Top-down derived modular and eclectic flavor symmetries

Saúl Ramos-Sánchez

Bethe Forum
Bonn

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From various collaborations with:

A. Baur, M. Kade, H.P. Nilles & P. Vaudrevange: 2001.01736, 2004.05200, 2008.07534, 2010.13798, 2012.09586 & 2104.03981

Y. Almumin, M-C. Chen, V. Knapp-Pérez, M. Ramos-Hamud, M. Ratz & S. Shukla: 1909.06910, 2102.11286 & 2108.02240

The flavor puzzle

and its potential solutions

Flavor puzzle

Despite the great success of the SM

- Need to explain
  - three flavors of SM particles
  - observed mass hierarchies
  - observed quark and lepton mixing *textures*
  - CP violation in CKM and PMNS
  - neutrino physics
  - ...

$$\begin{pmatrix} 0.974 & 0.224 & 0.0039 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.039 & 1.019 \end{pmatrix}_{CKM} , \qquad \begin{pmatrix} 0.829 & 0.539 & 0.147 \\ 0.493 & 0.584 & 0.645 \\ 0.262 & 0.607 & 0.75 \end{pmatrix}_{PMNS}$$

$m_{u_i} \sim 2.16, 1270, 172900$ MeV

$m_{d_i} \sim 4.67, 93, 4180$ MeV

$\Delta m^2_{21} = 7.4 \cdot 10^{-5}, \Delta m^2_{31(23)} \approx 2.5 \cdot 10^{-3}$ eV^2

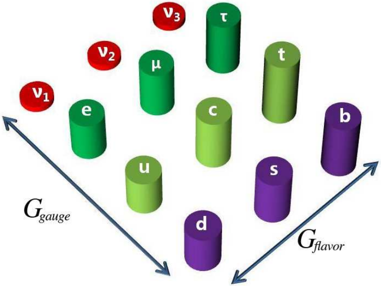
$m_{e_i} \sim 0.511, 105.7, 1776.9$ MeV

normal ordering

Approaches towards solving the flavor puzzle

<u>Traditional</u>: discrete non-Abelian flavor symmetries $G_{traditional}$ lead to models for quarks and leptons with great fits, $\theta_{13} \neq 0$,... requiring careful choice of flavon sector and flavon vevs

see reviews by Ishimori, Kobayashi, Ohki, Okada, Shimizu, Tanimoto (2010); Feruglio, Romanino (2019)



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flavon vev $alignment$ is very challenging ☹

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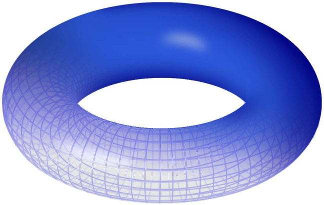
Modular: Yukawa couplings are modular forms $Y = Y(T)$ Feruglio (2017)

$$Y(T) \to Y(\gamma T) = (cT + d)^{n_Y} \rho_Y(\gamma) Y(T), \quad \gamma \in \Gamma = \mathrm{SL}(2, \mathbb{Z}), \rho_Y \in \Gamma_N$$



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- $\Gamma_N \cong S_3, A_4, S_4, A_5$ for $N = 2, 3, 4, 5$

$n_Y \in 2\mathbb{Z}$

$\Rightarrow$ 9 ν observables (m_ν, θ_{ij}, phases) by fixing 3 parameters!

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- 4-fold cover $\widetilde{\Gamma}_4 \cong [96,67], \widetilde{\Gamma}_8 \cong [768, 1085324], \widetilde{\Gamma}_{12} \cong [2304, \ldots]$

$n_Y \in \mathbb{Z}/2 \quad \to \quad$ metaplectic

Liu,Ding(2019); Liu,Yau,Qu,Ding(2020)

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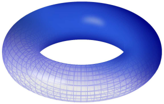
Bottom-up modular flavor symmetries: notes and questions

- Modulus T and modular transformations based on $\mathbb{T}^2$ torus



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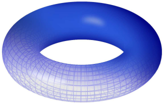

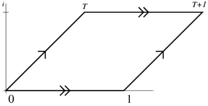

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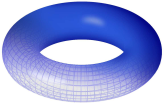

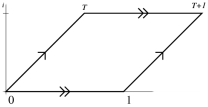

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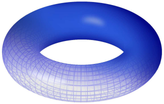

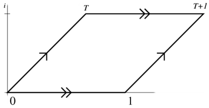

origin? *internal* torus? 2 extra dimensions?

- Several successful fits, mainly of lepton sector, but also quarks

$T \sim$ self-dual points, free W parameters + n_Y, n_ϕ, $\rho_Y(\gamma), \rho_\phi(\gamma)$

Is there a way to fix some of these parameters?

See talks by Feruglio, King, Petcov, Penedo, Titov, Ding

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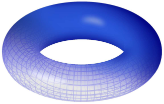

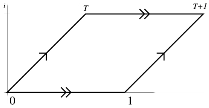

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- Low-energy SUSY (typically with $M_{pl} \to \infty$) assumed

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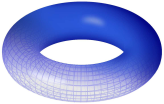

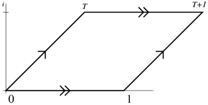

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modular forms without SUSY?

- Canonical Kähler potential $K_{ij} = \delta_{ij}$

additional terms/free parameters?

Kähler problem

Challenge: Kähler potential not fixed by modular flavor symmetries

Chen, SRS, Ratz (2019)

Demanding modular invariance only:

$$K = \alpha_0 \quad \underbrace{(\phi\bar{\phi})_{\mathbf{1}}}_{\text{canonical term}}$$

with $\qquad \alpha_0 = c_0(-\mathrm{i}T + \mathrm{i}\bar{T})^{n_\phi}$

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Demanding modular invariance only:

$$K = \alpha_0 (\phi\bar{\phi})_{\mathbf{1}} + \underbrace{\sum_k}_{\text{sum over singlets}} \alpha_k \underbrace{\left(\phi Y \bar{\phi} \bar{Y}\right)_{\mathbf{1},k}}_{\text{non-canonical terms}}$$

with $\alpha_0 = c_0(-\mathrm{i}T + \mathrm{i}\bar{T})^{n_\phi}$ and $\alpha_k = c_k(-\mathrm{i}T + \mathrm{i}\bar{T})^{n_\phi + n_Y}$

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and $\mathcal{O}(\alpha_0) \sim \mathcal{O}(\alpha_k)$

All α_k coefficients are "*new*" $\rightarrow$ modify predictions!

Kähler problem in Feruglio's simplest A_4 model

Take $\Gamma_3 \cong A_4$ and $n_Y = 1 = -n_\phi$ for $\phi = L$

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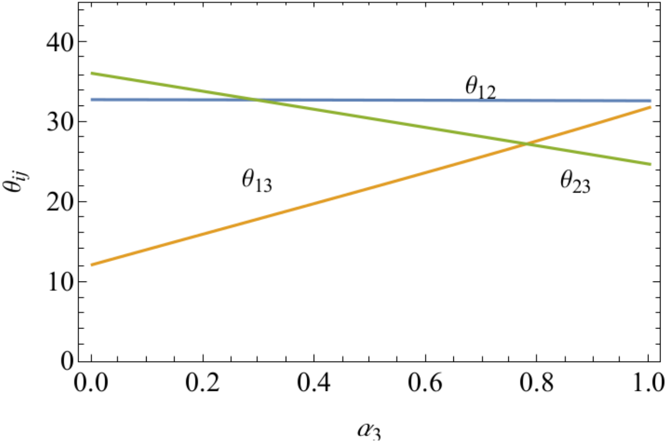

neutrino mixing angles depending on values of α_3

Chen, SRS, Ratz (2019)

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Ideas towards solutions based on or inspired by string theory

