Path from Lattice QCD to Neutrinoless Double-Beta Decay Amplitude

Saurabh Kadam

with

Zohreh Davoudi

PRD 102, 114521 (2020)
PRL 126, 152003 (2021)
PRD 105, 094502 (2022)

Outline

- Motivation

  Why is it needed?

- Formalism

  What are the tools?

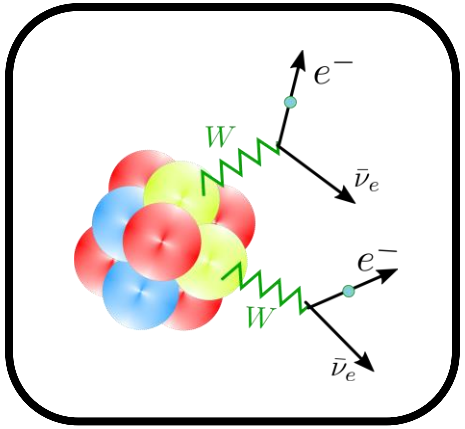
- Results

  How to use them?

Double β Decays

Two neutrino double beta decay ($2\nu\beta\beta$)

Eugene Wigner, Goeppert-Mayer (1935)

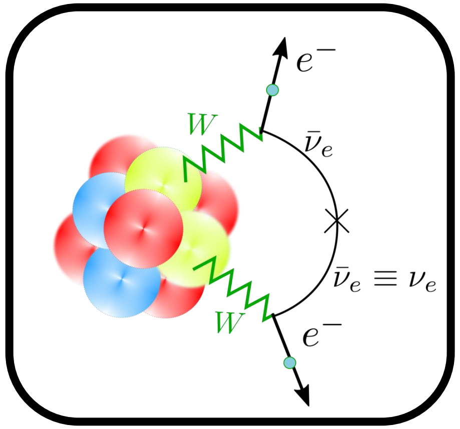


- Standard model (SM) process
- Extremely rare and has been observed

$$T_{1/2}(2\nu\beta\beta) \sim 10^{20}y$$

Neutrinoless double beta decay ($0\nu\beta\beta$)

Racah (1937)
Furry (1939)



- Total lepton number is violated
- Beyond the standard model (BSM) process
- Searches for it are ongoing
- Neutrinos are their own anti-particles

Avignone, Elliott and Engel
Reviews of Modern Physics, 80 (2008)

Double β Decays

Two neutrino double beta decay $(2\nu\beta\beta)$

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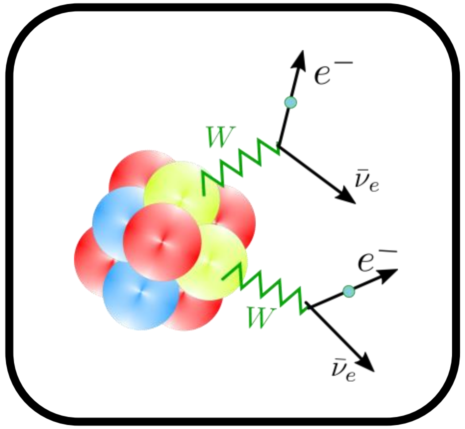


- Standard model (SM) process
- Extremely rare and has been observed

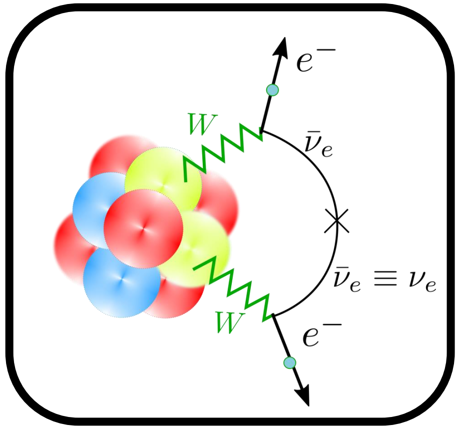
$$T_{1/2}(2\nu\beta\beta) \sim 10^{20}y$$

- Dominant background for $0\nu\beta\beta$ search experiments
- $2\nu\beta\beta$ can also probe potential BSM scenarios

Deppisch, Graf, and Šimkovic
PhysRevLett.125.171801

Neutrinoless double beta decay $(0\nu\beta\beta)$

Racah (1937)
Furry (1939)



- Total lepton number is violated
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Avignone, Elliott and Engel
Reviews of Modern Physics, 80 (2008)

What can possibly be responsible for a $0\nu\beta\beta$ decay?

- Minimal deviation from the SM:

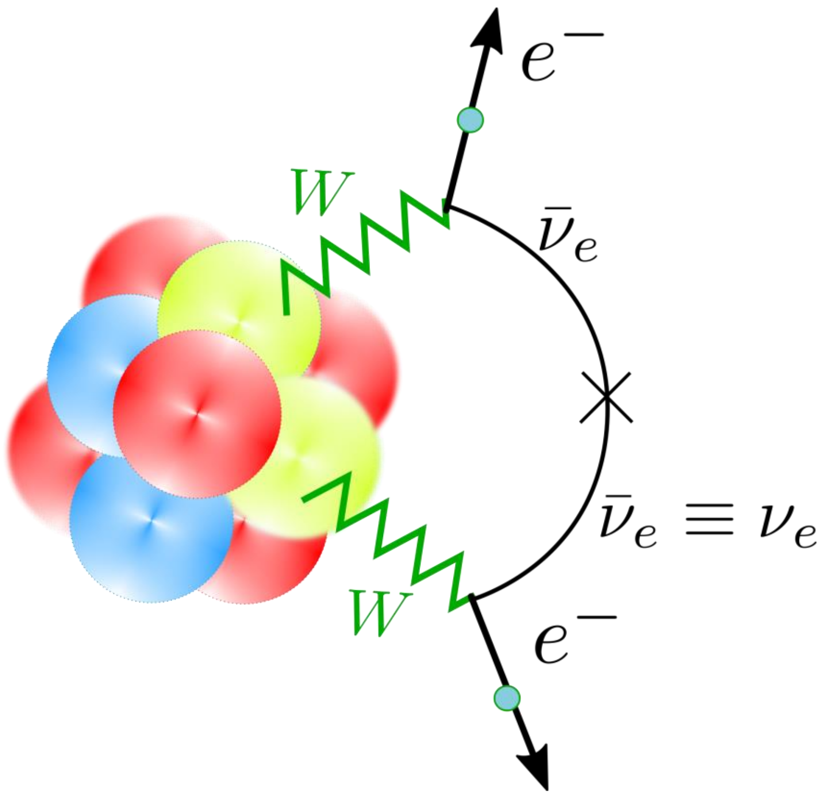
  Light neutrino exchange scenario

- SM neutrinos are promoted to Majorana neutrinos

- Effective Majorana mass

$$m_{\beta\beta} = \sum_k m_k U_{ek}^2$$

Needs accurate constraints



Obtaining Effective Majorana Mass from Experiments

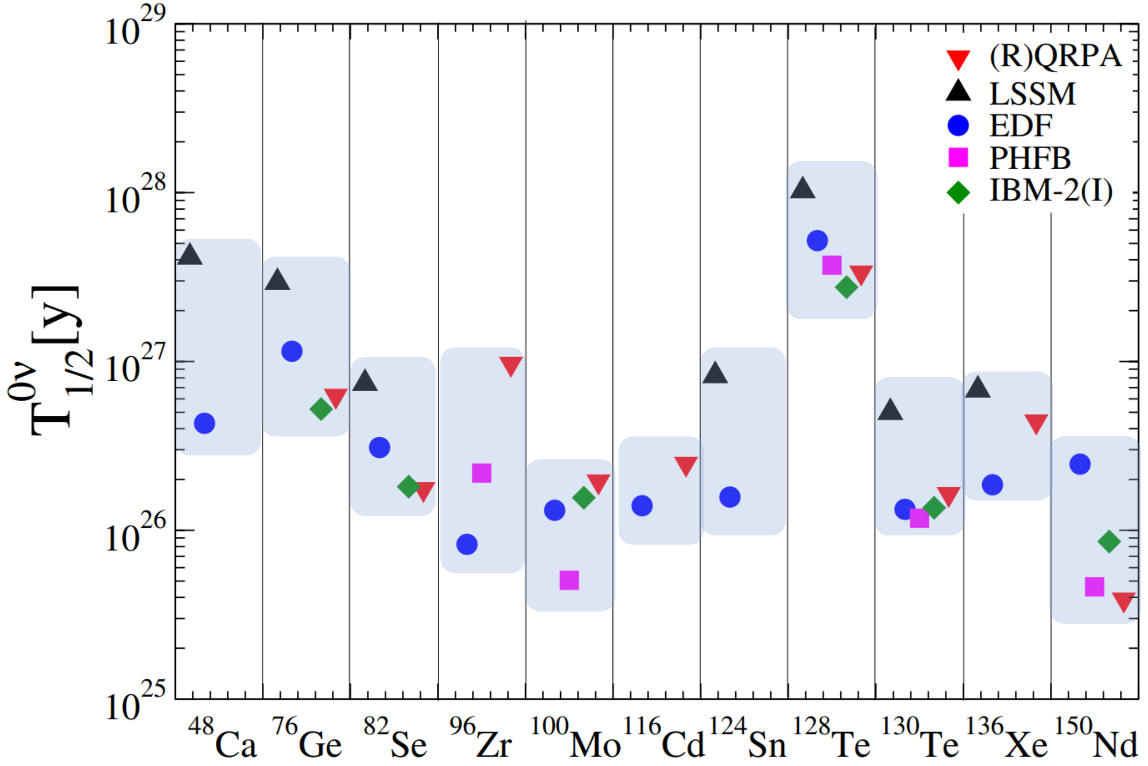
Phase space factor

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\,|M_{0\nu}|^2\,\langle m_{\beta\beta}\rangle^2$$

