

Modular Invariance (in flavor physics): a BU perspective



Bethe Center for
Theoretical Physics

Bethe Forum

Modular Flavor Symmetries

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motivations for modular invariance in flavour sector



a menu for the next talks



some thoughts aloud

The flavour puzzle

3 generations

QUARKS			GAUGE BOSONS	
mass → ≈3 MeV/c ²	= 1.275 GeV/c ²	= 173.07 GeV/c ²	0 0 1	= 126 GeV/c ²
charge → 2/3	2/3 1/2	2/3 1/2	g	Higgs boson
spin → 1/2				
up	charm	top	gluon	
≈4.8 MeV/c ²	= 95 MeV/c ²	= 4.18 GeV/c ²	0 0 1	
-1/3 1/2	-1/3 1/2	-1/3 1/2	γ	
down	strange	bottom	photon	
0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	0 1	
-1 1/2	-1 1/2	-1 1/2	Z	
electron	muon	tau	Z boson	
<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	±1 1	
0 1/2	0 1/2	0 1/2	W	
ν _e electron neutrino	ν _μ muon neutrino	ν _τ tau neutrino	W boson	

Image credit: Wikipedia

- gauge and Higgs boson masses
- all gauge interactions of 3 families of quark and lepton masses (15 × 3 Weyl spinors)



- 3 gauge couplings
- 2 masses (G_F, m_H)

fermion masses and mixing angles require (up to) 22 additional parameters [fermion bilinears]

- 6+6 masses
- 3+3 mixing angles
- 1+3 phases

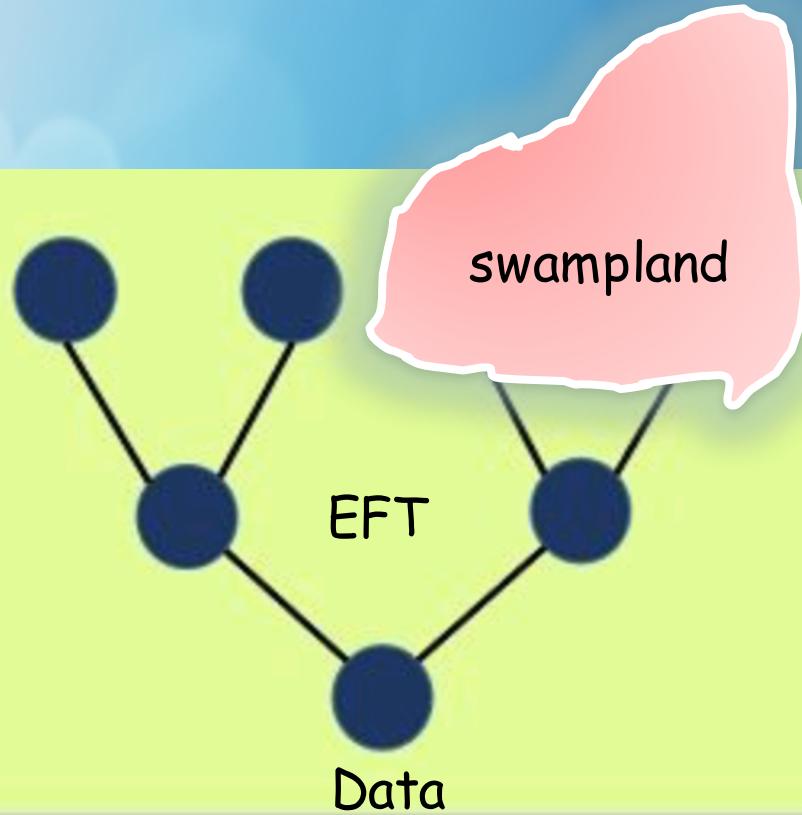
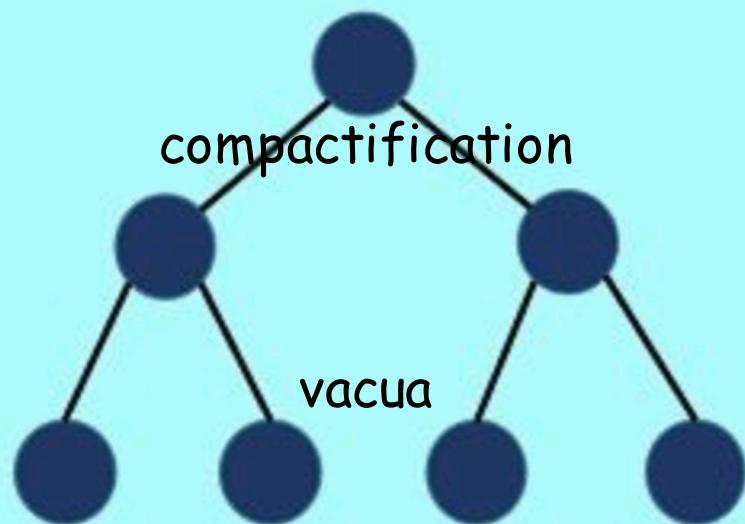
$$\mathcal{L}_Y = - \Psi^c \mathcal{Y} \Phi \Psi \quad \mathcal{L}_\nu = - \frac{1}{\Lambda} (\Phi \Psi) \mathcal{W} (\Phi \Psi)$$

$$m_{ij} = y_{ij} v$$

$$m_{\nu ij} = w_{ij} v^2 / \Lambda$$

loosely constrained by gauge symmetry

String Theory



Top-down Approach Vs Bottom-up Approach

Flavour Symmetries

“traditional” Flavour Symmetries

$$g \in G_{fl}$$

$$\Psi \xrightarrow{g} \varrho(g) \Psi$$



$$\varrho(g)^{c*} \gamma \varrho(g)^+ = \gamma$$

$$\Psi = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

can be combined with CP

$$\Psi \xrightarrow{CP} X_{CP} \Psi^*$$

example: Isospin SU(2) in strong interactions $m_p = m_n$

flavour symmetries of this type are necessarily broken

e.g.
largest
flavour symmetry
of the quark
sector

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$
$$u^c \quad c^c \quad t^c$$
$$d^c \quad s^c \quad b^c$$

$$U(3)^3$$

[up to anomalies]

broken down
to $U(1)_B \times U(1)_Y$

no constraint on
quark masses/mixing angles

Symmetry Breaking

τ_α

symmetry breaking sector:
set of dimensionless, gauge invariant
scalar fields, charged under G_{fl}

[τ_α stands for $\langle \tau_\alpha \rangle / \Lambda_F$ where the scale Λ_F has been set to 1]

lowest order
Lagrangian
parameters

higher
dimensional
operators

<- many free parameters

$$m_{ij}(\tau) = m_{ij}^0 + m_{ij}^{1\alpha} \tau_\alpha + m_{ij}^{1\bar{\alpha}} \bar{\tau}_{\bar{\alpha}} + m_{ij}^{2\alpha\beta} \tau_\alpha \tau_\beta + \dots$$

vacuum alignment
in SB sector

SUSY breaking effects
RGE corrections
($\Lambda_{\text{UV}}, m_{\text{SUSY}}, \tan\beta$)

huge number of models: G_{fl} continuous/discrete, global/local,.....
no baseline model in bottom-up approach

reviews:

Ishimori, Kobayashi, Ohki, Shimizu, Okada, Tanimoto , 1003.3552;
King, Luhn, 1301.1340;
King, Merle, Morisi, Shimizu, Tanimoto, 1402.4271;
King, 1701.04413
Hagedorn, 1705.00684;
F.F., Romanino 1912.06028

:

:

usual path:

choose G_f

assign f to G_f multiplets

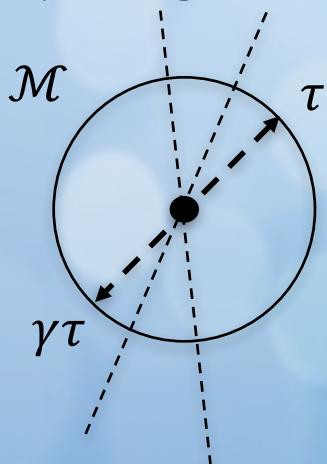
look for some SB sector
and adjust $\langle \tau_\alpha \rangle$

the crucial sector
relegated to the last step

can we reverse the logic?

$\tau \in \mathcal{M}$ = moduli space
parametrizes possible vacua

$\tau \in \mathcal{M}$ = {lines of the plane
passing through the origin}



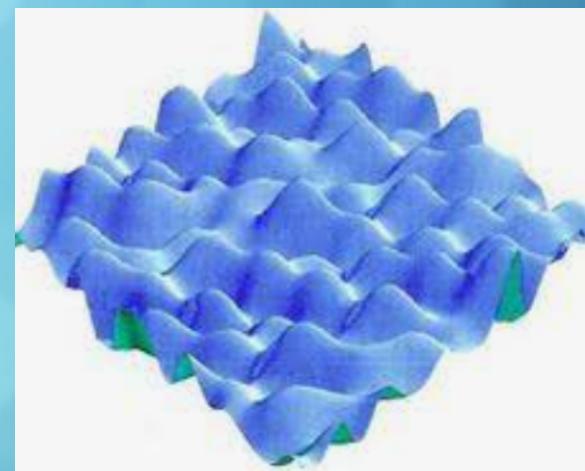
$$\mathcal{L}_{IR}(Y(\gamma\tau), \varphi') = \mathcal{L}_{IR}(Y(\tau), \varphi)$$

is a gauge symmetry

$$\begin{cases} \tau \rightarrow \gamma\tau \\ \varphi' = \xi(\gamma) \varphi \end{cases}$$

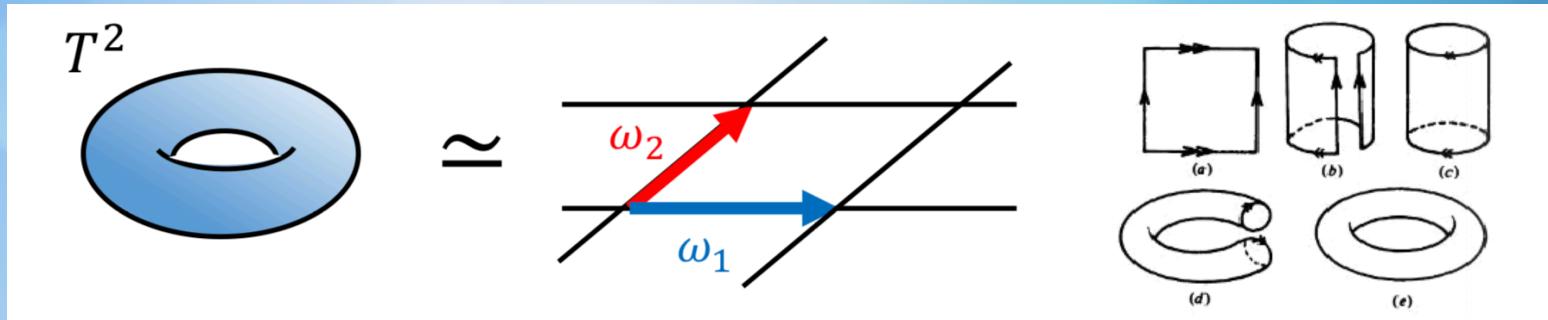
flavour symmetry G_f
action on matter fields

derived from \mathcal{M} now



a less trivial example

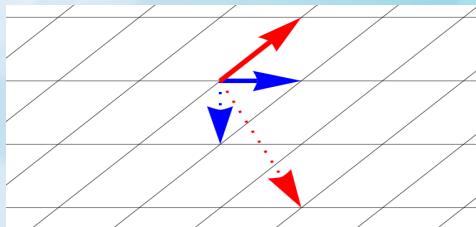
$\tau \in \mathcal{M} = \{\text{classes of conformally equivalent metrics on the torus}\}$



tori
parametrized by

$$\mathcal{M} = \left\{ \tau = \frac{\omega_2}{\omega_1} \mid \operatorname{Im}(\tau) > 0 \right\}$$

lattice left
invariant by
modular
transformations:



$$\tau \rightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d} \in SL(2, \mathbb{Z})$$

■

$$\mathcal{M} = \left\{ \tau = \frac{\omega_2}{\omega_1} \mid \operatorname{Im}(\tau) > 0 \right\}$$