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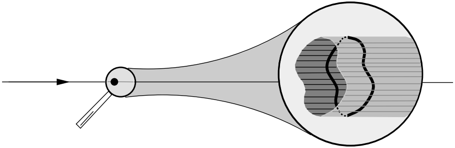
- Eclectic and quasi-eclectic pictures *à la* bottom–up

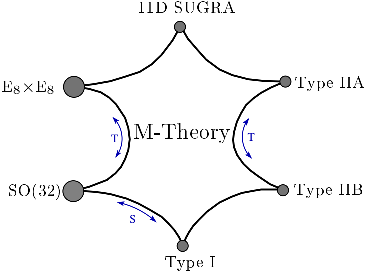

All about
STRINGS
We have resources for all
string-related topics

Strings

1970's: particles → strings



80-90's: 5 theories of superstrings (+branes)



quantum consistency
(no anomalies, no ghosts, no tachyons):

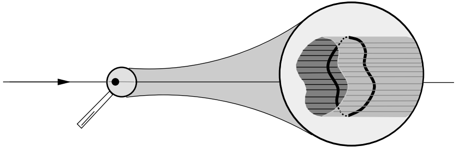
→

- * graviton included ☺
- * gauge bosons ☺
- * supersymmetry ☺☹
- * 10 dimensions ☹

  perhaps $\mathbb{M}^{10} = X^6 \times \mathbb{M}^4$

Stringy features for 4D model

particles $\longleftrightarrow$ strings



- SUSY & 10D space-time
- matter fields get all their properties from string features
- field couplings arise from string interactions

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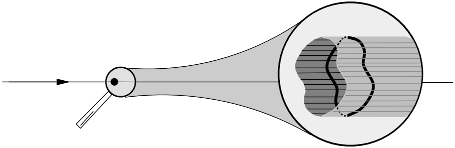

- SUSY & 10D space-time
  $\rightarrow$ compactify 6D on spaces with shapes and sizes set by moduli
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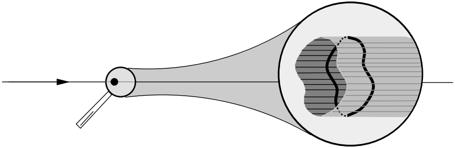
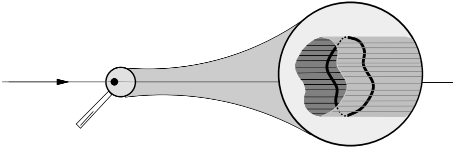


- SUSY & 10D space-time
  $\rightarrow$ compactify 6D on spaces with shapes and sizes set by moduli
    $\rightarrow$ geometric discrete symmetries & modular symmetries from $6D$
- matter fields get all their properties from string features
- field couplings arise from string interactions

Stringy features for 4D model

particles $\longleftrightarrow$ strings



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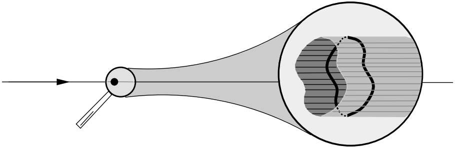

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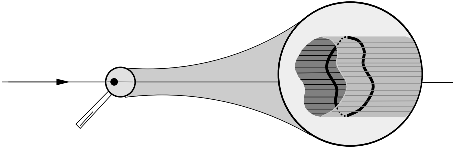

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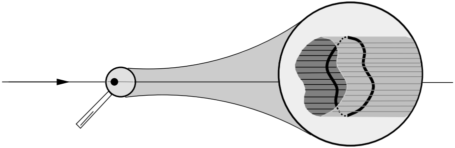

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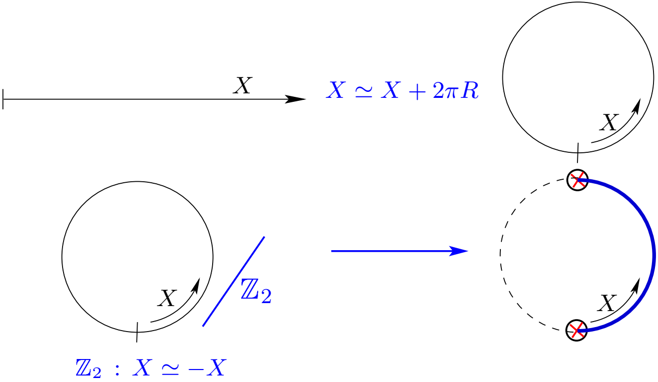
  - $\rightarrow$ couplings are modular forms with fixed properties

Heterotic Orbifolds

(in bosonic formulation)

1D $\mathbb{S}^1/\mathbb{Z}_2$ orbifold



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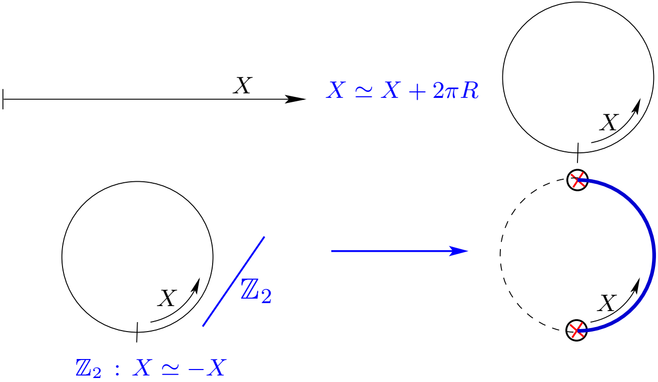


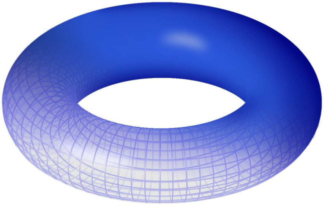
In general, an orbifold $\mathcal{O} := \mathbb{M}/S$

with a d-dimensional manifold $\mathbb{M}$

space group $S = \{(\Theta, \lambda) \mid \Theta :$ rotation in d-dim$, \lambda :$ translation$\}$

e.g. $\mathbb{S}^1/\mathbb{Z}_2 \cong \mathbb{R}/S$ with $S = \langle(-1, 2\pi R)\rangle \to X \simeq -1X + 2\pi R m$

2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifold with heterotic strings



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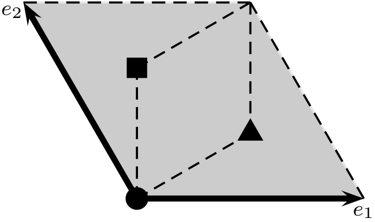

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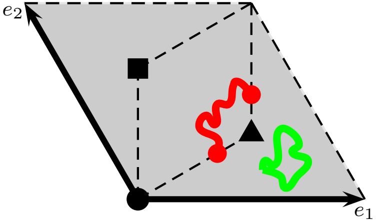
Matter at low energies arise from *closed strings*:

some are **free** *and some are* **fixed** *in compact space*



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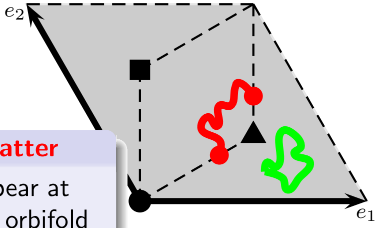

localized twisted matter

matter fields Φ_{n_i} appear at definite points in the orbifold

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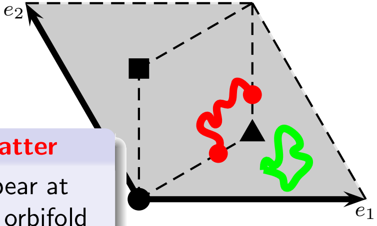


localized twisted matter

matter fields Φ_{n_i} appear at definite points in the orbifold

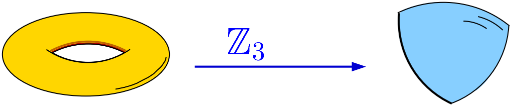
all matter properties are determined by the space group S:

- global and gauge symmetries, charges/representations,
- target-space modular properties (weights n_i and representations),...