Nuclear Matrix Element (NME) in Light Neutrino Exchange Scenario

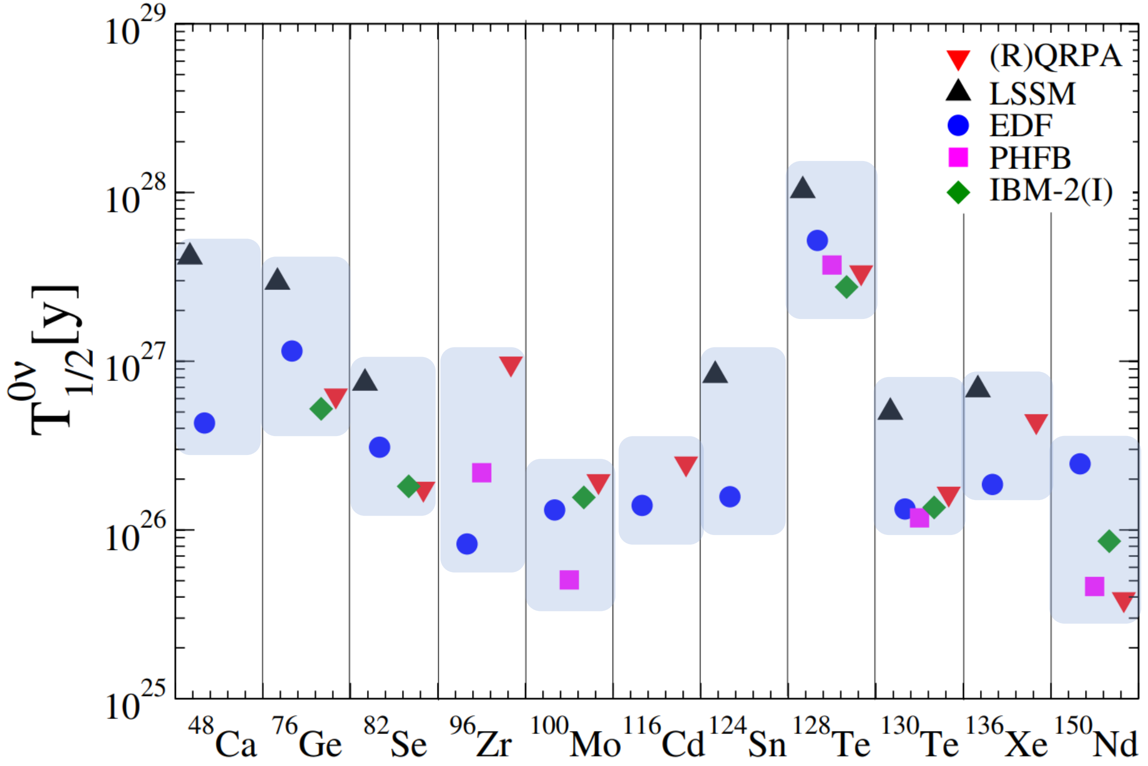
Half-lives from different methods for NME calculations

$$\langle m_{\beta\beta}\rangle = 0.05\ \text{eV}$$

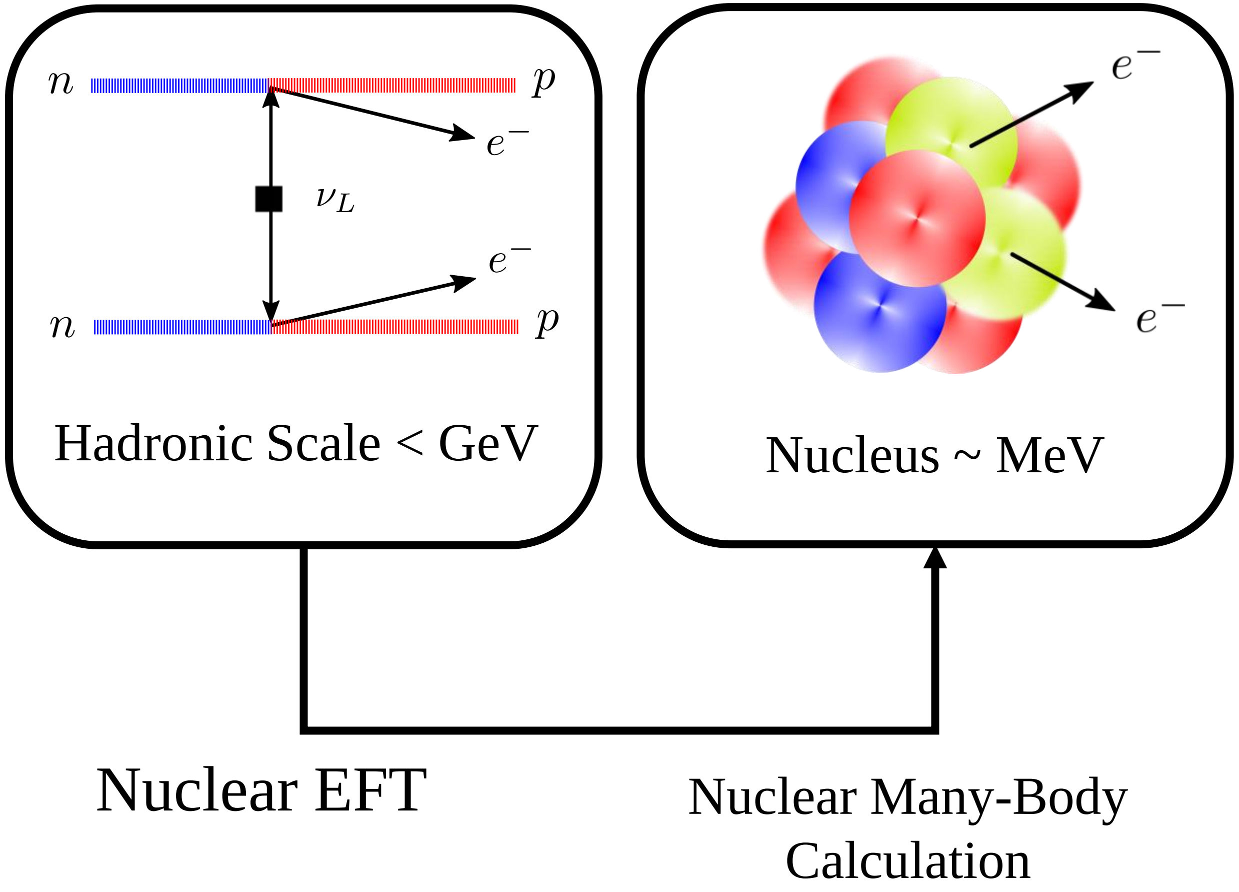


Vergados, Ejiri and Simkovic, Rep. Prog. Phys. 75 106301 (2012)

What is the source of these uncertainties in NME?



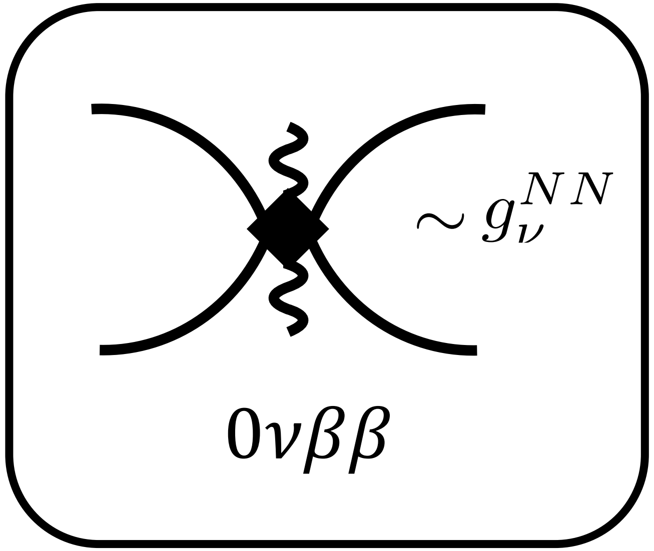
Nuclear Matrix Elements

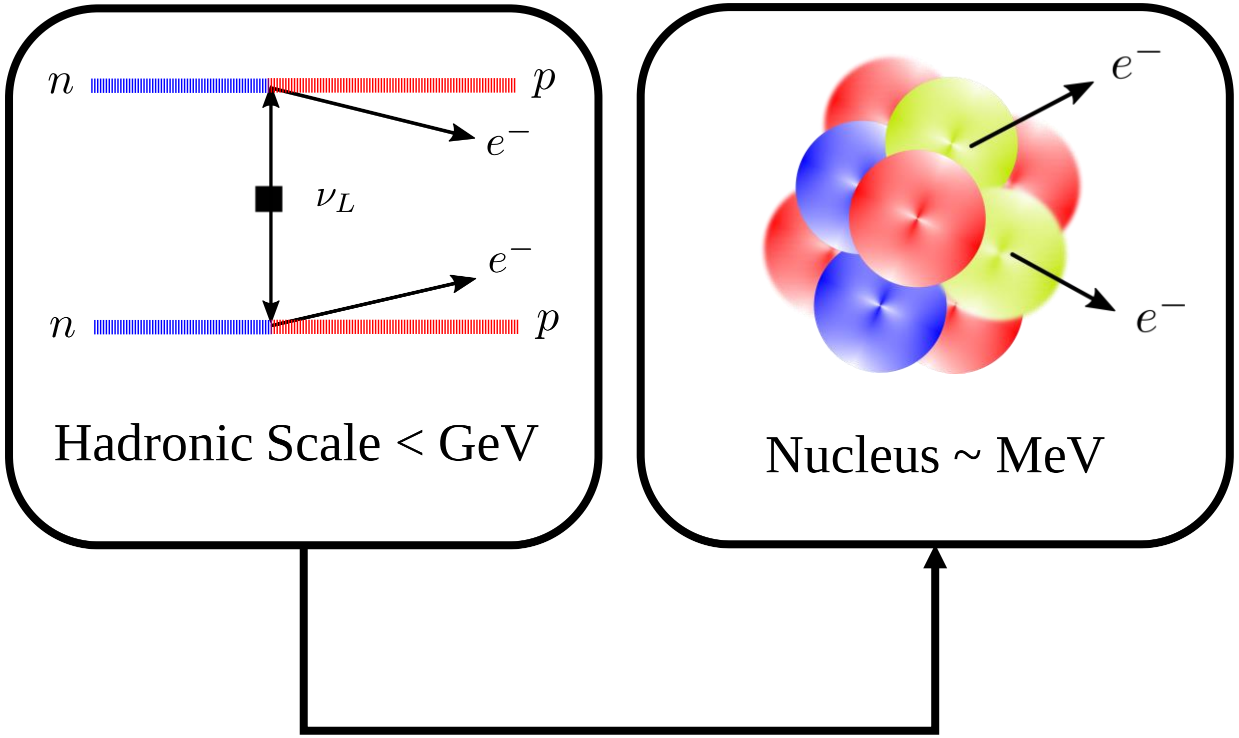


Low Energy Constants (LECs)

