Fermion mass **hierarchies** from residual modular symmetries and modulus **stabilisation**

in collaboration with S.T. Petcov, P.P. Novichkov

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FCT Fundação para a Ciência e a Tecnologia

DF Departamento de Física Técnico Lisboa

CFTP Centro de Física Teórica de Partículas

europa.eu European Union

QREN Quadro de Referência Estratégico Nacional

COMPETE

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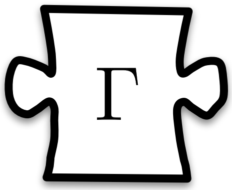
(CFTP, Lisbon)

Bethe Forum workshop on
“Modular Flavor Symmetries”, Bonn

4 May 2022

Modular symmetry cheat sheet (1/3)



$$\Gamma \equiv SL(2,\mathbb{Z}) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a,b,c,d \in \mathbb{Z},\ \det \gamma = 1 \right\}$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$S^2 = R, \quad (ST)^3 = R^2 = \mathbb{1}, \quad RT = TR$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} :$$

$$\tau \to -1/\tau$$

inver**S**ion

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} :$$

$$\tau \to \tau + 1$$

Translation

$$R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} :$$

$$\tau \to \tau$$

Redundant

but can affect fields...

Modular symmetry cheat sheet (2/3)

automorphy factor

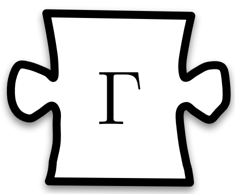
$$\psi \to \boxed{(c\tau + d)^{-k}}\ \boxed{\rho(\gamma)}\ \psi$$

Weight $k \in \mathbb{Z}$

"Almost trivial" representation of the modular group

$$\rho\big(\Gamma(N)\big) = \mathbb{1}$$
$$\rho\big(T\,\Gamma(N)\big) = \rho(T)$$
$$\rho\big(S\,\Gamma(N)\big) = \rho(S)$$
$$\ldots$$

Feruglio, 1706.08749

$\Gamma(N) \subset$ 

Principal congruence subgroup of level *N*

$$\Gamma(N) \equiv \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

$\rho(\gamma)$ **is effectively a representation of** $\Gamma'_N \equiv \Gamma/\Gamma(N)$

Modular symmetry cheat sheet (3/3)

Invariance of the superpotential

$$W \sim g(Y(\tau)\,\psi_1 \ldots \psi_n)_{\mathbf{1}}$$

$$\begin{cases} k_Y = k_1 + \ldots + k_n \\ \rho_Y \otimes \rho_1 \otimes \ldots \otimes \rho_n \supset \mathbf{1} \end{cases}$$

$$\psi \to (c\tau + d)^{-k} \; \rho(\gamma)\, \psi$$

$$Y(\tau) \to (c\tau + d)^{k_Y} \; \rho_Y(\gamma)\, Y(\tau)$$

Lowest-weight modular forms

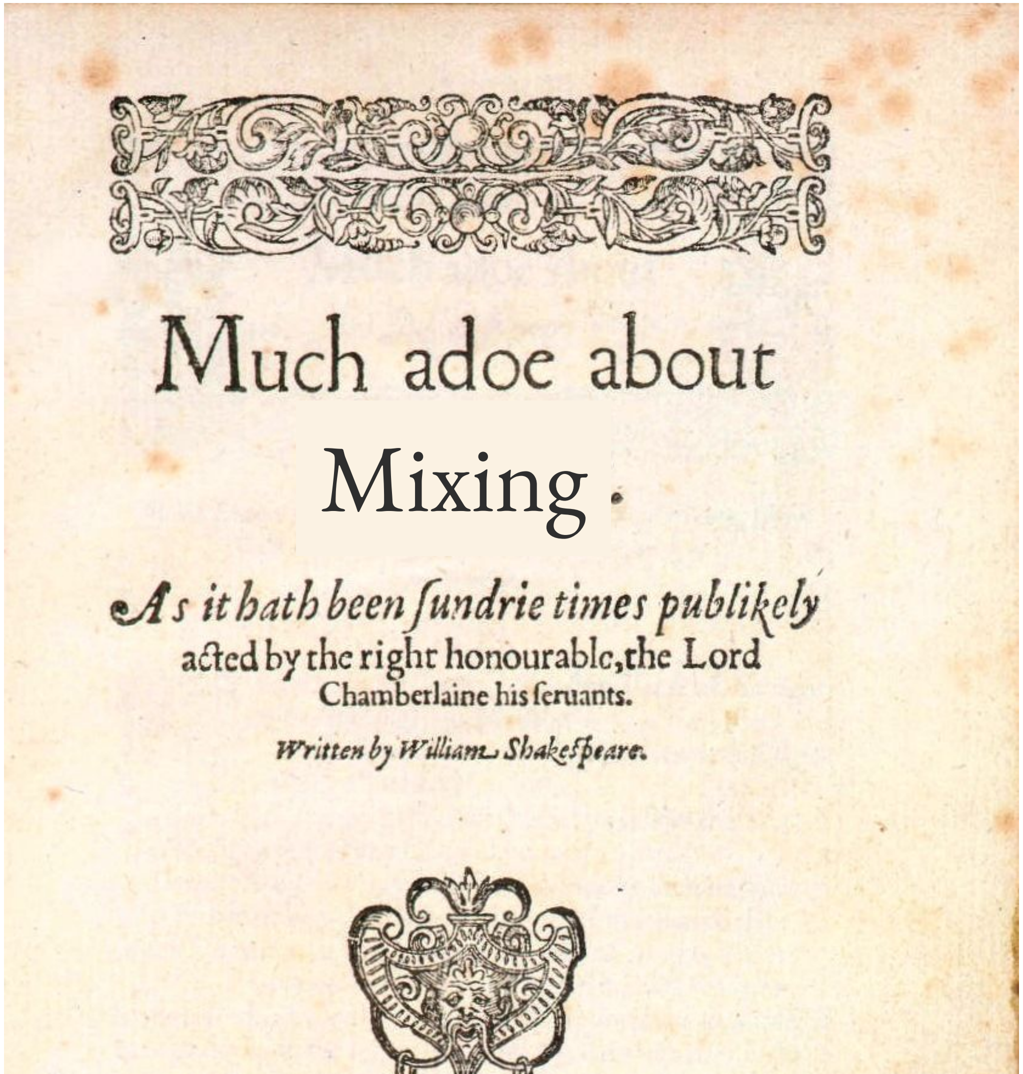
$\Gamma_N^{(\prime)}$	$Y_{\mathbf{r}}^{(k)}$
$\Gamma_2 \simeq S_3$	$Y_{\mathbf{2}}^{(2)}$
$\Gamma'_4 \simeq S'_4$	$Y_{\hat{\mathbf{3}}}^{(1)}$
$\Gamma_4 \simeq S_4$	$Y_{\mathbf{2}}^{(2)}, Y_{\mathbf{3}'}^{(2)}$
$\Gamma'_3 \simeq A'_4$	$Y_{\hat{\mathbf{2}}}^{(1)}$
$\Gamma_3 \simeq A_4$	$Y_{\mathbf{3}}^{(2)}$
$\Gamma'_5 \simeq A'_5$	$Y_{\hat{\mathbf{6}}}^{(1)}$
$\Gamma_5 \simeq A_5$	$Y_{\mathbf{3}}^{(2)}, Y_{\mathbf{3}'}^{(2)}, Y_{\mathbf{5}}^{(2)}$

Pieces of a puzzle
(a personal view)

- explanation of mass hierarchies?
- clear explanation of mixing?
- use TD to fix Kahler and irreps?
- phenomenology beyond masses and mixing?
- modular symmetry breaking as the only source of CPV?
- do away with SUSY?

I. Fermion mass hierarchies from residual modular symmetries

Mass hierarchies from modular symmetry?



Mass hierarchies from modular symmetry?

- Usually fermion mass hierarchies are put in **by hand**: hierarchies (or cancellations) between superpotential parameters

  e.g. $\gamma \ll \alpha \ll \beta$ [see talks by Serguey, Arsenii]

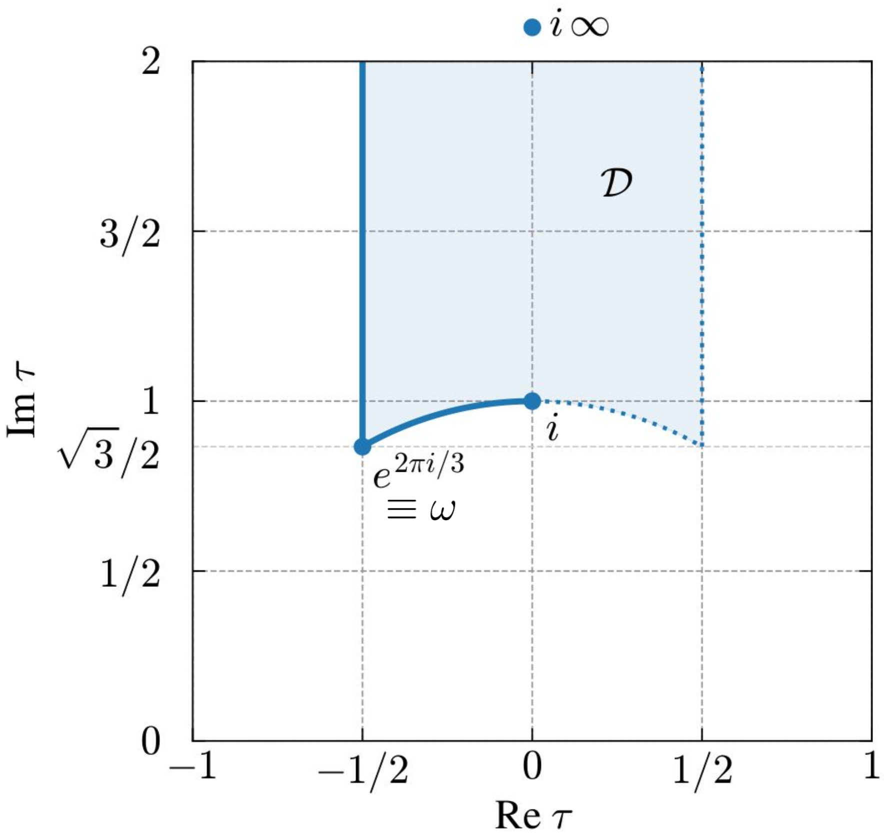
- **Other approaches** - new (weighted) scalars which enter the mass matrices a la Froggatt-Nielsen. Weights are analogous to FN charges

  [Steve King's talk]

  Criado, Feruglio, King, 1908.11867
  King, King, 2002.00969

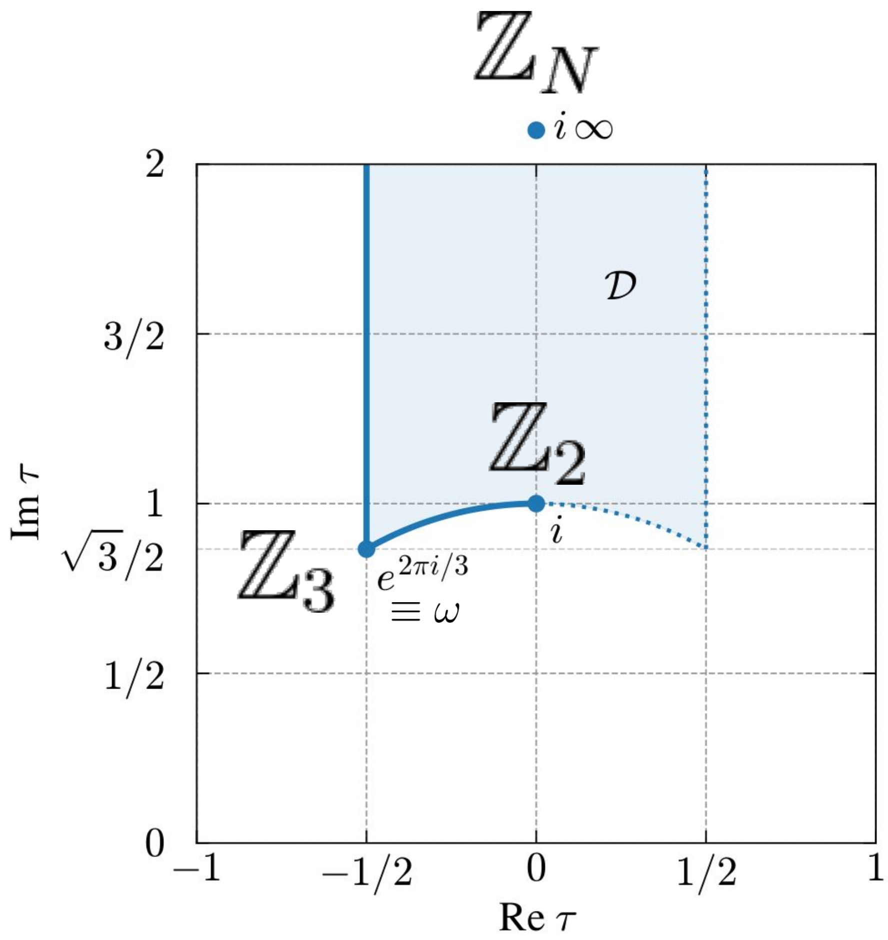
- **Our approach** - No new scalars, mechanism uses **only τ**, common weights across generations (unlike FN charges)

