Eclectic flavor symmetry breakdown

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based on: AB, H.P. Nilles, A. Trautner, S. Ramos–Sánchez, P.K.S. Vaudrevange – 2112.06940
AB, H.P. Nilles, A. Trautner, S. Ramos–Sánchez, P.K.S. Vaudrevange – 22xx.xxxxx

Outline:

INTRODUCTION

Motivation: How to obtain hierarchies?

Hierarchies can be obtained from softly broken symmetries

–> see other talks, especially Penedo

This talk:
Illustrate the breakdown of eclectic flavor groups for a benchmark model derived from a $\mathbb{T}^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ orbifold compactification of heterotic string theory.

THE ECLECTIC FLAVOR SYMMETRY OF $\mathbb{T}^2/\mathbb{Z}_3$

Consider flavor symmetries of $\mathbb{T}^2/\mathbb{Z}_3$ as two-dimensional elliptic fibration of $\mathbb{T}^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$.

Outer automorphisms of corresponding Narain space group yield the following symmetries:

- a $\Delta(54)$ traditional flavor symmetry
- an $\mathrm{SL}(2,\mathbb{Z})_T$ modular symmetry which acts as a $\Gamma'_3 \cong T'$ finite modular symmetry
- a $\mathbb{Z}_9^{\mathrm{R}}$ discrete R-symmetry as a remnant of $\mathrm{SL}(2,\mathbb{Z})_U$
- a $\mathbb{Z}_2^{\mathcal{CP}}$ $\mathcal{CP}$-like transformation

In total we have: —> see Ramos-Sánchez and Trautners talk

$$G_{\mathrm{eclectic}} \;=\; G_{\mathrm{traditional}} \cup G_{\mathrm{modular}} \cup G_{\mathrm{R}} \cup \mathcal{CP}$$

The eclectic group of this setting is of order 3888 and given by

$$G_{\mathrm{eclectic}} \;=\; \Omega(2) \rtimes \mathbb{Z}_2^{\mathcal{CP}}\,, \qquad \text{where} \qquad \Omega(2) \cong [1944, 3448]$$

MASSLESS MATTER FIELDS AND THEIR CHARGES UNDER FLAVOR SYMMETRY

sector	matter fields	eclectic flavor group $\Omega(2)$								
		modular T' subgroup				traditional $\Delta(54)$ subgroup				$\mathbb{Z}_9^R$
	Φ_n	irrep $\boldsymbol{s}$	$\rho_{\boldsymbol{s}}(\mathrm{S})$	$\rho_{\boldsymbol{s}}(\mathrm{T})$	n	irrep $\boldsymbol{r}$	$\rho_{\boldsymbol{r}}(\mathrm{A})$	$\rho_{\boldsymbol{r}}(\mathrm{B})$	$\rho_{\boldsymbol{r}}(\mathrm{C})$	R
bulk	Φ_0	$\mathbf{1}$	1	1	0	$\mathbf{1}$	1	1	$+1$	0
	Φ_{-1}	$\mathbf{1}$	1	1	-1	$\mathbf{1}'$	1	1	-1	3
θ	$\Phi_{-2/3}$	$\mathbf{2}'\oplus\mathbf{1}$	$\rho(\mathrm{S})$	$\rho(\mathrm{T})$	$-2/3$	$\mathbf{3}_2$	$\rho(\mathrm{A})$	$\rho(\mathrm{B})$	$+\rho(\mathrm{C})$	1
	$\Phi_{-5/3}$	$\mathbf{2}'\oplus\mathbf{1}$	$\rho(\mathrm{S})$	$\rho(\mathrm{T})$	$-5/3$	$\mathbf{3}_1$	$\rho(\mathrm{A})$	$\rho(\mathrm{B})$	$-\rho(\mathrm{C})$	-2
θ^2	$\Phi_{-1/3}$	$\mathbf{2}''\oplus\mathbf{1}$	$(\rho(\mathrm{S}))^*$	$(\rho(\mathrm{T}))^*$	$-1/3$	$\bar{\mathbf{3}}_1$	$\rho(\mathrm{A})$	$(\rho(\mathrm{B}))^*$	$-\rho(\mathrm{C})$	2
	$\Phi_{+2/3}$	$\mathbf{2}''\oplus\mathbf{1}$	$(\rho(\mathrm{S}))^*$	$(\rho(\mathrm{T}))^*$	$+2/3$	$\bar{\mathbf{3}}_2$	$\rho(\mathrm{A})$	$(\rho(\mathrm{B}))^*$	$+\rho(\mathrm{C})$	5
super-potential	$\mathcal{W}$	$\mathbf{1}$	1	1	-1	$\mathbf{1}'$	1	1	-1	3

$$\rho(\mathrm{S}) = \frac{\mathrm{i}}{\sqrt{3}}\begin{pmatrix}1&1&1\\1&\omega^2&\omega\\1&\omega&\omega^2\end{pmatrix}, \quad \rho(\mathrm{T}) = \begin{pmatrix}\omega^2&0&0\\0&1&0\\0&0&1\end{pmatrix}, \quad \rho(\mathrm{A}) = \begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}, \quad \rho(\mathrm{B}) = \begin{pmatrix}1&0&0\\0&\omega&0\\0&0&\omega^2\end{pmatrix}, \quad \rho(\mathrm{C}) = -\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}, \quad \text{with} \quad \omega = \mathrm{e}^{2\pi\mathrm{i}/3}\,.$$