A first hint of (geometric) flavor symmetries

- $\mathbb{T}^2/\mathbb{Z}_3$

T^2:



triangular pillow $\rightarrow$ symmetry of a triangle S_3

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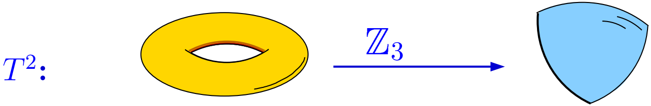

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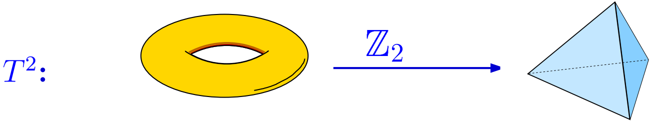

tetrahedron* $\rightarrow$ symmetry of a tetrahedron S_4

* for $\langle U \rangle = e^{\pi \mathrm{i}/3}$, but other values possible

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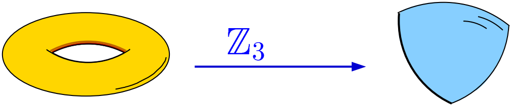

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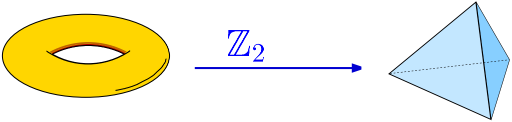

tetrahedron* $\rightarrow$ symmetry of a tetrahedron S_4

(Analogous results for $\mathbb{T}^2/\mathbb{Z}_K$, $K > 2$, unless background fields (Wilson lines) are included)

Towards the *eclectic* flavor picture

Use Narain formalism: split string in independent components

$$X(\tau, \sigma) = X_R(\sigma - \tau) + X_L(\sigma + \tau)$$

Groot-Nibbelink, Vaudrevange (2017)

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What are the outer automorphisms of $S_{Narain} = \{g\}$?

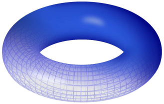
$$Out(S_{Narain}) = \left\{h = (\Sigma, t) \notin S_{Narain} \mid hgh^{-1} \in S_{Narain}\right\}$$

Rotations: $h_\Sigma = (\Sigma, 0) \to O(2,2;\mathbb{Z})$, Translations: $h_t = (\mathbb{1}_4, t)$

Baur, Nilles, Trautner, Vaudrevange (2019)

Towards the *eclectic* picture: what $Out(S_{Narain})$ is

String 2D toroidal compactifications have two moduli: T, U



$$G = \frac{\operatorname{Im} T}{\operatorname{Im} U} \begin{pmatrix} 1 & \operatorname{Re} U \\ \operatorname{Re} U & |U|^2 \end{pmatrix}, \qquad B = \operatorname{Re} T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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Recall: in $\mathrm{SL}(2,\mathbb{Z})$ $\qquad T \xrightarrow{\mathrm{S}} -\frac{1}{T}, \qquad T \xrightarrow{\mathrm{T}} T+1$

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$\mathrm{SL}(2,Z)_T = \langle \mathrm{S}_T, \mathrm{T}_T \rangle, \quad \mathrm{SL}(2,Z)_U = \langle \mathrm{S}_U, \mathrm{T}_U \rangle$ ☺

M: mirror symmetry ☺

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Nilles, Ratz, Trautner, Vaudrevange (2018); Novichkov, Penedo, Petcov, Titov (2019)

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Further, $\{h_t\}$ don't change T, U, but do transform fields
$\rightarrow$ traditional symmetry ☺

Common origin of modular and traditional flavor

Baur, Nilles, Trautner, Vaudrevange (1901.03251, 1908.00805)

In fact: flavoring is better with strings! ☺

$Out(S_{Narain}) \supset$ traditional & modular symmetries

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Next: demand $\mathbb{Z}_K$ orbifold invariance, act on fields, couplings

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Modular weights n, representations and couplings of Φ_n not *ad hoc*! ☺

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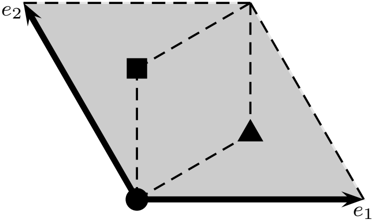
Example $\mathbb{T}^2/\mathbb{Z}_3$: must fix U to $\langle U \rangle = \omega = e^{2\pi \mathrm{i}/3} \to$ broken $\mathrm{SL}(2,\mathbb{Z})_U$



$$\mathrm{SL}(2,\mathbb{Z})_U \;\to\; \mathbb{Z}_9^R \quad \text{due to } n \in \{-5/3, -1, -2/3, -1/3, 0, 2/3\}$$

and $\Phi_n \xrightarrow{\gamma_U} \exp\{2\pi \mathrm{i} R/9\}\Phi_n$ con $R = 3(-n+\alpha)$

Common origin of modular and traditional flavor

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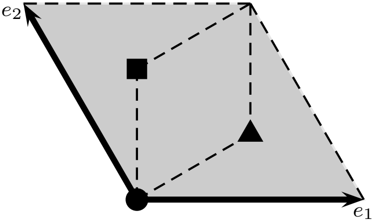
Example $\mathbb{T}^2/\mathbb{Z}_3$: $\langle U\rangle = \omega \quad\Rightarrow\quad \mathrm{SL}(2,\mathbb{Z})_U \to \mathbb{Z}_9^R$



Lauer, Mas, Nilles (1989)

By using CFT formalism, inspect $\mathrm{SL}(2,\mathbb{Z})_T$ on the triplet of matter fields:

$$h_\Sigma : \rho(\mathrm{S}_T) = \frac{\mathrm{i}}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \rho(\mathrm{T}_T) = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rho(\mathrm{S}_T)$ and $\rho(\mathrm{S}_T)$ build the reps. $\mathbf{2}' \oplus \mathbf{1}$ of modular group $\Gamma_3' = T'$ ☺

$$\Phi_{n=-2/3,-5/3} \xrightarrow{\mathrm{S}_T} (-T)^n \rho(\mathrm{S}_T)\Phi_n, \qquad \Phi_n \xrightarrow{\mathrm{T}_T} \rho(\mathrm{T}_T)\Phi_n$$

Ibáñez, Lüst (1992)

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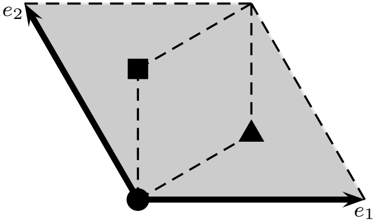

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$$h_t : \rho(\mathrm{A}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \ \rho(\mathrm{B}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \ \rho(\mathrm{C}) = \rho(\mathrm{S}_T^2)$$

$\rho(\mathrm{A})$, $\rho(\mathrm{B})$ and $\rho(\mathrm{C})$ build the reps $\mathbf{3}_2$ and $\mathbf{3}_1$ of traditional flavor group $\Delta(54)$ for $\Phi_{-2/3}$ and $\Phi_{-5/3}$

cf. also in Kobayashi, Plöger, Nilles, Raby, Ratz (2006)