Residual modular symmetries



- The **fundamental domain** is enough
- **Any τ** breaks the modular symmetry

Residual modular symmetries



- The **fundamental domain** is enough
- **Any τ** breaks the modular symmetry
- At special values of τ, some **residual symmetry** remains

Key idea:

some couplings vanish as we approach a symmetric point

Corrections to vanishing couplings

$$\tau = \tau_{\text{sym}}$$

$$M \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\psi^c \, M \, \psi$$

Key idea:

some couplings vanish as we approach a symmetric point

Corrections to vanishing couplings

$$\tau = \tau_{\text{sym}}$$

$$\epsilon \sim |\tau - \tau_{\text{sym}}| > 0$$

$$M \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \rightarrow \quad M \sim \begin{pmatrix} 1 & \epsilon^{\cdots} & \epsilon^{\cdots} \\ \epsilon^{\cdots} & \epsilon^{\cdots} & \epsilon^{\cdots} \\ \epsilon^{\cdots} & \epsilon^{\cdots} & \epsilon^{\cdots} \end{pmatrix}$$

$$\psi^c \, M \, \psi$$

In the vicinity of the sym. point, the couplings are $\mathcal{O}(\epsilon^l)$

Key idea:

some couplings vanish as we approach a symmetric point

Decompositions under residual groups (determine $\mathcal{O}(\epsilon^l)$)

τ_{sym}	Residual sym.	Possible powers ϵ^l
i	$\mathbb{Z}_2$	$l = 0, 1$
ω	$\mathbb{Z}_3$	$l = 0, 1, 2$
$i\infty$	$\mathbb{Z}_N$	$l = 0, 1, \ldots, N$

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$i\infty$	$\mathbb{Z}_N$	$l = 0, 1, \ldots, N$

Feruglio, Gherardi, Romanino, Titov, 2101.08718 (for A_4, m_e=0)

$$\psi^c\, M\, \psi$$

$$\psi \xrightarrow{\gamma} (c\tau + d)^{-k} \rho(\gamma)\, \psi$$

$$\psi^c \xrightarrow{\gamma} (c\tau + d)^{-k^c} \rho^c(\gamma)\, \psi^c$$

$$M(\tau) \xrightarrow{\gamma} M(\gamma\tau) = (c\tau + d)^K \rho^c(\gamma)^* M(\tau) \rho(\gamma)^\dagger$$

$$\psi \rightsquigarrow \mathbf{1}_{\ldots} \oplus \mathbf{1}_{\ldots} \oplus \mathbf{1}_{\ldots}$$

$$\psi^c \rightsquigarrow \mathbf{1}_{\ldots} \oplus \mathbf{1}_{\ldots} \oplus \mathbf{1}_{\ldots}$$

In general, depend on weights

Determined for all $N \leqslant 5$

Example: hierarchical mass matrix (A5)

$$\psi \sim (\mathbf{3}, k)$$
$$\psi^c \sim (\mathbf{3}', k^c)$$

$$\Rightarrow$$

Under the residual group of

$$\tau_{\text{sym}} = i\infty$$

$$\psi \rightsquigarrow 1_0 \oplus \mathbf{1}_1 \oplus \mathbf{1}_4$$
$$\psi^c \rightsquigarrow 1_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_3$$

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For $\psi^c M \psi$, we expect:

$$M \sim \begin{pmatrix} 1 & \epsilon^4 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & \epsilon & \epsilon^3 \end{pmatrix}$$

with $\epsilon = e^{-2\pi \, \text{Im} \, \tau/5}$

$\Rightarrow$

fermion spectrum

$$\sim (1, \epsilon, \epsilon^4)$$ ✔

Indeed the case, provided enough modular forms contribute to *M*

(otherwise, $m_e = 0$)

Example: hierarchical mass matrix (A5)

$$\psi \sim (\mathbf{3}, k) \quad \Rightarrow$$

Under the residual group of

$$\tau_{\text{sym}} = i\infty$$

Not like Froggatt-Nielsen. Instead, it is an **improvement**!

Explicit example at weight 2

$$W \supset \sum_s \alpha_s \left(Y_{\mathbf{5}}^{(5,2)}(\tau) \psi^c \psi \right)_{\mathbf{1},s} \quad \Rightarrow \quad M(\tau) = \alpha \begin{pmatrix} \sqrt{3} Y_1 & Y_5 & Y_2 \\ Y_4 & -\sqrt{2} Y_3 & -\sqrt{2} Y_5 \\ Y_3 & -\sqrt{2} Y_2 & -\sqrt{2} Y_4 \end{pmatrix}_{Y_{\mathbf{5}}^{(5,2)}}$$

$$(Y_1, Y_2, Y_3, Y_4, Y_5) \simeq \mathcal{N} \left(-1/\sqrt{6},\, q,\, 3q^2,\, 4q^3,\, 7q^4 \right)$$

with $\epsilon = e^{-2\pi \,\mathrm{Im}\, \tau / 5}$

Indeed the case, provided enough modular forms contribute to *M*

(otherwise, $m_e = 0$)

Scan of possible mass patterns

Performed for 3 generations, for all $N \leqslant 5$

e.g. fermion spectra for multiplets of modular A5

$\mathbf{r}$	$\mathbf{r}^c$	$\tau \simeq \omega$, $k + k^c \equiv 0$	$\tau \simeq \omega$, $k + k^c \equiv 1$	$\tau \simeq \omega$, $k + k^c \equiv 2$	$\tau \simeq i\infty$
$\mathbf{3}$	$\mathbf{3}$	$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(1,1,1)$
$\mathbf{3}$	$\mathbf{3}'$	$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(1,\epsilon,\epsilon^4)$
$\mathbf{3}'$	$\mathbf{3}'$	$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(1,1,1)$
$\mathbf{3}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	$(1,\epsilon,\epsilon^2)$	$(1,\epsilon,\epsilon^2)$	$(1,\epsilon,\epsilon^2)$	$(1,\epsilon,\epsilon^4)$
$\mathbf{3}'$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	$(1,\epsilon,\epsilon^2)$	$(1,\epsilon,\epsilon^2)$	$(1,\epsilon,\epsilon^2)$	$(1,\epsilon^2,\epsilon^3)$
$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	$(1,1,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$	$(\epsilon,\epsilon,\epsilon)$	$(1,1,1)$

Scan of possible mass patterns

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$\mathbf{3}$	$\mathbf{3}$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$
$\mathbf{3}$	$\mathbf{3}'$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, \epsilon, \epsilon^4)$
$\mathbf{3}'$	$\mathbf{3}'$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$
$\mathbf{3}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$	$(1, \epsilon, \epsilon^2)$	$(1, \epsilon, \epsilon^2)$	$(1, \epsilon, \epsilon^2)$	$(1, \epsilon, \epsilon^4)$
$\mathbf{3}'$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$	$(1, \epsilon, \epsilon^2)$	$(1, \epsilon, \epsilon^2)$	$(1, \epsilon, \epsilon^2)$	$(1, \epsilon^2, \epsilon^3)$
$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$	$(1, 1, 1)$	$(\epsilon^2, \epsilon^2, \epsilon^2)$	$(\epsilon, \epsilon, \epsilon)$	$(1, 1, 1)$

Promising hierarchical patterns

N	Γ'_N	Pattern	Sym. point	Viable $\mathbf{r} \otimes \mathbf{r}^c$
2	S_3	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
3	A'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
			$\tau \simeq i\infty$	
4	S'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
		$(1, \epsilon, \epsilon^3)$	$\tau \simeq i\infty$	
5	A'_5	$(1, \epsilon, \epsilon^4)$	$\tau \simeq i\infty$	

Promising hierarchical patterns

N	Γ'_N	Pattern	Sym. point	Viable $\mathbf{r} \otimes \mathbf{r}^c$
2	S_3	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{2} \oplus \mathbf{1}^{(\prime)}] \otimes [\mathbf{1} \oplus \mathbf{1}^{(\prime)} \oplus \mathbf{1}']$
3	A'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{1}_a \oplus \mathbf{1}_a \oplus \mathbf{1}'_a] \otimes [\mathbf{1}_b \oplus \mathbf{1}_b \oplus \mathbf{1}''_b]$
			$\tau \simeq i\infty$	$[\mathbf{1}_a \oplus \mathbf{1}_a \oplus \mathbf{1}'_a] \otimes [\mathbf{1}_b \oplus \mathbf{1}_b \oplus \mathbf{1}''_b]$ with $\mathbf{1}_a \neq (\mathbf{1}_b)^*$
4	S'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{3}_a$, or $\mathbf{2} \oplus \mathbf{1}^{(\prime)}$, or $\hat{\mathbf{2}} \oplus \hat{\mathbf{1}}^{(\prime)}] \otimes [\mathbf{1}_b \oplus \mathbf{1}_b \oplus \mathbf{1}'_b]$
		$(1, \epsilon, \epsilon^3)$	$\tau \simeq i\infty$	$\mathbf{3} \otimes [\mathbf{2} \oplus \mathbf{1}$, or $\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}']$, $\mathbf{3}' \otimes [\mathbf{2} \oplus \mathbf{1}'$, or $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}']$, $\hat{\mathbf{3}}' \otimes [\hat{\mathbf{2}} \oplus \hat{\mathbf{1}}$, or $\hat{\mathbf{1}} \oplus \hat{\mathbf{1}} \oplus \hat{\mathbf{1}}']$, $\hat{\mathbf{3}} \otimes [\hat{\mathbf{2}} \oplus \hat{\mathbf{1}}'$, or $\hat{\mathbf{1}} \oplus \hat{\mathbf{1}}' \oplus \hat{\mathbf{1}}']$
5	A'_5	$(1, \epsilon, \epsilon^4)$	$\tau \simeq i\infty$	$\mathbf{3} \otimes \mathbf{3}'$

Promising hierarchical patterns (try leptons)

N	Γ'_N	Pattern	Sym. point	Viable $\mathbf{r} \otimes \mathbf{r}^c$
2	S_3	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
3	A'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$ $\tau \simeq i\infty$	
4	S'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
		$(1, \epsilon, \epsilon^3)$	$\tau \simeq i\infty$	$\hat{\mathbf{3}}' \otimes (\hat{\mathbf{2}} \oplus \hat{\mathbf{1}})$
5	A'_5	$(1, \epsilon, \epsilon^4)$	$\tau \simeq i\infty$	$\mathbf{3} \otimes \mathbf{3}'$

$L \sim (\hat{\mathbf{2}} \oplus \hat{\mathbf{1}}, 2),\ E^c \sim (\hat{\mathbf{3}}', 2),\ N^c \sim (\mathbf{3}, 1)$

8 parameters

$L \sim (\mathbf{3}, 3),\ E^c \sim (\mathbf{3}', 1),\ N^c \sim (\hat{\mathbf{2}}, 2)$

8 parameters

Masses are OK :)

Promising hierarchical patterns (try leptons)

N	Γ'_N	Pattern	Sym. point	Viable $\mathbf{r} \otimes \mathbf{r}^c$
2	S_3	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
3	A'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$ $\tau \simeq i\infty$	
4	S'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	
		$(1, \epsilon, \epsilon^3)$	$\tau \simeq i\infty$	$\hat{\mathbf{3}}' \otimes (\hat{\mathbf{2}} \oplus \hat{\mathbf{1}})$
5	A'_5	$(1, \epsilon, \epsilon^4)$	$\tau \simeq i\infty$	$\mathbf{3} \otimes \mathbf{3}'$

$L \sim (\hat{\mathbf{2}} \oplus \hat{\mathbf{1}}, 2),\ E^c \sim (\hat{\mathbf{3}}', 2),\ N^c \sim (\mathbf{3}, 1)$

8 parameters

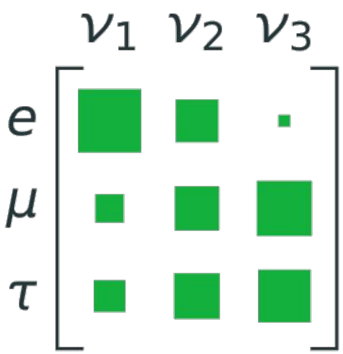
$L \sim (\mathbf{3}, 3),\ E^c \sim (\mathbf{3}', 1),\ N^c \sim (\hat{\mathbf{2}}, 2)$

8 parameters

Masses are OK, but mixing is tuned :(

Wrong PMNS in the symmetric limit: parameters are driven into cancellations

How to avoid fine-tuning (in the lepton sector)



$$\xrightarrow{\tau \to \tau_{\text{sym}}} \begin{bmatrix} \star & \star & 0 \\ \star & \star & \star \\ \star & \star & \star \end{bmatrix} \text{ or } \begin{bmatrix} \star & \star & \star \\ \star & \star & \star \\ \star & \star & \star \end{bmatrix}$$

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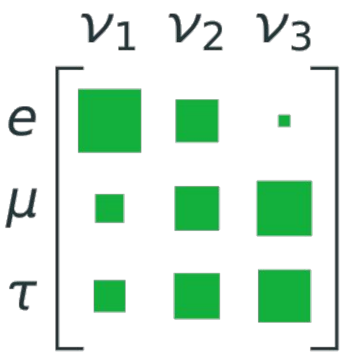

$$\xrightarrow{\tau \to \tau_{\mathrm{sym}}} \begin{bmatrix} \star & \star & 0 \\ \star & \star & \star \\ \star & \star & \star \end{bmatrix} \text{ or } \begin{bmatrix} \star & \star & \star \\ \star & \star & \star \\ \star & \star & \star \end{bmatrix}$$

