
Hadronic interactions from lattice QCD with $O(a)$ improved Wilson fermions

Hartmut Wittig

in collaboration with:

J. Green, A. Hanlon, P. Junnarkar, M. Padmanath, S. Paul & Baryon Scattering Collaboration (BaSc)

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Outline

Introduction: Dibaryons in phenomenology and experiment

Lattice formalism and the Mainz dibaryon project

The H dibaryon: Lattice results

Nucleon-nucleon scattering

Charmed tetraquarks

Summary & Conclusions

The H Dibaryon

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Perhaps a Stable Dihyperon*

R. L. Jaffe†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of -2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the H are discussed.

- MIT bag model predicts dihyperon state (H) with $I = 0$, $S = -2$, $J^P = 0^+$ and mass $m_H = 2150 \text{ MeV}$, which must decay weakly
- “Nagara event”: Observation of a ${}_{\Lambda\Lambda}^6\text{He}$ double-hypernucleus

Binding energy: $B_{\Lambda\Lambda} = 7.25 \pm 0.19 \left({}^{+0.18}_{-0.11} \right) \text{ MeV}$

Interpreted as sequential weak decay of ${}_{\Lambda\Lambda}^6\text{He}$: $m_H > 2m_{\Lambda} - B_{\Lambda\Lambda} = 2223.7 \text{ MeV}$ @ 90% CL

Hyperon-hyperon interactions and the H Dibaryon

Hyperon-hyperon interactions

- Relevant for the physics of (double) hyper nuclei, neutron-rich matter, neutron stars
- No experimental information available ($\Lambda\Lambda$ -scattering not viable experimentally)

Deeply bound $udsuds$ state (“sexaquark”) proposed / discussed as dark matter candidate:
[G.R. Farrar, A. Strumia et al.,...]

$$m_H < 2(m_p + m_e) = 1877.6 \text{ MeV} \quad \Rightarrow \quad H \text{ dibaryon absolutely stable}$$

$$m_H > 2(m_p + \text{B.E.}) = 1860 \text{ MeV} \quad \Rightarrow \quad \text{Nuclei absolutely stable}$$

Recall: $2m_\Lambda = 2230 \text{ MeV}$

$$m_H = 2150 \text{ MeV} \quad (\text{Jaffe's bag model estimate})$$

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda} = 2223.7 \text{ MeV} \quad (\text{Nagara event})$$

→ Scenario requires very large binding energy of $\approx 360 \text{ MeV}$

Dibaryons in Lattice QCD

Flavour structure of two octet baryons:

$$\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27})_S \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}})_A$$

- H dibaryon lies in **1**-dimensional irrep of $SU(3)_{\text{flavour}}$
- Upon $SU(3)$ -symmetry breaking, **8** and **27** mix with singlet
- Singlet, octet and **27**plet operators constructed from linear combinations of $\Lambda\Lambda$, $\Sigma\Sigma$ and $N\Xi$ operators

e.g.
$$[\mathbf{1}] = -\sqrt{\frac{1}{8}}[\Lambda\Lambda]^{I=0} + \sqrt{\frac{3}{8}}[\Sigma\Sigma]^{I=0} + \sqrt{\frac{4}{8}}[N\Xi_s]^{I=0}$$

Other dibaryons:

- Dineutron lies in **27** irrep
- Deuteron lies in $\overline{\mathbf{10}}$ irrep with $J^P = 1^+$

Dibaryons in Lattice QCD

Interpolating operators for the H dibaryon

Hexaquark operators (inspired by Jaffe's original bag model calculation):

$$[rstuvw] = \epsilon_{ijk} \epsilon_{lmn} \left(s^a C \gamma_5 P_+ t^b \right) \left(v^l C \gamma_5 P_+ w^m \right) \left(r^k C \gamma_5 P_+ u^n \right)$$

$$H^{(1)} = \frac{1}{48} ([sudsud] - [udusds] - [dudsus])$$

$$H^{(27)} = \frac{1}{48 \sqrt{3}} (3[sudsud] + [udusds] - [dudsus])$$

Momentum-projected two-baryon operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk} \left(s^i C \gamma_5 P_+ t^j \right) r_\alpha^k$$

$$(BB)(P; t) = \sum_x e^{-ip_1 \cdot x} B_1(\mathbf{x}, t) (C \gamma_5 P_+) \sum_y e^{-ip_2 \cdot y} B_2(\mathbf{y}, t), \quad \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$$

→ project onto $(BB)^{(1)}, (BB)^{(8)}, (BB)^{(27)}$

Dibaryons in Lattice QCD

Interpolating operators for other dibaryon channels

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk} \left(s^i C \gamma_5 P_+ t^j \right) r_\alpha^k$$

Spin-1 interpolator:

$$(BB)_i(\mathbf{p}_1, \mathbf{p}_2) = \sum_{\mathbf{x}} e^{-i\mathbf{p}_1 \cdot \mathbf{x}} B_1(\mathbf{x}, t) (C \gamma_i P_+) \sum_{\mathbf{y}} e^{-i\mathbf{p}_2 \cdot \mathbf{y}} B_2(\mathbf{y}, t)$$

Deuteron:

$$(BB)_{i; T_1^+}^{(n)} = \frac{1}{N} \sum_{\mathbf{p}; p^2=n} (BB)_i(-\mathbf{p}, \mathbf{p})$$

Dibaryons in Lattice QCD: Correlator matrices and GEVP

Consider set on N_{op} interpolating operators for a given hadron:

Correlation matrix: $C_{ij}(\mathbf{P}, \tau) = \langle O_i(\mathbf{P}, t) O_j(\mathbf{P}, t')^\dagger \rangle, \quad \tau = t - t', \quad i, j = 1, \dots, N_{\text{op}}$

- Variational method: solve Generalised Eigenvalue Problem (GEVP)

$$\mathbf{C}(t_1) v_n(t_1, t_0) = \lambda_n(t_1, t_0) \mathbf{C}(t_0) v_n(t_1, t_0)$$

$$w_n^\dagger(t_1, t_0) \mathbf{C}(t_1) = \lambda_n(t_1, t_0) w_n^\dagger(t_1, t_0) \mathbf{C}(t_0), \quad n = 1, \dots, N_{\text{op}}$$

- Project onto approximately diagonal correlator: $\Lambda_{mn}(t) = w_n^\dagger \mathbf{C}(t) v_m$
- Compute the effective n^{th} energy level: $E_n^{\text{eff}}(t) = \frac{1}{\Delta t} \ln \frac{\Lambda_{nn}(t)}{\Lambda_{nn}(t + \Delta t)}$
- Distillation: quark propagator with Laplace-Heaviside (LapH) smearing:

$$\mathcal{S} D^{-1} \mathcal{S}, \quad \mathcal{S}^{(t)}(\mathbf{x}, \mathbf{y}) = \sum_{k=1}^{N_{\text{LapH}}} V^{(k)}(\mathbf{x}, t) \otimes V^{(k)}(\mathbf{y}, t)^\dagger \quad (V^{(k)} : k^{\text{th}} \text{ eigenvector of Laplacian } \Delta)$$

Dibaryons in Lattice QCD: Finite-volume quantisation

Two-particle scattering: $\det \left(\tilde{\mathcal{K}}^{-1}(p^2) - B(p^2, L) \right) = 0$

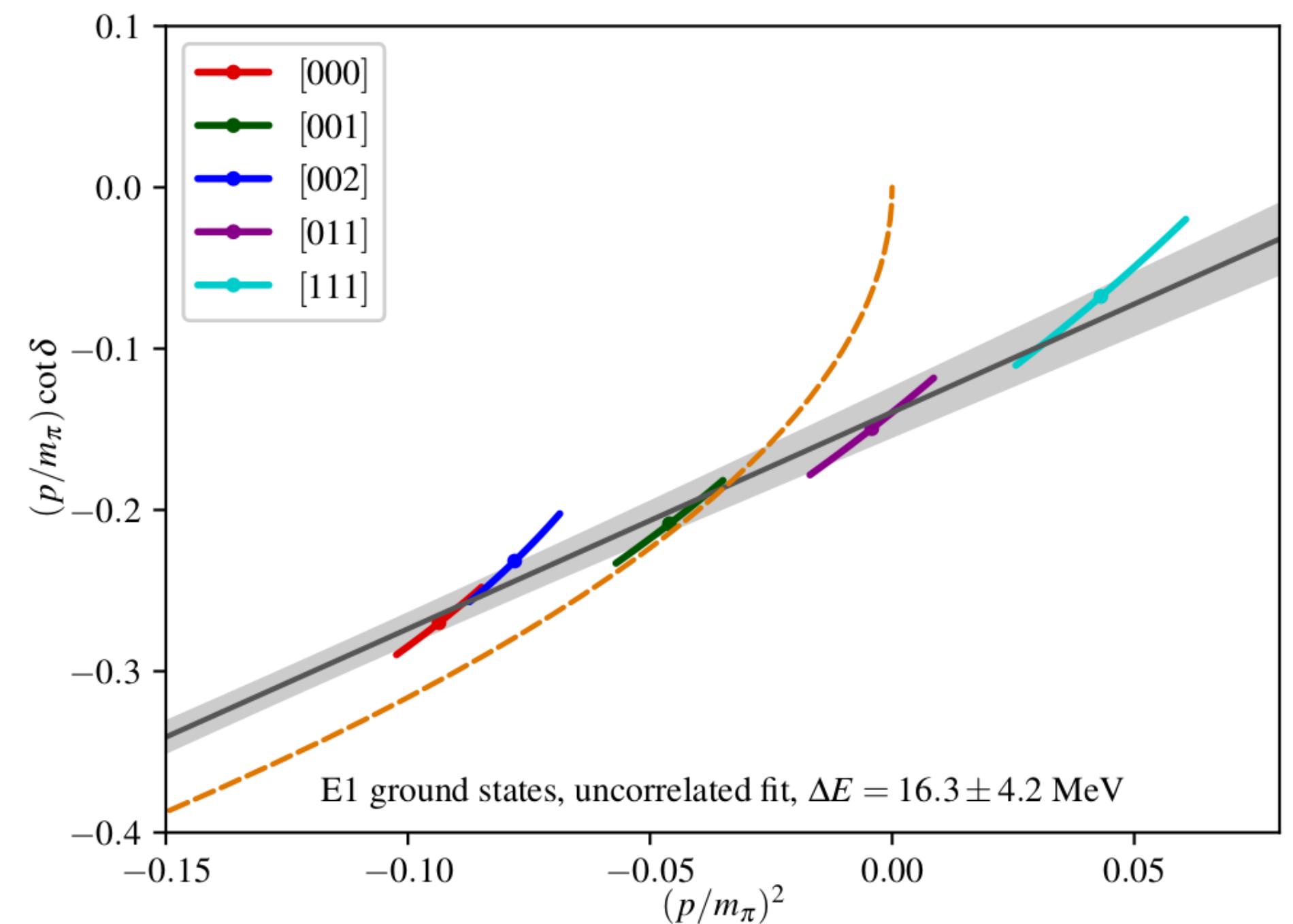
S-wave: $p \cot \delta(p) = \frac{2}{\gamma L \sqrt{\pi}} Z_{00}^D(1, q^2), \quad q = pL/2\pi, \quad Z_{00}^D(1, q^2) = \frac{1}{\sqrt{4\pi}} \left\{ \sum_{q^2 \neq n^2}^{\Lambda_n} \frac{1}{q^2 - n^2} - 4\pi \Lambda_n \right\}$

Scattering momentum: $p^2 = \frac{1}{4}(E^2 - \mathbf{P} \cdot \mathbf{P}) - m_\Lambda^2$
(H dibaryon)

Pole of scattering amplitude: $\mathcal{A} \propto \frac{1}{p \cot \delta(p) - ip}$

Fit to effective range expansion:

$$\Rightarrow p \cot \delta_0(p) = A + Bp^2 + \dots \stackrel{!}{=} -\sqrt{-p^2}$$



The “Mainz” Dibaryon project

Past and present members:

Anthony Francis, Jeremy Green, Andrew Hanlon, Parikshit Junnarkar, Padmanath Madanagopalan, Chuan Miao, Srijit Paul, Tom Rae, H.W.

Based on CLS ensembles with $N_f = 2$ and $N_f = 2 + 1$ flavours of $O(a)$ improved Wilson quarks

Methodology:

- Variational method: point-to-all ($N_f = 2$) and timeslice-to-all propagators
- Exact distillation: timeslice-to-all propagators *[Peardon et al., PRD 80 (2009) 054506]*
- Finite-volume quantisation
- Various dibaryon channels — extension to charmed tetraquarks

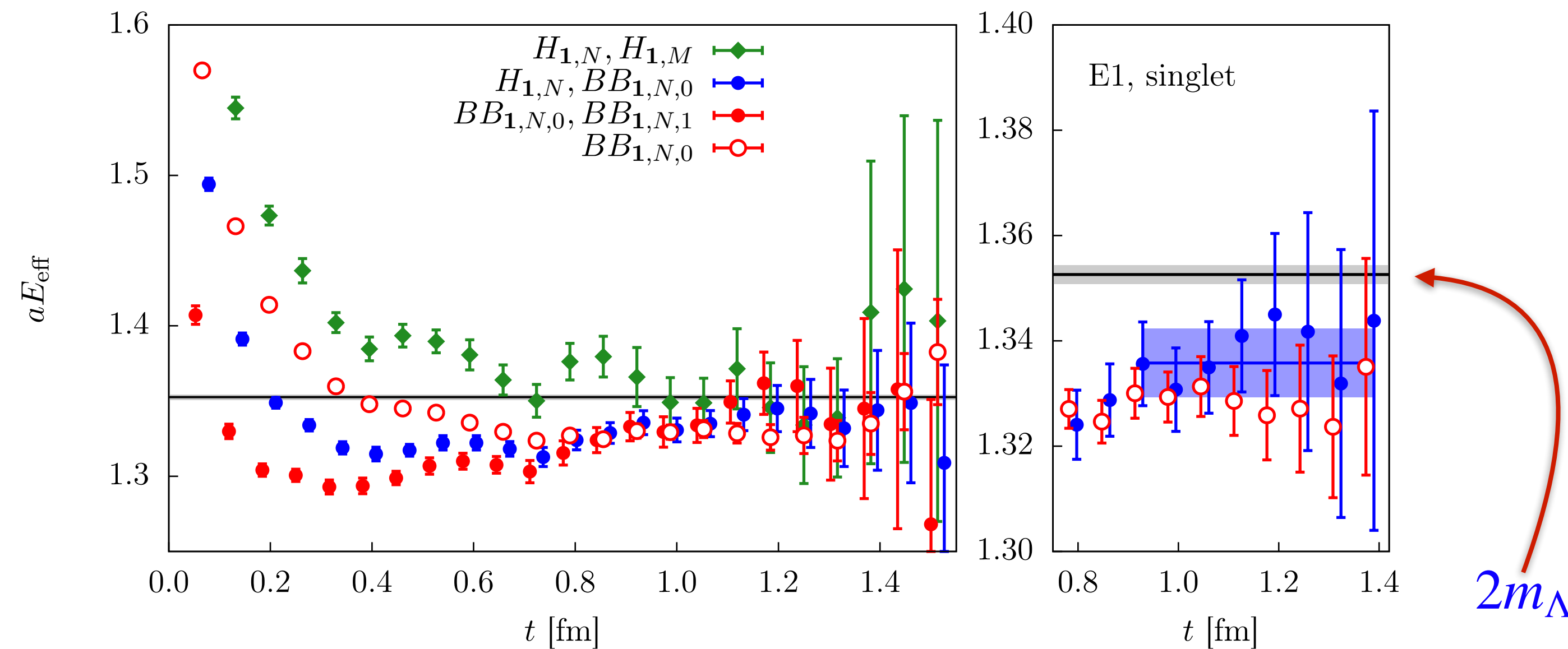
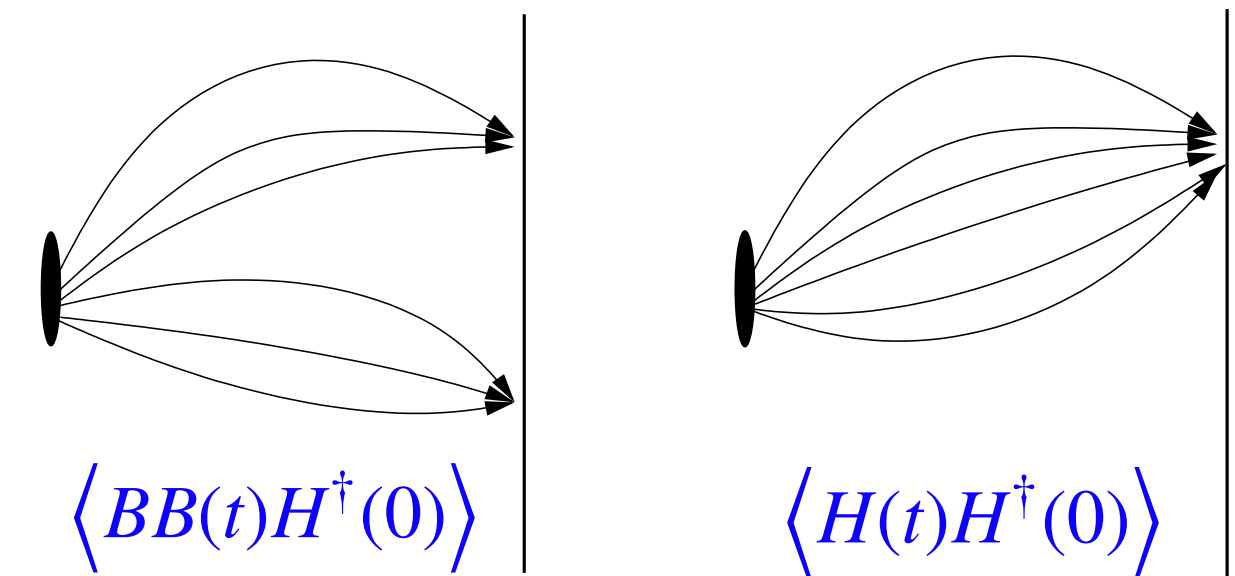
Collaboration within “Baryon Scattering” (BaSc) Collaboration