■

$$G_f = SL(2, \mathbb{Z})$$

■

$$\varphi' = (c\tau + d)^{-k_\varphi} \rho_\varphi(\gamma) \varphi$$

unitary representation
of the finite modular group

$SL(2, \mathbb{Z}_N)$

$N = 1, 2, 3, \dots$

$\mathcal{N}=1$ SUSY modular invariant theories

Yukawa interactions in $\mathcal{N}=1$ global SUSY [extension to $\mathcal{N}=1$ SUGRA straightforward]

$$S = \int d^4x d^2\theta w(\tau, \varphi) + h.c + \text{kinetic terms}$$

invariance satisfied by "minimal" Kahler potential

$$w(\tau, \varphi) = \sum_n Y_{I_1 \dots I_n}(\tau) \varphi^{(I_1)} \dots \varphi^{(I_n)}$$

field-dependent Yukawa couplings

invariance of $w(\Phi)$ guaranteed by an holomorphic $Y_{I_1 \dots I_n}(\tau)$ such that

$$Y_{I_1 \dots I_n}(\gamma\tau) = (c\tau + d)^{k_Y(n)} \rho(\gamma) Y_{I_1 \dots I_n}(\tau)$$

1. $k_Y(n) - k_{I_1} - \dots - k_{I_n} = 0$

2. $\rho \times \rho^{I_1} \times \dots \times \rho^{I_n} \supset 1$

modular forms
of level N and weight k_Y

form a linear space $\mathcal{M}_k(\Gamma_N)$
of finite dimension

Example

$$\Gamma_3 \approx A_4$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow (c\tau + d)^{-1} \rho(\gamma) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$k_\nu = +1$

~ 3 of Γ_3

$$w(\tau, \nu) = m_0 \nu Y(\tau) \nu + h.c.$$

modular form of level 3
 $k = +2$ and $\rho \subset 3 + 1 + 1' + 1''$

$$\begin{aligned} d(\mathcal{M}_2(\Gamma_3)) &= 3 \\ \rho &= 3 \end{aligned}$$

$$m(\tau) = m_0 \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

[F.F. 1706.08749]

Yukawas completely determined in terms of τ up to an overall constant

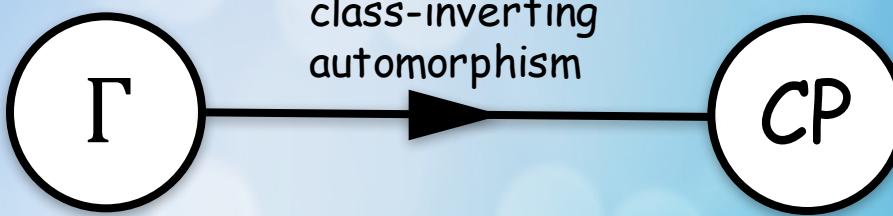
no corrections from higher order operators in the exact SUSY limit

one puzzle, many pieces





Gui-Jun Ding
Serguey Petcov
Arsenii Titov
Andreas Trautner



a unique CP law consistent
with the modular group
 $[\text{Im}(\tau) > 0]$

$$\tau \rightarrow -\tau^*$$

[Novichkov, Penedo, Petcov and Titov 1905.11970
Baur, Nilles, Trautner and Vaudrevange, 1901.03251]

[up to modular transformations]



outer automorphism of $SL(2, \mathbb{Z})$

$$S \rightarrow S \quad T \rightarrow T^{-1}$$

CP on matter multiplets

$$\varphi^{(I)} \rightarrow X_{CP} \varphi^{(I)*}$$

$X_{CP} = \mathbb{I}$ not restrictive if
 S and T symmetric
[canonical CP basis]

[in such a basis]
 CP invariance



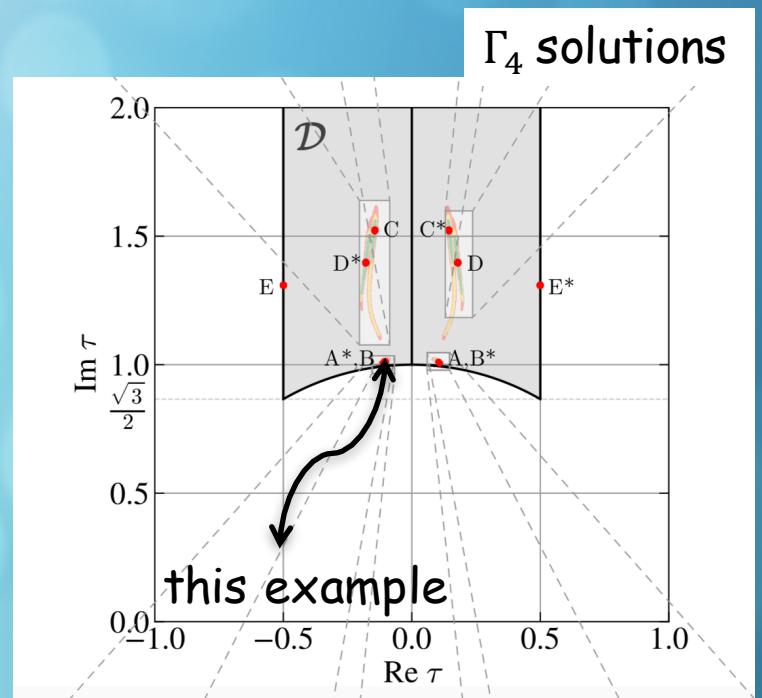
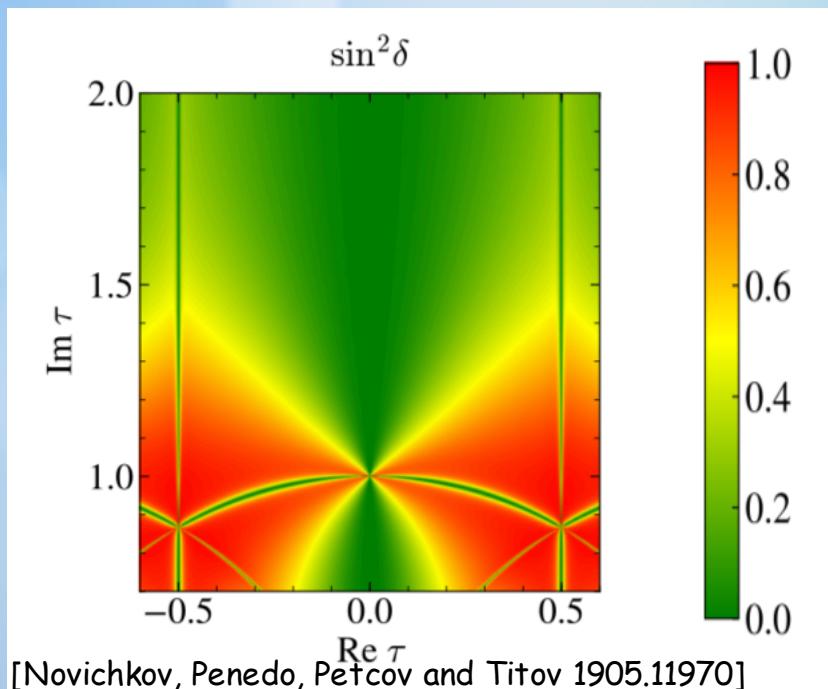
$$g_i^* = g_i$$

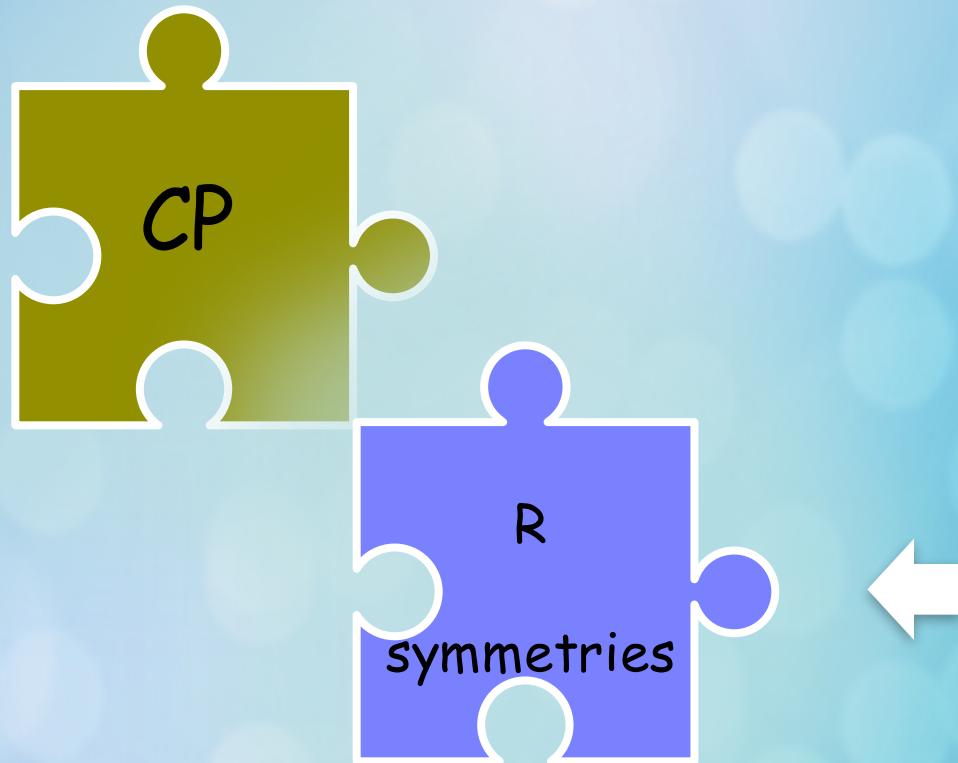
on Lagrangian parameters g_i

CP conserved $\leftrightarrow \tau$ imaginary or
at the border of the fundamental
domain

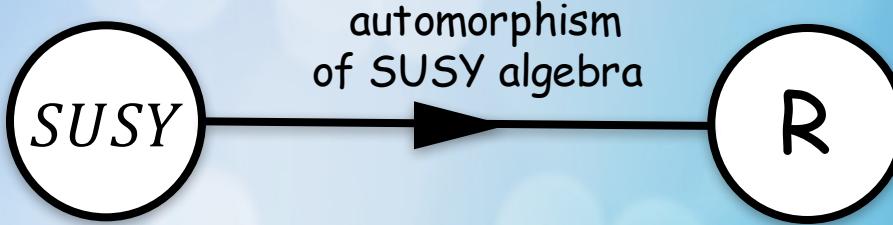
otherwise CP spontaneously broken
by $\langle \tau \rangle$

[P. P. Novichkov, J. T. Penedo,
S. T. Petcov and A. V. Titov,
1811.04933 and 1812.02158]





Andreas Trautner
Saul Ramos Sanchez



$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu \quad Q_\alpha \rightarrow e^{i\sigma} Q_\alpha \quad \bar{Q}_{\dot{\beta}} \rightarrow e^{-i\sigma} \bar{Q}_{\dot{\beta}} \quad U(1)_R \text{ in N=1}$$

both in rigid and in local N=1 SUSY

$$w \rightarrow e^{i2\sigma} w$$

Example $R = S^2 \in SL(2, \mathbb{Z})$

$$w(R\tau) = \pm w(\tau)$$

$R\tau = \tau$ [Nilles, Ramos-Sanchez and Vaudrevange, 2004.0520]0

both signs allowed
minus sign can be compensated by $d^2\theta$

can be generalized?

