A light hybrid resonance from lattice QCD

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hadspec
hadspec.org

Multihadron dynamics in a box
Bethe Forum Bonn
August 2022

UNIVERSITY OF CAMBRIDGE

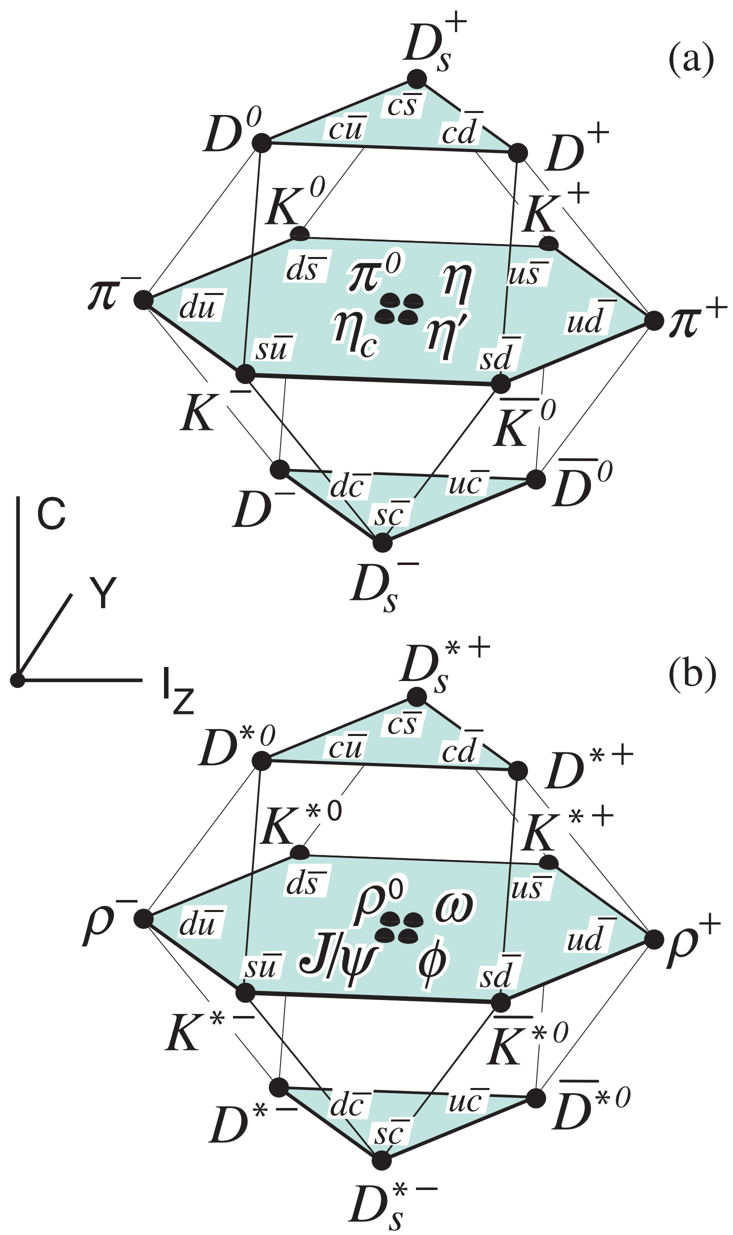
THE ROYAL SOCIETY

quark model

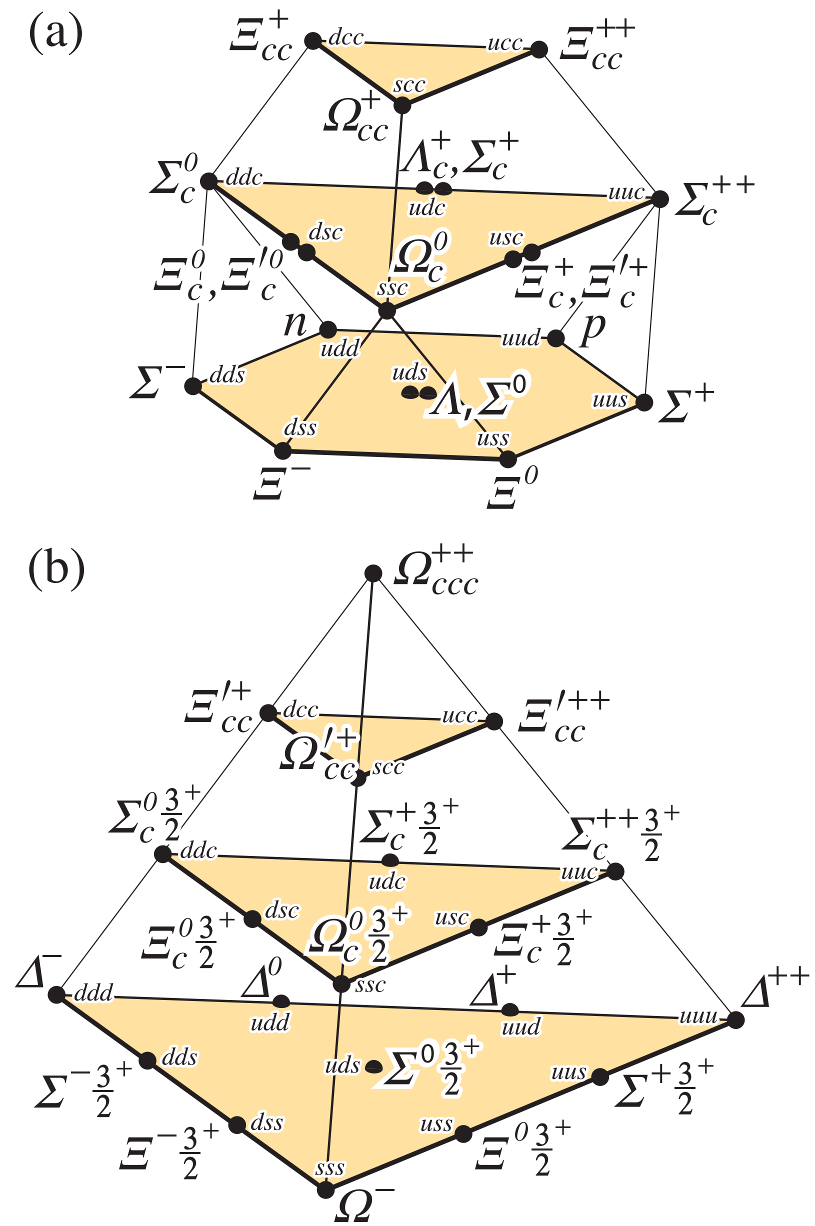
- "conventional" quark model states built from $q\bar{q}$ and qqq

[PDG quark model review]

mesons



baryons



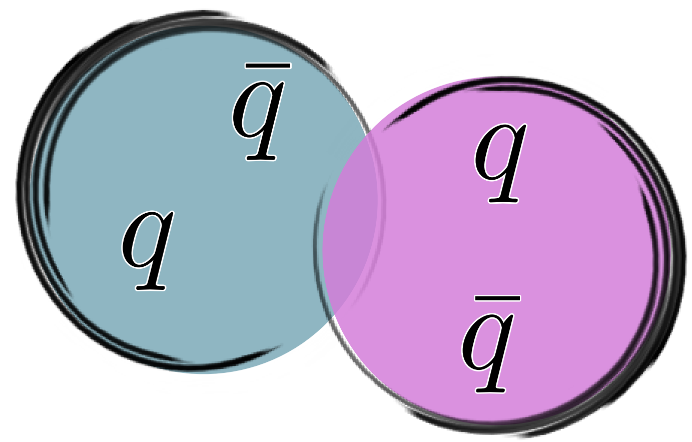
impossible to construct:

$$J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}$$

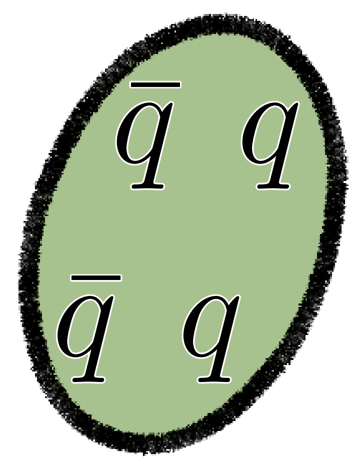
beyond the quark model:

many ways to make a colour singlet

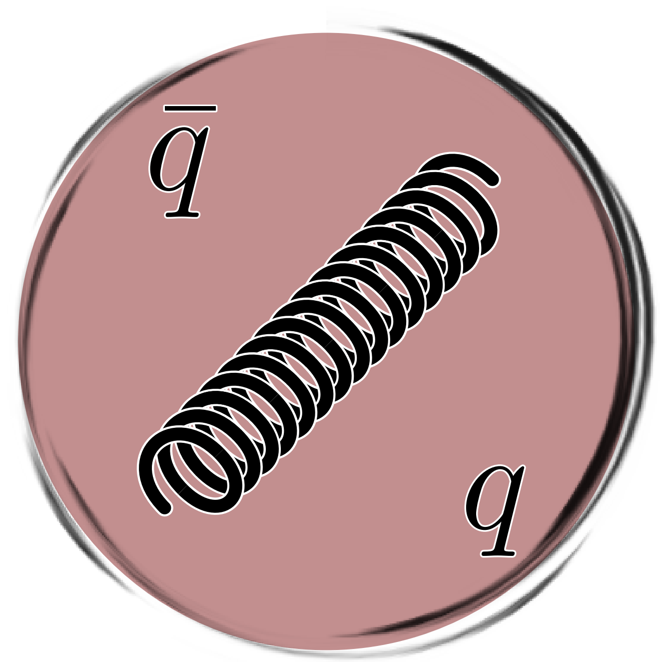
- molecules



- tetraquarks



- hybrids



beyond the quark model:

many ways to make a colour singlet

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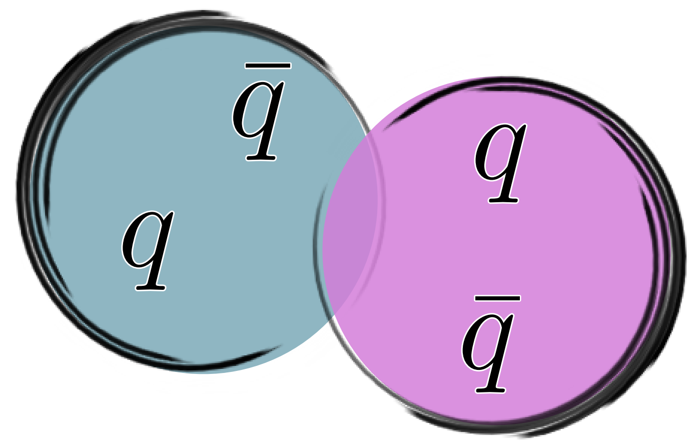
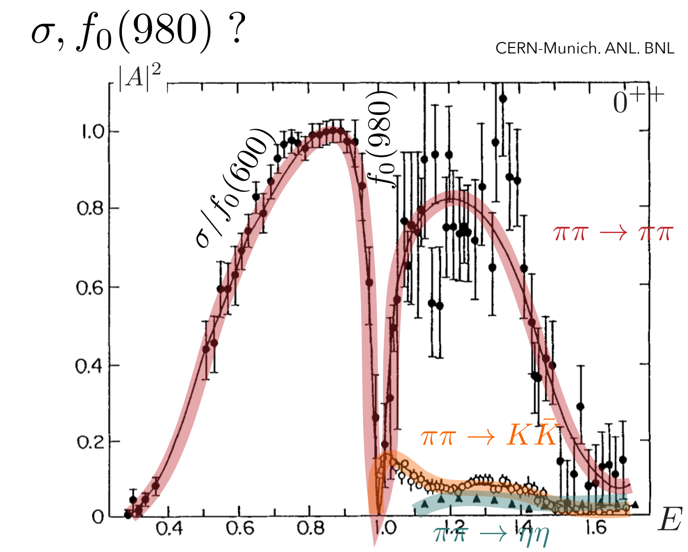
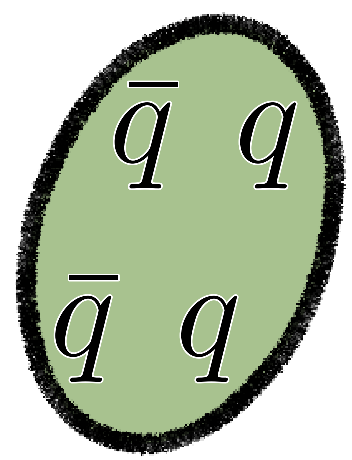
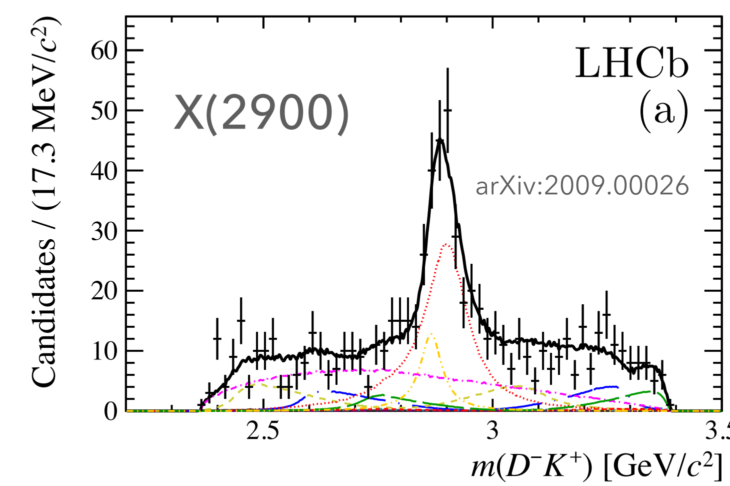


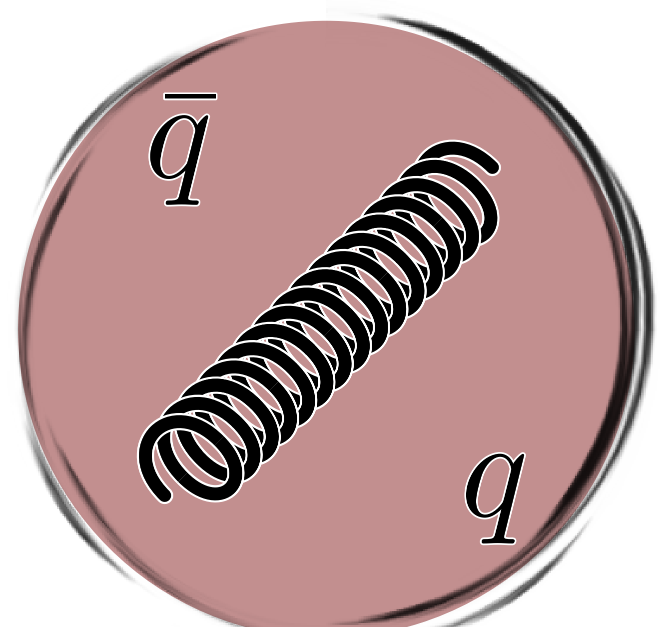


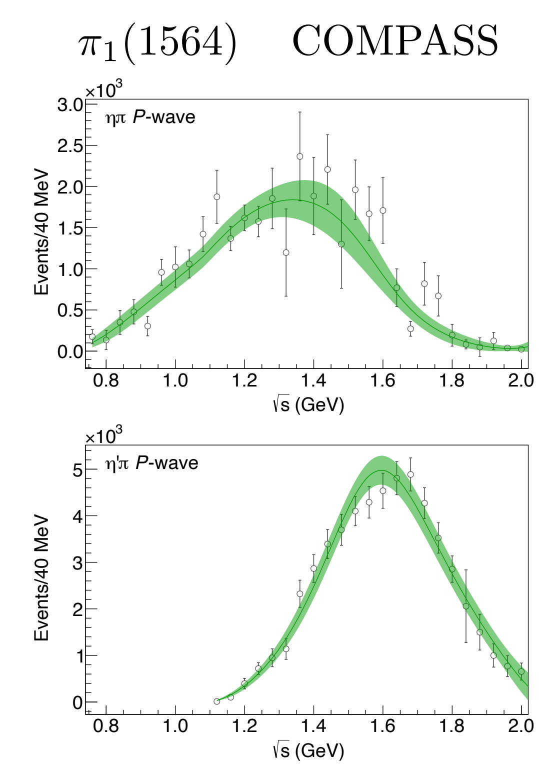
- tetraquarks



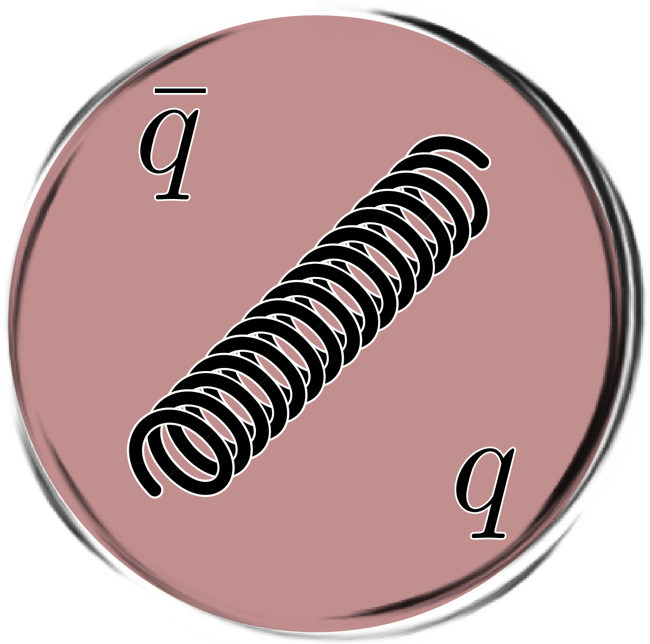


- hybrids

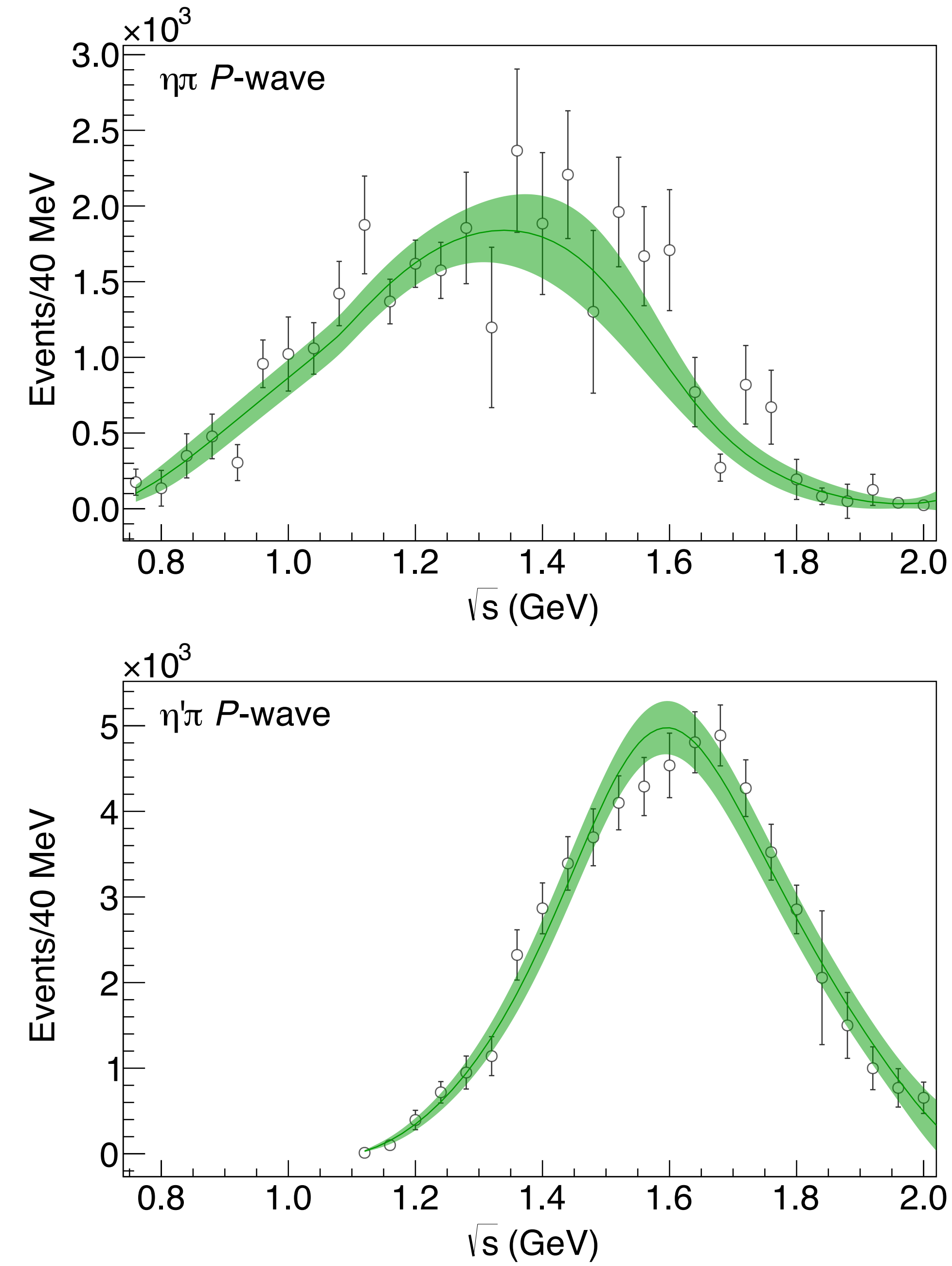




$\pi_1 \quad J^{PC} = 1^{-+}$



COMPASS arXiv: 1408.4286, PLB 740 (2015) 303-311

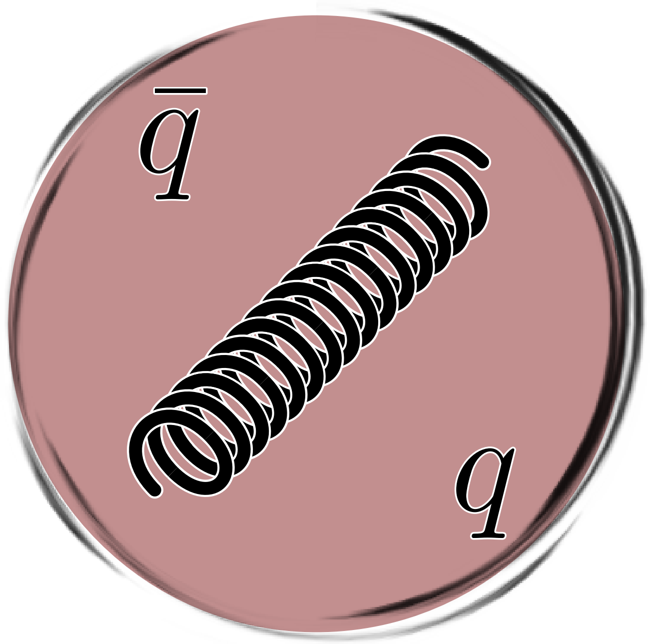


COMPASS PWA sees peaks at different masses:
are there two resonances or one?