- Stochastic LapH on large physical volumes *[Morningstar et al., PRD 83 (2011) 114505]*
- Alternative discretisations: exponentiated Clover, domain wall

The H Dibaryon: Pilot study in 2-flavour QCD

- Pion masses match earlier calculations by NPLQCD and HALQCD
- Point-to-all propagators: asymmetric GEVP
- Hexaquark operators have poor overlap onto ground state
- Distillation: much better signal
- Finite-volume quatisation yields smaller binding energy

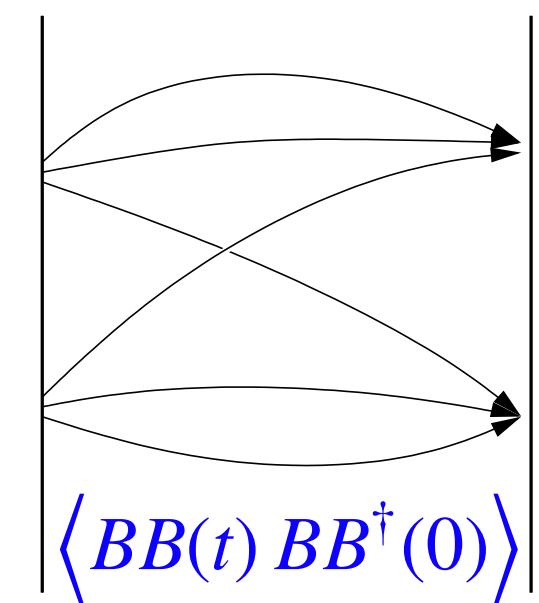
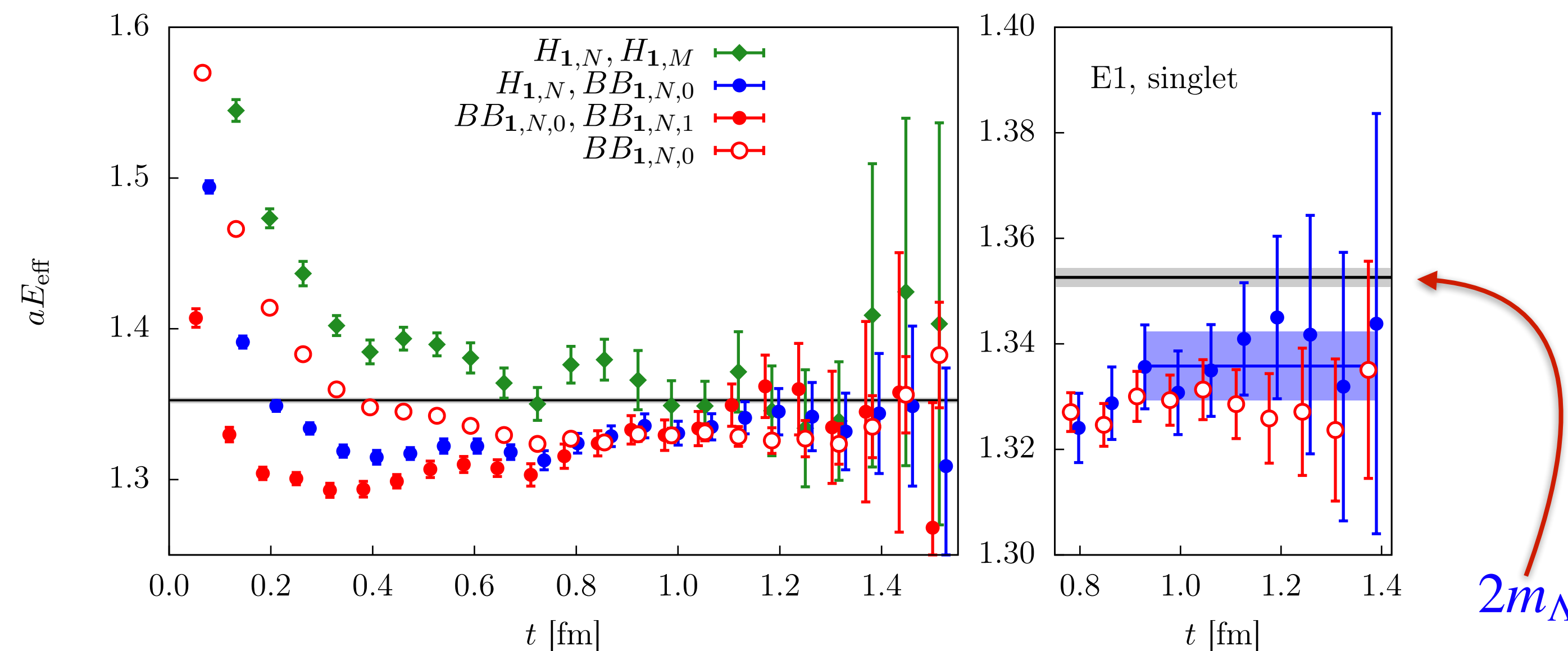
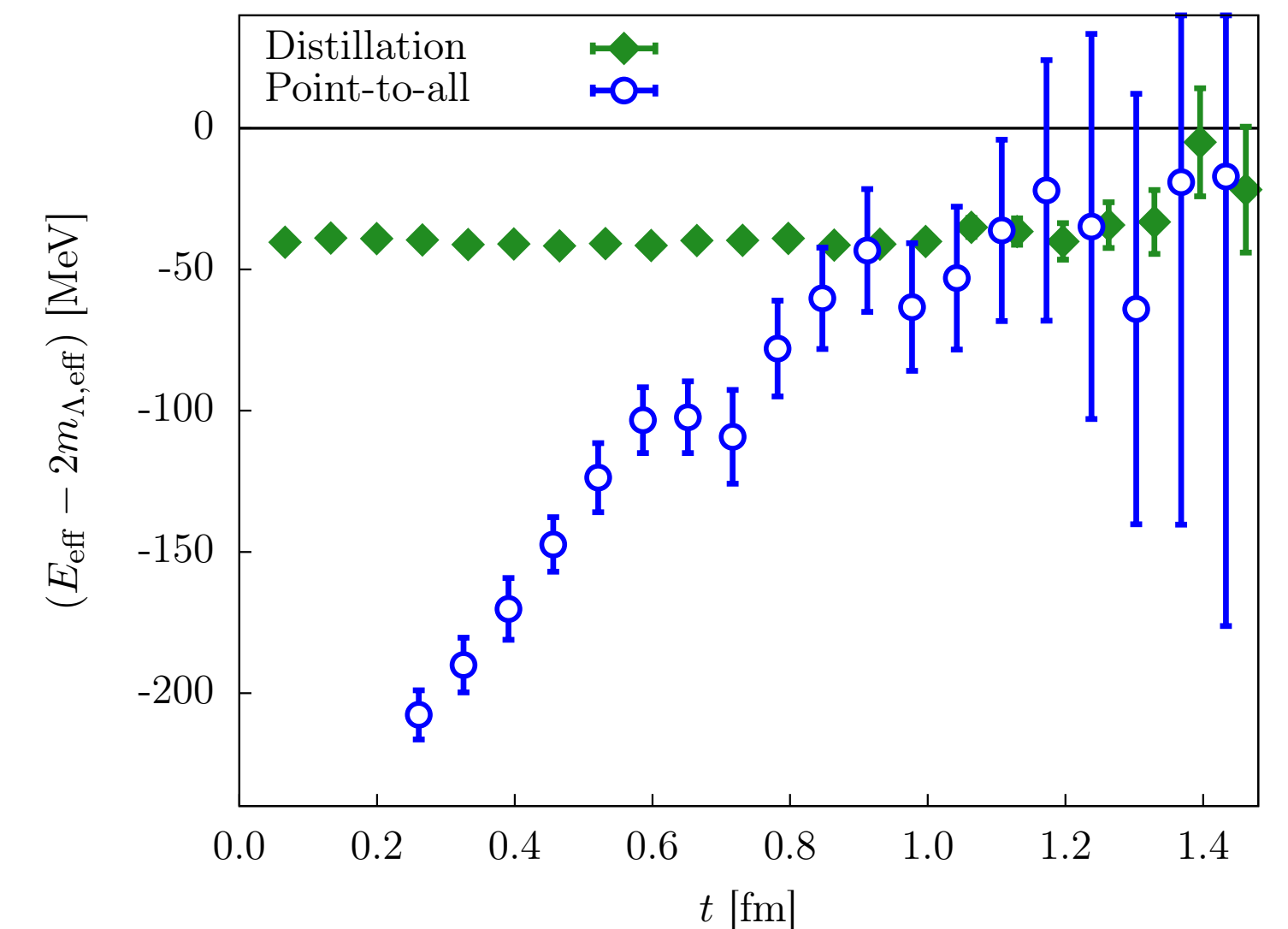
[Francis et al., PRD 99 (2019) 074505]