Reyimuaji, Romanino, 1801.10530

1. $\begin{cases} L \sim 1 \oplus 1 \oplus 1 \\ E^c \sim 1 \oplus \mathbf{r} \not\supset 1 \end{cases}$

2. $\begin{cases} L \sim \mathbf{1} \oplus \mathbf{1} \oplus \bar{\mathbf{1}} \\ E^c \sim \bar{\mathbf{1}} \oplus \mathbf{r} \not\supset \mathbf{1}, \bar{\mathbf{1}} \end{cases}$

3. $m_e = m_\mu = m_\tau = 0$

4. $m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = 0$

for mixing near symmetric points, see also Okada, Tanimoto, 2009.14242

Promising hierarchical patterns (leptons)

N	Γ'_N	Pattern	Sym. point	Viable $\mathbf{r}_{E^c} \otimes \mathbf{r}_L$	Case
2	S_3	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{2} \oplus \mathbf{1}^{(\prime)}] \otimes [\mathbf{1} \oplus \mathbf{1}^{(\prime)} \oplus \mathbf{1}']$	1 or 4
3	A'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{1}_a \oplus \mathbf{1}_a \oplus \mathbf{1}'_a] \otimes [\mathbf{1}_b \oplus \mathbf{1}_b \oplus \mathbf{1}''_b]$	2
			$\tau \simeq i\infty$	$[\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}'] \otimes [\mathbf{1}'' \oplus \mathbf{1}'' \oplus \mathbf{1}']$, $[\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}''] \otimes [\mathbf{1}' \oplus \mathbf{1}' \oplus \mathbf{1}'']$	2
4	S'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{3}_a, \text{ or } \mathbf{2} \oplus \mathbf{1}^{(\prime)}, \text{ or } \hat{\mathbf{2}} \oplus \hat{\mathbf{1}}^{(\prime)}] \otimes [\mathbf{1}_b \oplus \mathbf{1}_b \oplus \mathbf{1}'_b]$	1 or 4
5	A'_5	–	–	–	–

1. $\begin{cases} L \sim 1 \oplus 1 \oplus 1 \\ E^c \sim 1 \oplus \mathbf{r} \not\supset 1 \end{cases}$

2. $\begin{cases} L \sim \mathbf{1} \oplus \mathbf{1} \oplus \bar{\mathbf{1}} \\ E^c \sim \bar{\mathbf{1}} \oplus \mathbf{r} \not\supset \mathbf{1}, \bar{\mathbf{1}} \end{cases}$

3. $m_e = m_\mu = m_\tau = 0$

4. $m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = 0$

Promising hierarchical patterns (leptons)

N	Γ'_N	Pattern	Sym. point	Viable $\mathbf{r}_{E^c} \otimes \mathbf{r}_L$	Case
2	S_3	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$		1 or 4
3	A'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$		2
			$\tau \simeq i\infty$		2
4	S'_4	$(1, \epsilon, \epsilon^2)$	$\tau \simeq \omega$	$[\mathbf{3}_a \quad] \otimes [\mathbf{1}_b \oplus \mathbf{1}_b \oplus \mathbf{1}'_b]$	1 or 4
5	A'_5	–	–	–	–

1. $\begin{cases} L \sim 1 \oplus 1 \oplus 1 \\ E^c \sim 1 \oplus \mathbf{r} \not\supset 1 \end{cases}$

2. $\begin{cases} L \sim \mathbf{1} \oplus \mathbf{1} \oplus \bar{\mathbf{1}} \\ E^c \sim \bar{\mathbf{1}} \oplus \mathbf{r} \not\supset \mathbf{1}, \bar{\mathbf{1}} \end{cases}$

3. $m_e = m_\mu = m_\tau = 0$

4. $m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = 0$

Example: lepton model close to ω

Only S_4' model from a scan requiring minimal # params., $m_e > 0$, and Dirac phase within 2σ range (otherwise unconstrained):

$$L \sim (\hat{\mathbf{1}} \oplus \hat{\mathbf{1}} \oplus \hat{\mathbf{1}}', 2),\ E^c \sim (\hat{\mathbf{3}}, 4),\ N^c \sim (\mathbf{3}', 1)$$

Superpotential:

$$W = \left[\alpha_1 \left(Y^{(4,6)}_{\mathbf{3}',1} E^c L_1\right)_{\mathbf{1}} + \alpha_3 \left(Y^{(4,6)}_{\mathbf{3}',1} E^c L_2\right)_{\mathbf{1}} + \alpha_4 \left(Y^{(4,6)}_{\mathbf{3}',2} E^c L_2\right)_{\mathbf{1}} + \alpha_5 \left(Y^{(4,6)}_{\mathbf{3}} E^c L_3\right)_{\mathbf{1}}\right] H_d$$
$$+ \left[g_1 \left(Y^{(4,3)}_{\hat{\mathbf{3}}} N^c L_1\right)_{\mathbf{1}} + g_2 \left(Y^{(4,3)}_{\hat{\mathbf{3}}} N^c L_2\right)_{\mathbf{1}} + g_3 \left(Y^{(4,3)}_{\hat{\mathbf{3}}'} N^c L_3\right)_{\mathbf{1}}\right] H_u$$
$$+ \Lambda \left(Y^{(4,2)}_{\mathbf{2}} (N^c)^2\right)_{\mathbf{1}} .$$

[gCP imposed, see talk by Arsenii next]

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Only S_4' model from a scan requiring minimal # params., $m_e > 0$, and Dirac phase within 2σ range (otherwise unconstrained):

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$$M_e \propto \begin{pmatrix} 1 & \alpha - 2\beta & 2\sqrt{3} i\gamma \\ \sqrt{3}\epsilon & \sqrt{3}(\alpha + 2\beta)\epsilon & 2i\gamma\epsilon \\ \frac{5}{2}\epsilon^2 & \left(\frac{5}{2}\alpha - \beta\right)\epsilon^2 & -\frac{5}{\sqrt{3}} i\gamma\epsilon^2 \end{pmatrix}$$

$$|\epsilon| \simeq 2.8 \left| \frac{\tau - \omega}{\tau - \omega^2} \right|$$

$$M_\nu \propto \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & a \\ 1 & a & 2i\sqrt{\frac{2}{3}} b \end{pmatrix}$$

Example: lepton model close to ω

Only S_4' model from a scan requiring minimal # params., $m_e > 0$, and Dirac phase within 2σ range (otherwise unconstrained):

$$L \sim (\hat{\mathbf{1}} \oplus \hat{\mathbf{1}} \oplus \hat{\mathbf{1}}', 2),\ E^c \sim (\hat{\mathbf{3}}, 4),\ N^c \sim (\mathbf{3}', 1)$$

$$M_e \propto \begin{pmatrix} 1 & \alpha - 2\beta & 2\sqrt{3}i\gamma \\ \sqrt{3}\epsilon & \sqrt{3}(\alpha + 2\beta)\epsilon & 2i\gamma\epsilon \\ \frac{5}{2}\epsilon^2 & \left(\frac{5}{2}\alpha - \beta\right)\epsilon^2 & -\frac{5}{\sqrt{3}}i\gamma\epsilon^2 \end{pmatrix}$$

$$|\epsilon| \simeq 2.8 \left|\frac{\tau - \omega}{\tau - \omega^2}\right| \sim \left|\tau - e^{2\pi i/3}\right|$$

$$M_\nu \propto \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & a \\ 1 & a & 2i\sqrt{\frac{2}{3}}b \end{pmatrix}$$

$$u \equiv \frac{\tau - \omega}{\tau - \omega^2}$$

$|u|$ quantifies the deviation of τ from the left cusp (the original ϵ)

Example: lepton model close to ω

Only S_4' model from a scan requiring minimal # params., $m_e > 0$, and Dirac phase within 2σ range (otherwise unconstrained):

$$L \sim (\hat{\mathbf{1}} \oplus \hat{\mathbf{1}} \oplus \hat{\mathbf{1}}', 2),\ E^c \sim (\hat{\mathbf{3}}, 4),\ N^c \sim (\mathbf{3}', 1)$$

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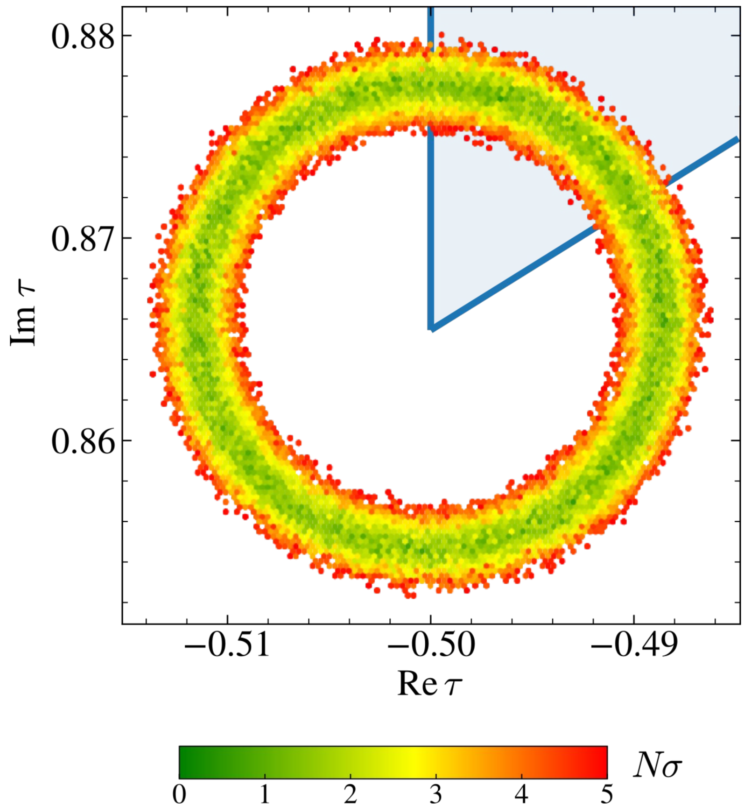
$$|\epsilon| \simeq 2.8 \left| \frac{\tau - \omega}{\tau - \omega^2} \right|$$

$$M_\nu \propto \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & a \\ 1 & a & 2i\sqrt{\frac{2}{3}} b \end{pmatrix}$$

$\lvert\epsilon\rvert \simeq 0.02$	$\alpha = 2.45 \pm 0.44$
$a = 1.5 \pm 0.15$	$\beta = 2.14 \pm 0.32$
$b = 2.22 \pm 0.17$	$\gamma = 0.91 \pm 0.05$

Example: lepton model close to ω

$$|\epsilon| \simeq 0.02 \;\Leftrightarrow\; |u| \simeq 0.007$$



$$m_e = \mathcal{O}(\epsilon^2)$$
$$m_\mu = \mathcal{O}(\epsilon)$$
$$m_\tau = \mathcal{O}(1)$$

✔

$$\text{NO}, \quad m_{\nu_1} = 0 \qquad \delta \simeq \pi$$
$$m_{\beta\beta} = (1.44 \pm 0.33)\ \text{meV}$$

Naturally allows for **hierarchies**, **large mixing**, and some **predictivity**

Summary I

Summary I

- Fermion **mass hierarchies** can naturally arise if τ is in the vicinity of a point of residual symmetry,

$$\tau_{\rm sym} = \omega, i\infty, (i)$$

- This mechanism works without flavons.

- **Natural lepton mixing** can also arise in such models. Requiring no fine-tuning in the whole lepton sector is remarkably restrictive.

- As seen in the model and anticipated from the hierarchical patterns, $|u| \simeq 0.007$ is required. Ad hoc?

II. Modulus stabilisation

Simplest modular-invariant potentials?