Nuclear Matrix Elements

Cirigliano, Dekens, de Vries, Hoferichter, Mereghetti
Phys. Rev. Lett. 126, 172002 (2021)





Nuclear EFT

Nuclear Many-Body Calculation

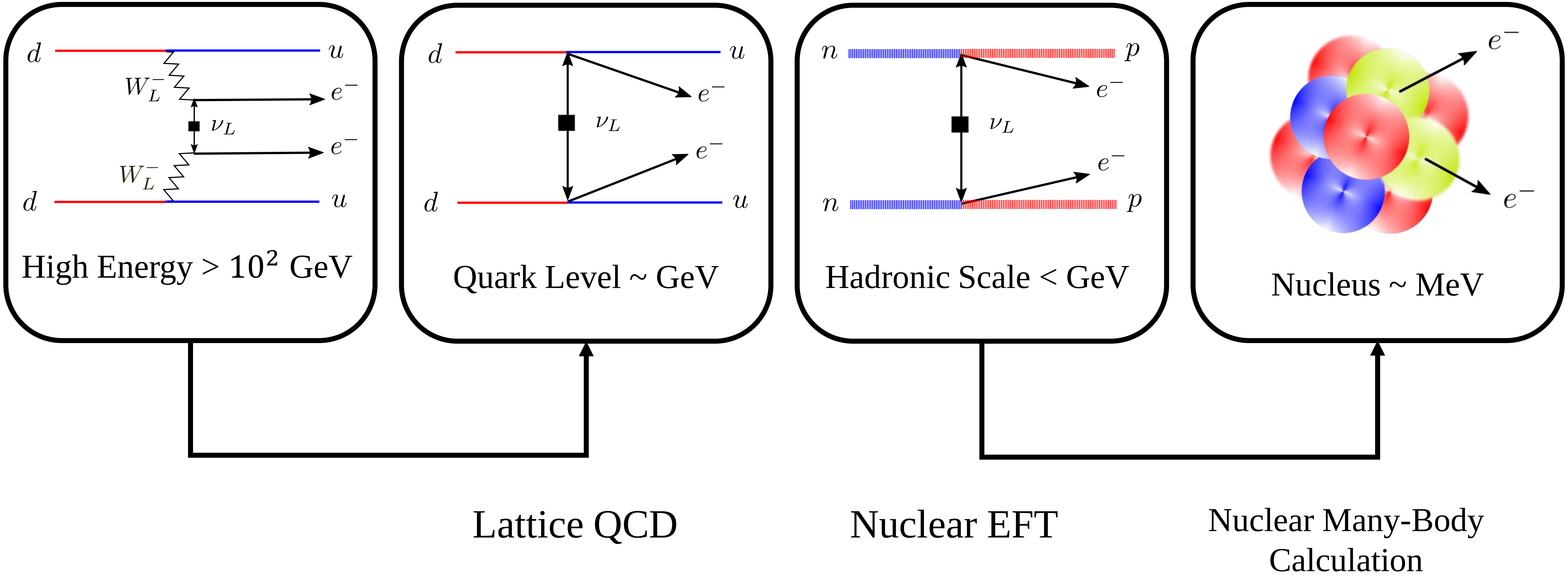
- Undertermined LEC g_ν^{NN} in pionless EFT
- LO $0\nu\beta\beta$ amplitude remains unknown
- An indirect estimate of g_ν^{NN} suggests an enhancement of ~ 40% in NME for Ca nucleus

Cirigliano, Dekens, de Vries, Hoferichter, Mereghetti
Phys. Rev. Lett. 126, 172002

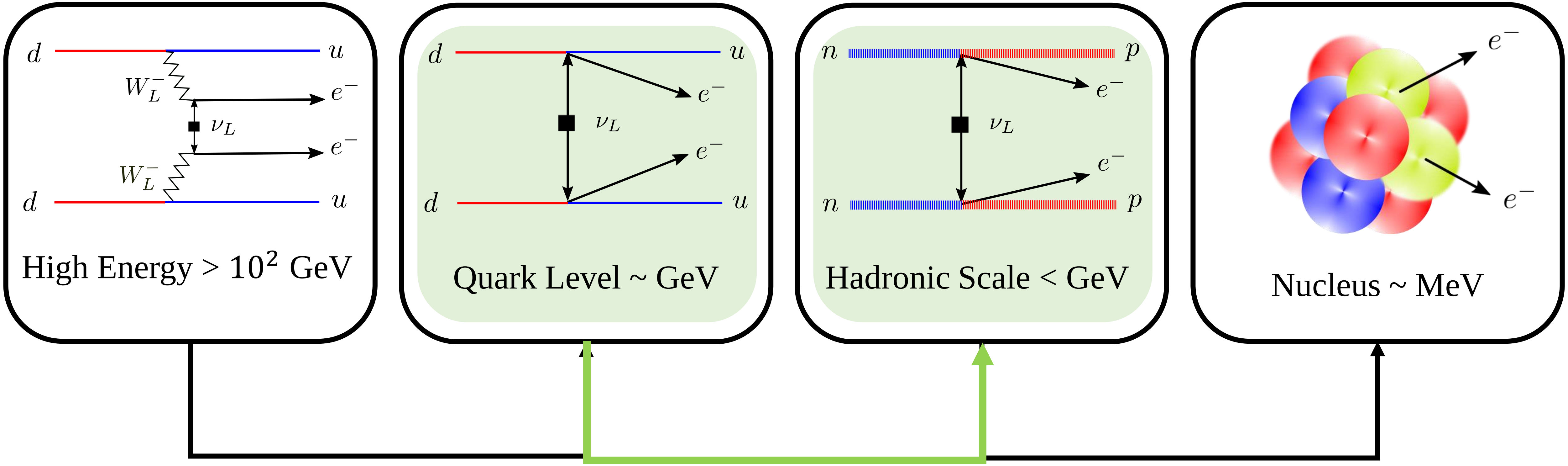
Wirth, Yao, Hergert
PhysRevLett.127.242502
(2021)

Jokiniemi, Soriano,Menendez
j.physletb.2021.136720
(2021)

Nuclear Matrix Elements



Nuclear Matrix Elements



Lattice QCD

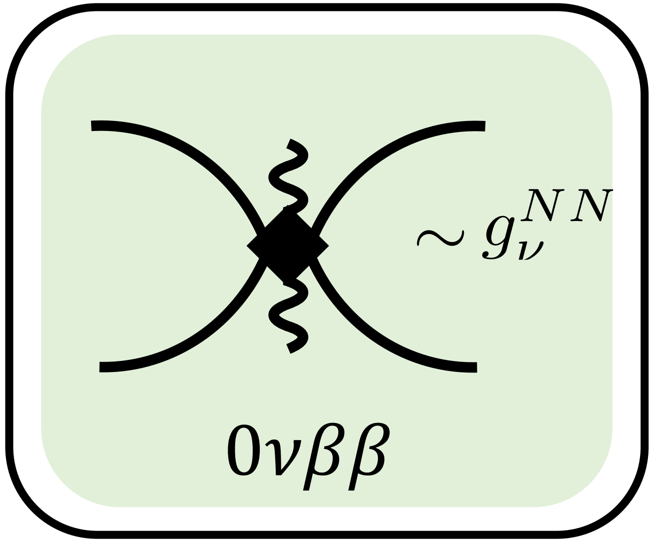
Nuclear EFT

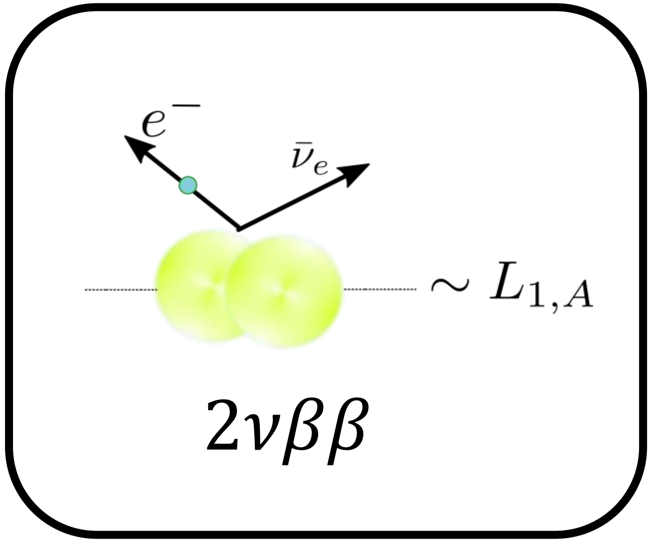
Matching EFT Amplitude to
Lattice QCD to constrain LECs