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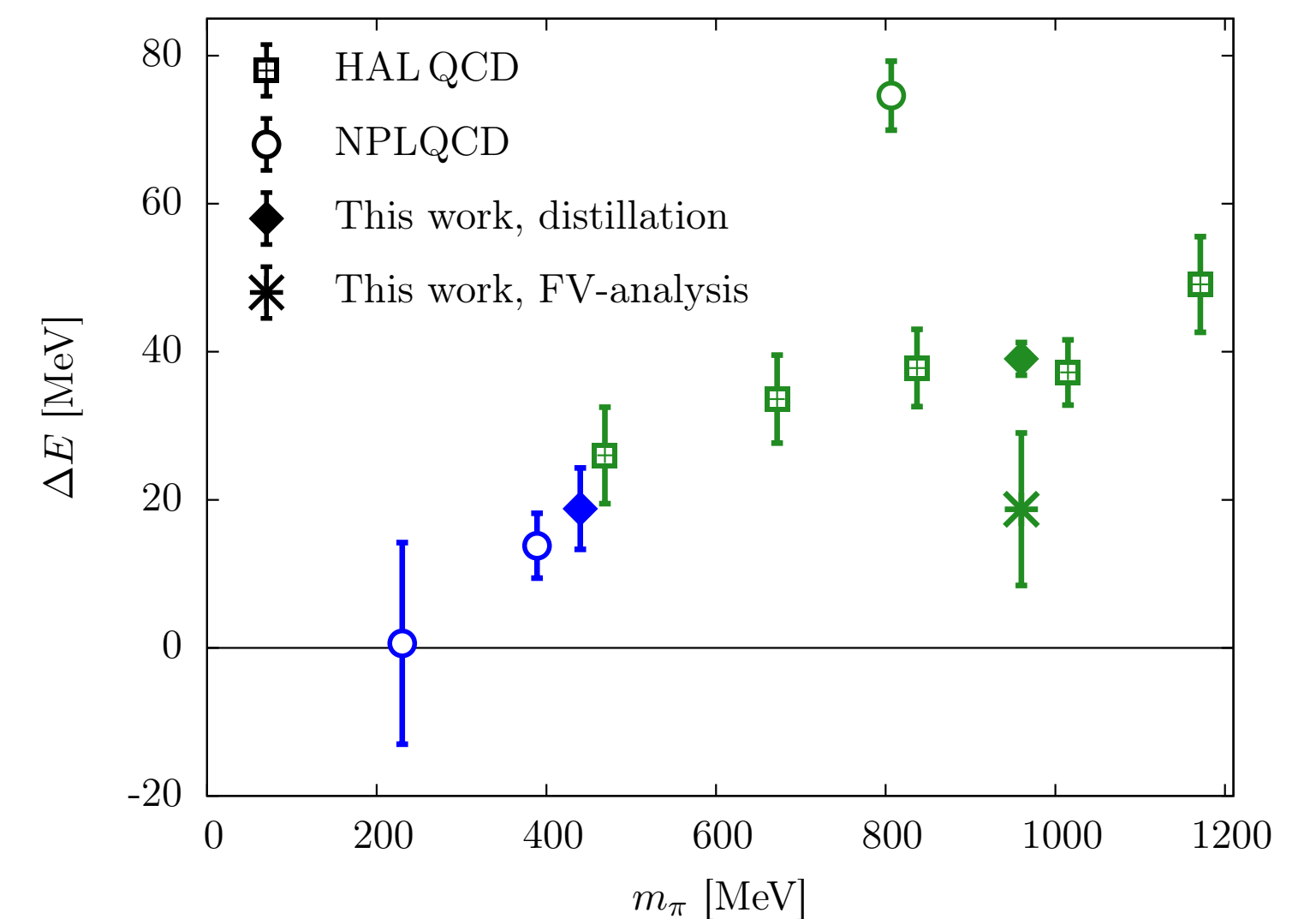
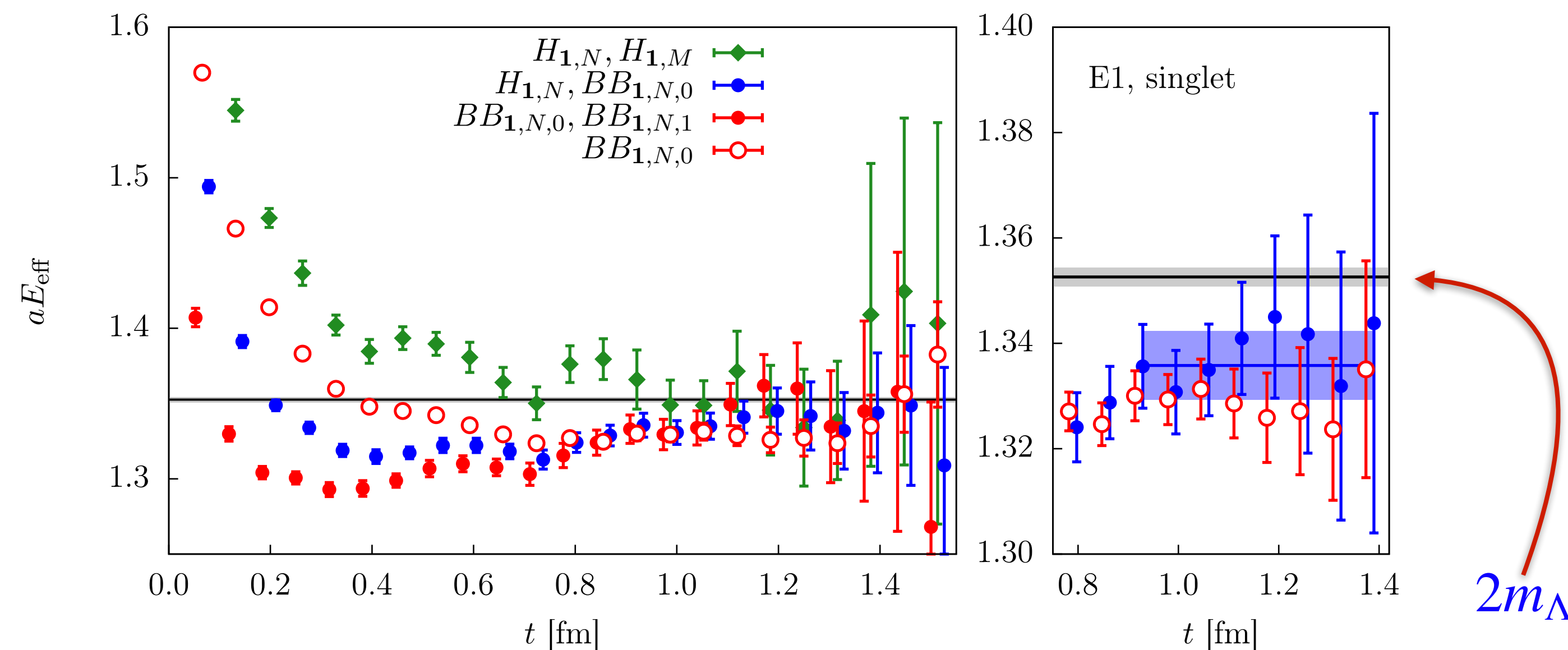
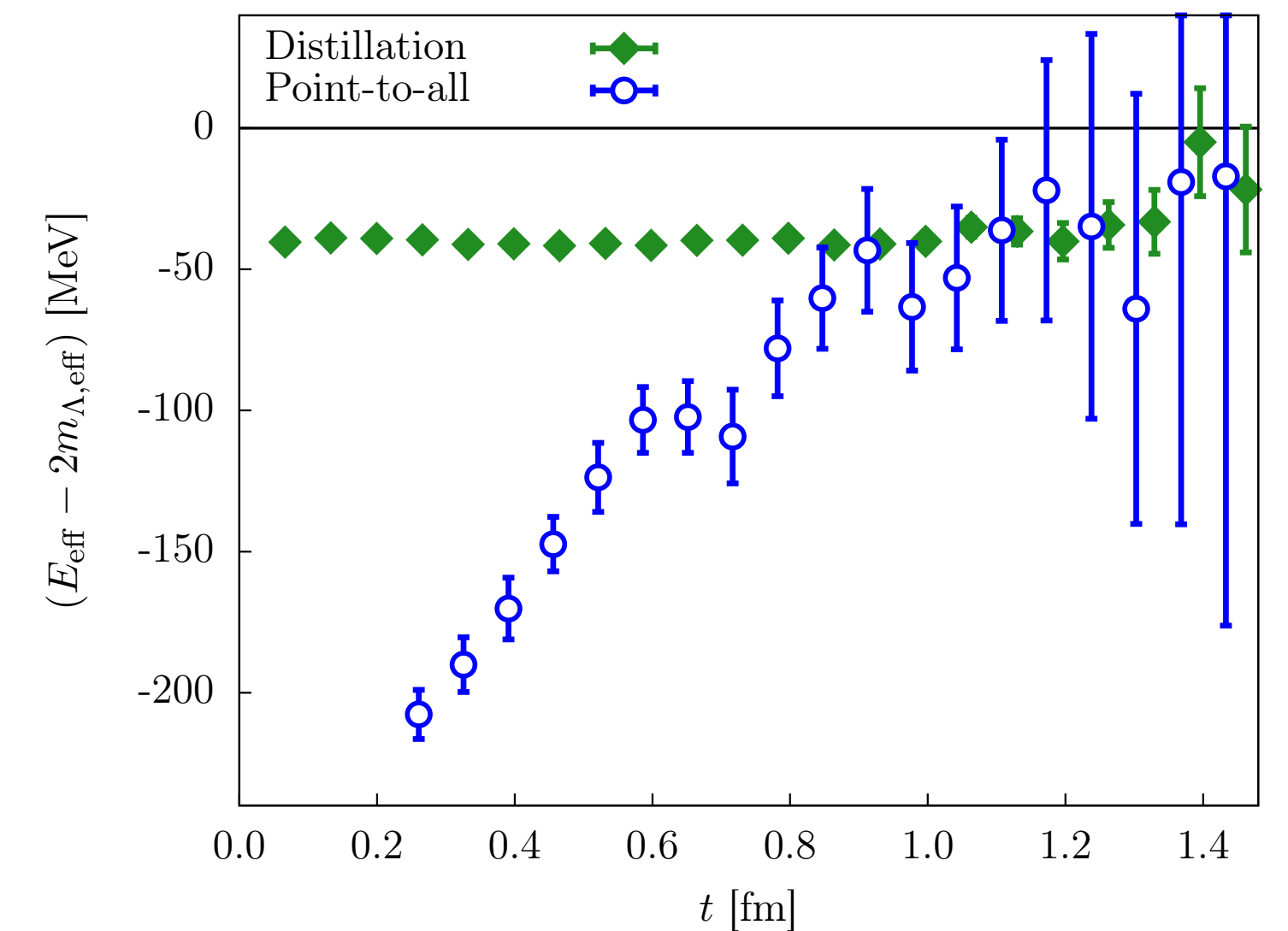
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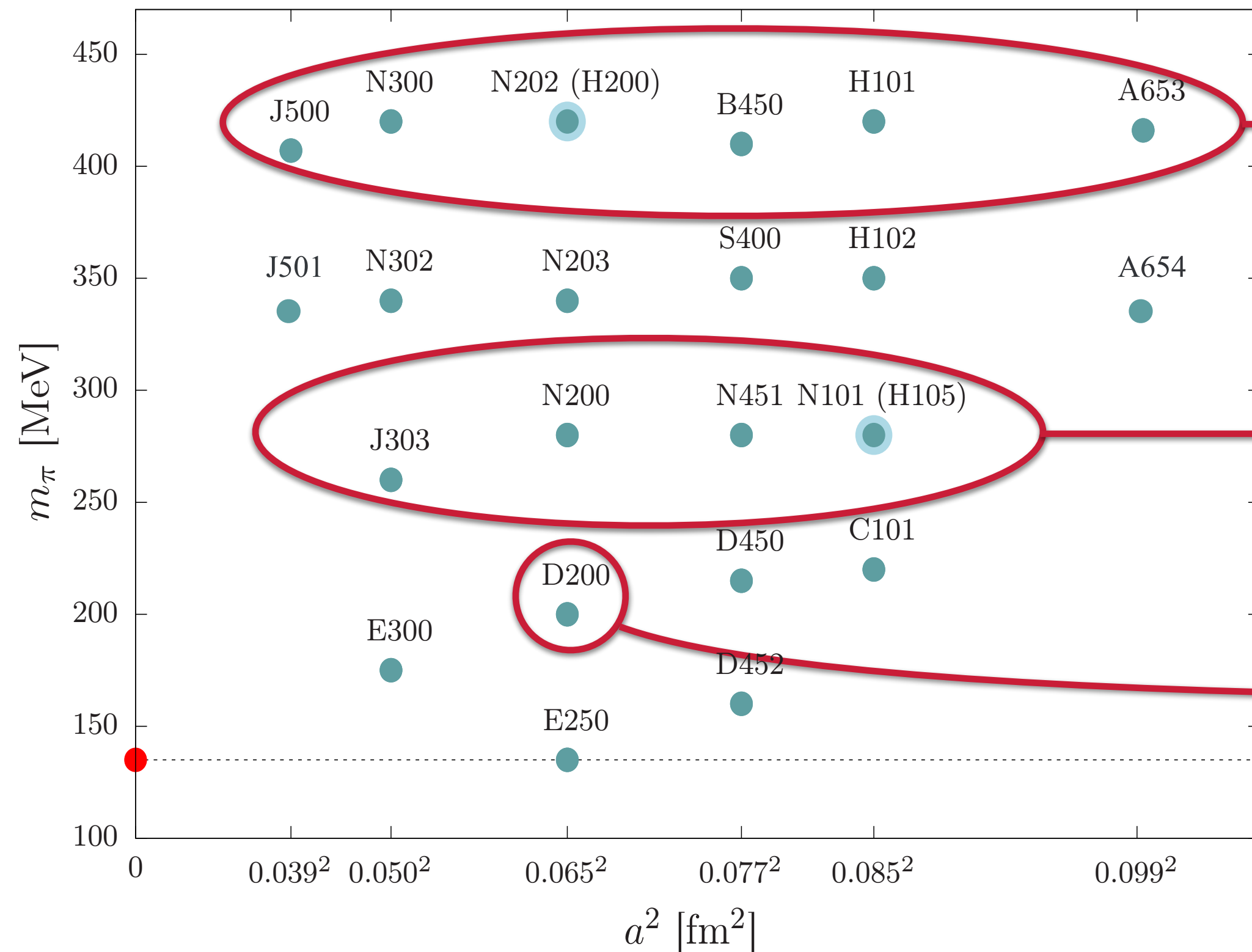
[Francis et al., PRD 99 (2019) 074505]



Dibaryons in QCD with 3 and 2 + 1 flavours

Use CLS ensembles with $N_f = 2 + 1$ flavours of $O(a)$ improved Wilson quarks

- Six lattice spacings: $a = 0.099 - 0.039$ fm; pion masses $m_\pi = 130 - 420$ MeV
- Timeslice-to-all propagators; chiral trajectory: $\text{Tr } M_q = \text{const.} \Leftrightarrow \frac{1}{2}m_\pi^2 + m_K^2 \approx \text{const.}$



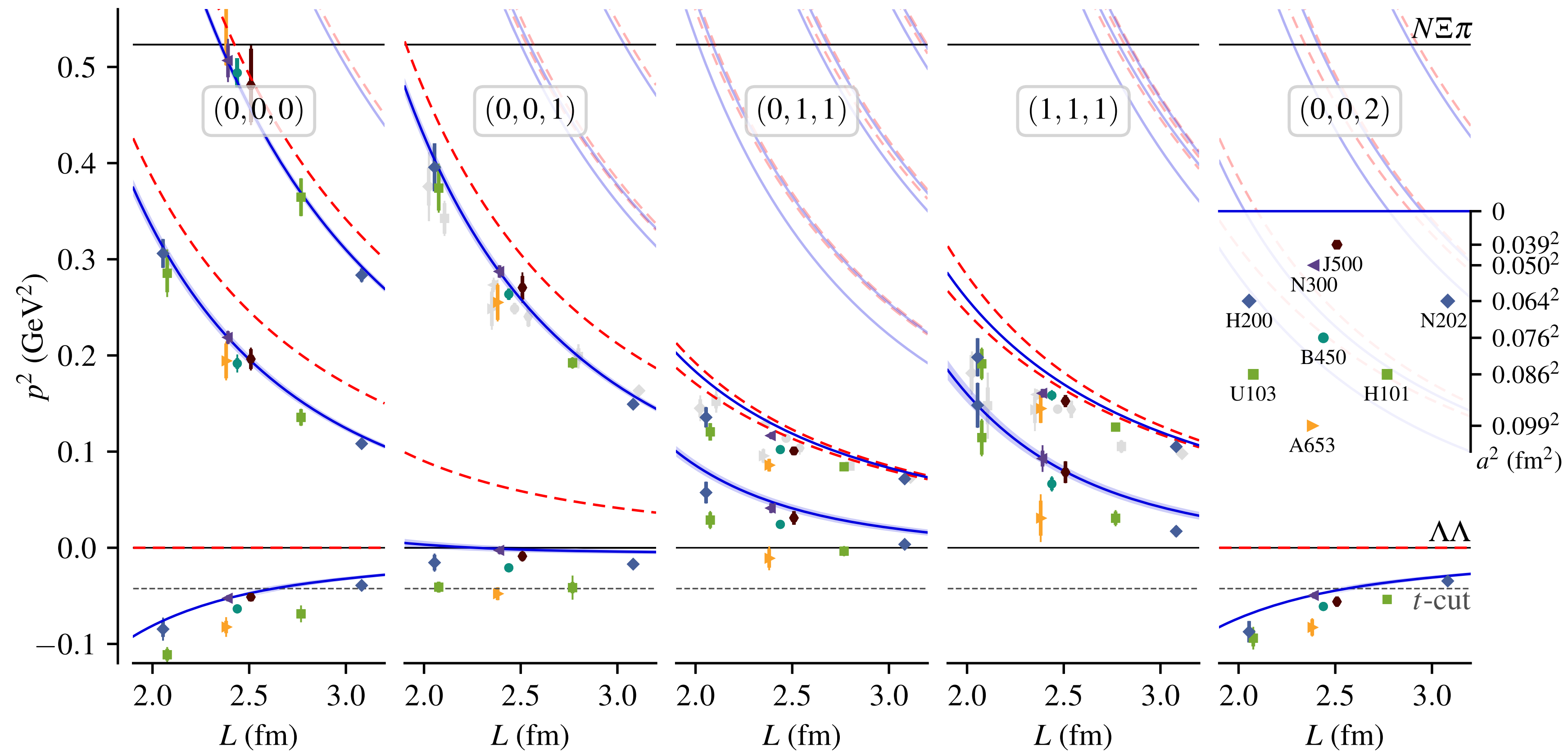
Scaling test at $m_\pi = m_K \approx 420$ MeV,
SU(3)-symmetric point (published)

Scaling test at $m_\pi \approx 280$ MeV, broken SU(3)
(ongoing)

Stochastic LapH at $m_\pi \approx 200$ MeV, — in collab. with BaSc
(see talk by John Bulava (FRI) on $N\pi$ scattering lengths)

H Dibaryon at the $SU(3)$ -symmetric point

Scattering momenta in finite volume in different frames:



— : interacting spectrum in continuum limit

- - - : non-interacting levels

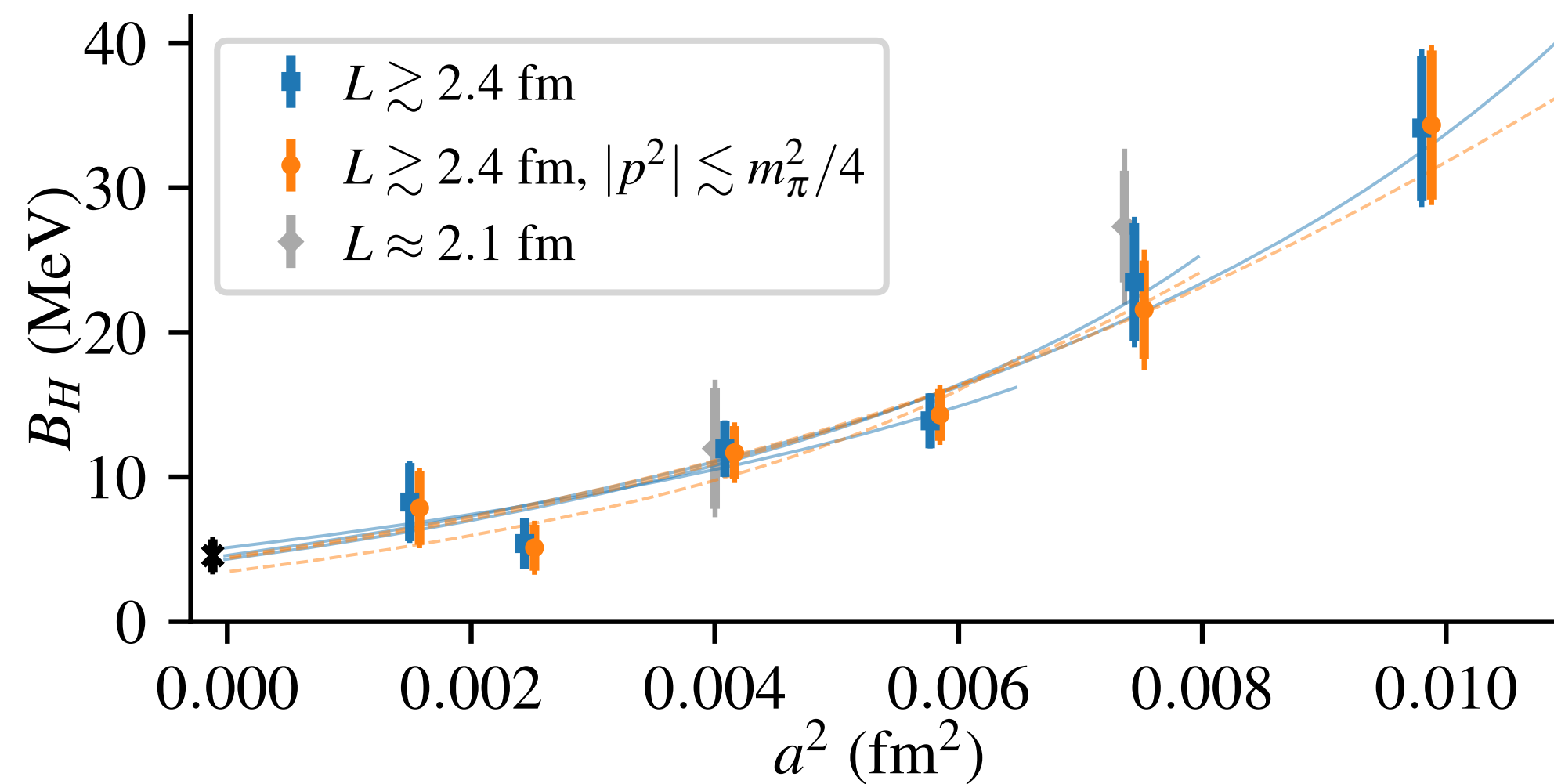
H Dibaryon at the SU(3)-symmetric point

Continuum extrapolation

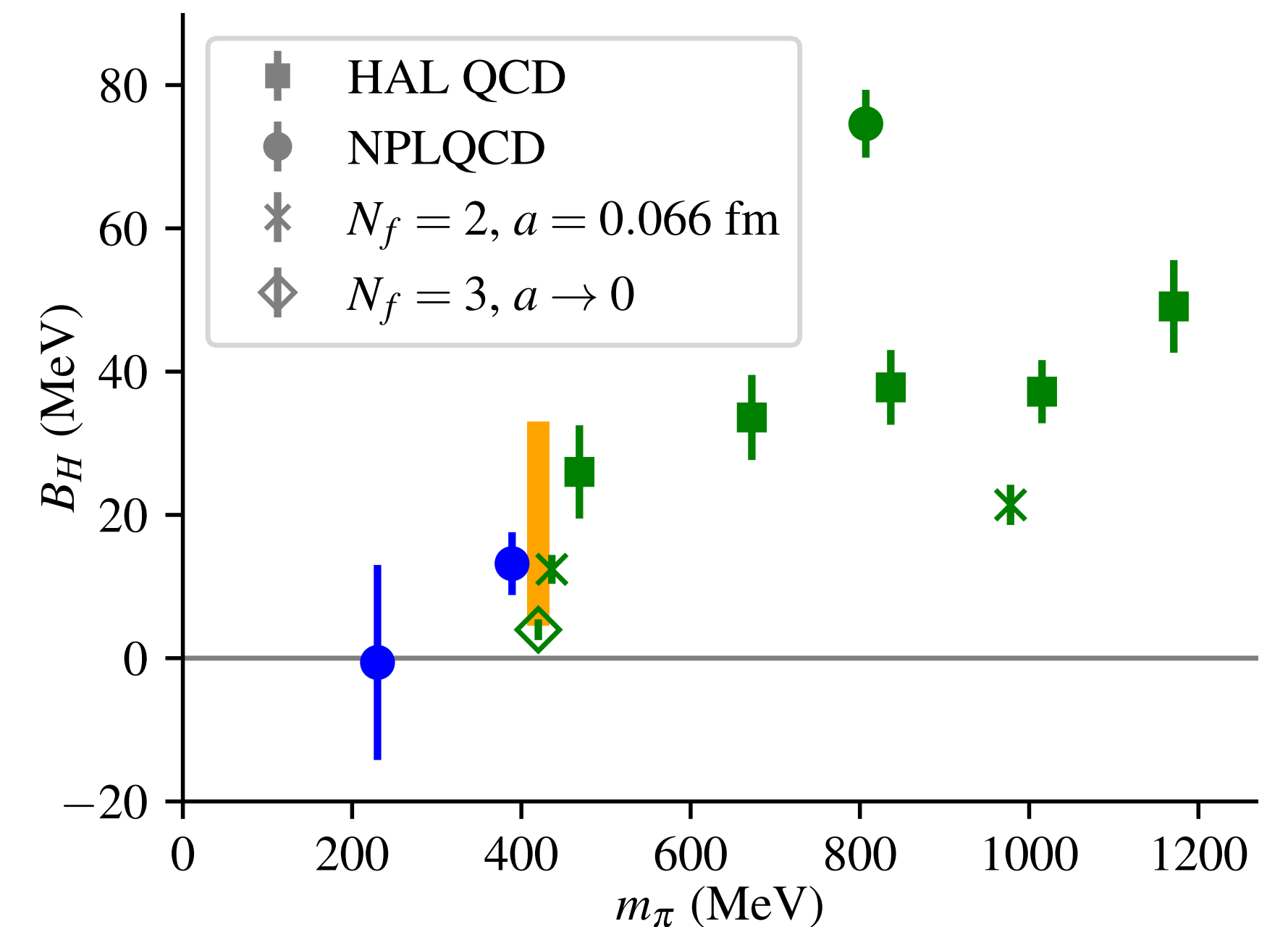
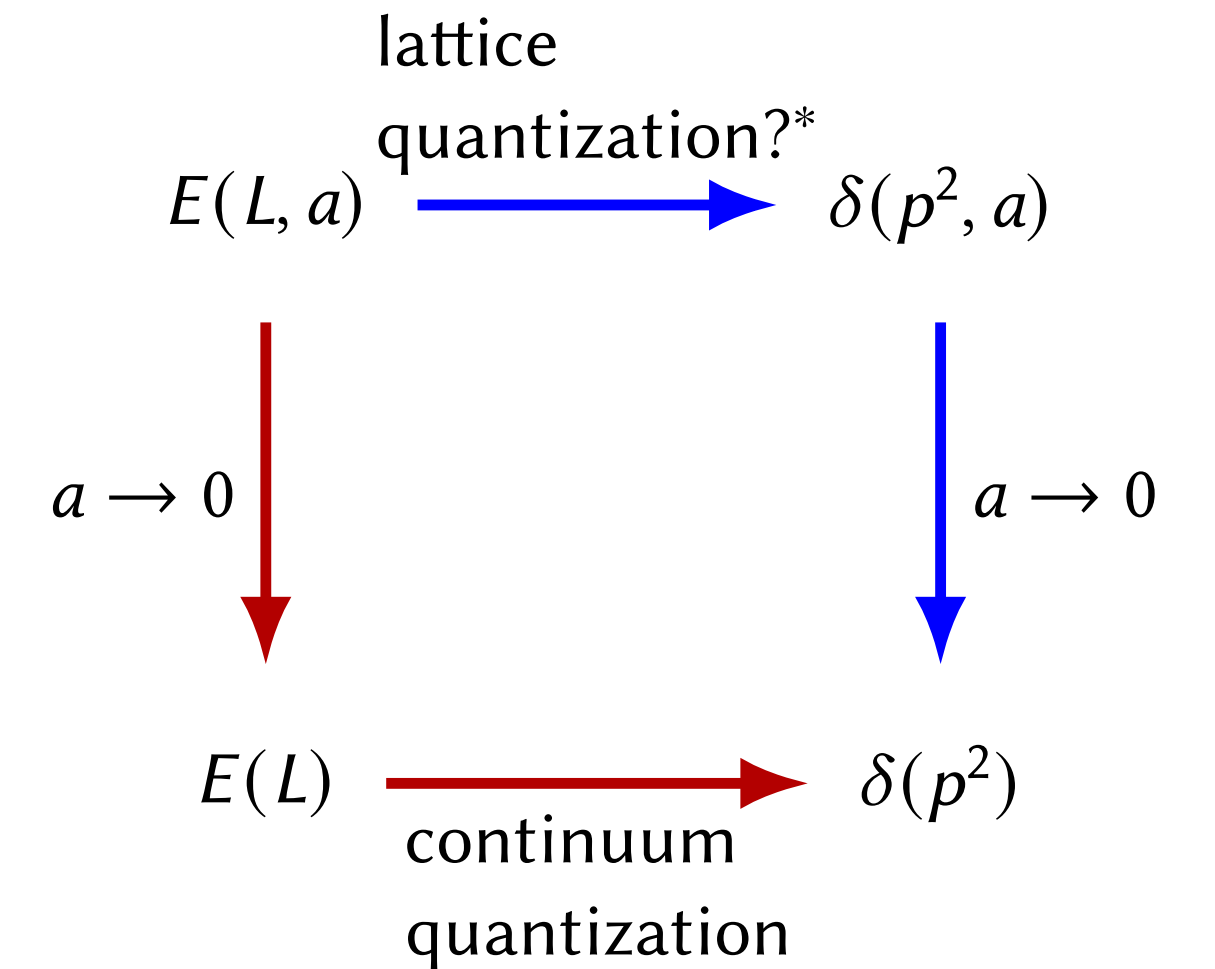
Finite-volume quantisation condition only valid in continuum limit

Perform combined fit of $p \cot \delta(p)$ in both p^2 and a :

$$p \cot \delta(p) = \sum_{i=0}^{N-1} c_i p^{2i} \stackrel{!}{=} -\sqrt{-p^2}, \quad c_i = c_{i0} + c_{i1} a^2$$

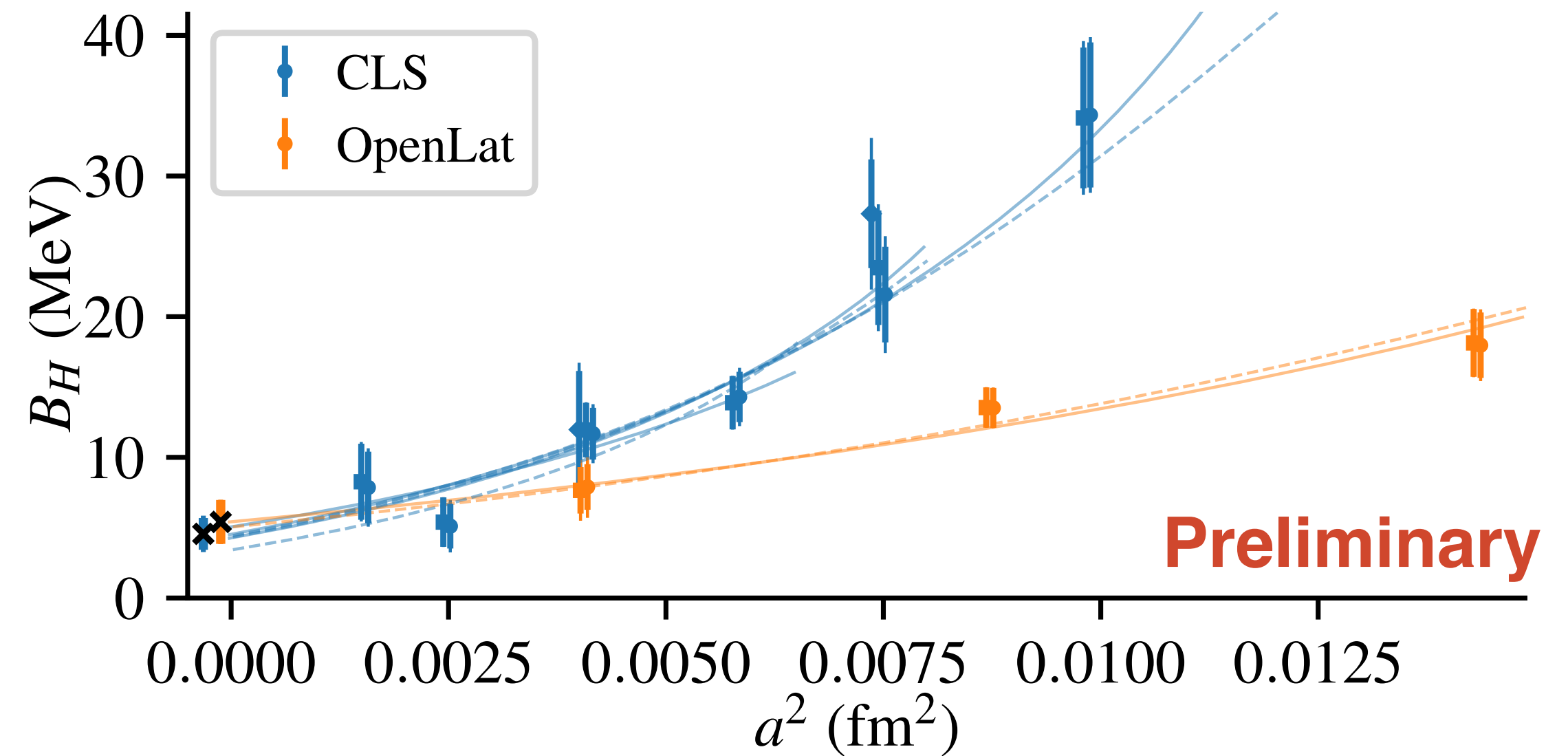
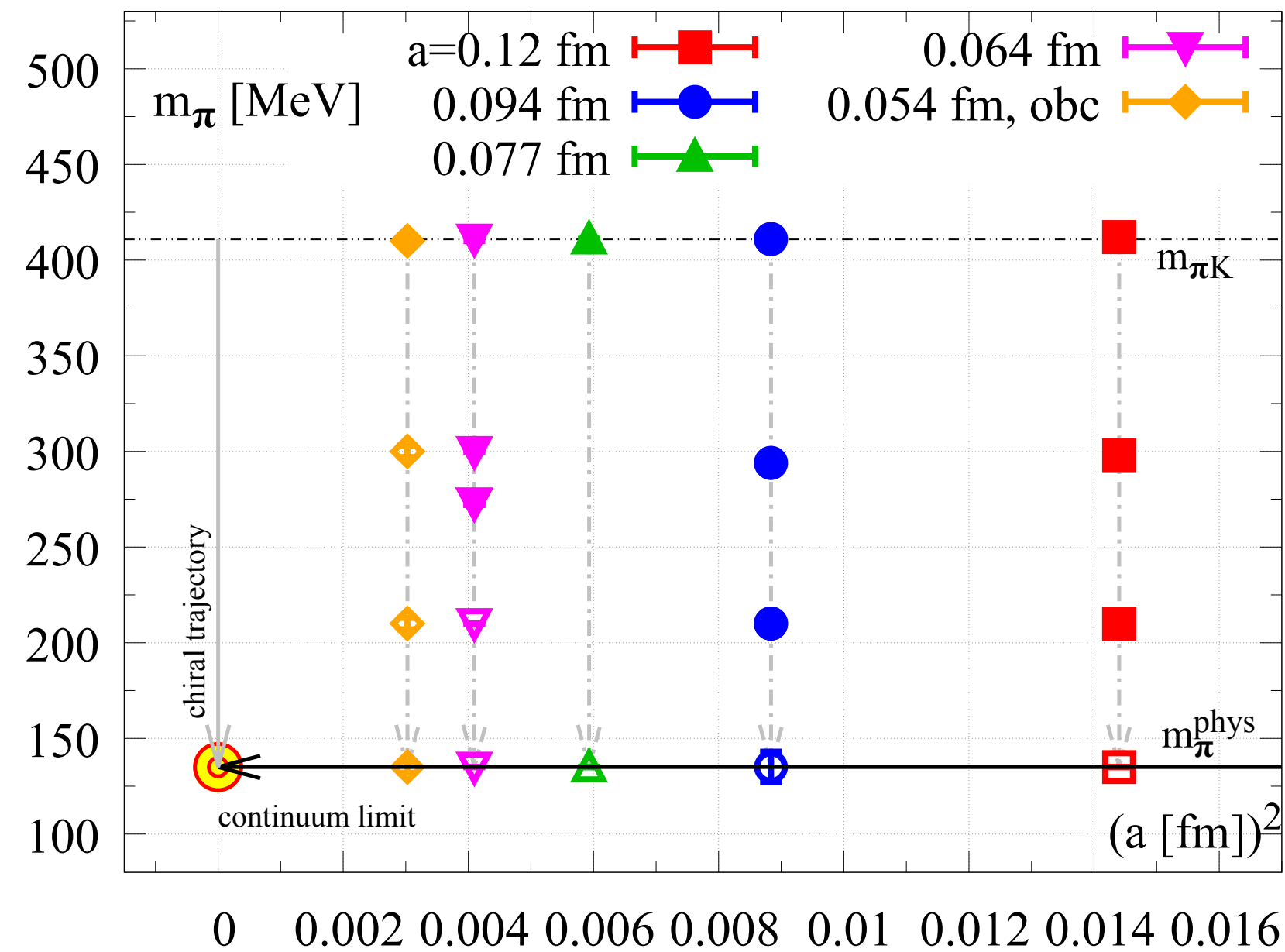


$$\Rightarrow B_H^{N_f=3} = 4.56 \pm 1.13 \pm 0.63 \text{ MeV}$$



H Dibaryon at the SU(3)-symmetric point

Cross-check using Stabilised Wilson Fermions (OpenLat)



$$D_{ee} + D_{oo} = (4 + m_0) \exp \left\{ \frac{c_{sw}}{4 + m_0} \frac{i}{4} \sigma_{\mu\nu} \widehat{F}_{\mu\nu} \right\},$$

