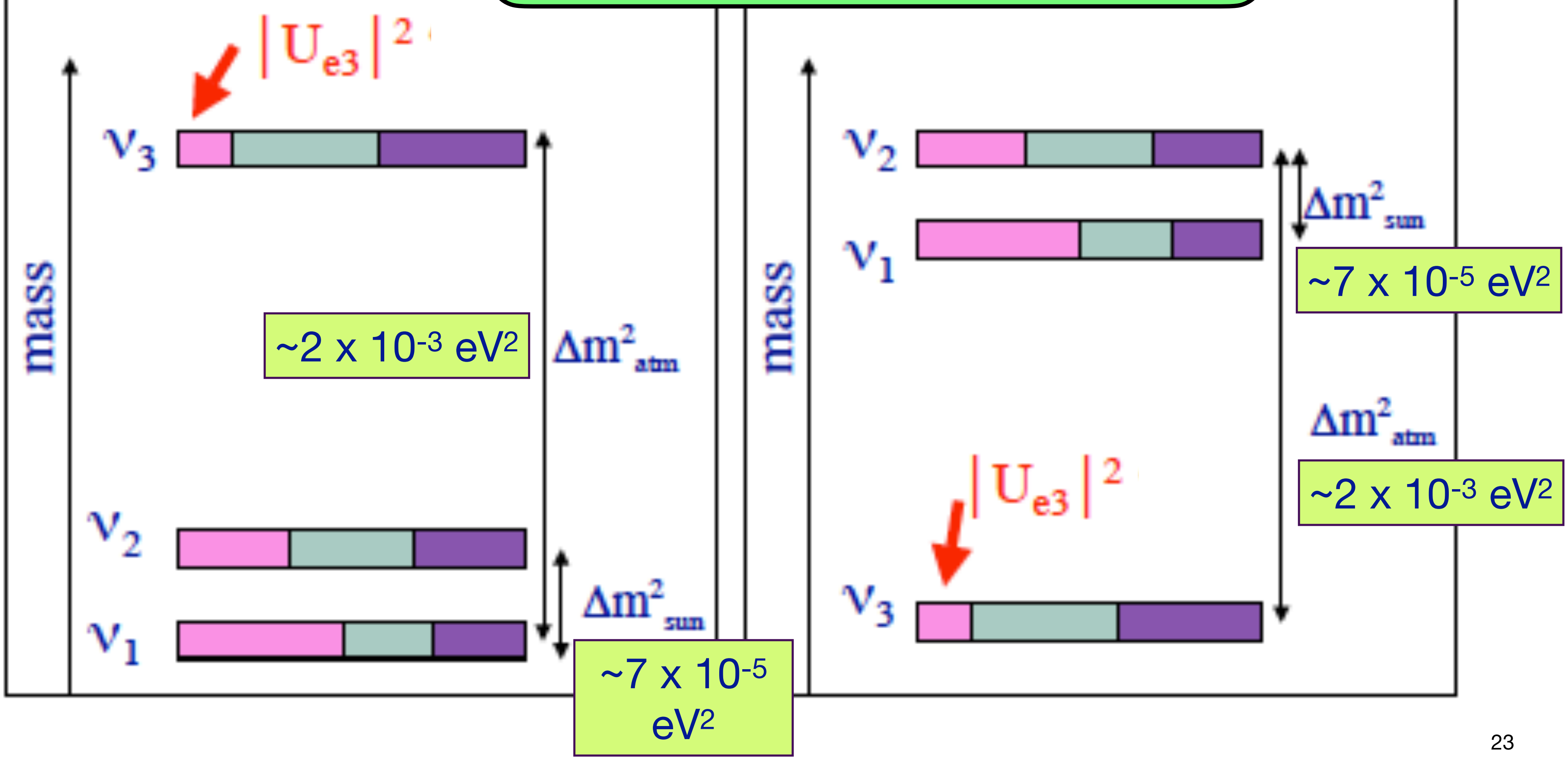
Where Do We Stand?



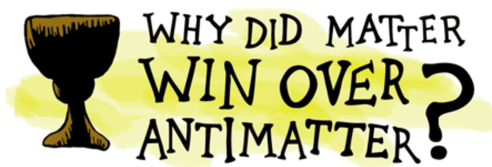
Where Do We Stand?

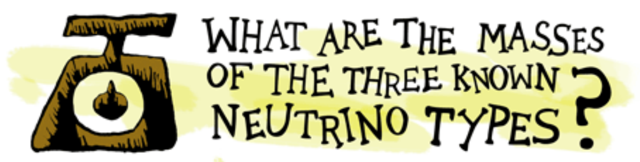
Normal Ordering

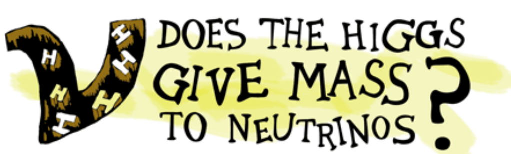
$$\sin^2\theta_{23} \sim \frac{1}{2}, \quad \sin^2\theta_{12} \sim \frac{1}{3}, \quad \sin\ \ \theta_{13} \sim \frac{1}{6}$$



Open Questions - Neutrino Properties

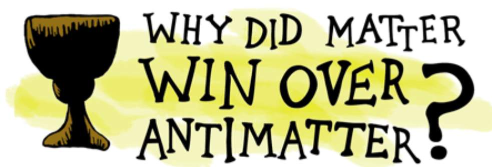


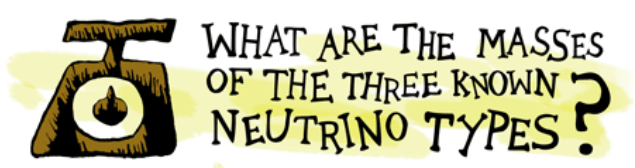


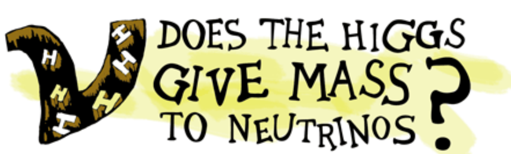


- ☞ Majorana vs Dirac?
- ☞ CP violation in lepton sector?
- ☞ Absolute mass scale of neutrinos?
- ☞ Mass ordering: sign of (Δm_{13}^2)?
- ☞ Sterile neutrino(s)?
- ☞ Precision: $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?

Open Questions - Neutrino Properties







- ☞ Majorana vs Dirac?
- ☞ CP violation in lepton sector?
- ☞ Absolute mass scale of neutrinos?
- ☞ Mass ordering: sign of (Δm_{13}^2)?
- ☞ Sterile neutrino(s)?
- ☞ Precision: $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?