JPAC: COMPASS data can be described by a single resonance pole m~1564 MeV, Γ~500 MeV
arXiv:1810.04171 PRL122, 042002 (2019)

similar result: COMPASS+Crystal Barrel data, B. Kopf et al - arXiv: 2008.11566, Γ~400 MeV

$\pi_1 \quad J^{PC} = 1^{-+}$



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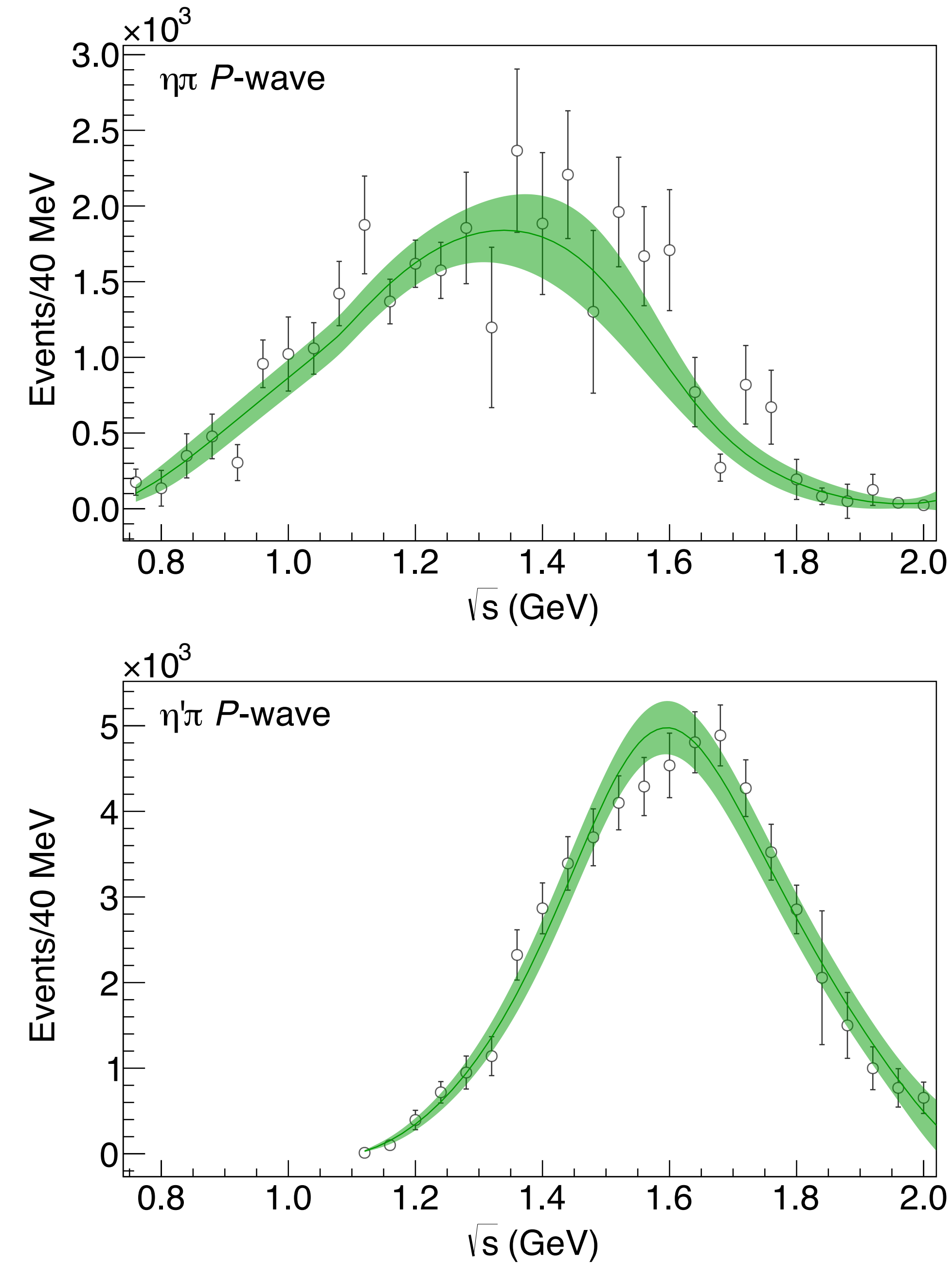
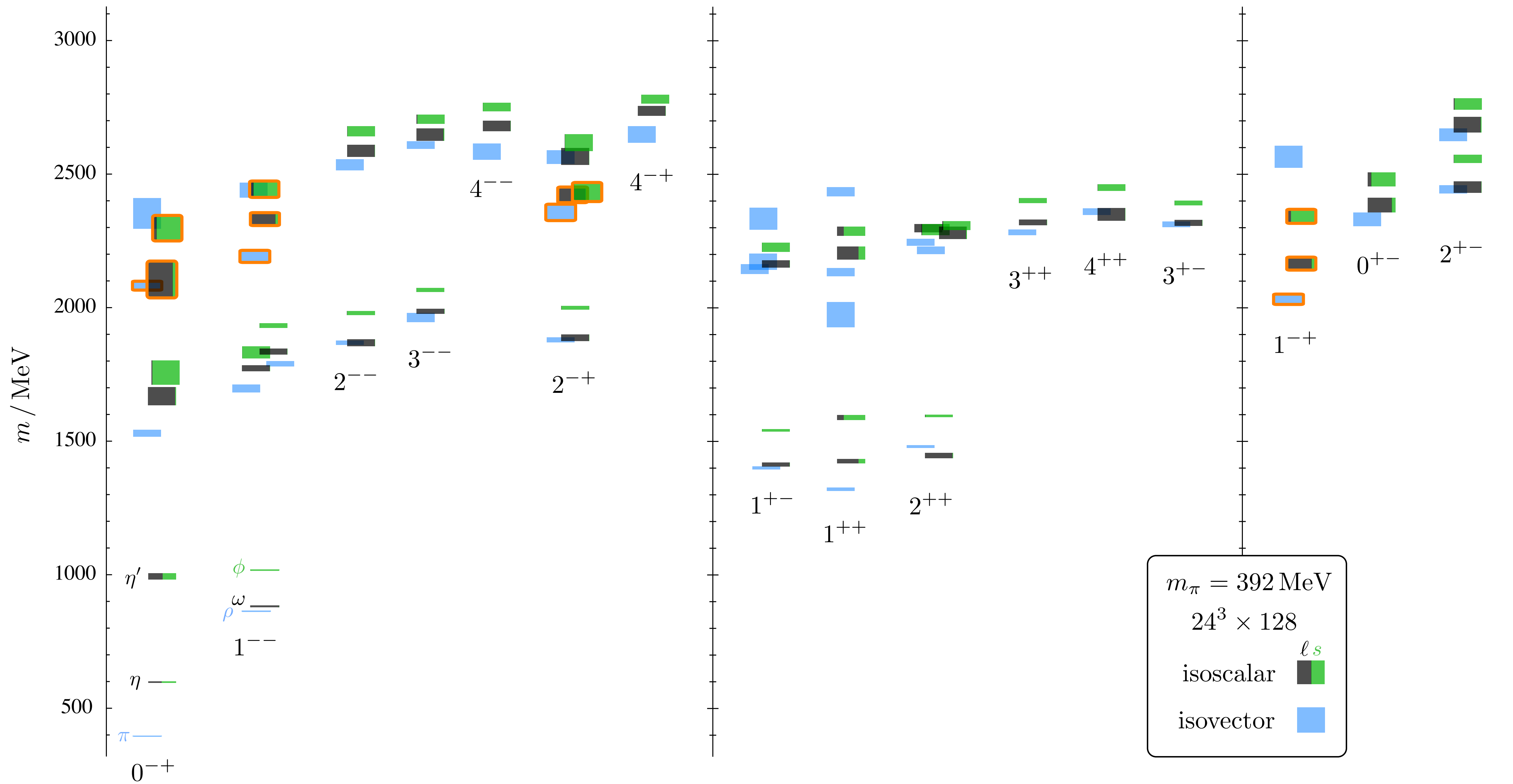


GlueX at Jefferson Lab is collecting data

Hadron Spectrum Collaboration:
spectra from local $q\bar{q}$ constructions

- hybrids found at ***all*** masses from **light** to bottom

Dudek, Edwards, Guo, Thomas - 1309.2608



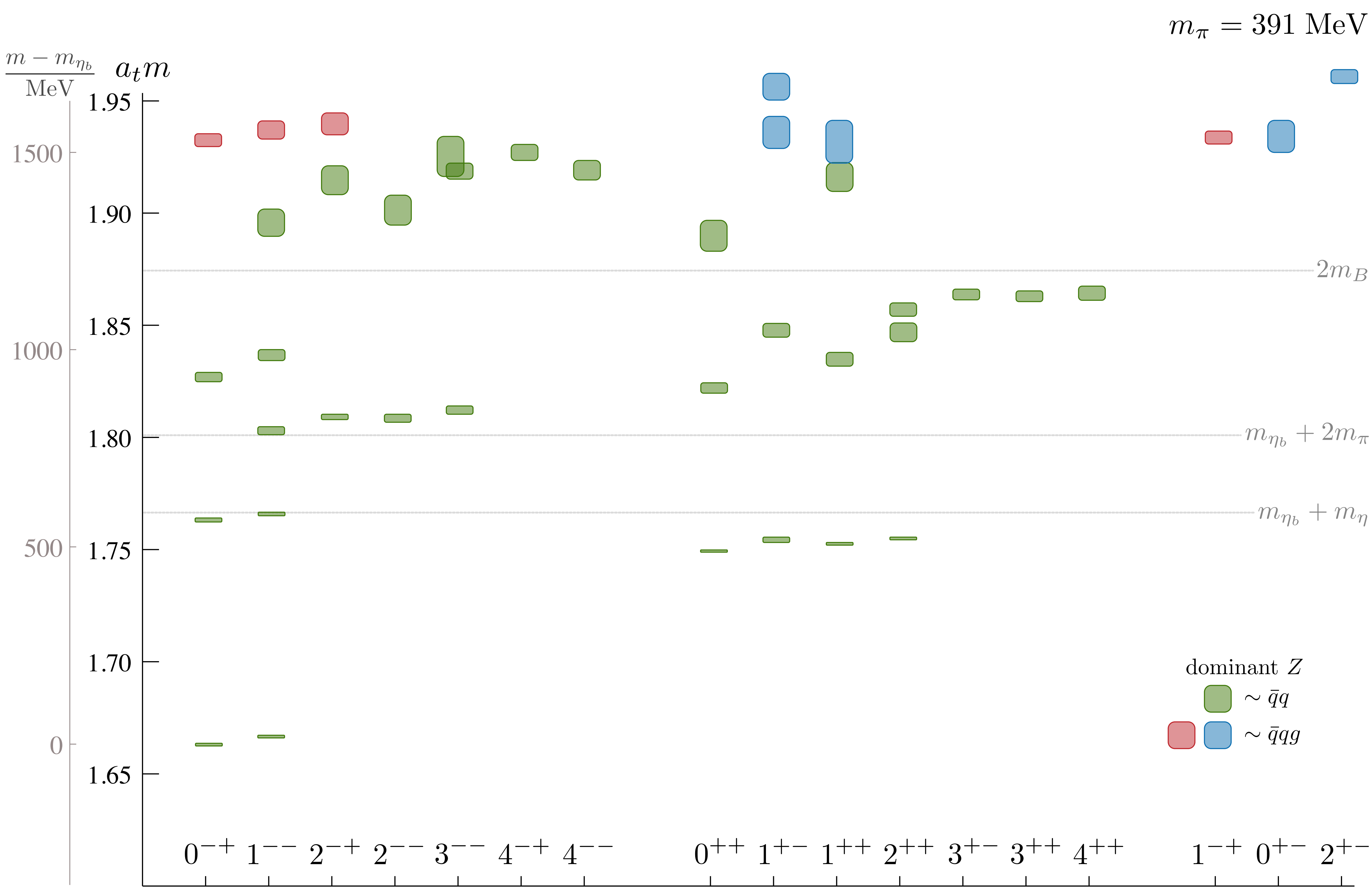
similar states can be identified for half-integer spin

- see https://arxiv.org/abs/1201.2349

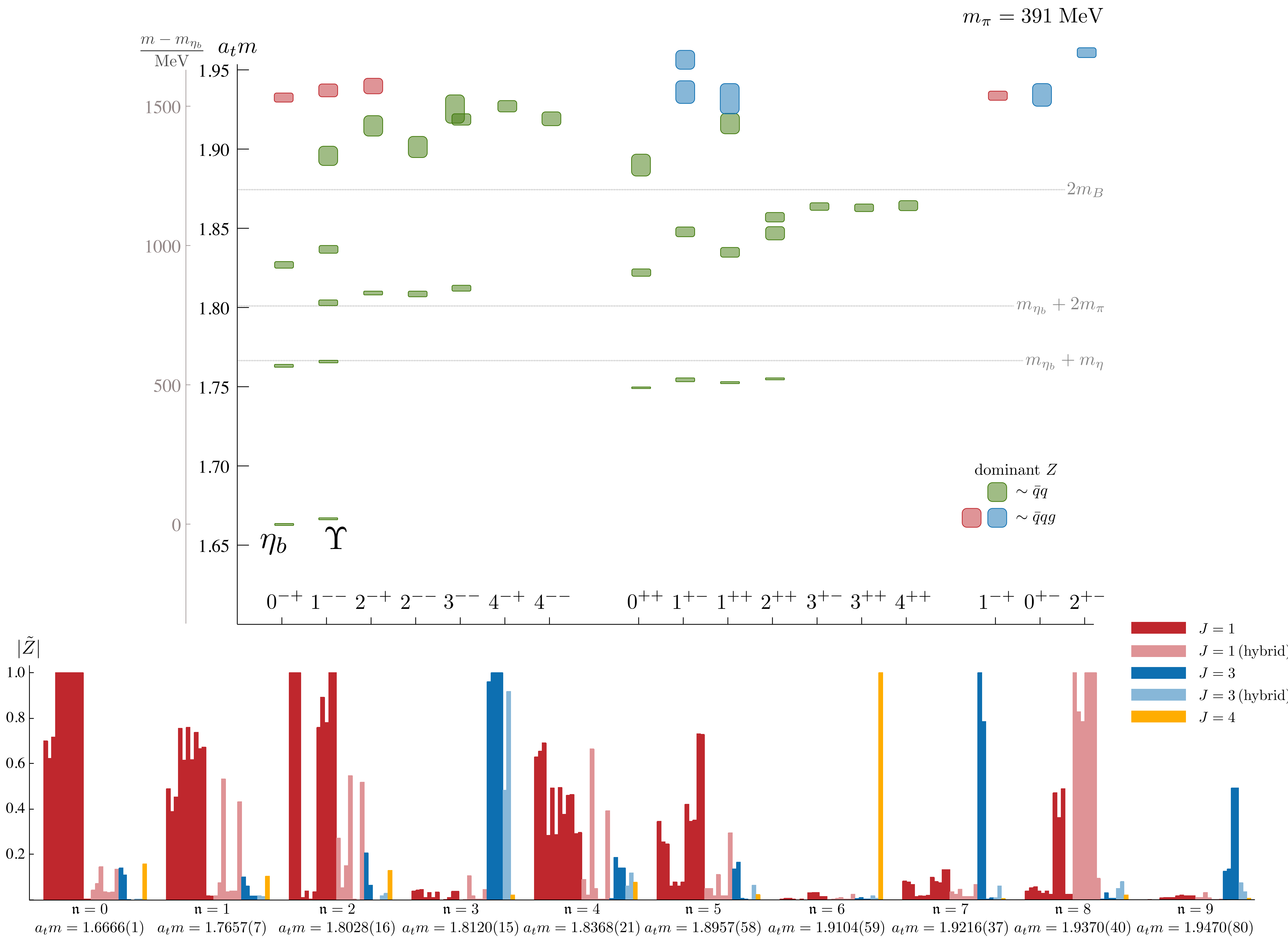
Hadron Spectrum Collaboration:
spectra from local $q\bar{q}$ constructions

- hybrids found at ***all*** masses from light to **bottom**

Sinéad Ryan & DW - 2008.02656



evidence for hybrids from lattice QCD



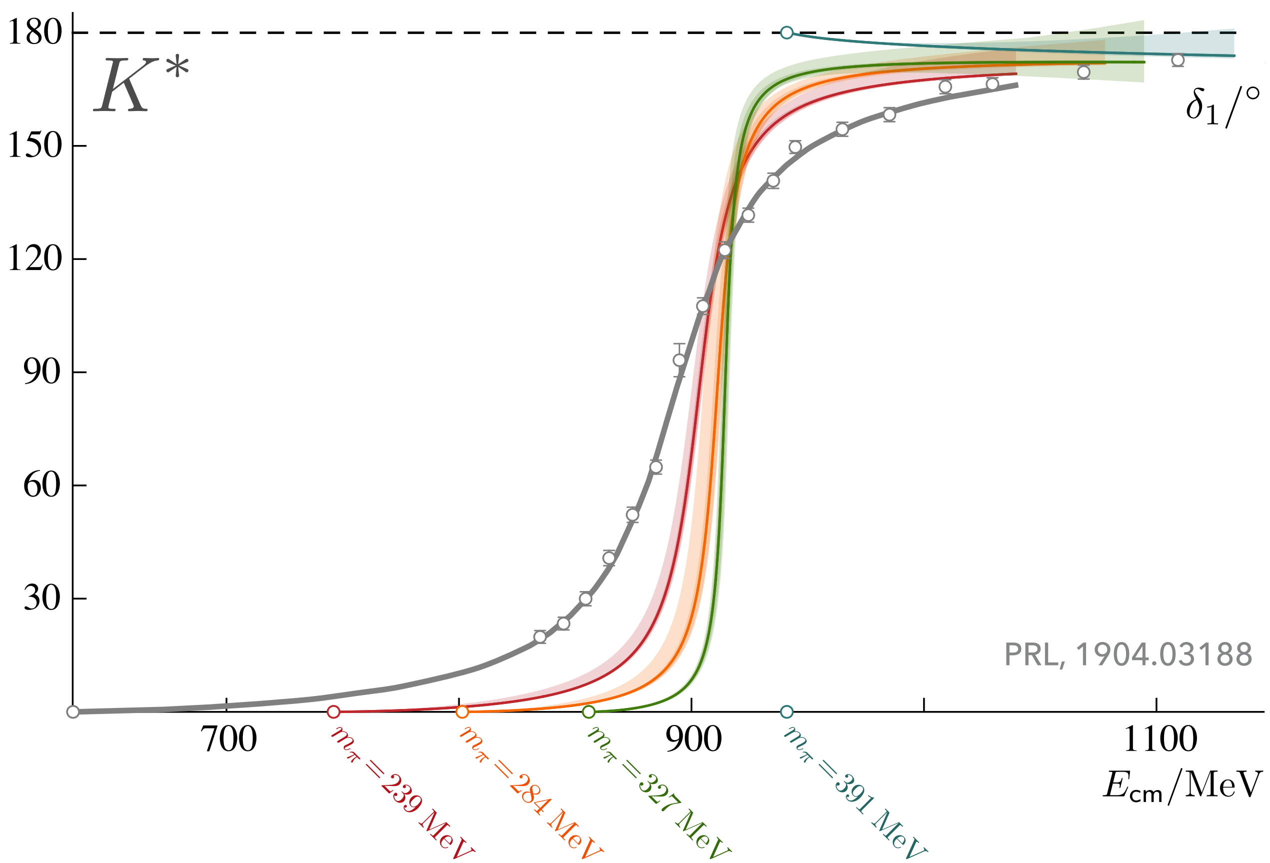
it's very challenging to study the $\pi_1(1564)$ using anything like a physical pion mass

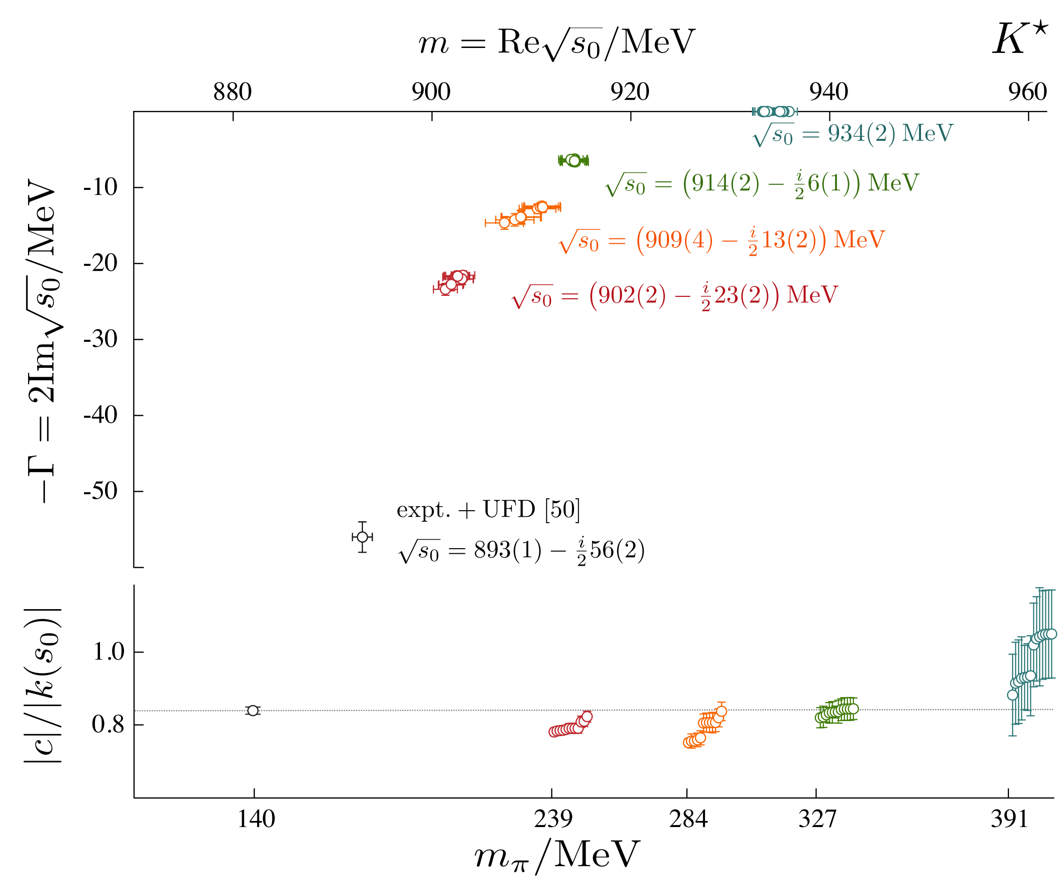
- use heavier-than-physical pions

what does this tell us?