Nuclear Many-Body Calculation

Snowmass:
0vbb: A Roadmap for Matching Theory to Experiment
arXiv:2203.12169

Constraining LECs from Lattice QCD





Davoudi and Kadam
Phys. Rev. Lett. 126, 152003 (2021)

Davoudi and Kadam
Phys. Rev. D 102, 114521 (2020)

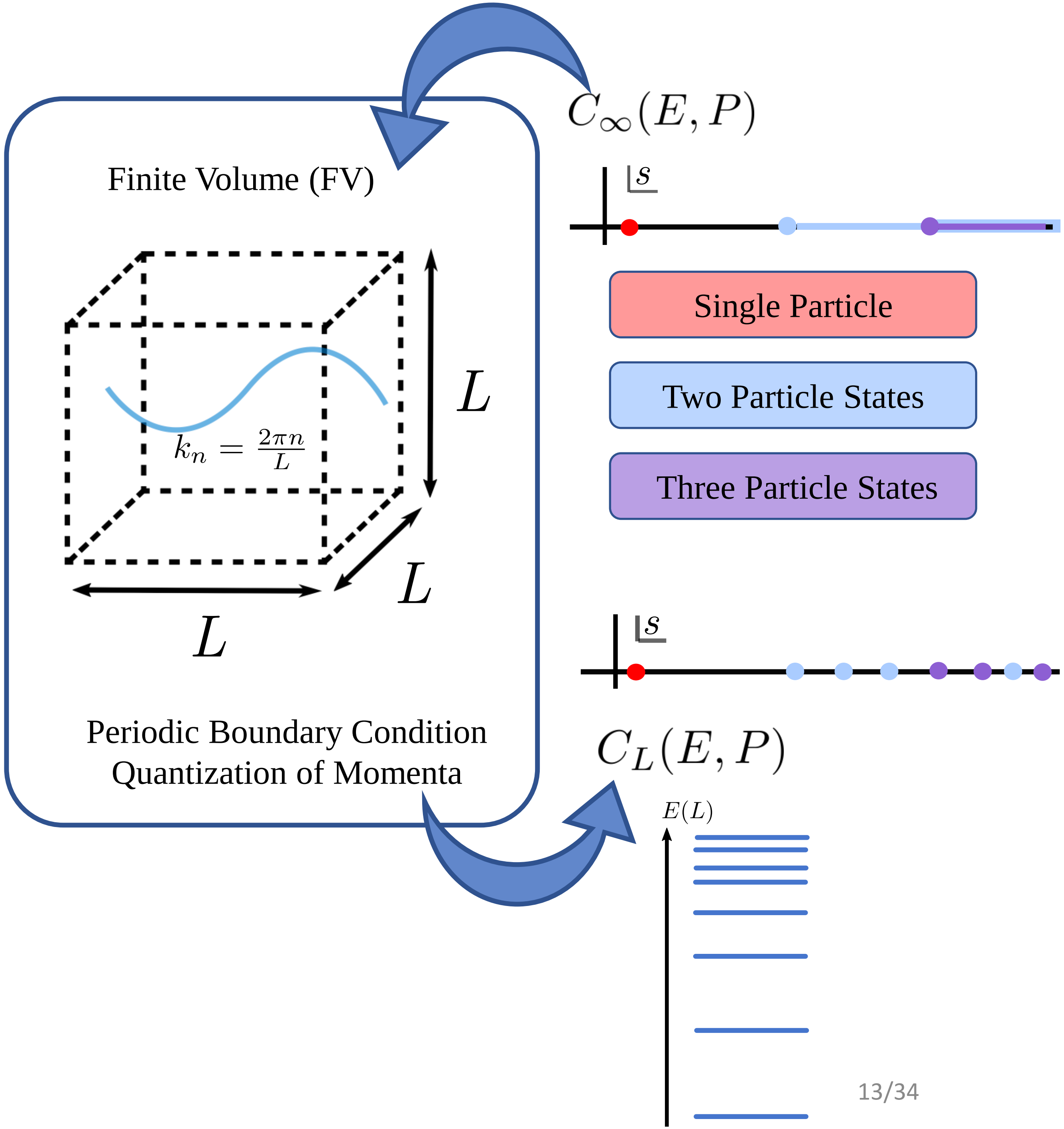
Lattice QCD

QCD Formulated on

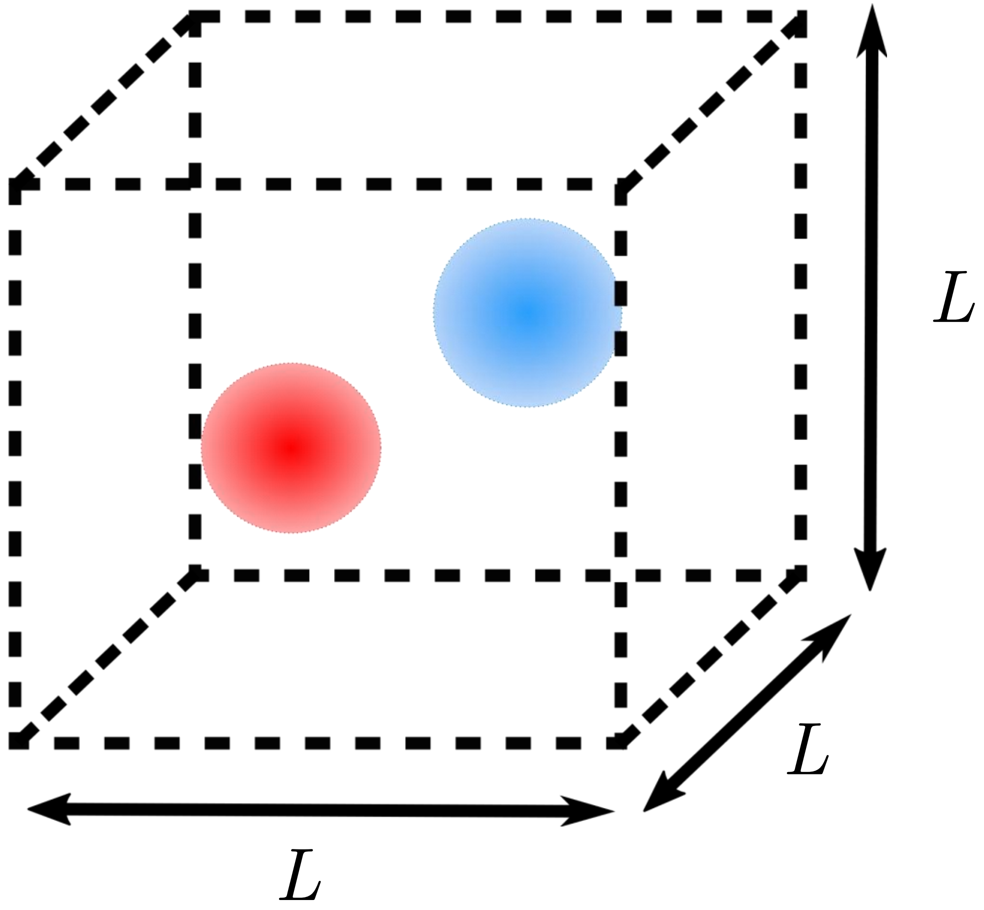
- ❖ Discrete Euclidean Spacetime Grid
- ❖ Lattice spacing a
- ❖ Finite Volume L^3
- ❖ Monte-Carlo Sampling

Finite vs. Infinite Volume Physics

- ❖ Branch cuts are replaced with poles
- ❖ Corresponding energies give the FV spectrum
- ❖ Position of poles is related to the scattering amplitude



Assumptions

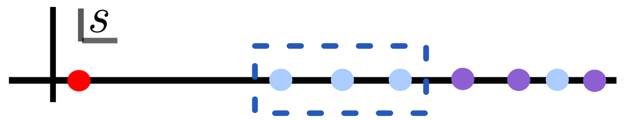


- Lattice spacing $a \to 0$
- Ignore discretization effects

- Infinite temporal extent
- Energy is a continuous variable

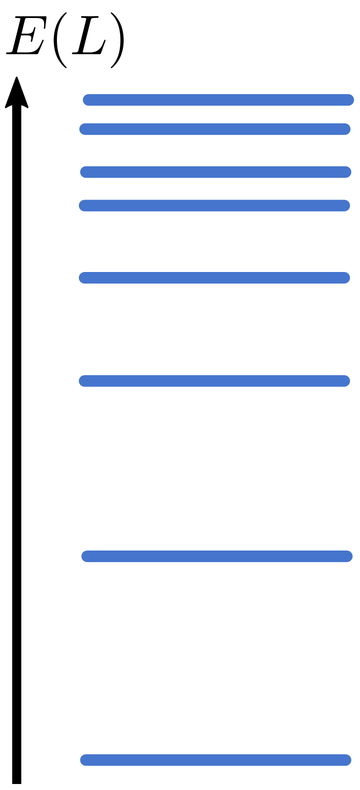
- In COM frame $P = (E, \mathbf{0})$
- Energy below three particle threshold

$C_L(E)$



Two Nucleon Scattering Amplitude

?