MSSM Charge Assignments in Explicit Models

Model	ℓ	$\bar{e}$	$\bar{\nu}$	q	$\bar{u}$	$\bar{d}$	H_u	H_d	flavons
A	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	Φ_0	Φ_0	$\Phi_{-2/3,-1}$
B	$\Phi_{-1/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-1/3}$	Φ_{-1}	Φ_0	$\Phi_{-2/3,-1}$
C	$\Phi_{-2/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-2/3}$	Φ_{-1}	Φ_{-1}	$\Phi_{-1/3,-1}$
D	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{\pm 2/3,0}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	Φ_0	$\Phi_{-1,0}$	$\Phi_{\pm 2/3,-1}$
E	$\Phi_{-2/3,-1/3}$	$\Phi_{-2/3,0}$	$\Phi_{0,-2/3,-1/3,-5/3}$	$\Phi_{-1,-2/3}$	$\Phi_{-2/3}$	$\Phi_{0,-2/3}$	Φ_0	Φ_0	$\Phi_{-2/3,-1/3,-5/3,-1}$

For a more detailed classification of these ~ 1000 models of type $\mathrm{A}-\mathrm{E}$ see:

[Olguin-Trejo, Pérez-Martínez, Ramos-Sánchez: 1808.06622]

MSSM CHARGE ASSIGNMENTS IN EXPLICIT MODELS

Model	ℓ	$\bar{e}$	$\bar{\nu}$	q	$\bar{u}$	$\bar{d}$	H_u	H_d	flavons
A	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	Φ_0	Φ_0	$\Phi_{-2/3,-1}$
B	$\Phi_{-1/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-1/3}$	Φ_{-1}	Φ_0	$\Phi_{-2/3,-1}$
C	$\Phi_{-2/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-2/3}$	Φ_{-1}	Φ_{-1}	$\Phi_{-1/3,-1}$
D	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{\pm 2/3,0}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	Φ_0	$\Phi_{-1,0}$	$\Phi_{\pm 2/3,-1}$
E	$\Phi_{-2/3,-1/3}$	$\Phi_{-2/3,0}$	$\Phi_{0,-2/3,-1/3,-5/3}$	$\Phi_{-1,-2/3}$	$\Phi_{-2/3}$	$\Phi_{0,-2/3}$	Φ_0	Φ_0	$\Phi_{-2/3,-1/3,-5/3,-1}$

For example, superpotential of model of type A:

$$\mathcal{W} \;=\; \hat{Y}^{(1)}(T)\left\{\phi^0\left[\phi^0_{\mathrm{u}}H_{\mathrm{u}}\bar{u}q\varphi_{\mathrm{u}}+\phi^0_{\mathrm{d}}H_{\mathrm{d}}\bar{d}q\varphi_{\mathrm{e}}+\phi^0_{\mathrm{e}}H_{\mathrm{d}}\bar{e}\ell\varphi_{\mathrm{e}}+H_{\mathrm{u}}\bar{\nu}\ell\varphi_{\nu}\right]+\phi^0_{\mathrm{M}}\bar{\nu}\bar{\nu}\varphi_{\mathrm{e}}\right\}\;.$$

MSSM CHARGE ASSIGNMENTS IN EXPLICIT MODELS

Model	ℓ	$\bar{e}$	$\bar{\nu}$	q	$\bar{u}$	$\bar{d}$	H_u	H_d	flavons
A	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	Φ_0	Φ_0	$\Phi_{-2/3,-1}$
B	$\Phi_{-1/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-2/3}$	$\Phi_{-1/3}$	Φ_{-1}	Φ_0	$\Phi_{-2/3,-1}$
C	$\Phi_{-2/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-2/3}$	Φ_{-1}	Φ_{-1}	$\Phi_{-1/3,-1}$
D	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{\pm 2/3,0}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	$\Phi_{-1/3}$	Φ_0	$\Phi_{-1,0}$	$\Phi_{\pm 2/3,-1}$
E	$\Phi_{-2/3,-1/3}$	$\Phi_{-2/3,0}$	$\Phi_{0,-2/3,-1/3,-5/3}$	$\Phi_{-1,-2/3}$	$\Phi_{-2/3}$	$\Phi_{0,-2/3}$	Φ_0	Φ_0	$\Phi_{-2/3,-1/3,-5/3,-1}$

For example, superpotential of model of type A:

$$\mathcal{W} = \hat{Y}^{(1)}(T)\left\{\phi^0\left[\phi^0_{\mathrm{u}} H_{\mathrm{u}}\bar{u}q\varphi_{\mathrm{u}} + \phi^0_{\mathrm{d}} H_{\mathrm{d}}\bar{d}q\varphi_{\mathrm{e}} + \phi^0_{\mathrm{e}} H_{\mathrm{d}}\bar{e}\ell\varphi_{\mathrm{e}} + H_{\mathrm{u}}\bar{\nu}\ell\varphi_\nu\right] + \phi^0_{\mathrm{M}}\bar{\nu}\bar{\nu}\varphi_{\mathrm{e}}\right\} .$$

Observations

- Yukawas are not only integer weight, but all of them are exactly 1.
- Fields with same modular weight transform under same representation of flavor symmetry.

  –> also appears in other TD constructions –> see Otsukas talk

  E.g. 2201.04505: “In our scenarios, wavefunctions [...] are modular forms.”

Sources of Breaking

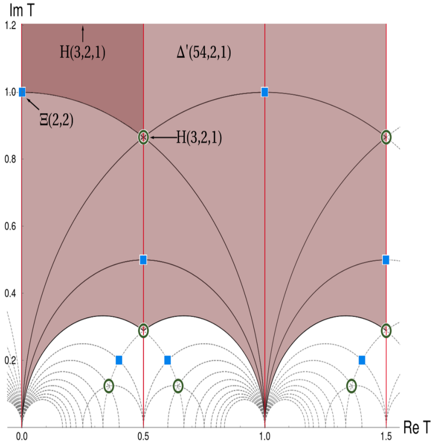
$$\boxed{G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}} \cup G_{\text{R}} \cup \mathcal{CP}}$$