a suite of current/upcoming experiments to address these puzzles

Experimental Precision

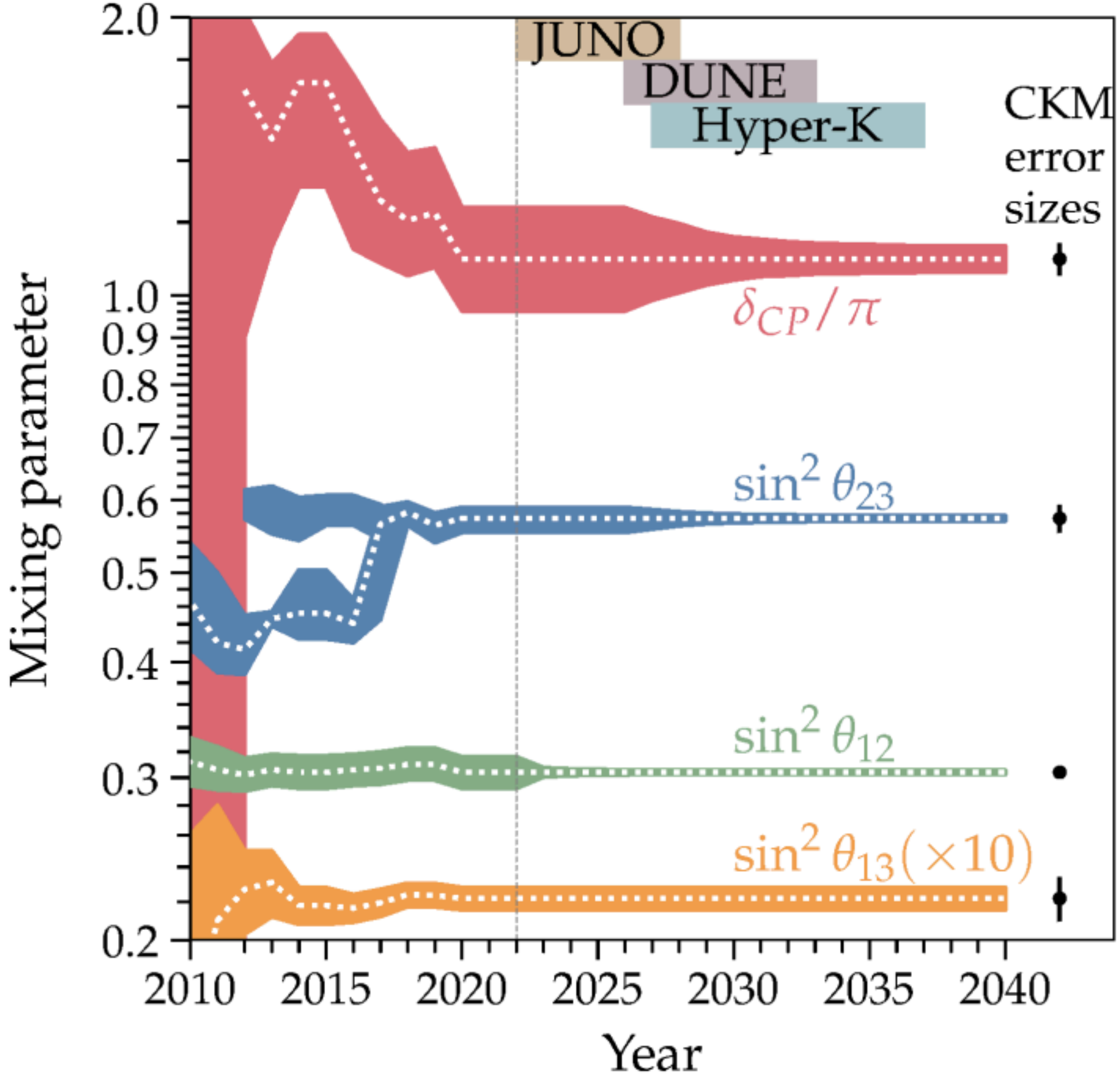
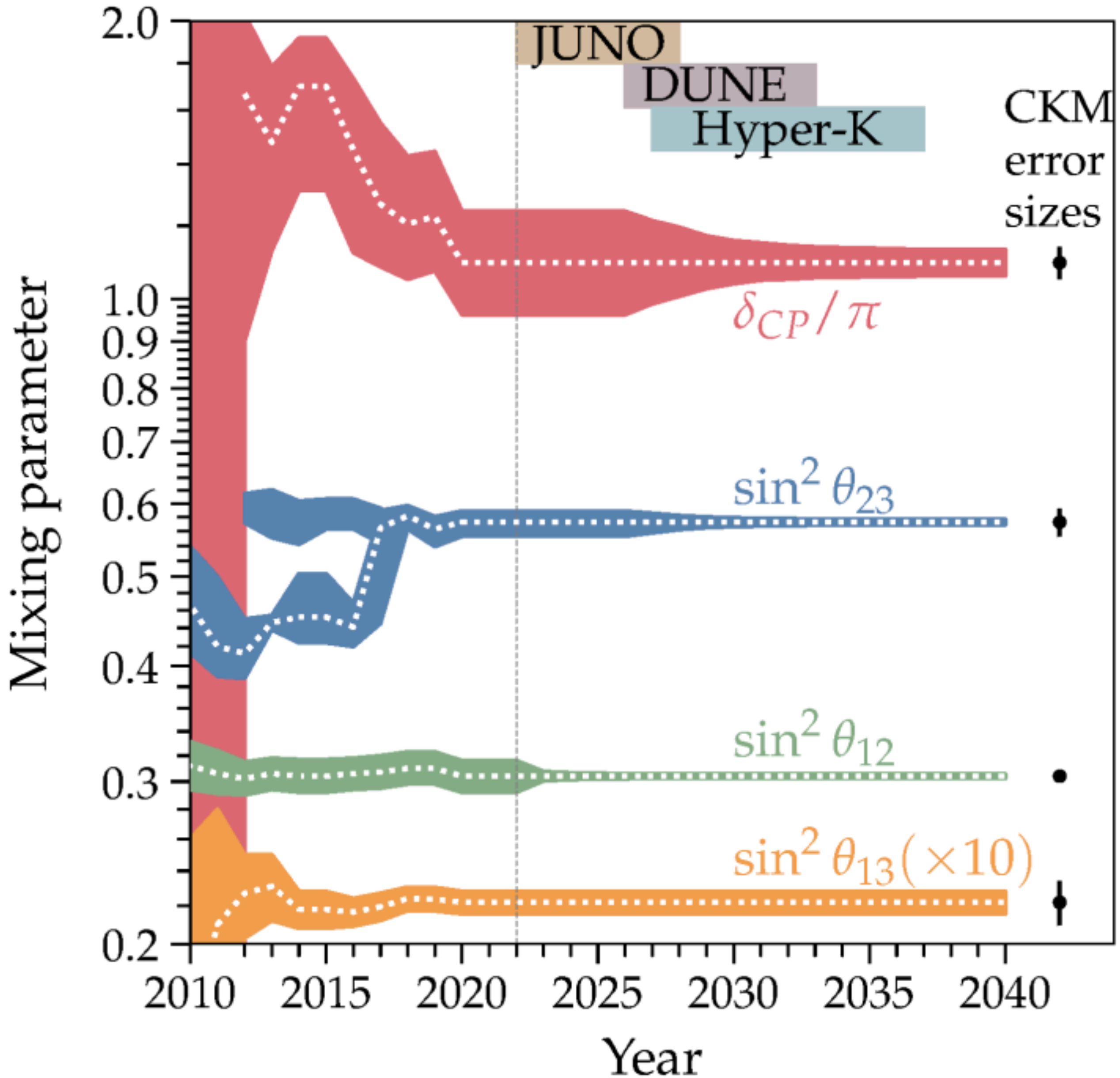


Figure Credit: Song, Li, Argüelles, Bustamante, Vincent (2020)

Experimental Precision

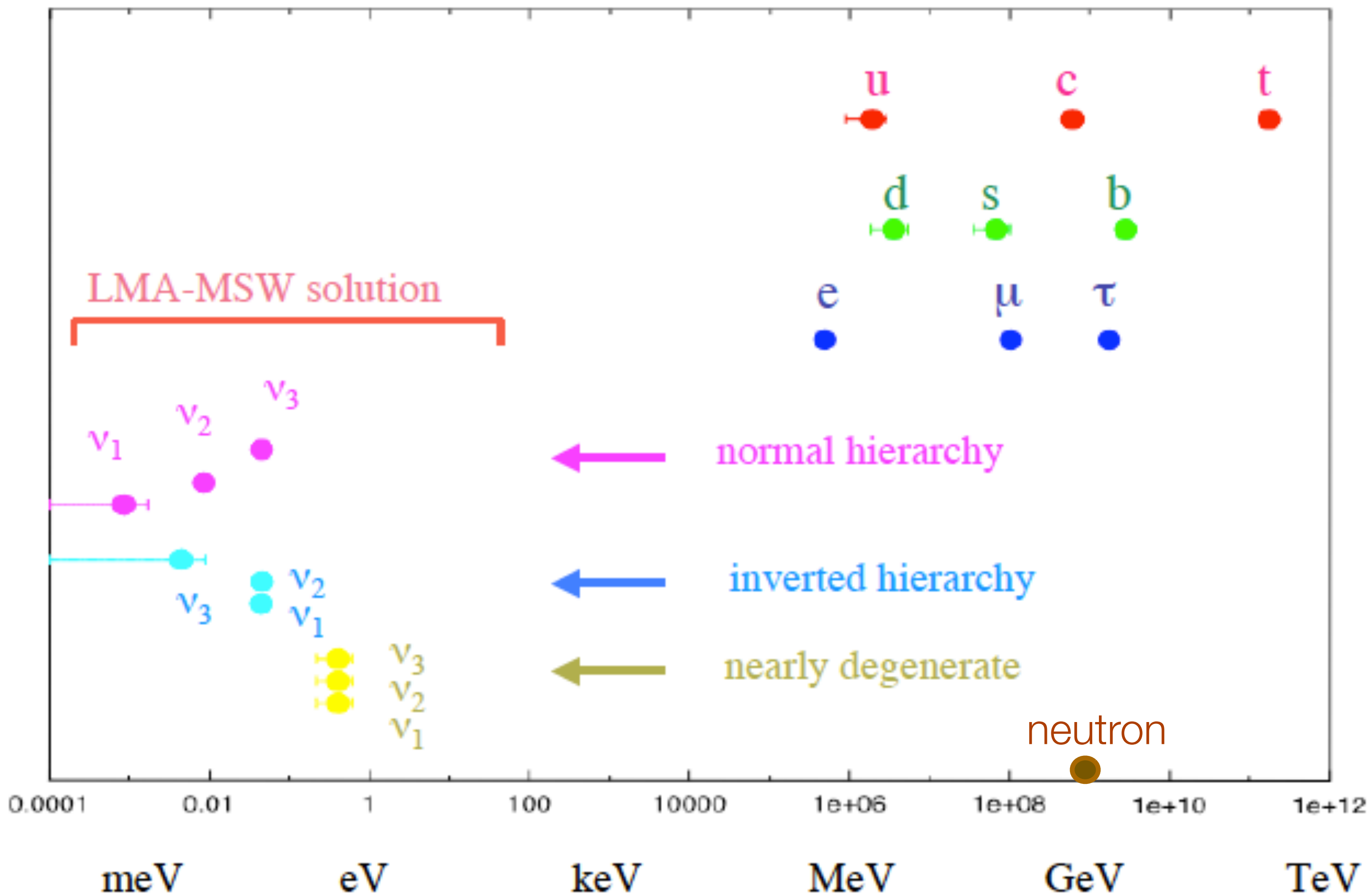


Are theoretical precision compatible with experimental precision?