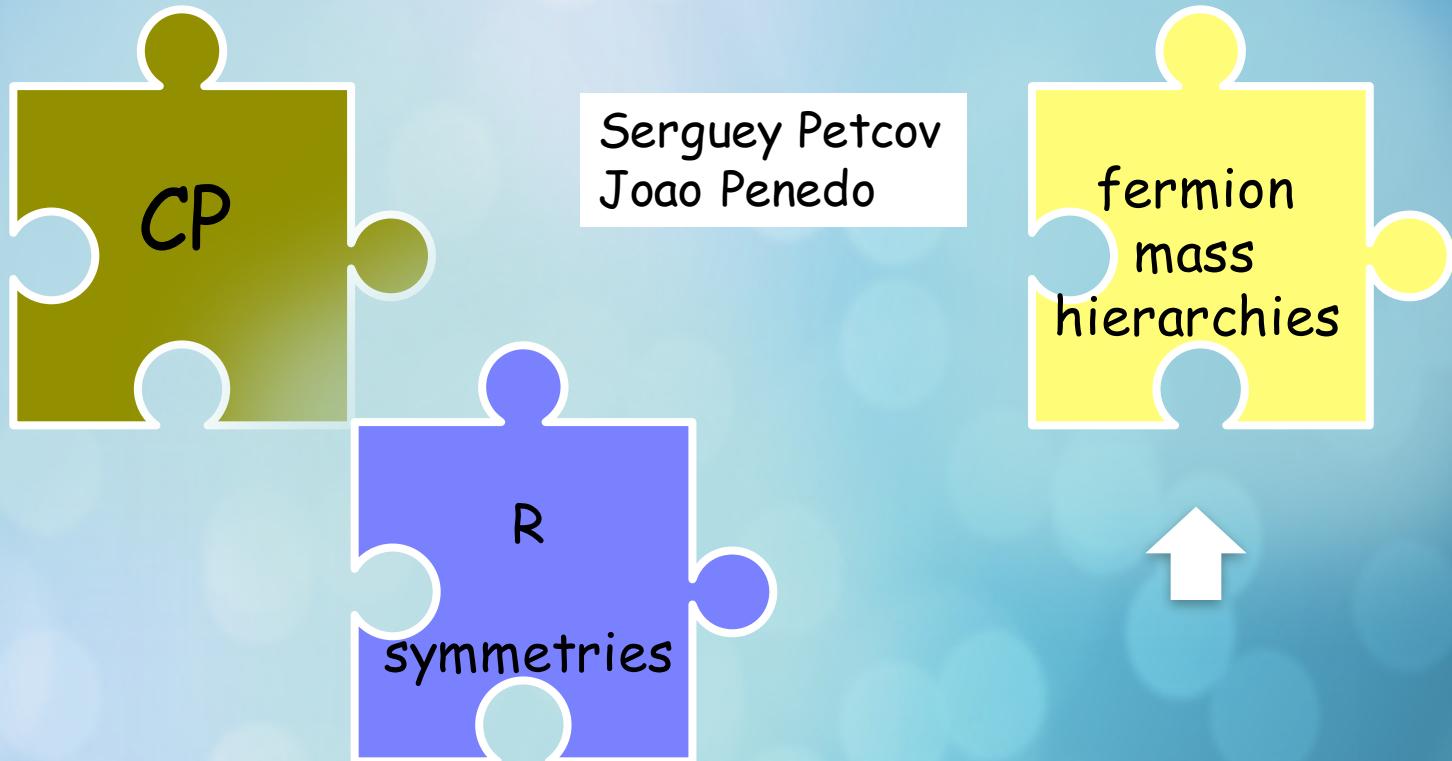
$$w(\gamma\tau, \theta_\gamma) = e^{i2\sigma_\gamma} w(\tau, \theta) \quad \text{compensated by}$$

$$d^2\theta_\gamma = e^{-i2\sigma_\gamma} d^2 \theta$$

moreover phase of $w(\tau, \theta)$ irrelevant in local SUSY

$$\mathcal{G} = K(\tau, \bar{\tau}) + \log |w(\tau)|^2$$

still unexploited
in BU model building



close to fixed points the theory is special

fixed points

$$\bar{\gamma} \tau_{FP} = \tau_{FP}$$

τ_{FP}	$\bar{\gamma}$	
$i\infty$	T	R
i	S	R
$\omega = e^{i\frac{2\pi}{3}}$	ST	R

$\bar{\gamma}$ transformation can be linearized

$$u = u(\tau) \quad u(\tau_{FP}) = 0$$

$$\Phi^{(I)} = K^{(I)}(u) \varphi^{(I)}$$



F., Gherardi, Romanino and Titov, arXiv:2101.08718

$$u(\bar{\gamma}\tau) = e^{i\frac{2\pi}{p}} u(\tau)$$

$$\Phi^{(I)} \xrightarrow{\bar{\gamma}} \Omega^{(I)}(\bar{\gamma}) \Phi^{(I)}$$

in terms of the new variables

$$\Phi^{cT} m(u) \Phi$$

close to the FP

we can expand

$m(u)$ in powers of u

$$m_{ij}(u) = m_{ij}^{(0)} + m_{ij}^{(1)} u + m_{ij}^{(2)} u^2 + \dots$$

$$m^{(n)}(0) = e^{-i\frac{2\pi n}{p}} \Omega_c^*(\bar{\gamma}) m^{(n)}(0) \Omega^+(\bar{\gamma})$$

exactly as in linearly realized Z^N symmetries, spontaneously broken by u

rank of $m_{ij}^{(0)}$ can be <3



hierarchical mass spectrum

Example: $N = 5$ and $\tau_{FP} = i\infty$

Novichkov, Penedo and Petcov, 2102.07488

$$\begin{aligned}\varphi &\sim (3, k) \\ \varphi^c &\sim (3', k^c)\end{aligned}$$

decomposition
under Z^5

$$\begin{aligned}\varphi &\sim 1_0 \oplus 1_1 \oplus 1_4 \\ \varphi^c &\sim 1_0 \oplus 1_2 \oplus 1_3\end{aligned}$$

$$m(u) \propto \begin{pmatrix} 1 & u^4 & u \\ u^3 & u^2 & u^4 \\ u^2 & u & u^3 \end{pmatrix} \quad u = e^{-i2\pi\tau/5}$$

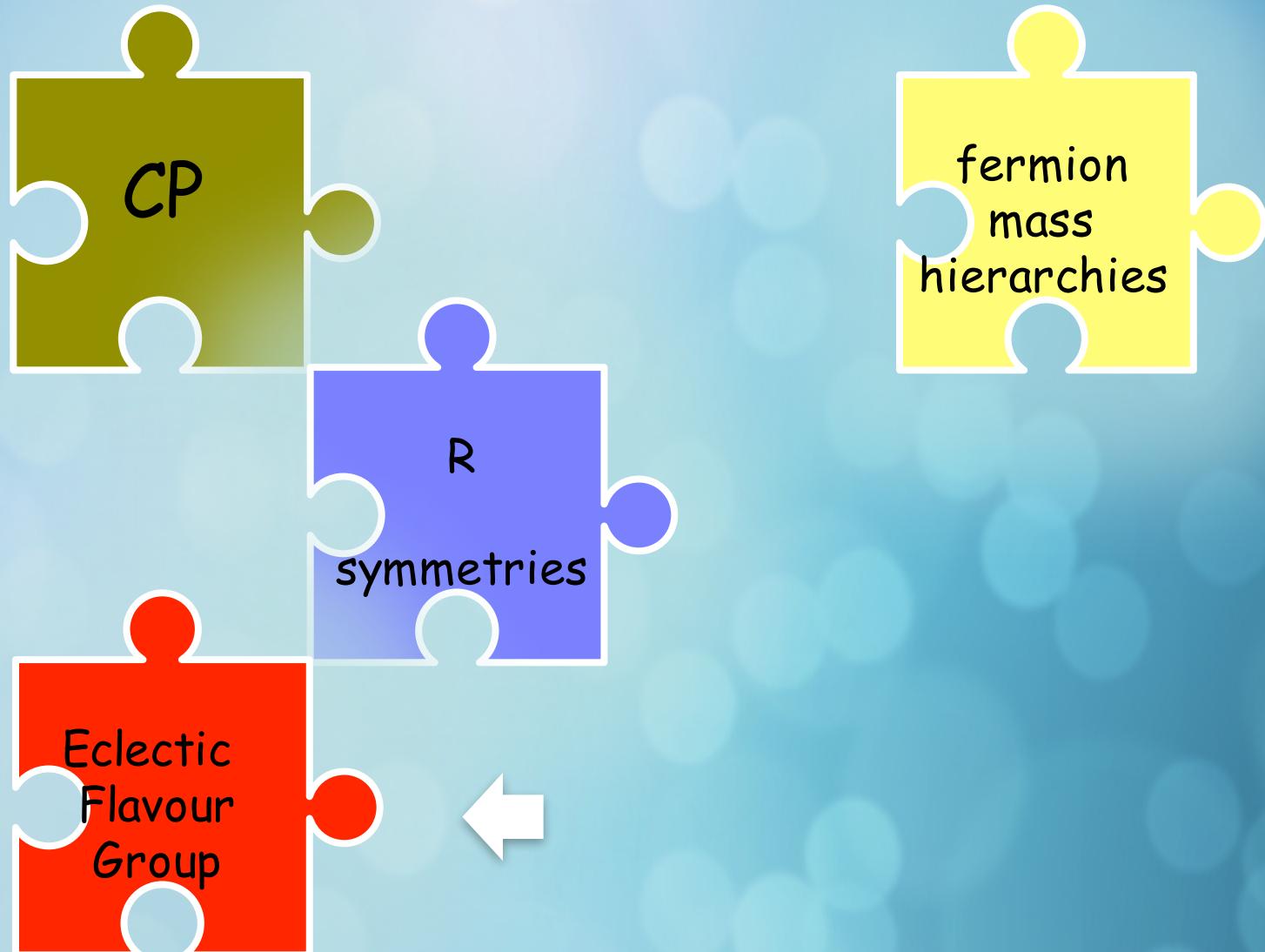
$$(m_1, m_2, m_3) \approx (1, u, u^4)$$

traditional symmetry

$$\tau_{FP} \xrightarrow{\bar{\gamma}} \bar{\gamma} \tau_{FP} = \tau_{FP}$$

modular symmetry

$$\tau_{FP} \xrightarrow{\gamma} \gamma \tau_{FP} \equiv \frac{a\tau+b}{c\tau+d}$$



Saul Ramos Sanchez

which is the most general framework including Γ_N ?

[Nilles, Ramos-Sanchez and Vaudrevange 2001,01736]

look for G_{ecl} including
new transformations
leaving τ invariant

$$\tau \xrightarrow{\gamma} \gamma\tau \equiv \frac{a\tau+b}{c\tau+d}$$

$$\tau \xrightarrow{g} \tau$$

$$\varphi \xrightarrow{\gamma} (c\tau + d)^k \rho(\gamma) \varphi$$

$$\varphi \xrightarrow{g} \varrho(g) \varphi$$

$$g \in G_{fl}$$

consistency condition

$$\rho(\gamma) \varrho(g) \rho(\gamma^{-1}) = \varrho(u_\gamma(g))$$

G_{fl} is a normal
subgroup in G_{ecl}

interesting case: modular group as outer automorphism of
traditional flavour group

not all G_{fl} suitable to such
an extension

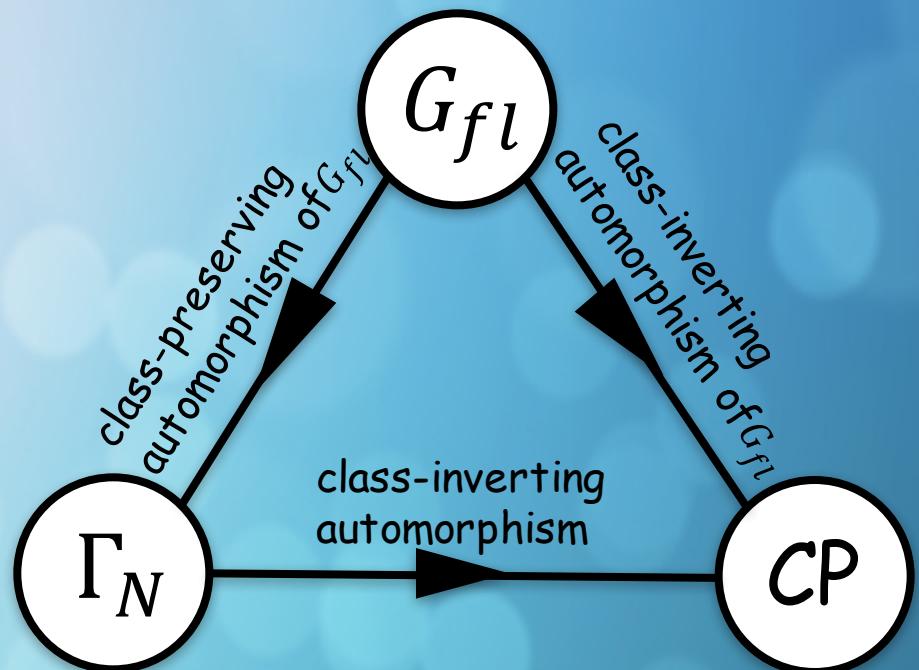
model building just born

Chen, Knapp-Perez, Ramos-Hamud, Ramos-Sanchez,
Ratz and Shukla, 2108.02240

can be combined with CP
(and R-symmetries)

unification of flavour, CP and
modular symmetries

at fixed points, $\bar{\gamma} \tau_{FP} = \tau_{FP}$ and
 $\bar{\gamma}$ becomes "traditional":
enhanced traditional G_{fl}