- eg K*(892)

$$t \sim \frac{c^2}{s_0 - s}$$





- pole coupling hardly changes
- similar for rho, b_1, f_2

$$m_\pi = 391 \text{ MeV}$$

$$\pi_1 \to \pi b_1 \to \pi\pi\omega$$
$$\to \pi\pi\phi$$
$$\to \pi\pi\pi\eta$$
$$\to \pi K \bar{K}$$

a problem for another day

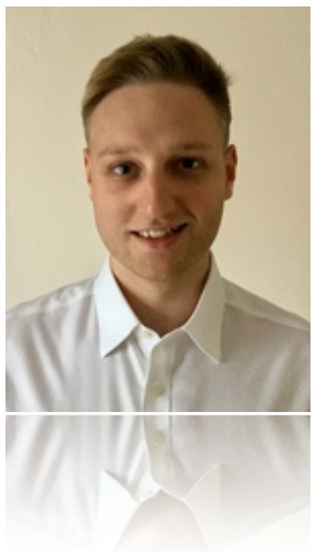
$$m_\pi = 688 \text{ MeV}$$

$$m_u = m_d = m_s$$
$$m_\pi = m_K = m_{\eta^8}$$

much simpler
fewer channels for a first attempt

simple counting 3*700 MeV = 2100 MeV
- 3 body is pushed off to higher energies

Woss Dudek Edwards Thomas



JSA thesis prize
PANDA thesis prize

arXiv:2009.10034

Decays of an exotic 1^{-+} hybrid meson resonance in QCD

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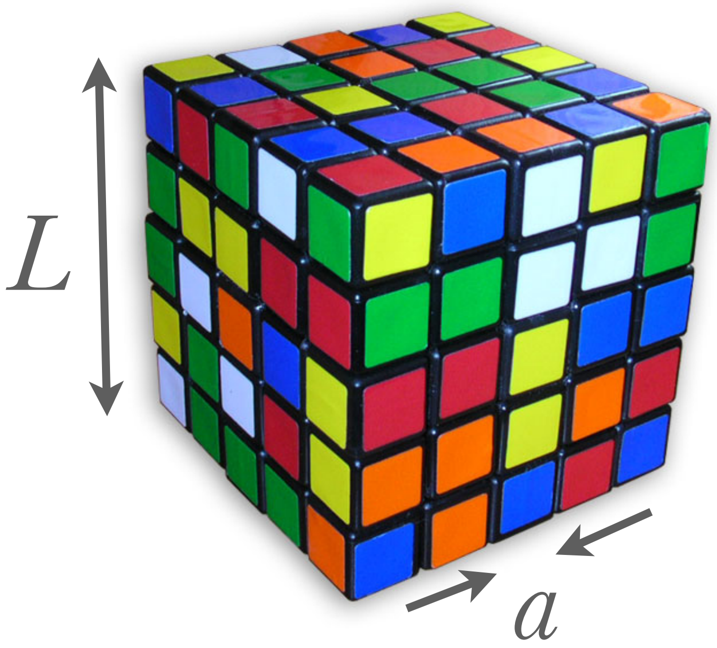
(for the Hadron Spectrum Collaboration)
[1] *DAMTP, University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge, CB3 0WA, UK*
[2] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[3] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
(Dated: 21 September 2020)

We present the first determination of the hadronic decays of the lightest exotic $J^{PC} = 1^{-+}$ resonance in lattice QCD. Working with SU(3) flavor symmetry, where the up, down and strange quark masses approximately match the physical strange-quark mass giving $m_\pi \sim 700$ MeV, we compute finite-volume spectra on six lattice volumes which constrain a scattering system featuring eight coupled channels. Analytically continuing the scattering amplitudes into the complex energy plane, we find a pole singularity corresponding to a narrow resonance which shows relatively weak coupling to the open pseudoscalar–pseudoscalar, vector–pseudoscalar and vector–vector decay channels, but large couplings to at least one kinematically-closed axial-vector–pseudoscalar channel. Attempting a simple extrapolation of the couplings to physical light-quark mass suggests a broad π_1 resonance decaying dominantly through the $b_1\pi$ mode with much smaller decays into $f_1\pi$, $\rho\pi$, $\eta'\pi$ and $\eta\pi$. A large total width is potentially in agreement with the experimental $\pi_1(1564)$ candidate state, observed in $\eta\pi$, $\eta'\pi$, which we suggest may be heavily suppressed decay channels.



anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=12, 14, 16, 18, 20, 24
m_π = 688 MeV

this study - total momentum zero irreps only
sufficient energy levels from 6 volumes
moving frames have a rich, dense spectrum

operators used:

$\bar{\psi}\Gamma\overleftrightarrow{D}\ldots\overleftrightarrow{D}\psi$ local qq-like constructions

$$(\bar{\psi}\boldsymbol{\Gamma}\psi)_i = \underbrace{\epsilon_{ijk}(\bar{\psi}\gamma_j\psi)\,B_k}_{1^{--}\otimes 1^{+-}\to 1^{-+}},$$

includes hybrid-like constructions

$$B_k \propto \epsilon_{kpq}[\overleftrightarrow{D}_p, \overleftrightarrow{D}_q]$$

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\,\Omega_\pi(\vec{p}_2)$$

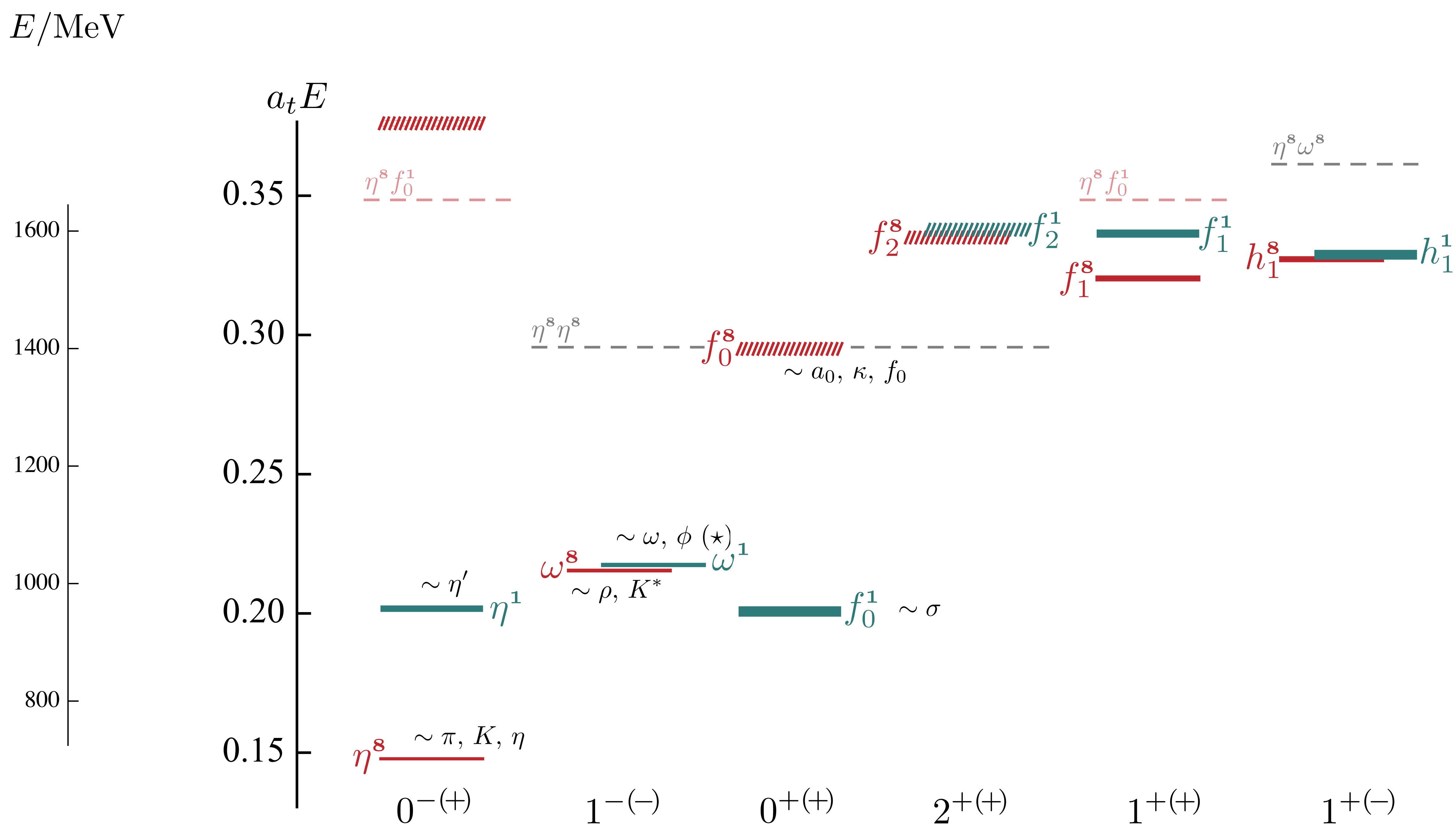
two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

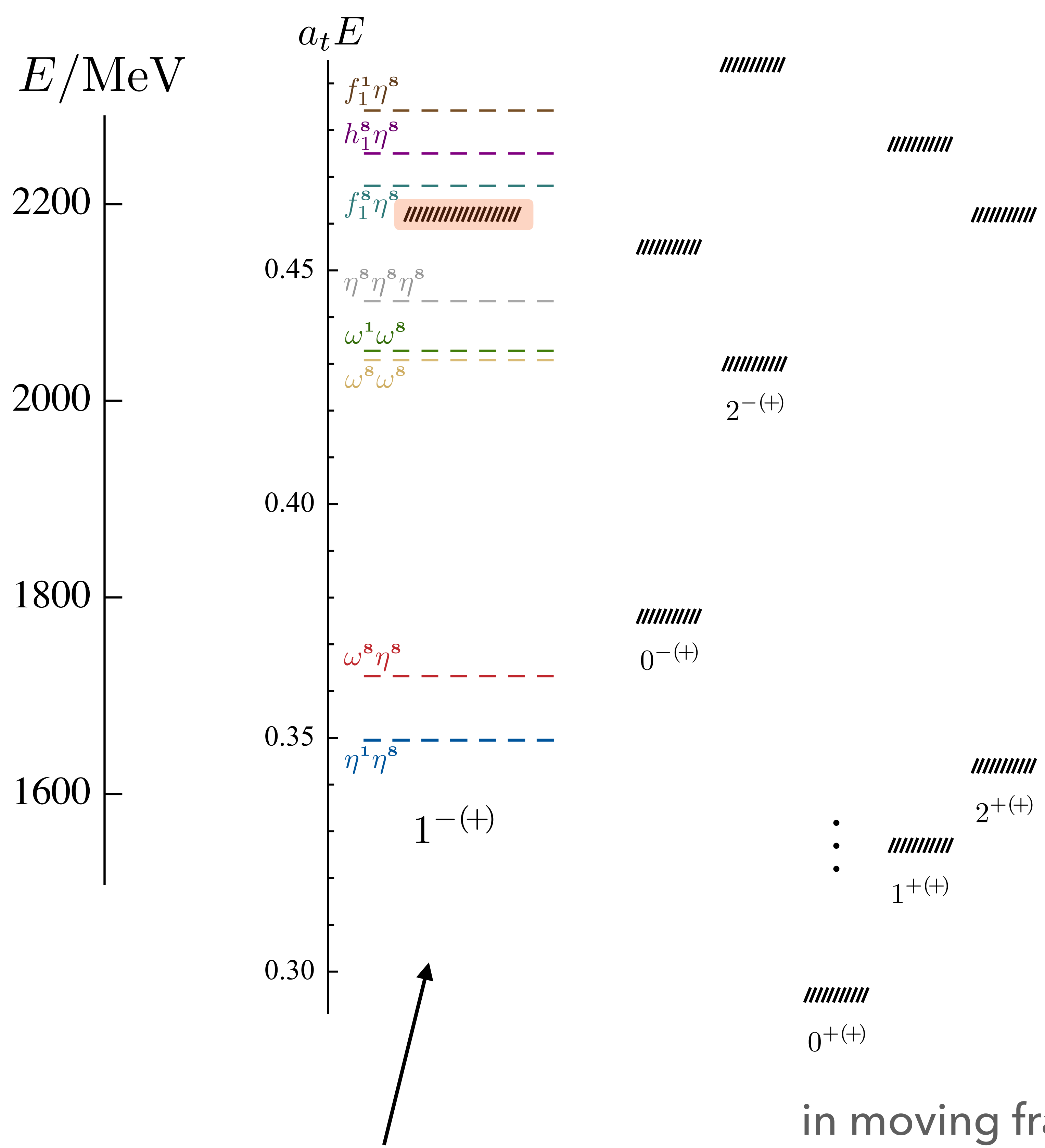
uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many wick contractions

- we compute a large correlation matrix
- then use GEVP to extract energies


E/MeV
1600
1400
1200
1000
800
$a_t E$
0.35
0.30
0.25
0.20
0.15
$\eta^8 f_0^1$
$\eta^8\eta^8$
f_0^8
$\sim a_0,\ \kappa,\ f_0$
f_2^8
f_2^1
$\eta^8 f_0^1$
f_1^1
f_1^8
$\eta^8\omega^8$
h_1^8
h_1^1
$\sim \omega,\ \phi\ (\star)$
ω^8
ω^1
$\sim \rho,\ K^*$
$\sim \eta'$
η^1
f_0^1
$\sim \sigma$
η^8
$\sim \pi,\ K,\ \eta$
$0^{-(+)}$
$1^{-(-)}$
$0^{+(+)}$
$2^{+(+)}$
$1^{+(+)}$
$1^{+(-)}$

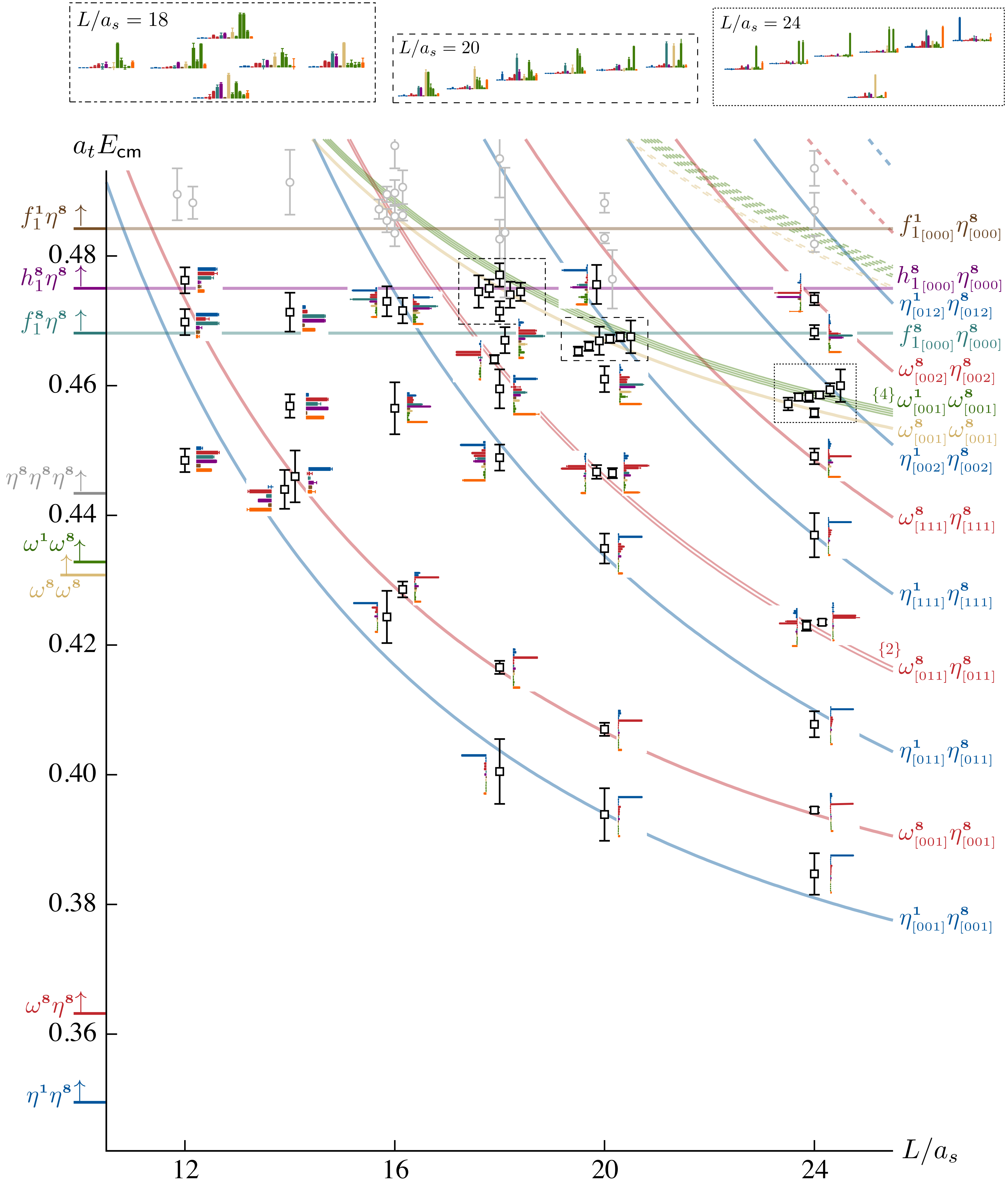
David Wilson (Cambridge)

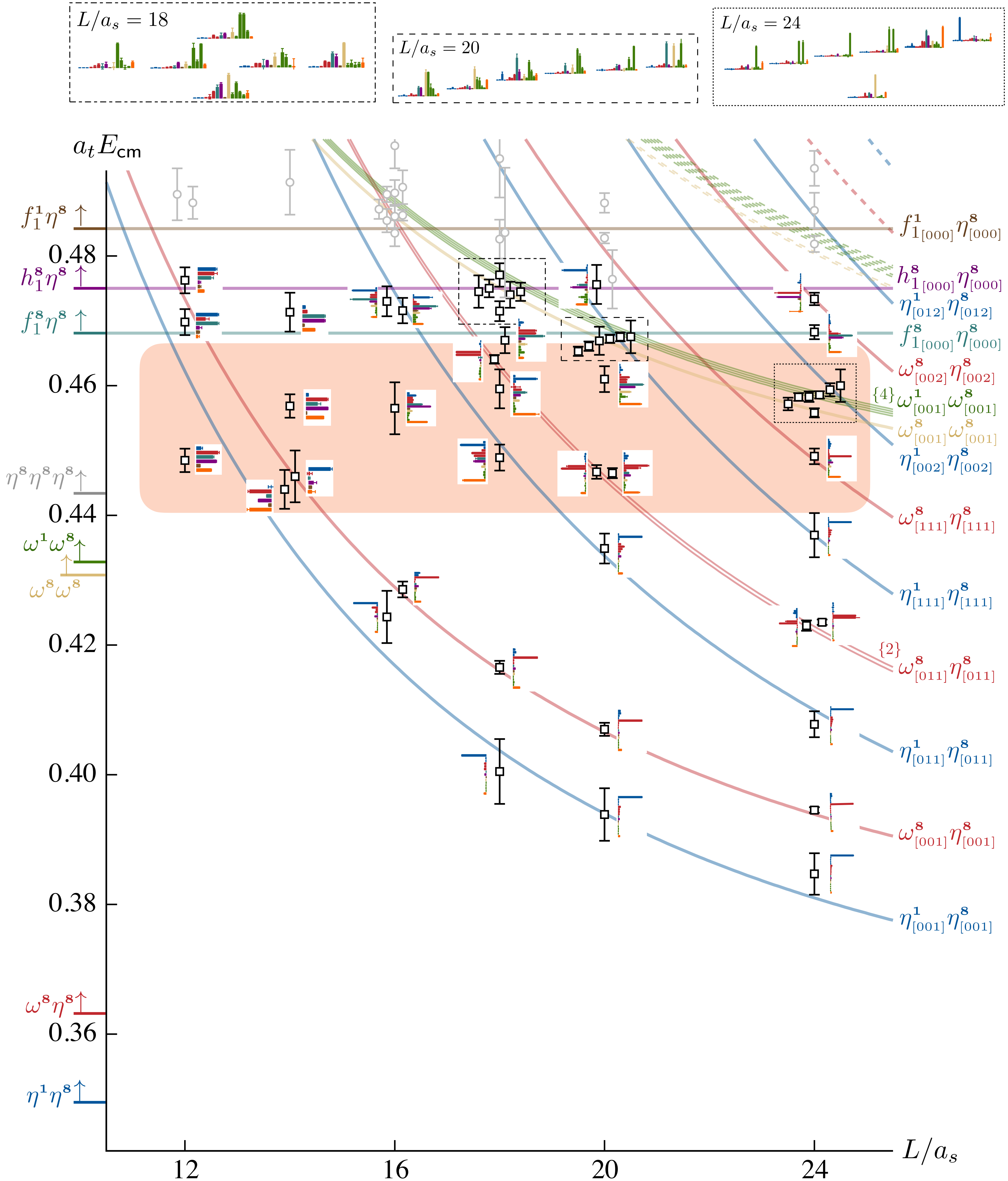


no $\eta^8\eta^8$ because of bose symmetry

in moving frames

- many other resonances to consider


$L/a_s = 18$
$L/a_s = 20$
$L/a_s = 24$
$a_t E_{\mathsf{cm}}$
$f_1^1\eta^8$ ↑
$h_1^8\eta^8$ ↑
$f_1^8\eta^8$ ↑
$\eta^8\eta^8\eta^8$ ↑
$\omega^1\omega^8$ ↑
$\omega^8\omega^8$
$\omega^8\eta^8$ ↑
$\eta^1\eta^8$ ↑
0.48
0.46
0.44
0.42
0.40
0.38
0.36
12
16
20
24
L/a_s
$f^1_{1[000]}\eta^8_{[000]}$
$h^8_{1[000]}\eta^8_{[000]}$
$\eta^1_{[012]}\eta^8_{[012]}$
$f^8_{1[000]}\eta^8_{[000]}$
$\omega^8_{[002]}\eta^8_{[002]}$
{4}
$\omega^1_{[001]}\omega^8_{[001]}$
$\omega^8_{[001]}\omega^8_{[001]}$
$\eta^1_{[002]}\eta^8_{[002]}$
$\omega^8_{[111]}\eta^8_{[111]}$
$\eta^1_{[111]}\eta^8_{[111]}$
{2}
$\omega^8_{[011]}\eta^8_{[011]}$
$\eta^1_{[011]}\eta^8_{[011]}$
$\omega^8_{[001]}\eta^8_{[001]}$
$\eta^1_{[001]}\eta^8_{[001]}$