Figure Credit: Song, Li, Argüelles, Bustamante, Vincent (2020)

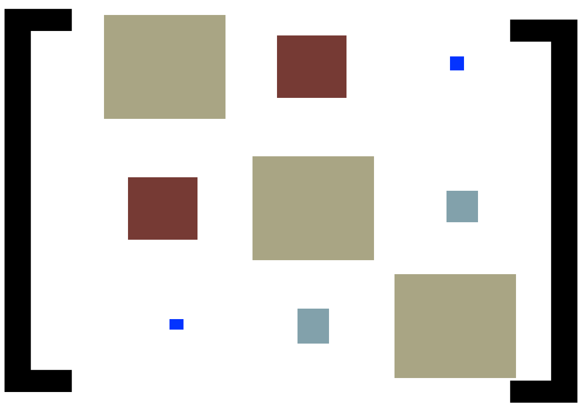
Open Questions - Theoretical

☞ Smallness of neutrino mass: $m_\nu \ll m_{e,\,u,\,d}$

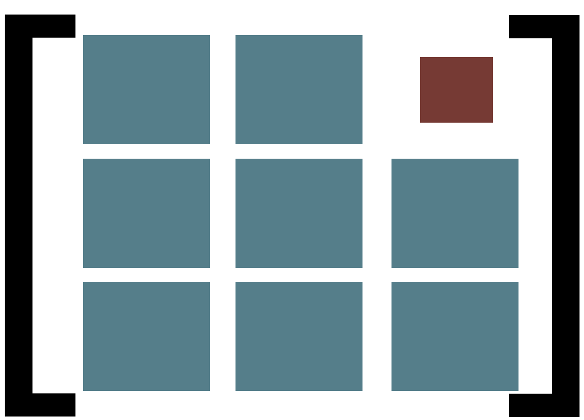


Open Questions - Theoretical

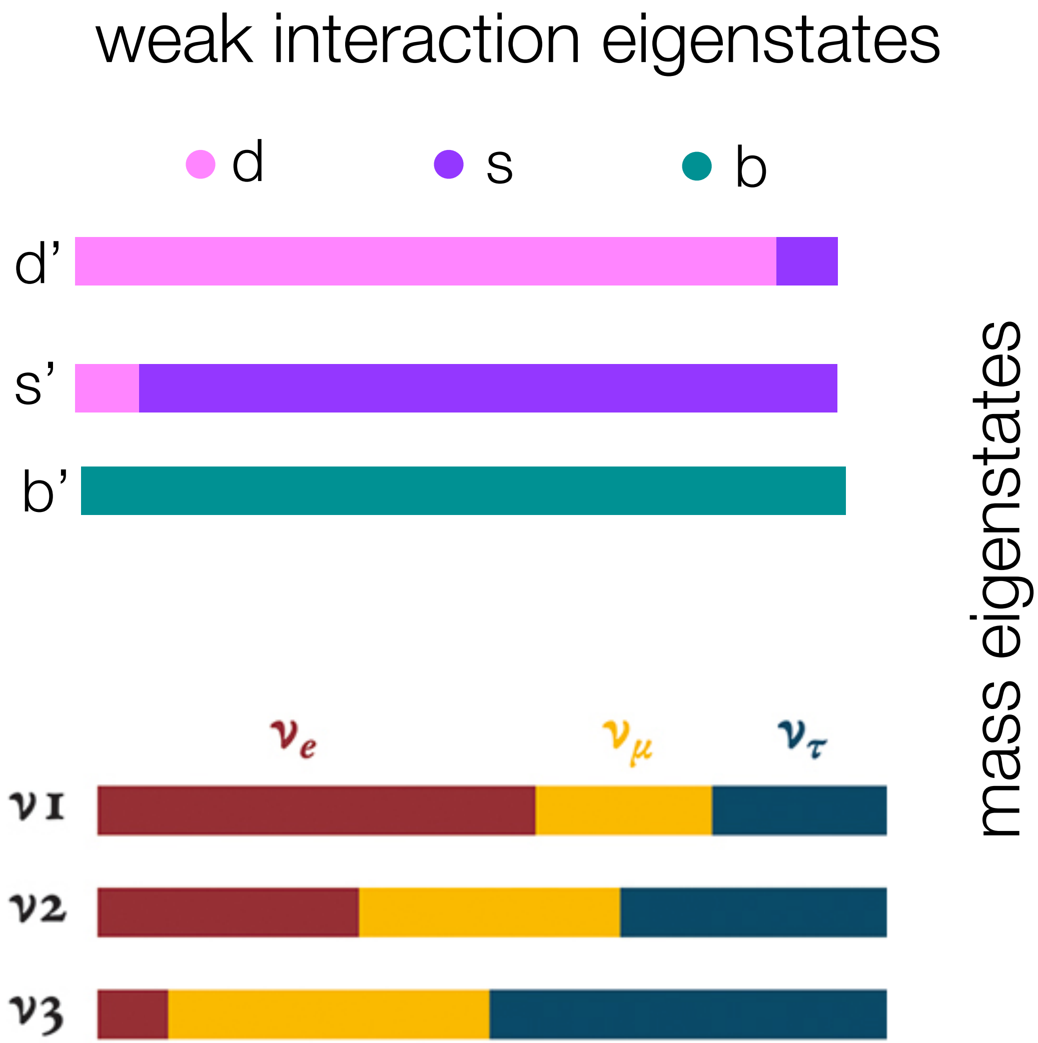
☞ **Flavor structure:**

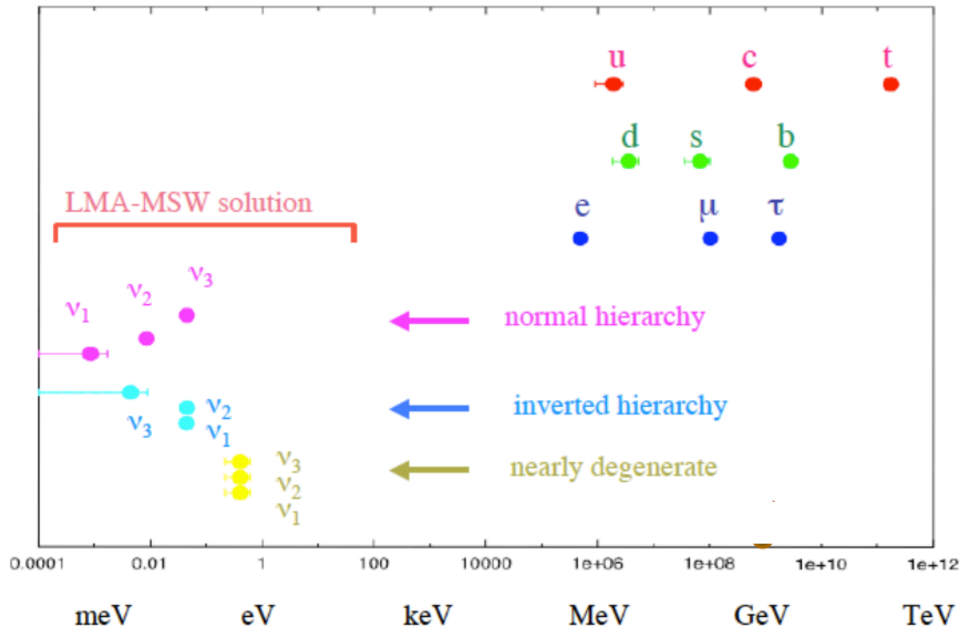


quark mixing

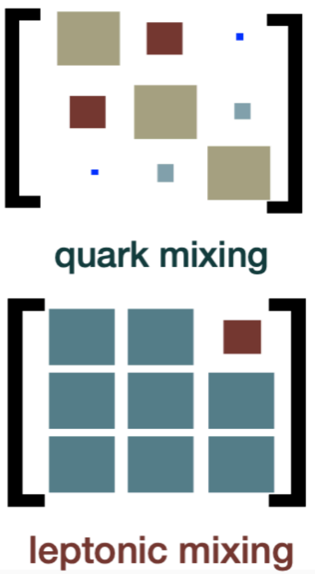


leptonic mixing





☞ Flavor structure:



Fermion mass and hierarchy problem

➟ Dominant fraction (22 out of 28) of free parameters in SM

Where do fermion mass hierarchy, flavor mixing, and CP violation come from?

Where do fermion mass hierarchy, flavor mixing, and CP violation come from?

Is there a simpler organization principle?

Where do fermion mass hierarchy, flavor mixing, and CP violation come from?

Is there a simpler organization principle?

Where do neutrinos get their masses from?

Where do fermion mass hierarchy, flavor mixing, and CP violation come from?

Is there a simpler organization principle?

Where do neutrinos get their masses from?

Is it the Higgs or something else that gives neutrino masses?

Why are neutrinos light? Seesaw Mechanism

- Adding the right-handed neutrinos:

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$$m_\nu \sim m_{light} \sim \frac{m_D^2}{M_R} << m_D$$