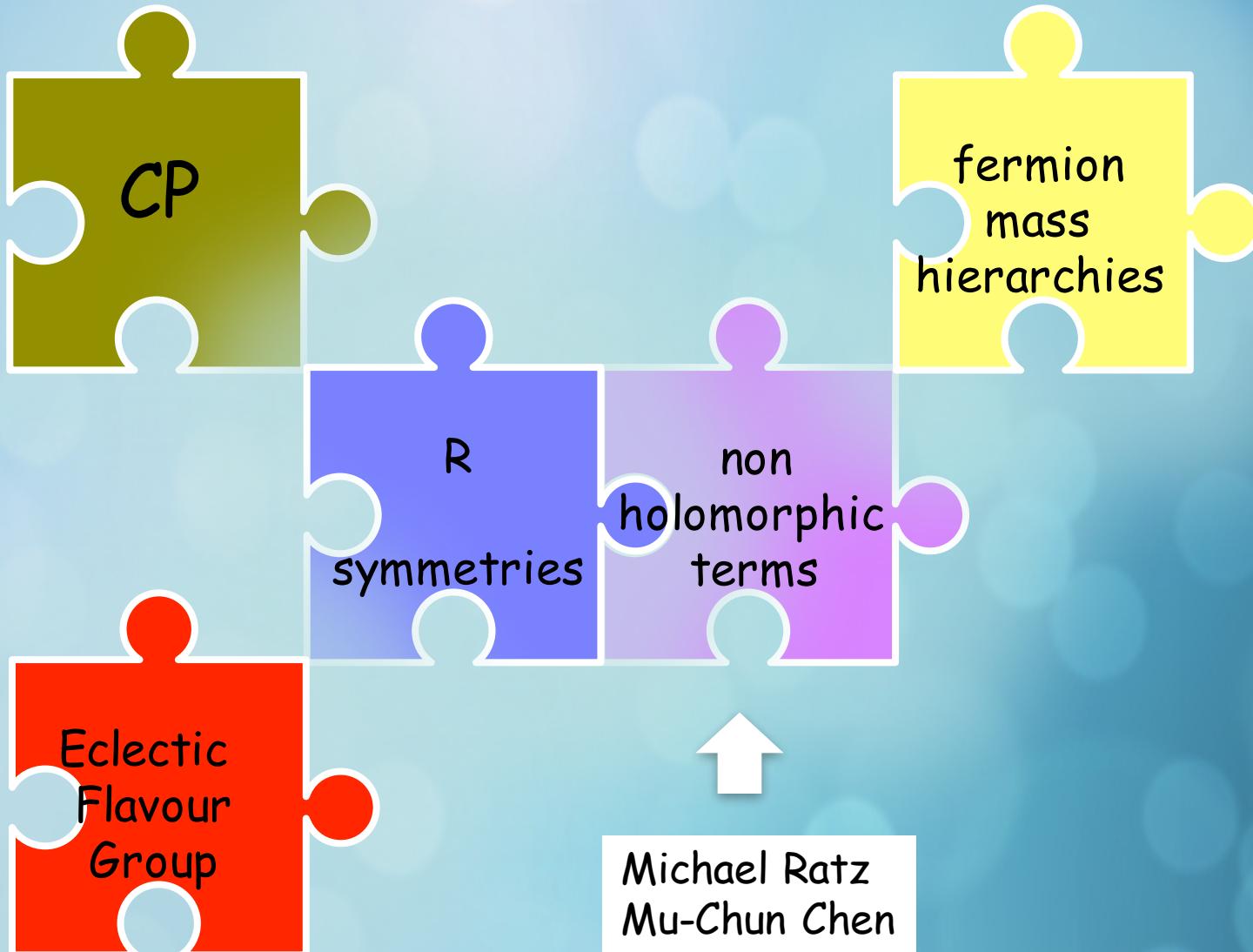
G_{ecl} has a rich representation content

sector	fields		osc.		modular T' subgroup			traditional $\Delta(54)$ subgroup		
	Φ_n		irrep s	$\rho_s(S)$	$\rho_s(T)$	n	irrep r	$\rho_r(A)$	$\rho_r(B)$	$\rho_r(C)$
θ	$\Phi_{-2/3}$	no	$\mathbf{2}' \oplus \mathbf{1}$	$\rho(S)$	$\rho(T)$	$-2/3$	$\mathbf{3}_2$	$\rho(A)$	$\rho(B)$	$-\rho(C)$
	$\Phi_{-5/3}$	yes	$\mathbf{2}' \oplus \mathbf{1}$	$\rho(S)$	$\rho(T)$	$-5/3$	$\mathbf{3}_1$	$\rho(A)$	$\rho(B)$	$+\rho(C)$

G_{ecl} shown to restrict the allowed Kahler potential

[Nilles, Ramos-Sanchez and Vaudrevange 2004.05200]

weak point: G_{ecl} requires additional flavons
could it be that traditional G_{fl} are only an accident of orbifold description?



$$K = -h \log(-i\tau + i\bar{\tau}) + \sum_{I,p} c_p^{(I)} (-i\tau + i\bar{\tau})^{(pk_H - k_I)} \varphi^{(I)+} [H(\tau, \bar{\tau})]^p \varphi^{(I)}$$

$$H(\gamma\tau, \gamma\bar{\tau}) = |c\tau + d|^{2k_H} \rho^{(I)}(\gamma) H(\tau, \bar{\tau}) \rho^{(I)+}(\gamma) \quad \text{easy to build from MF}$$

$p = 0$



flavour-universal
minimal kinetic terms

$p \neq 0$



flavour-nonuniversal
kinetic terms

$$\varphi^{(I)} \rightarrow z_{(I)}^{1/2}(\tau, \bar{\tau}) \varphi^{(I)}$$

[F.F. 1706.08749
Chen, Ramon-Sanchez,
Ratz 1909.06910]

any p is allowed by EFT
no suppression for large p
lack of power counting

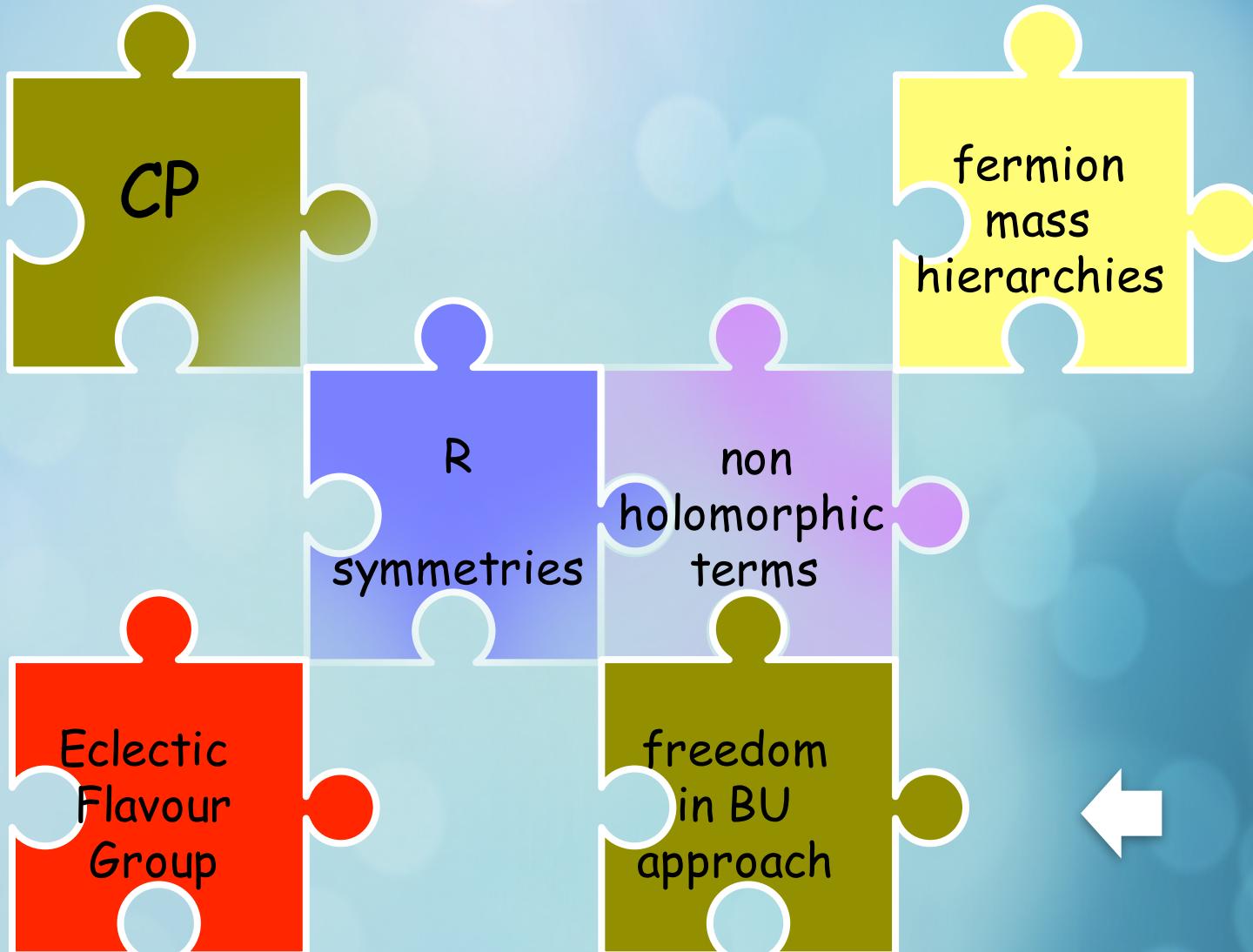


any additional rule from TD
to control K ?

e.g. asymptotic requirement such as

$$\lim_{Im \tau \rightarrow \infty} K = K_{min}$$

fermion masses
are non-holomorphic



field space	G_{fl}	$\text{SL}(2, \mathbb{Z}_N)$	R	CP
φ	$\varrho_\varphi(g)$	$k_\varphi \quad \rho_\varphi(\gamma)$	r_φ	X_φ^{CP}

even in BU approach, some constraints apply

$$\rho_\varphi(\gamma) \varrho_\varphi(g) \rho_\varphi(\gamma^{-1}) = \varrho_\varphi(u_\gamma(g))$$