Common origin of modular and traditional flavor

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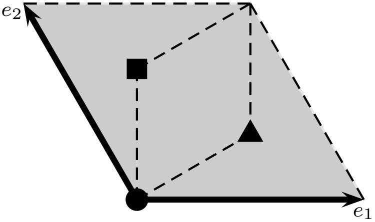

first eclectic flavor symmetry: traditional + modular flavor

$$G_{\text{traditional}} \cup G_{\text{modular}} \cong (\Delta(54) \cup \mathbb{Z}_9^R) \cup T' \cong \Omega(2) = [1944, 3448]$$

with $\mathcal{CP}$: $\Omega(2) \rtimes \mathbb{Z}_2^{\mathcal{CP}} \cong [3888, ...]$

Baur, Nilles, Trautner, Vaudrevange (2019); Nilles, SRS, Vaudrevange (2020)

Common origin of modular and traditional flavor

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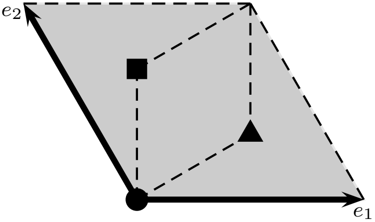


	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-5/3}$	$\Phi_{-1/3}$	$\Phi_{2/3}$
$\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\mathbf{3}_1$	$\bar{\mathbf{3}}_1$	$\bar{\mathbf{3}}_2$
T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$
$\mathbb{Z}_9^R$	0	3	1	-2	2	5

and $\mathbb{Z}_2^{\mathcal{CP}}$: $\quad \Phi_n \xrightarrow{\mathcal{CP}} \overline{\Phi}_n \qquad \forall n$

Baur, Nilles, Trautner, Vaudrevange (2019); Nilles, SRS, Vaudrevange (2020)

Modular forms as couplings in $\mathbb{T}^2/\mathbb{Z}_3$

Yukawa coupling coefficients $\hat{Y}$ are modular forms!

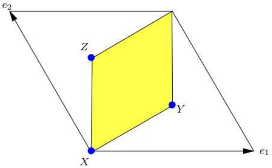
modular forms $\hat{Y}_{\boldsymbol{s}}^{(n_Y)}$	eclectic flavor group $\Omega(1)$							
	modular T' subgroup				traditional $\Delta(54)$ subgroup			
	irrep $\boldsymbol{s}$	$\rho_{\boldsymbol{s}}(\mathrm{S})$	$\rho_{\boldsymbol{s}}(\mathrm{T})$	n_Y	irrep $\boldsymbol{r}$	$\rho_{\boldsymbol{r}}(\mathrm{A})$	$\rho_{\boldsymbol{r}}(\mathrm{B})$	$\rho_{\boldsymbol{r}}(\mathrm{C})$
$\hat{Y}_{\mathbf{2''}}^{(1)}$	$\mathbf{2''}$	$\rho_{\mathbf{2''}}(\mathrm{S})$	$\rho_{\mathbf{2''}}(\mathrm{T})$	1	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}}^{(4)}$	$\mathbf{1}$	1	1	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1'}}^{(4)}$	$\mathbf{1'}$	1	ω	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{3}}^{(4)}$	$\mathbf{3}$	$\rho_{\mathbf{3}}(\mathrm{S})$	$\rho_{\mathbf{3}}(\mathrm{T})$	4	$\mathbf{1}$	1	1	1

$$\hat{Y}_{\mathbf{2''}}^{(1)} := \begin{pmatrix} \hat{Y}_1(T) \\ \hat{Y}_2(T) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \eta(3T)^3/\eta(T) \\ \eta(T/3)^3/\eta(T) \end{pmatrix}$$

No arbitrary modular weights n_Y nor representations $\boldsymbol{s}$! ☺

Superpotential and Kähler in $\mathbb{T}^2/\mathbb{Z}_3$

Restricted superpotential Baur,Nilles,Trautner,SRS,Vaudrevange(2021-22), see talks by Baur & Trautner



$$\Rightarrow \mathcal{W} \supset c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\quad \Phi^{i}_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

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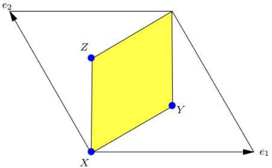

$$\Rightarrow \mathcal{W} \supset c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3+Y_1\,Y_2\,Y_3+Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3+X_1\,Y_3\,Z_2+X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2+X_2\,Y_3\,Z_1+X_3\,Y_2\,Z_1\big)\Big]\,,$$

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More interestingly

$$K = -\log(-\mathrm{i}T+\mathrm{i}T) + \textstyle\sum_i \Big[(-\mathrm{i}T+\mathrm{i}T)^{-2/3} + (-\mathrm{i}T+\mathrm{i}T)^{1/3}|\hat{Y}^{(1)}_{\mathbf{2''}}|^2 + \ldots\Big]\,|\Phi^{i}_{-2/3}|^2$$

$+$ suppressed corrections with flavon fields

Only canonical terms are allowed

$\rightarrow$ predictivity of bottom-up models with Γ'_N recovered! ☺

Nilles, SRS, Vaudrevange (2004.05200)

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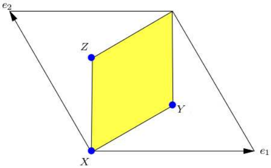

$$\Rightarrow \mathcal{W} \supset c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\quad \Phi^i_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

More interestingly

$$K = -\log(-\mathrm{i}T + \mathrm{i}\bar{T}) + \textstyle\sum_i \Big[(-\mathrm{i}T + \mathrm{i}\bar{T})^{-2/3} + (-\mathrm{i}T + \mathrm{i}\bar{T})^{1/3}|\hat{Y}^{(1)}_{\mathbf{2''}}|^2 + \ldots\Big]\,|\Phi^i_{-2/3}|^2$$

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Only canonical terms are allowed (due to traditional symmetry)

$\rightarrow$ predictivity of bottom-up models with Γ'_N recovered! ☺

Nilles, SRS, Vaudrevange (2004.05200)

Flavor-unification enhancement

$\gamma\, T_{fp} \stackrel{!}{=} T_{fp} \quad \Rightarrow \quad G_{\text{stabilizer}} = \{\gamma\} \subset G_{\text{modular}}$ is traditional symmetry

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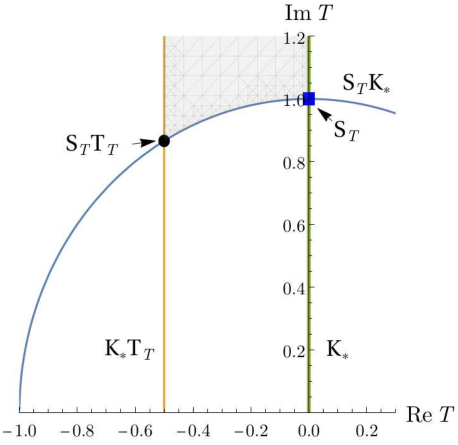

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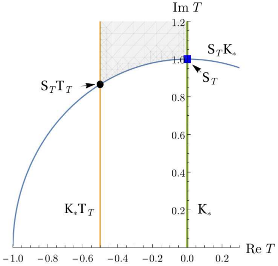
enhanced $G_{\text{traditional}} = G_{\text{traditional}} \cup G_{\text{stabilizer}}$ @ $T = T_{fp}$

$\langle T\rangle$	stabilizer generators		unified flavor symmetries	
	non $\mathcal{CP}$-like	$\mathcal{CP}$-like	without $\mathcal{CP}$	with $\mathcal{CP}$
i	S_T	K_*	$\Xi(2,2) \cong [324,111]$	$[648,548]$
ω	$\mathrm{S}_T\mathrm{T}_T$	$\mathrm{K}_*\mathrm{T}_T$	$H(3,2,1) \cong [486,125]$	$[972,469]$
$\mathrm{Re}\langle T\rangle = 0$		K_*	$\Delta'(54,2,1) \cong [162,44]$	$[324,125]$
$\mathrm{Re}\langle T\rangle = {}^{-1}/{}_{2}$		$\mathrm{K}_*\mathrm{T}_T$	$\Delta'(54,2,1) \cong [162,44]$	$[324,125]$
$\lvert\langle T\rangle\rvert = 1$		$\mathrm{S}_T\mathrm{K}_*$	$\Delta'(54,2,1) \cong [162,44]$	$[324,125]$