- Studied by Cvetič, Font, Ibáñez, Lüst and Quevedo (1991)
  $\mathcal{N} = 1$ SUGRA

$$K(\tau, \overline{\tau}) = -\Lambda_K^2 \log(2 \text{ Im } \tau)$$

$$G(\tau, \overline{\tau}) = \kappa^2 K(\tau, \overline{\tau}) + \log \left| \kappa^3 W(\tau) \right|^2$$

$$\kappa^2 = 8\pi / M_P^2$$

- Superpotential has modular weight $-\mathfrak{n} = -1, -2, -3, \ldots$

$$W(\tau) = \Lambda_W^3 \frac{H(\tau)}{\eta(\tau)^{2\mathfrak{n}}}$$

$$\mathfrak{n} = \kappa^2 \Lambda_K^2$$

- Simplified model, independent of the level *N*

Modular-invariant potentials

$$W(\tau) = \Lambda_W^3 \frac{H(\tau)}{\eta(\tau)^{2\mathfrak{n}}}$$

$$V = e^{\kappa^2 K}\left(K^{i\bar{j}} D_i W D_{\bar{j}} W^* - 3\kappa^2 |W|^2\right)$$

Modular-invariant potentials

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$$V = e^{\kappa^2 K}\left(K^{i\bar{j}} D_i W D_{\bar{j}} W^* - 3\kappa^2 |W|^2\right)$$

$$V(\tau,\bar{\tau}) = \frac{\Lambda_V^4}{(2\,\mathrm{Im}\,\tau)^{\mathfrak{n}} |\eta(\tau)|^{4\mathfrak{n}}} \left[\left| iH'(\tau) + \frac{\mathfrak{n}}{2\pi} H(\tau) \hat{G}_2(\tau,\bar{\tau})\right|^2 \frac{(2\,\mathrm{Im}\,\tau)^2}{\mathfrak{n}} - 3|H(\tau)|^2\right]$$

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$$V = e^{\kappa^2 K}\left(K^{i\bar{j}} D_i W D_{\bar{j}} W^* - 3\kappa^2 |W|^2\right)$$

$$\Lambda_V = \left(\kappa^2 \Lambda_W^6\right)^{1/4}$$

$$V(\tau,\bar{\tau}) = \frac{\Lambda_V^4}{(2\,\mathrm{Im}\,\tau)^{\mathfrak{n}} |\eta(\tau)|^{4\mathfrak{n}}} \left[\left| iH'(\tau) + \frac{\mathfrak{n}}{2\pi} H(\tau) \hat{G}_2(\tau,\bar{\tau})\right|^2 \frac{(2\,\mathrm{Im}\,\tau)^2}{\mathfrak{n}} - 3|H(\tau)|^2\right]$$

$$\hat{G}_2(\tau,\bar{\tau}) = G_2(\tau) - \frac{\pi}{\mathrm{Im}\,\tau}$$

$$\frac{\eta'(\tau)}{\eta(\tau)} = \frac{i}{4\pi} G_2(\tau)$$

Modular-invariant potentials

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$$\mathfrak{n} = 3$$

$$V(\tau, \bar{\tau}) = \frac{\Lambda_V^4}{8(\mathrm{Im}\,\tau)^3 |\eta|^{12}} \left[\frac{4}{3} \left| iH' + \frac{3}{2\pi} H \hat{G}_2 \right|^2 (\mathrm{Im}\,\tau)^2 - 3|H|^2 \right]$$

The superpotential

$$W(\tau) = \Lambda_W^3 \frac{H(\tau)}{\eta(\tau)^6}$$

$$V(\tau,\overline{\tau}) = \frac{\Lambda_V^4}{8(\operatorname{Im}\tau)^3|\eta|^{12}} \left[\frac{4}{3}\left|iH' + \frac{3}{2\pi}H\hat{G}_2\right|^2 (\operatorname{Im}\tau)^2 - 3|H|^2\right]$$

- Most general holomorphic $H(\tau)$ (except at $i\infty$) Cvetič et al (1991)

$$H(\tau) = (j(\tau) - 1728)^{m/2}\, j(\tau)^{n/3}\, \mathcal{P}(j(\tau))$$

$m, n = 0, 1, 2, \ldots$

$$j = \left(\frac{72}{\pi^2}\frac{\eta\eta'' - 3\eta'^2}{\eta^{10}}\right)^3 = \left[\frac{72}{\pi^2\eta^6}\left(\frac{\eta'}{\eta^3}\right)'\right]^3$$

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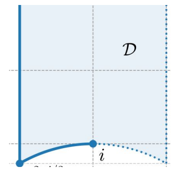
$$\mathcal{P}(j) = 1$$

simplest choice

- This potential is **modular-** and **CP-invariant** (also for some other $P(j)$'s)
- Everything can be expressed in terms of η and its derivatives...

q- and u-expansions of η



$$|q| \leq e^{-\sqrt{3}\pi} \simeq 0.004$$

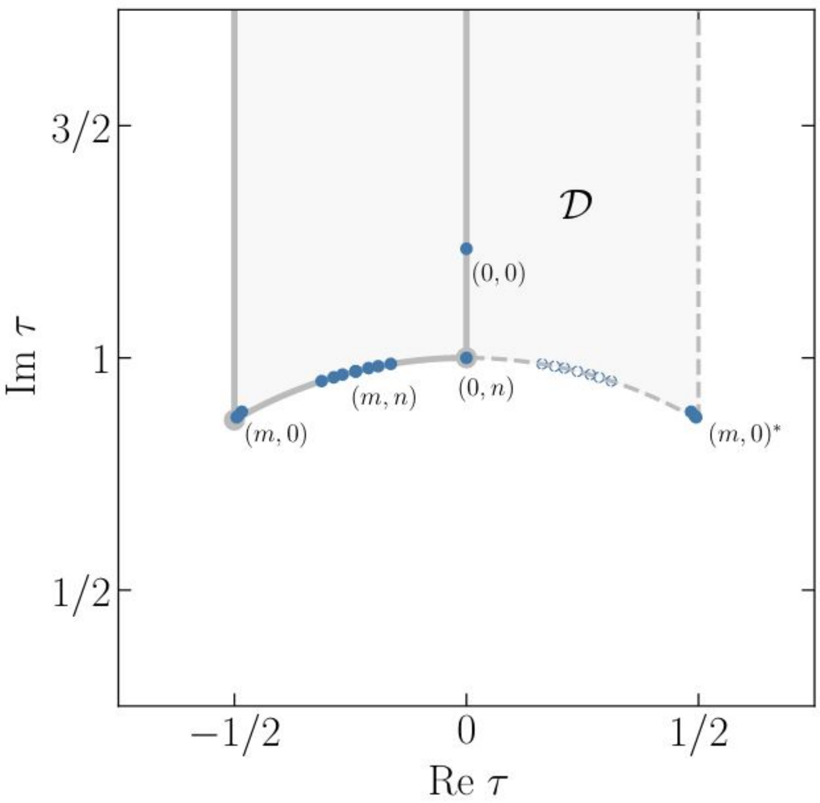
$$\eta = q^{1/24} \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{3n^2-n}{2}} = q^{1/24} \left(1 - q - q^2 + q^5 + q^7 - q^{12} - q^{15} + \mathcal{O}(q^{22})\right)$$

$$u \equiv \frac{\tau - \omega}{\tau - \omega^2} \qquad \tilde{\eta}(u) \equiv \frac{\eta(u)}{\sqrt{1-u}}$$

$$u \xrightarrow{ST} \omega^2 u \qquad \tilde{\eta}(u) \xrightarrow{ST} \tilde{\eta}(u)$$

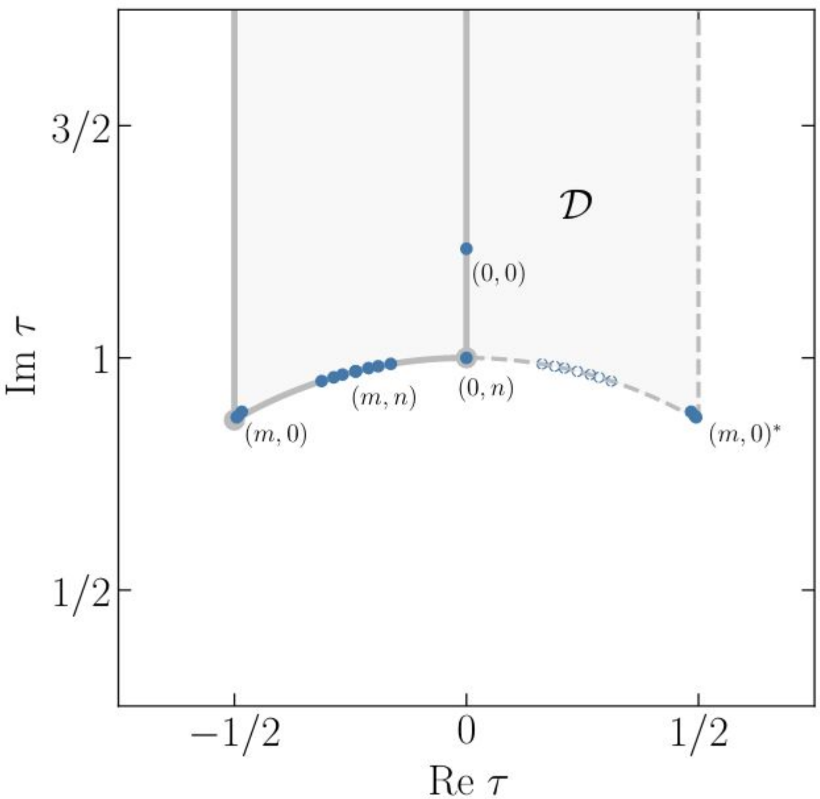
$$\begin{aligned} \tilde{\eta}(u) &\simeq e^{-i\pi/24} \left(0.800579 - 0.573569 u^3 - 0.780766 u^6 - 0.150007 u^9\right) + \mathcal{O}(u^{12}) \\ &\equiv e^{-i\pi/24} \left(\tilde{\eta}_0 + \tilde{\eta}_3 u^3 + \tilde{\eta}_6 u^6 + \tilde{\eta}_9 u^9\right) + \mathcal{O}(u^{12}) \,, \end{aligned}$$

Global minima for (*m*,*n*)-potentials

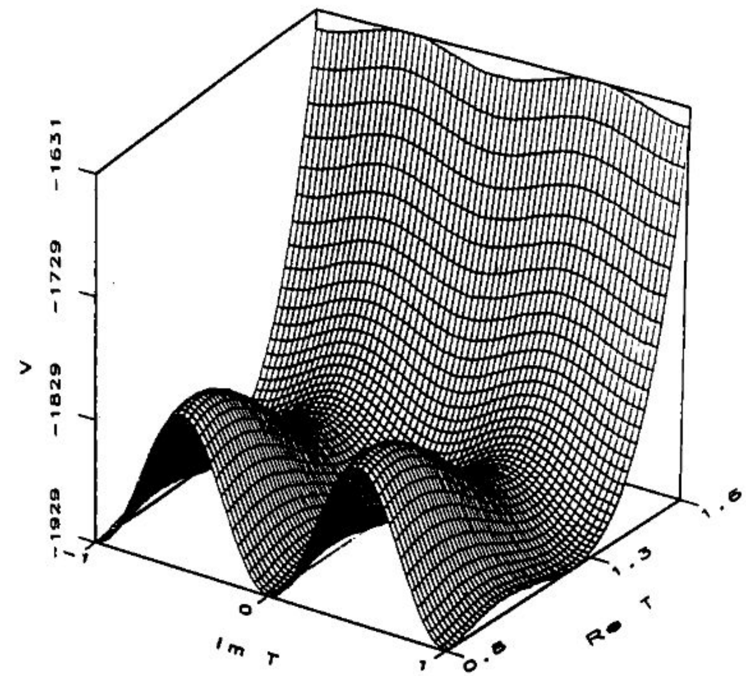


"(...) we conjecture that all extrema of V entirely lie on [the boundary]." — Cvetič et al.

Global minima for (*m*,*n*)-potentials



(0,0) ✓



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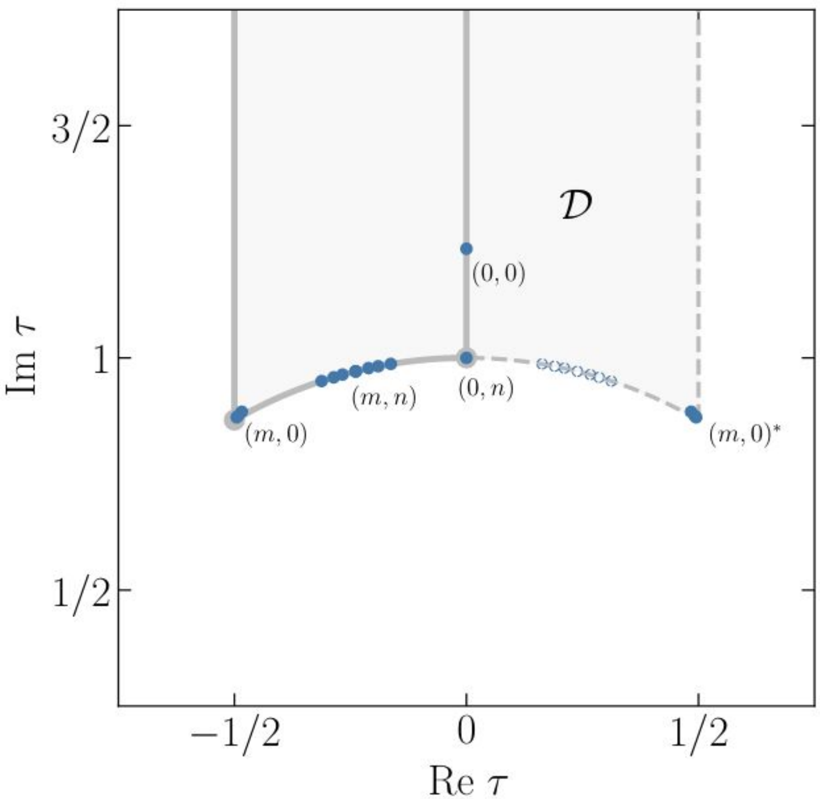
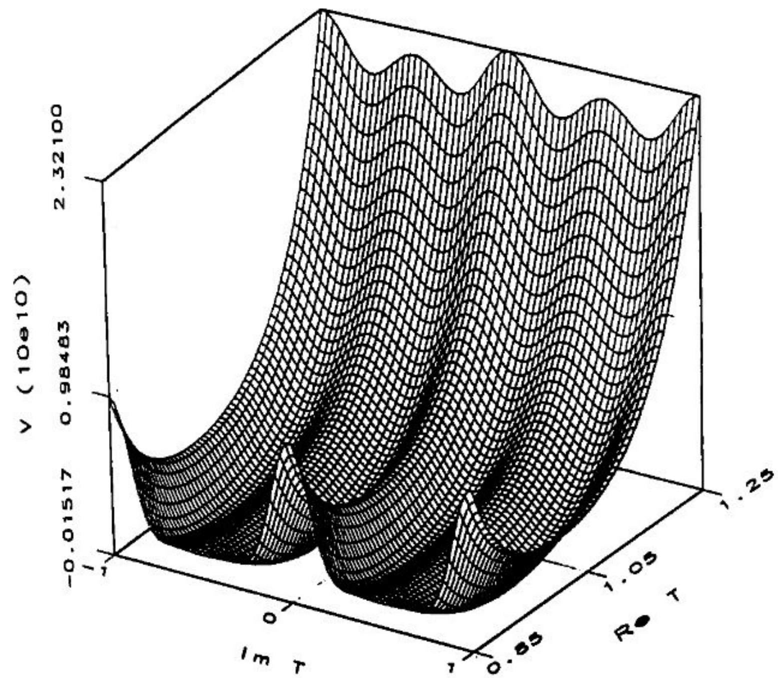
Global minima for (*m*,*n*)-potentials



