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Generalised CP symmetry in the bottom-up modular invariance approach to flavour

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Outline

- ▶ Flavour symmetry and generalised CP (GCP)
- ▶ Modular symmetry
- ▶ GCP transformations consistent with modular symmetry
- ▶ Implications of GCP for modular-invariant theories
- ▶ Example

Flavour symmetry and generalised CP

A rigid SUSY theory with a flavour symmetry G_f

$$\psi(x) \xrightarrow{g} \rho_{\mathbf{r}}(g)\psi(x), \quad g \in G_f$$

Chiral superfield

Irrep of G_f

Non-Abelian finite group

Generalised CP (GCP) transformation

$$\psi(x) \xrightarrow{CP} X_{\mathbf{r}} \bar{\psi}(x_P)$$

Unitary matrix acting
on flavour space

Hermitian conjugate
superfield

$$x = (t, \mathbf{x}) \quad x_P = (t, -\mathbf{x})$$

Canonical CP transformation

$$X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$$

Flavour symmetry and generalised CP

The form of $X_{\mathbf{r}}$ is constrained by G_f

$$\psi(x) \xrightarrow{CP} X_{\mathbf{r}} \bar{\psi}(x_P) \xrightarrow{g} X_{\mathbf{r}} \rho_{\mathbf{r}}^*(g) \bar{\psi}(x_P) \xrightarrow{CP^{-1}} X_{\mathbf{r}} \rho_{\mathbf{r}}^*(g) X_{\mathbf{r}}^{-1} \psi(x)$$

This defines the **consistency condition**

$$X_{\mathbf{r}} \rho_{\mathbf{r}}^*(g) X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g') , \quad g, g' \in G_f$$

Feruglio, Hagedorn, Ziegler, 1211.5560
Holthausen, Lindner, Schmidt, 1211.6953
M.-C. Chen et al., 1402.0507

- Satisfied for all \mathbf{r} simultaneously
- For a given \mathbf{r} , $X_{\mathbf{r}}$ is defined up to a G_f transformation and an overall phase
- Realises a *homomorphism* $v(g) = g'$ of G_f ; for faithful \mathbf{r} , an *automorphism*
- $v(g) = g'$ must be *class-inverting* w.r.t. G_f ($g', g^{-1} \in$ same conjugacy class)
- *Outer automorphism* (there is no $h \in G_f$ s.t. $g' = h^{-1} g h$)

Flavour symmetry and generalised CP

If $X = X^T$, s.t. $\psi(x) \xrightarrow{CP^2} \psi(x)$, the full symmetry group

$$G_{CP} = G_f \rtimes H_{CP}, \quad H_{CP} \simeq Z_2^{CP}$$

Feruglio, Hagedorn, Ziegler, 1211.5560

$G_f = S_3, A_4, S_4, A_5$ can be generated by two transformations satisfying

$$\langle S, T \mid S^2 = (ST)^3 = T^N = I \rangle, \quad N = 2, 3, 4, 5$$

In a **symmetric basis** for S and T , where $\rho_{\mathbf{r}}(S) = \rho_{\mathbf{r}}^T(S)$ and $\rho_{\mathbf{r}}(T) = \rho_{\mathbf{r}}^T(T)$,
 $X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$ up to inner automorphisms, i.e., $X_{\mathbf{r}} = \rho_{\mathbf{r}}(g)$, $g \in G_f$

Holthausen, Lindner, Schmidt, 1211.6953

Ding, King, Stuart, 1307.4212

Li, Ding, 1503.03711

Modular symmetry

$$\tau \rightarrow \gamma\tau \equiv \frac{a\tau + b}{c\tau + d} \quad a, b, c, d \in \mathbb{Z} \quad ad - bc = 1$$

$$\bar{\Gamma} = \langle S, T \mid S^2 = (ST)^3 = I \rangle \simeq PSL(2, \mathbb{Z})$$

$$\tau \xrightarrow{S} -\frac{1}{\tau}$$

duality

discrete shift symmetry

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Infinite normal subgroups of $SL(2, \mathbb{Z})$, $N = 2, 3, 4, \dots$