Semi-realistic orbifold models

A model is semi-realistic (or MSSM-like) if it exhibits:

- $\mathcal{G}_{4D} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathcal{G}_{hidden} \times \mathrm{U}(1)'^z$ gauge group
- $\mathcal{G}_{hidden}$ admits gaugino condensates due to little hidden matter
- 3 families of quarks & leptons
- 2 (or more) Higgs doublets
- $\mathrm{U}(1)_Y$ admits traditional unification at some M_{GUT}, i.e.
$$\sin^2 \vartheta_w(M_{GUT}) = 3/8$$
- The Yukawa of at least one up-type quark is trilinear (no flavons)
- All (most?) exotics can acquire masses $\sim M_s$

Promising models with electic $\Omega(2)$

[Olguín-Trejo, Pérez-Martínez, SRS (2018)]

symmetry	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-I	$\mathbb{Z}_6$-II	$\mathbb{Z}_6$-II	$\mathbb{Z}_6$-II	$\mathbb{Z}_6$-II
geometry	2	3	1	2	1	2	3	4
# models	149	27	30	30	363	349	353	356
symmetry	$\mathbb{Z}_7$	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-I	$\mathbb{Z}_8$-II	$\mathbb{Z}_8$-II	$\mathbb{Z}_{12}$-I	$\mathbb{Z}_{12}$-I
geometry	1	1	2	3	1	2	1	2
# models	1	268	246	389	2,023	505	556	555
symmetry	$\mathbb{Z}_{12}$-II	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$
geometry	1	1	2	3	5	6	7	8
# models	363	205	369	444	42	401	76	25
symmetry	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_2$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$
geometry	9	10	12	(1,1)	(1,6)	(2,1)	(2,4)	(3,1)
# models	27	21	3	10,580	86	6,158	328	22,305
symmetry	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_4$	$\mathbb{Z}_2\times\mathbb{Z}_6$-I	$\mathbb{Z}_2\times\mathbb{Z}_6$-I
geometry	(4,1)	(5,1)	(6,1)	(7,1)	(8,1)	(9,1)	1	2
# models	4,519	2,116	3,246	2,667	911	2,142	583	353
symmetry	$\mathbb{Z}_3\times\mathbb{Z}_3$	$\mathbb{Z}_3\times\mathbb{Z}_3$	$\mathbb{Z}_3\times\mathbb{Z}_3$	$\mathbb{Z}_3\times\mathbb{Z}_3$	$\mathbb{Z}_3\times\mathbb{Z}_3$	$\mathbb{Z}_3\times\mathbb{Z}_6$	$\mathbb{Z}_3\times\mathbb{Z}_6$	$\mathbb{Z}_4\times\mathbb{Z}_4$
geometry	(1,1)	(1,4)	(2,1)	(3,1)	(4,1)	1	2	1
# models	1,108	8	1,952	6	215	4,493	540	28,649
symmetry	$\mathbb{Z}_4\times\mathbb{Z}_4$	$\mathbb{Z}_4\times\mathbb{Z}_4$	$\mathbb{Z}_4\times\mathbb{Z}_4$	$\mathbb{Z}_6\times\mathbb{Z}_6$				
geometry	2	3	4	1				
# models	9,853	5,522	4,730	3,696				

http://stringpheno.fisica.unam.mx/stringflavor

121,246

semi-realistic models!

Messages from eclectic flavor symmetries

Eclectic flavor symmetries

$G_{\text{eclectic}} = G_{\text{modular}} \cup G_{\text{traditional}}$ with $G_{\text{modular}} \subset Out(G_{\text{traditional}})$

Also happens in models based on magnetized tori. See Ohki, Uemura, Watanabe (2020); Otsuka's talk(?)

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Many semi-realistic models → viable phenomenology? see talk by Alex Baur

For $\mathbb{T}^2/\mathbb{Z}_K$, $K \neq 2$, $\mathrm{SL}(2,\mathbb{Z})_U$ broken to a discrete R symmetry

What happens with $K = 2$?

Siegel modular flavor group

from string theory

Baur, Kade, Nilles, SRS, Vaudrevange: 2008.07534, 2012.09586, 2104.03981

Orbifold $\mathbb{T}^2/\mathbb{Z}_2$

Translational outer automorphisms of S_{Narain}: $G_{\text{traditional}} = D_8 \times D_8/\mathbb{Z}_2$

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Translational outer automorphisms of S_{Narain}: $G_{\text{traditional}} = D_8 \times D_8/\mathbb{Z}_2$

G_{modular} contains *everything* because $\langle U \rangle =$ free

$h_\Sigma =$	S_U	T_U	S_T	T_T	M	K_*
$U \xrightarrow{h_\Sigma}$	$-1/U$	$U+1$	U	U	T	$-\bar{U}$
$T \xrightarrow{h_\Sigma}$	T	T	$-1/T$	$T+1$	U	$-\bar{T}$

$$G_{\text{modular}} = (S_3^T \times S_3^U) \rtimes \mathbb{Z}_4^M \rtimes \mathbb{Z}_2^{\mathcal{CP}} \cup \mathbb{Z}_4^R$$

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$T \xrightarrow{h_\Sigma}$	T	T	$-1/T$	$T+1$	U	$-\bar{T}$

$$G_{\text{modular}} = (S_3^T \times S_3^U) \rtimes \mathbb{Z}_4^M \rtimes \mathbb{Z}_2^{\mathcal{CP}} \cup \mathbb{Z}_4^R$$

$\Phi_{(n_T,n_U)} =$	$\Phi_{(0,0)}$	$\Phi_{(-1,-1)}$	$\Phi_{(-1/2,-1/2)}$	$\Phi_{(-3/2,\,1/2)}$	$\Phi_{(\,1/2,-3/2)}$	$\hat{Y}^{(2)}_{\mathbf{4}_3}$	$\mathcal{W}$
$D_8 \times D_8/\mathbb{Z}_2$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{1}_0$	$\mathbf{1}_0$
$S_3^T \times S_3^U$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{4}_1$	$(\mathbf{4}_1 \oplus \mathbf{4}_1)$		$\mathbf{4}_3$	$\mathbf{1}_0$
n_T	0	-1	$-1/2$	$-3/2$	$1/2$	2	-1
n_U	0	-1	$-1/2$	$1/2$	$-3/2$	2	-1
$\mathbb{Z}_4^R$	0	2	3	1	1	0	$2 \mod 4$

Modular transformations in $\mathbb{T}^2/\mathbb{Z}_2$

Observation: if T and U are included in a modulus matrix

$$\Omega := \begin{pmatrix} T & 0 \\ 0 & U \end{pmatrix} \qquad \text{subject to} \quad \operatorname{Im}\Omega > 0$$

all modular transformations (w/o K_*) are 4×4 matrices

$$\mathcal{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(4,\mathbb{Z}) \qquad \text{e.g.} \quad \mathcal{M}_{(\gamma_T,\gamma_U)} = \begin{pmatrix} a_U & 0 & b_U & 0 \\ 0 & a_T & 0 & b_T \\ c_U & 0 & d_U & 0 \\ 0 & c_T & 0 & d_T \end{pmatrix}$$

with

$$\mathrm{Sp}(4,\mathbb{Z}) = \{\mathcal{M} \in \mathbb{Z}^{4\times4} | \mathcal{M}^T J \mathcal{M} = J\} \quad \text{and} \quad J = \begin{pmatrix} 0 & \mathbb{1}_2 \\ -\mathbb{1}_2 & 0 \end{pmatrix}$$