(1,1) ✔



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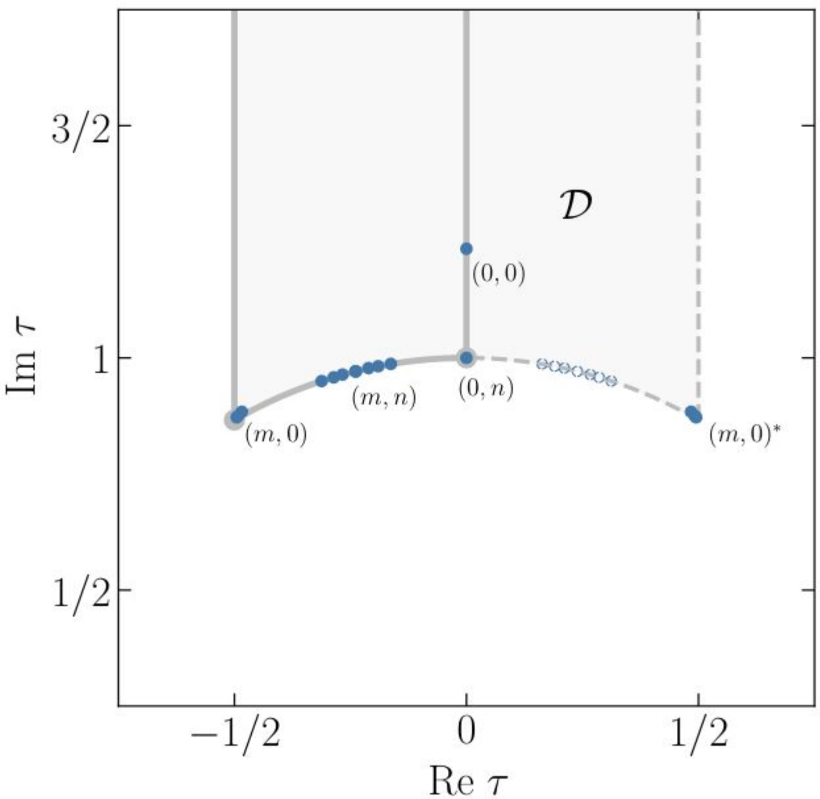
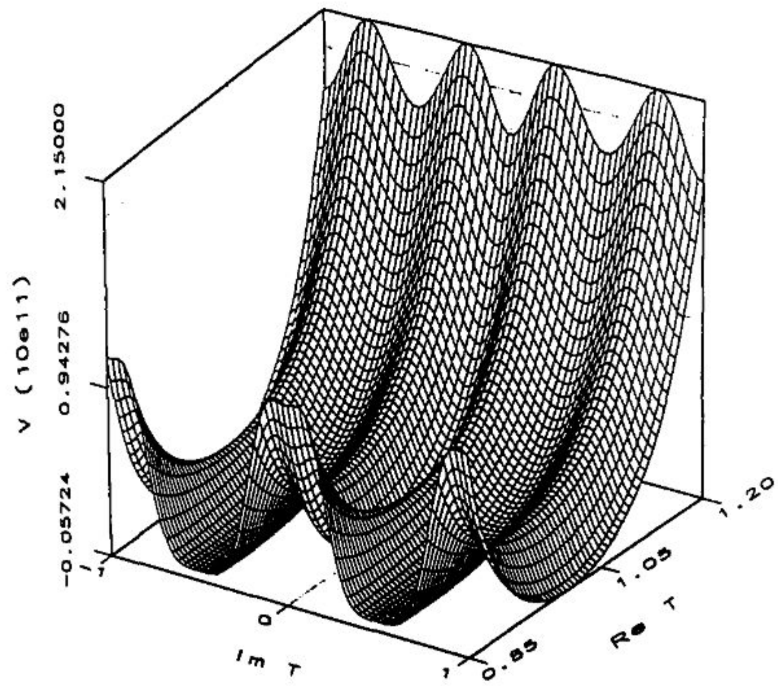
Global minima for (*m*,*n*)-potentials



(0,3) ✓



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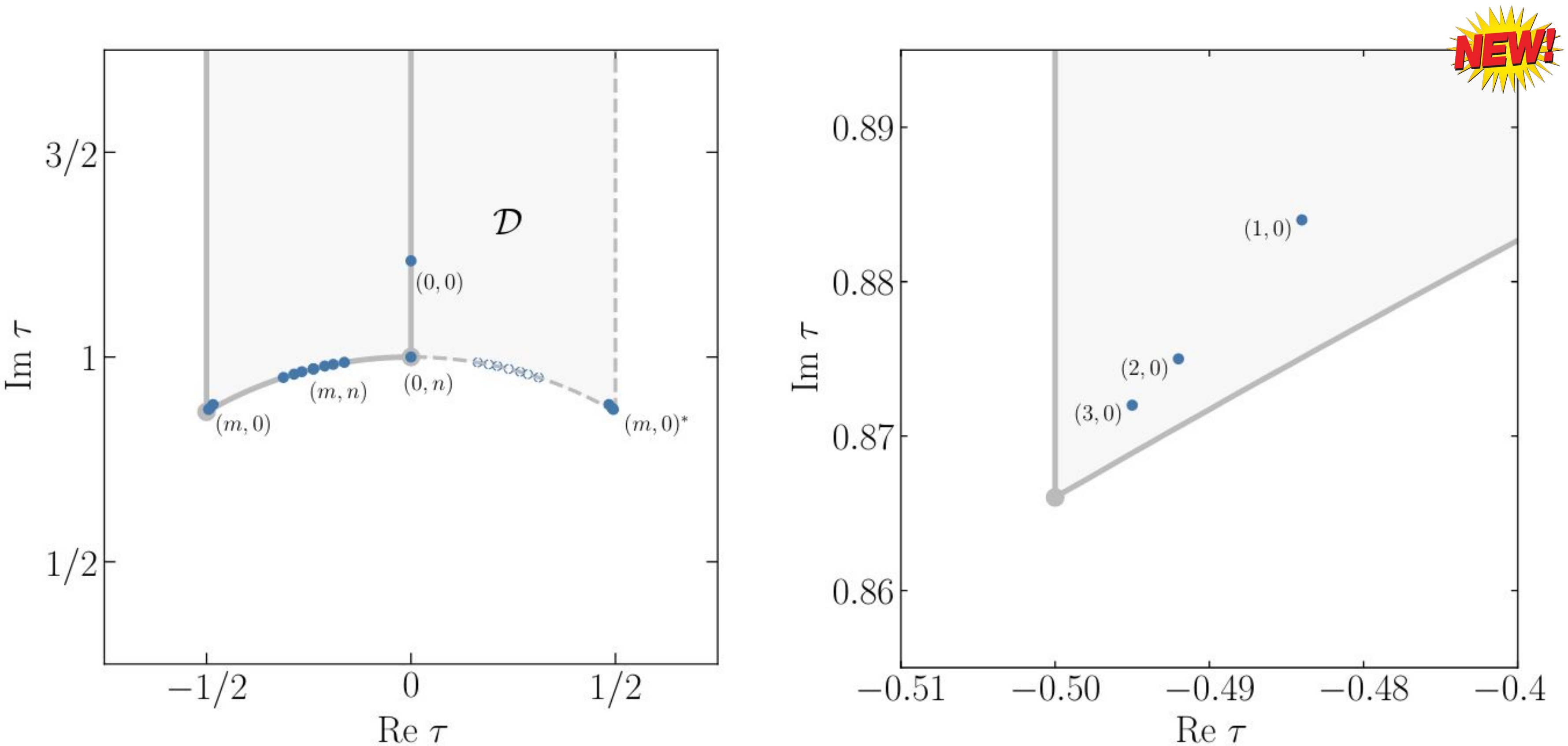

⚠

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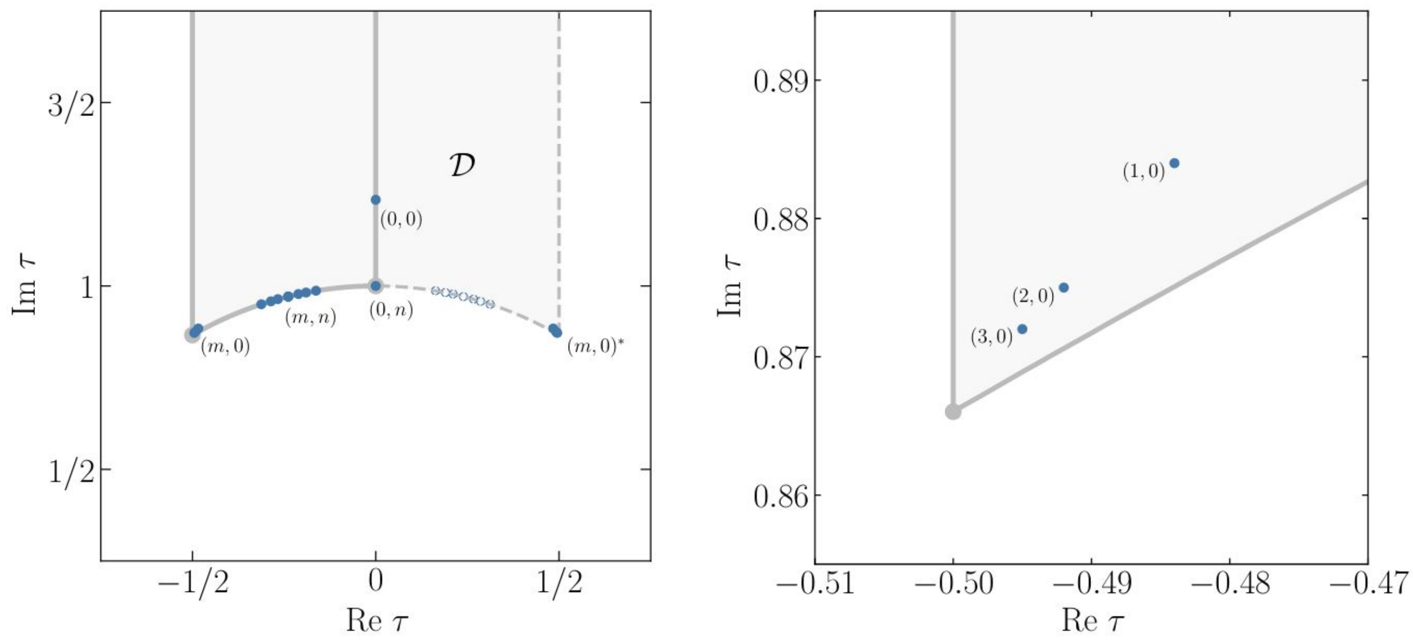

(0, 0) is a single minimum at $\tau \simeq 1.2i$ on the imaginary axis, corresponding to the case $m = n = 0$;

(0, *n*) is a single minimum at the symmetric point $\tau = i$ attained when $m = 0$, $n \neq 0$;

(*m*, 0) and (*m*, 0)* are a pair of degenerate minima for each $m \neq 0$ and $n = 0$: $(m, 0)$ is located in the vicinity of the left cusp $\tau = \omega$, approaching this symmetric point as m increases, while $(m, 0)^*$ is its CP-conjugate;

(*m*, *n*) is a series of minima on the unit arc, corresponding to $m \neq 0$, $n \neq 0$; these minima shift towards $\tau = \omega$ $(\tau = i)$ along the arc as m (n) grows.