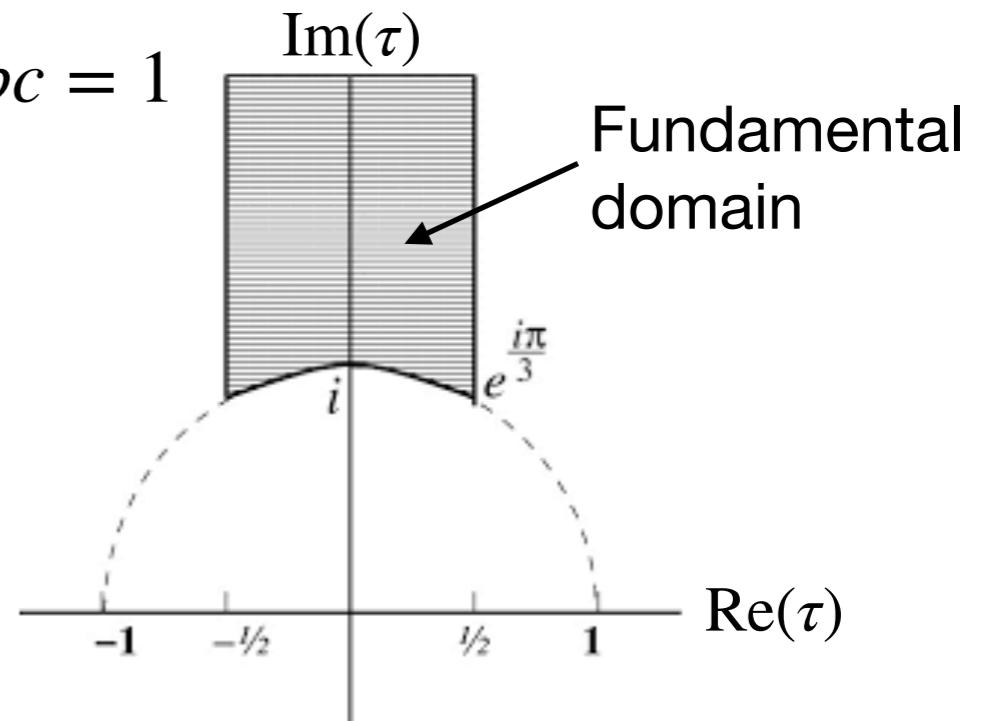
$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

Principal congruence subgroups of the modular group

$$\bar{\Gamma}(2) \equiv \Gamma(2)/\{I, -I\} \quad \bar{\Gamma}(N) \equiv \Gamma(N), \quad N > 2$$

Finite modular groups

$$\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$$



Modular forms

Holomorphic functions transforming under $\bar{\Gamma}(N)$ as

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \gamma \in \bar{\Gamma}(N)$$

k is weight
non-negative even integer

N is level
natural number

Modular forms of weight k and level N form a linear space $\mathcal{M}_k(\bar{\Gamma}(N))$ of finite dimension. We can choose a basis in this space s.t. $F(\tau) \equiv (f_1(\tau), f_2(\tau), \dots)^T$ transforms as

$$F(\gamma\tau) = (c\tau + d)^k \rho_{\mathbf{r}}(\tilde{\gamma}) F(\tau), \quad \gamma \in \bar{\Gamma}$$

\mathbf{r} is a unitary representation of Γ_N

$\tilde{\gamma}$ represents the equivalence class of γ in Γ_N

Feruglio, 1706.08749

Modular-invariant SUSY theories

$\mathcal{N} = 1$ rigid SUSY matter action

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} K(\tau, \bar{\tau}, \psi, \bar{\psi}) + \int d^4x d^2\theta W(\tau, \psi) + \int d^4x d^2\bar{\theta} \bar{W}(\bar{\tau}, \bar{\psi})$$

Ferrara, Lust, Shapere, Theisen, PLB **225** (1989) 363

Ferrara, Lust, Theisen, PLB **233** (1989) 147

$$\begin{cases} \tau \rightarrow \frac{a\tau + b}{c\tau + d} \\ \psi_i \rightarrow (c\tau + d)^{-k_i} \rho_{\mathbf{r}_i}(\tilde{\gamma}) \psi_i \end{cases} \Rightarrow \begin{cases} W(\tau, \psi) \rightarrow W(\tau, \psi) \\ K(\tau, \bar{\tau}, \psi, \bar{\psi}) \rightarrow K(\tau, \bar{\tau}, \psi, \bar{\psi}) + f_K(\tau, \psi) + \bar{f}_K(\bar{\tau}, \bar{\psi}) \end{cases}$$

Feruglio, 1706.08749

$$W(\tau, \psi) = \sum_n \sum_{\{i_1, \dots, i_n\}} \sum_s g_{i_1 \dots i_n, s} \left(Y_{i_1 \dots i_n, s}(\tau) \psi_{i_1} \dots \psi_{i_n} \right)_{\mathbf{1}, s}$$

$$Y(\tau) \xrightarrow{\gamma} (c\tau + d)^{k_Y} \rho_{\mathbf{r}_Y}(\tilde{\gamma}) Y(\tau)$$

$$k_Y = k_{i_1} + \dots + k_{i_n}$$

$$\mathbf{r}_Y \otimes \mathbf{r}_{i_1} \otimes \dots \otimes \mathbf{r}_{i_n} \supset \mathbf{1}$$

CP transformation of the modulus

Consistency condition chain $CP \rightarrow \gamma \rightarrow CP^{-1}$ on a superfield

$$\psi(x) \xrightarrow{CP} X_{\mathbf{r}} \bar{\psi}(x_P) \xrightarrow{\gamma} (c\tau^* + d)^{-k} X_{\mathbf{r}} \rho_{\mathbf{r}}^*(\gamma) \bar{\psi}(x_P) \xrightarrow{CP^{-1}} (c\tau_{CP^{-1}}^* + d)^{-k} X_{\mathbf{r}} \rho_{\mathbf{r}}^*(\gamma) X_{\mathbf{r}}^{-1} \psi(x)$$