Including K_*, we need

$$\mathrm{GSp}(4,\mathbb{Z}) = \{\mathcal{M} \in \mathbb{Z}^{4\times4} | \mathcal{M}^T J \mathcal{M} = \pm J\}$$

Modular transformations of $\mathbb{T}^2/\mathbb{Z}_2$ vs $\mathrm{Sp}(4,\mathbb{Z})$

symmetry	$\mathrm{Sp}(4,\mathbb{Z})$	$\mathrm{O}_{\hat\eta}(2,2,\mathbb{Z})$	transformation of moduli
$\mathrm{SL}(2,\mathbb{Z})_T$	$\mathcal{M}_{(\mathrm{S},\mathbb{1}_2)}$	S_T	$T \to -\frac{1}{T}$ $U \to U$
	$\mathcal{M}_{(\mathrm{T},\mathbb{1}_2)}$	T_T	$T \to T+1$ $U \to U$
$\mathrm{SL}(2,\mathbb{Z})_U$	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{S})}$	S_U	$T \to T$ $U \to -\frac{1}{U}$
	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{T})}$	T_U	$T \to T$ $U \to U+1$
Mirror	$\mathcal{M}_\times$	M	$T \to U$ $U \to T$
?	$\mathcal{M}\binom{\ell}{m}$	?	
$\mathcal{CP}$-like	$\mathcal{M}_* \in \mathrm{GSp}(4,\mathbb{Z})$	K_*	$T \to -\bar{T}$ $U \to -\bar{U}$

Modular transformations of $\mathbb{T}^2/\mathbb{Z}_2$ vs $\mathrm{Sp}(4,\mathbb{Z})$

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$\mathrm{SL}(2,\mathbb{Z})_T$	$\mathcal{M}_{(\mathrm{S},\mathbb{1}_2)}$	S_T	$T\to-\frac{1}{T}$ $U\to U$
	$\mathcal{M}_{(\mathrm{T},\mathbb{1}_2)}$	T_T	$T\to T+1$ $U\to U$
$\mathrm{SL}(2,\mathbb{Z})_U$			
Mirror			
?	$\mathcal{M}\binom{\ell}{m}$	?	
$\mathcal{CP}$-like	$\mathcal{M}_*\in\mathrm{GSp}(4,\mathbb{Z})$	K_*	$T\to-\bar T$ $U\to-\bar U$

Include continuous Wilson-line modulus Z

$$\Omega=\begin{pmatrix} T & Z \\ Z & U\end{pmatrix},\qquad Z=-a_2+Ua_1,\, a_i\in\mathbb{R}$$

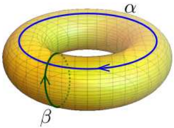
Demand Wilson-line shift $a_1\to a_1+\ell,\, a_2\to a_2+m$

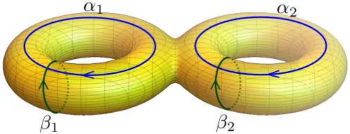
$\ell,m\in\mathbb{Z}$

Modular transformations of $\mathbb{T}^2/\mathbb{Z}_2$ vs $\mathrm{Sp}(4,\mathbb{Z})$

symmetry	$\mathrm{Sp}(4,\mathbb{Z})$	$\mathrm{O}_{\hat\eta}(2,3,\mathbb{Z})$	transformation of moduli
$\mathrm{SL}(2,\mathbb{Z})_T$	$\mathcal{M}_{(\mathrm{S},\mathbb{1}_2)}$	S_T	$T\to-\frac{1}{T}$ $U\to U-\frac{Z^2}{T}$ $Z\to-\frac{Z}{T}$
	$\mathcal{M}_{(\mathrm{T},\mathbb{1}_2)}$	T_T	$T\to T+1$ $U\to U$ $Z\to Z$
$\mathrm{SL}(2,\mathbb{Z})_U$	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{S})}$	S_U	$T\to T-\frac{Z^2}{U}$ $U\to-\frac{1}{U}$ $Z\to-\frac{Z}{U}$
	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{T})}$	T_U	$T\to T$ $U\to U+1$ $Z\to Z$
Mirror	$\mathcal{M}_\times$	M	$T\to U$ $U\to T$ $Z\to Z$
Wilson line shift	$\mathcal{M}\binom{\ell}{m}$	$\mathrm{W}\binom{\ell}{m}$	$T\to T+m\,(m\,U+2\,Z-\ell)$ $U\to U$ $Z\to Z+m\,U-\ell$
$\mathcal{CP}$-like	$\mathcal{M}_*\in$ $\mathrm{GSp}(4,\mathbb{Z})$	K_*	$T\to-\bar{T}$ $U\to-\bar{U}$ $Z\to-\bar{Z}$

Origin of the Siegel modular flavor group





$\mathrm{Sp}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})$	$\mathrm{Sp}(4,\mathbb{Z})$
$\mathrm{GSp}(2,\mathbb{Z}) \cong \mathrm{GL}(2,\mathbb{Z})$	$\mathrm{GSp}(4,\mathbb{Z})$
γ	$\mathcal{M}$
T	$\Omega(T,U,Z)$
Γ_N	$\Gamma_{2,N}$
Φ_n	Φ_{n_T,n_U}
modular forms	Siegel modular forms

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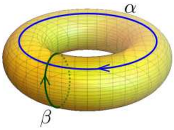

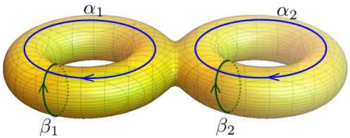

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Φ_n	Φ_{n_T,n_U}
modular forms	Siegel modular forms

Extend to $n_T \neq n_U$, new modular weight associated with Z?

Find out the exact form of *all* transformations

Compare with compactifications on CY

Ishiguro, Kobayashi, Otsuka (2021)

Metaplectic flavor symmetries

in the top–down approach

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see e.g. Liu, Yao, Qu, Ding (2020)

- $\widetilde{\Gamma} = \mathrm{Mp}(2,\mathbb{Z})$: metaplectic group = double cover of $\mathrm{SL}(2,\mathbb{Z})$

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- Note that Φ_n has $n \in \mathbb{Z}/2$!

Finite metaplectic flavor symmetries

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$$\widetilde{\Gamma}_{4N} = \frac{\mathrm{Mp}(2,\mathbb{Z})}{\widetilde{\Gamma}(4N)}, \qquad \widetilde{\Gamma}(4N) : \text{metaplectic congruence subgroup}$$

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Metaplectic modular forms $Y^{(n=1/2)}$ transform as

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D–brane compactifications and magnetized tori

- Many semi-realistic models based on intersecting D–branes

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- Modular properties of fields reveal modular flavor symmetries ☺

  Kobayashi, Otsuka (2019-21); Ohki, Uemura, Watanabe (2020); Ishiguro, Kikuchi, Ogawa, Uchida, Kobayashi, Otsuka,. . .