$$m_{heavy} \sim M_R$$

For $m_{\nu_3} \sim \sqrt{\Delta m^2_{atm}}$

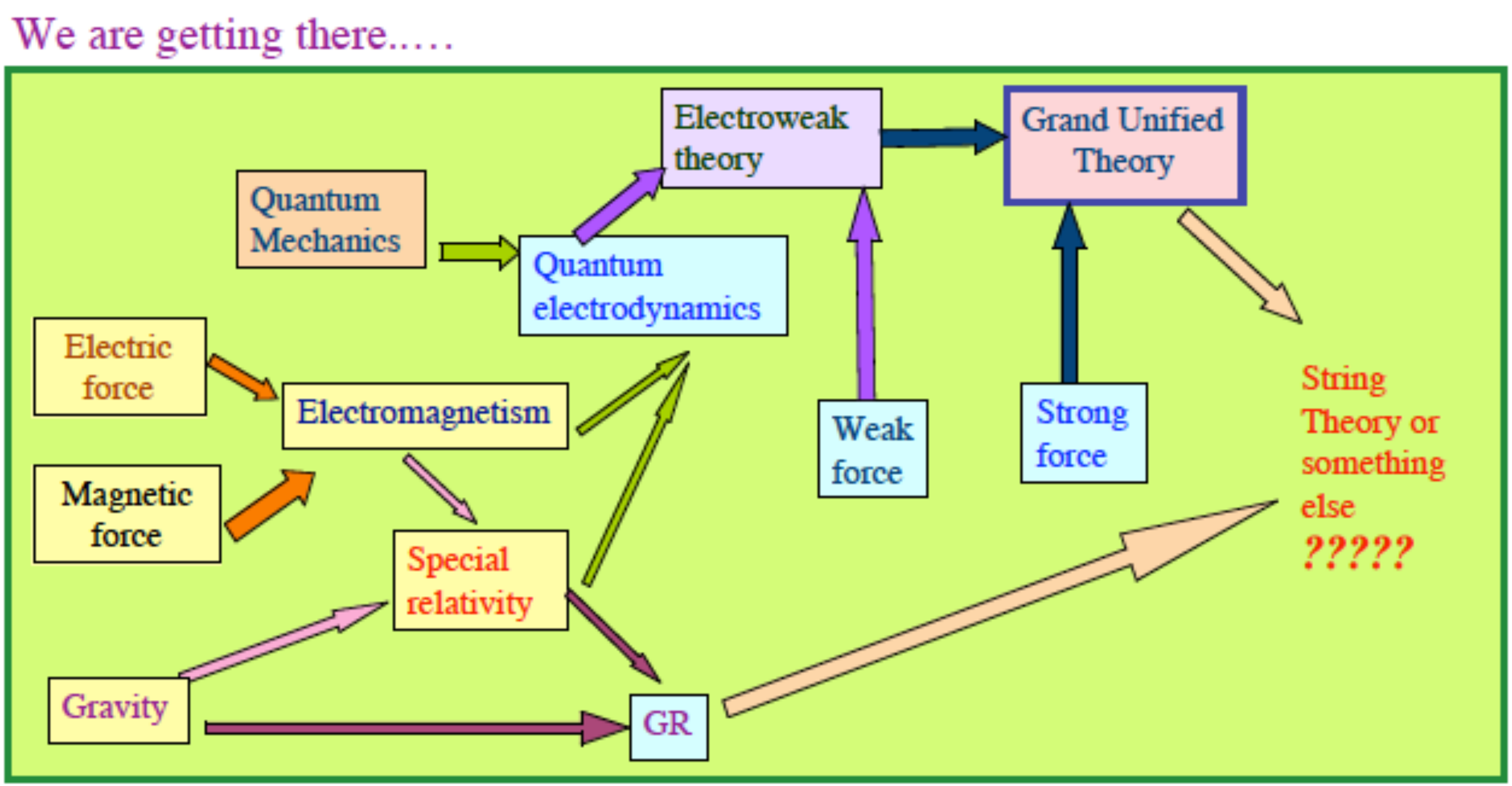
If $m_D \sim m_t \sim 180 \ \ GeV$

$\Rightarrow$ $\boxed{M_R \ \sim \ 10^{15} \ GeV \ \ (\text{GUT !!})}$

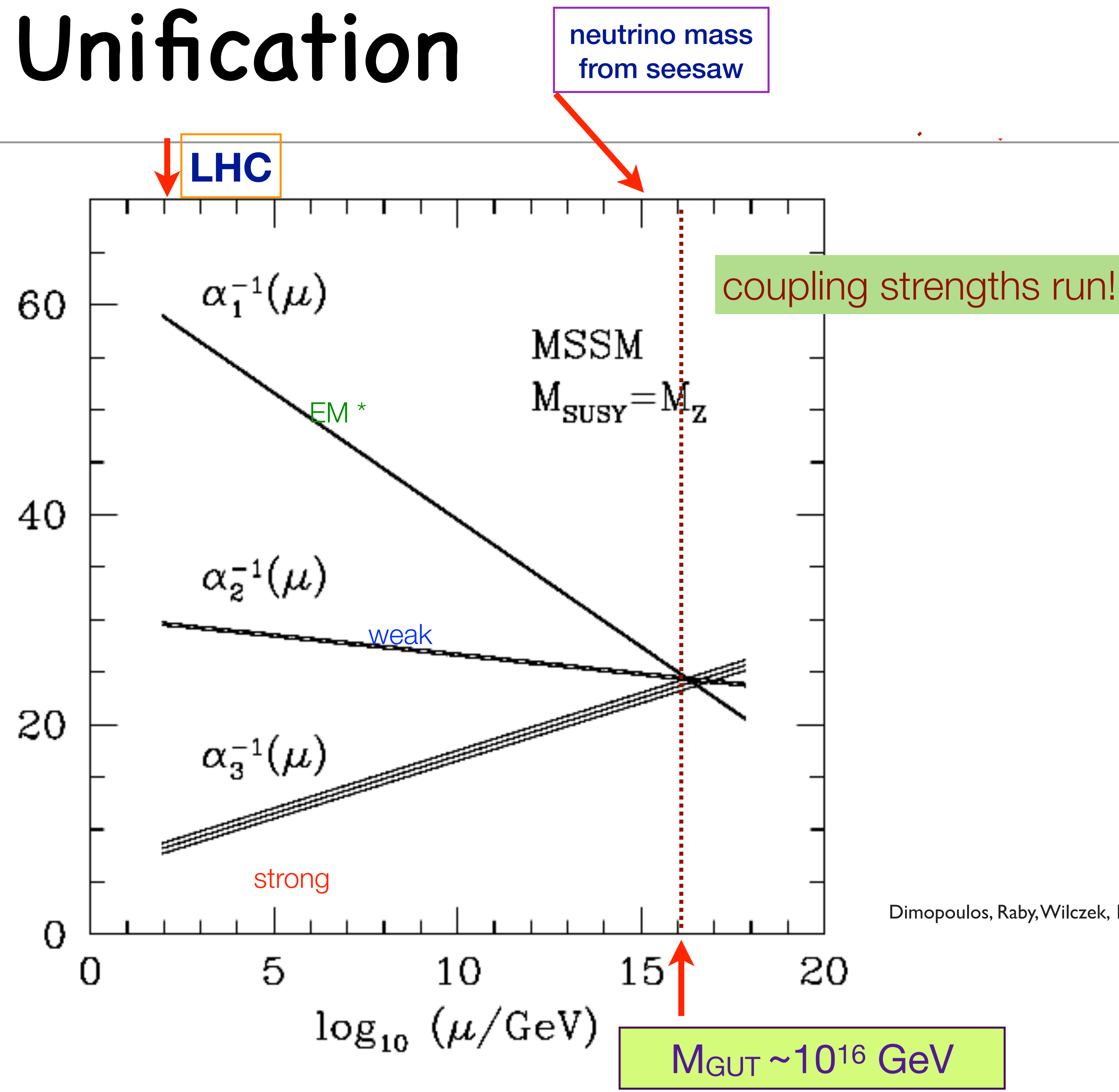
Minkowski, 1977; Yanagida, 1979; Gell-Mann, Ramond, Slansky, 1979; Mohapatra, Senjanovic, 1981

Ultimate Goal of Grand Unification

- Maxwell: electric and magnetic forces are different aspects of electromagnetism
- Einstein: early attempt to unify electric force and gravity



Grand Unification

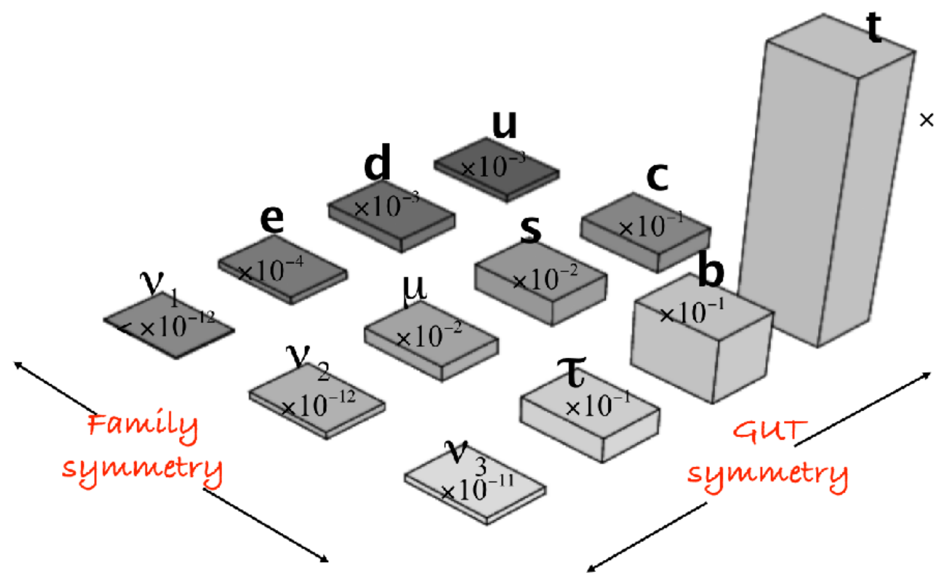


Dimopoulos, Raby, Wilczek, 1981

Symmetry Relations

Grand Unified Theories: GUT symmetry

Quarks ↔ Leptons



[Figure Credit: King, Luhn, arXiv:1301.1340]