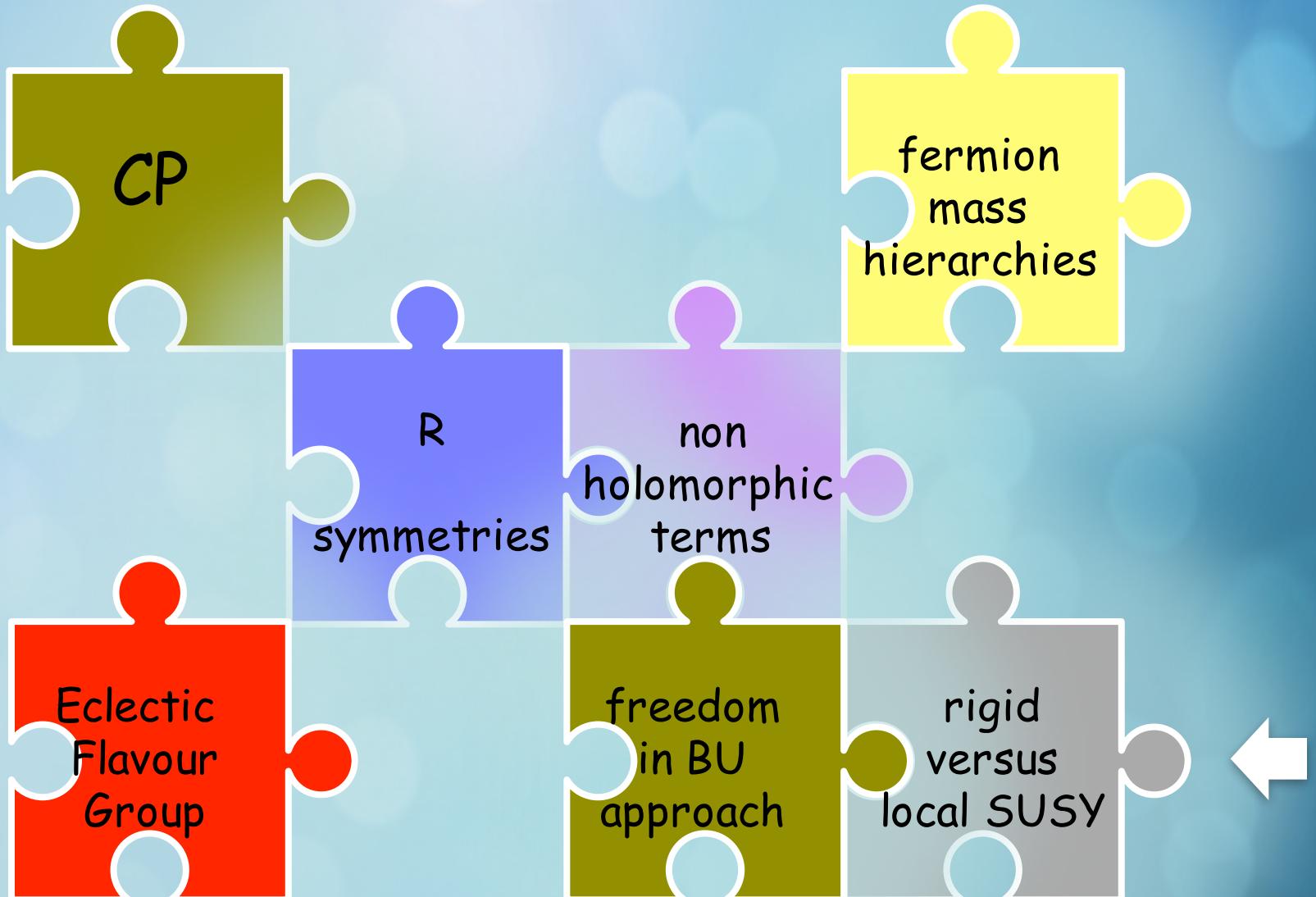
$$X_\varphi^{CP} \varrho_\varphi^*(g) X_\varphi^{CP-1} = \varrho_\varphi(u_{CP}(g))$$

$$X_\varphi^{CP} \rho_\varphi^*(\gamma) X_\varphi^{CP-1} = \rho_\varphi(u_{CP}(\gamma))$$

but a large freedom still remains

$$\text{SL}(2, \mathbb{Z}_N), k_\varphi, r_\varphi, \dots$$

are there general rules from TD, that restrict this freedom?



rigid SUSY

$$\begin{cases} w(\Phi) \xrightarrow{\gamma} w(\Phi) \\ K(\Phi, \bar{\Phi}) \xrightarrow{\gamma} K(\Phi, \bar{\Phi}) + f(\Phi) + \overline{f(\Phi)} \end{cases}$$

local SUSY

$$\mathcal{G} = K(\Phi, \bar{\Phi}) + \log|w(\Phi)|^2$$

modular invariance now requires

$$\begin{cases} w(\Phi) \xrightarrow{\gamma} e^{i\alpha_\gamma} e^{-f(\Phi)} w(\Phi) \\ K(\Phi, \bar{\Phi}) \xrightarrow{\gamma} K(\Phi, \bar{\Phi}) + f(\Phi) + \overline{f(\Phi)} \end{cases}$$

$$K_{min}(\Phi, \bar{\Phi})$$

$$f(\Phi) = \log (c\tau + d)^h$$

$$w(\Phi) \xrightarrow{\gamma} e^{i\alpha_\gamma} (c\tau + d)^{-h} w(\Phi)$$

a solution for the rigid case

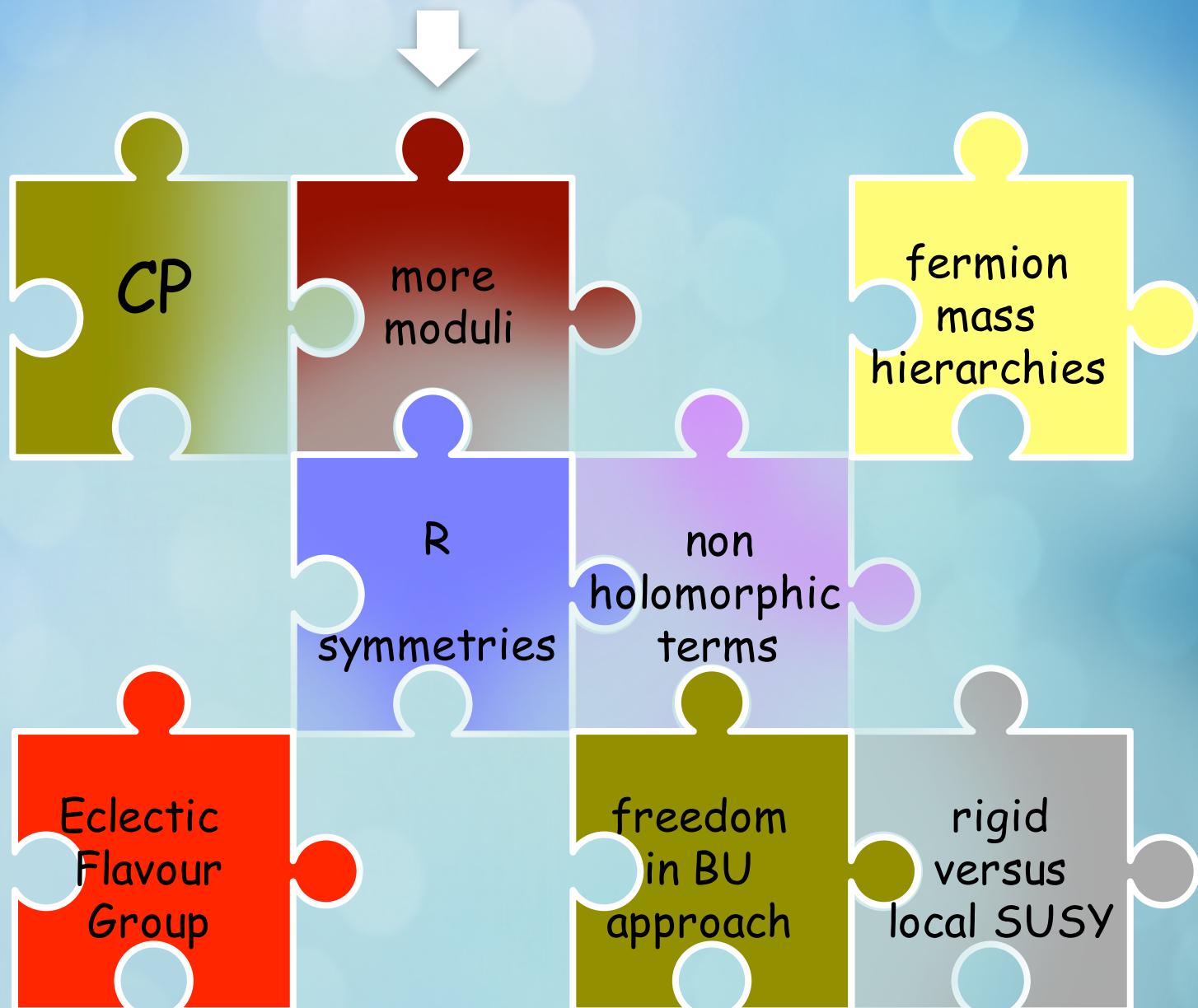
$$w(\Phi) = \varphi^c \mathcal{Y}(\tau) \varphi$$

can always be converted
into a solution for the local case
by shifting the matter weights

$$k_\varphi \rightarrow k_\varphi + h/2$$

$$k_{\varphi^c} \rightarrow k_{\varphi^c} + h/2$$

unless a TD selection rule forbids it!



String compactifications down to 4D naturally produce many moduli

in a BU description the natural generalization of Modular Invariance
is Symplectic Modular Invariance

[Ding, F., Liu 2010.07952]

(non-compact)
moduli space

$$\mathcal{M} = G/K$$

$$G = Sp(2g, \mathbb{R}) \quad K = U(g)$$

$$\mathcal{M} = \{\tau \in GL(g, \mathbb{C}) \mid \tau^t = \tau, Im(\tau) > 0\}$$

Siegel upper half plane

complex dimension $g(g + 1)/2$

action of G on τ

$$\tau \rightarrow \gamma\tau = (A\tau + B)(C\tau + D)^{-1}$$

$$\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

candidate flavour group

$$\Gamma = Sp(2g, \mathbb{Z}) \quad \text{Siegel modular group}$$

$$\varphi^{(I)} \rightarrow \det(C\tau + D)^{k_I} \rho^I(\gamma) \varphi^{(I)}$$

automorphy factor

unitary representation of
finite Siegel modular group

$$\Gamma / G_d(n) = \Gamma_{g,n}$$

genus level

minimal Kahler potential

superpotential

$$w(\tau, \varphi) = \sum_n Y_{I_1 \dots I_n}(\tau) \varphi^{(I_1)} \dots \varphi^{(I_n)}$$

Invariance of $w(\Phi)$ guaranteed by an holomorphic $Y_{I_1 \dots I_n}(\tau)$ such that

$$Y_{I_1 \dots I_n}(\gamma\tau) = j(\gamma, \tau)^{k_Y(n)} \rho(\gamma) Y_{I_1 \dots I_n}(\tau) \quad \gamma \in \Gamma$$

1. $k_Y(n) + k_{I_1} + \dots + k_{I_n} = 0$

2. $\rho \times \rho^{I_1} \times \dots \times \rho^{I_n} \supset 1$

special case of automorphic
forms for G, K, Γ

modular forms
of Γ/G_d and weight k_Y

form a linear space $\mathcal{M}_k(\Gamma/G_d)$
of finite dimension

new features

at genus
 $g = 2$

$$\mathcal{M} = \left\{ \tau = \begin{pmatrix} \tau_1 & \tau_3 \\ \tau_3 & \tau_2 \end{pmatrix} \mid \det(Im(\tau)) > 0, \text{tr}(Im(\tau)) > 0 \right\}$$

[de Medeiros Varzielas, King and Zhou,
1906.02208]

$\tau_3 = 0$ $\Gamma \approx SL(2, \mathbb{Z}) \otimes SL(2, \mathbb{Z})$ $\mathcal{M} \approx$ 2 factorized tori