The (*m*,*0*) family of potentials

- *u*-expand (*m*,*0*) potentials to analyse them near the left cusp

$$V_{m,0} = \Lambda_V^4 \, \frac{1728^m}{\sqrt{3}\,\tilde{\eta}_0^{12}} \left\{ -1 - 2\,|u|^2 + \left(A_m^2 - 3\right)|u|^4 \right\} + \mathcal{O}(|u|^6)$$

$$A_m \equiv \frac{864\,|\tilde{\eta}_3|^3}{\pi^6\,\tilde{\eta}_0^{27}}\, m + \frac{6\,|\tilde{\eta}_3|}{\tilde{\eta}_0}$$

$$\simeq 68.78\, m + 4.30$$

The (*m*,0) family of potentials

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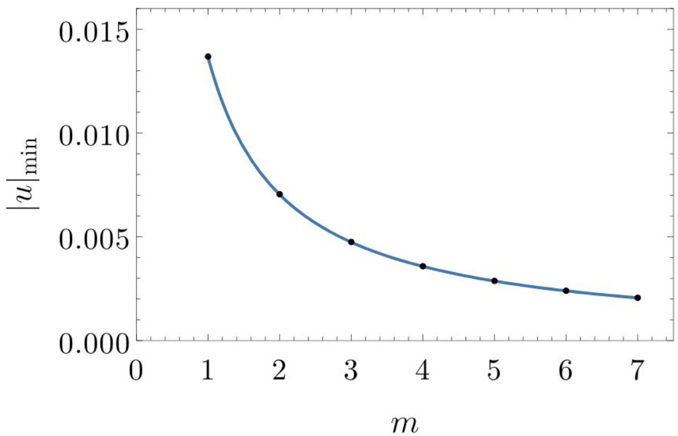
$$A_m \equiv \frac{864\,|\tilde{\eta}_3|^3}{\pi^6\,\tilde{\eta}_0^{27}}\,m + \frac{6\,|\tilde{\eta}_3|}{\tilde{\eta}_0} \simeq 68.78\,m + 4.30$$

- Mexican hat potential
  (cusp is a maximum!)



$$|u|_{\min} \simeq \left(A_m^2 - 3\right)^{-1/2} \simeq A_m^{-1} = \frac{0.0145}{m + 0.0625}$$

The (*m*,*0*) family of potentials

(phase dependence)

$$u = |u| e^{i\phi}$$

- *u*-expanding to higher order shows dependence on $\phi \in [-\pi/3, 0]$

$$V_{m,0} \propto -1 - 2\,|u|^2 + \left(A_m^2 - 3\right)|u|^4 + \left(-4 + 2A_m^2 + B_m^2 \cos 6\phi\right)|u|^6 + 2A_m B_m^2 \cos 3\phi\,|u|^7 + \left(-5 + 3A_m^2 + 2B_m^2 \cos 6\phi\right)|u|^8 + \mathcal{O}(|u|^9)$$

$$B_m^2 \equiv \frac{864\,|\tilde{\eta}_3|^3}{\pi^6\,\tilde{\eta}_0^{27}}\, m \left[\frac{864\,|\tilde{\eta}_3|^3}{\pi^6\,\tilde{\eta}_0^{27}}(m-2) + \frac{3\left(31\,\tilde{\eta}_3^2 - 10\tilde{\eta}_0\tilde{\eta}_6\right)}{\tilde{\eta}_0|\tilde{\eta}_3|}\right] + \frac{6\left(7\tilde{\eta}_3^2 - 2\tilde{\eta}_0\tilde{\eta}_6\right)}{\tilde{\eta}_0^2}$$

$$\simeq 4730.60\, m^2 - 2069.73\, m + 33.26\,.$$

The (*m*,*0*) family of potentials

(phase dependence)

$$u = |u| e^{i\phi}$$

- *u*-expanding to higher order shows dependence on $\phi \in [-\pi/3, 0]$

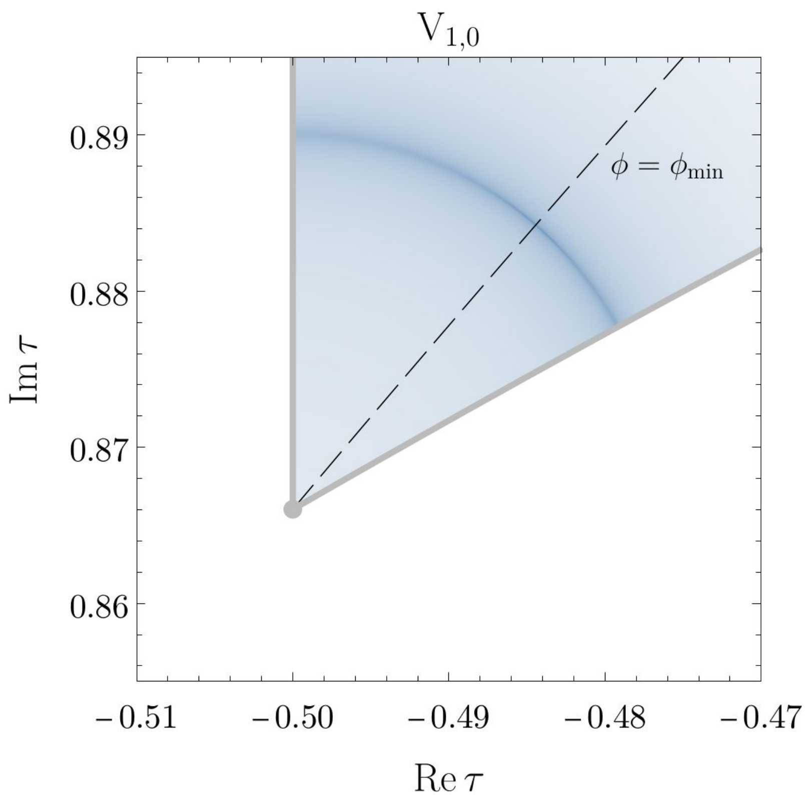
$$V_{m,0} \propto -1 - 2\,|u|^2 + \left(A_m^2 - 3\right)|u|^4 + \left(-4 + 2A_m^2 + B_m^2 \cos 6\phi\right)|u|^6 \\ + 2A_m B_m^2 \cos 3\phi\, |u|^7 + \left(-5 + 3A_m^2 + 2B_m^2 \cos 6\phi\right)|u|^8 + \mathcal{O}(|u|^9)$$

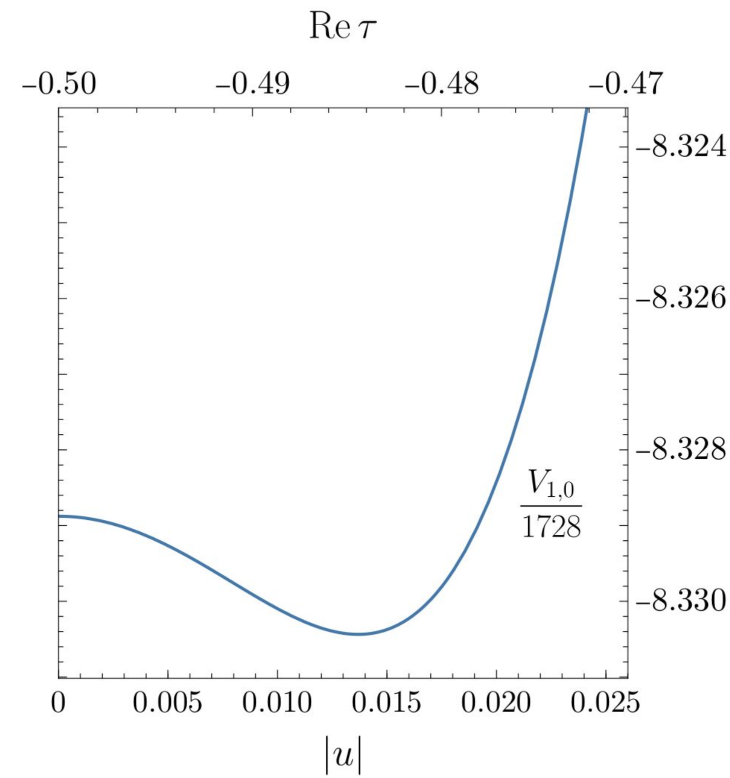
$$B_m^2 \equiv \frac{864\,|\tilde{\eta}_3|^3}{\pi^6\,\tilde{\eta}_0^{27}}\, m \left[\frac{864\,|\tilde{\eta}_3|^3}{\pi^6\,\tilde{\eta}_0^{27}}(m-2) + \frac{3\left(31\,\tilde{\eta}_3^2 - 10\tilde{\eta}_0\tilde{\eta}_6\right)}{\tilde{\eta}_0|\tilde{\eta}_3|}\right] + \frac{6\left(7\tilde{\eta}_3^2 - 2\tilde{\eta}_0\tilde{\eta}_6\right)}{\tilde{\eta}_0^2} \\ \simeq 4730.60\, m^2 - 2069.73\, m + 33.26\,.$$

- Phase of *u* mostly determined by $|u|^6$ and $|u|^7$ terms

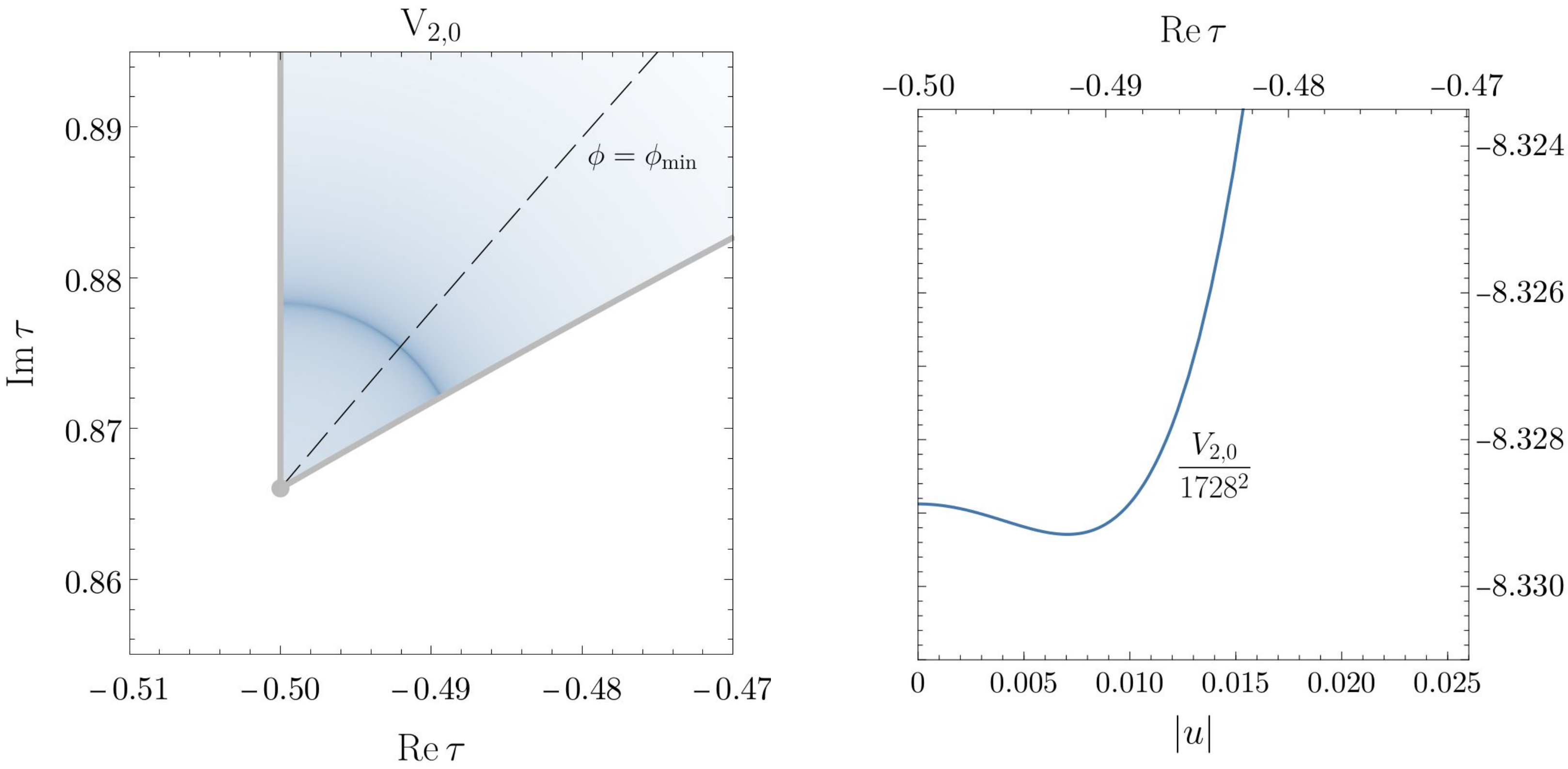
$$\phi_{\min} \simeq -\frac{2\pi}{9} = -40^\circ$$

The (m,0) family of potentials (m = 1)