Internal components of matter fields

Almumin, Chen, Knapp-Pérez, SRS, Ratz, Shukla (2021); Tatsuta (2021)

"Wave-functions" $\psi^{j,M}$: solutions to the Dirac equation on a torus background with M magnetic fluxes

$$\psi^{j,M}(z,T) = (2M\,\mathrm{Im}\,T)^{1/4}\, e^{\pi \mathrm{i} M z \frac{\mathrm{Im}\,z}{\mathrm{Im}\,T}}\,\vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (Mz, MT)$$

$$0 \leq j \leq M-1 \qquad \Rightarrow \qquad M \text{ zero modes or ``flavors''}$$

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M solutions build generators of an M-dim vector space, whose elements transform as

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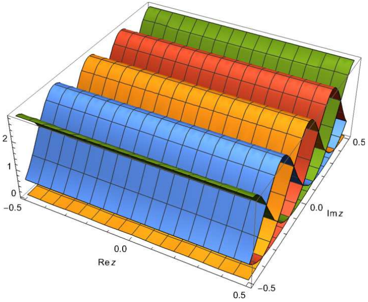
Valid for all M, also $M=3$ ☺

Intersecting wavefunctions & couplings



$$Y_{ijk}(\tilde{\zeta}, T) = g \int d^2 z \psi^{i,M_1} \psi^{j,M_2} (\psi^{k,M_3})^* \propto \vartheta \left[\begin{array}{c} \frac{\text{fluxes}}{\lambda} \\ 0 \end{array} \right] (\tilde{\zeta}/d, \lambda T)$$

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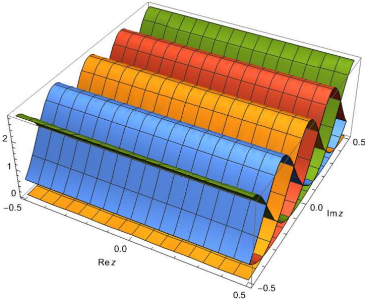

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wich are representations of $\widetilde{\Gamma}_{2\lambda}$ ☺

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From top-down to bottom-up

eclectic flavor symmetries

Eclectic flavor groups

Key observation: T' is subgroup of $Out(\Delta(54))$ ☺

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- Verify whether there is a third (class-inverting) outer automorphism acting as a $\mathbb{Z}_2$ $\mathcal{CP}$-like transformation to further enhance the eclectic flavor symmetry

Eclectic flavor groups

flavor group $\mathcal{G}_{\text{fl}}$	GAP ID	$\text{Aut}(\mathcal{G}_{\text{fl}})$	finite modular groups		eclectic flavor group
Q_8	[8, 4]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\mathbb{Z}_3 \times \mathbb{Z}_3$	[9, 2]	$\text{GL}(2,3)$	without $\mathcal{CP}$	S_3	$\Delta(54)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$	[108, 17]
A_4	[12, 3]	S_4	without $\mathcal{CP}$	S_3 S_4	S_4 S_4
			with $\mathcal{CP}$	–	–
T'	[24, 3]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\Delta(27)$	[27, 3]	[432, 734]	without $\mathcal{CP}$	S_3 T'	$\Delta(54)$ $\Omega(1)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$ $\text{GL}(2,3)$	[108, 17] [1296, 2891]
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Nilles, SR-S, Vaudrevange (2001.01736)

Quasi-Electic realization

of a simple lepton model

Quasi-eclectic picture $A_4 \times \Gamma_3 \to A_4$

Chen, Knapp-Pérez, Ramos-Hamud, SRS, Ratz, Shukla (2021)

	$(E_1^{\mathcal{C}}, E_2^{\mathcal{C}}, E_3^{\mathcal{C}})$	L	H_d	H_u	χ	φ	S_χ	S_φ	Y
$A_4^{\text{traditional}}$	$(\mathbf{1}_0, \mathbf{1_2}, \mathbf{1_1})$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
Γ_3	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$
modular weights	$(1,1,1)$	-1	0	0	0	0	0	0	2

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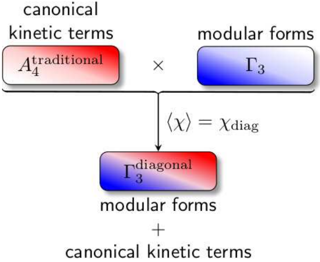

$$m_\nu = \frac{v_u^2 \varepsilon_1}{\sqrt{3}\Lambda} \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

phenomenology like Feruglio's first model

In summary

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Concluding remarks

- Finite modular flavor symmetries are great, but have open questions
- Toroidal orbifold compactifications of string theory reveal an
  $eclectic$ flavor structure = traditional $\cup$ modular symmetries
- In string models: moduli, modular weights, representations, charges of Φ and Y
- In $\mathbb{T}^2\mathbb{Z}_2$: natural to include Siegel flavor groups with 3rd modulus = Wilson line
- In magnetized tori: metaplectic flavor symmetries $\widetilde{\Gamma}_{4N}$ are direct
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- $Eclectic$ and $Quasi-eclectic$ flavors appear in bottom-up
- In string models, more useful constraints:
  matter modular weights, representations and charges defined by compactification

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To work on

- pheno & *eclectic* breakdown

  see Baur's talk
- $\mathcal{CP}$ and $\mathcal{CP}$ violation ?

  see Trautner's talk
- moduli stabilization ?
- complete Siegel picture ?
- non-supersymmetric constructions ?

Just in case...

Backup slides

Some things we *don't* know

- Need to explain
  - three flavors of SM particles
  - observed mass patterns
  - observed quark and lepton mixing *textures*
  - CKM, PMNS phases
  - neutrino physics
  - suppression of proton decay
  - baryogenesis
  - CP violation
  - origin of dark matter
  - ...

- Many proposed non-Abelian flavor (discrete) symmetries that (can) answer some of these questions

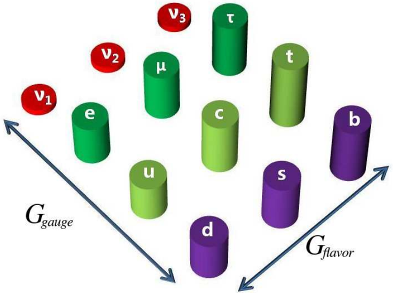
We do know: flavor symmetries...

- Extension of the group of symmetries of SM particles

$$G_{\mathsf{SM}} \times G_{flavor}$$

Typically $G_{flavor} \subset \mathrm{SU}(3)$

- Matter transforms under G_{flavor}, relating families



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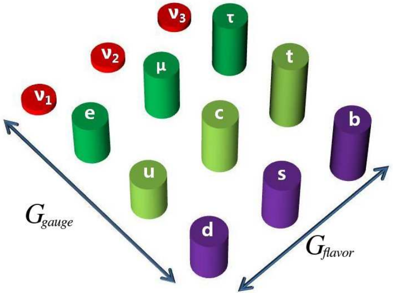

More technically, the Lagrangian is invariant under

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \xrightarrow{g} \rho(g) \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \qquad \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}, \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

for (reducible or non-reducible triplet) matrix reps. $\rho(g), g \in G_{flavor}$

$\Rightarrow$ mixtures of quarks in V_{CKM} and of leptons in U_{PMNS}

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$$S_3, D_4, Q_8, A_4, T_7, S_4, T', \Delta(27), \Delta(54), A_5, \Sigma(168), \ldots$$

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How to proceed with *traditional* flavor symmetries

- Take your favorite traditional flavor symmetry G_{flavor}

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- Choose your favorite representations for quark and lepton fields

e.g. quark doublets Q as $\mathbf{3}$ or $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ of $A_4, \ldots$

- Write your G_{flavor}-invariant Lagrangian $\mathcal{L}$ or superpotential W

$$\text{e.g. } \mathcal{L} \supset -y^u_{ij}\phi^* Q^i \bar{u}^j - y^d_{ij}\phi\, Q^i \bar{d}^j - y^e_{ij}\phi^* L^i \bar{e}^j - \frac{\lambda_{ij}}{\Lambda} L_i \phi\, \bar{L}_j \phi^*$$

- Introduce some flavon field s in some nontrivial representation, then give it a vev e.g. $\langle s \rangle = v_s(1, 0, \ldots)^T$ to break G_{fl}
- EW breakdown with $\langle \phi \rangle \neq 0$
- Diagonalize quark and lepton matrices to compute V_{CKM} and U_{PMNS} and adjust couplings and vevs to data