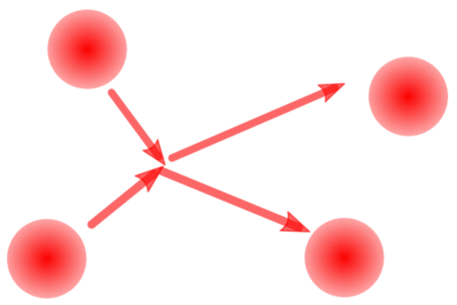


Discrete Finite Volume Spectrum

Evaluate

$$C_L(E) = \int_L d^3x \int dx_0\, e^{iEx_0} \left[\langle 0|\, T[B(x)B^\dagger(0)]\, |0\rangle\right]_L$$

Nucleon Interpolating Operators



$i\mathcal{M}_{NN\to NN}$

Infinite Volume Scattering Amplitude

Goal: Identify poles from the FV effects

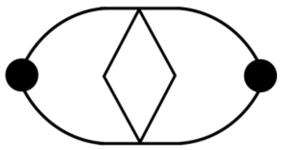
FV method

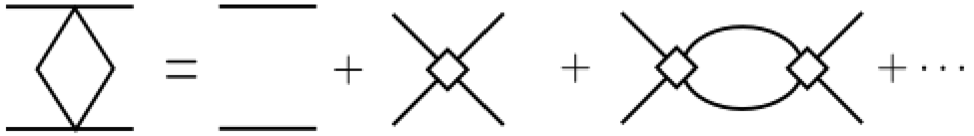
Lellouch, and Luscher (LL) (2001), Commun. Math. Phys. 219,

Kim, Sachrajda, and Sharpe (2005), Nucl. Phys. B727

$$C_L(E) = \int_L d^3x \int dx_0 \, e^{iEx_0} \left[\langle 0 | \, T[B(x)B^\dagger(0)] \, | 0 \rangle \right]_L$$

Evaluate the correlation function non-perturbatively

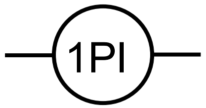
$C_L =$ 



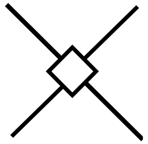
Which diagrams give singular sums in $\vec{k}$?
If particles in summed loops can go on-shell

Same as Infinite Volume

One particle irreducible diagrams

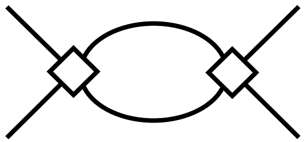


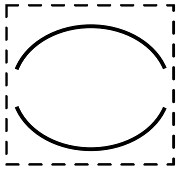
2 → 2 Bethe-Salpeter Kernel



Power Law Difference

Two particle loop in s - channel



 $\sim \quad I_0^V(E) = I_0^\infty(E) + F_0(E)$

Poles in $C_L(E)$ are identified using $F_0(E)$

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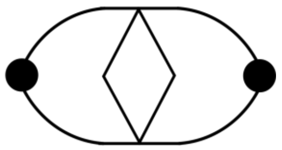
Lellouch, and Luscher (LL) (2001), Commun. Math. Phys. 219,

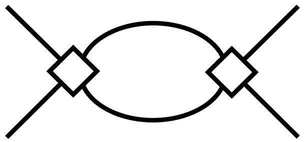
Kim, Sachrajda, and Sharpe (2005), Nucl. Phys. B727

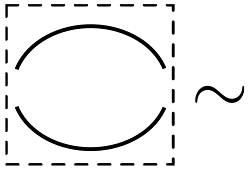
$$C_L(E) = \int_L d^3x \int dx_0\, e^{iEx_0} \left[\langle 0|\, T[B(x)B^\dagger(0)]\, |0\rangle\right]_L$$

Evaluate the correlation function in EFT non-perturbatively

$C_L =$ 

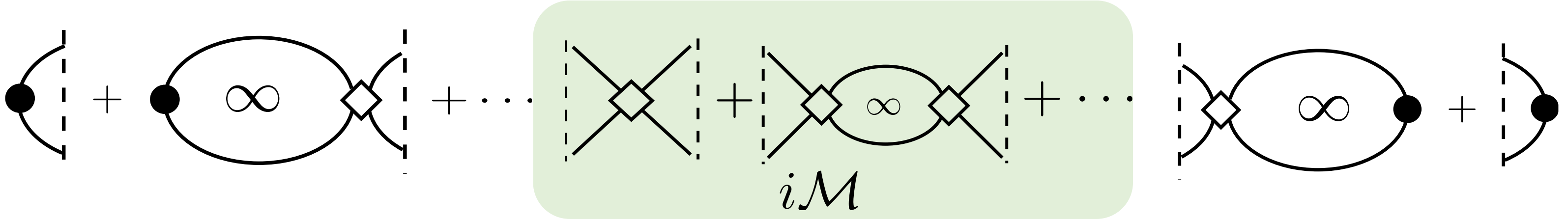
Identify and isolate FV corrections





$$I_0^V(E) = I_0^\infty(E) + F_0(E)$$

Rearrange to get Infinite volume 2 → 2 amplitude

$C_L =$ 

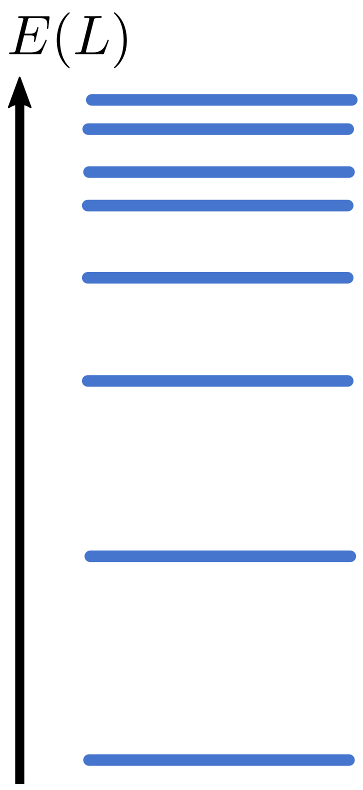
Geometric sum over F_0 terms

$$C_L \sim \frac{1}{F_0^{-1} + \mathcal{M}}$$

Poles of $C_L(E)$ are identified by

$$\det\left[F_0^{-1}(E_n) + \mathcal{M}(E_n)\right] = 0$$

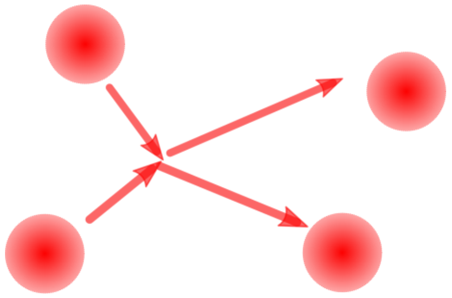
Two Nucleon Scattering Amplitude



Finite Volume Two Particle Spectrum

Luscher's Quantization Condition

$$\det\left[F_0^{-1}(E_n) + \mathcal{M}(E_n)\right] = 0$$



$$i\mathcal{M}_{NN\to NN}$$

Infinite Volume Scattering Amplitude

Luscher Commun. Math. Phys. 104, 177 (1986)
Luscher Commun. Math. Phys. 105, 153 (1986)
Kim, Sachrajda, and Sharpe, Nucl. Phys. B727 (2005)

FV Formalism

Review: Davoudi, Detmold, Shanahan, Orginos, Parreno, Savage, Wagman physrep.2020.10.004

Extended towards electro-weak current ($\mathcal{J}$) interactions:

- Formalism for generalized $0 + \mathcal{J} \to 2$ and $1 + \mathcal{J} \to 2$ processes.

  From LQCD: $\gamma^* \to \pi\pi$ and $\pi\gamma^* \to \pi\pi$ amplitudes.

  Briceno, Hansen, and Walker-Loud (2015) Phys. Rev. D 91, 034501
  Briceno and Hansen (2015),Phys. Rev. D92 (7), 074509
  Feng et al. (2015) Phys. Rev. D91 (5), 054504
  Briceno et al. (2015a) Phys. Rev. Lett. 115, 242001

- Formalism for $2 + \mathcal{J} \to 2$ processes.

  Value of $L_{1,A}$ from LQCD via studying pp fusion $pp \to de^+\nu$ process.

  Briceño and Davoudi (2013) Phys. Rev. D 88, 094507
  Briceno and Hansen (2016) Phys. Rev. D94 (1), 013008
  NPLQCD Collaboration Phys. Rev. Lett. 119 (6) (2017) 62002.

- Formalism for $1 + 2\,\mathcal{J} \to 1$ processes.

  $2\nu\beta\beta$ matrix elements (MEs) at $m_\pi \sim 800$ MeV

  Briceño, Davoudi, Hansen, Schindler and Baroni Phys. Rev. D 101, 014509
  NPLQCD Collaboration Phys. Rev. D 96, 054505.

- $K_L - K_S$ mass difference

  Christ, Izubuchi, Sachrajda, Soni, and Yu (RBC and UKQCD Collaborations) Phys. Rev. D 88, 014508 (2013)

- Formalism for $1 + 2\,\mathcal{J} \to 1$ processes with massless leptonic propagators

  Christ, Feng, Jin, and Sachrajda Phys. Rev. D 103, 014507 (2021)
  Feng, Jin, Wang, and Zhang Phys. Rev. D 103, 034508 (2021)

- Formalism for $2 + 2\,\mathcal{J} \to 2$ processes for $2\nu\beta\beta$ and $0\nu\beta\beta$.