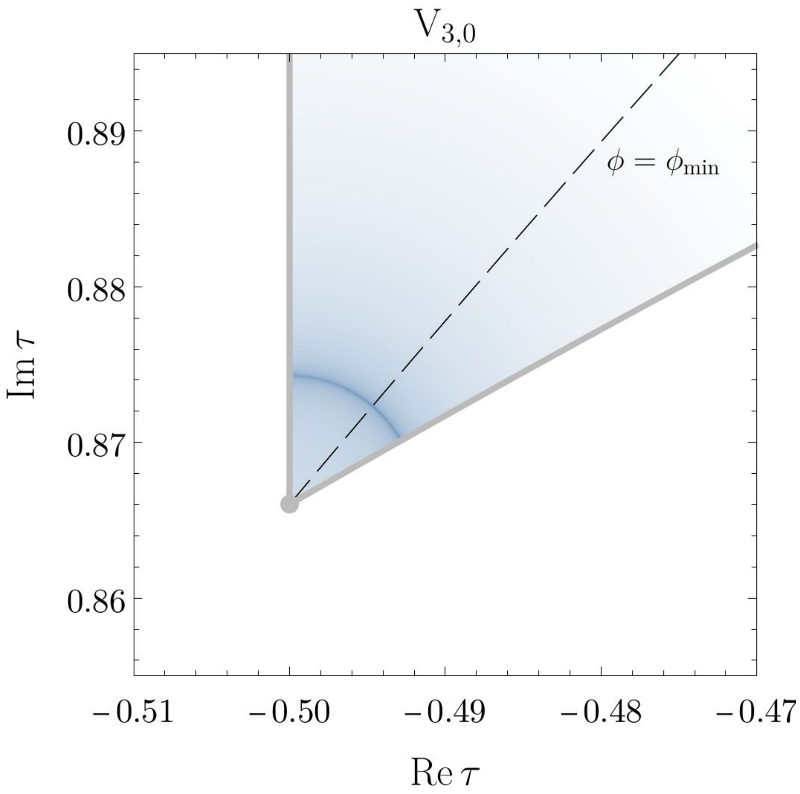


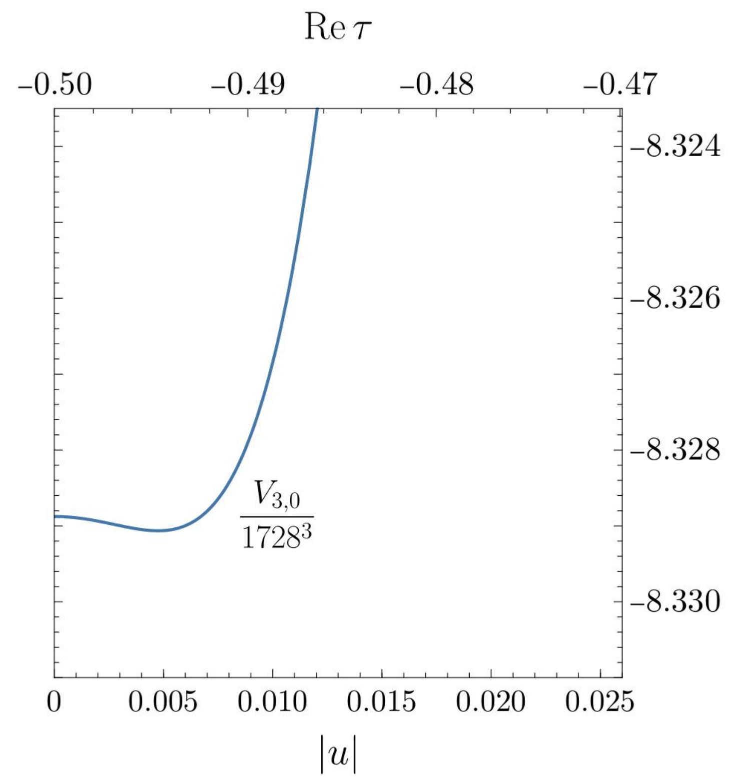


The (*m*,0) family of potentials (*m* = 2)

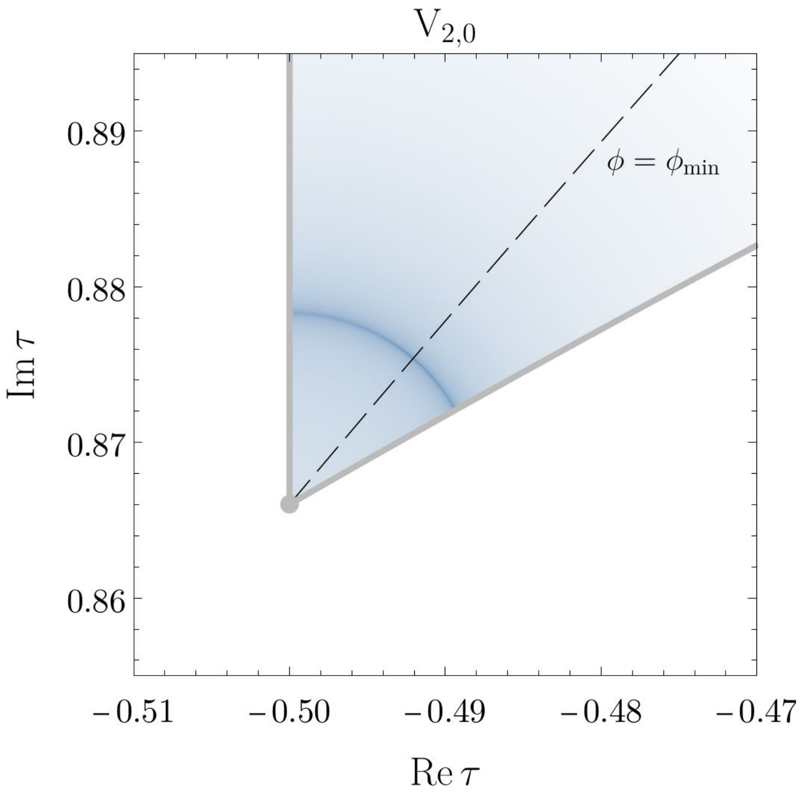


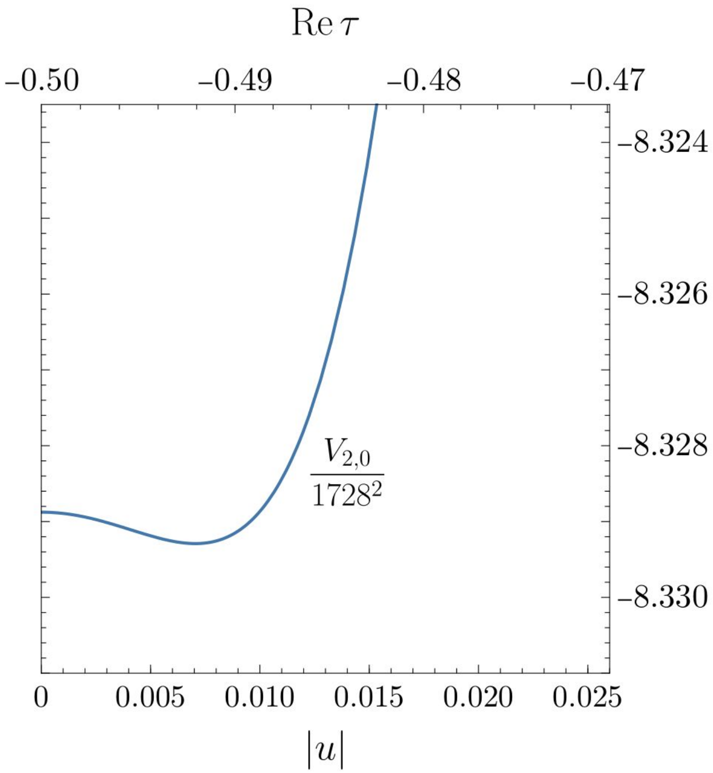
The (*m*,*0*) family of potentials (*m* = *3*)





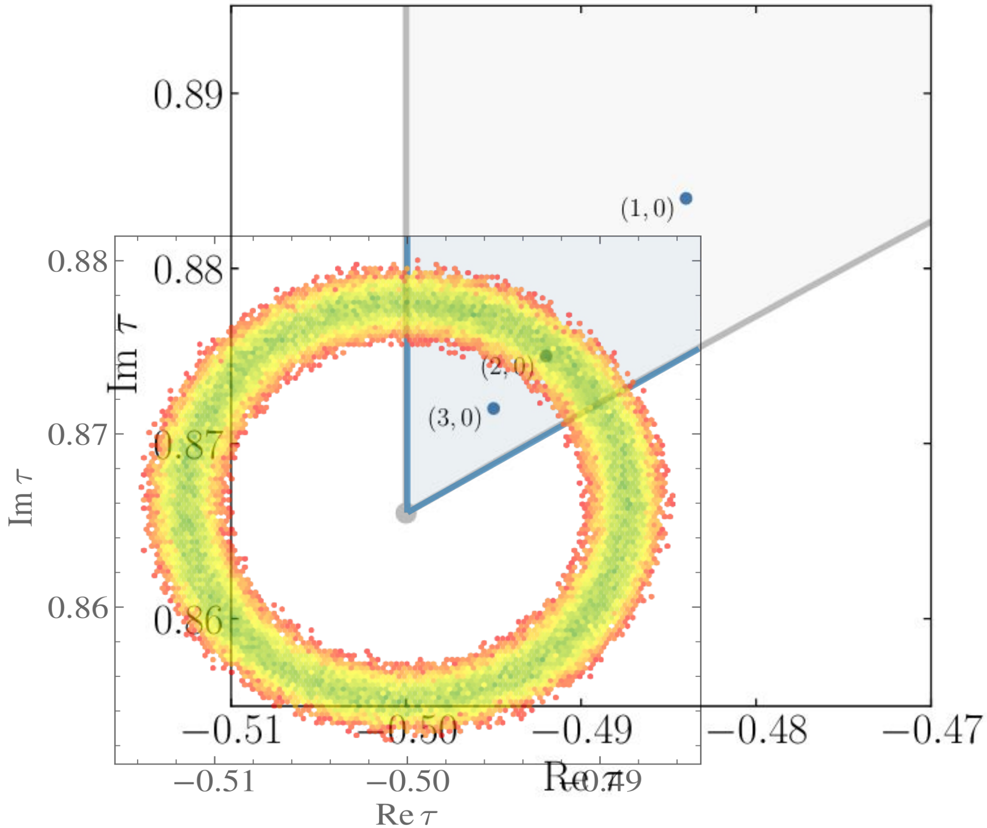
The (m,0) family of potentials (m = 2)





$$|\epsilon| \simeq 0.02 \Leftrightarrow |u| \simeq 0.007$$

Matching puzzle pieces?



$$|\epsilon| \simeq 0.02 \Leftrightarrow |u| \simeq 0.007$$

The global SUSY limit (a comment)

$$\mathfrak{n} = \kappa^2 \Lambda_K^2 \to 0$$

$$K(\tau,\bar\tau) = -\Lambda_K^2 \log(2\,\mathrm{Im}\,\tau)$$

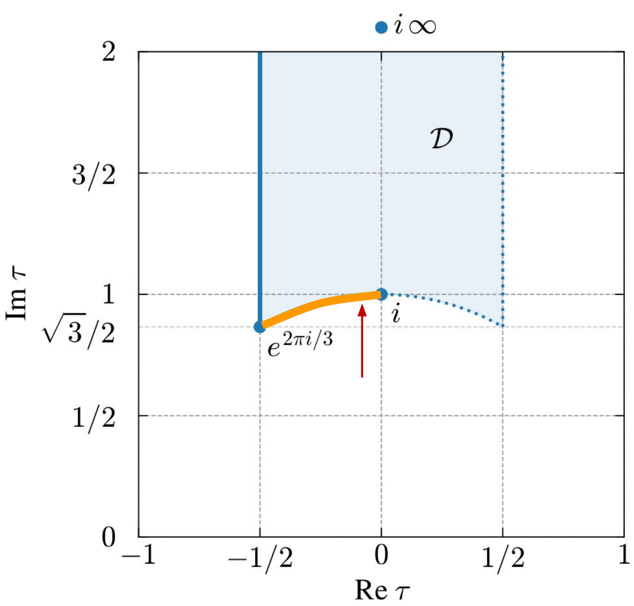
$$\kappa^2 = 8\pi/M_P^2$$

$$W(\tau) = \Lambda_W^3 H(\tau)$$

$$H(\tau) = (j(\tau)-1728)^{m/2}\, j(\tau)^{n/3}\, \mathcal{P}(j(\tau))$$

$$V(\tau,\bar\tau) = \frac{4\Lambda_W^6}{\Lambda_K^2} (\mathrm{Im}\,\tau)^2 \left|H'(\tau)\right|^2$$

- Global minima are zeros of *H*’
- non-trivial $\mathcal{P}(j)$ can be engineered to produce minima at arbitrary points in the fundamental domain



Summary II

Summary II

- There are simple potentials for modulus stabilisation, which are independent of the level N
- Novel **CP-breaking minima** are found, located **in the vicinity** of (but not directly on) the cusps
- The found deviation $|u|$ matches the BU requirement "My favourite because it requires no tuning at all"


Vielen Dank!
POSTAMT


Food for thought

Natural normalisation of modular forms?

- Often, several modular multiplets provide **independent contributions** to the mass matrices

$$W = \left[\alpha_1 \left(Y^{(4,6)}_{\mathbf{3}',1} E^c L_1\right)_{\mathbf{1}} + \alpha_3 \left(Y^{(4,6)}_{\mathbf{3}',1} E^c L_2\right)_{\mathbf{1}} + \alpha_4 \left(Y^{(4,6)}_{\mathbf{3}',2} E^c L_2\right)_{\mathbf{1}} + \alpha_5 \left(Y^{(4,6)}_{\mathbf{3}} E^c L_3\right)_{\mathbf{1}}\right] H_d$$
$$+ \left[g_1 \left(Y^{(4,3)}_{\hat{\mathbf{3}}} N^c L_1\right)_{\mathbf{1}} + g_2 \left(Y^{(4,3)}_{\hat{\mathbf{3}}} N^c L_2\right)_{\mathbf{1}} + g_3 \left(Y^{(4,3)}_{\hat{\mathbf{3}}'} N^c L_3\right)_{\mathbf{1}}\right] H_u$$
$$+ \Lambda \left(Y^{(4,2)}_{\mathbf{2}} (N^c)^2\right)_{\mathbf{1}} .$$

- Modular forms are **arbitrarily normalised**

$$(Y_1, Y_2, Y_3, Y_4, Y_5) \simeq \mathcal{N}\left(-1/\sqrt{6},\, q,\, 3q^2,\, 4q^3,\, 7q^4\right)$$

- Is there a canonical way to fix the magnitude of these normalisations?
  Example: build all multiplets from lowest weight one(s) via tensor products, using some "canonically normalized" CGCs (a la Quantum Mechanics)

Larger fundamental domains?