$\tau_3 \neq 0$ $\mathcal{M} \approx$ generic Riemann surface of genus 2

the finite Siegel modular groups $\Gamma_{2,n}$ are big: $|\Gamma_{2,2} \approx S_6| = 720, |\Gamma_{2,3}| = 51840,$

already $\Gamma_{2,2} \approx S_6$ has no three-dimensional irreps

rich pattern of invariant spaces, $H\tau = \tau$, having 0, 1 or 2 complex dimensions
here the flavour group can be restrict to the normalizer $N(H)$

$$\Gamma \rightarrow N(H) = \{\gamma \in \Gamma \mid \gamma^{-1}H\gamma = H\}$$

for instance

$$\tau_1 = \tau_2 \quad \Omega = \left\{ \tau = \begin{pmatrix} \tau_1 & \tau_3 \\ \tau_3 & \tau_1 \end{pmatrix} \mid \tau \in G/K \right\}$$

working here $\Gamma_{2,2}$ is projected into the smaller group $S_4 \times Z_2$

Orbifolds from $Sp(4, \mathbb{Z})$ and
their modular symmetries

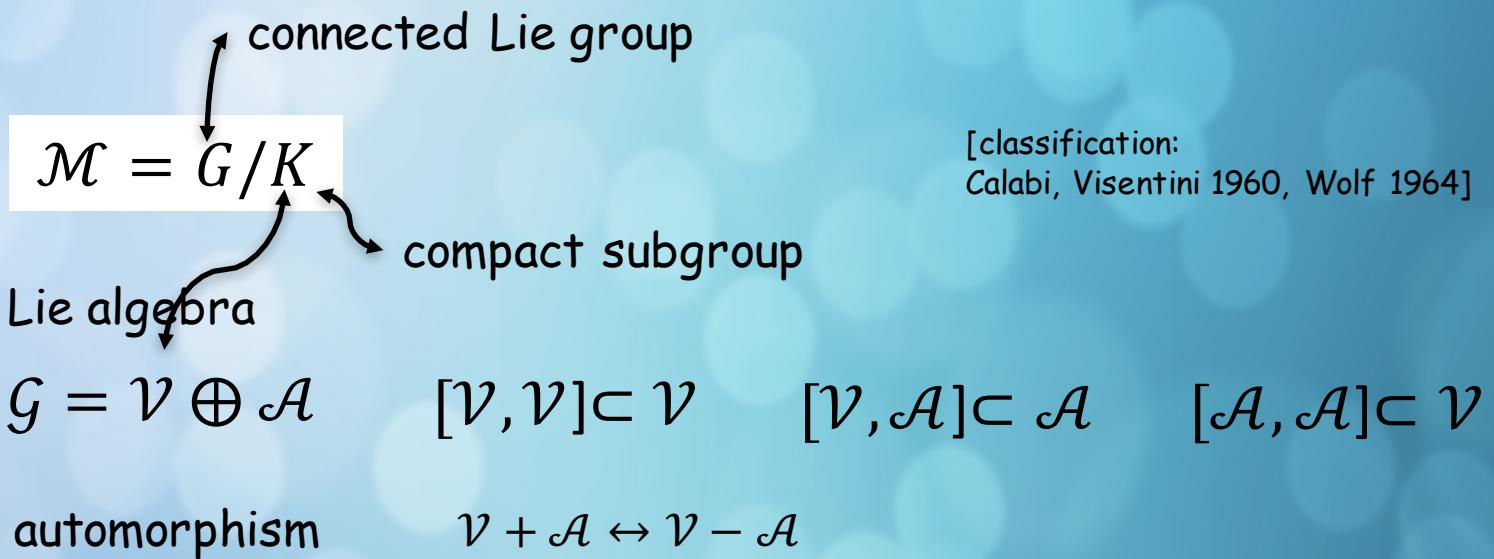
Nilles, Ramos-Sanchez, Trautner and Vaudrevange,
2105.08078

HSS = Hermitian Symmetric Spaces

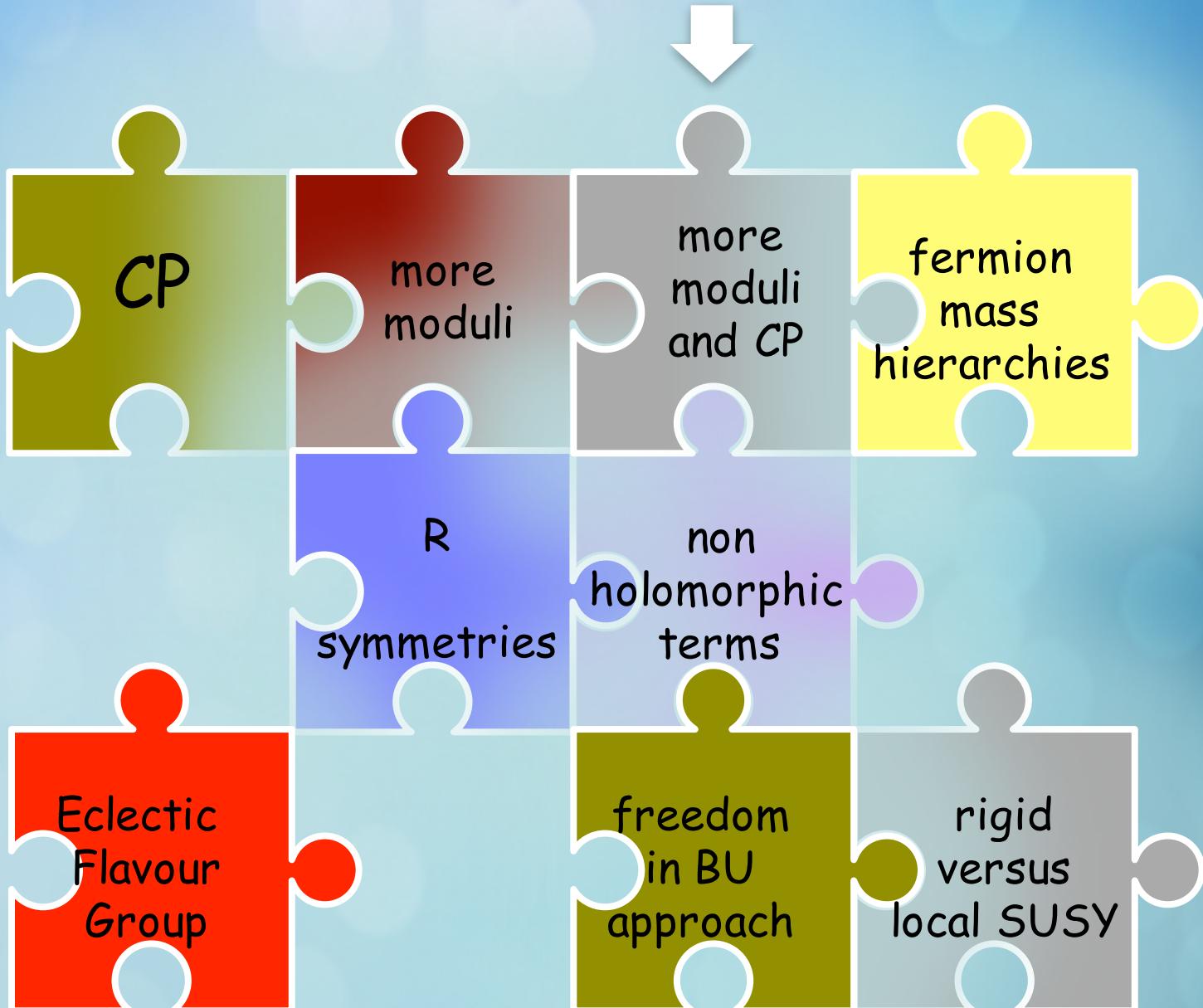
HSS are Kähler

non-compact HSS as moduli space in sugra and compactified strings

related to automorphic forms, building blocks of Yukawa couplings



flavour symmetry: a **discrete subgroup** Γ of G



\mathcal{CP} invariance in $Sp(2g, \mathbb{Z})$

[Ding, F., Liu 2102.06716]

\mathcal{CP} belongs to $\text{Out}(\Gamma)$

Ishiguro, Kobayashi and Otsuka, 2107.00487]

$$\mathcal{CP} \gamma \mathcal{CP}^{-1} = u(\gamma)$$

$$\gamma \in \Gamma = Sp(2g, \mathbb{Z})$$



$$\tau \rightarrow \tau_{CP} = -\tau^* \quad \text{up to } \text{In}(\Gamma)$$

[moduli]

$$\begin{cases} \varphi^{(I)} \rightarrow \det(C\tau + D)^{k_I} \rho^I(\gamma) \varphi^{(I)} \\ \varphi^{(I)} \xrightarrow{CP} X_I \bar{\varphi}^{(I)}(x_P) \end{cases}$$

[matter fields]

$$\begin{cases} Y(\gamma\tau) = \det(C\tau + D)^{k_Y} \rho_Y(\gamma) Y(\tau) \\ Y^a(\tau) \xrightarrow{CP} Y^a(-\tau^*) = \lambda_b^a X_Y Y^{b*}(\tau) \end{cases}$$

[modular forms]

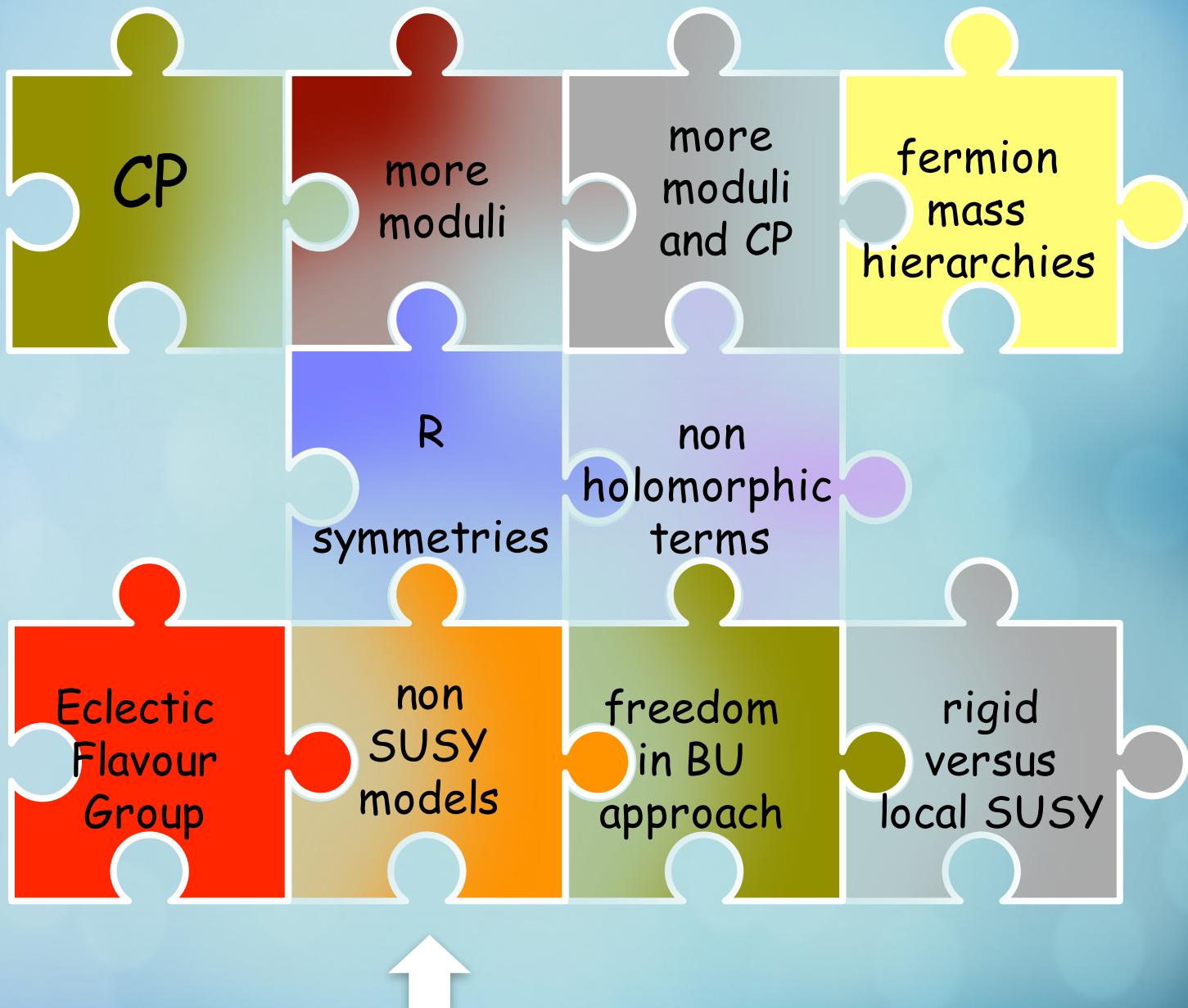
$$X_I \rho^{I*}(\gamma) X_I^{-1} = \chi(\gamma)^g k_I \rho_Y(u(\gamma))$$

[Nilles, Ramos-Sanchez and Vaudrevange 2001.01736]
 Baur, Nilles, Trautner and Vaudrevange, 1901.03251,
 H. Ohki, S. Uemura and R. Watanabe, 2003.04174]