1. VEV of modulus, $\langle T \rangle$ —> see Penedos talk
   Special points with enhanced symmetry:

$$\begin{aligned}
\langle T \rangle &= \mathrm{i} & \Rightarrow \quad & \Xi(2,2) \cong [324, 111] \\
\langle T \rangle &= \omega, -\omega^2 & \Rightarrow \quad & H(3,2,1) \cong [486, 125] \\
\langle T \rangle &= \mathrm{i}\infty, 1 & \Rightarrow \quad & H(3,2,1) \cong [486, 125]
\end{aligned}$$

2. VEVs of flavons:

$$\begin{aligned}
\langle \Phi_{-2/3} \rangle &\sim \mathbf{3}_2, & \langle \Phi_{-5/3} \rangle &\sim \mathbf{3}_1, & \langle \Phi_{-1} \rangle &\sim \mathbf{1}', \\
\langle \Phi_{-1/3} \rangle &\sim \bar{\mathbf{3}}_2, & \langle \Phi_{+2/3} \rangle &\sim \bar{\mathbf{3}}_1.
\end{aligned}$$

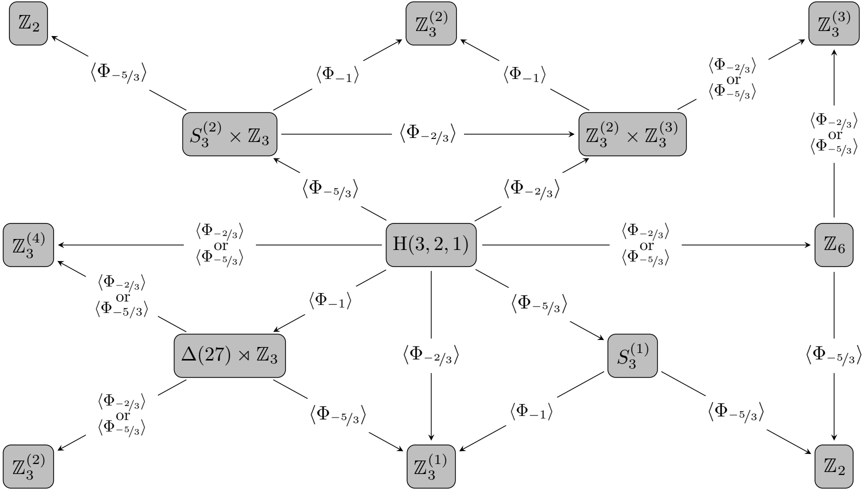


BREAKDOWN OF $H(3,2,1)$ AT $\langle T\rangle = \omega$

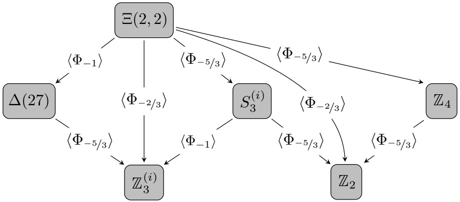
$H(3,2,1)$ subgroup	branchings $\Phi_{-2/3}$	branchings $\Phi_{-5/3}$	subgroup generator(s)	corresponding vevs $\langle\Phi_{-2/3}\rangle$	corresponding vevs $\langle\Phi_{-5/3}\rangle$
$\mathrm{S}_3^{(2)}\times\mathbb{Z}_3$	$\mathbf{1}_1'\oplus\mathbf{2}_\omega$	$\mathbf{1}_1\oplus\mathbf{2}_\omega$	$\mathrm{AC},\mathrm{B}^2\mathrm{A}^2,\mathrm{R}(\mathrm{ST})^4$	–	$(1,\omega^2,1)^\mathrm{T}$
$\mathbb{Z}_3^{(2)}\times\mathbb{Z}_3^{(3)}$	$\mathbf{1}\oplus\mathbf{1}_{\omega^2,1}\oplus\mathbf{1}_{\omega,\omega^2}$	$\mathbf{1}\oplus\mathbf{1}_{\omega^2,1}\oplus\mathbf{1}_{\omega,\omega^2}$	$\mathrm{B}^2\mathrm{A}^2,\mathrm{R}(\mathrm{ST})^4$	$(1,\omega^2,1)^\mathrm{T}$	$(1,\omega^2,1)^\mathrm{T}$ (preserves $\mathrm{S}_3^{(2)}\times\mathbb{Z}_3$)
$\mathbb{Z}_3^{(2)}$	$\mathbf{1}\oplus\mathbf{1}_{\omega^2}\oplus\mathbf{1}_\omega$	$\mathbf{1}\oplus\mathbf{1}_{\omega^2}\oplus\mathbf{1}_\omega$	$\mathrm{B}^2\mathrm{A}^2$	$(1,\omega^2,1)^\mathrm{T}\oplus\langle\Phi_{-1}\rangle$	$(1,\omega^2,1)^\mathrm{T}\oplus\langle\Phi_{-1}\rangle$
$\mathbb{Z}_3^{(3)}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{\omega^2}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{\omega^2}$	$\mathrm{R}(\mathrm{ST})^4$	$(-\omega^2,1,0)^\mathrm{T}$ $+\alpha(-\omega^2,0,1)^\mathrm{T}$	$(-\omega^2,1,0)^\mathrm{T}$ $+\alpha(-\omega^2,0,1)^\mathrm{T}$
$\mathrm{S}_3^{(1)}$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	C,A	–	$(1,1,1)^\mathrm{T}$
$\mathbb{Z}_3^{(1)}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	A	$(1,1,1)^\mathrm{T}$	$(1,1,1)^\mathrm{T}\oplus\langle\Phi_{-1}\rangle$
$\mathbb{Z}_6$	$\mathbf{1}\oplus\mathbf{1}_{-1}\oplus\mathbf{1}_{-\omega}$	$\mathbf{1}\oplus\mathbf{1}_{-1}\oplus\mathbf{1}_\omega$	$\mathrm{CR}^2(\mathrm{ST})^8$	$(0,1,-1)^\mathrm{T}$	$(-2\omega^2,1,1)^\mathrm{T}$
$\mathbb{Z}_3^{(3)}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{\omega^2}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{\omega^2}$	$\mathrm{R}(\mathrm{ST})^4$	$(-\omega^2,1,0)^\mathrm{T}$ $+\alpha(-\omega^2,0,1)^\mathrm{T}$	$(-\omega^2,1,0)^\mathrm{T}$ $+\alpha(-\omega^2,0,1)^\mathrm{T}$
$\mathbb{Z}_3^{(4)}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	$\mathrm{BR}(\mathrm{ST})^2$	$(b^*,\omega,a)^\mathrm{T}$	$(1,a,b)^\mathrm{T}$
$\mathbb{Z}_2$	$\mathbf{1}\oplus\mathbf{1}_{-1}\oplus\mathbf{1}_{-1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{-1}$	C	$(0,1,-1)^\mathrm{T}$ (preserves $\mathbb{Z}_6$)	$(1,0,0)^\mathrm{T}$ $+\alpha(0,1,1)^\mathrm{T}$