$$D = \begin{pmatrix} D_{ee} & D_{eo} \\ D_{oe} & D_{oo} \end{pmatrix}$$

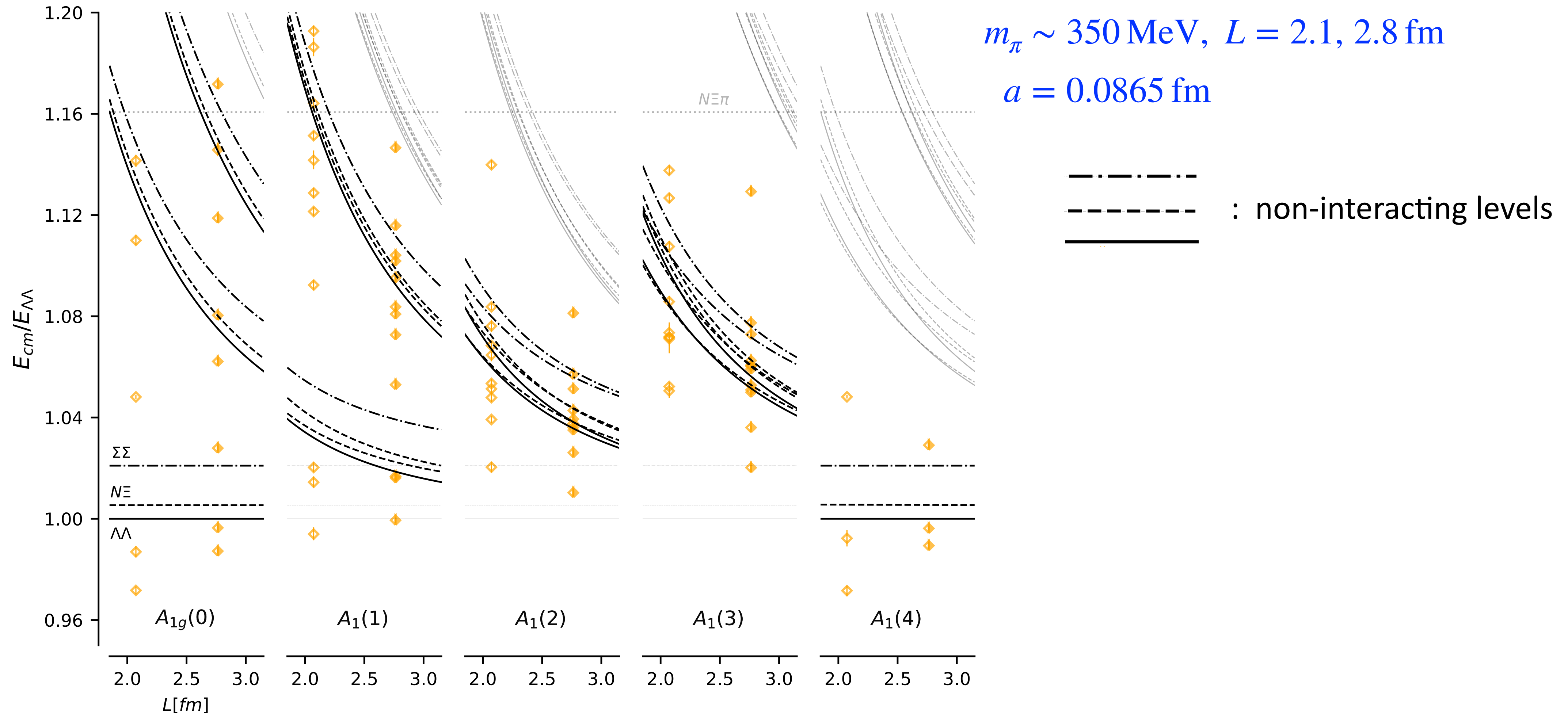
$$B_H^{N_f=3} = \begin{cases} 4.56 \pm 1.13 \pm 0.63 \text{ MeV} & \text{CLS} \\ 5.41 \pm 1.56 \pm 0.24 \text{ MeV} & \text{OpenLat} \end{cases}$$

(CLS: systematic error includes fit error, plus cut in a , L and p^2
OpenLat: systematic error from fit uncertainty only)

The H Dibaryon away from the SU(3)-symmetric point

Finite-volume energy levels at decreasing pion mass:

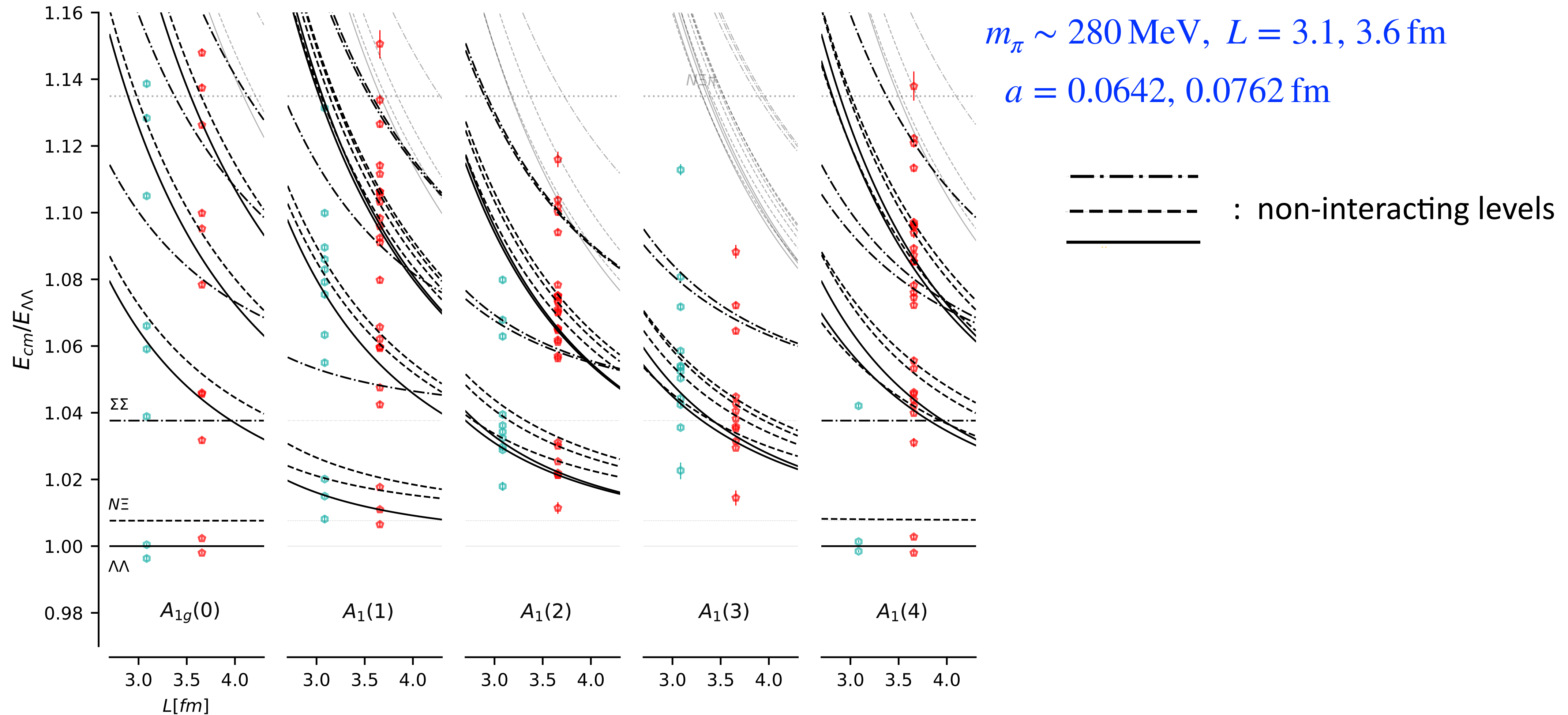
[M. Padmanath et al., arXiv:2111.11541]



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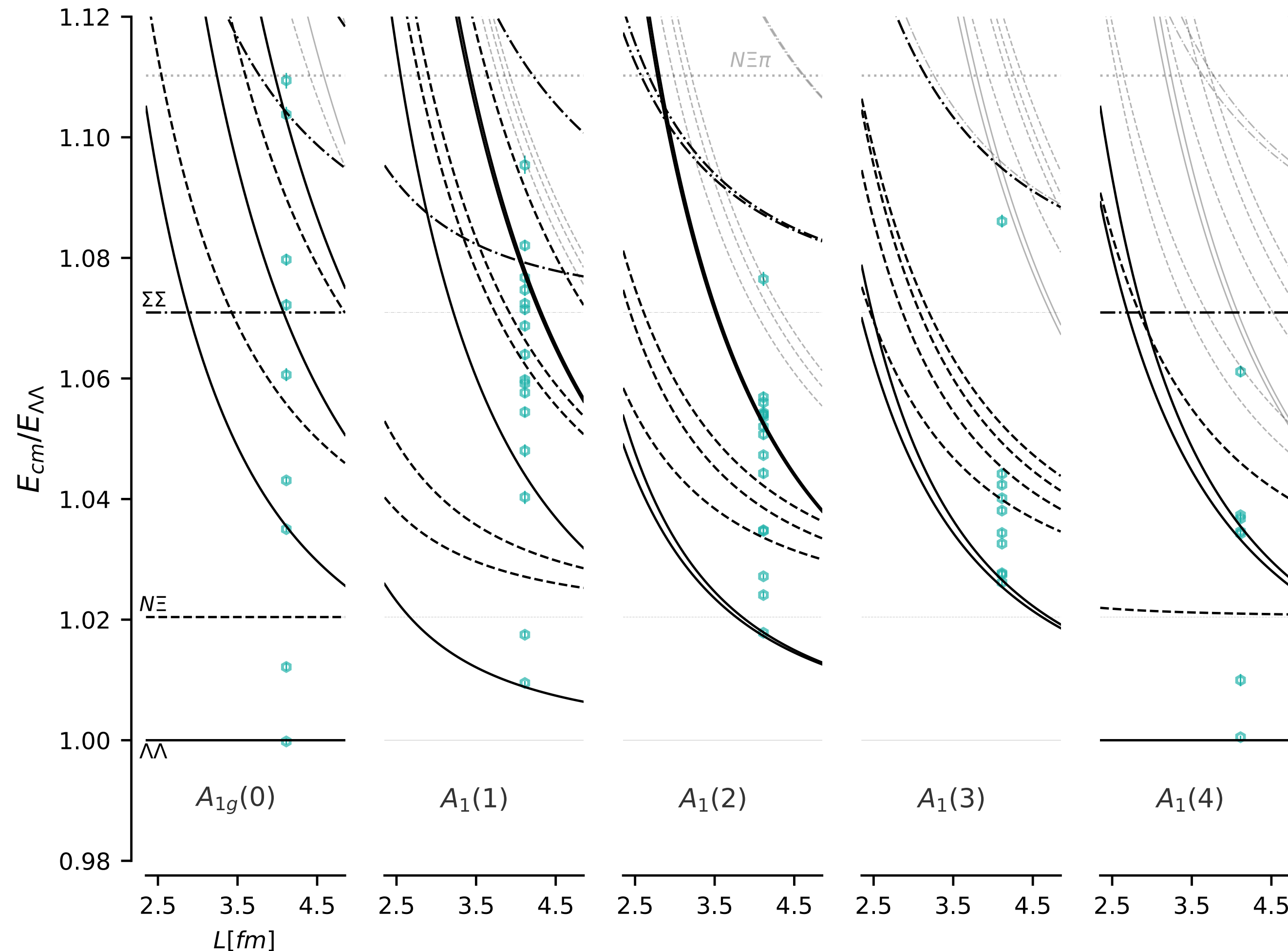
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The H Dibaryon away from the SU(3)-symmetric point

Finite-volume energy levels at decreasing pion mass:

[M. Padmanath et al., arXiv:2111.11541]



$m_\pi \sim 200$ MeV, $L = 4.2$ fm

$a = 0.0642$ fm

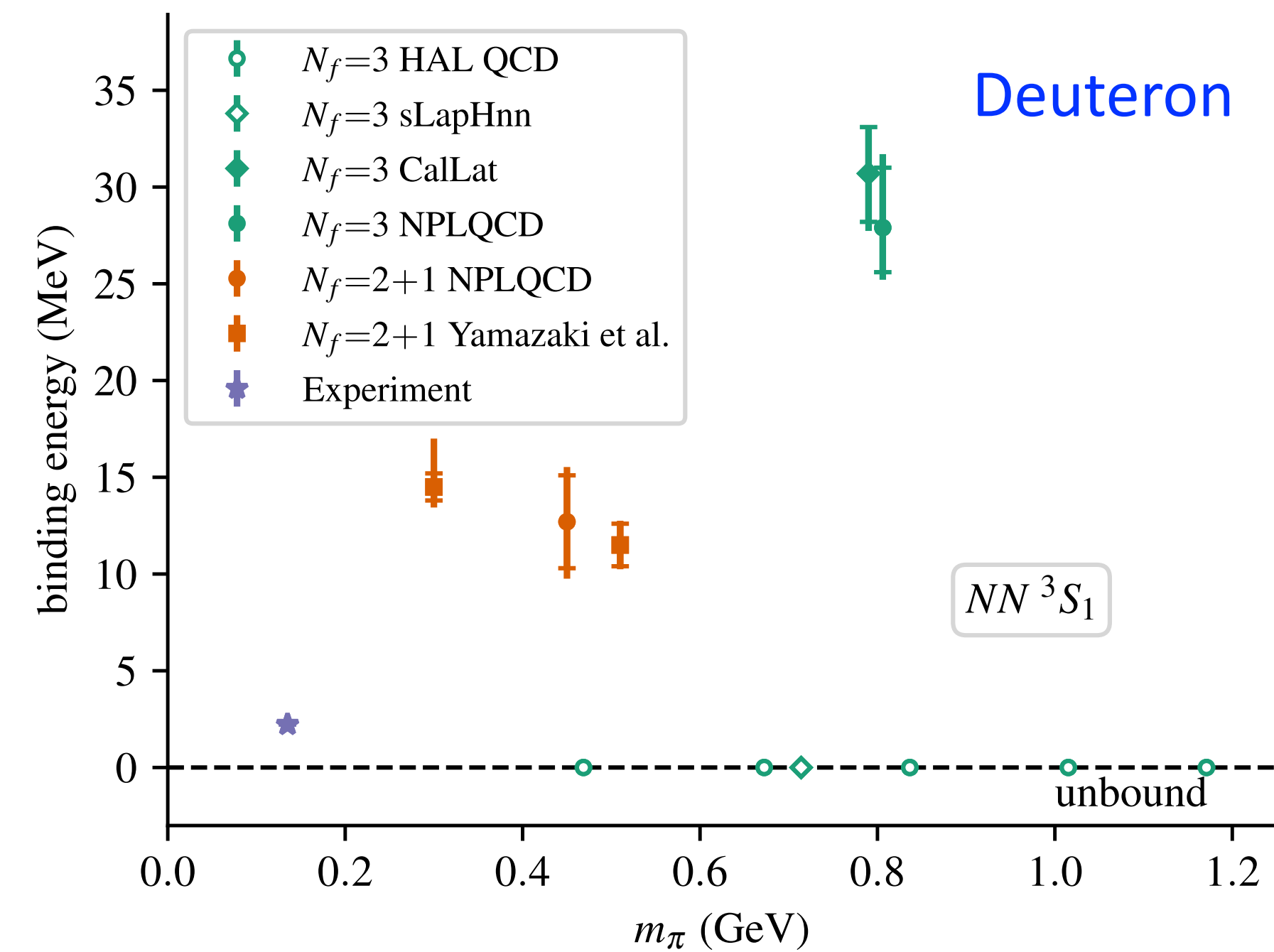
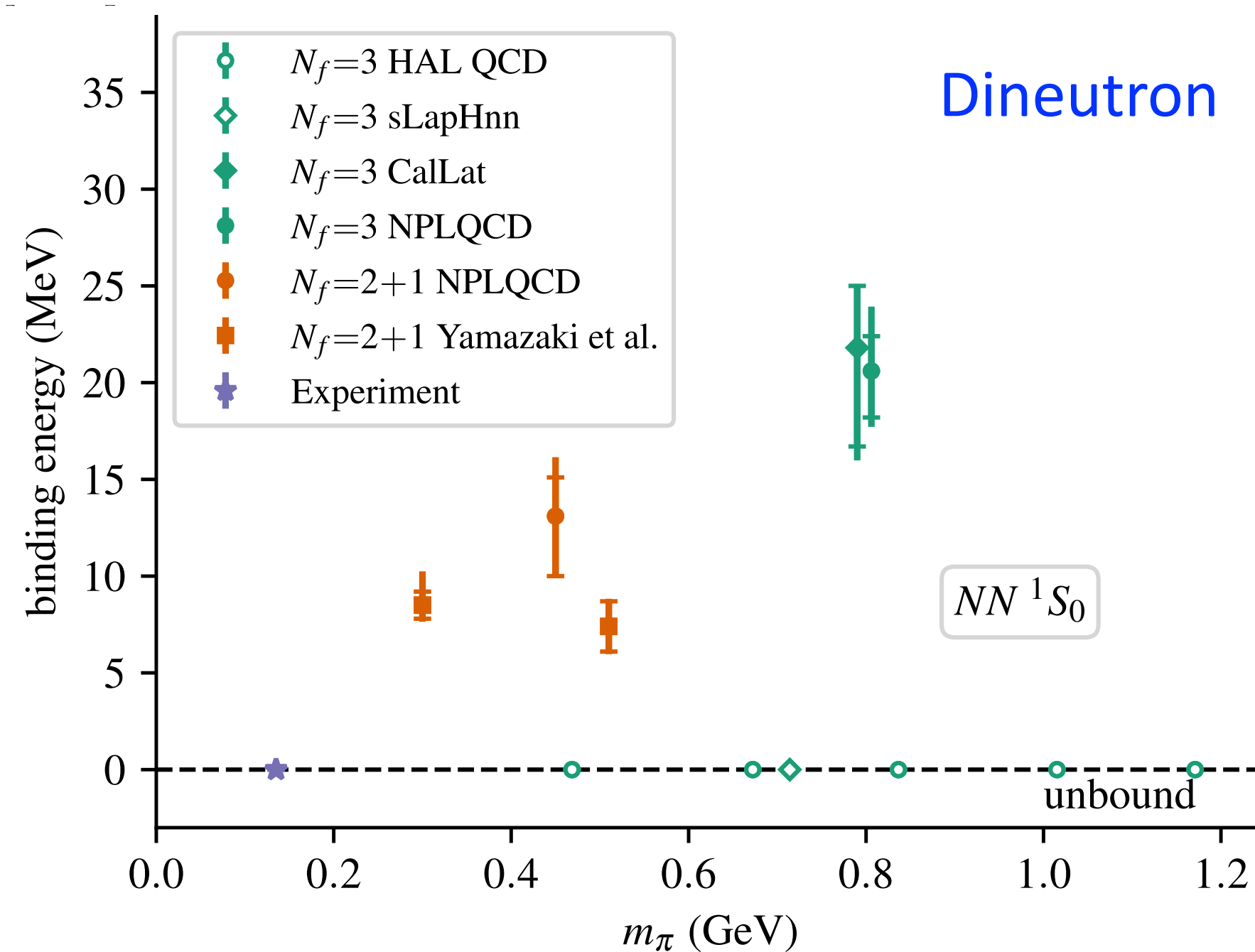
— : non-interacting levels