$\overbrace{\hspace{15em}}$
 $(c'\tau + d')^{-k} \rho_{\mathbf{r}}(\gamma')$

$\tau_{CP^{-1}}$ is the result of applying CP^{-1} to the modulus

$$X_{\mathbf{r}} \rho_{\mathbf{r}}^*(\gamma) X_{\mathbf{r}}^{-1} = \left(\frac{c'\tau + d'}{c\tau_{CP^{-1}}^* + d} \right)^{-k} \rho_{\mathbf{r}}(\gamma')$$



l.h.s. is independent of τ constant (assuming $k \neq 0$)

$$\frac{c'\tau + d'}{c\tau_{CP^{-1}}^* + d} = \frac{1}{\lambda^*} \quad \lambda \in \mathbb{C} \quad \text{and} \quad |\lambda| = 1 \quad (\lambda, c', d' \text{ depend on } \gamma)$$

$$\gamma = S : \quad c = 1 \quad d = 0 \quad C \equiv c'(S) \quad D \equiv d'(S) \quad \Lambda \equiv \lambda(S)$$

$$\tau \xrightarrow{CP^{-1}} \tau_{CP^{-1}} = \Lambda(C\tau^* + D) \quad \tau \xrightarrow{CP} \tau_{CP} = \frac{1}{C}(\Lambda\tau^* - D)$$

CP transformation of the modulus

$CP \rightarrow T \rightarrow CP^{-1}$ on the modulus

$$\tau \xrightarrow{CP} \frac{1}{C} (\Lambda \tau^* - D) \xrightarrow{T} \frac{1}{C} (\Lambda (\tau^* + 1) - D) \xrightarrow{CP^{-1}} \underbrace{\tau + \frac{\Lambda}{C}}_{\text{modular transformation}}$$

$$\begin{cases} \Lambda/C \in \mathbb{Z} \\ |\Lambda| = 1 \end{cases} \Rightarrow |C| = 1, \text{ i.e., } C = \pm 1 \Rightarrow \Lambda = \pm 1$$

Choosing $C = -1$ and $\Lambda = +1$, s.t. $\text{Im } \tau_{CP} > 0$, we obtain

$$\tau_{CP} = n - \tau^* \quad n \in \mathbb{Z}$$

$CP \rightarrow S \rightarrow CP^{-1}$ on the modulus does not lead to further constraints

It is always possible to redefine the CP transformation in such a way that $n = 0$

$$\boxed{\tau \xrightarrow{CP} -\tau^*}$$

This CP transformation appears in the TD approach:

Baur, Nilles, Trautner, Vaudrevange, 1901.03251; 1908.00805

Acharya et al, hep-th/9506143; Dent, hep-ph/0105285; Giedt, hep-ph/0204017

Extended modular group

$CP \rightarrow \gamma \rightarrow CP^{-1}$ on the modulus

$$\tau \xrightarrow{CP} -\tau^* \xrightarrow{\gamma} -\frac{a\tau^* + b}{c\tau^* + d} \xrightarrow{CP^{-1}} \frac{a\tau - b}{-c\tau + d}$$

Outer automorphism of $\bar{\Gamma}$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow u(\gamma) \equiv CP\gamma CP^{-1} = \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$$

$$u(S) = S \quad u(T) = T^{-1}$$

Extended modular group

$$\bar{\Gamma}^* = \left\langle \tau \xrightarrow{S} -1/\tau, \quad \tau \xrightarrow{T} \tau + 1, \quad \tau \xrightarrow{CP} -\tau^* \right\rangle \simeq \bar{\Gamma} \rtimes Z_2^{CP}$$

$$\bar{\Gamma}^* \simeq PGL(2, \mathbb{Z}) \text{ with } CP = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad \text{if} \quad ad - bc = 1 \quad \text{and} \quad \tau \rightarrow \frac{a\tau^* + b}{c\tau^* + d} \quad \text{if} \quad ad - bc = -1$$

CP transformation of superfields

$$\psi(x) \xrightarrow{CP} X_{\mathbf{r}} \bar{\psi}(x_P)$$
$$X_{\mathbf{r}} \rho_{\mathbf{r}}^*(\gamma) X_{\mathbf{r}}^{-1} = \left(\frac{c'\tau + d'}{c\tau_{CP^{-1}}^* + d} \right)^{-k} \rho_{\mathbf{r}}(\gamma') = (\pm 1)^k \rho_{\mathbf{r}}(\gamma')$$

can be chosen +1
in $\bar{\Gamma}$ invariant theory

$X_{\mathbf{r}} \rho_{\mathbf{r}}^*(\gamma) X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(\gamma')$ (as in the traditional flavour symmetry case)

$u(\gamma)$ is unique: $S \rightarrow S$ and $T \rightarrow T^{-1}$

For each \mathbf{r} , $X_{\mathbf{r}}$ is fixed (up to an overall phase)