$L/a_s = 18$
$L/a_s = 20$
$L/a_s = 24$
$a_t E_{cm}$
$f_1^1 \eta^8$
$h_1^8 \eta^8$
$f_1^8 \eta^8$
$\eta^8\eta^8\eta^8$
$\omega^1\omega^8$
$\omega^8\omega^8$
$\omega^8\eta^8$
$\eta^1\eta^8$
0.48
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12
16
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L/a_s
$f^1_{1\,[000]}\eta^8_{[000]}$
$h^8_{1\,[000]}\eta^8_{[000]}$
$\eta^1_{[012]}\eta^8_{[012]}$
$f^8_{1\,[000]}\eta^8_{[000]}$
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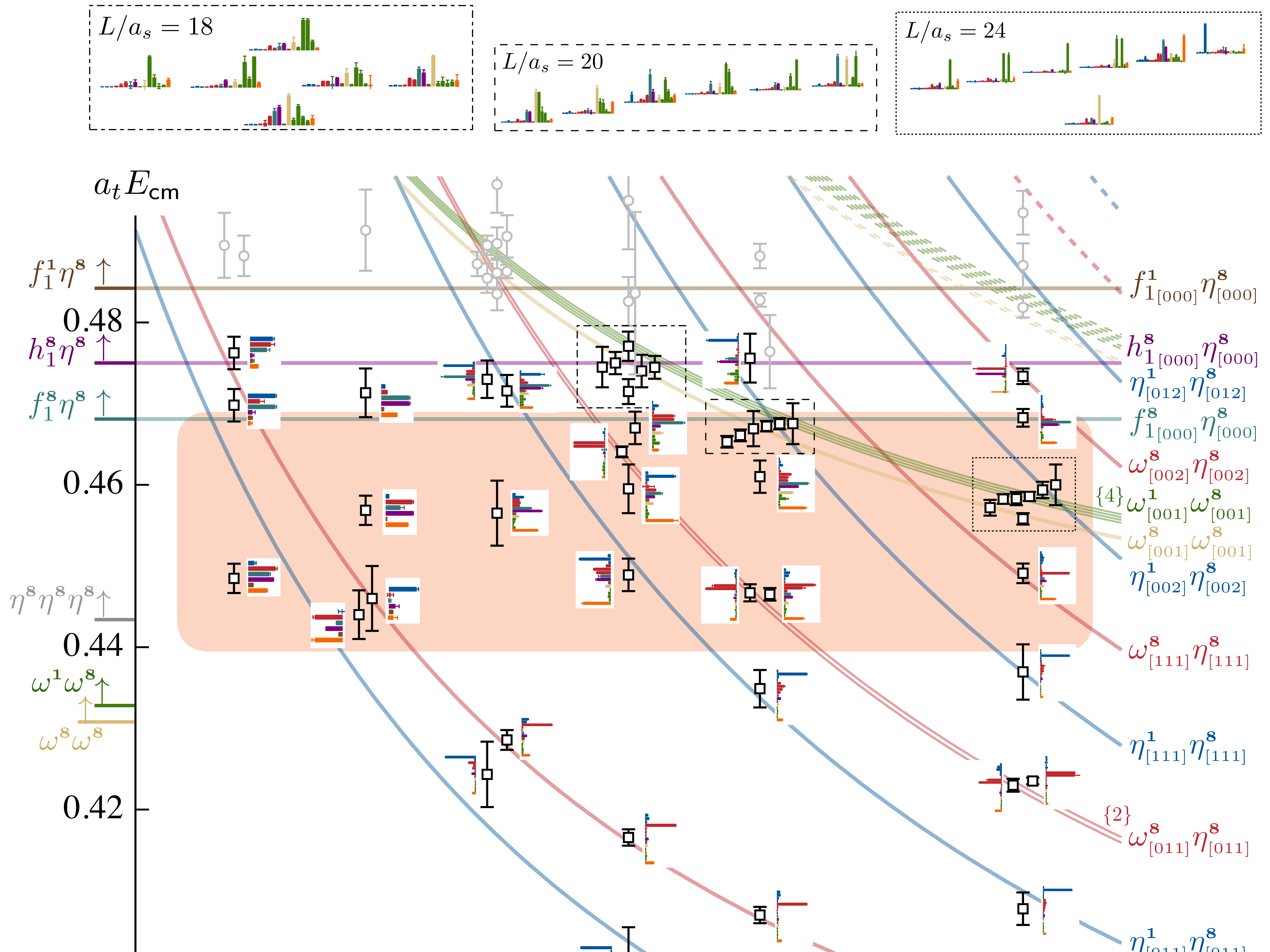
{4}
$\omega^1_{[001]}\omega^8_{[001]}$
$\omega^8_{[001]}\omega^8_{[001]}$
$\eta^1_{[002]}\eta^8_{[002]}$
$\omega^8_{[111]}\eta^8_{[111]}$
$\eta^1_{[111]}\eta^8_{[111]}$
{2}
$\omega^8_{[011]}\eta^8_{[011]}$
$\eta^1_{[011]}\eta^8_{[011]}$
$\omega^8_{[001]}\eta^8_{[001]}$
$\eta^1_{[001]}\eta^8_{[001]}$



$L/a_s = 18$
$L/a_s = 20$
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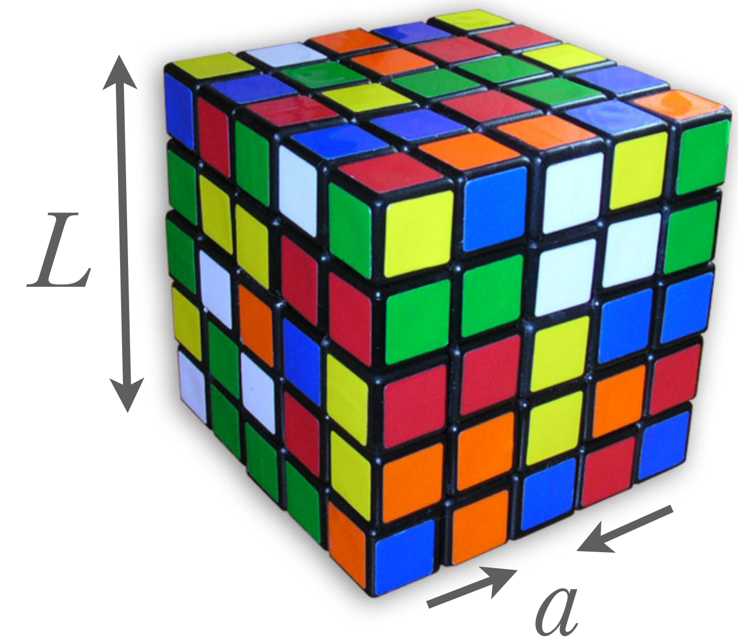
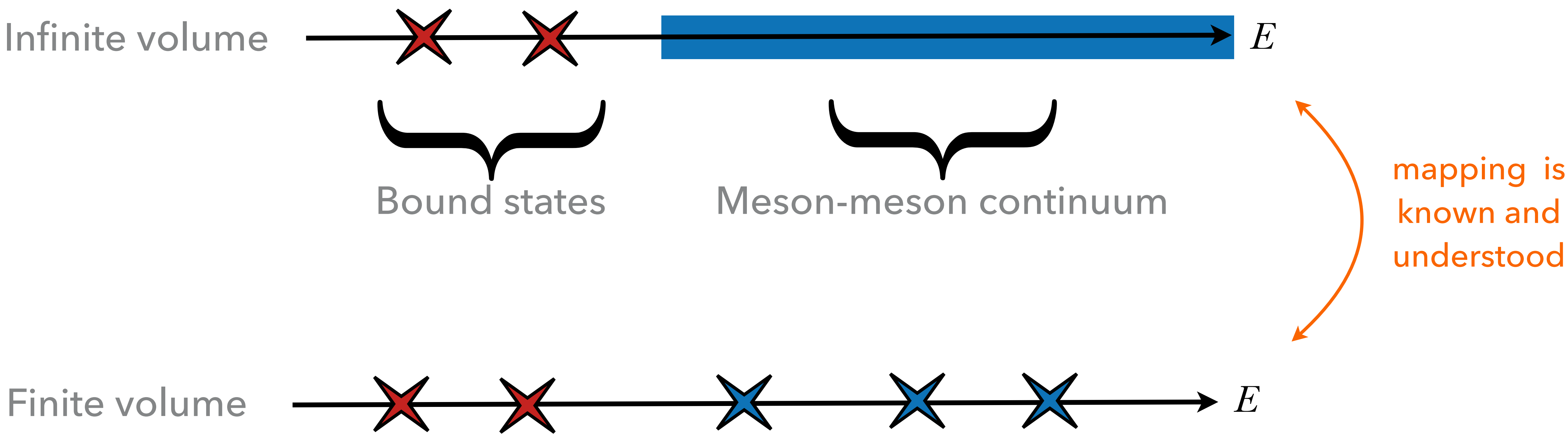
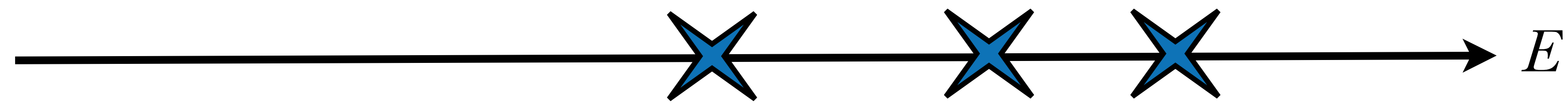
$a_t E_{\mathsf{cm}}$
0.48
0.46
0.44
0.42
$f_1^{\mathbf{1}}\eta^{\mathbf{8}}$
$h_1^{\mathbf{8}}\eta^{\mathbf{8}}$
$f_1^{\mathbf{8}}\eta^{\mathbf{8}}$
$\eta^{\mathbf{8}}\eta^{\mathbf{8}}\eta^{\mathbf{8}}$
$\omega^{\mathbf{1}}\omega^{\mathbf{8}}$
$\omega^{\mathbf{8}}\omega^{\mathbf{8}}$
$f^{\mathbf{1}}_{1[000]}\eta^{\mathbf{8}}_{[000]}$
$h^{\mathbf{8}}_{1[000]}\eta^{\mathbf{8}}_{[000]}$
$\eta^{\mathbf{1}}_{[012]}\eta^{\mathbf{8}}_{[012]}$
$f^{\mathbf{8}}_{1[000]}\eta^{\mathbf{8}}_{[000]}$
$\omega^{\mathbf{8}}_{[002]}\eta^{\mathbf{8}}_{[002]}$
{4} $\omega^{\mathbf{1}}_{[001]}\omega^{\mathbf{8}}_{[001]}$
$\omega^{\mathbf{8}}_{[001]}\omega^{\mathbf{8}}_{[001]}$
$\eta^{\mathbf{1}}_{[002]}\eta^{\mathbf{8}}_{[002]}$
$\omega^{\mathbf{8}}_{[111]}\eta^{\mathbf{8}}_{[111]}$
$\eta^{\mathbf{1}}_{[111]}\eta^{\mathbf{8}}_{[111]}$
{2} $\omega^{\mathbf{8}}_{[011]}\eta^{\mathbf{8}}_{[011]}$



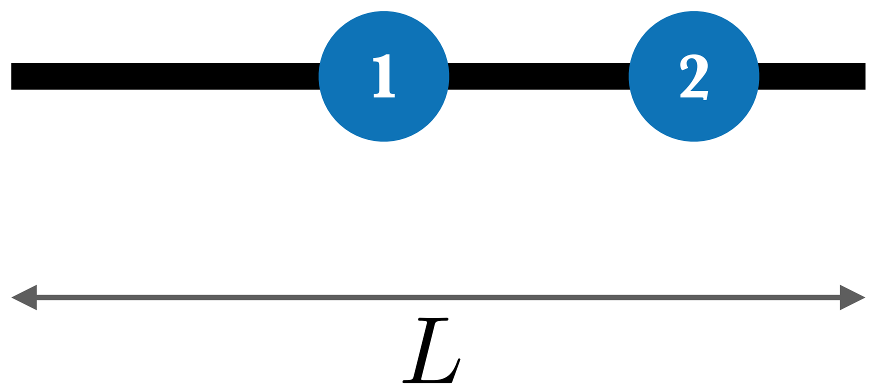


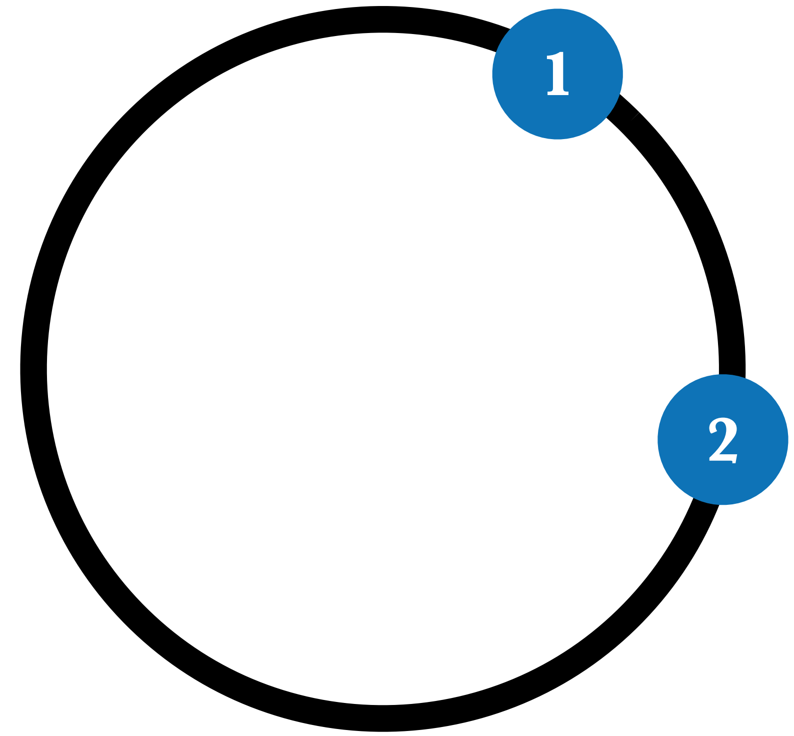
Momentum is quantised - no continuum $\vec{p} = \frac{2\pi}{L}\vec{n}$

Lüscher et al



1-dimensional QM, periodic BC, two particles, no interactions

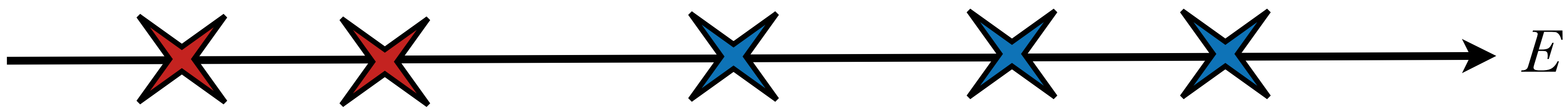




momentum is quantised: $p_i = \frac{2\pi n_i}{L}$

two particle energies are discrete:

$$E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}}$$

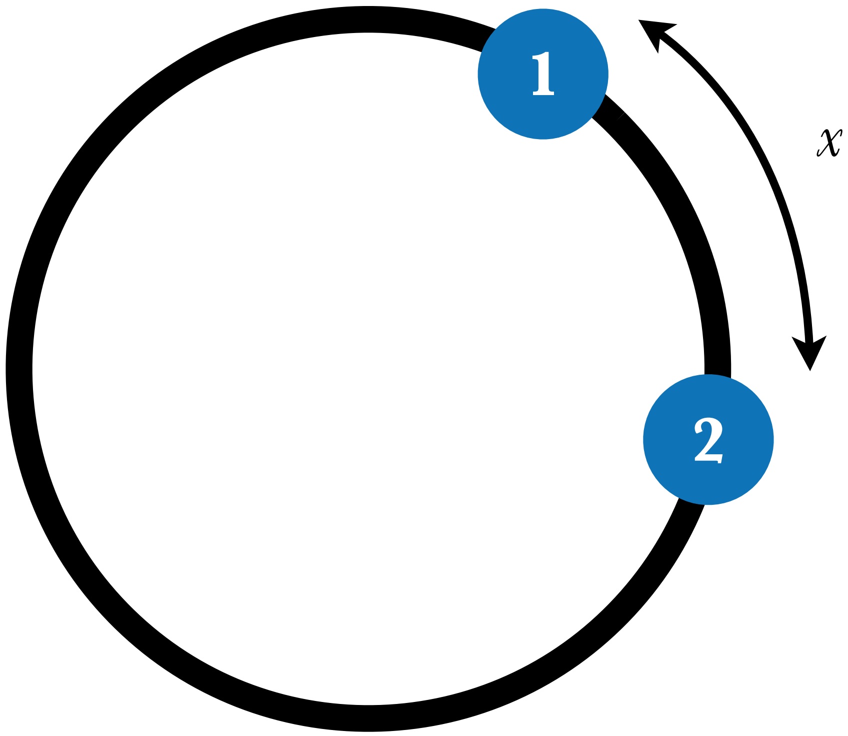


1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via Lüscher's method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

Direct extension of the elastic quantisation condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering *t*-matrix

known finite-volume functions

Many extensions of the original Lüscher formalism to moving frames, unequal masses, etc

Quantisation condition for an arbitrary t-matrix of coupled (pseudo)scalars - all in agreement
Hansen & Sharpe 2012, Briceño & Davoudi 2012, Guo et al 2012

Quantisation condition generalised to scattering of particles with non-zero spin for arbitrary scattering amplitudes (the one used here):
Briceño, arXiv:1401.3312, PRD 89 (2014) 7, 074507

$$\det\left[\boldsymbol{D}(E_{\mathsf{cm}})\right] = 0$$

$$\boldsymbol{D}(E_{\mathsf{cm}}) = \mathbf{1} + i\boldsymbol{\rho}(E_{\mathsf{cm}})\cdot\boldsymbol{t}(E_{\mathsf{cm}})\cdot\left(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E_{\mathsf{cm}},L)\right)$$

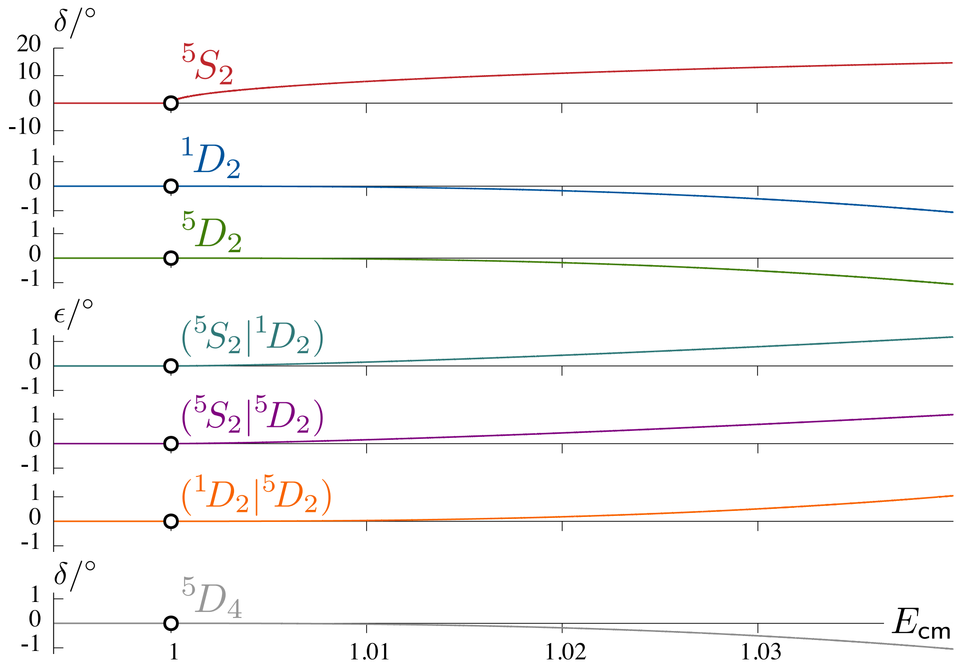
$$\boldsymbol{S}(E_{\mathsf{cm}}) = \mathbf{1} + 2i\sqrt{\boldsymbol{\rho}(E_{\mathsf{cm}})}\cdot\boldsymbol{t}(E_{\mathsf{cm}})\cdot\sqrt{\boldsymbol{\rho}(E_{\mathsf{cm}})}$$

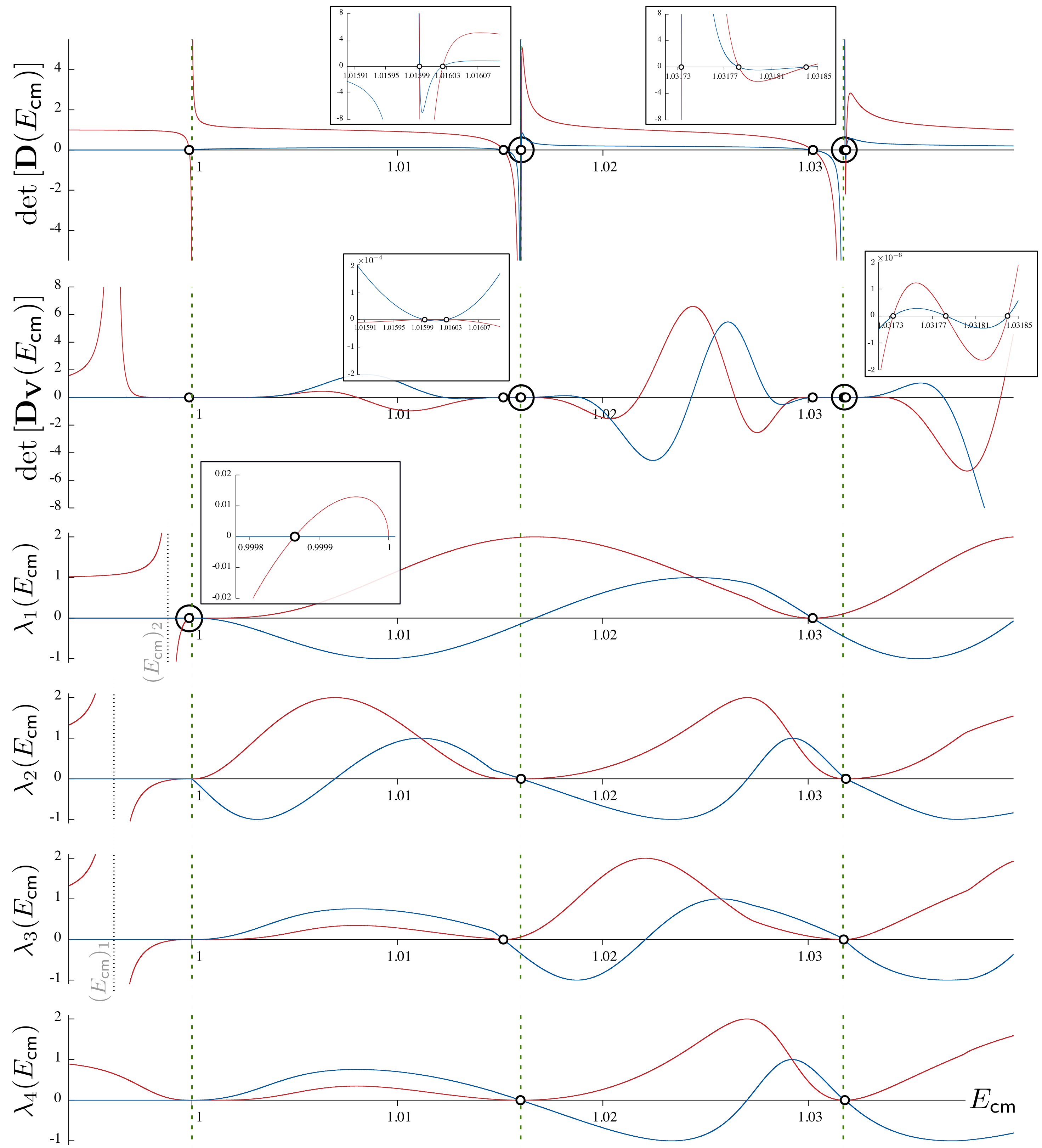
$$\boldsymbol{D}_{\boldsymbol{V}}(E_{\mathsf{cm}}) = \mathbf{1} + \boldsymbol{S}\cdot\boldsymbol{V}$$

$$\boldsymbol{V} = (1+i\boldsymbol{\mathcal{M}})(1-i\boldsymbol{\mathcal{M}})^{-1}$$

$$\det\left[\boldsymbol{D}_{\boldsymbol{V}}(E_{\mathsf{cm}})\right] = \prod_{p=1}^{n}\lambda_p(E_{\mathsf{cm}})$$