- Resolve a dense spectrum of energy levels at several pion masses
- Next: perform amplitude analysis

Nucleon-nucleon interactions

[See Jeremy Green's talk at Lattice 2022]

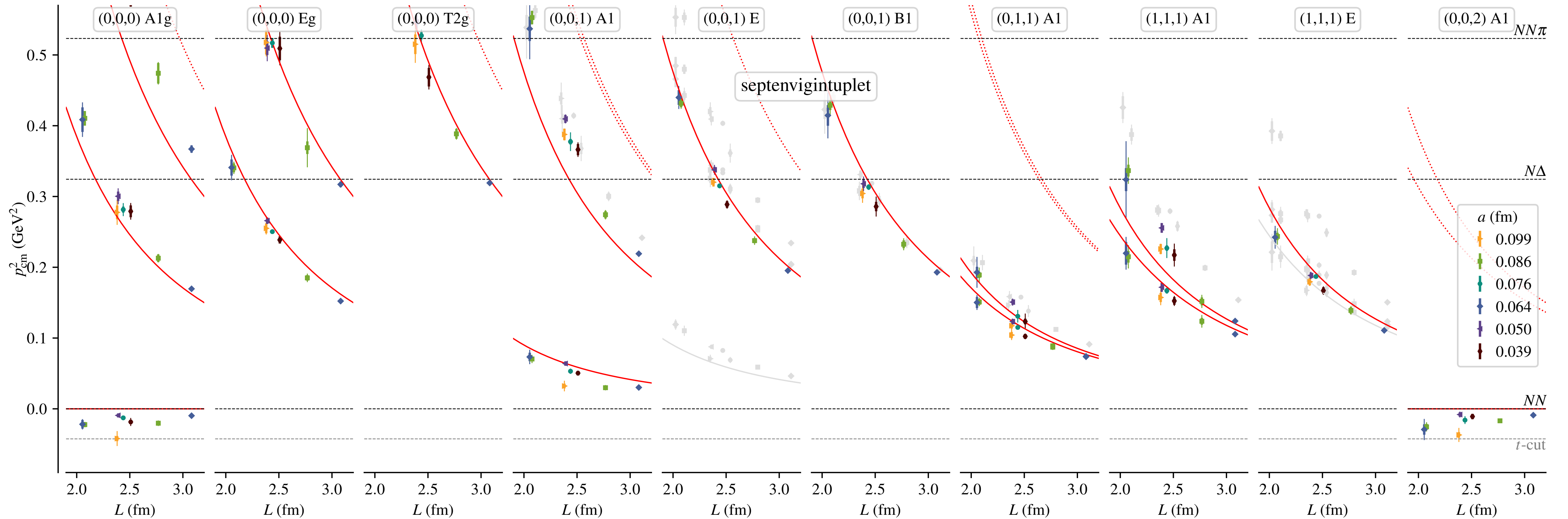
Inconclusive results on existence of bound states at unphysical pion masses:



Study the dineutron and deuteron channels at SU(3)-symmetric point

- Employ distillation and symmetric GEVP
- Study dependence on lattice spacing

27-plet (NN , $I = 1$): spin-0 spectrum

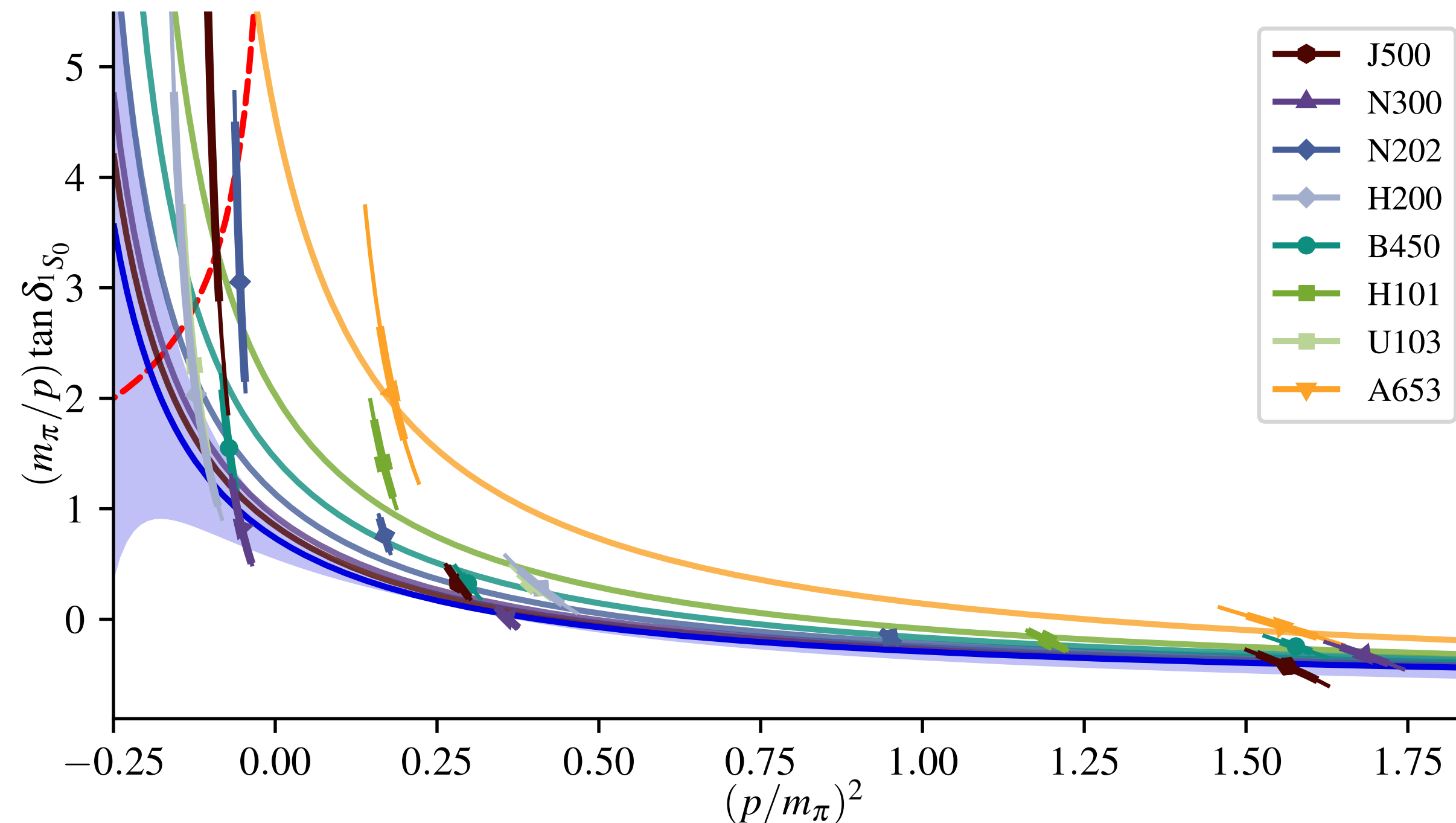


- Spin-1 states (grey) identified by overlaps
- Quantisation condition factorises in spin; 1S_0 and 1D_2 are relevant

— : non-interacting levels

27-plet (NN , $I = 1$): spin-0

Phase shift analysis: 1S_0

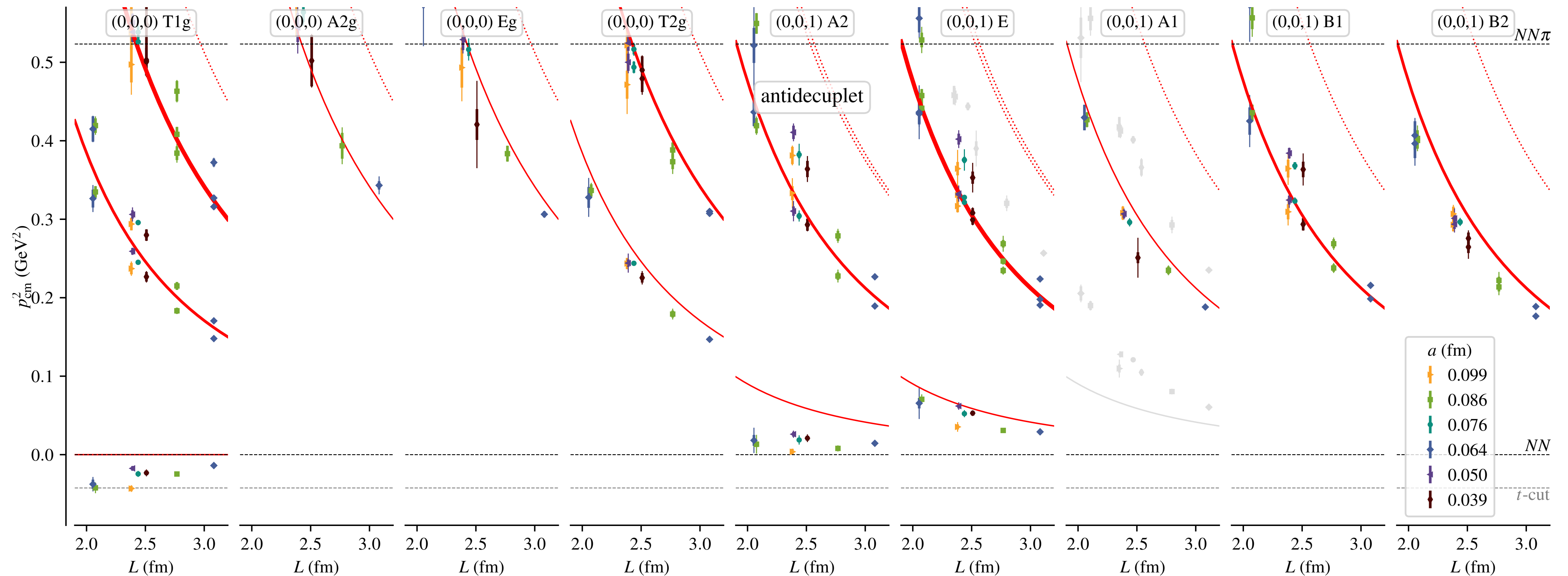


- Levels from rest frame and first moving frame
- Fit to rational function:

$$p \cot \delta(p) = \frac{c_0 + c_1 p^2}{1 + c_2 p^2}$$

- Observe virtual bound state
- Phase shift decreases towards continuum limit
 - Discretisation effects enhance baryon-baryon interactions

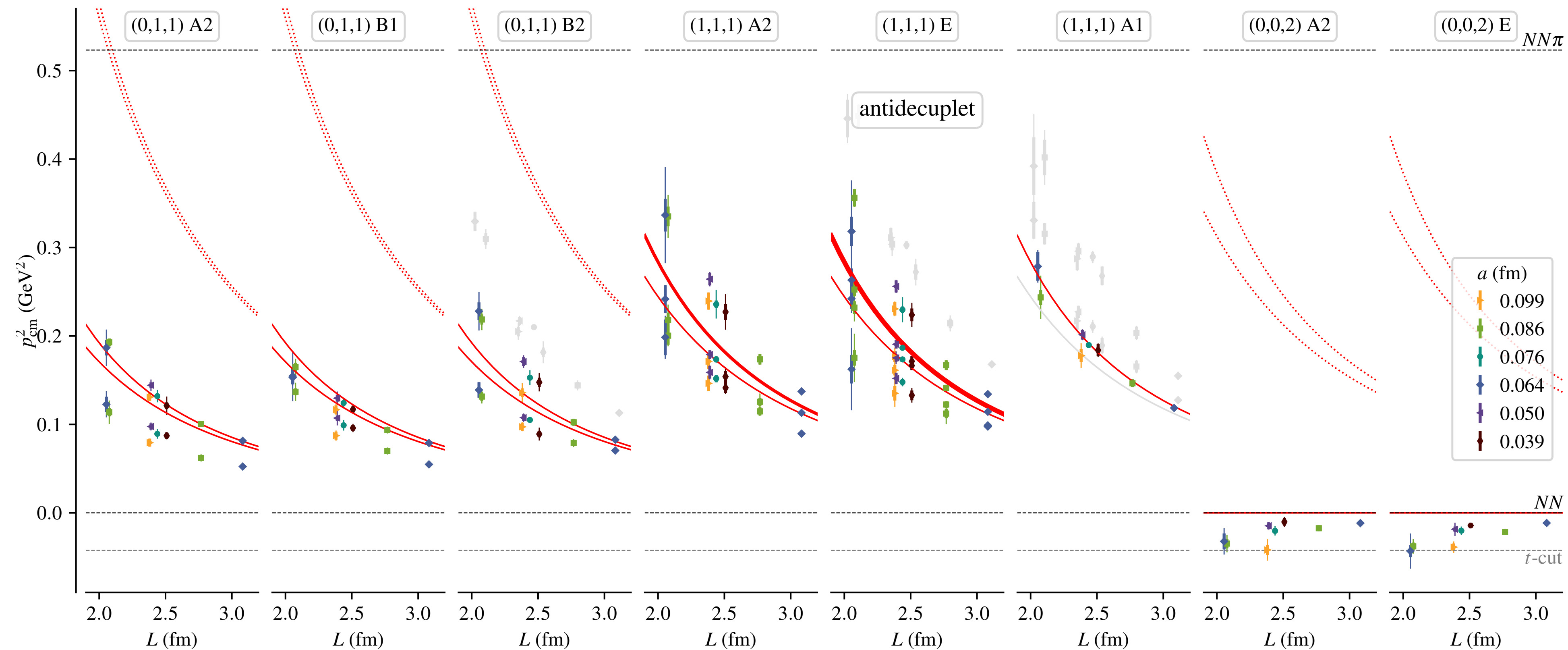
Anti-decuplet ($NN, I = 0$): spin-1 spectrum