Symmetric basis

$$\rho_{\mathbf{r}}^*(S) = \rho_{\mathbf{r}}^\dagger(S) = \rho_{\mathbf{r}}(S^{-1}) = \rho_{\mathbf{r}}(S) \quad \rho_{\mathbf{r}}^*(T) = \rho_{\mathbf{r}}^\dagger(T) = \rho_{\mathbf{r}}(T^{-1})$$

$$X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$$

CP transformation of modular forms

$$Y(\tau) \xrightarrow{CP} Y(-\tau^*) = X_{\mathbf{r}} Y^*(\tau)$$

It can be shown that

$$X_{\mathbf{r}}^T Y^*(-\tau^*) \xrightarrow{\gamma} (c\tau + d)^k \rho_{\mathbf{r}}(\gamma) [X_{\mathbf{r}}^T Y^*(-\tau^*)]$$

i.e. $X_{\mathbf{r}}^T Y^*(-\tau^*)$ transforms as a modular form

If there exists a *unique modular form multiplet* at a certain level N , weight k and representation \mathbf{r} , which is the case for $N = 2, 3, 4, 5$ and $k = 2$, then

$$Y(\tau) = z X_{\mathbf{r}}^T Y^*(-\tau^*) \quad z \in \mathbb{C}$$

Representing $Y(\tau) = Y(-(-\tau^*)^*)$, it follows that

$$X_{\mathbf{r}} X_{\mathbf{r}}^* = |z|^2 \mathbb{I}_{\mathbf{r}}$$

- $z = e^{i\phi}$
- $X_{\mathbf{r}} = X_{\mathbf{r}}^T$

CP transformation: summary

Three ingredients

- $\tau \xrightarrow{CP} -\tau^*$
- $\psi(x) \xrightarrow{CP} X_{\mathbf{r}} \bar{\psi}(x_P)$
- $Y(\tau) \xrightarrow{CP} X_{\mathbf{r}} Y^*(\tau)$

In a **symmetric basis** for representation matrices of generators S and T of Γ_N

$$X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$$

Implication of CP for couplings

$$W \supset \sum_s g_s \left(Y_s(\tau) \psi_1 \dots \psi_n \right)_{\mathbf{1},s} \quad \overline{W} \supset \sum_s g_s^* \overline{\left(Y_s(\tau) \psi_1 \dots \psi_n \right)_{\mathbf{1},s}}$$

In a **symmetric basis** ($X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$)

$$g_s \left(Y_s(\tau) \psi_1 \dots \psi_n \right)_{\mathbf{1},s} \xrightarrow{CP} g_s \left(Y_s^*(\tau) \bar{\psi}_1 \dots \bar{\psi}_n \right)_{\mathbf{1},s} = g_s \overline{\left(Y_s(\tau) \psi_1 \dots \psi_n \right)_{\mathbf{1},s}}$$

reality of Clebsch-Gordan coefficients
(holds for $N \leq 5$)

$$g_s = g_s^*$$

Implication of CP for mass matrices

$$W_L = \sum_s g_s \left(Y_s(\tau) E^c L H_d \right)_{\mathbf{1},s} = \sum_s g_s \lambda_{ij}^s(\tau) E_i^c L_j H_d \equiv \lambda_{ij}(\tau) E_i^c L_j H_d$$

$$M_e = v_d \lambda^\dagger \text{ (in the left-right convention)}$$

In a symmetric basis ($X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$)

$$M_e(-\tau^*) = M_e^*(\tau)$$

$$M_e(-\tau^*) = v_d \sum_s g_s^* (\lambda^s)^\dagger(-\tau^*) = v_d \sum_s g_s^* (\lambda^s)^T(\tau)$$

$$M_e^*(\tau) = \left(v_d \sum_s g_s^* (\lambda^s)^\dagger(\tau) \right)^* = v_d \sum_s g_s (\lambda^s)^T(\tau)$$

CP-conserving values of the modulus

Obviously $\tau \xrightarrow{CP} -\tau^* = \tau$ ($\text{Re } \tau = 0$) preserves CP

$$M_{e,\nu}(\tau) = M_{e,\nu}^*(\tau) \Rightarrow \sin \delta = \sin \alpha_{21} = \sin \alpha_{31} = 0$$

However, τ and $\gamma\tau$ are physically equivalent, and CP is expected to be preserved for

$$\tau \xrightarrow{CP} -\tau^* = \gamma\tau$$

Indeed, it can be checked that

$$\text{obs} [M_{e,\nu}(\tau)] = \text{obs} [M_{e,\nu}(\gamma\tau)] = \text{obs} [M_{e,\nu}^*(\tau)]$$