  Davoudi and Kadam Phys. Rev. D 102, 114521 (2020)
  Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

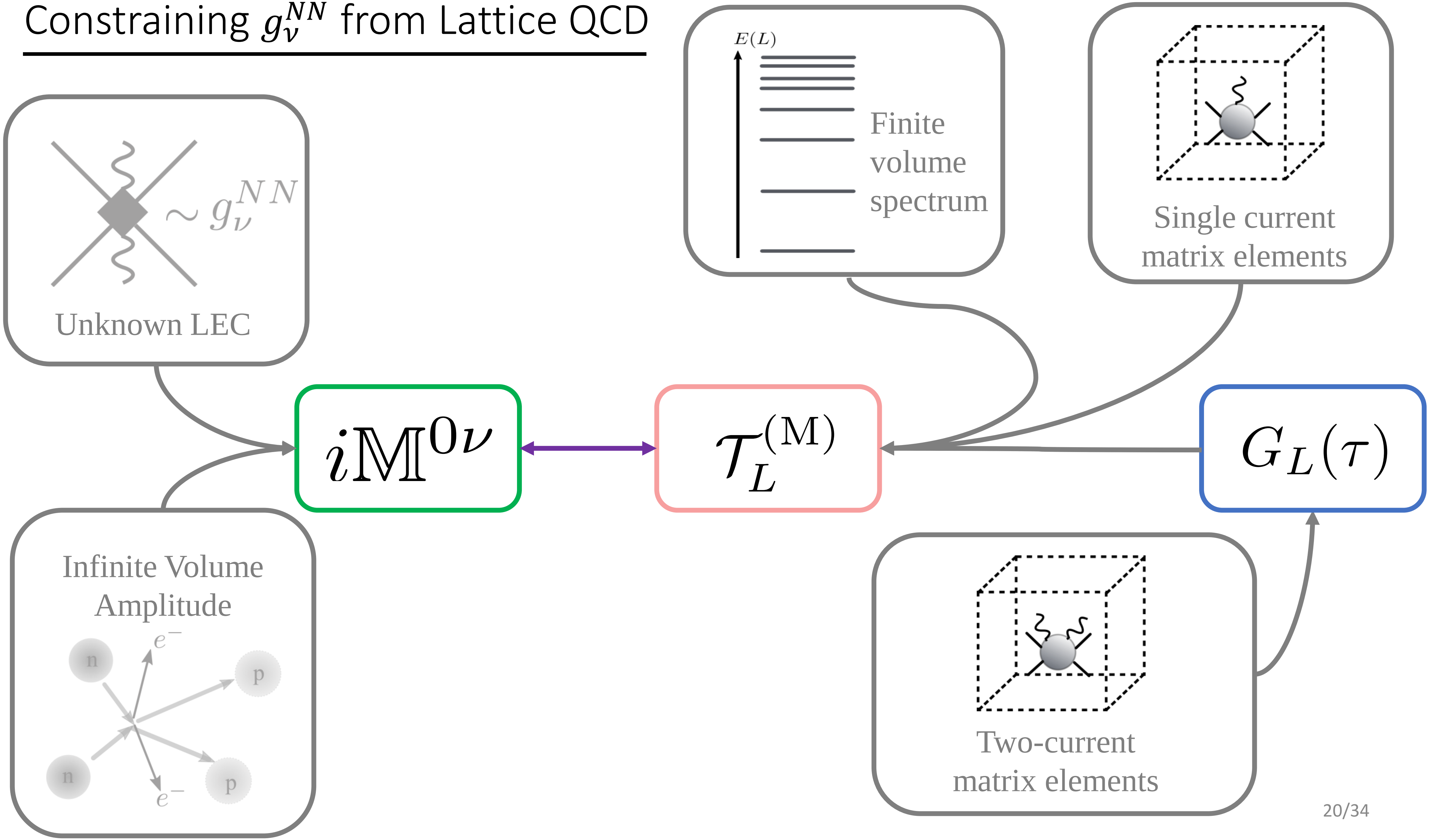
- Light sterile neutrino contribution to $\pi^- \to \pi^+ e^- e^-$ from LQCD at the physical pion mass

  Tuo, Feng, and Jin arXiv:2206.00879

- $\pi^- \to \pi^+ e^- e^-$ from LQCD at m_π in 300-430 MeV

  Detmold and Murphy arXiv:2004.07404
  Detmold, Jay, Murphy, Oare, and Shanahan arXiv:2208.05322

Constraining g_ν^{NN} from Lattice QCD



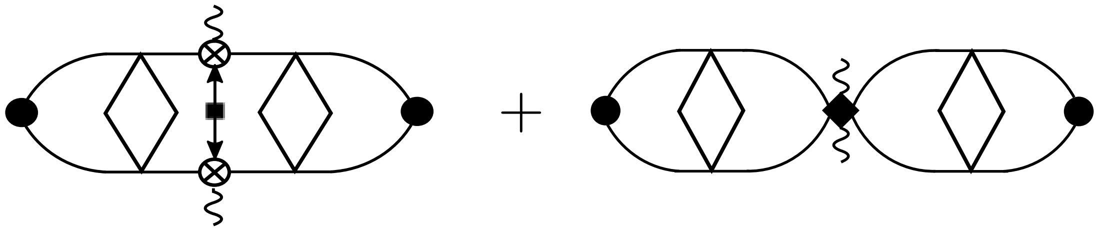
$0\nu\beta\beta$ Decay

Finite Volume

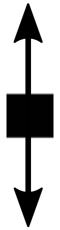
For details see:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iE_1 z_0} \left[\langle E_{n_f} | \, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)] \, | E_{n_i} \rangle \right]_L$$

Evaluate the correlation function in EFT non-perturbatively

$C_L^{0\nu} =$ 

Sum over quantized momenta

With neutrino propagator  $\sim \frac{1}{L^3} \sum_{\mathbf{q}\neq\mathbf{0}} \frac{m_{\beta\beta}}{|\mathbf{q}|^2}$

IR regulated by removing zero mode

$0\nu\beta\beta$ Decay

Finite Volume

For details see:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iE_1 z_0} \left[\langle E_{n_f} | \, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)] \, | E_{n_i}\rangle\right]_L$$

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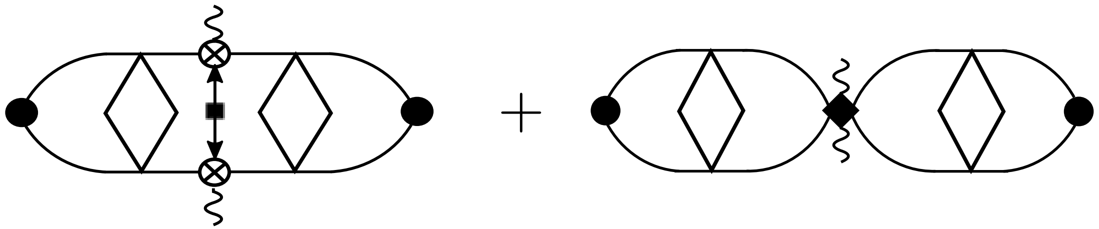
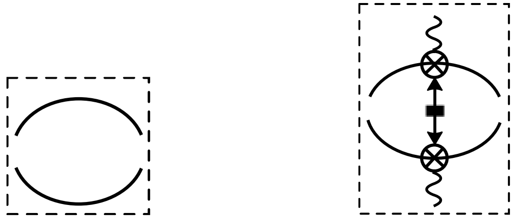
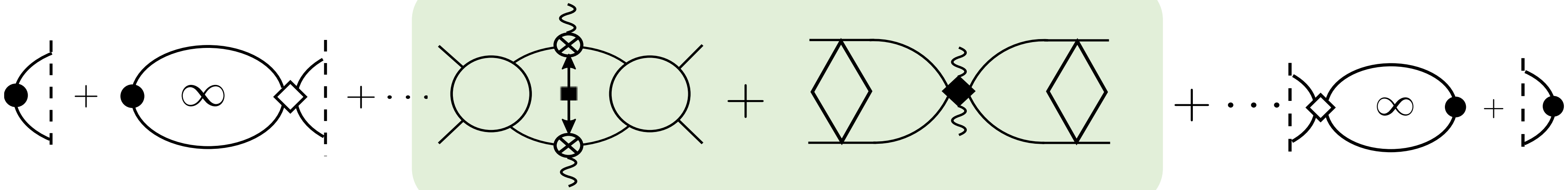
$C_L^{0\nu} =$ 

Identify and isolate FV corrections



$F_0(E_i)$ δJ^{V} $F_0(E_f)$

Rearrange to get infinite volume quantities



$i\mathbb{M}^{0\nu}$

Constraining g_ν^{NN} from Lattice QCD

For details see:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

$$L^6 \left|\mathcal{T}_L^{(\mathrm{M})}\right|^2 = \left|\mathcal{R}^*(E_{n_f})\right| \left|i\mathbb{M}^{0\nu}\right|^2 \left|\mathcal{R}^*(E_{n_i})\right|$$

Lellouch Luscher residue matrix

$$\mathcal{R}(E_n) = \lim_{E\to E_n} \frac{(E-E_n)}{F_0^{-1}+\mathcal{M}}$$

Constraining g_ν^{NN} from Lattice QCD

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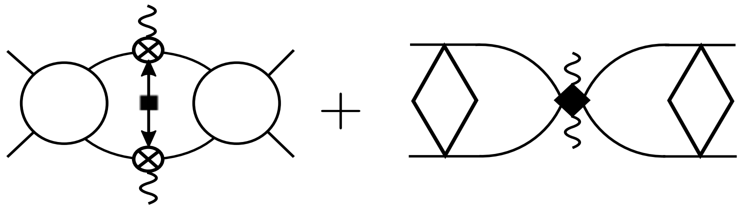
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

$$L^6 \left|\mathcal{T}_L^{(\mathrm{M})}\right|^2 = \left|\mathcal{R}^*(E_{n_f})\right| \left|i\mathbb{M}^{0\nu}\right|^2 \left|\mathcal{R}^*(E_{n_i})\right|$$