- Spin-0 states (grey) identified by overlaps
- 3S_1 , 3D_1 , 3D_2 and 3D_3 can be relevant

— : non-interacting levels
(thickness proportional to degeneracy)

Anti-decuplet ($NN, I = 0$): spin-1 spectrum

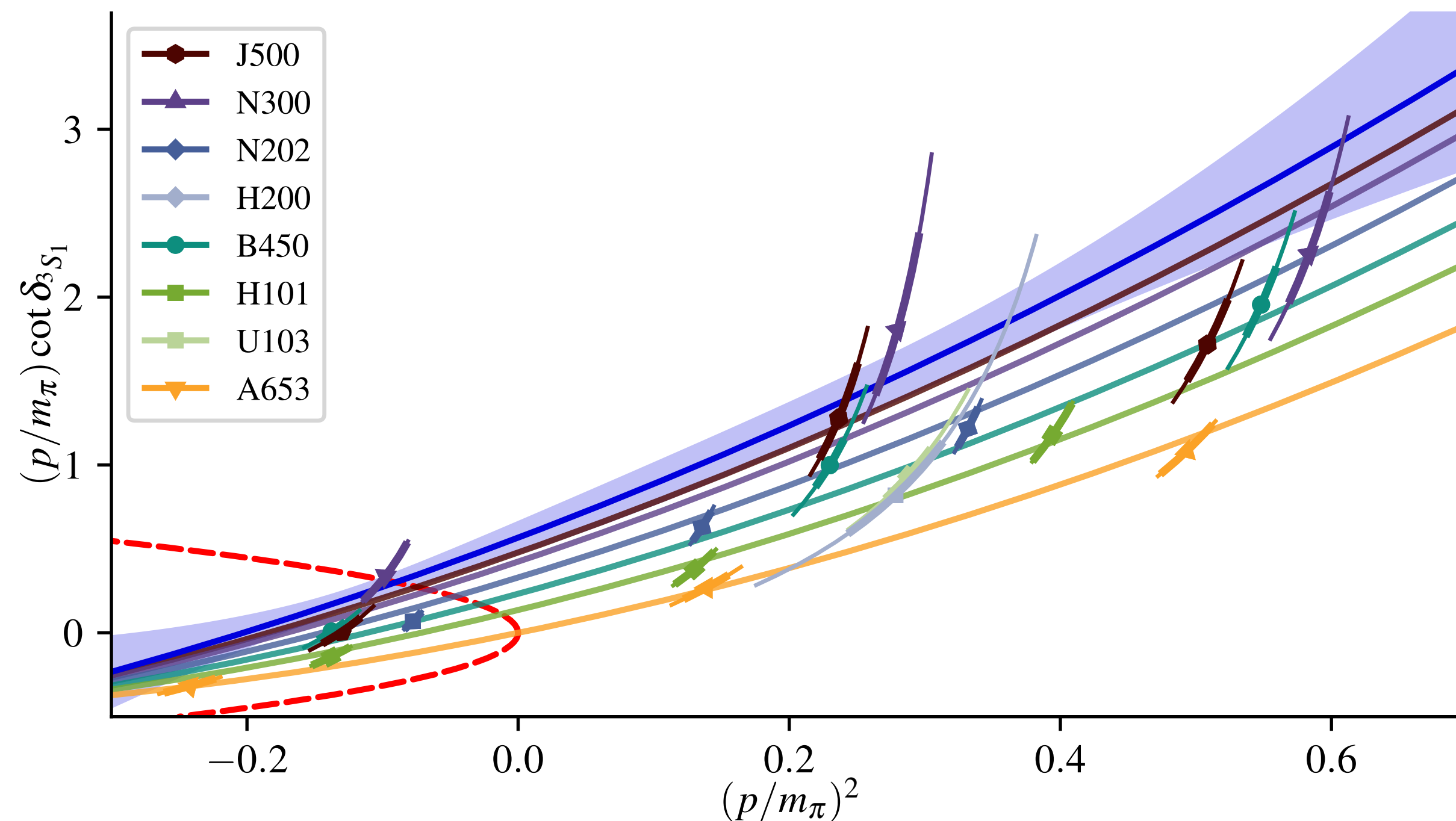


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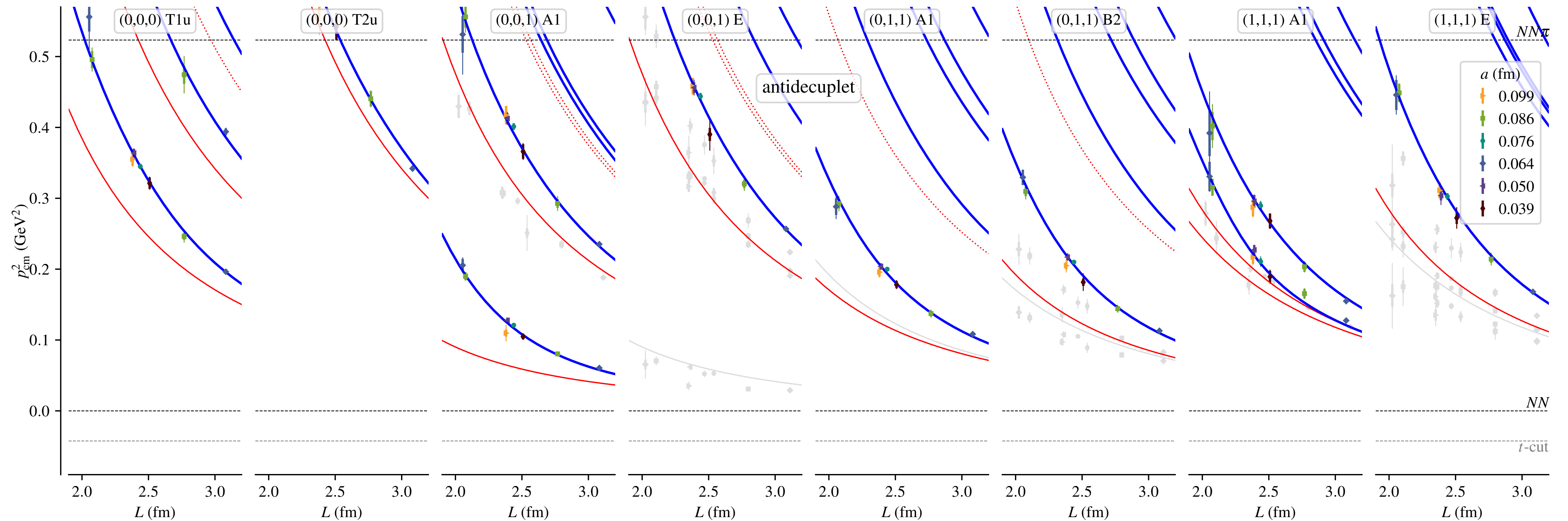
Phase shift analysis: 3S_1



- Helicity-averaged levels from first two moving frames
- Neglect mixing with 3D_1

- Observe virtual bound state $\rightarrow$ Deuteron not bound at $m_\pi = m_K \sim 420 \text{ MeV}$
- Phase shift decreases towards continuum limit

Anti-decuplet ($NN, I = 0$): spin-0 spectrum



— : 4-parameter fit to 70 energy levels

— : non-interacting levels

$$p^3 \cot \delta_{1P_1} = c_0 + c_1 p^2, \quad p^3 \cot \delta_{1F_3} = c_2 + c_3 p^8$$

(good fit quality without terms describing lattice artefacts)

Coupled partial waves, e.g. ${}^3S_1 - {}^3D_1$

[See Jeremy Green's talk at Lattice 2022]

Ansatz for K -matrix: Blatt-Biedenharn parameterisation

$$\tilde{K}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix} \begin{pmatrix} \cos \epsilon_1 & -\sin \epsilon_1 \\ \sin \epsilon_1 & \cos \epsilon_1 \end{pmatrix} \begin{pmatrix} p \cot \delta_{1\alpha} & 0 \\ 0 & p \cot \delta_{1\beta} \end{pmatrix} \begin{pmatrix} \cos \epsilon_1 & \sin \epsilon_1 \\ -\sin \epsilon_1 & \cos \epsilon_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix}$$

Assume $\delta_{1\beta} = 0 \rightarrow \epsilon_1$ causes splitting of helicity states

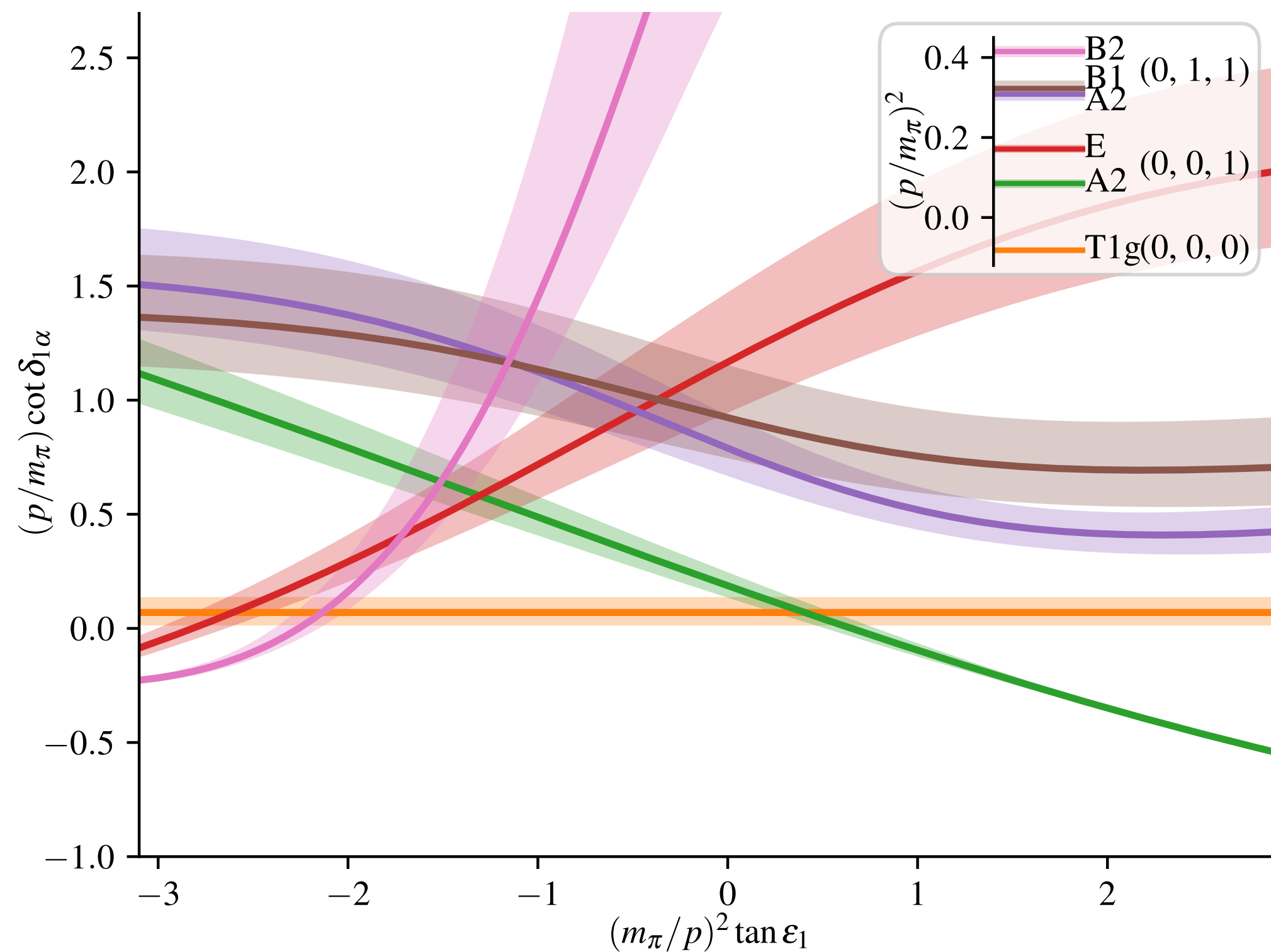
Energy levels impose

$$p \cot \delta_{1\alpha} = \frac{B_{00} + (B_{01} + B_{10})x + B_{11}x^2}{1 + p^4 x^2}, \quad x = p^{-2} \tan \epsilon_1$$

Fit spectrum on N202 to

$$p \cot \delta_{1\alpha} = c_1 + c_2 p^2$$

$$p^{-2} \tan \epsilon_1 = c_3.$$

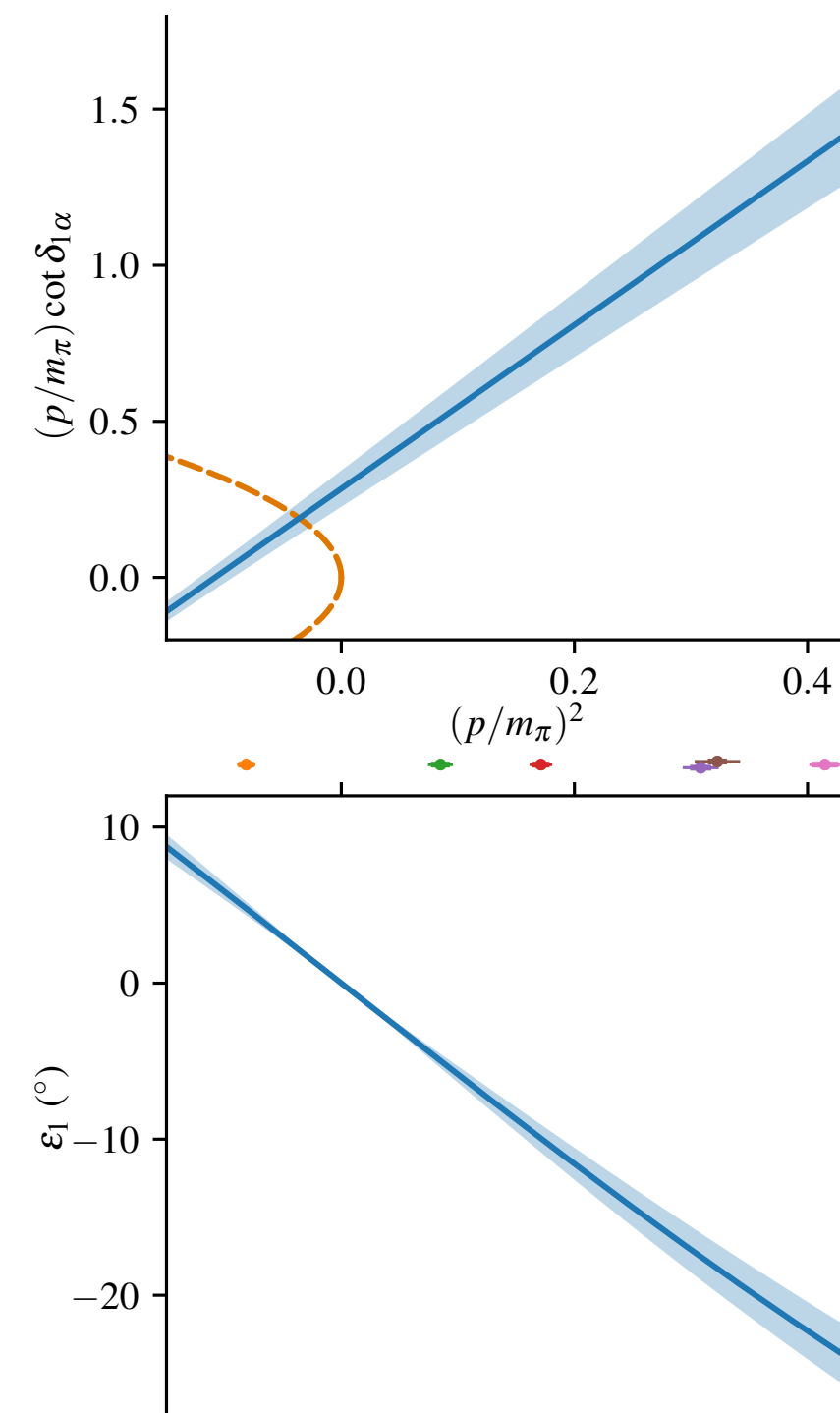
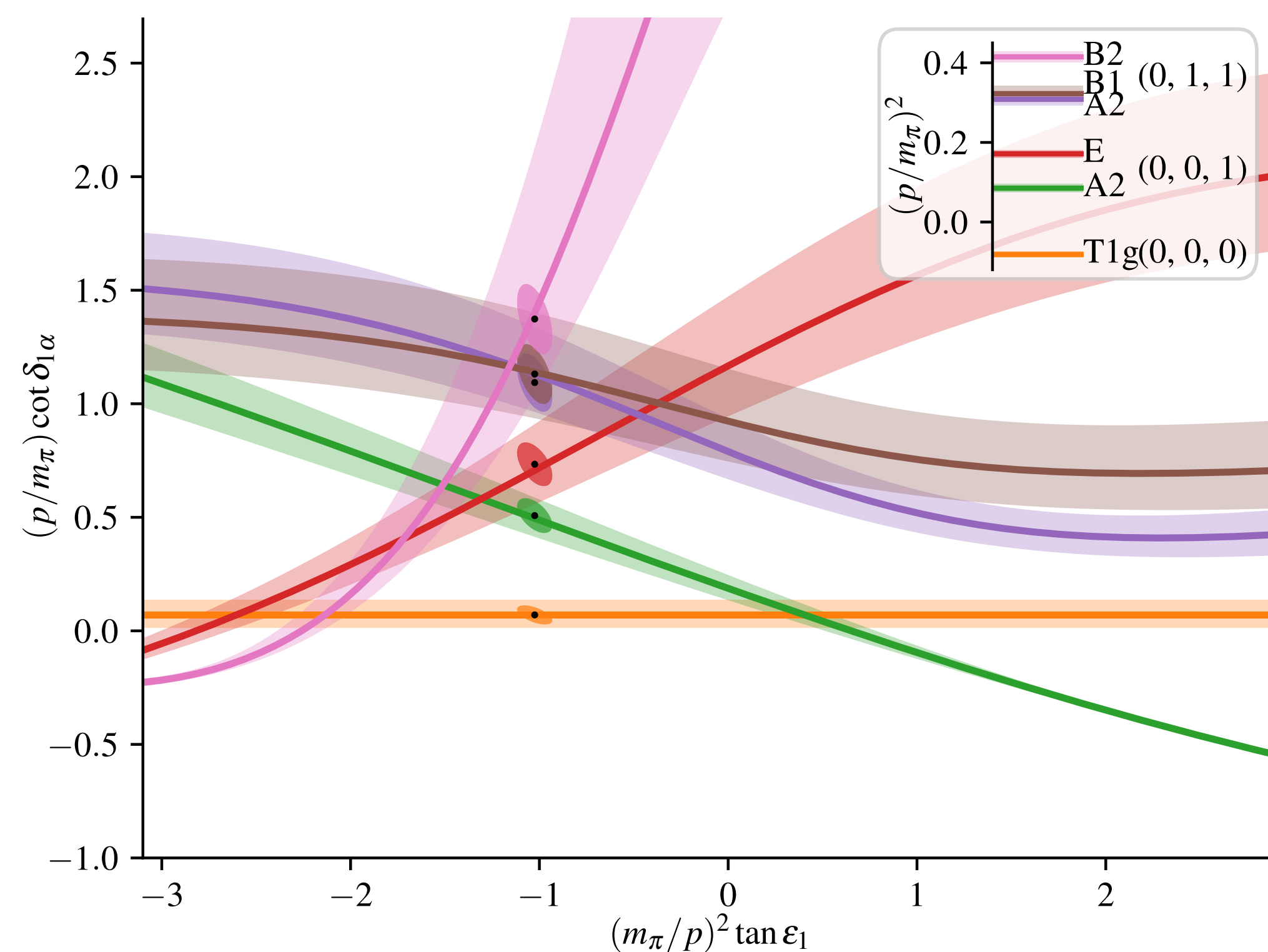