\mathcal{CP} violation as a property of the vacuum

[Novichkov, Penedo, Petcov and Titov
 1905.11970]

[Baur, Kade, Nilles, Ramos-Sanchez,
 Vaudrevange 2012.09586]



modular invariance



compactification of 2 ED
on a torus
SUSY not mandatory

SUSY



holomorphicity

$y(\tau)$

modular forms
of level N and weight k_y

form a linear space $\mathcal{M}_k(\Gamma_N)$
of finite dimension

non SUSY framework

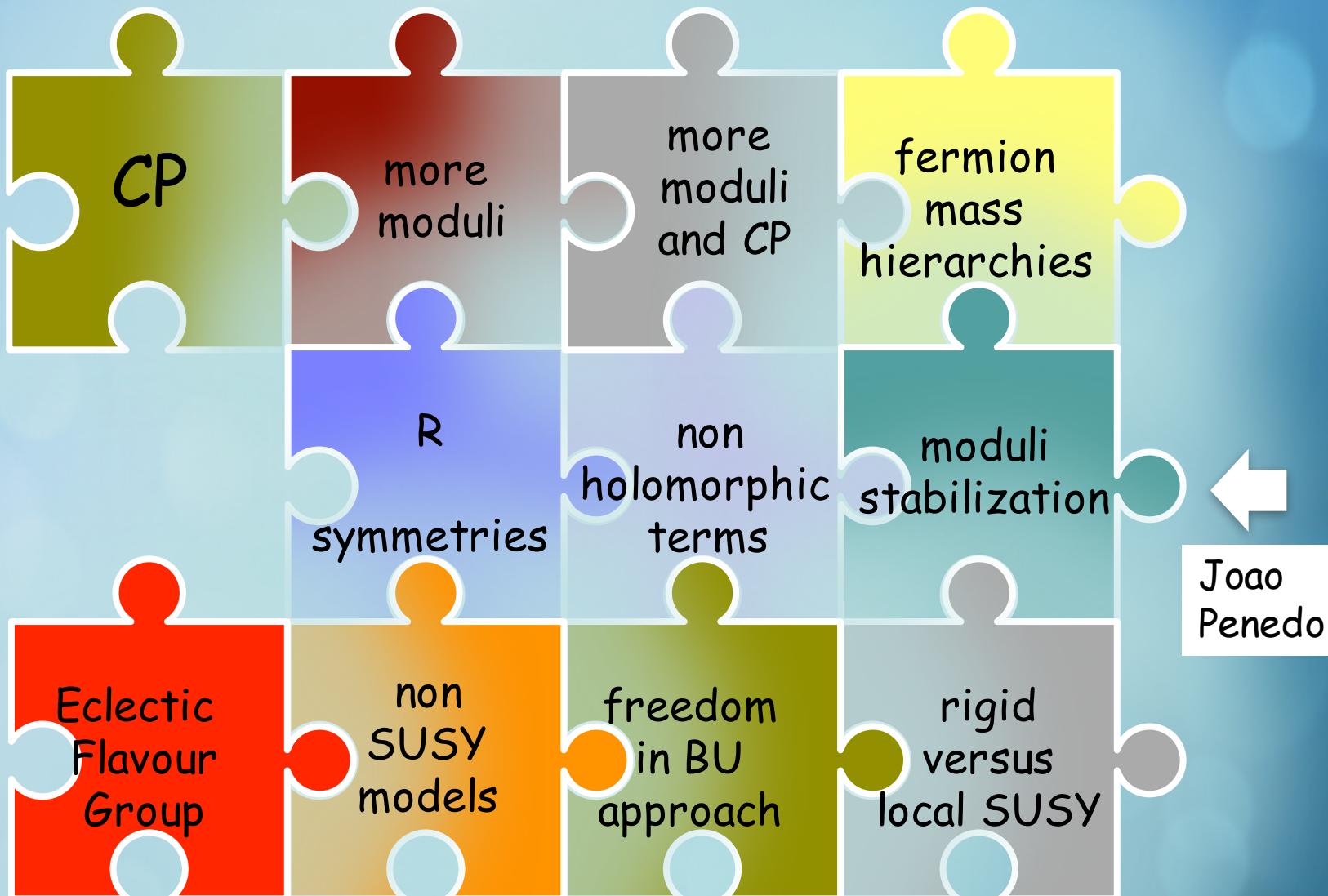


non-holomorphic
forms?

$y(\tau, \bar{\tau})$



“accidental”
holomorphicity



Joao
Penedo

moduli stabilization

what determines the value of τ ?

- anthropic selection
- cosmological evolution
- extrema of $V(\tau)$

extrema of $V(\tau)$ at the border of the fundamental region and along the $Im(\tau)$ axis ?

agrees with Michel's argument

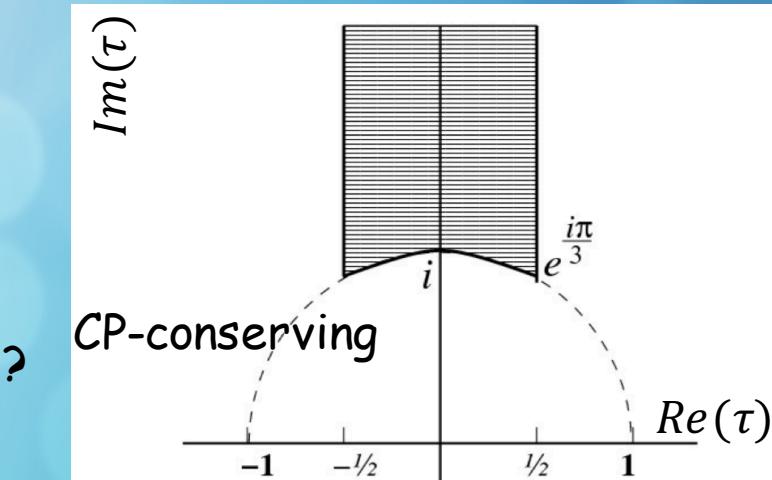
Michel, L. (1971), Comptes Rendus Acad. Sci **272**, 433.

$$f(-x) = f(x)$$

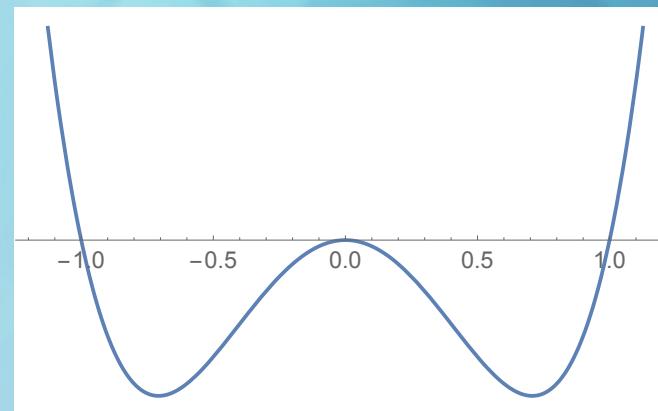
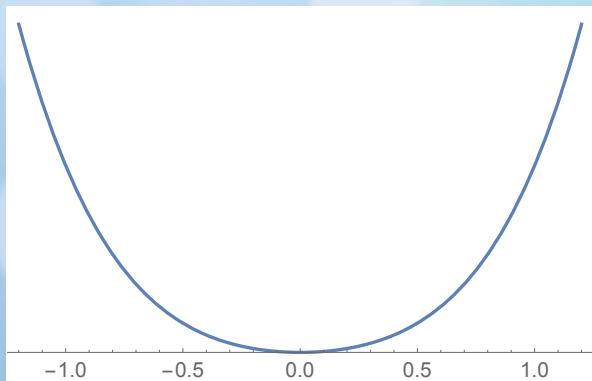
1.

$$x_{FP} = 0$$

always an extremum of $f(x)$



[Cvetic, Font, Ibáñez, Lust and Quevedo,
Nucl.Phys.B 361 (1991) 194]

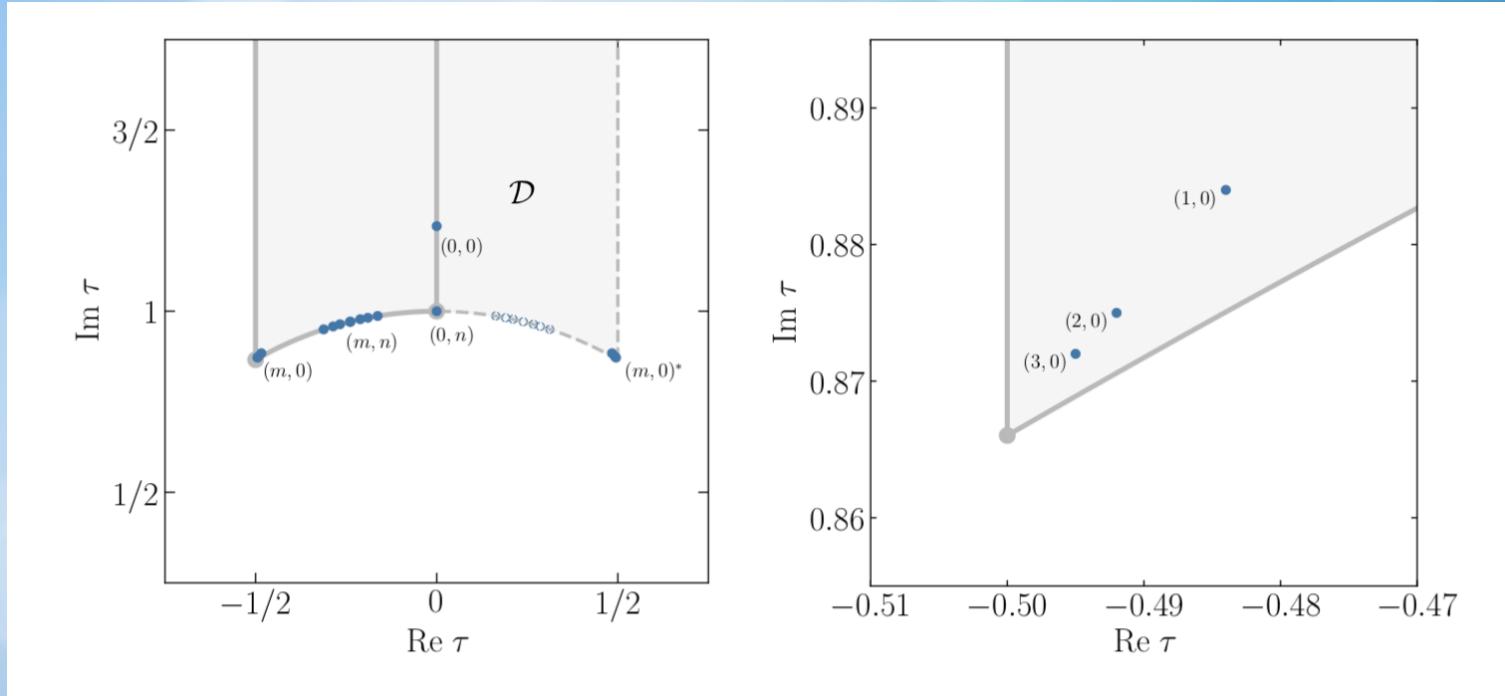


2.