modular invariance CP invariance

CP-conserving values of the modulus

Fundamental domain of $\bar{\Gamma}$

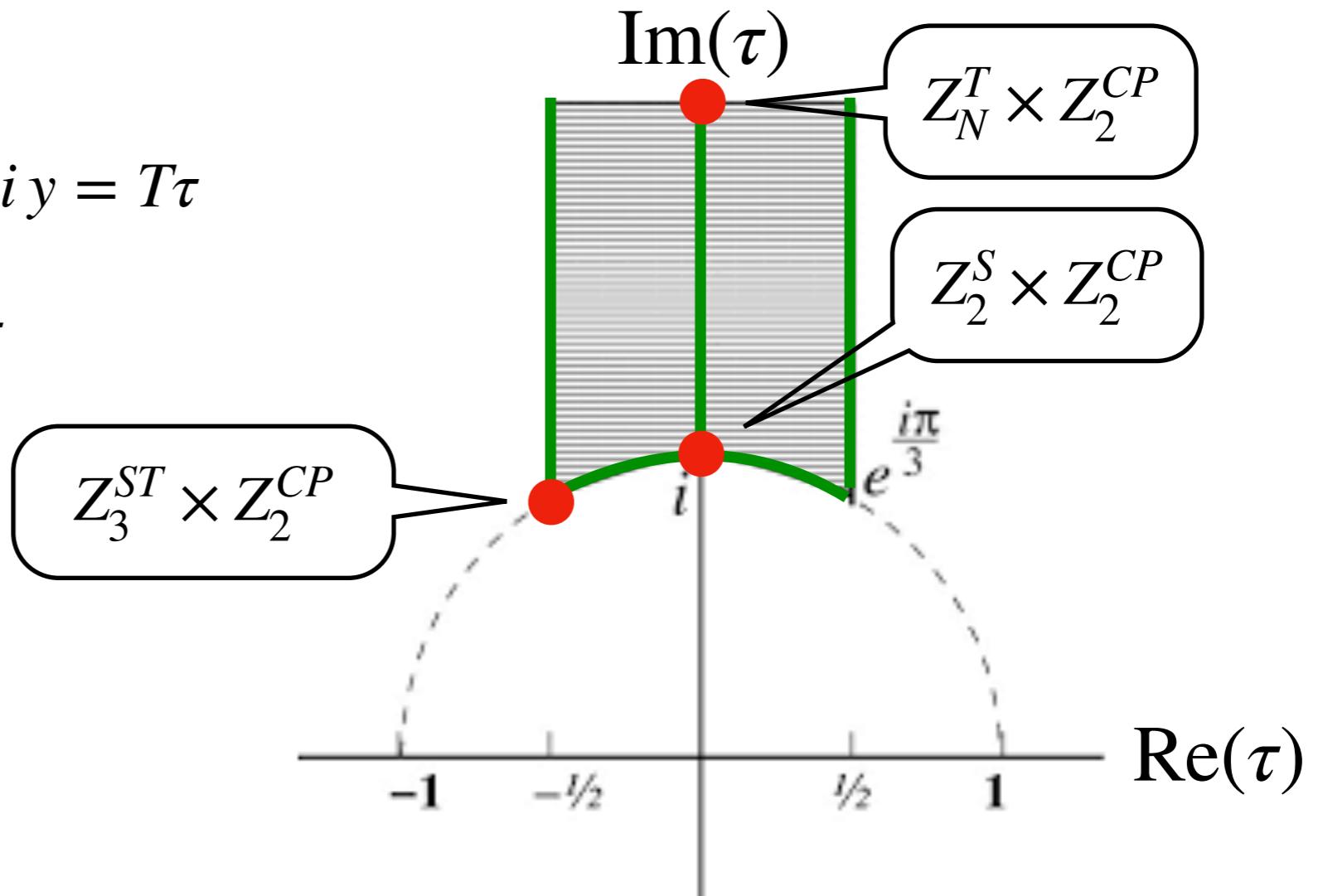
$$\mathcal{D} = \left\{ \tau \in \mathbb{C} : \operatorname{Im} \tau > 0, |\operatorname{Re} \tau| \leq \frac{1}{2}, |\tau| \geq 1 \right\}$$

Under CP the interior is mapped to itself

$$\tau = iy \xrightarrow{CP} iy$$

$$\tau = -\frac{1}{2} + iy \xrightarrow{CP} \frac{1}{2} + iy = T\tau$$

$$\tau = e^{i\varphi} \xrightarrow{CP} -e^{-i\varphi} = S\tau$$



Modular S_4 models with seesaw type I

Novichkov, Penedo, Petcov, AT, 1811.04933

	E_1^c	E_2^c	E_3^c	N^c	L	H_d	H_u
$SU(2)_L \times U(1)_Y$	(1, +1)	(1, +1)	(1, +1)	(1, 0)	(2, -1/2)	(2, -1/2)	(2, +1/2)
$\Gamma_4 \cong S_4$	1 or 1'	1 or 1'	1 or 1'	3 or 3'	3 or 3'	1	1
k_I	k_1	k_2	k_3	k_N	k_L	0	0

$$W = \sum_{i=1}^3 \alpha_i \left(E_i^c L F_{E_i}(\tau) \right)_1 H_d + g \left(N^c L F_N(\tau) \right)_1 H_u + \Lambda \left(N^c N^c F_M(\tau) \right)_1$$