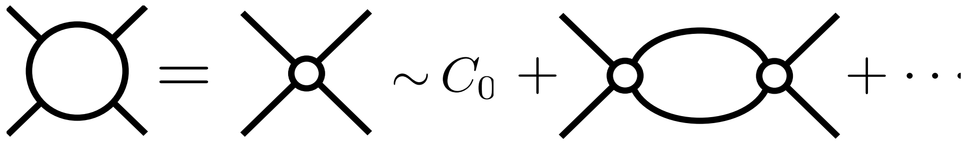
Lellouch Luscher residue matrix

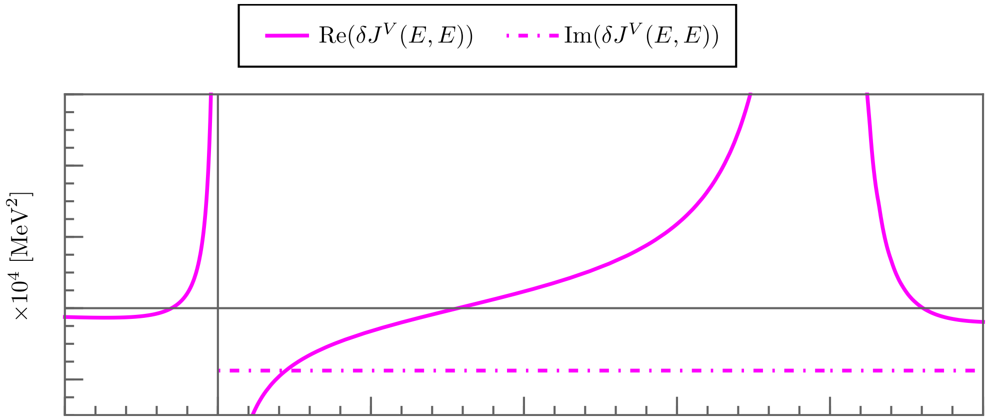
$$\mathcal{R}(E_n) = \lim_{E\to E_n} \frac{(E-E_n)}{F_0^{-1}+\mathcal{M}}$$

$$i\mathbb{M}^{0\nu} = i\mathcal{M}_{nn\to pp}^{(\mathrm{Int.})} - m_{\beta\beta}(1+3g_A^2)\mathcal{M}_{nn}\,\delta J^V\,\mathcal{M}_{pp}$$

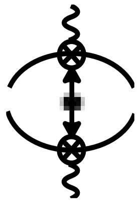
$i\mathcal{M}_{nn\to pp}^{(\mathrm{Int.})} =$ 

Physical LO two-nucleon amplitude

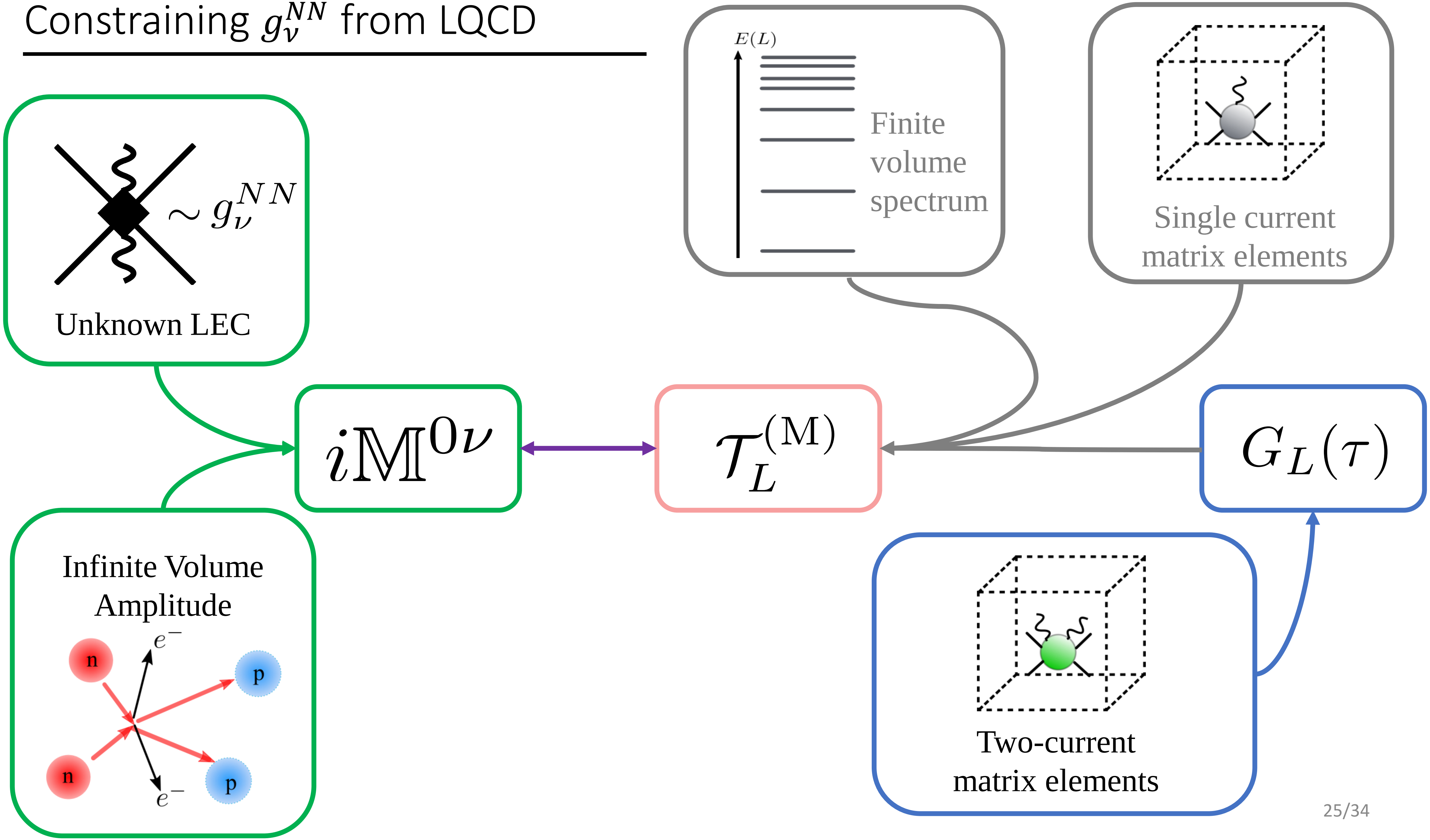




Finite volume corrections

 $\sum_{\vec{k}} = \int_{\vec{k}} + \delta J^V$

Constraining g_ν^{NN} from LQCD



$$\mathcal{T}_L^{(\mathrm{M})}$$

Minkowski Signature
Correlation Function

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iEz_0} \left[\langle E_{n_f}|\, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)]\, |E_{n_i}\rangle\right]_L$$

?

$$G_L(\tau)$$

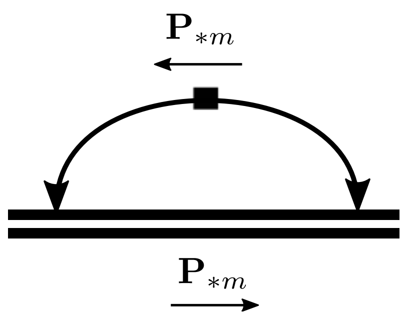
Euclidean Time
Four-point Correlation
Function from LQCD

$$G_L(\tau) = \int_L d^3z \left[\langle E_f, L|\, T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau, z)\, S_\nu^E(\tau, z) \mathcal{J}^{(\mathrm{E})}(0)]\, |E_i, L\rangle\right]_L ,$$

Plugging back the missing time integral

$$\mathcal{T}_L^{(\mathrm{E})} \overset{?}{=} \int d\tau e^{E\tau} G_L(\tau) \quad \sim \int_0^\infty d\tau\, e^{-(|\mathbf{P}_{*m}| + E_{*m} - E_*)\tau}$$

$$E_* = E_i - E$$



$\mathcal{T}_L^{(\mathrm{M})}$

Minkowski Signature Correlation Function

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iEz_0} \left[\langle E_{n_f}|\, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)]\, |E_{n_i}\rangle\right]_L$$

?

$G_L(\tau)$

Euclidean Time Four-point Correlation Function from LQCD

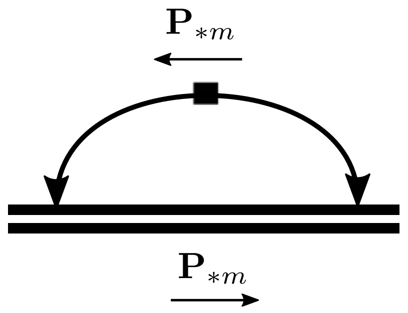
$$G_L(\tau) = \int_L d^3z \left[\langle E_f, L|\, T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau, z)\, S_\nu^E(\tau, z) \mathcal{J}^{(\mathrm{E})}(0)]\, |E_i, L\rangle\right]_L ,$$

Plugging back the missing time integral

$$\mathcal{T}_L^{(\mathrm{E})} \stackrel{?}{=} \int d\tau e^{E\tau} G_L(\tau) \quad \sim \int_0^\infty d\tau\, e^{-(|\mathbf{P}_{*m}|+E_{*m}-E_*)\tau}$$

Diverges for $|\mathbf{P}_{*m}| + E_{*m} < E_*$

$$E_* = E_i - E$$



Diverges for intermediate states that can go on-shell

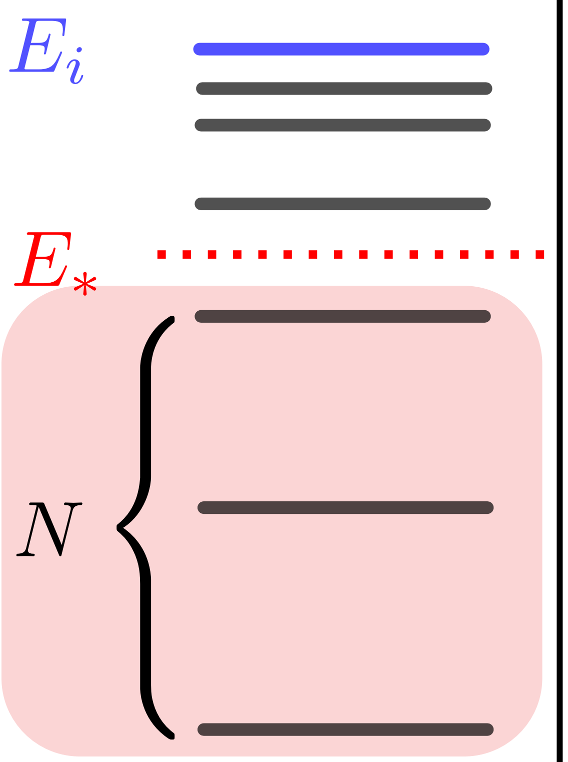
Need to remove these divergences for analytic continuation!!