Family Symmetry:

e-family ↔ muon-family ↔ tau-family

Symmetry Relations

Symmetry $\Rightarrow$ relations among parameters

$\Rightarrow$ reduction in number of fundamental parameters

Symmetry Relations

Symmetry $\Rightarrow$ relations among parameters

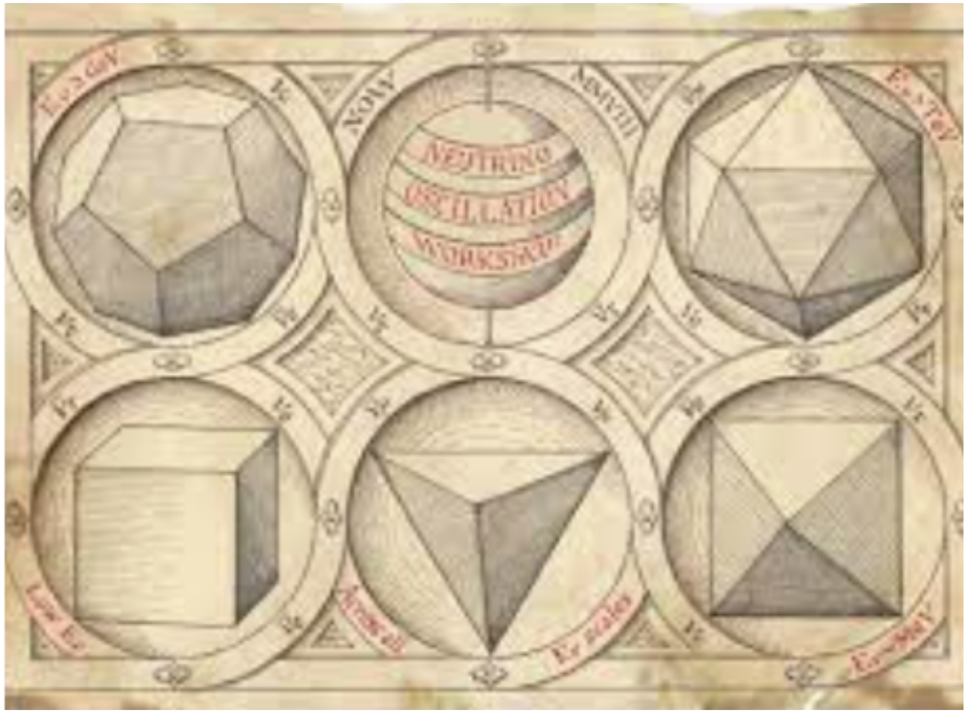
$\Rightarrow$ reduction in number of fundamental parameters

Symmetry $\Rightarrow$ experimentally testable

correlations among physical observables

Origin of Flavor Mixing and Mass

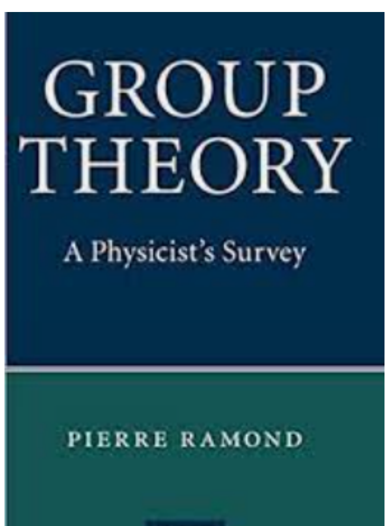
- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T' (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6
  - T_{13}



By Eligio Lisi

Origin of Flavor Mixing and Mass

- Recently, models based on discrete family symmetry groups have been constructed
    - A_4 (tetrahedron)
    - T' (double tetrahedron)
    - S_3 (equilateral triangle)
    - S_4 (octahedron, cube)
    - A_5 (icosahedron, dodecahedron)
    - Δ_{27}
    - Q_6
    - T_{13}



By Eligio Lisi

Tri-bimaximal Neutrino Mixing

- **Latest Global Fit (3σ)**

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$ $[\theta^{lep}_{23} \sim 49.2°]$

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$ $[\theta^{lep}_{12} \sim 33.4°]$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$ $[\theta^{lep}_{13} \sim 8.57°]$

- **Tri-bimaximal Mixing Pattern**

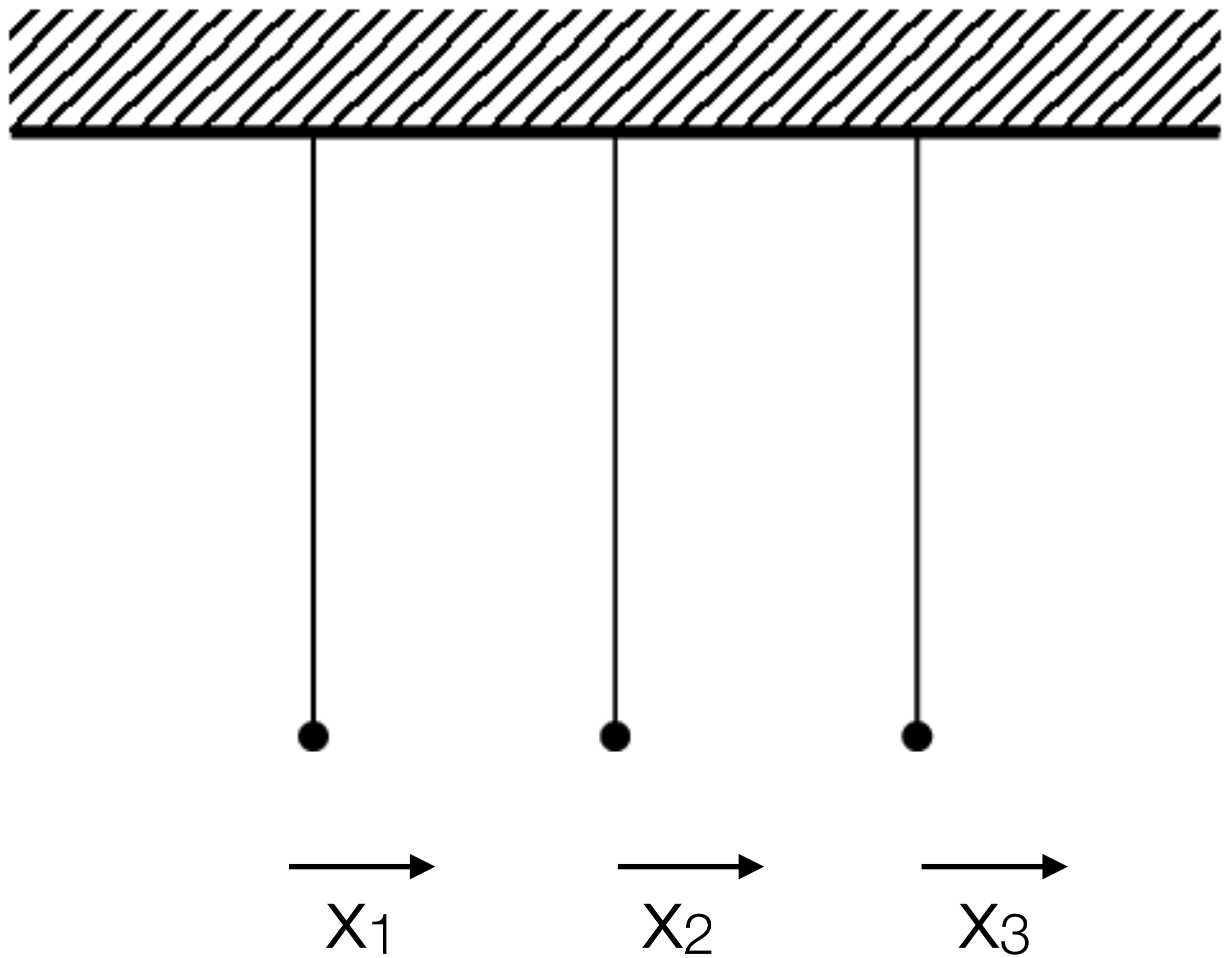
Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

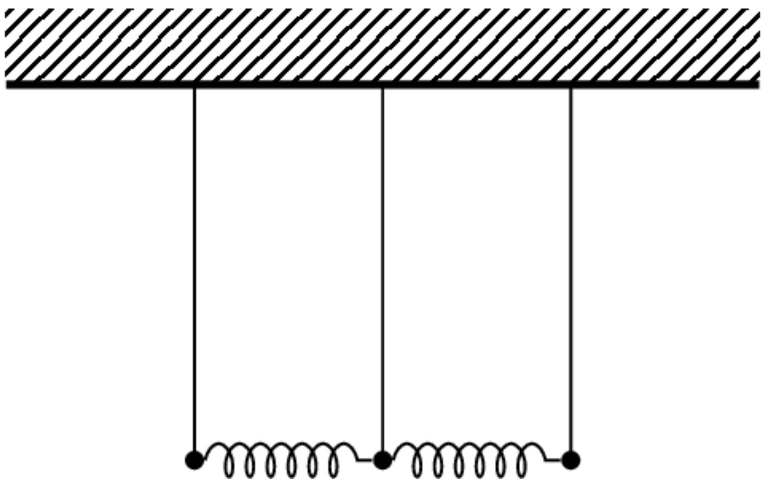
$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