- Despite working with representations of the quotients, our theories are **fully modular invariant**
- To have canonical kinetic terms,

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad \Rightarrow \quad g_i \to (c\tau + d)^{-k_{Y_i}} g_i$$

- e.g. in a particular model,

$$(\tau,\ \beta/\alpha,\ \gamma/\alpha,\ g'/g,\ \ldots,\ \Lambda'/\Lambda,\ \ldots) \to$$
$$\left(\frac{a\tau + b}{c\tau + d},\ (c\tau + d)^{-2}\,\beta/\alpha,\ (c\tau + d)^{-2}\,\gamma/\alpha,\ g'/g,\ \ldots,\ \Lambda'/\Lambda,\ \ldots\right)$$

these different parameter sets lead to the same observables

see section 4 of Novichkov, JP, Petcov, Titov, 1811.04933

Universality of simple potential?

Kahler effects on hierarchies?

Backup slides

Decompositions under residual groups: A5’

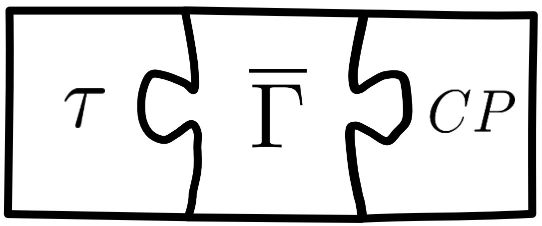
$\mathbf{r}$	$\mathbb{Z}_4^S\,(\tau = i)$	$\mathbb{Z}_3^{ST} \times \mathbb{Z}_2^R\,(\tau = \omega)$	$\mathbb{Z}_5^T \times \mathbb{Z}_2^R\,(\tau = i\infty)$
$\mathbf{1}$	$\mathbf{1}_k$	$\mathbf{1}_k^{\pm}$	$\mathbf{1}_0^{\pm}$
$\hat{\mathbf{2}}$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+3}$	$\mathbf{1}_{k+1}^{\mp} \oplus \mathbf{1}_{k+2}^{\mp}$	$\mathbf{1}_2^{\mp} \oplus \mathbf{1}_3^{\mp}$
$\hat{\mathbf{2}}'$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+3}$	$\mathbf{1}_{k+1}^{\mp} \oplus \mathbf{1}_{k+2}^{\mp}$	$\mathbf{1}_1^{\mp} \oplus \mathbf{1}_4^{\mp}$
$\mathbf{3}$	$\mathbf{1}_k \oplus \mathbf{1}_{k+2} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k^{\pm} \oplus \mathbf{1}_{k+1}^{\pm} \oplus \mathbf{1}_{k+2}^{\pm}$	$\mathbf{1}_0^{\pm} \oplus \mathbf{1}_1^{\pm} \oplus \mathbf{1}_4^{\pm}$
$\mathbf{3}'$	$\mathbf{1}_k \oplus \mathbf{1}_{k+2} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k^{\pm} \oplus \mathbf{1}_{k+1}^{\pm} \oplus \mathbf{1}_{k+2}^{\pm}$	$\mathbf{1}_0^{\pm} \oplus \mathbf{1}_2^{\pm} \oplus \mathbf{1}_3^{\pm}$
$\mathbf{4}$	$\mathbf{1}_k \oplus \mathbf{1}_k \oplus \mathbf{1}_{k+2} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k^{\pm} \oplus \mathbf{1}_k^{\pm} \oplus \mathbf{1}_{k+1}^{\pm} \oplus \mathbf{1}_{k+2}^{\pm}$	$\mathbf{1}_1^{\pm} \oplus \mathbf{1}_2^{\pm} \oplus \mathbf{1}_3^{\pm} \oplus \mathbf{1}_4^{\pm}$
$\hat{\mathbf{4}}$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+3} \oplus \mathbf{1}_{k+3}$	$\mathbf{1}_k^{\mp} \oplus \mathbf{1}_k^{\mp} \oplus \mathbf{1}_{k+1}^{\mp} \oplus \mathbf{1}_{k+2}^{\mp}$	$\mathbf{1}_1^{\mp} \oplus \mathbf{1}_2^{\mp} \oplus \mathbf{1}_3^{\mp} \oplus \mathbf{1}_4^{\mp}$
$\mathbf{5}$	$\mathbf{1}_k \oplus \mathbf{1}_k \oplus \mathbf{1}_k \oplus \mathbf{1}_{k+2} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k^{\pm} \oplus \mathbf{1}_{k+1}^{\pm} \oplus \mathbf{1}_{k+1}^{\pm} \oplus \mathbf{1}_{k+2}^{\pm} \oplus \mathbf{1}_{k+2}^{\pm}$	$\mathbf{1}_0^{\pm} \oplus \mathbf{1}_1^{\pm} \oplus \mathbf{1}_2^{\pm} \oplus \mathbf{1}_3^{\pm} \oplus \mathbf{1}_4^{\pm}$
$\hat{\mathbf{6}}$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+3} \oplus \mathbf{1}_{k+3} \oplus \mathbf{1}_{k+3}$	$\mathbf{1}_k^{\mp} \oplus \mathbf{1}_k^{\mp} \oplus \mathbf{1}_{k+1}^{\mp} \oplus \mathbf{1}_{k+1}^{\mp} \oplus \mathbf{1}_{k+2}^{\mp} \oplus \mathbf{1}_{k+2}^{\mp}$	$\mathbf{1}_0^{\mp} \oplus \mathbf{1}_0^{\mp} \oplus \mathbf{1}_1^{\mp} \oplus \mathbf{1}_2^{\mp} \oplus \mathbf{1}_3^{\mp} \oplus \mathbf{1}_4^{\mp}$

Details of the model fit

Model	Section 4.2 (S_4')
$\mathrm{Re}\,\tau$	$-0.496^{+0.009}_{-0.016}$
$\mathrm{Im}\,\tau$	$0.877^{+0.0023}_{-0.024}$
α_2/α_1	—
α_3/α_1	$2.45^{+0.44}_{-0.42}$
α_4/α_1	$-2.37^{+0.36}_{-0.3}$
α_5/α_1	$1.01^{+0.06}_{-0.06}$
g_2/g_1	$1.5^{+0.15}_{-0.14}$
g_3/g_1	$2.22^{+0.17}_{-0.15}$
$v_d\,\alpha_1$, GeV	$4.61^{+1.32}_{-1.33}$
$v_u^2\,g_1/\Lambda$, eV	$0.268^{+0.057}_{-0.063}$
$\epsilon(\tau)$	$0.0186^{+0.0028}_{-0.0023}$
CL mass pattern	$(1, \epsilon, \epsilon^2)$
max(BG)	0.848

m_e/m_μ	$0.00475^{+0.00061}_{-0.00052}$
m_μ/m_τ	$0.0556^{+0.0136}_{-0.0116}$
r	$0.0298^{+0.00196}_{-0.0023}$
δm^2, 10^{-5} eV^2	$7.38^{+0.35}_{-0.44}$
$\lvert\Delta m^2\rvert$, 10^{-3} eV^2	$2.48^{+0.05}_{-0.04}$
$\sin^2\theta_{12}$	$0.304^{+0.039}_{-0.036}$
$\sin^2\theta_{13}$	$0.0221^{+0.0019}_{-0.002}$
$\sin^2\theta_{23}$	$0.539^{+0.0522}_{-0.099}$
m_1, eV	0
m_2, eV	$0.0086^{+0.0002}_{-0.00026}$
m_3, eV	$0.0502^{+0.00046}_{-0.00043}$
$\Sigma_i m_i$, eV	$0.0588^{+0.0002}_{-0.0002}$
$\lvert\langle m\rangle\rvert$, eV	$0.00144^{+0.00035}_{-0.00033}$
δ/π	$1 \pm \mathcal{O}(10^{-6})$
α_{21}/π	0
α_{31}/π	$1 \pm \mathcal{O}(10^{-5})$
$N\sigma$	0.563

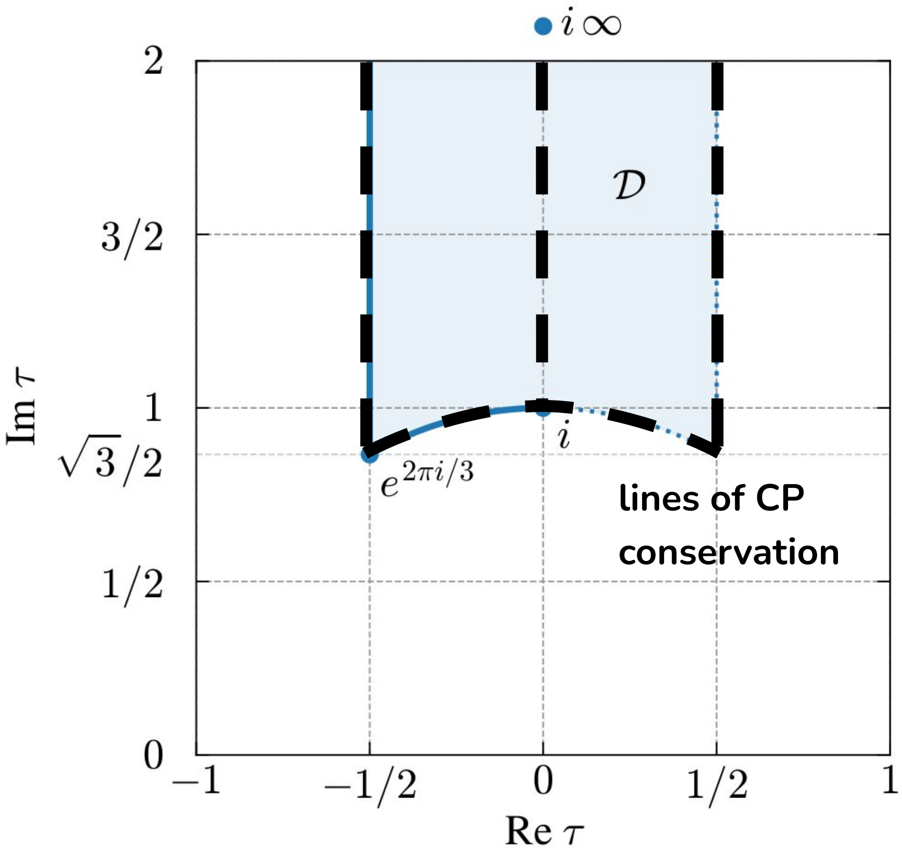
Combining modular and CP symmetries



$$\tau \xrightarrow{\text{CP}} -\tau^*$$

$$\psi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}}^{\text{CP}}\, \overline{\psi}(x_{\text{P}})$$

$$Y(\tau) \xrightarrow{\text{CP}} X_{\mathbf{r}}^{\text{CP}}\, Y^*(\tau)$$



Constraints on the Kähler potential?

- **Kähler** not constrained by the symmetry.
- Under a modular transformation, invariant up to:

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) \to K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) + f(\chi_i; \tau) + f(\overline{\chi}_i; \overline{\tau})$$

- Minimal choice:

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) = -h\,\Lambda_0^2 \log(-i(\tau - \overline{\tau})) + \sum_i \frac{|\chi_i|^2}{(-i(\tau - \overline{\tau}))^{k_i}}$$

should be justified from the top-down

Chen, Ramos-Sánchez and Ratz, 1909.06910

- Further constraints may arise from combining modular group + traditional finite flavour symmetry

Nilles, Ramos-Sanchez, Vaudrevange, 2004.05200

SUSY breaking effects?

- **RGEs & threshold corrections** need to be considered, depend on $\tan\beta$ and unknown SUSY spectrum
- **SUSY-breaking** corrections can be made negligible via separation of scales (power counting argument)
- Under reasonable conditions, predictions may be unaffected

Feruglio and Criado, 1807.01125

Extrema at $\tau = i, \omega$

Gonzalo, Ibáñez and Uranga, 1812.06520

	$V\,(T=1)$	Type of Extrema	H	$\frac{dH}{dT}$	SUSY
$m>1$	$V=0$	Min	0	0	Yes
$m=1$	$\frac{1}{T_I^3\lvert\eta\rvert^{12}}\left\{\lvert a\rvert^2\lvert C\rvert^2\right\}>0$	Max $-2.57<\frac{H'''}{H'}<-1.57$ SP $\frac{H'''}{H'}<-2.57$ or $\frac{H'''}{H'}>-1.57$	0	$\neq 0$	No
$m=0$	$\propto\frac{\lvert P(0)\rvert^2}{T_I^3\lvert\eta\rvert^{12}}\{-3\}<0$	Min $\left\lvert\frac{H''}{H}+1.19\right\rvert>\frac{3}{2}$ Max $-\frac{3}{4}<\frac{H''}{H}+1.19<\frac{3}{4}$ SP (Saddle Point) if else	$\neq 0$	0	Yes

Table 2. Classification of the extrema found at $T=i$.

	$V\,(T=\rho)$	Type of Extrema	H	$\frac{dH}{dT}$	SUSY
$n>1$	$V=0$	Minimum	0	0	Yes
$n=1$	$\frac{1}{\lvert\eta\rvert^{12}}\left\{\frac{4}{3}\lvert\mathcal{P}(1728)\rvert^2\lvert D\rvert^2\right\}>0$	Maximum	0	$\neq 0$	No
$n=0$	$\propto\frac{1728^m\lvert\mathcal{P}(1728)\rvert^2}{T_I^3\lvert\eta\rvert^{12}}\{-3\}<0$	Maximum	$\neq 0$	0	Yes

Table 3. Classification of the extrema found at $T=\rho$.

No, there is no tuning in choosing this form of the superpotential (arguably)

$$H(\tau) \propto (J(\tau) - 1)^{m/2}$$

Subset of all possible $H(\tau)$ which vanish only at the symmetric point $\tau=i$ (itself distinguished by modular symmetry)

$$J(\tau) \equiv j(\tau)/1728$$