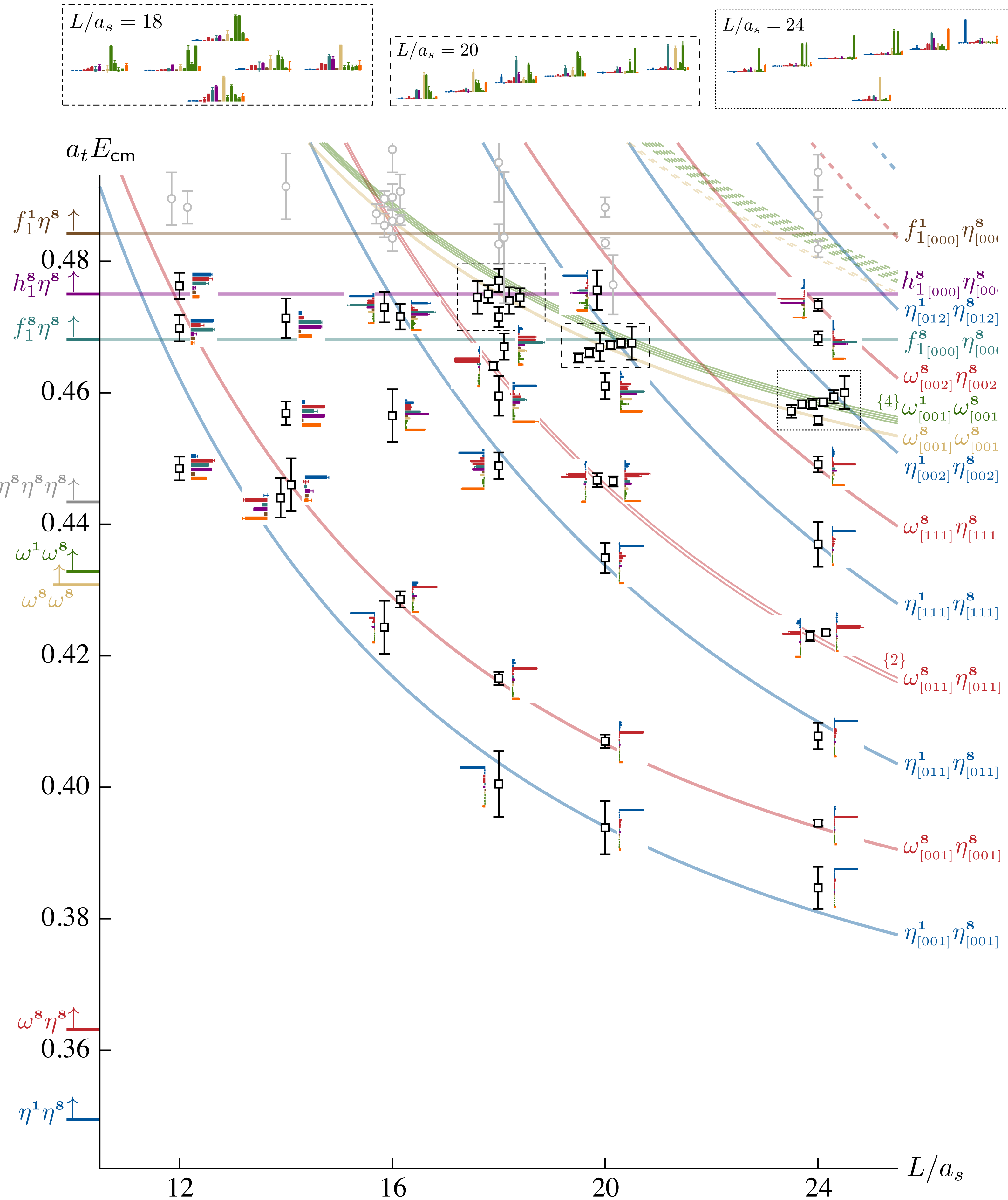
$$\boldsymbol{D}_{\boldsymbol{V}}(E_{\mathsf{cm}})\,\boldsymbol{v}^{(p)}(E_{\mathsf{cm}}) = \lambda_p(E_{\mathsf{cm}})\,\boldsymbol{v}^{(p)}(E_{\mathsf{cm}})$$





an eigenvalue decomposition simplifies root finding in systems with degenerate levels

Woss, DW, Dudek 2001.08474



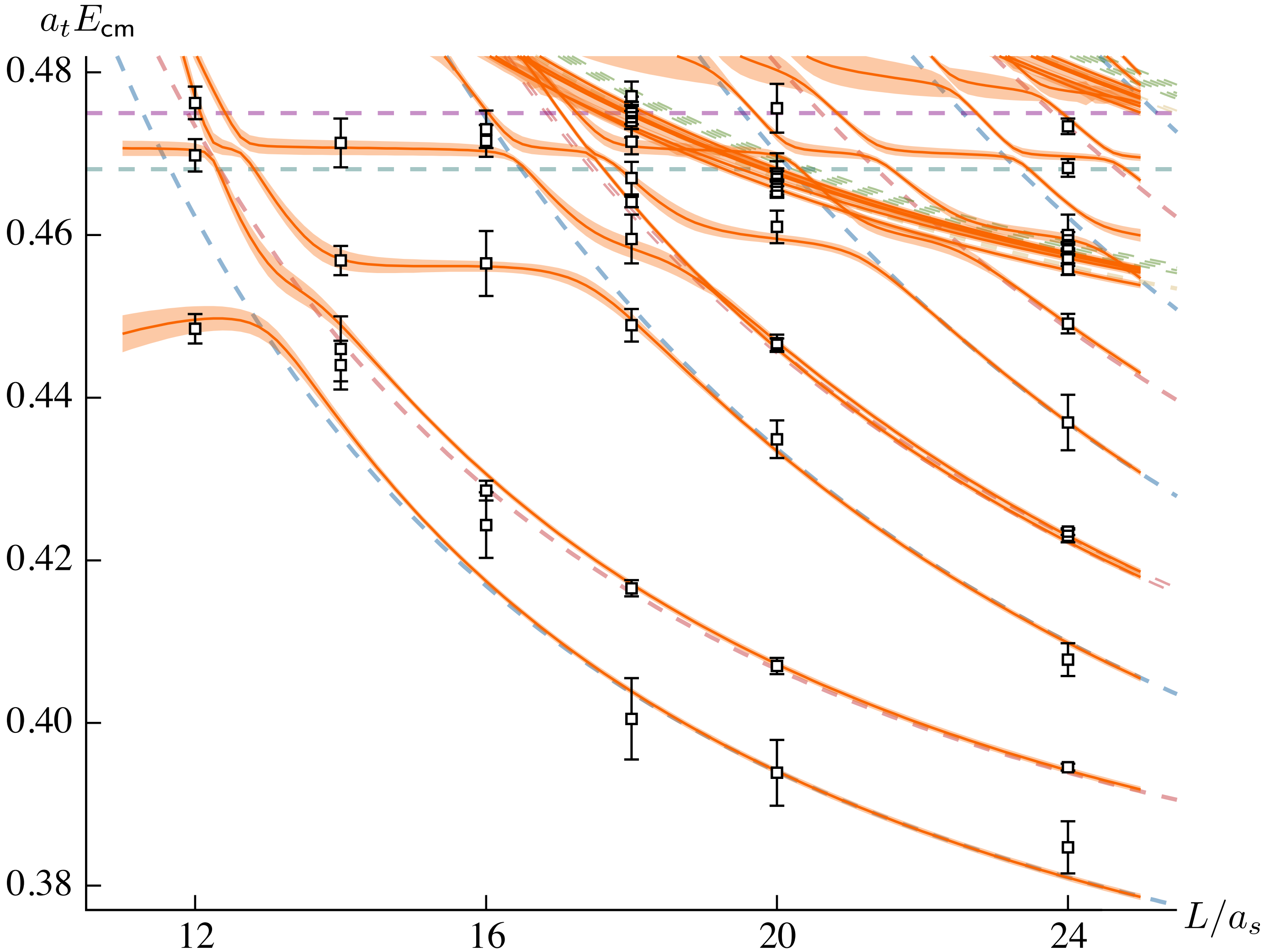
partial waves

$1^{-(+)}$	$\eta^1\eta^8\{^1P_1\}$
	$\omega^8\eta^8\{^3P_1\}$
	$\omega^8\omega^8\{^3P_1\}$, $\omega^1\omega^8\{^1P_1, {}^3P_1, {}^5P_1\}$
	$f_1^8\eta^8\{^3S_1\}$, $h_1^8\eta^8\{^3S_1\}$
$3^{-(+)}$	$\eta^1\eta^8\{^1F_3\}$
	$\omega^8\eta^8\{^3F_3\}$
	$\omega^1\omega^8\{^5P_3\}$

K-matrix parametrisation

$$t^{-1} = K^{-1} + I$$

- pole coupled in 1^{-+}
- various constants



partial waves

$1^{-(+)}$	$\eta^1\eta^8\{^1P_1\}$ $\omega^8\eta^8\{^3P_1\}$ $\omega^8\omega^8\{^3P_1\}$, $\omega^1\omega^8\{^1P_1, {}^3P_1, {}^5P_1\}$ $f_1^8\eta^8\{^3S_1\}$, $h_1^8\eta^8\{^3S_1\}$
$3^{-(+)}$	$\eta^1\eta^8\{^1F_3\}$ $\omega^8\eta^8\{^3F_3\}$ $\omega^1\omega^8\{^5P_3\}$

K-matrix parametrisation

$$t^{-1} = K^{-1} + I$$

- pole coupled in 1^{-+}
- various constants

$$\boldsymbol{K}_{VV}(s) = \begin{bmatrix} \gamma_{\omega^8\omega^8\{^3P_1\}} & 0 & 0 & 0 \\ 0 & \gamma_{\omega^1\omega^8\{^1P_1\}} & 0 & 0 \\ 0 & 0 & \gamma_{\omega^1\omega^8\{^3P_1\}} & 0 \\ 0 & 0 & 0 & \gamma_{\omega^1\omega^8\{^5P_1\}} \end{bmatrix}$$

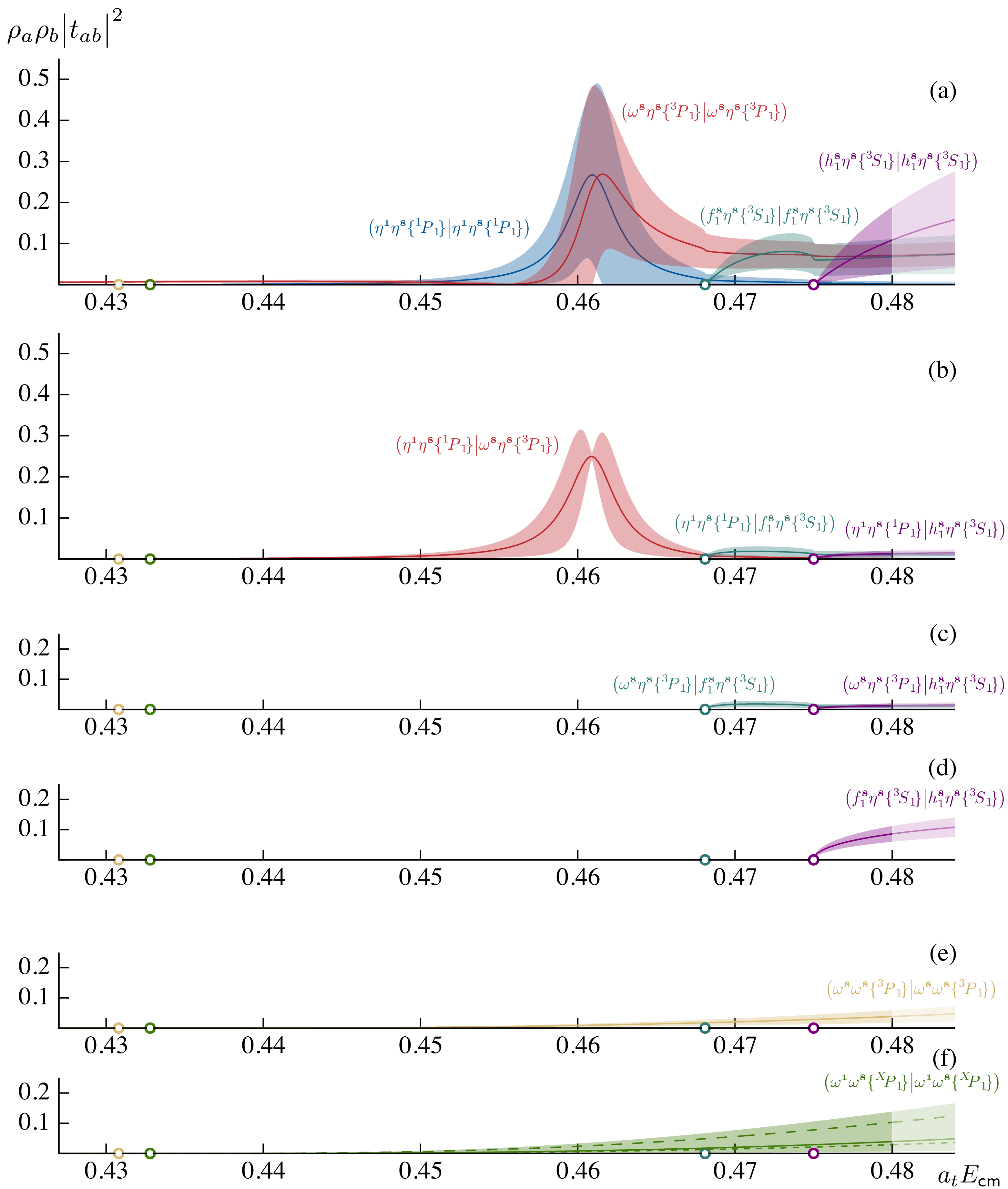
vector-vector appears decoupled

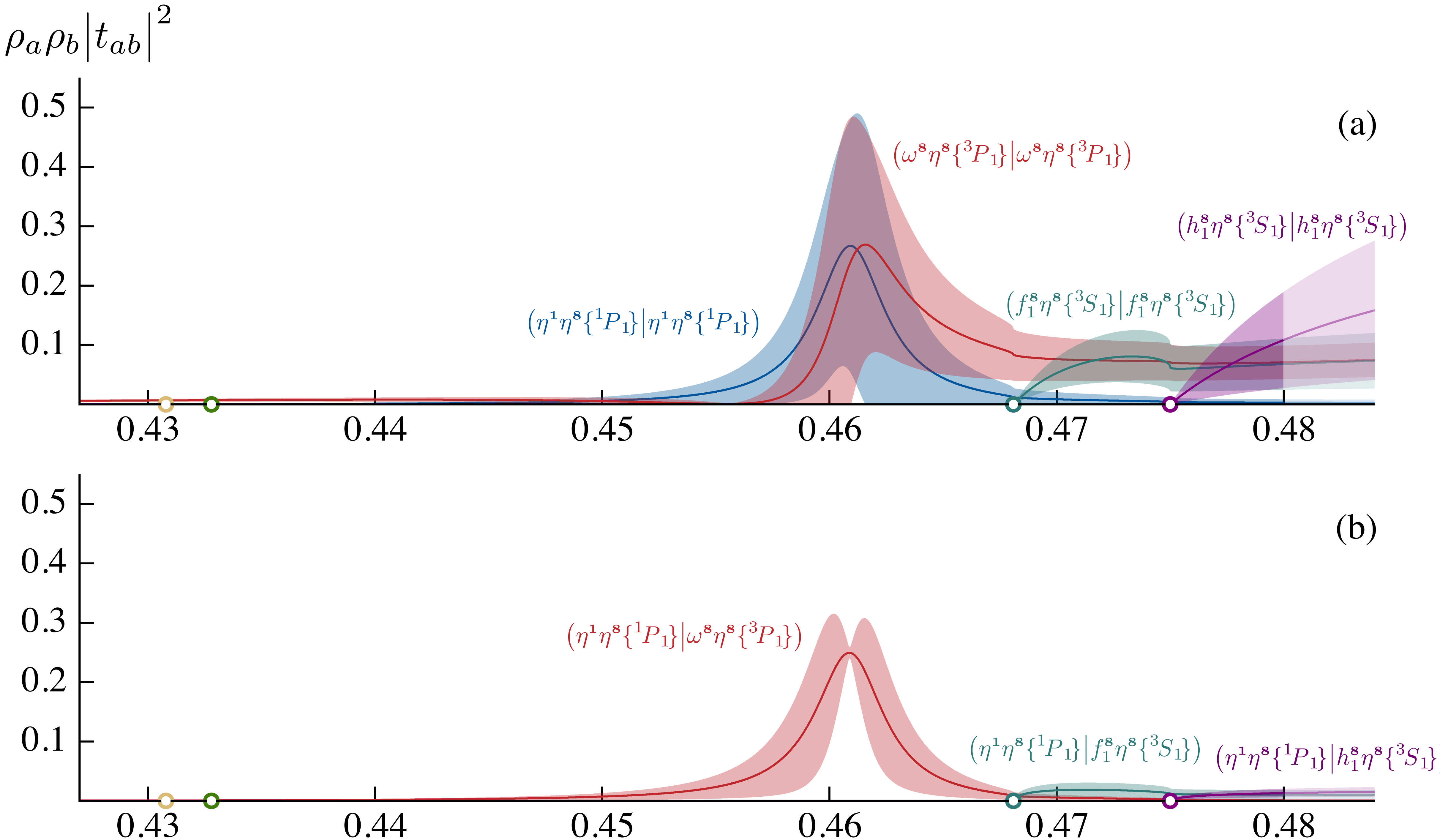
$$\boldsymbol{K}_{\not{VV}}(s) = \frac{\boldsymbol{g}\,\boldsymbol{g}^T}{m^2 - s} + \begin{bmatrix} \gamma_{\eta^1\eta^8\{^1P_1\}} & 0 & 0 & 0 \\ 0 & \gamma_{\omega^8\eta^8\{^3P_1\}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

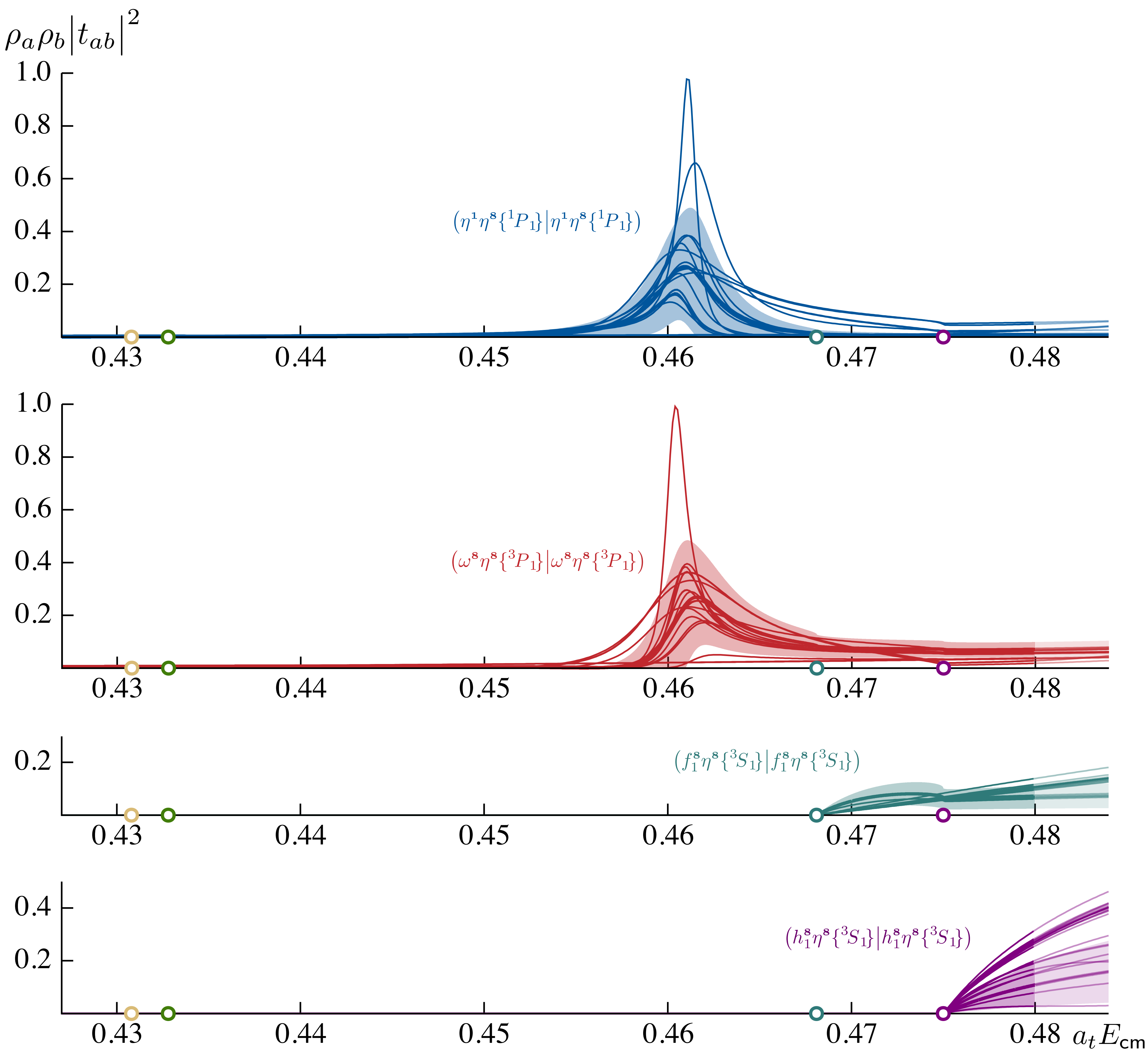
these channels have larger mixing

$$\boldsymbol{g} = \left(g_{\eta^1\eta^8\{^1P_1\}}, g_{\omega^8\eta^8\{^3P_1\}}, g_{f_1^8\eta^8\{^3S_1\}}, g_{h_1^8\eta^8\{^3S_1\}} \right)$$