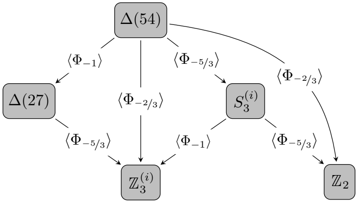
Where $a := -1+\eta-\eta^4+\eta^5-\eta^8$ and $b := -\eta+\eta^4-\eta^8$ with $\eta := \mathrm{e}^{2\pi\mathrm{i}/18}$ and $\omega = \mathrm{e}^{2\pi\mathrm{i}/3}$.

BREAKDOWN OF $H(3,2,1)$ AT $\langle T\rangle = \omega$ AND $\langle T\rangle = \mathrm{i}\infty$



BREAKDOWN OF $\Xi(2,2)$ AND $\Delta(54)$





$\Xi(2,2)$ subgroup	branchings $\Phi_{-2/3}$	$\Phi_{-5/3}$	subgroup generator(s)	corresponding vevs $\langle\Phi_{-2/3}\rangle$	$\langle\Phi_{-5/3}\rangle$
$S_3^{(1)}$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	A, C	–	$(1,1,1)^T$
$\mathbb{Z}_3^{(1)}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	A	$(1,1,1)^T$	$(1,1,1)^T\oplus\langle\Phi_{-1}\rangle$
$S_3^{(2)}$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	ABA, C	–	$(\omega,1,1)^T$
$\mathbb{Z}_3^{(2)}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$	ABA	$(\omega,1,1)^T$	$(\omega,1,1)^T\oplus\langle\Phi_{-1}\rangle$
$\mathbb{Z}_4$	$\mathbf{1}_{-1}\oplus\mathbf{1}_i\oplus\mathbf{1}_{-i}$	$\mathbf{1}\oplus\mathbf{1}_{-1}\oplus\mathbf{1}_i$	S^3	–	$(1+\sqrt{3},1,1)^T$
$\mathbb{Z}_2$	$\mathbf{1}\oplus\mathbf{1}_{-1}\oplus\mathbf{1}_{-1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{-1}$	C	$(0,1,-1)^T$	$(1,0,0)^T$ $+\alpha(0,1,1)^T$

$\Delta(54)$ subgroup	Φ_{-1}	branchings $\Phi_{-2/3}$	$\Phi_{-5/3}$	subgroup generator(s)	corresponding vevs $\langle\Phi_{-2/3}\rangle$	$\langle\Phi_{-5/3}\rangle$
$\Delta(27)$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	A, B	$(0,0,0)^T\oplus\langle\Phi_{-1}\rangle$	$(0,0,0)^T\oplus\langle\Phi_{-1}\rangle$
$S_3^{(1)}$	$\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	A, C	–	$(1,1,1)^T$
$\mathbb{Z}_3^{(1)}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$		A	$(1,1,1)^T$	$(1,1,1)^T\oplus\langle\Phi_{-1}\rangle$
$S_3^{(2)}$	$\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	B, C	–	$(1,0,0)^T$
$\mathbb{Z}_3^{(2)}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$		B	$(1,0,0)^T$	$(1,0,0)^T\oplus\langle\Phi_{-1}\rangle$
$S_3^{(3)}$	$\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	ABA, C	–	$(\omega,1,1)^T$
$\mathbb{Z}_3^{(3)}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$		ABA	$(\omega,1,1)^T$	$(\omega,1,1)^T\oplus\langle\Phi_{-1}\rangle$
$S_3^{(4)}$	$\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{2}$	$\mathbf{1}\oplus\mathbf{2}$	AB^2A, C	–	$(\omega^2,1,1)^T$
$\mathbb{Z}_3^{(4)}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}_\omega\oplus\mathbf{1}_{\omega^2}$		AB^2A	$(\omega^2,1,1)^T$	$(\omega^2,1,1)^T\oplus\langle\Phi_{-1}\rangle$
$\mathbb{Z}_2$	$\mathbf{1}_{-1}$	$\mathbf{1}\oplus\mathbf{1}_{-1}\oplus\mathbf{1}_{-1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}_{-1}$	C	$(0,1,-1)^T$	$(1,0,0)^T$ $+\alpha(0,1,1)^T$

$\mathcal{CP}$

To preserve the $\mathcal{CP}$-like transformation a flavon vev has to fulfill:

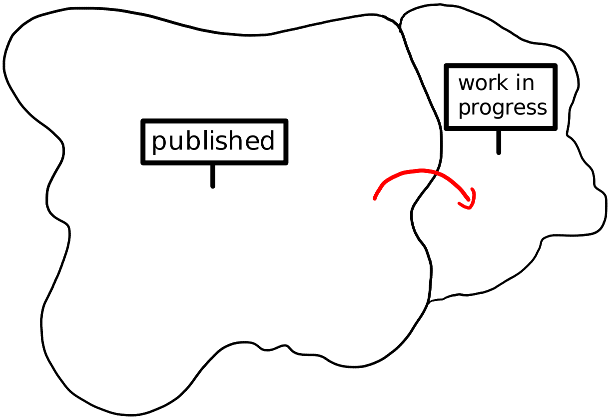
$$\rho(g)\, U\, \langle\bar{\Phi}\rangle \overset{!}{=} \langle\Phi\rangle\,, \qquad g \in \Omega(2)$$

where

$$\begin{aligned} \langle T\rangle &= \mathrm{i} &&: U = \mathrm{id}\,, \\ \langle T\rangle &= \omega &&: U = \rho(T)^*\,, \\ \langle T\rangle &= 1 &&: U = \rho(T^2)\,. \end{aligned}$$

- Subtle question of rephasing $\langle\Phi\rangle$ with a global phase. $\to$ model dependent
- Modulo the global choice of phase, all flavon vevs listed in the tables above have $\mathcal{CP}$-like stabilizers.
- Nonetheless, $\mathcal{CP}$ will be broken once the flavon and/or modulus vevs are deflected away from their symmetry enhances points.

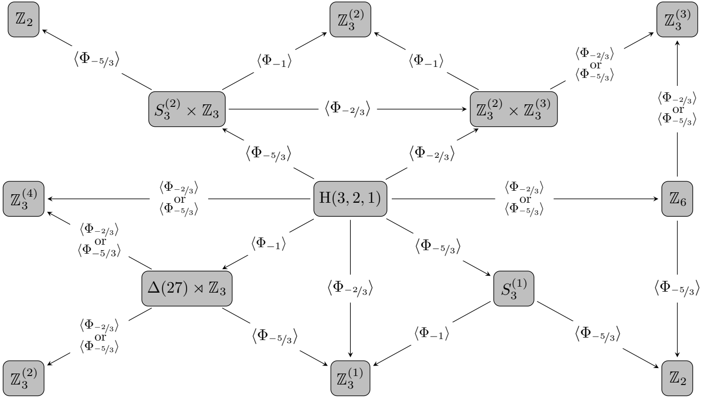
Disclaimer



Example Model

Model of type A: $$\mathcal{W} \;=\; \hat{Y}^{(1)}(T)\left\{\phi^0\left[\phi^0_\mathrm{u} H_\mathrm{u}\bar{u}q\varphi_\mathrm{u} + \phi^0_\mathrm{d} H_\mathrm{d}\bar{d}q\varphi_\mathrm{e} + \phi^0_\mathrm{e} H_\mathrm{d}\bar{e}\ell\varphi_\mathrm{e} + H_\mathrm{u}\bar{\nu}\ell\varphi_\nu\right] + \phi^0_\mathrm{M}\bar{\nu}\bar{\nu}\varphi_\mathrm{e}\right\}\,.$$

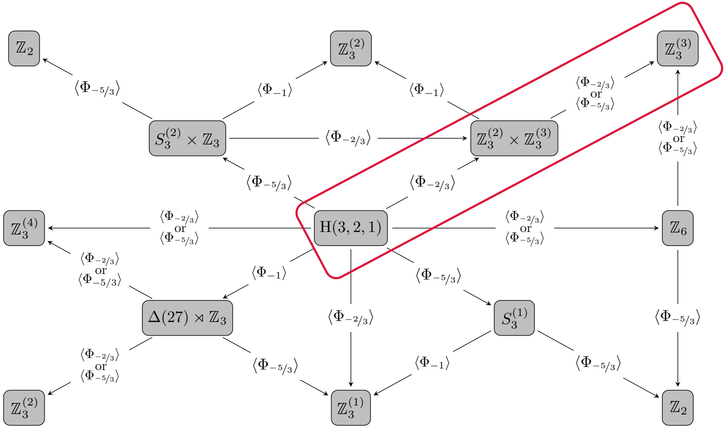
Breaking pattern at $\langle T\rangle = \mathrm{i}\infty$:



EXAMPLE MODEL

Model of type A: $$\mathcal{W} \;=\; \hat{Y}^{(1)}(T)\left\{\phi^0\left[\phi^0_{\mathrm{u}} H_{\mathrm{u}}\bar{u}q\varphi_{\mathrm{u}} + \phi^0_{\mathrm{d}} H_{\mathrm{d}}\bar{d}q\varphi_{\mathrm{e}} + \phi^0_{\mathrm{e}} H_{\mathrm{d}}\bar{e}\ell\varphi_{\mathrm{e}} + H_{\mathrm{u}}\bar{\nu}\ell\varphi_{\nu}\right] + \phi^0_{\mathrm{M}}\bar{\nu}\bar{\nu}\varphi_{\mathrm{e}}\right\}\;.$$

Breaking pattern at $\langle T\rangle = \mathrm{i}\infty$:



Hierarchies from Residual Symmetries

$$
\begin{tikzcd}
 & & & & \emptyset \\
\Omega(2) \arrow[r, "\langle T \rangle = \mathrm{i}\infty"] & H(3,2,1) \arrow[r, "\langle \varphi_{\mathrm{e}} \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}"] & \mathbb{Z}_3^{(2)} \times \mathbb{Z}_3^{(3)} \arrow[r] & \mathbb{Z}_3^{(3)} \arrow[ur] \arrow[dr] & \\
 & & & & \emptyset
\end{tikzcd}
$$