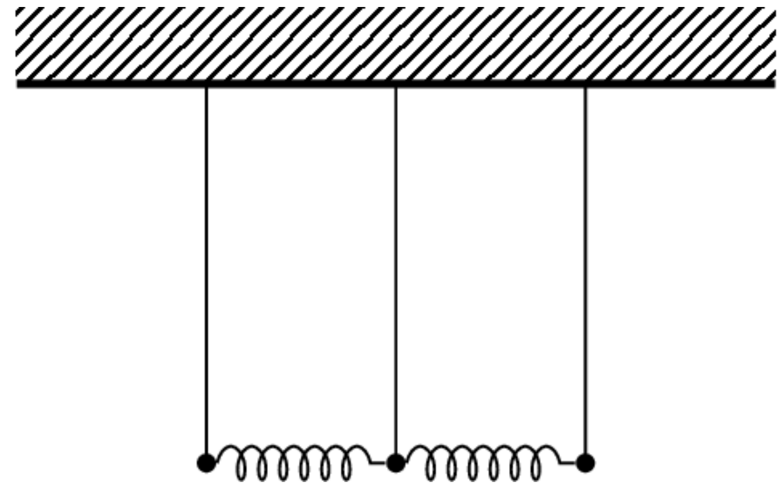
TBM and Coupled Pendulums



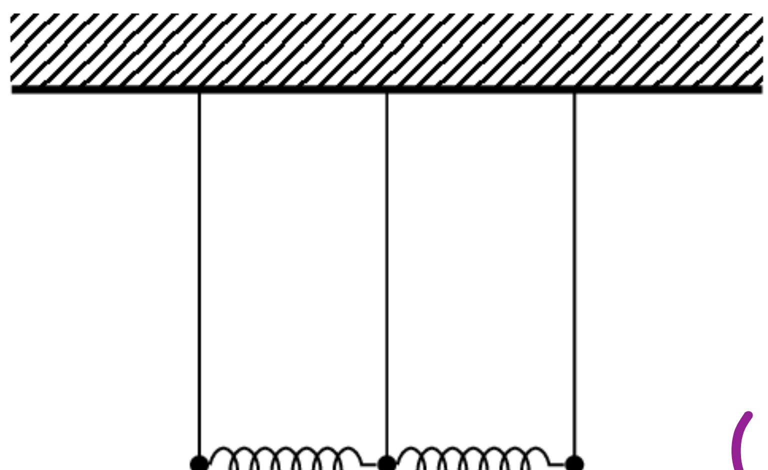
TBM and Coupled Pendulums



(1, 0, -1)



(1, 1, 1)



(1, -2, 1)

Tri-bimaximal Neutrino Mixing

- **Latest Global Fit (3σ)**

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$ [$\theta^{lep}_{23} \sim 49.2°$]

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$ [$\theta^{lep}_{12} \sim 33.4°$]

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$ [$\theta^{lep}_{13} \sim 8.57°$]

- **Tri-bimaximal Mixing Pattern**

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

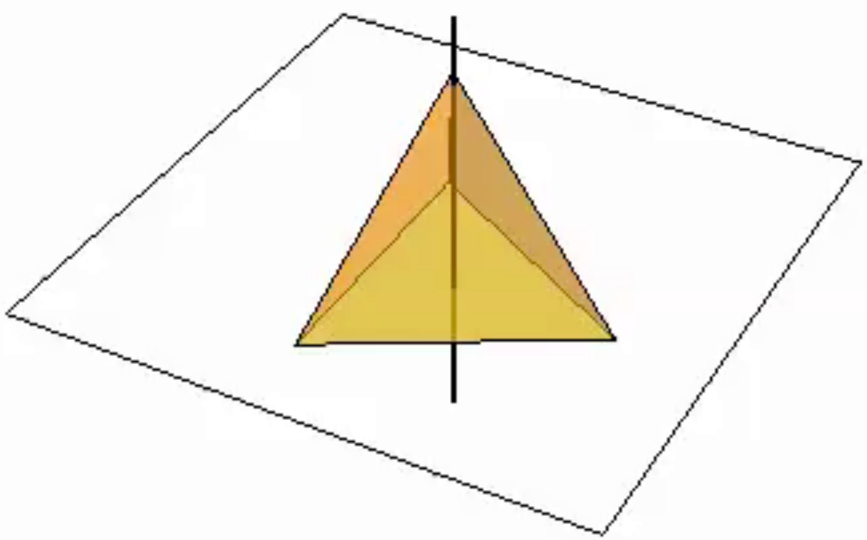
$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

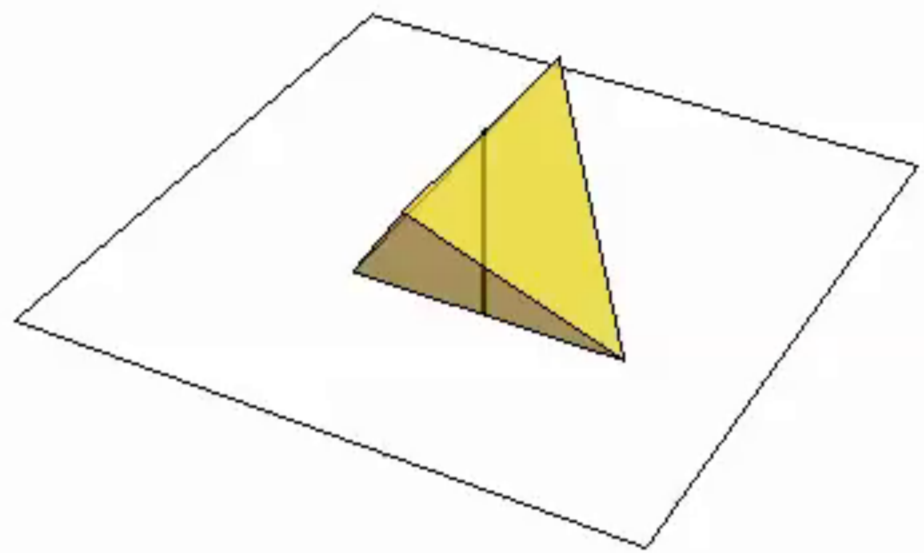
TBM from A4 Group

T: (1234) $\rightarrow$ (2314)

S: (1234) $\rightarrow$ (4321)

$$\boxed{S^2 = 1, \quad (ST)^3 = 1, \quad T^3 = 1}$$





Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields

2 free parameters

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

relative strengths $\Rightarrow$ CG's

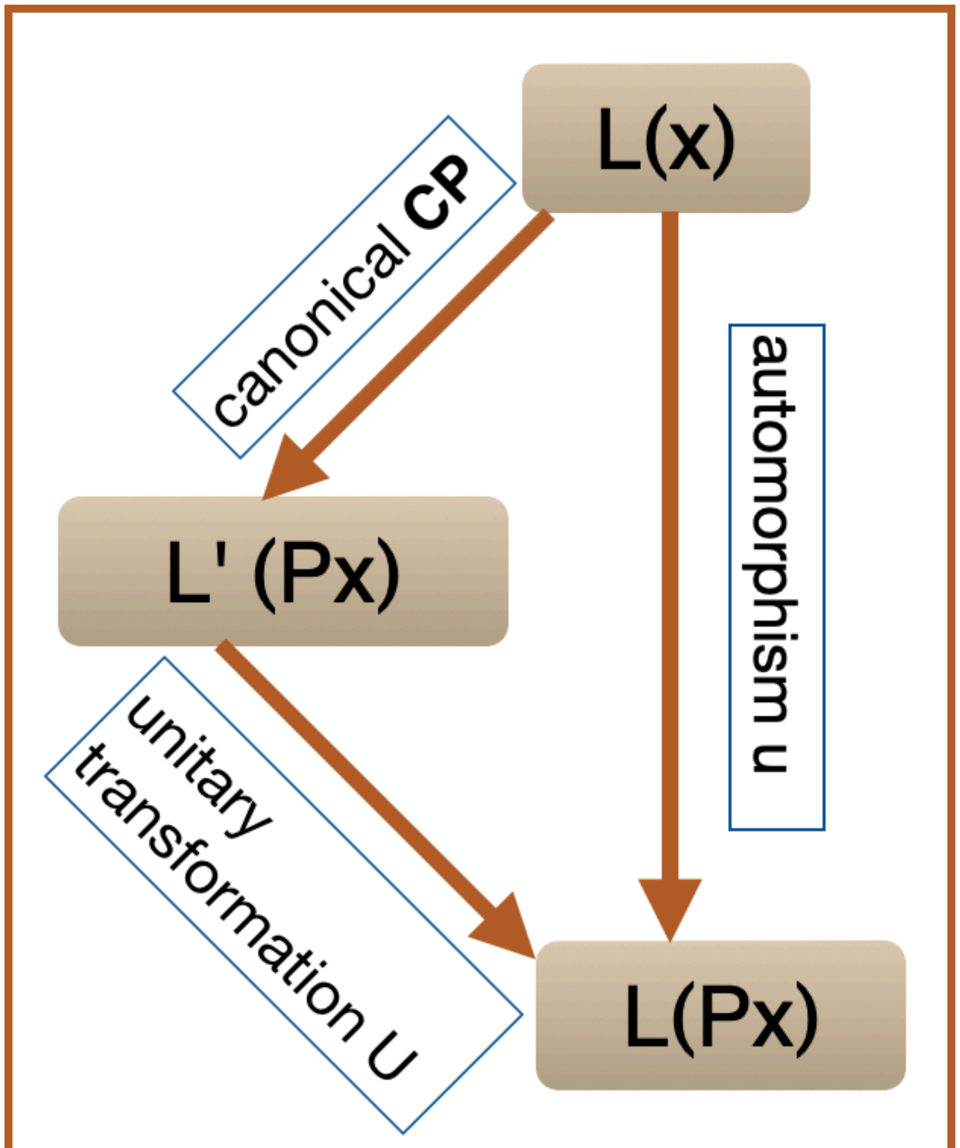
- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Neutrino Mixing Angles from Group Theory