Modular

Flavor Symmetries

Modular symmetries as flavor symmetries

"Simplest" modular group: $\mathrm{SL}(2,\mathbb{Z})$

$$\gamma \in \mathrm{SL}(2,\mathbb{Z}): \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad \det\gamma = ad - bc = 1$$

generators: $$\mathrm{S} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathrm{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

general *presentation*: $$\left\langle \mathrm{S}, \mathrm{T} \,|\, \mathrm{S}^4 = (\mathrm{ST})^3 = \mathbb{1}, \quad \mathrm{S}^2\mathrm{T} = \mathrm{TS}^2 \right\rangle$$

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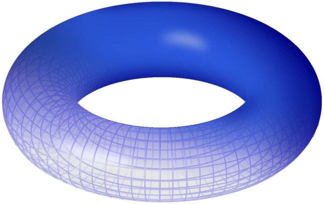
Describe deformations of a torus $\mathbb{T}^2$ defined by a modulus T



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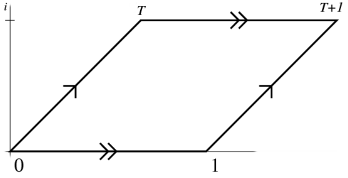

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Congruence modular subgroups: $\Gamma(N) \subset \mathrm{SL}(2,\mathbb{Z})$

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e.g. Liu, Ding (2019)

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Finite modular subgroups: $\Gamma_N \cong \mathrm{PSL}(2,\mathbb{Z})/\bar{\Gamma}(N)$ $(\mathrm{PSL}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})/\{\pm\mathbb{1}\})$

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e.g. de Adelhaart, Feruglio, Hagedorn (2011)

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Superfields build reps. of Γ_N or Γ'_N; transform as

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Admissible iff

$$W(\Phi_{n_1},\ldots) \xrightarrow{\gamma} (cT+d)^{-1}\,\mathbb{1}\,W(\Phi_{n_1},\ldots), \qquad \text{i.e. } n_Y+\sum n_i=-1, \quad \prod \rho(\gamma)=\mathbb{1}$$

Note the nontrivial $automorphy\ factor$ $(cT+d)^{-1}$ $\rightarrow$ W covariant

How to proceed with *modular* flavor symmetries

- Take your favorite symmetry: $G_{mod} = \Gamma_N \in \{S_3, A_4, S_4, A_5, \ldots\}$
- Choose your favorite representations $\rho(\gamma)$ for quark and lepton fields

e.g. quark doublets Q as $\mathbf{3}$ or $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ of $\Gamma_3 \cong A_4, \ldots$

- Pick your favorite modular weights n_i and n_Y
- Write your G_{mod}-covariant superpotential W

$$\text{e.g. } W \supset \hat{Y}^u H_u Q \bar{u} + \hat{Y}^d H_d Q \bar{d} + \hat{Y}^e H_d L \bar{e} + \frac{\hat{Y}}{\Lambda} L H_u \, L H_u$$

- Take your favorite inv. Kähler potential K; typical choice $K = \sum |\Phi_{n_i}|^2$

MANY other modular invariant K possible! - Chen, SR-S, Ratz (1909.06910)

- Choose a $\langle T \rangle \neq 0 \quad \rightarrow \quad$ nontrivial rep. of $\hat{Y}(\langle T \rangle)$ breaks G_{mod}
- EW breakdown with $\langle H_u \rangle, \langle H_d \rangle \neq 0$
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models for quarks and models for leptons exist

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Perhaps strings could offer a solution ?

Some candidate flavor symmetries: the old picture

- Orbifold $\mathcal{O} = \mathbb{R}^6/S \leftarrow$ space group: rotations, reflexions and shifts
- Localized states are subject to two kinds of "geometric" symmetries

  A: permutation symmetries among fixed points $\rightarrow S_n$

  B: stringy selection rules for their interactions

  $$\rightarrow D_S = \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \mathbb{Z}_{N_3} \times \cdots$$

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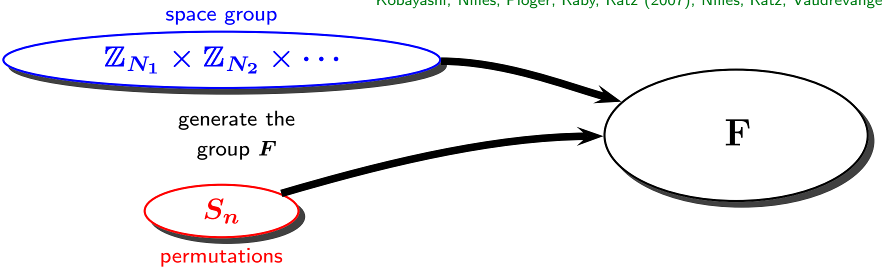

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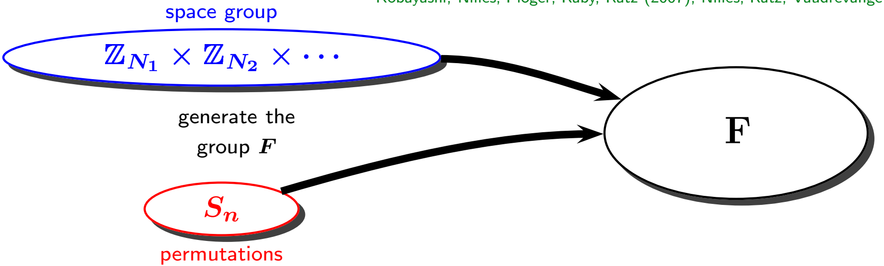

$$\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \cdots \cup S_n = F$$

Abelian   non-Abelian   non-Abelian!

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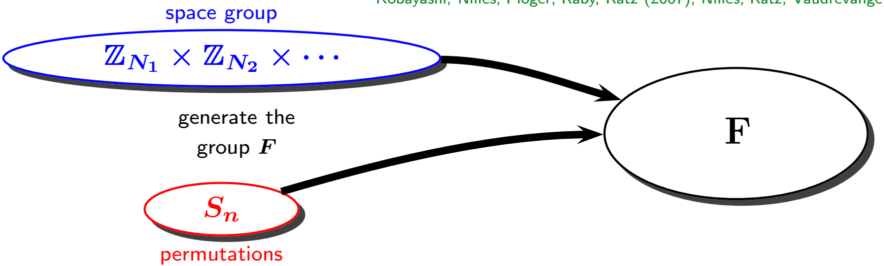
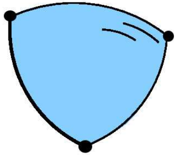


$$\underbrace{\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \cdots}_{\text{Abelian}} \rtimes \underbrace{S_n}_{\text{non-Abelian}} = \underbrace{F}_{\text{non-Abelian!}}$$

Some candidate flavor symmetries: the *old* picture

$\mathbb{T}^2/\mathbb{Z}_3$



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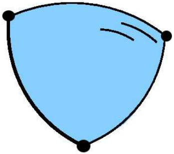

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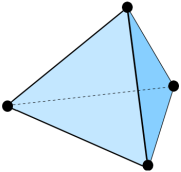

$$\rightarrow \quad F = D_4 \times D_4/\mathbb{Z}_2$$

Eclectic flavor symmetries

Nilles, SR-S, Vaudrevange (2001.01736)

Eclectic flavor groups

Key observation: T' is an outer automorphism group of $\Delta(54)$ ☺

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$$(u_S)^4 = (u_S \circ u_T)^3 = (u_T)^N = \mathbb{1}, \qquad u_S^2 \circ u_T = u_T \circ u_S^2$$

Eclectic flavor groups

Key observation: T' is an outer automorphism group of $\Delta(54)$ ☺

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- Eclectic group $\cong$ multiplicative closure of G_{fl} and G_{mod}