Modular invariance imposes the following constraints on the weights:

$$\begin{cases} k_{\alpha_i} = k_i + k_L \\ k_g = k_N + k_L \\ k_\Lambda = 2k_N \end{cases} \Leftrightarrow \begin{cases} k_i = k_{\alpha_i} - k_g + k_\Lambda/2 \\ k_L = k_g - k_\Lambda/2 \\ k_N = k_\Lambda/2 \end{cases}$$

$$W = \lambda_{ij}(\tau) E_i^c L_j H_d + \mathcal{Y}_{ij}(\tau) N_i^c L_j H_u + \frac{1}{2} M_{ij}(\tau) N_i^c N_j^c$$

$$M_e = v_d \lambda^\dagger \quad M_\nu = -v_u^2 \mathcal{Y}^T M^{-1} \mathcal{Y}$$

Modular S_4 models with seesaw type I

Novichkov, Penedo, Petcov, AT, 1811.04933

Charged leptons: $(k_{\alpha_1}, k_{\alpha_2}, k_{\alpha_3}) = (2, 4, 4)$

Neutrinos: $(k_\Lambda, k_g) = (0, 2)$

$$W = \alpha \left(E_1^c L Y_{\mathbf{3}'}^{(2)} \right)_1 H_d + \beta \left(E_2^c L Y_{\mathbf{3}}^{(4)} \right)_1 H_d + \gamma \left(E_3^c L Y_{\mathbf{3}'}^{(4)} \right)_1 H_d$$

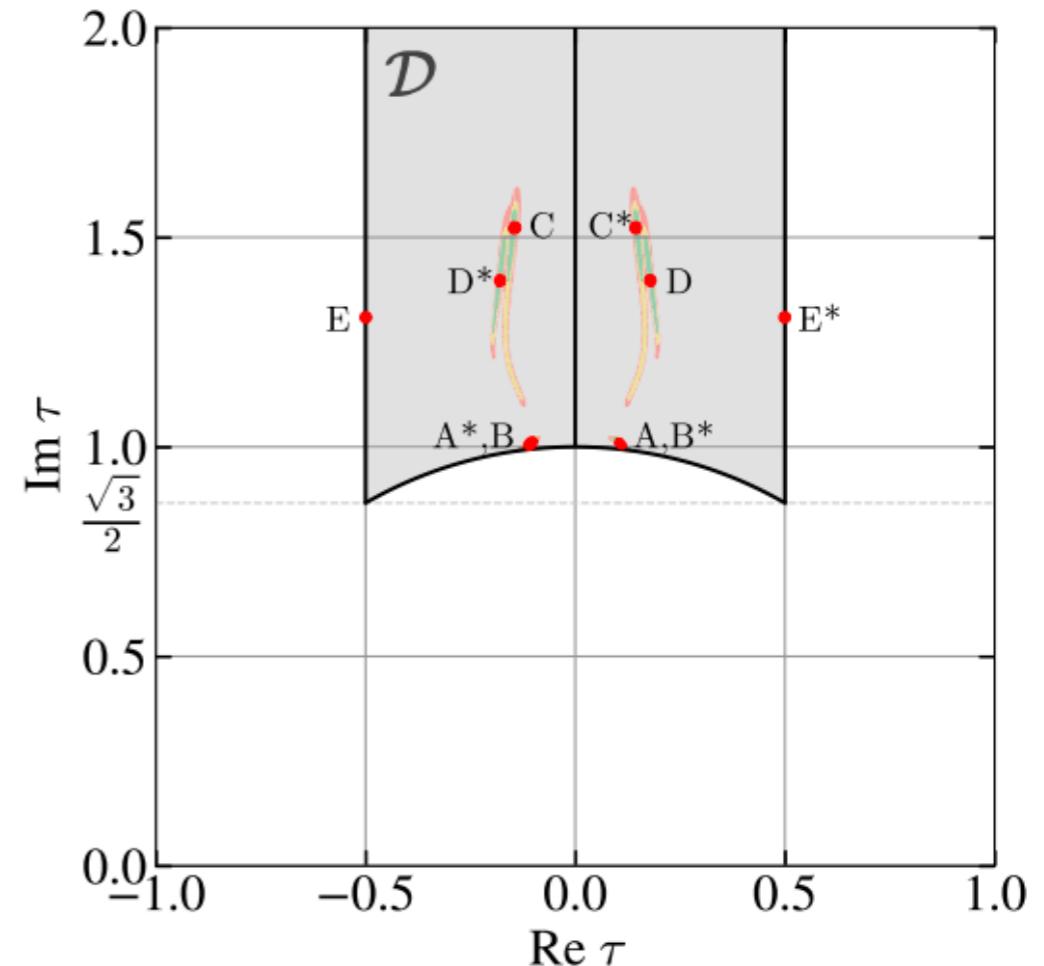
No CP

$$+ g \left(N^c L Y_{\mathbf{2}}^{(2)} \right)_1 H_u + \text{g' complex} \left(N^c L Y_{\mathbf{3}'}^{(2)} \right)_1 H_u + \Lambda \left(N^c N^c \right)_1$$