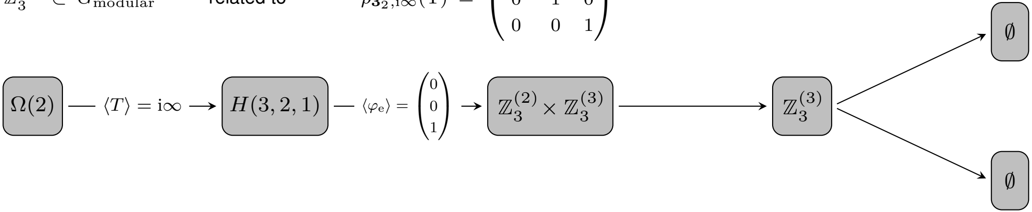
Hierarchies from Residual Symmetries

$\mathbb{Z}_3^{(2)} \subset G_{\mathrm{traditional}}$ related to $\rho_{\mathbf{3}_2,\mathrm{i}\infty}(\mathrm{ABA}^2) = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\mathbb{Z}_3^{(3)} \subset G_{\mathrm{modular}}$ related to $\rho_{\mathbf{3}_2,\mathrm{i}\infty}(\mathrm{T}) = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Symmetry broken if:

$\rho(g)\,\langle\varphi_{\mathrm{e}}\rangle \neq \langle\varphi_{\mathrm{e}}\rangle$



HIERARCHIES FROM RESIDUAL SYMMETRIES

$$\langle\varphi_{\mathrm{e}}\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}\lambda_1\\\lambda_2\\0\end{pmatrix}$$

$$\lambda_1, \lambda_2 \ll 1$$

$\mathbb{Z}_3^{(2)} \subset G_{\mathrm{traditional}}$ related to $\rho_{\mathbf{3}_2,\mathrm{i}\infty}(\mathrm{ABA}^2) = \begin{pmatrix}\omega & 0 & 0\\0 & \omega^2 & 0\\0 & 0 & 1\end{pmatrix}$

$\mathbb{Z}_3^{(3)} \subset G_{\mathrm{modular}}$ related to $\rho_{\mathbf{3}_2,\mathrm{i}\infty}(\mathrm{T}) = \begin{pmatrix}\omega^2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$

Symmetry broken if:

$\rho(g)\,\langle\varphi_{\mathrm{e}}\rangle \neq \langle\varphi_{\mathrm{e}}\rangle$

$$\begin{tikzcd}
& & & & \boxed{\emptyset} \\
\boxed{\Omega(2)} \arrow[r, "\langle T\rangle = \mathrm{i}\infty"] & \boxed{H(3,2,1)} \arrow[r, "\langle\varphi_{\mathrm{e}}\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}"] & \boxed{\mathbb{Z}_3^{(2)} \times \mathbb{Z}_3^{(3)}} \arrow[r] & \boxed{\mathbb{Z}_3^{(3)}} \arrow[ur] \arrow[dr] & \\
& & & & \boxed{\emptyset}
\end{tikzcd}$$

Hierarchies from Residual Symmetries

$$\langle\varphi_{\mathrm{e}}\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ 0 \end{pmatrix}$$

$$\lambda_1, \lambda_2 \ll 1$$

$\mathbb{Z}_3^{(2)} \subset G_{\mathrm{traditional}}$ related to $\rho_{\mathbf{3}_2, \mathrm{i}\infty}(\mathrm{ABA}^2) = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

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 & & & & \emptyset
\end{tikzcd}$$

HIERARCHIES FROM RESIDUAL SYMMETRIES

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$$\boxed{\Omega(2)} \xrightarrow{\langle T\rangle = \mathrm{i}\infty} \boxed{H(3,2,1)} \xrightarrow{\langle\varphi_\mathrm{e}\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}} \boxed{\mathbb{Z}_3^{(2)}\times\mathbb{Z}_3^{(3)}} \xrightarrow{\langle\varphi_\mathrm{e}\rangle = \begin{pmatrix}0\\\lambda_2\\1\end{pmatrix}} \boxed{\mathbb{Z}_3^{(3)}}$$

$$\boxed{\mathbb{Z}_3^{(3)}} \xrightarrow{\langle\varphi_\mathrm{e}\rangle = \begin{pmatrix}\lambda_1\\\lambda_2\\1\end{pmatrix}} \boxed{\emptyset}$$

$$\boxed{\mathbb{Z}_3^{(3)}} \xrightarrow{\epsilon = \mathrm{e}^{2\pi\mathrm{i}\langle T\rangle/3} \neq 0} \boxed{\emptyset}$$

HIERARCHIES FROM RESIDUAL SYMMETRIES

$$\langle\varphi_\mathrm{e}\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}\lambda_1\\\lambda_2\\0\end{pmatrix}$$

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$\mathbb{Z}_3^{(3)} \subset G_\mathrm{modular}$ related to $\rho_{\mathbf{3}_2,\mathrm{i}\infty}(\mathrm{T}) = \begin{pmatrix}\omega^2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

Symmetry broken if:

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$$\boxed{\Omega(2)} \xrightarrow{\langle T\rangle = \mathrm{i}\infty} \boxed{H(3,2,1)} \xrightarrow{\langle\varphi_\mathrm{e}\rangle = \left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)} \boxed{\mathbb{Z}_3^{(2)}\times\mathbb{Z}_3^{(3)}} \xrightarrow{\langle\varphi_\mathrm{e}\rangle = \left(\begin{smallmatrix}0\\\lambda_2\\1\end{smallmatrix}\right)} \boxed{\mathbb{Z}_3^{(3)}}$$

$$\boxed{\mathbb{Z}_3^{(3)}} \xrightarrow{\langle\varphi_\mathrm{e}\rangle = \left(\begin{smallmatrix}\lambda_1\\\lambda_2\\1\end{smallmatrix}\right)} \boxed{\emptyset} \qquad \boxed{\mathbb{Z}_3^{(3)}} \xrightarrow{\epsilon = \mathrm{e}^{2\pi\mathrm{i}\langle T\rangle/3} \neq 0} \boxed{\emptyset}$$