Group Theoretical Origin of CP Violation

M-CC, Mahanthappa (2009); M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)



complex CGs ⇨ G and physical CP transformations do not always commute

Class-inverting outer automorphism

⇕

Physical CP

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

Discrete (flavor) symmetry G

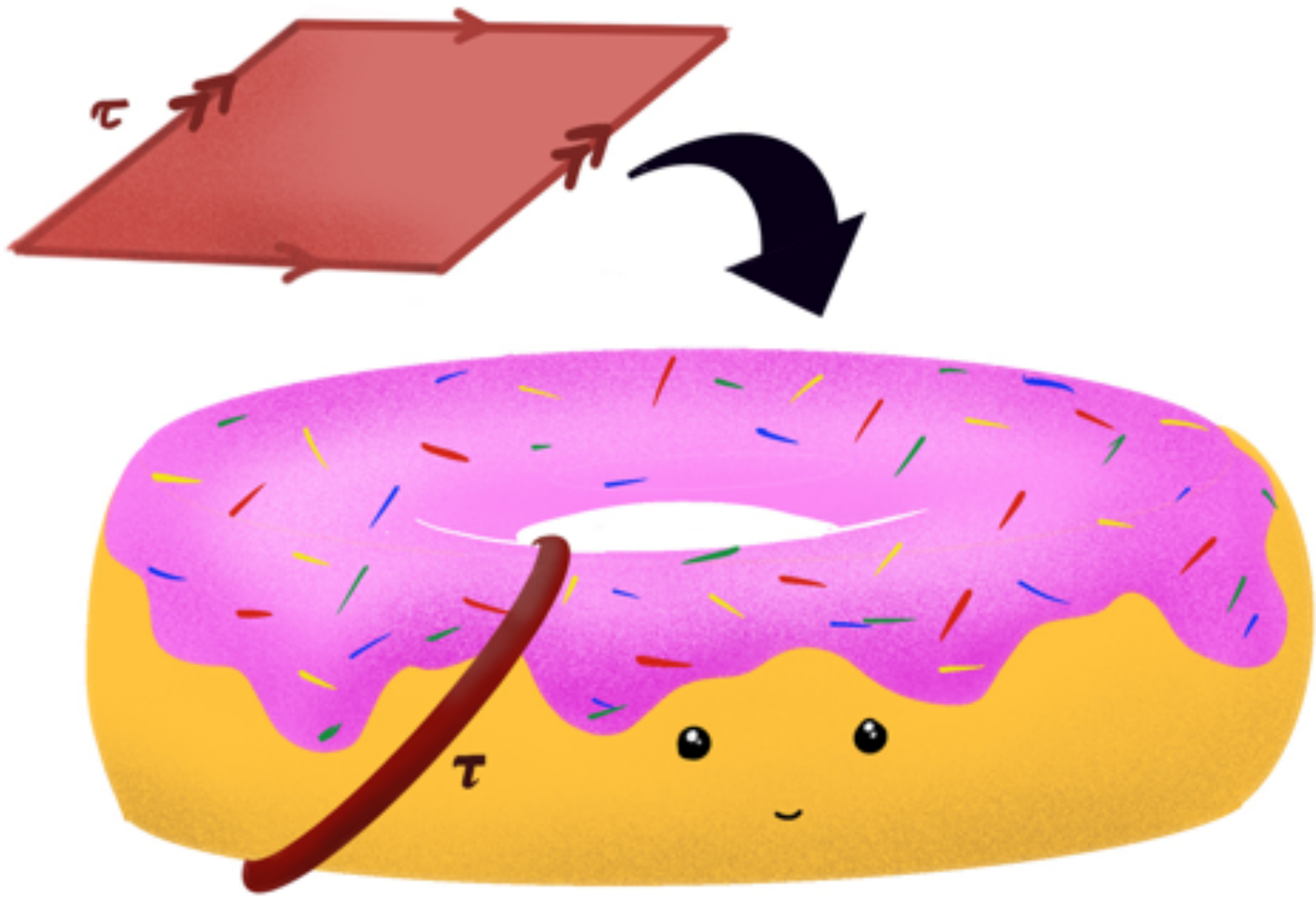
Type II: one can impose a physical CP transformation

Type I groups G_{I}:
generic settings based on G_{I} do not allow for a physical CP transformation

Type II A groups $G_{\mathrm{II\ A}}$:
there is a CP basis in which all CG's are real

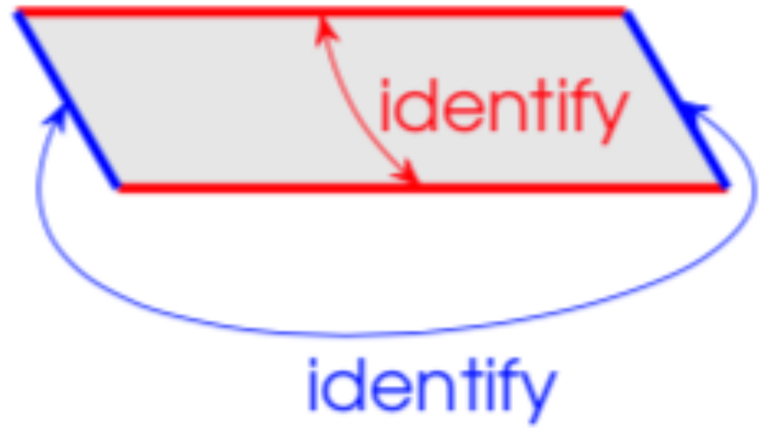
Type II B groups $G_{\mathrm{II\ B}}$:
there is no basis in which all CG's are real

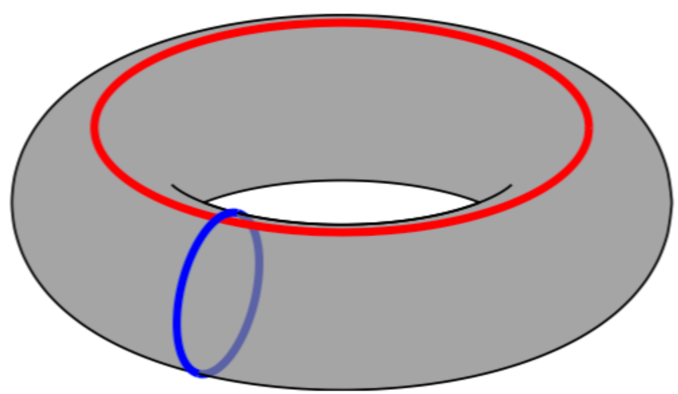
Modular Flavor Symmetries



Donuts = TORI

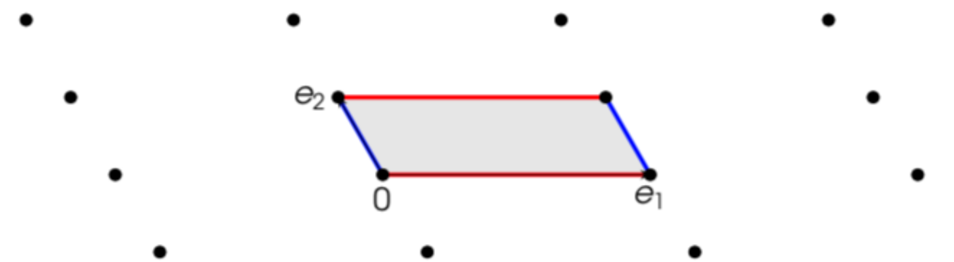
constructed
from
parallelogram





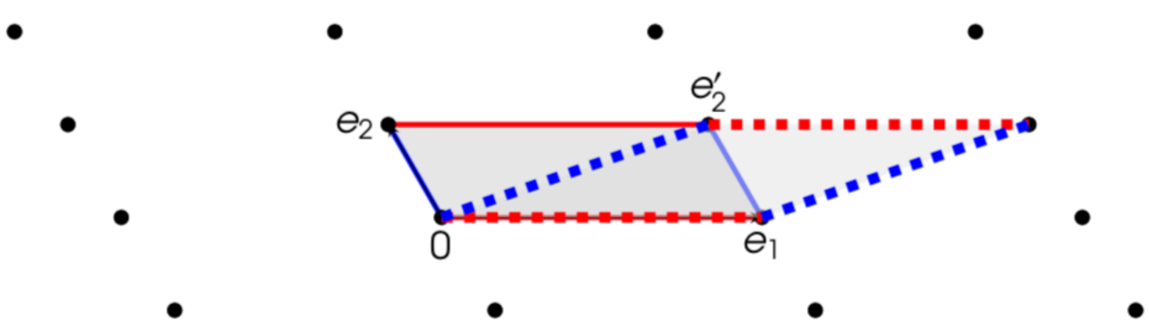
two cycles

Modular Symmetries



edges ⇒ lattice basis vectors

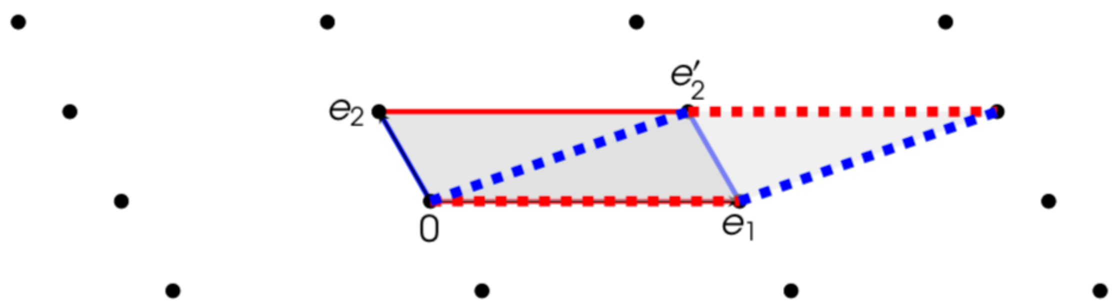
points in plane identified if differ by a lattice translation



Equivalent TORI related by Modular Symmetries

Modular Symmetries

- TORI: fundamental domain not unique



- Basis Vectors are related:

$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \overset{\gamma}{\mapsto} \begin{pmatrix} e_2' \\ e_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