extrema at FPs are more natural than elsewhere

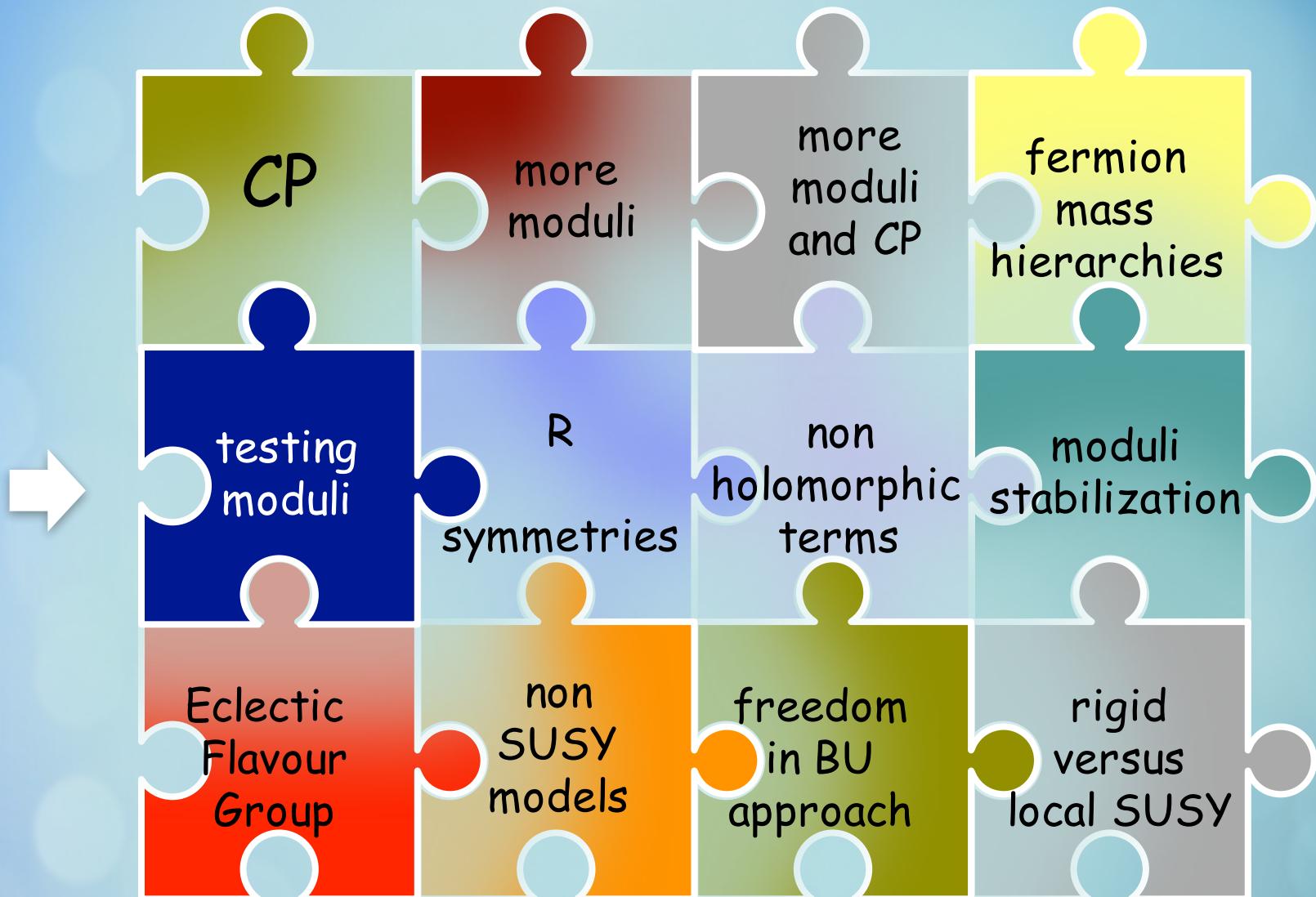
$$f(x) = a x^2 + b x^4 \quad \text{minimum at } x \neq 0 \text{ requires } a < 0, b > 0$$

[Novichkov, Penedo and Petcov, 2201.02020]



CP-violating minima from local SUSY

what conditions are required on $V(\tau)$?



tests of modulus couplings

G-J. Ding, FF,
2003.13448

non standard neutrino
interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f} \bar{\sigma}^\mu \partial_\mu f + \frac{1}{2} \partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2$$

$$- (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2} \nu (m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha) \nu + h.c. + \dots$$

$$\tau = \langle \tau \rangle + \frac{\varphi_u + i \varphi_v}{\sqrt{2}}$$

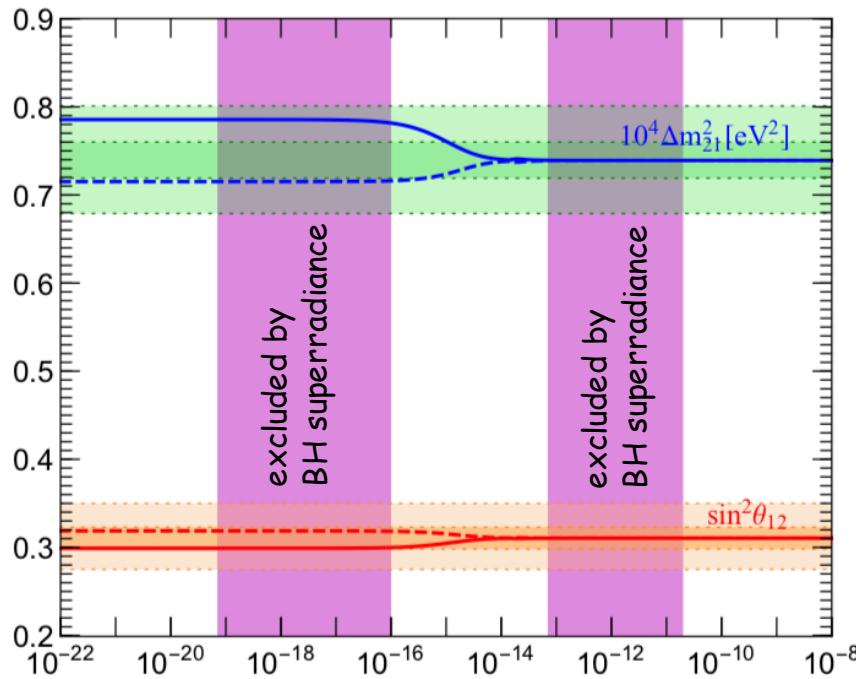


in medium with non-zero
electron number density

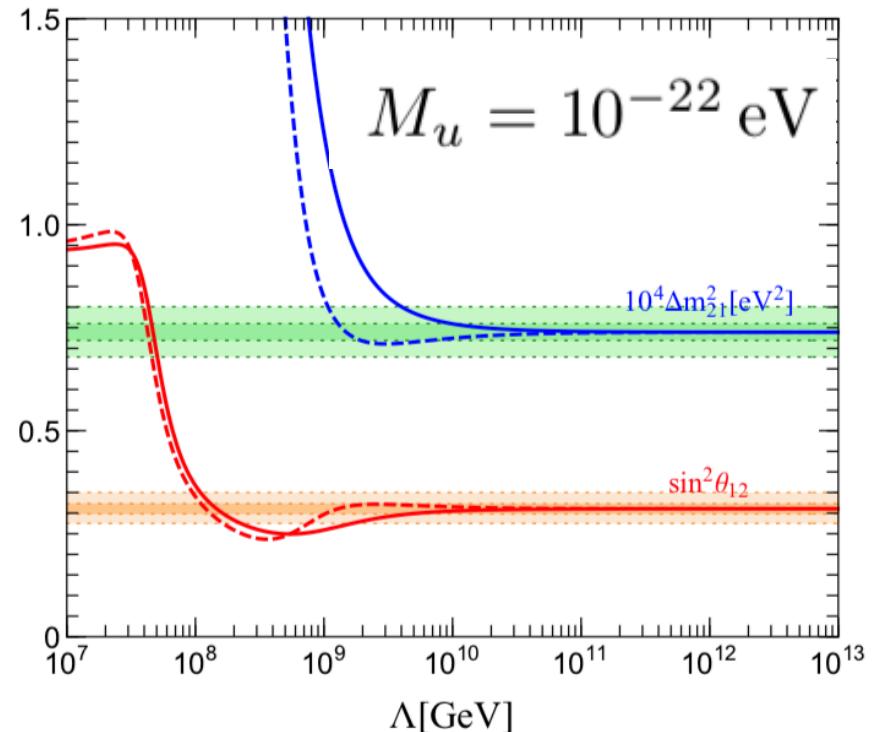
small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\text{Re}(\mathcal{Z}^e) \mathcal{Z}^\nu}{M^2(R)},$$

in the sun:



$$\Lambda = 5 \times 10^9 \text{ GeV} \quad \begin{matrix} M_u [\text{eV}] \\ \text{[modulus VEV]} \end{matrix}$$



CP

more moduli

more moduli
and CP

fermion
mass
hierarchies

testing
moduli

R

symmetries

non
holomorphic
terms

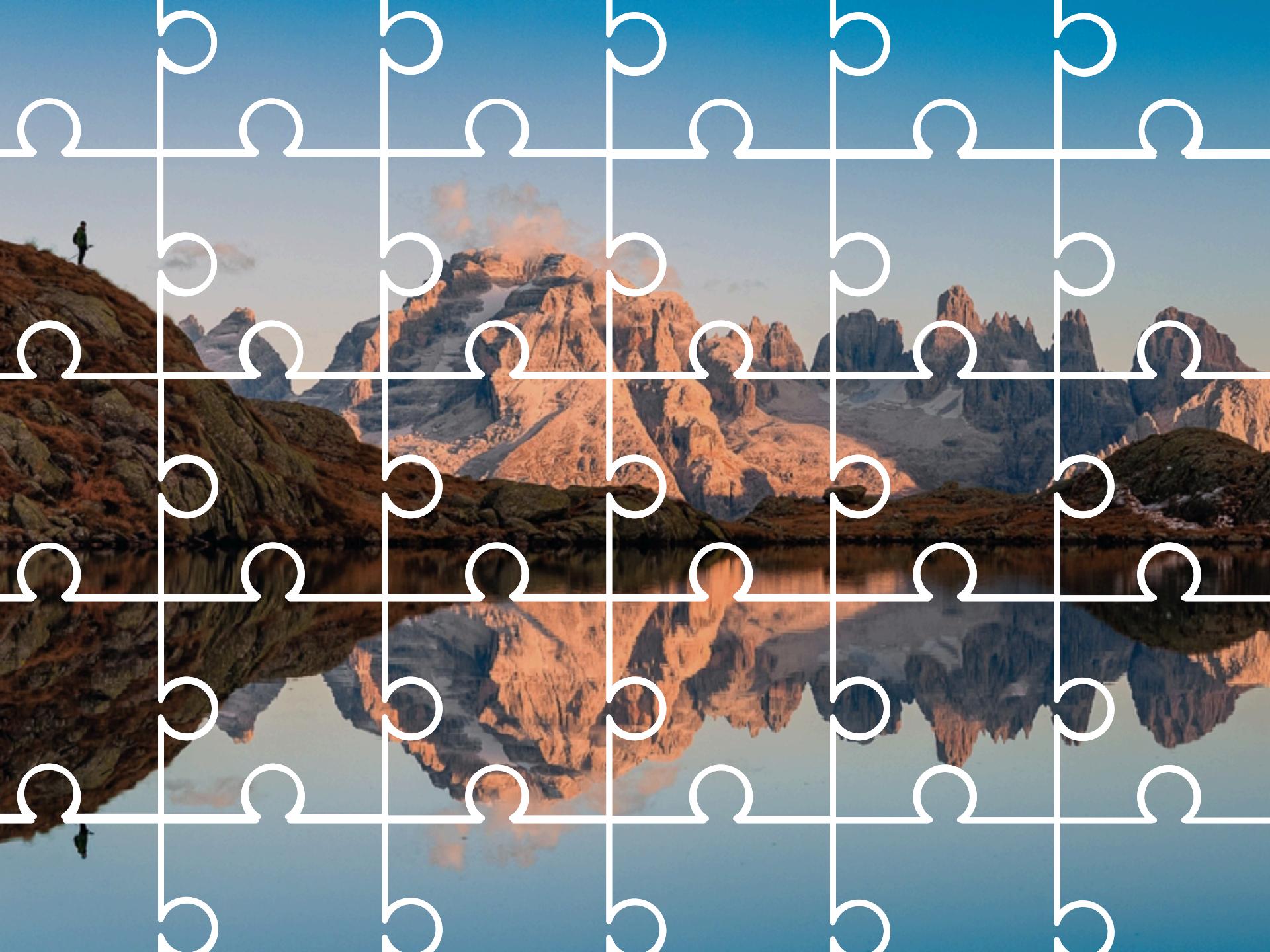
moduli
stabilization

Eclectic
Flavour
Group

non
SUSY
models

freedom
in BU
approach

rigid
versus
local SUSY



**THANK
YOU!**

back-up slides