Soft breaking of $\mathbb{Z}_3^{(2)}\times\mathbb{Z}_3^{(3)}$ controlled by: $\lambda_2, \lambda_1, \epsilon$

$$W_\mathrm{e} = c_\mathrm{e}\phi^0\phi_\mathrm{e}^0 H_\mathrm{d}\left(\hat{Y}^{(1)}(T)\,\bar{e}\,\ell\,\varphi_\mathrm{e}\right)_{\mathbf{1}'}$$

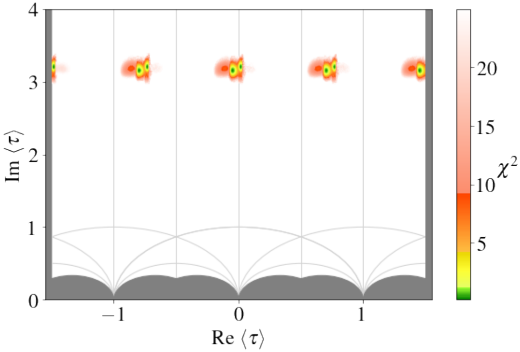
for $\lambda_2 \gg \lambda_1 \gg \epsilon$: $\quad \dfrac{m_\mathrm{e}}{m_\mu} \approx \left|\dfrac{\lambda_1}{\lambda_2}\right|, \quad \dfrac{m_\mu}{m_\tau} \approx |\lambda_2|$

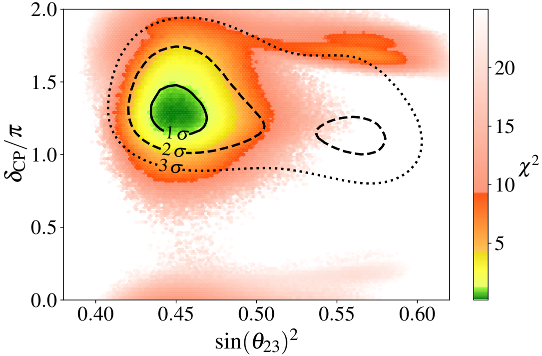
for $\lambda_2 \gg \epsilon \gg \lambda_1$: $\quad \dfrac{m_\mathrm{e}}{m_\mu} \approx 9\left|\dfrac{\epsilon^2}{\lambda_2^2}\right|, \quad \dfrac{m_\mu}{m_\tau} \approx |\lambda_2|$

Results of Fit (preliminary results)

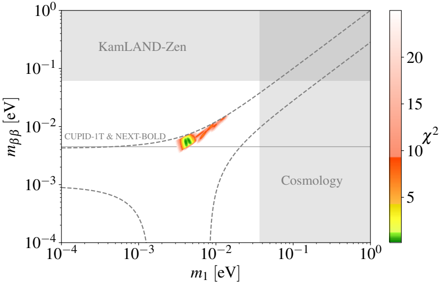
	model			experiment (NuFit v5.1)		
	best fit	1σ range	3σ range	best fit	1σ range	3σ range
m_e/m_μ	0.00478	$0.00461 \to 0.00499$	$0.00422 \to 0.00538$	0.0048	$0.0046 \to 0.0050$	
m_μ/m_τ	0.0568	$0.0523 \to 0.0607$	$0.0434 \to 0.0697$	0.0565	$0.0520 \to 0.610$	
$\sin^2\theta_{12}$	0.302	$0.293 \to 0.315$	$0.274 \to 0.334$	0.304	$0.292 \to 0.316$	$0.269 \to 0.343$
$\sin^2\theta_{13}$	0.02256	$0.02187 \to 0.02304$	$0.02069 \to 0.02423$	0.02246	$0.02184 \to 0.02308$	$0.02060 \to 0.02435$
$\sin^2\theta_{23}$	0.445	$0.436 \to 0.467$	$0.413 \to 0.592$	0.450	$0.434 \to 0.469$	$0.408 \to 0.603$
δ^{l}_{CP}/π	1.27	$1.16 \to 1.46$	$0.82 \to 1.94$	1.28	$1.14 \to 1.48$	$0.80 \to 1.94$
η_1/π	0.0274	$0.0180 \to 0.0342$	$0.00001 \to 0.0414$	-	-	-
η_2/π	0.994	$0.993 \to 0.996$	$0.970 \to 0.999998$	-	-	-
J_{CP}	-0.025	$-0.033 \to -0.016$	$-0.035 \to 0.018$	-0.026	$-0.033 \to -0.014$	$-0.034 \to 0.000$
J_{CP}^{max}	0.0335	$0.0339 \to 0.0341$	$0.0318 \to 0.0352$	0.0336	$0.0329 \to 0.0341$	$0.0317 \to 0.0353$
$\Delta m_{21}^2/10^{-5}$ [eV^2]	7.43	$7.35 \to 7.49$	$7.21 \to 7.65$	7.42	$7.22 \to 7.63$	$6.82 \to 8.04$
$\Delta m_{31}^2/10^{-3}$ [eV^2]	2.508	$2.487 \to 2.533$	$2.437 \to 2.587$	2.510	$2.483 \to 2.537$	$2.430 \to 2.593$
m_1 [eV]	0.0042	$0.0039 \to 0.0049$	$0.0034 \to 0.0130$	< 0.037	-	-
m_2 [eV]	0.0096	$0.0095 \to 0.0099$	$0.0092 \to 0.0156$	-	-	-
m_3 [eV]	0.0503	$0.0501 \to 0.0505$	$0.0496 \to 0.0519$	-	-	-
$\sum_i m_i$ [eV]	0.0640	$0.0636 \to 0.0652$	$0.0628 \to 0.0804$	< 0.110	-	-
$m_{\beta\beta}$ [eV]	0.0055	$0.0046 \to 0.0065$	$0.0040 \to 0.0145$	< 0.061	-	-
m_β [eV]	0.0099	$0.0097 \to 0.0102$	$0.0094 \to 0.0158$	< 1.1	-	-
χ^2	0.15					