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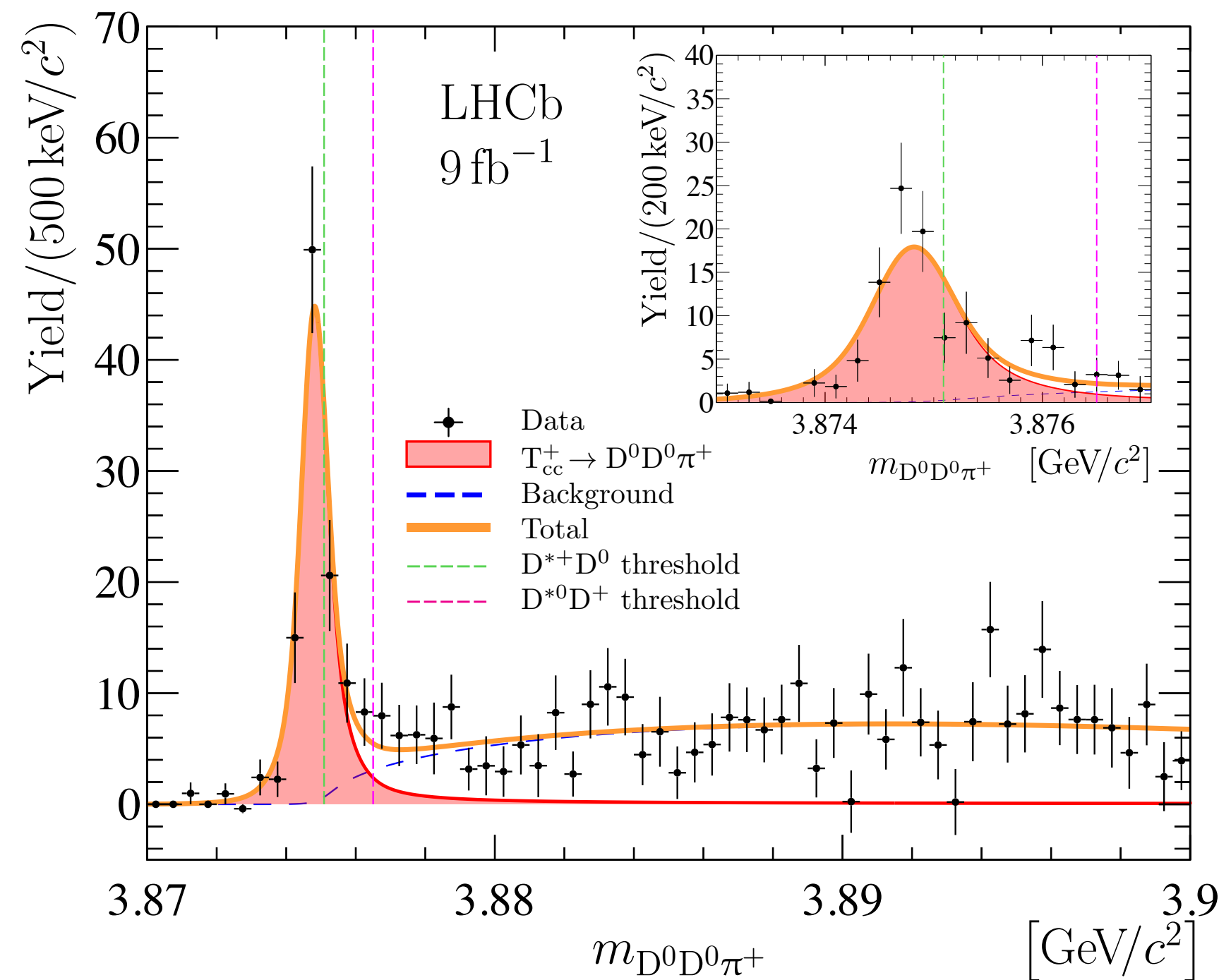
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Sign of ϵ_1 opposite to experiment

Charmed tetraquarks

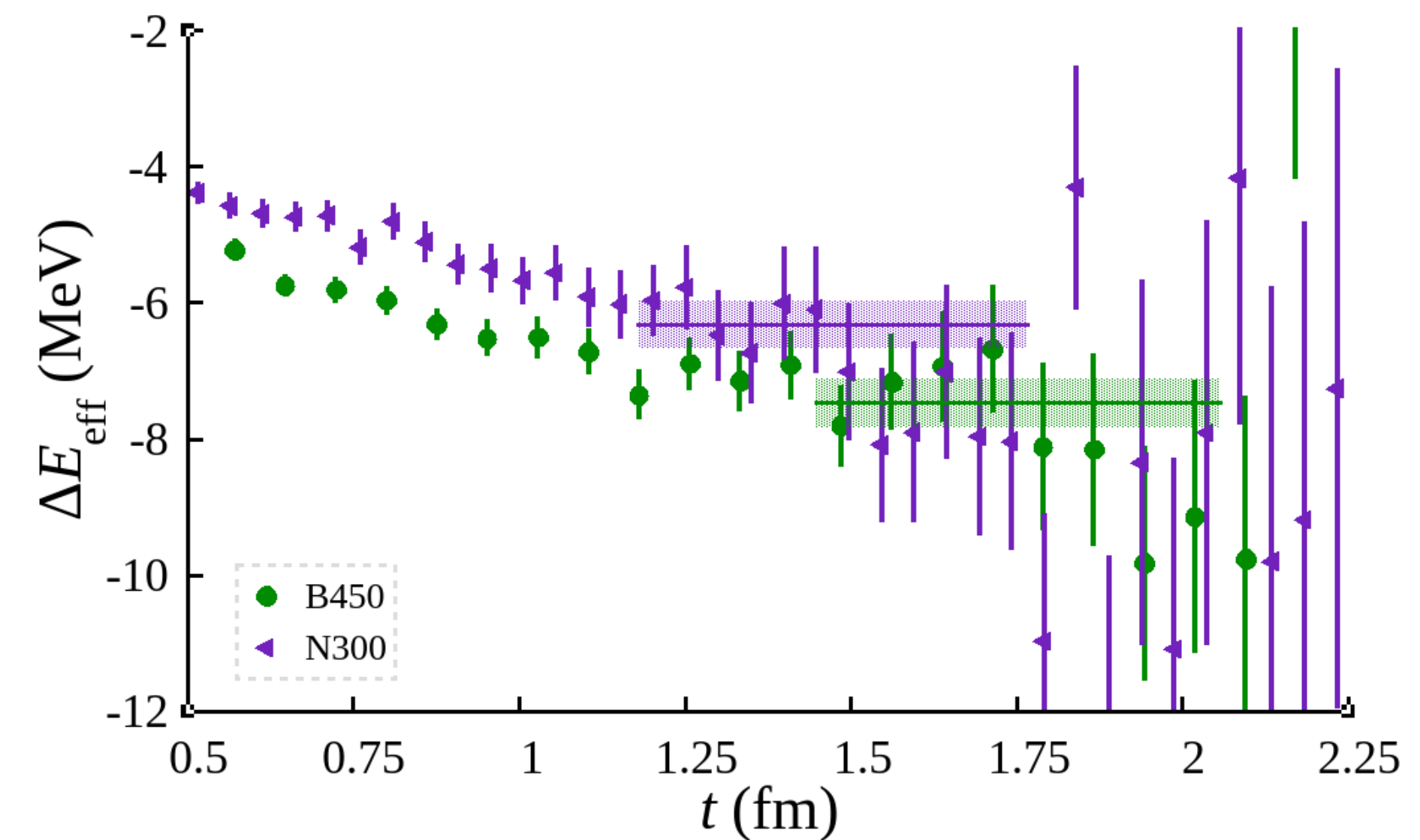
LHCb: observation of doubly charmed tetraquark T_{cc}^+ with $I = 0$, $J^P = 1^+$ close to $D^{*+}D^0$ threshold



$$\delta m_{\text{pole}} = -360 \pm 40_{-0}^{+4} \text{ keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2_{-14}^{+0} \text{ keV}$$

- Lattice QCD: discretisation effects may be large for heavy quark systems
- Perform scaling test for different lattice spacings



Charmed tetraquarks: Finite-volume energy levels

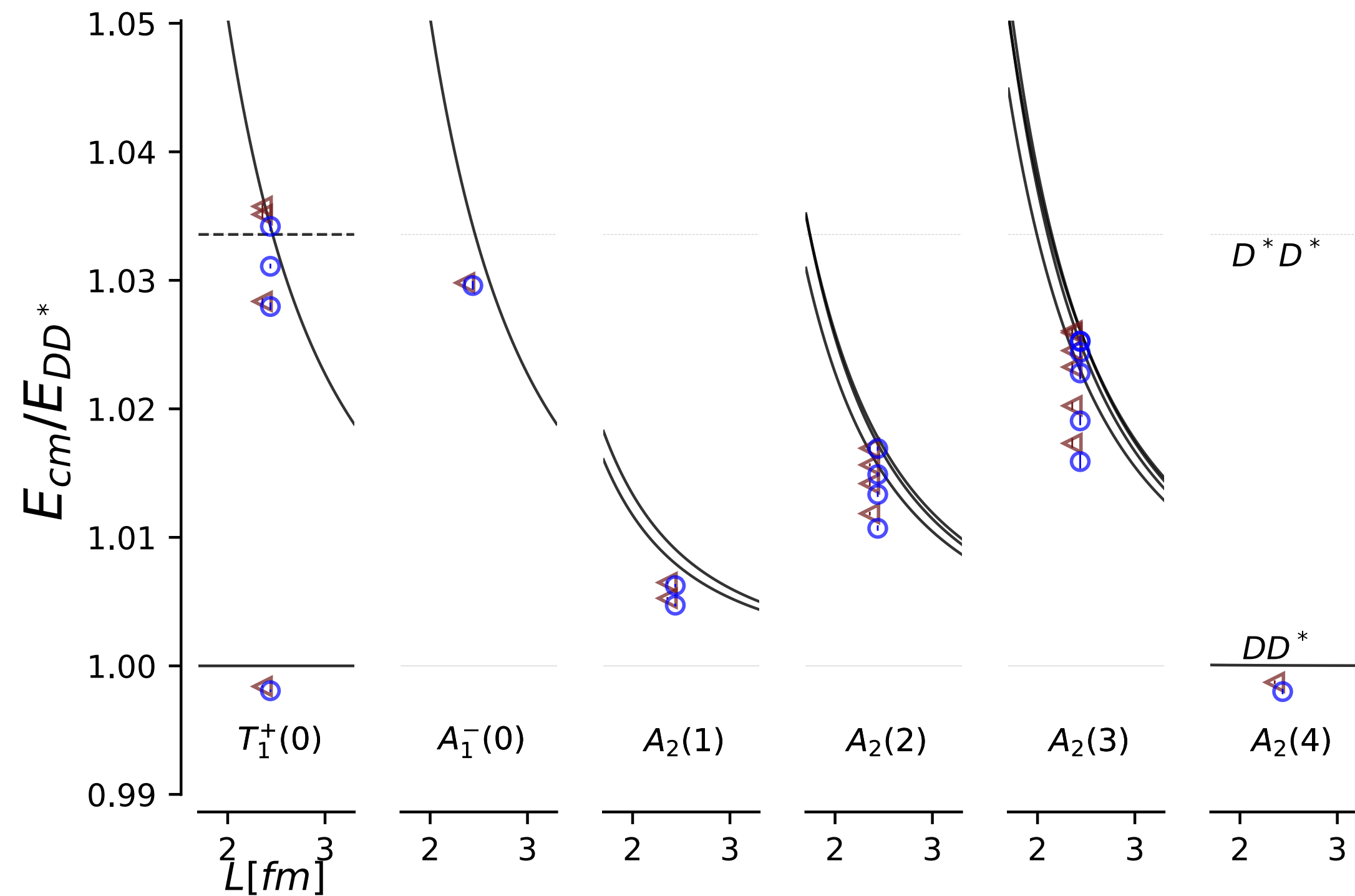
Perform scaling test in SU(3)-symmetric limit

$$m_\pi \sim 420 \text{ MeV}, L = 2.4 \text{ fm}$$

$$a = 0.0498, 0.0762 \text{ fm}$$

- Preliminary amplitude analysis suggests virtual bound state
- Complementary to calculation at 280 MeV pion mass

[Padmanath & Prelovsek, PRL 129 (2022) 032002]



Conclusions — Outlook

- * Distillation, GEVP and Finite-volume Quantisation:
Detailed and precise studies of two-particle interactions
- * Discretisation effects are sizeable:
 - Binding energy of H dibaryon much smaller in continuum limit: $O(5 \text{ MeV})$
 - Lattice artefacts enhance strength of hadron-hadron interactions
 - Confirmed using different lattice actions
- * No bound states in dineutron and deuteron channels observed at $m_\pi = m_K \sim 420 \text{ MeV}$
- * SU(3)-flavour breaking makes amplitude analysis much more complicated
- * Mixing with higher partial waves under study

Thank you!