Solutions A and A*

Input parameters		Observables		Predictions	
Re τ	± 0.1045	m_e/m_μ	0.0048	m_1 [eV]	0.017
Im τ	1.0100	m_μ/m_τ	0.0565	m_2 [eV]	0.019
β/α	9.465	r	0.0299	m_3 [eV]	0.053
γ/α	0.0022	$\sin^2 \theta_{12}$	0.305	δ/π	± 1.31
Re (g'/g)	0.2330	$\sin^2 \theta_{13}$	0.0213	α_{21}/π	± 0.30
Im (g'/g)	± 0.4924	$\sin^2 \theta_{23}$	0.551	α_{31}/π	± 0.87
$v_d \alpha$ [MeV]	53.19	δm^2 [10^{-5} eV 2]	7.34	$ m_{ee} $ [eV]	0.017
$v_u^2 g^2 / \Lambda$ [eV]	0.0093	$ \Delta m^2 $ [10^{-3} eV 2]	2.455	$\sum_i m_i$ [eV]	0.090
		$N\sigma$	0.02	Ordering	NO

8 (5) parameters vs 12 (9) observables



Modular S_4 models with seesaw type I

Novichkov, Penedo, Petcov, AT, 1905.11970

Charged leptons: $(k_{\alpha_1}, k_{\alpha_2}, k_{\alpha_3}) = (2, 4, 4)$

Neutrinos: $(k_\Lambda, k_g) = (0, 2)$

$$W = \alpha \left(E_1^c L Y_{\mathbf{3}'}^{(2)} \right)_1 H_d + \beta \left(E_2^c L Y_{\mathbf{3}}^{(4)} \right)_1 H_d + \gamma \left(E_3^c L Y_{\mathbf{3}'}^{(4)} \right)_1 H_d$$

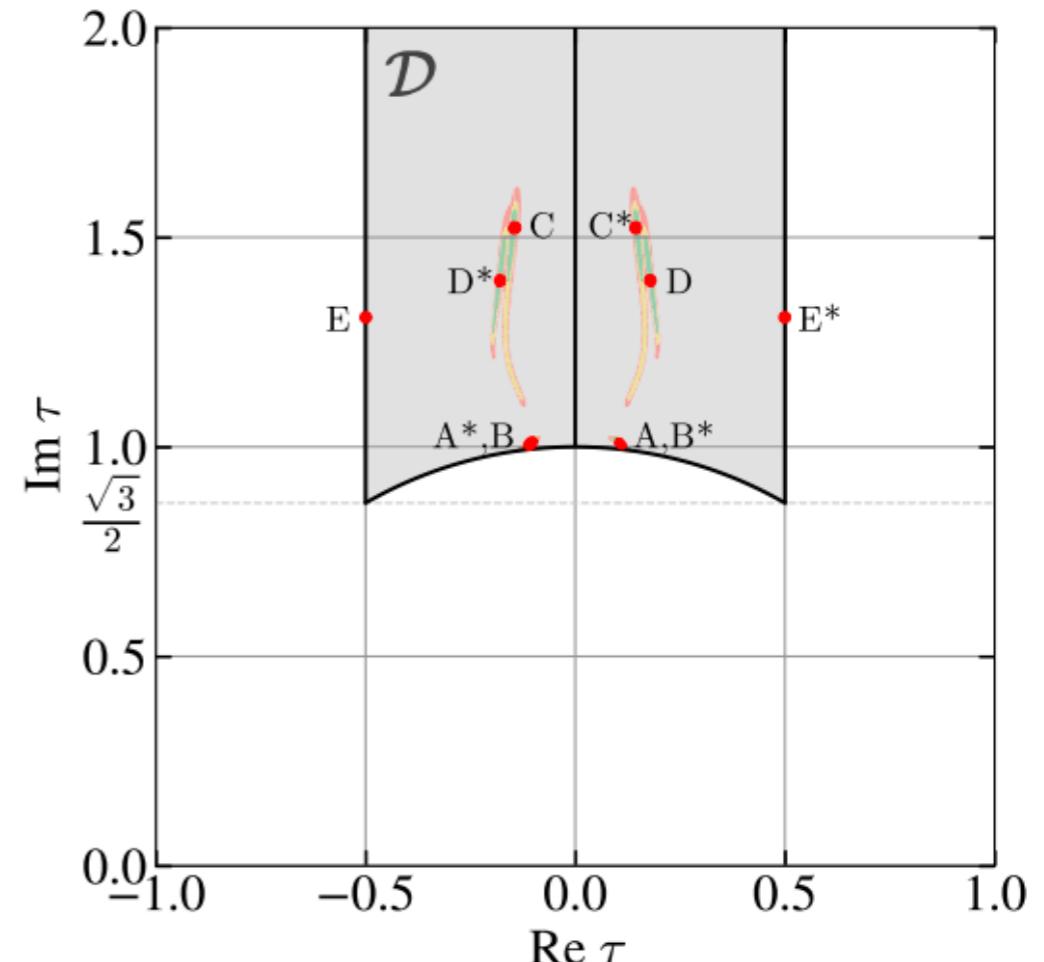
With CP

$$+ g \left(N^c L Y_{\mathbf{2}}^{(2)} \right)_1 H_u + \underbrace{g'}_{\text{real}} \left(N^c L Y_{\mathbf{3}'}^{(2)} \right)_1 H_u + \Lambda \left(N^c N^c \right)_1$$