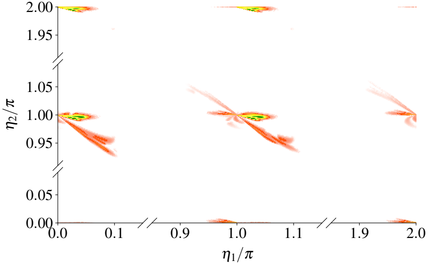
Results of Fit (preliminary results)

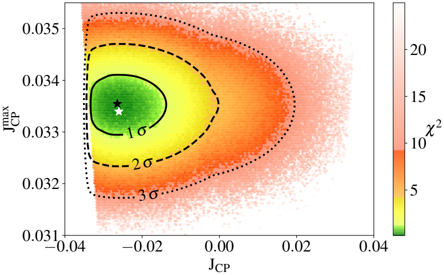


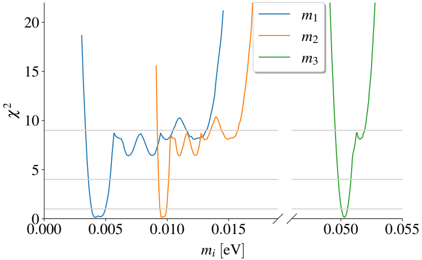


Predictions (preliminary results)









MODULAR SYMMETRIES AND KÄHLER CORRECTIONS

[Chen et al: 1909.06910]:

- Kähler potential does not necessarily needs to be canonical for modular flavor symmetries.
- Non-trivial Kähler metrics can strongly alter observables.

[Nilles et al: 2004.05200]:

- For an $\Omega(2)$ eclectic flavor symmetry, the Kähler potential is canonical at leading order. (due to traditional flavor symmetry)

[Baur et al: 22xx.xxxxx]:

- For $\Omega(2)$, the Kähler potential is not canonical at next to leading order.
- Kähler corrections allow for a fit of the quark sector.

KÄHLER POTENTIAL OF EXAMPLE MODEL

Leading order: —> [Nilles, Ramos-Sáchez, Vaudrevange: 2004.05200]

$$
\begin{aligned}
K \supset & -\log(-\mathrm{i}T + \mathrm{i}\bar{T}) \\
& + \sum_{\Psi} \left[(-\mathrm{i}T + \mathrm{i}\bar{T})^{-2/3} + (-\mathrm{i}T + \mathrm{i}\bar{T})^{1/3} |\hat{Y}^{(1)}(T)|^2\right] |\Psi|^2 \\
& + \sum_{\Phi} \left[(-\mathrm{i}T + \mathrm{i}\bar{T})^{-2/3} + (-\mathrm{i}T + \mathrm{i}\bar{T})^{1/3} |\hat{Y}^{(1)}(T)|^2\right] |\Phi|^2 \,.
\end{aligned}
$$

Next-to-leading order: —> [AB, Nilles, Ramos-Sánchez, Trautner, Vaudrevange: 22xx.xxxxx]

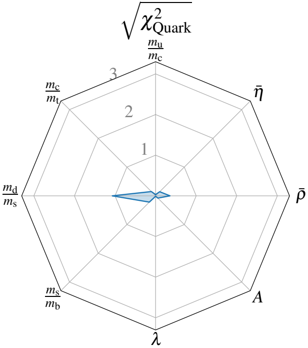
$$
K \supset \sum_{\Psi,\Phi} \left[(-\mathrm{i}T + \mathrm{i}\bar{T})^{-4/3} \sum_a |\Psi\Phi|^2_{\mathbf{1},a} + (-\mathrm{i}T + \mathrm{i}\bar{T})^{-1/3} \sum_a |\hat{Y}^{(1)}(T)\Psi\Phi|^2_{\mathbf{1},a}\right] \,.
$$

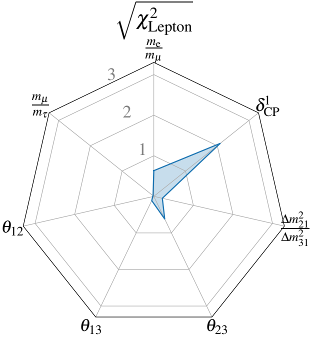
Ψ: Matter field, Φ: Flavon.

Results of Fitting Both Sectors Simultaneously (preliminary results)

observables	$\frac{m_u}{m_c}$	$\frac{m_c}{m_t}$	$\frac{m_d}{m_s}$	$\frac{m_s}{m_b}$	λ	A	$\bar{\rho}$	$\bar{\eta}$
model	0.0019	0.0028	0.057	0.018	0.2265	0.97	0.15	0.36
experiment	0.0019	0.0028	0.051	0.18	0.2265	0.79	0.14	0.36

observables	$\frac{m_e}{m_\mu}$	$\frac{m_\mu}{m_\tau}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	Δm^2_{21}	Δm^2_{31}	m_1	m_2	m_3	η_1	η_2	J_{CP}	J^{max}_{CP}	$\sum m_i$	m_β	$m_{\beta\beta}$
model	0.0019	0.056	0.303	0.0223	0.44	1.77	0.000075	0.0025	0.004	0.01	0.05	0.18	1.11	-0.022	0.0334	0.064	0.010	0.0067
experiment	0.0048	0.067	0.304	0.0225	0.45	1.28	0.000074	0.0025	< 0.037	-	-	-	-	-0.026	0.0336	< 0.110	< 1.1	< 0.061





SUMMARY

- Eclectic flavor symmetry is broken by
  1. Expectation values of moduli
  2. Expectation values of flavon fields
- Residual symmetries are common
- Approximate residual symmetries can generate hierarchies
- Fits with eclectic flavor symmetries are possible
- Kähler Corrections can play an important role in TD derived models

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Thank you!