$a, b, c, d \in \mathbb{Z}$

- Volume of fundamental domain the same $\Rightarrow$ $\det \gamma = 1$

Modular Symmetries

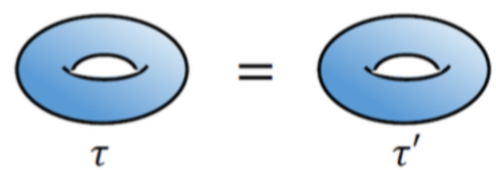
- Two basic transformations:

$$T\ :\ e_2 \mapsto e_2' = e_2 + e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S\ :\ e_1 \mapsto e_1' = e_2 \quad \text{and} \quad e_2 \mapsto e_2' = -e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

- In complex coordinates: modulus $\tau = e_2/e_1$

$$\tau \stackrel{S}{\longmapsto} \frac{-1}{\tau} \quad \text{and} \quad \tau \stackrel{T}{\longmapsto} \tau + 1$$



- S and T generate $\mathrm{SL}(2,\mathbb{Z})$ and satisfy

$$\boxed{S^2 = (S\,T)^3 = \mathbb{1}}$$

Modular Symmetries

- **Finite Modular Group (quotient group):** $\Gamma_N := \Gamma/\Gamma(N)$
  where principal congruence group $\Gamma(N)$ is

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \underbrace{\boxed{SL(2,\mathbb{Z})/\mathbb{Z}_2}}_{\Gamma} ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

- Generators of the quotient group Γ_N satisfy

$$\boxed{S^2 = 1, \quad (ST)^3 = 1, \quad T^N = 1}$$

- Some examples

$$\Gamma_2 \simeq S_3, \qquad \Gamma_3 \simeq A_4, \qquad \Gamma_4 \simeq S_4, \qquad \Gamma_5 \simeq A_5$$

Modular Symmetries

Feruglio (2017)

- Imposing modular symmetry Γ on the Lagrangian:

$$\mathscr{L} \supset \sum Y_{i_1,\, i_2,\, \dots\dots\, i_n} \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}$$

$$\tau \xmapsto{\gamma} \gamma\tau := \frac{a\tau + b}{c\tau + d} \,,$$

$$\Phi_j \xmapsto{\gamma} (c\tau + d)^{k_j} \boxed{\rho_{r_j}(\gamma)} \Phi_j \,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

k_i : integers

representation matrix of Γ_N

- Yukawa Couplings = Modular Forms at level "N" w/ weight "k"

$$f_i(\gamma\tau) = (c\tau + d)^{-k} \boxed{[\rho_N(\gamma)]_{ij}} f_j(\tau)$$

$k = k_{i_1} + k_{i_2} + \dots + k_{i_n}$

representation matrix of Γ_N

A Toy Modular A_4 Model

Feruglio (2017)

- Weinberg Operator $\qquad \mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\ Y\ (H_u \cdot L)]_1$
- Traditional A4 Flavor Symmetry
    - Yukawa Coupling Y $\rightarrow$ Flavon VEVs (A_4 triplet, 6 real parameters)

$$Y \to \langle\phi\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix}$$

- Modular A4 Flavor Symmetry
    - Yukawa Coupling Y $\rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$Y \to \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

Modular Forms

Feruglio (2017)

- Level (N) = 3, Weight (k) = 2, in terms of Dedekind eta-function

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right]\\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]\\
Y_3(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \quad .
\end{aligned}
$$

$$
\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad\qquad q \equiv e^{i2\pi\tau}
$$

A Toy Modular A_4 Model

Feruglio (2017)

- **Input Parameters:**

$$\boxed{\tau = 0.0111 + 0.9946i} \qquad v_u^2/\Lambda$$

- **Predictions:**

$$\frac{\Delta m_{sol}^2}{|\Delta m_{atm}^2|} = 0.0292$$

$$\sin^2\theta_{12} = 0.295 \qquad \sin^2\theta_{13} = 0.0447 \qquad \sin^2\theta_{23} = 0.651$$

$$\frac{\delta_{CP}}{\pi} = 1.55 \qquad \frac{\alpha_{21}}{\pi} = 0.22 \qquad \frac{\alpha_{31}}{\pi} = 1.80 \quad .$$

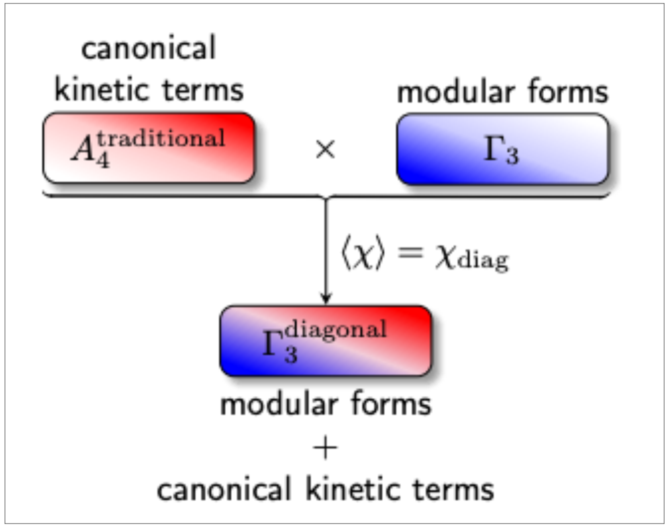
$$m_1 = 4.998 \times 10^{-2}\ eV \qquad m_2 = 5.071 \times 10^{-2}\ eV \qquad m_3 = 7.338 \times 10^{-4}\ eV$$

Modular Symmetries: Bottom-Up Meet Top-Down

- Bottom-Up:
  - reducing the number of parameters: in extreme case, entire neutrino mass matrix controlled by τ Feruglio (2017)
    - many interesting models based on modular flavor symmetries

      [Talks by Ferruccio Feruglio, Steve King, Serguey Petcov, João Penedo, Arsenii Titov, Gui-Jun Ding]
  - traditional NA flavor symmetries: corrections to kinetic terms -> sizable for NA discrete symmetries for leptons

    Leurer, Nir, Seiberg ('93); Dudas, Pokorski, Savoy ('95); M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
    - (Quasi-eclectic) setup with modular symmetries: corrections to kinetic terms can be under control ⟶ **reduction of theory uncertainty**



MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

Modular Symmetries: Bottom-Up Meet Top-Down

- Top-Down:
  - Modular flavor symmetries from strings
    - Calabi-Yau [Talk by Hajime Otsuka]
    - Modular Symmetries from magnetized tori e.g. Almumin, MCC, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)
    - Eclectic Flavor Symmetries e.g. Baur, Nilles, Trautner, Vaudrevange (2019)
      - Ingredients for reducing theoretical uncertainty [Talks by Saúl Ramos-Sanchez, Alexander Baur]
  - CP and other outer automorphisms in modular symmetries e.g. Baur, Nilles, Trautner, Vaudrevange (2019) [Talk by Andreas Trautner]

Acknowledgements

Yahya Almumin (UCI Grad)

Víctor Knapp-Pérez (Former UNAM UG; UCI Grad)

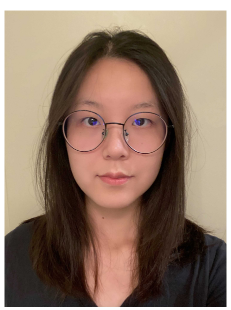
Adreja Mondol (UCI Grad)

Mario Ramos-Hamud (UNAM MS; Cambridge Grad 2022)

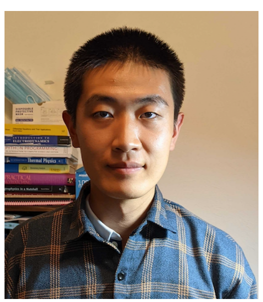
Shreya Shukla (UCI Grad)

Maximilian Fallbacher (former TUM Grad)

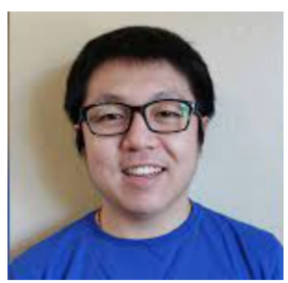
Andreas Trautner (MPI Heidelberg PD; former TUM Grad/ BCTP PD)



Murong Cheng (UIUC Grad; former UCI UG)

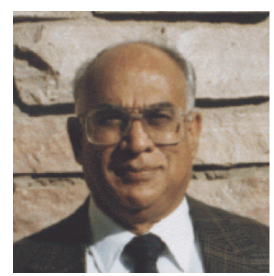


Martin Yulun Li (Virginia Tech Grad; former UCI UG)



Tommy Wen Chin (San Jose State U UG; Penn State Grad Fall'22)

Chinmayi Subramanya (IISER, Mohali, India UG)



K.T. Mahanthappa (CU Boulder)

Saúl Ramos-Sánchez (UNAM, Mexico)

Michael Ratz (UCI)

Outlook

- Fundamental origin of fermion mass & mixing patterns still unknown
  - It took decades to understand the gauge sector of SM
- Uniqueness of Neutrino masses offers exciting opportunities to explore BSM Physics
  - Many NP frameworks; addressing other puzzles
  - Early Universe (leptogenesis, non-thermal relic neutrinos)
- New Tools/insights: examples of pheno relevance of formal theories
  - Non-Abelian Discrete Flavor Symmetries
    - Deep connection between outer automorphisms and CP
  - Modular Flavor Symmetries
    - Enhanced predictivity of flavor models (enhanced theory precision)
    - Possible connection to string theories -> promising venue toward realistic theories
- **TD-BU: Having diverse perspectives drives intellectual excellence**