+ simple constants in 3^{-+}


$\rho_a \rho_b |t_{ab}|^2$
(a)
$(\omega^8\eta^8\{^3P_1\}|\omega^8\eta^8\{^3P_1\})$
$(h_1^8\eta^8\{^3S_1\}|h_1^8\eta^8\{^3S_1\})$
$(f_1^8\eta^8\{^3S_1\}|f_1^8\eta^8\{^3S_1\})$
$(\eta^1\eta^8\{^1P_1\}|\eta^1\eta^8\{^1P_1\})$
(b)
$(\eta^1\eta^8\{^1P_1\}|\omega^8\eta^8\{^3P_1\})$
$(\eta^1\eta^8\{^1P_1\}|f_1^8\eta^8\{^3S_1\})$
$(\eta^1\eta^8\{^1P_1\}|h_1^8\eta^8\{^3S_1\})$
(c)
$(\omega^8\eta^8\{^3P_1\}|f_1^8\eta^8\{^3S_1\})$
$(\omega^8\eta^8\{^3P_1\}|h_1^8\eta^8\{^3S_1\})$
(d)
$(f_1^8\eta^8\{^3S_1\}|h_1^8\eta^8\{^3S_1\})$
(e)
$(\omega^8\omega^8\{^3P_1\}|\omega^8\omega^8\{^3P_1\})$
(f)
$(\omega^1\omega^8\{^XP_1\}|\omega^1\omega^8\{^XP_1\})$
0.5 0.4 0.3 0.2 0.1
0.43 0.44 0.45 0.46 0.47 0.48
$a_t E_{\mathsf{cm}}$


$\rho_a \rho_b |t_{ab}|^2$
0.5
0.4
0.3
0.2
0.1
(a)
$(\omega^8\eta^8\{^3P_1\}|\omega^8\eta^8\{^3P_1\})$
$(h_1^8\eta^8\{^3S_1\}|h_1^8\eta^8\{^3S_1\})$
$(f_1^8\eta^8\{^3S_1\}|f_1^8\eta^8\{^3S_1\})$
$(\eta^1\eta^8\{^1P_1\}|\eta^1\eta^8\{^1P_1\})$
0.43
0.44
0.45
0.46
0.47
0.48
(b)
$(\eta^1\eta^8\{^1P_1\}|\omega^8\eta^8\{^3P_1\})$
$(\eta^1\eta^8\{^1P_1\}|f_1^8\eta^8\{^3S_1\})$
$(\eta^1\eta^8\{^1P_1\}|h_1^8\eta^8\{^3S_1\})$

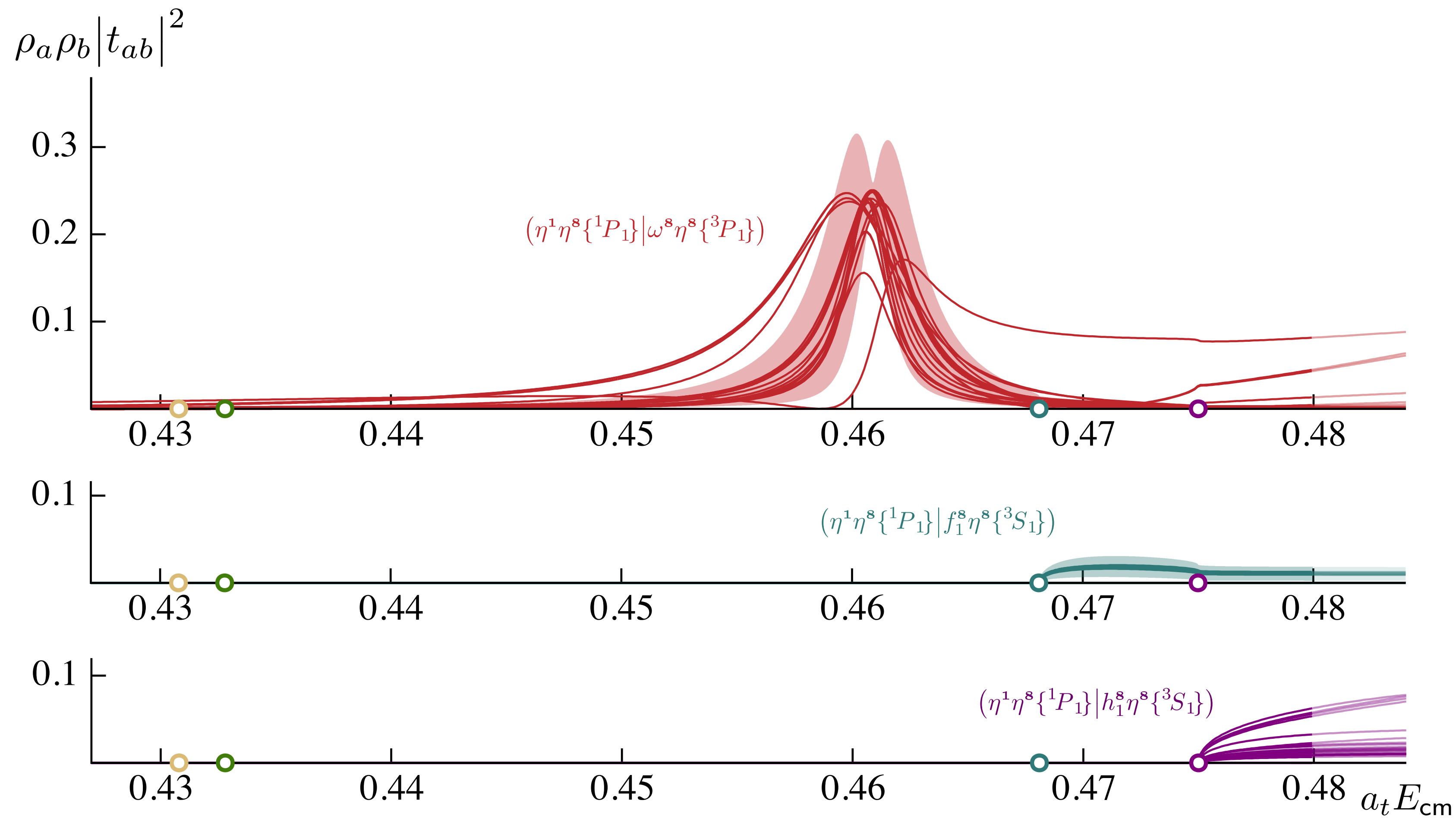

$\rho_a \rho_b |t_{ab}|^2$
1.0
0.8
0.6
0.4
0.2
$(\eta^1\eta^8\{^1P_1\}|\eta^1\eta^8\{^1P_1\})$
0.43
0.44
0.45
0.46
0.47
0.48
$(\omega^8\eta^8\{^3P_1\}|\omega^8\eta^8\{^3P_1\})$
$(f_1^8\eta^8\{^3S_1\}|f_1^8\eta^8\{^3S_1\})$
$(h_1^8\eta^8\{^3S_1\}|h_1^8\eta^8\{^3S_1\})$
$a_t E_{\mathsf{cm}}$

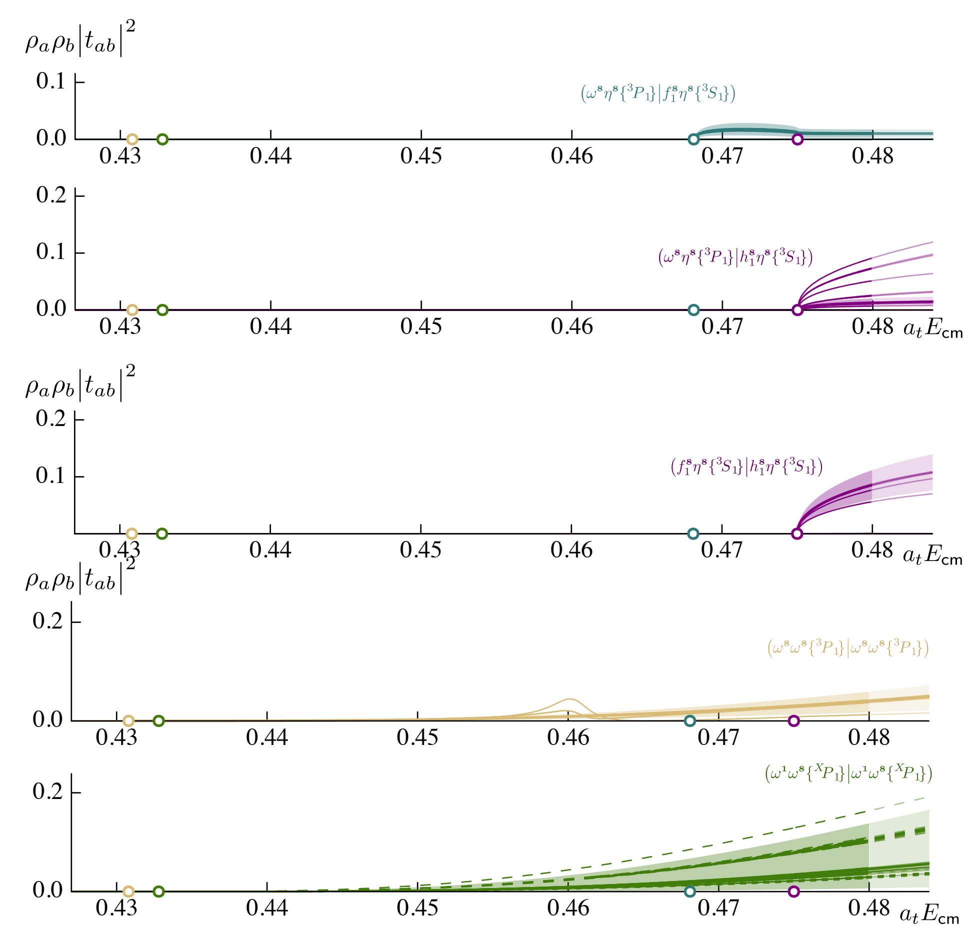
$$\eta^{\mathbf{1}}\eta^{\mathbf{8}}\{^1P_1\} \to \omega^{\mathbf{8}}\eta^{\mathbf{8}}\{^3P_1\}$$

perhaps more familiar :

$$\eta'\pi \to \rho\pi$$

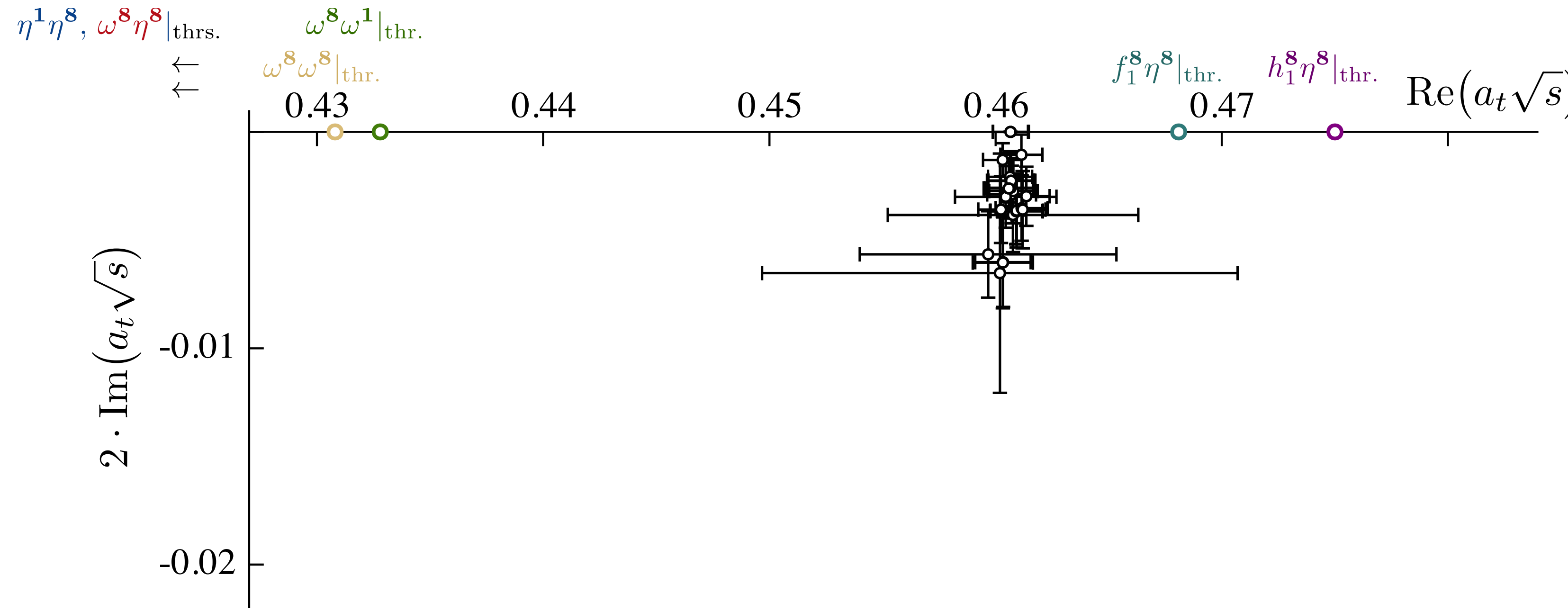
$$\to K^\star\bar{K}$$





in the region of a pole

$$t \sim \frac{c^2}{s_0 - s}$$



narrow resonance

$$a_t\sqrt{s} = 0.4606(26) \pm \tfrac{i}{2}0.0039(39)$$
$$\sqrt{s} = \big(2144(12) \pm \tfrac{i}{2}18(18)\big)\ \text{MeV}$$

small imaginary part,
consistent with zero in
some parameterisations

many sheets (2^n)

pole is located on "proximal" sheet
open channels: $\text{Im}k_i < 0$
closed channels: $\text{Im}k_i > 0$

in the region of a pole

$$t \sim \frac{c^2}{s_0 - s}$$

narrow resonance

$$a_t\sqrt{s} = 0.4606(26) \pm \tfrac{i}{2}0.0039(39)$$
$$\sqrt{s} = \big(2144(12) \pm \tfrac{i}{2}18(18)\big)\ \text{MeV}$$

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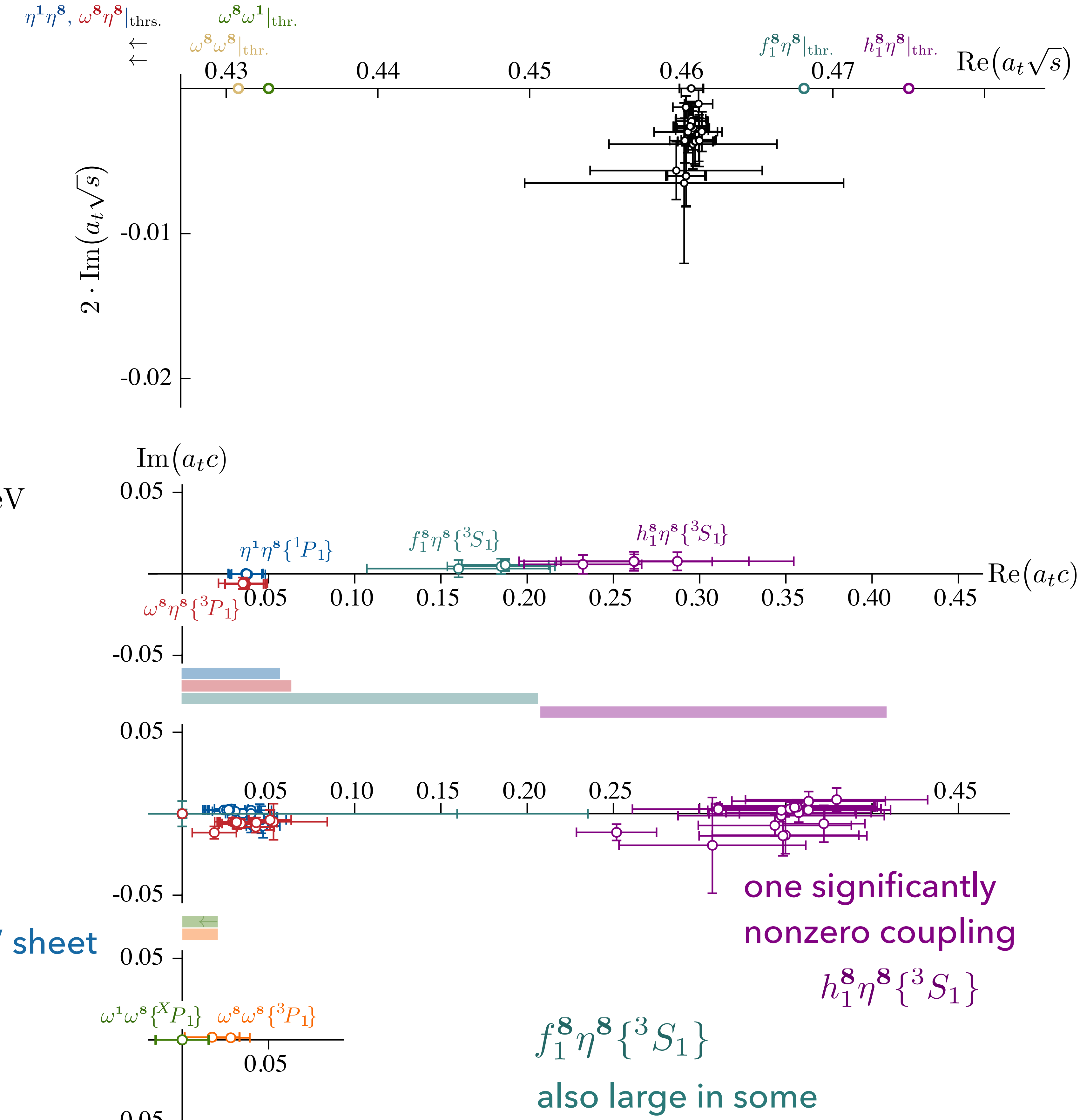
closed channels: $\text{Im}k_i > 0$



one significantly nonzero coupling $h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$ also large in some parameterisations

Flavour decomposition
- break apart SU(3) multiplets
- use CGs e.g. from de Swart (Rev. Mod. Phys. 35, 916 (1963))
- mixing angles needed for singlets taken from PDG

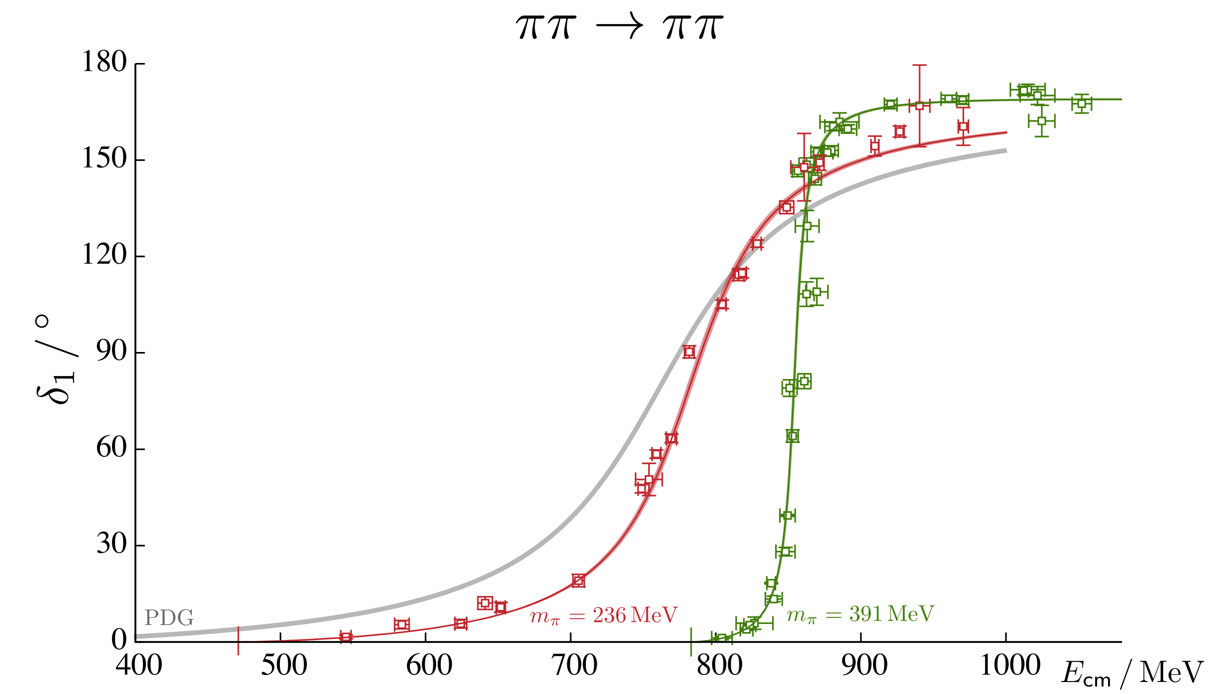
very heavy quarks
- crudely extrapolate to physical pions

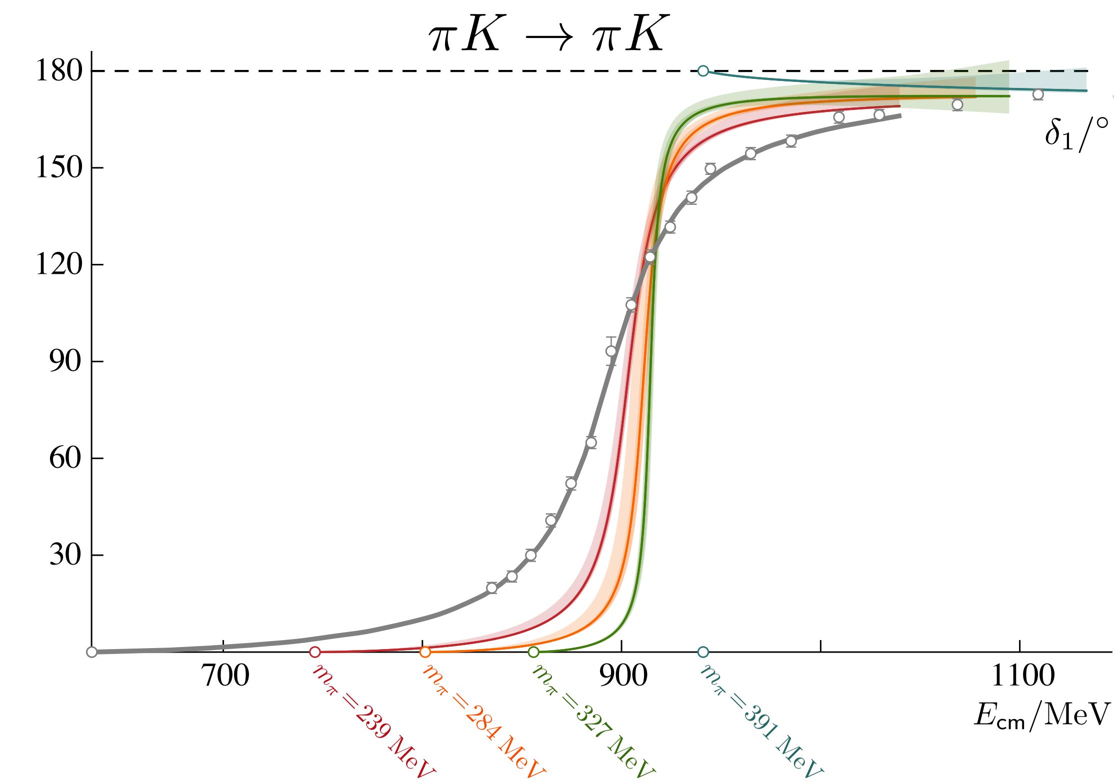
scale couplings:

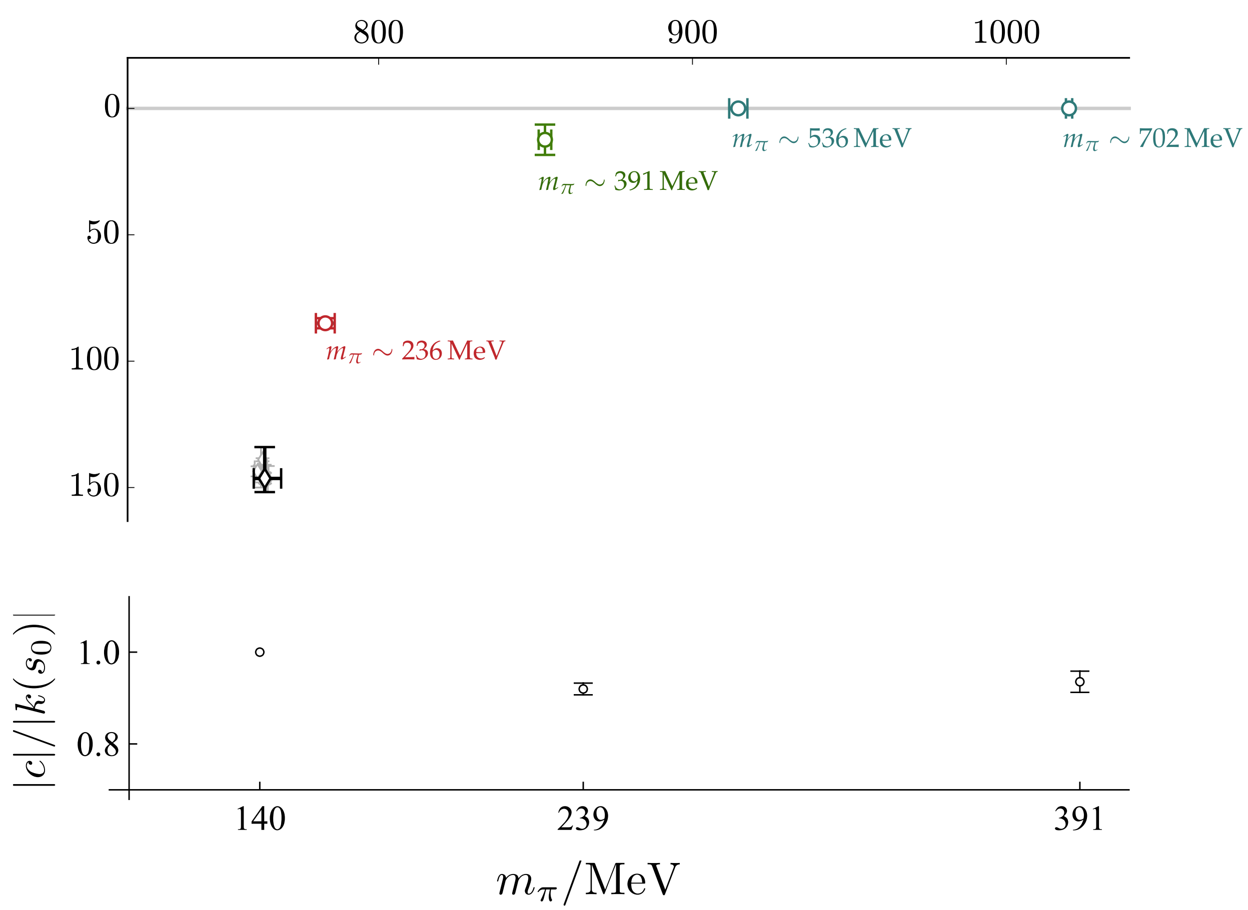
$$|c|^{\rm phys} = \left| \frac{k^{\rm phys}(m_R^{\rm phys})}{k(m_R)} \right|^{\ell} |c|$$

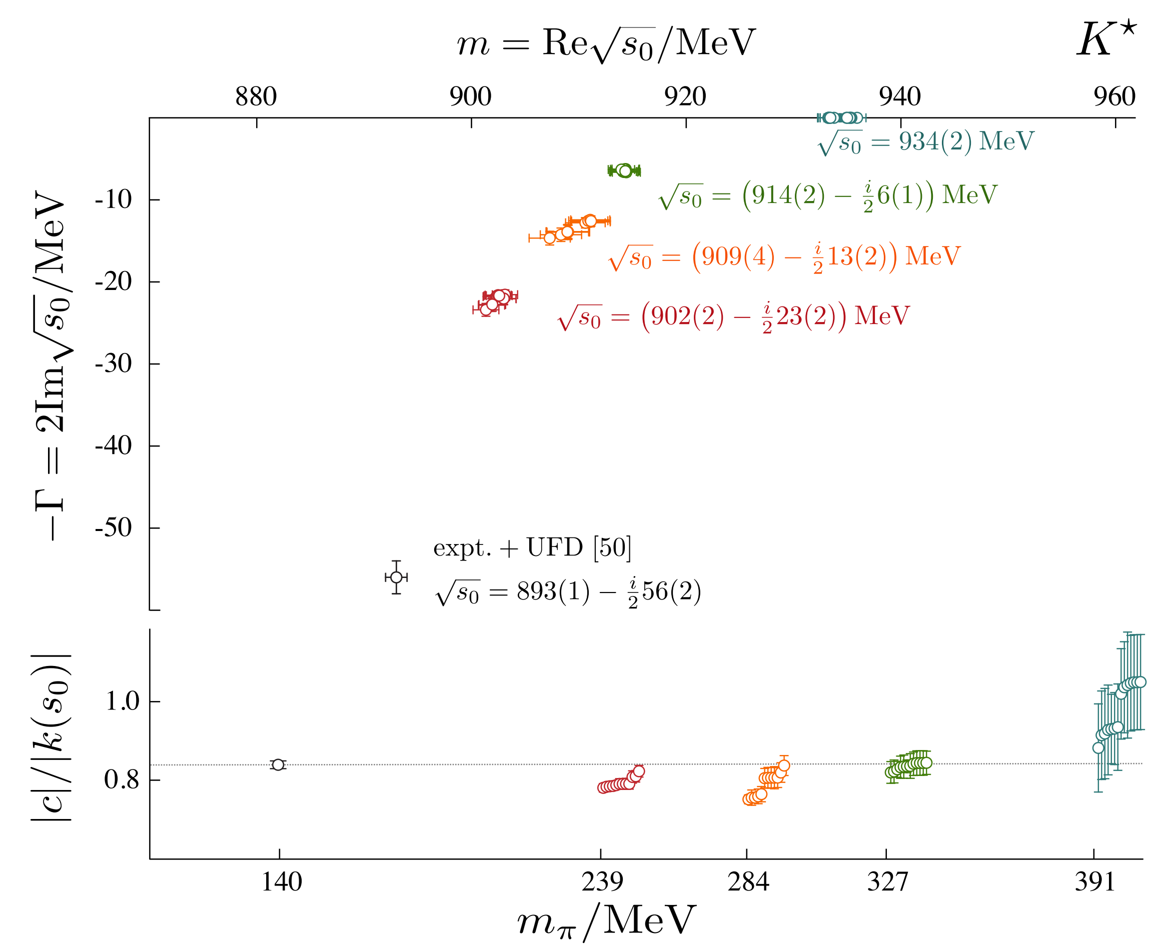
choose m_R=1563 MeV

pole couplings - vectors



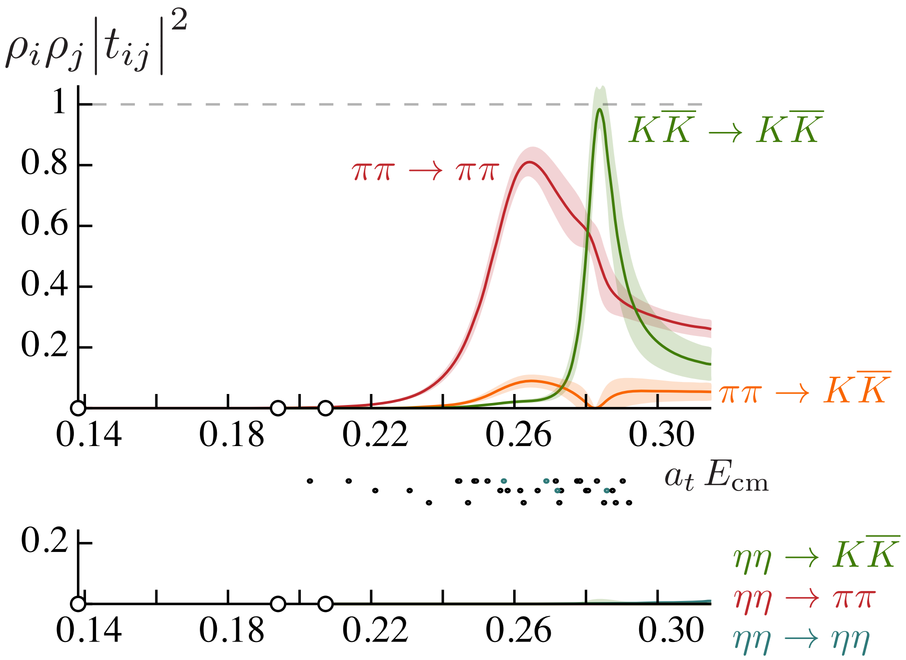


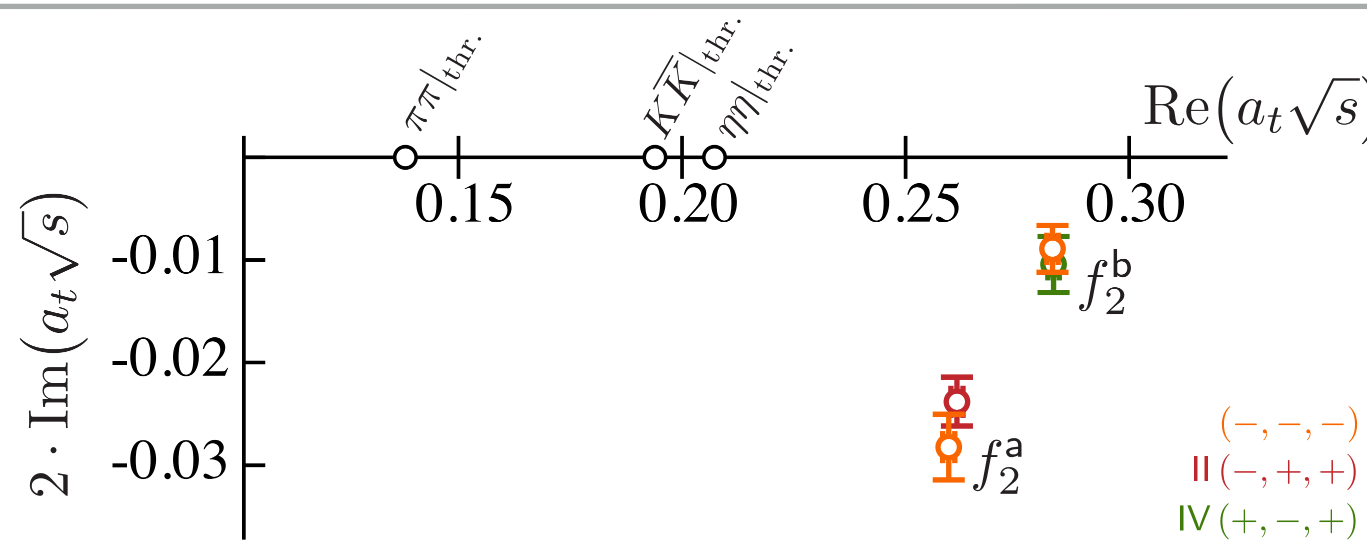


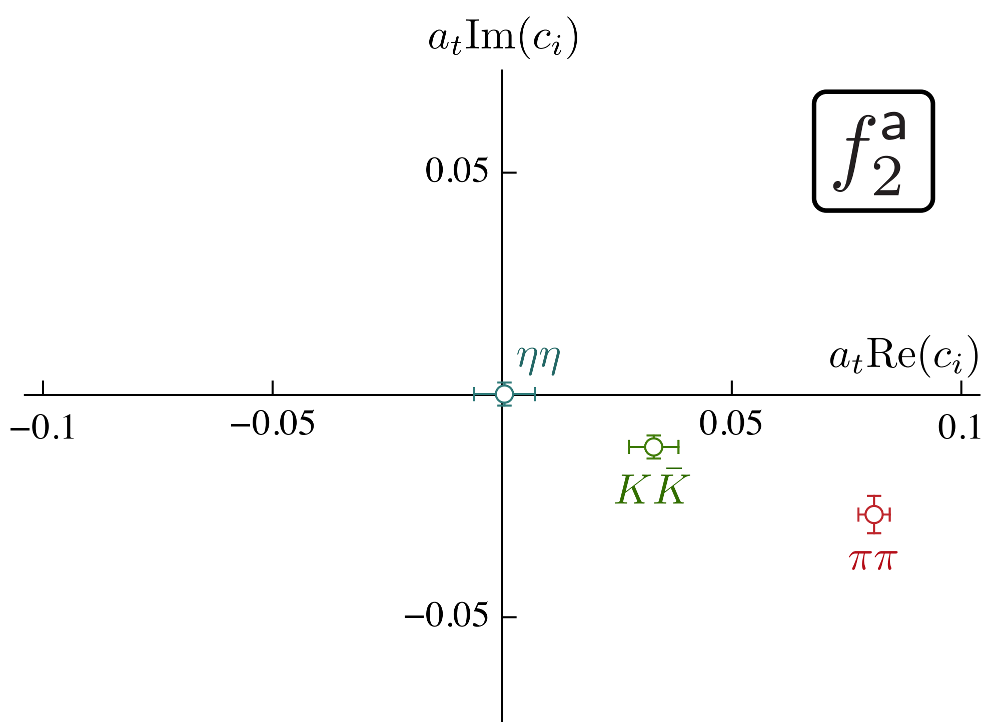


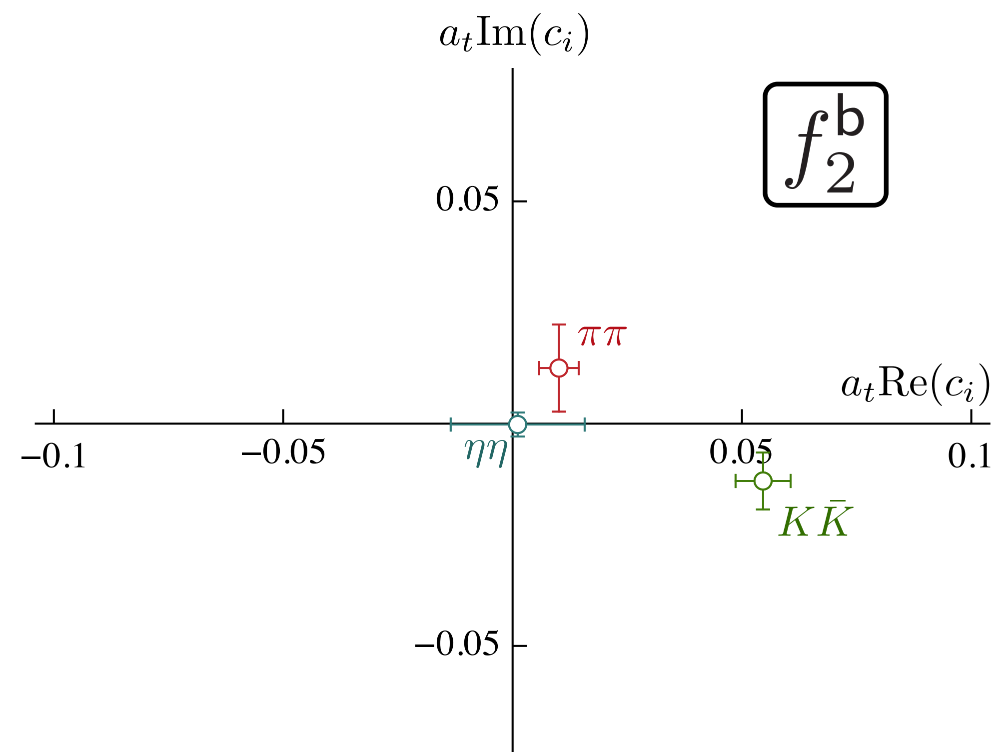
see also Arkaitz Rodas slides @ Lattice 2022

weak dependence on m_π even when K* appears as a bound state







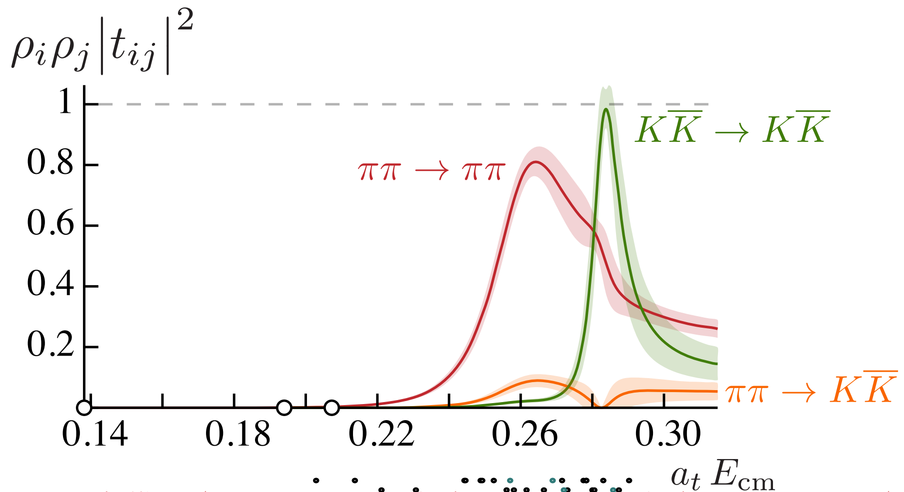


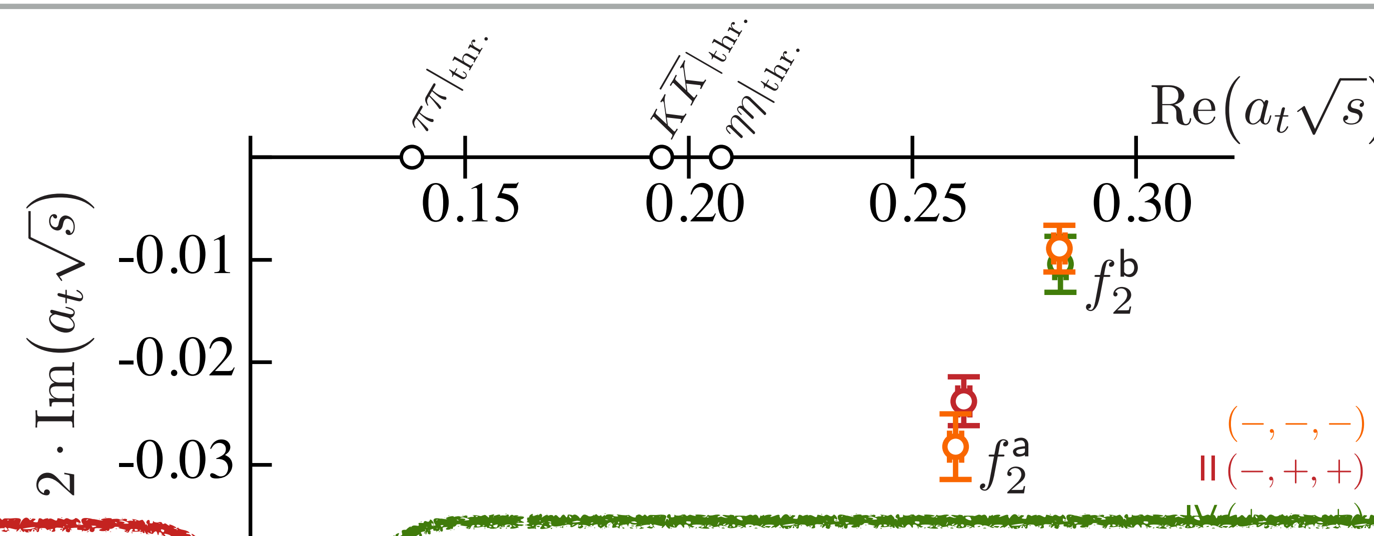
$f_2^{\mathsf{a}}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

$f_2^{\mathsf{b}}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)$ MeV

$\mathrm{Br}(f_2^{\mathsf{b}} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\mathsf{b}} \to K\overline{K}) \sim 92\%$





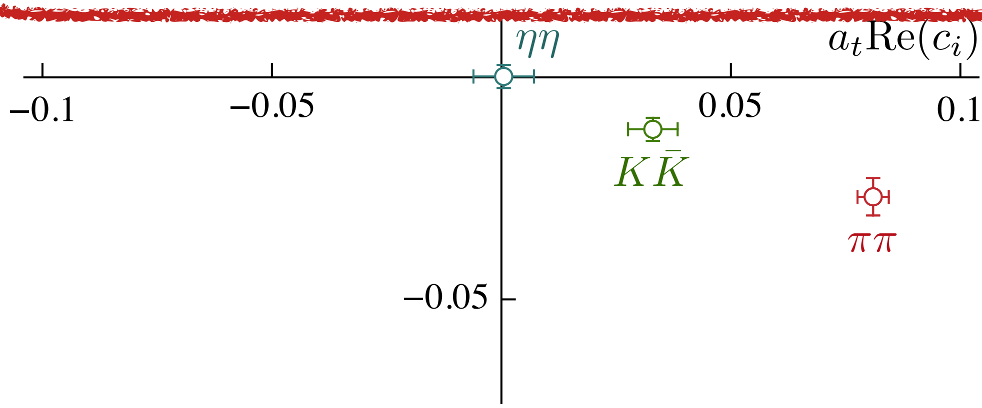
$f_2(1270)$ DECAY MODES

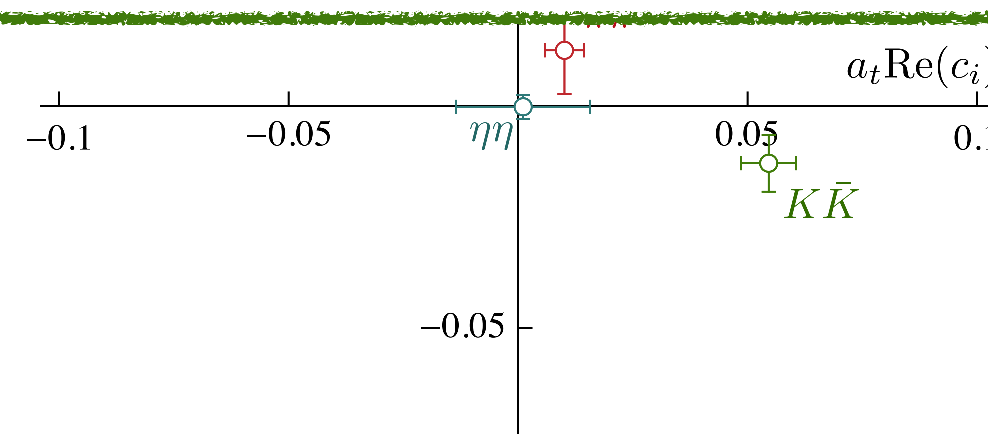
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ {}^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(\ 7.7\ {}^{+1.1}_{-3.2})$ %
Γ_3	$K\overline{K}$	$(\ 4.6\ {}^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	$(\ 2.8\ \pm 0.4)$ %

PDG2017

$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}$	$(88.7\ \pm 2.2)$ %
Γ_2	$\eta\eta$	$(10.4\ \pm 2.2)$ %
Γ_3	$\pi\pi$	$(\ 8.2\ \pm 1.5) \times 10^{-3}$
Γ_4	$K\overline{K}^*(892)+$ c.c.	