Solutions A and A*

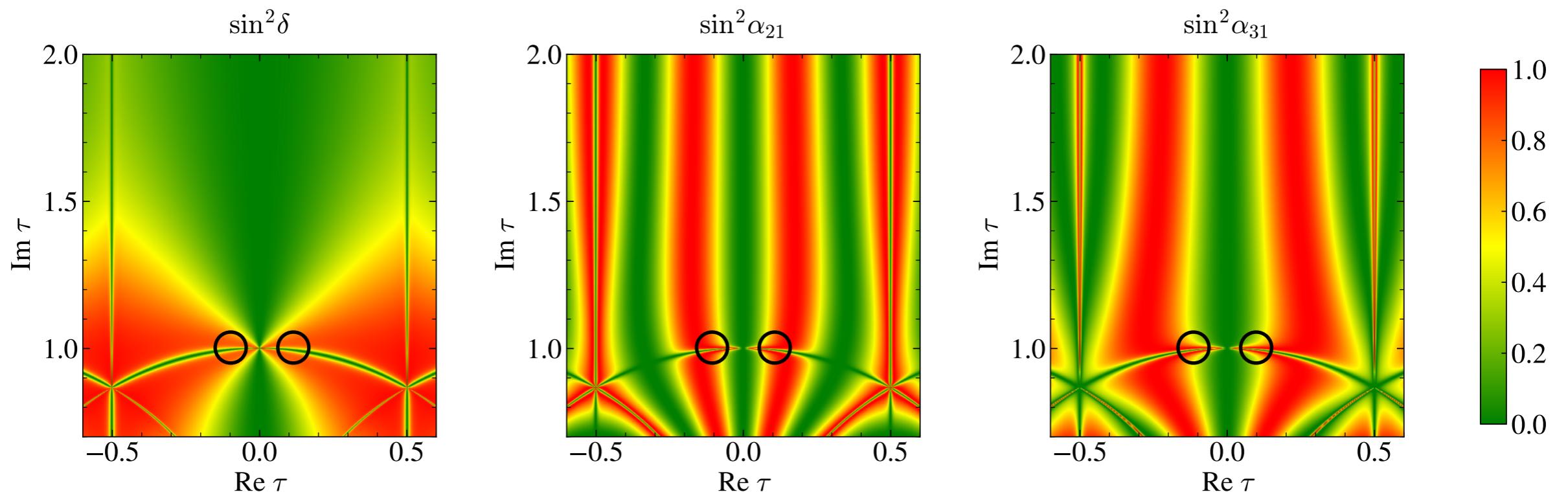
Input parameters		Observables		Predictions	
Re τ	± 0.0992	m_e/m_μ	0.0048	m_1 [eV]	0.012
Im τ	1.0160	m_μ/m_τ	0.0576	m_2 [eV]	0.015
β/α	9.348	r	0.0298	m_3 [eV]	0.051
γ/α	0.0022	$\sin^2 \theta_{12}$	0.305	δ/π	± 1.64
g'/g	-0.0209	$\sin^2 \theta_{13}$	0.0214	α_{21}/π	± 0.35
$v_d \alpha$ [MeV]	53.61	$\sin^2 \theta_{23}$	0.486	α_{31}/π	± 1.25
$v_u^2 g^2 / \Lambda$ [eV]	0.0135	δm^2 [10^{-5} eV 2]	7.33	$ m_{ee} $ [eV]	0.012
		$ \Delta m^2 $ [10^{-3} eV 2]	2.457	$\sum_i m_i$ [eV]	0.078
		$N\sigma$	1.01	Ordering	NO

7 (4) parameters vs 12 (9) observables



CP-conserving values of the modulus

A check: τ is varied, all other parameters are fixed to their b.f.v.



A small departure from the CP-conserving boundary of \mathcal{D} can lead to large CPV

Conclusions

- ▶ CP can be **consistently combined** with modular symmetry in the BU approach
 - $\tau \xrightarrow{CP} -\tau^*$
 - $\psi(x) \xrightarrow{CP} X_{\mathbf{r}} \bar{\psi}(x_P)$
 - $Y(\tau) \xrightarrow{CP} X_{\mathbf{r}} Y^*(\tau)$
- ▶ In a **symmetric basis** for $\rho_{\mathbf{r}}(S)$ and $\rho_{\mathbf{r}}(T)$, $X_{\mathbf{r}} = \mathbb{I}_{\mathbf{r}}$ and **couplings are real**
- ▶ Smaller number of free parameters => **higher predictive power**
- ▶ τ can be the **only source of CP violation**
- ▶ CP extends $\bar{\Gamma} \simeq PSL(2, \mathbb{Z})$ to $\bar{\Gamma}^* \simeq \bar{\Gamma} \rtimes \mathbb{Z}_2^{CP} \simeq PGL(2, \mathbb{Z})$

Backup slides

Correlations in CP-invariant model

Novichkov, Penedo, Petcov, AT, 1905.11970