$f_2^{\rm a}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

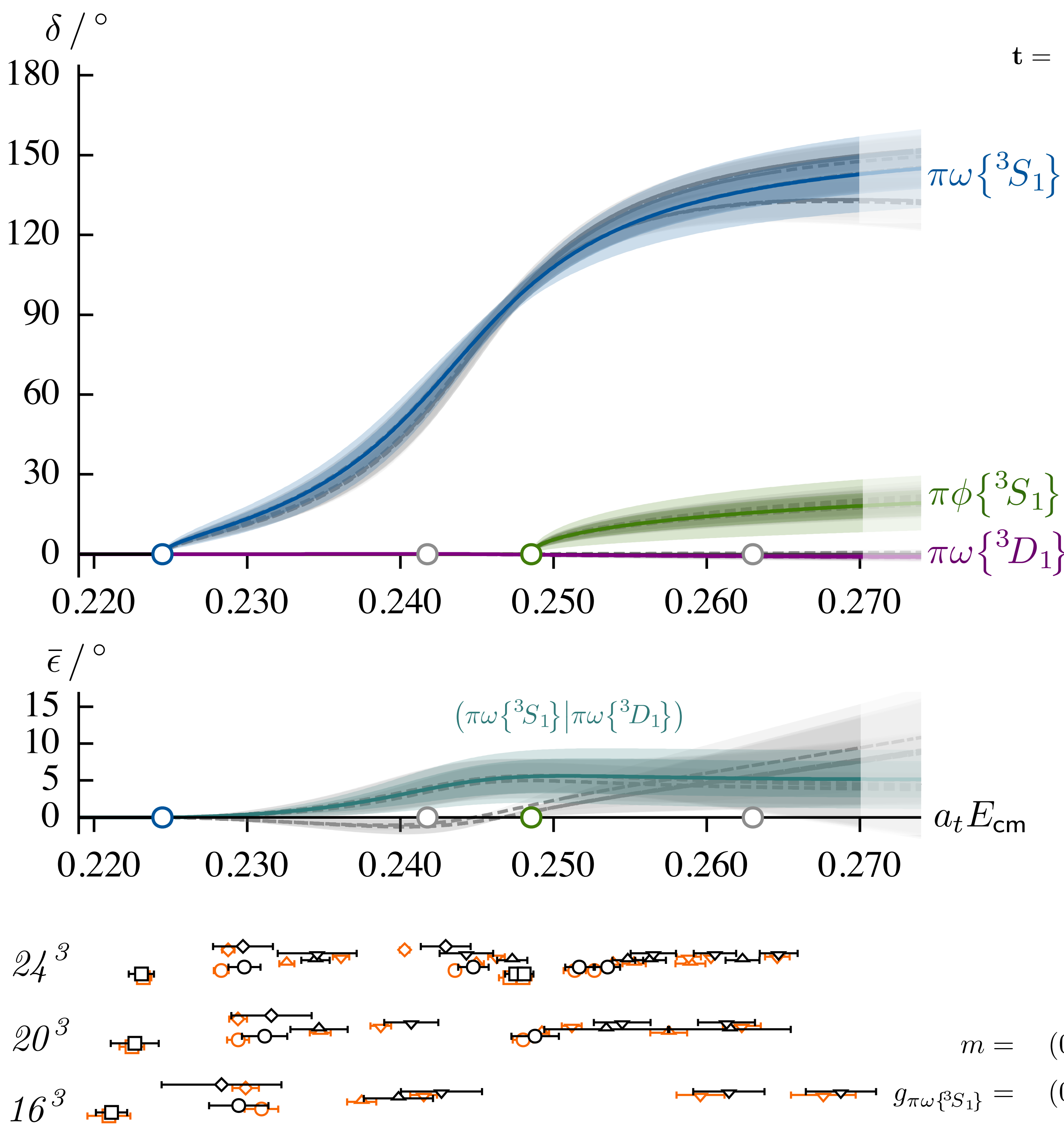
$\mathrm{Br}(f_2^{\rm a} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\rm a} \to K\overline{K}) \sim 12\%$

$f_2^{\rm b}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)$ MeV

$\mathrm{Br}(f_2^{\rm b} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\rm b} \to K\overline{K}) \sim 92\%$

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$

Woss et al, 1904.04136



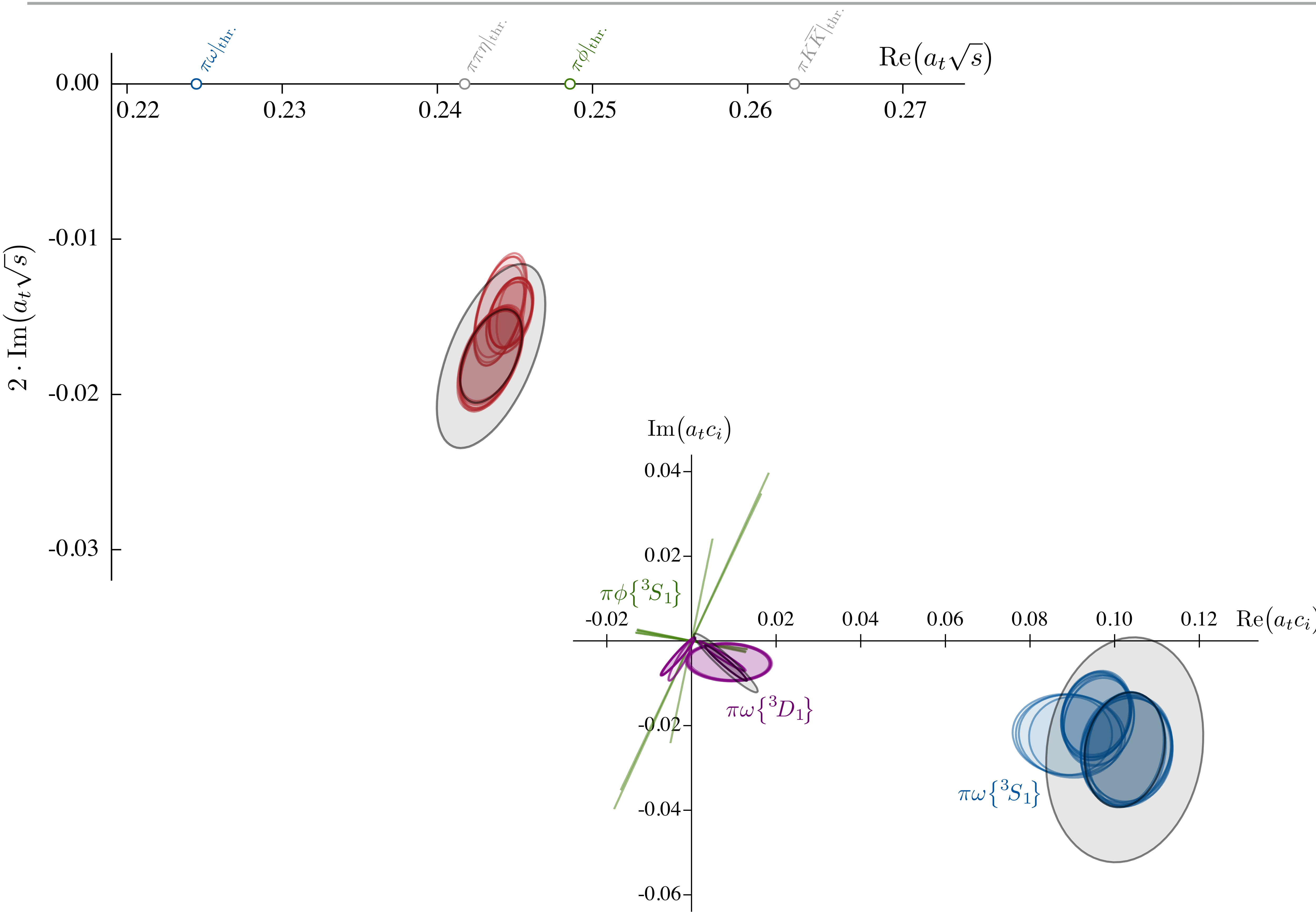
e.g.: three-channel K-matrix with a pole and constants:

$$[t^{-1}]_{\ell Ja,\,\ell' Jb} = (2k^{(a)})^{-\ell}\,[K^{-1}]_{\ell Ja,\,\ell' Jb}\,(2k^{(a)})^{-\ell} + \delta_{\ell'\ell} I_{ab}$$

$$K_{\ell Ja,\,\ell' Jb} = \frac{g_{\ell Ja}\, g_{\ell' Jb}}{m^2 - s} + \gamma_{\ell Ja,\,\ell' Jb}$$

$$\begin{aligned} m &= (0.2465 \pm 0.0007 \pm 0.0001)\cdot a_t^{-1} \\ g_{\pi\omega\{^3S_1\}} &= (0.106 \pm 0.007 \pm 0.007)\cdot a_t^{-1} \\ g_{\pi\omega\{^3D_1\}} &= (1.08 \pm 0.47 \pm 0.28)\cdot a_t \\ \gamma^{(0)}_{\pi\omega\{^3S_1\},\pi\omega\{^3S_1\}} &= -0.35 \pm 0.19 \pm 0.18 \\ \gamma^{(0)}_{\pi\phi\{^3S_1\},\pi\phi\{^3S_1\}} &= 0.90 \pm 0.24 \pm 0.27 \end{aligned} \qquad \begin{bmatrix} 1 & -0.05 & 0.05 & -0.01 & -0.23 \\ & 1 & 0.70 & -0.54 & -0.06 \\ & & 1 & -0.39 & -0.06 \\ & & & 1 & 0.22 \\ & & & & 1 \end{bmatrix}$$

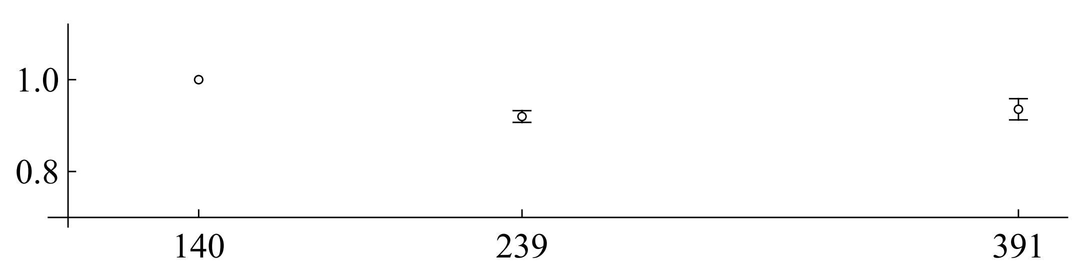
$$\chi^2/N_{\mathrm{dof}} = \frac{36.8}{36-5} = 1.19.$$


πω|thr.
ππη|thr.
πϕ|thr.
πKK̄|thr.
Re(a_t√s)
0.00
0.22
0.23
0.24
0.25
0.26
0.27
-0.01
-0.02
-0.03
2 · Im(a_t√s)
Im(a_t c_i)
0.04
0.02
-0.02
-0.04
-0.06
πϕ{³S₁}
-0.02
0.02
0.04
0.06
0.08
0.10
0.12
Re(a_t c_i)
πω{³D₁}
πω{³S₁}

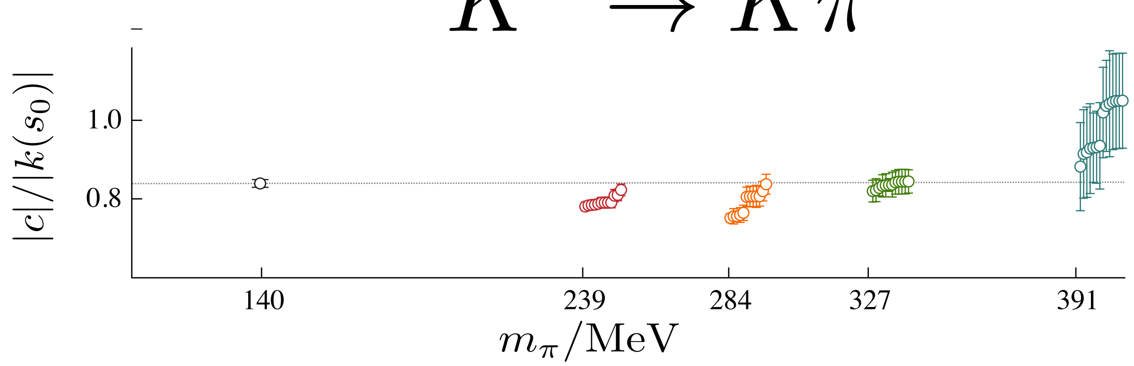
	scaled	PDG
$\lvert c(f_2 \to \pi\pi)\rvert$	$488(28)$	453^{+9}_{-4}
$\lvert c(f_2 \to K\overline{K})\rvert$	$139(27)$	$132(7)$
$\lvert c(f_2' \to \pi\pi)\rvert$	$103(32)$	$33(4)$
$\lvert c(f_2' \to K\overline{K})\rvert$	$321(50)$	$389(12)$
$\lvert c(b_1 \to \pi\omega)\rvert$	$564(114)$	$556(17)$

$$|c|^{\text{phys}} = \left|\frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)}\right|^{\ell} |c|$$

$\rho \to \pi\pi$



$K^* \to K\pi$



evidence for weakly varying couplings as a function of m_π in several cases

- seems reasonable to scale couplings to estimate properties of the π_1

consider I=1, I_z=+1 component π_1^+

begin with experimentally observed decay modes: $\eta\pi$, $\eta'\pi$

just one component:

$$\eta^{\mathbf{1}}\eta^{\mathbf{8}} \quad \mathbf{1} \times \mathbf{8} \to \mathbf{8}$$

$$\pi_1 \to \pi\eta_1$$

rotate η_1 to physical states:

$$\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \qquad \theta_P \sim -10^\circ$$

couplings are then:

$$\left|c(\pi_1 \to \eta\pi)\right| = \left|c_{\eta^{\mathbf{1}}\eta^{\mathbf{8}}} \sin\theta_P\right|$$

$$\left|c(\pi_1 \to \eta'\pi)\right| = \left|c_{\eta^{\mathbf{1}}\eta^{\mathbf{8}}} \cos\theta_P\right|$$

decay of $\eta'\pi$ > $\eta\pi$

coupling at m_π = 688 MeV
scale to lighter masses

Flavour decomposition

- break apart SU(3) multiplets
- use CGs from de Swart (Rev. Mod. Phys. 35, 916 (1963))
- mixing angles needed for singlets taken from PDG

$$\mathbf{8} \otimes \mathbf{8} \to \mathbf{1} \oplus \mathbf{8_1} \oplus \mathbf{8_2} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

$$\pi_1^{\mathbf{8}} \to \omega^{\mathbf{8}} \eta^{\mathbf{8}}$$

$$\text{eg}: \pi_1^+ \to \tfrac{1}{\sqrt{3}} \left(\pi^+ \rho^0 - \pi^0 \rho^+\right) + \tfrac{1}{\sqrt{6}} \left(K^+ \bar{K}^{*0} - \bar{K}^0 K^{*+}\right)$$

$$\left|c(\pi_1 \to \rho\pi)\right| = \sqrt{\tfrac{2}{3}} \left|c_{\omega^{\mathbf{8}}\eta^{\mathbf{8}}}\right|$$

$$\left|c(\pi_1 \to K^* \overline{K})\right| = \sqrt{\tfrac{1}{3}} \left|c_{\omega^{\mathbf{8}}\eta^{\mathbf{8}}}\right|$$

largest decay modes:

$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$$\mathbf{8}\otimes\mathbf{8}\to\mathbf{1}\oplus\mathbf{8_1}\oplus\mathbf{8_2}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

$h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$$-\sqrt{\tfrac{3}{10}}\left(K_{1A}^+\,\overline{K}^0+\overline{K}_{1A}^0\,K^+\right)+\tfrac{1}{\sqrt{5}}\left(a_1^+\eta_8+(f_1)_8\pi^+\right)$$

$$\begin{aligned}\left|c(\pi_1\to a_1\eta)\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\cos\theta_P\right|\\ \left|c(\pi_1\to a_1\eta')\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\sin\theta_P\right|\\ \left|c(\pi_1\to f_1(1285)\pi)\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\cos\theta_A\right|\\ \left|c(\pi_1\to f_1(1420)\pi)\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\sin\theta_A\right|.\end{aligned}$$

$$\tfrac{1}{\sqrt{6}}\left(K_{1B}^+\,\overline{K}^0-\overline{K}_{1B}^0\,K^+\right)+\tfrac{1}{\sqrt{3}}\left(b_1^+\pi^0-b_1^0\,\pi^+\right)$$

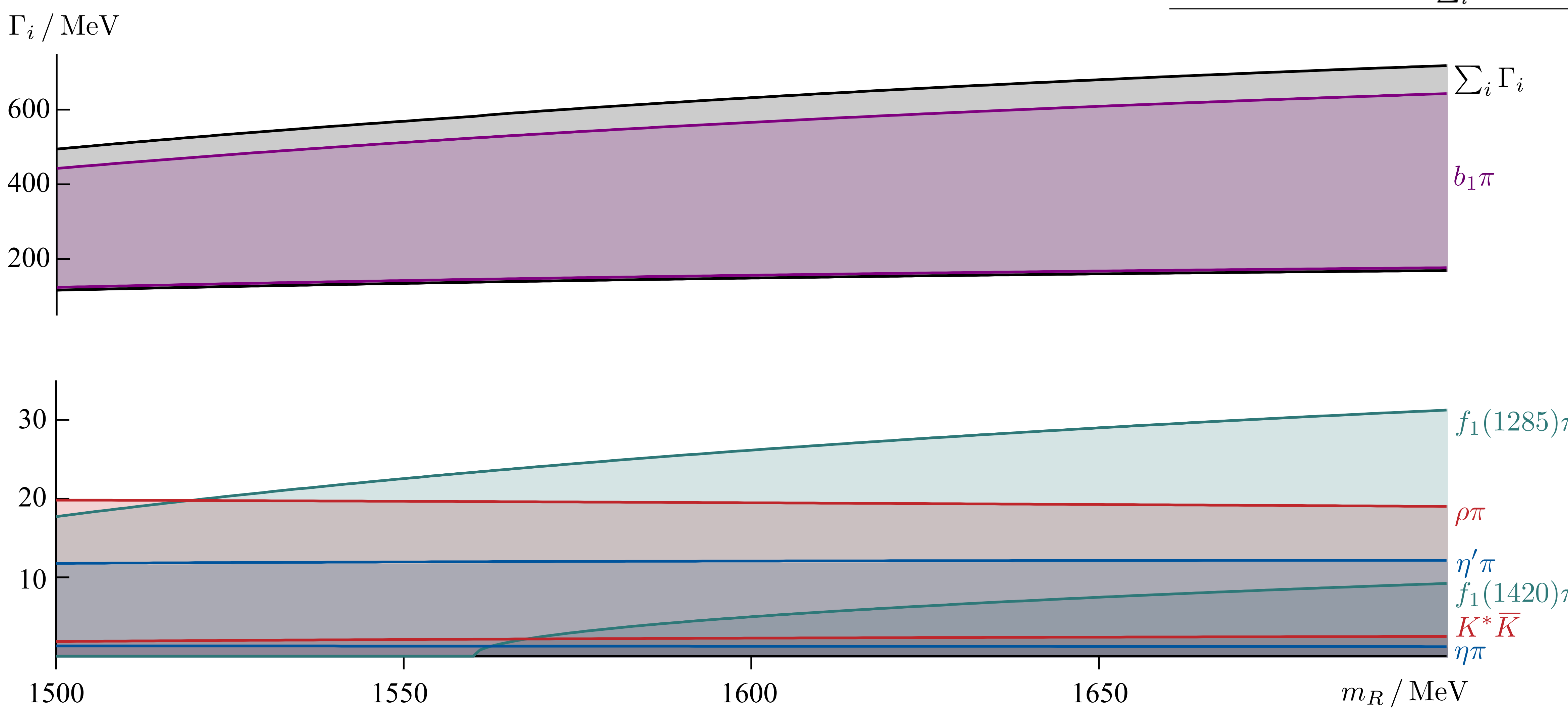
$$\left|c(\pi_1\to b_1\pi)\right| = \sqrt{\tfrac{2}{3}}\,\left|c_{h_1^{\mathbf{8}}\eta^{\mathbf{8}}}\right|$$

kaon-K_1 channels kinematically closed for $m \gtrsim 1500$ MeV

$$|c|^{\text{phys}} = \left|\frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)}\right|^{\ell} |c|$$

$$\Gamma(R \to i) = \frac{\left|c_i^{\text{phys}}\right|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}}) \quad \text{[PDG]}$$

	thr./MeV	$\lvert c_i^{\text{phys}}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\overline{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
			$\Gamma = \sum_i \Gamma_i = 139 \to 590$



For the first time, we have a QCD computation of a π_1 resonance

- a heavier than physical pion mass was used with $m_u=m_d=m_s$
- multibody decay modes are suppressed, only 2-body becomes relevant
- we find large coupling to a kinematically-closed axial-vector–pseudoscalar channel
- narrow resonance at m_π=688 MeV

Extrapolating to the experimentally-observed mass, we find

- the dominant decay mode appears to be $b_1\pi$
- in experiment this is a 5π final state
- current analyses of $\eta\pi$ and $\eta'\pi$ channels may be quite suppressed w.r.t. $b_1\pi$
- broad resonance

This SU(3) calculation has components that apply to the other elements of the octet

- but other components are expected to also contribute (eg singlet in η_1)
- nevertheless - there's likely to be a family of hybrids

Charmonium, bottomonium is another interesting place to look

- heavier quarks may make extrapolating to the experimental masses more straightforward