Removing Divergences from $G_L(\tau)$

Davoudi and Kadam
Phys. Rev. Lett. 126, 152003 (2021)

Where is the divergence coming from?
Two particle FV states which can go on-shell

$$|\mathbf{P}_{*m}| + E_{*m} < E_*$$



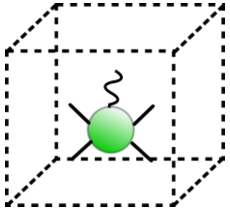
Finite Volume Two Particle Spectrum

Two-body spectrum to identify N low-lying states

Constructing divergent contributions

Spectral representation by integrating over time

$$\mathcal{T}_L^{(\mathrm{M})} = i \sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$

$c_m \sim$  Finite volume matrix elements of single hadronic current between the initial (final) and intermediate states

$$G_L^{<}(\tau) \equiv \sum_{m=0}^{N-1} c_m\, \theta(\tau)\, e^{-(|\mathbf{P}_{*m}| + E_{*m} - E_f)|\tau|}$$

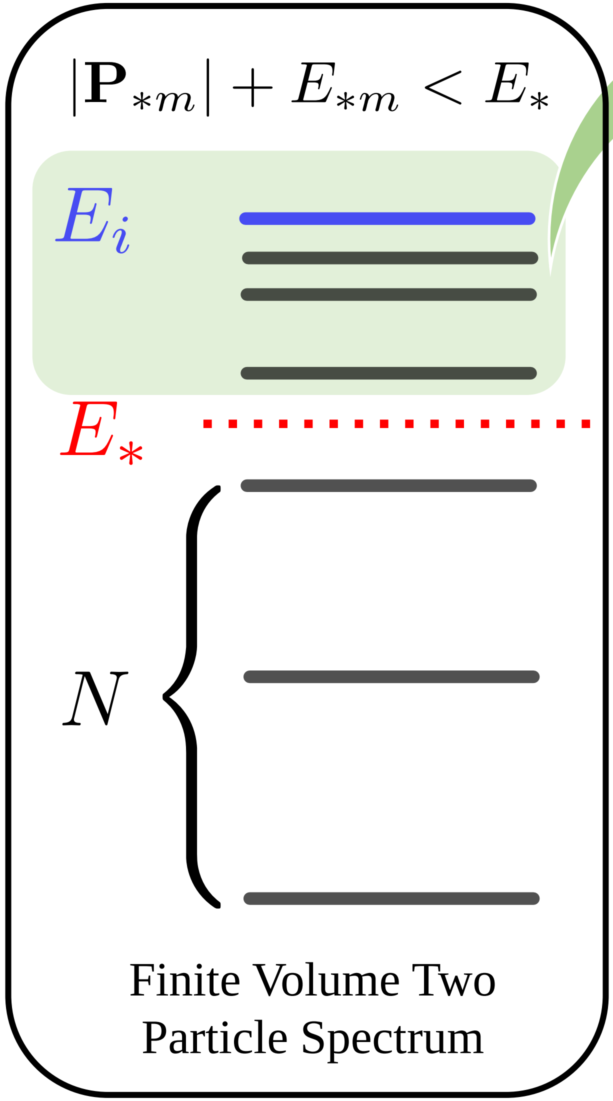
Can be analytically continued to Minkowski space

$G_L(\tau)$
Four-point Function From LQCD

$$\mathcal{T}_L^{(\mathrm{E})\geq} \equiv \int d\tau\, e^{E\tau} \left[G_L(\tau) - G_L^{<}(\tau)\right]$$

What do we want?

$$\mathcal{T}_L^{(\mathrm{M})} = i \sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$

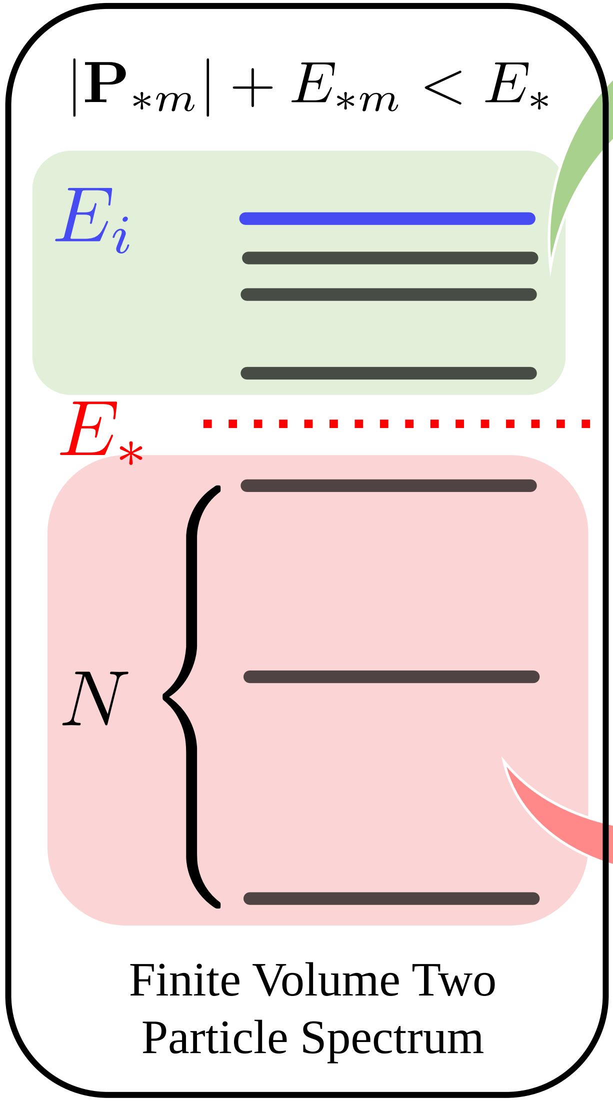


So far, we have

$$\mathcal{T}_L^{(\mathrm{E})\geq} \equiv \int d\tau\, e^{E\tau} \left[G_L(\tau) - G_L^{<}(\tau)\right]$$

What do we want?

$$\mathcal{T}_L^{(\mathrm{M})} = i\sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$



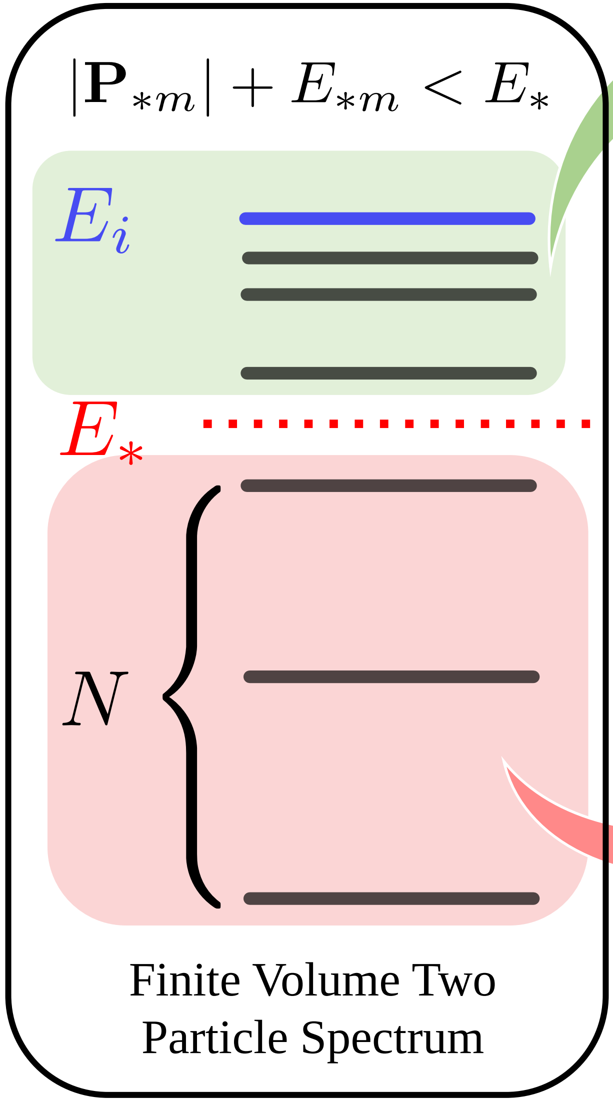
So far, we have

$$\mathcal{T}_L^{(\mathrm{E})\geq} \equiv \int d\tau\, e^{E\tau}\left[G_L(\tau) - G_L^{<}(\tau)\right]$$

Missing Piece

$$\mathcal{T}_L^{(\mathrm{E})<} \equiv \sum_{m=0}^{N-1} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}|}$$

$$\mathcal{T}_L^{(\mathrm{M})} = i\mathcal{T}_L^{(\mathrm{E})<} + i\mathcal{T}_L^{(\mathrm{E})\geq}$$



What do we want?

$$\mathcal{T}_L^{(\mathrm{M})} = i \sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$

So far, we have

$$\mathcal{T}_L^{(\mathrm{E})\geq} \equiv \int d\tau \, e^{E\tau} \left[G_L(\tau) - G_L^{<}(\tau) \right]$$

Missing Piece

$$\mathcal{T}_L^{(\mathrm{E})<} \equiv \sum_{m=0}^{N-1} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}|}$$

$$\mathcal{T}_L^{(\mathrm{M})} = i\mathcal{T}_L^{(\mathrm{E})<} + i\mathcal{T}_L^{(\mathrm{E})\geq}$$

For $L = 8$ fm

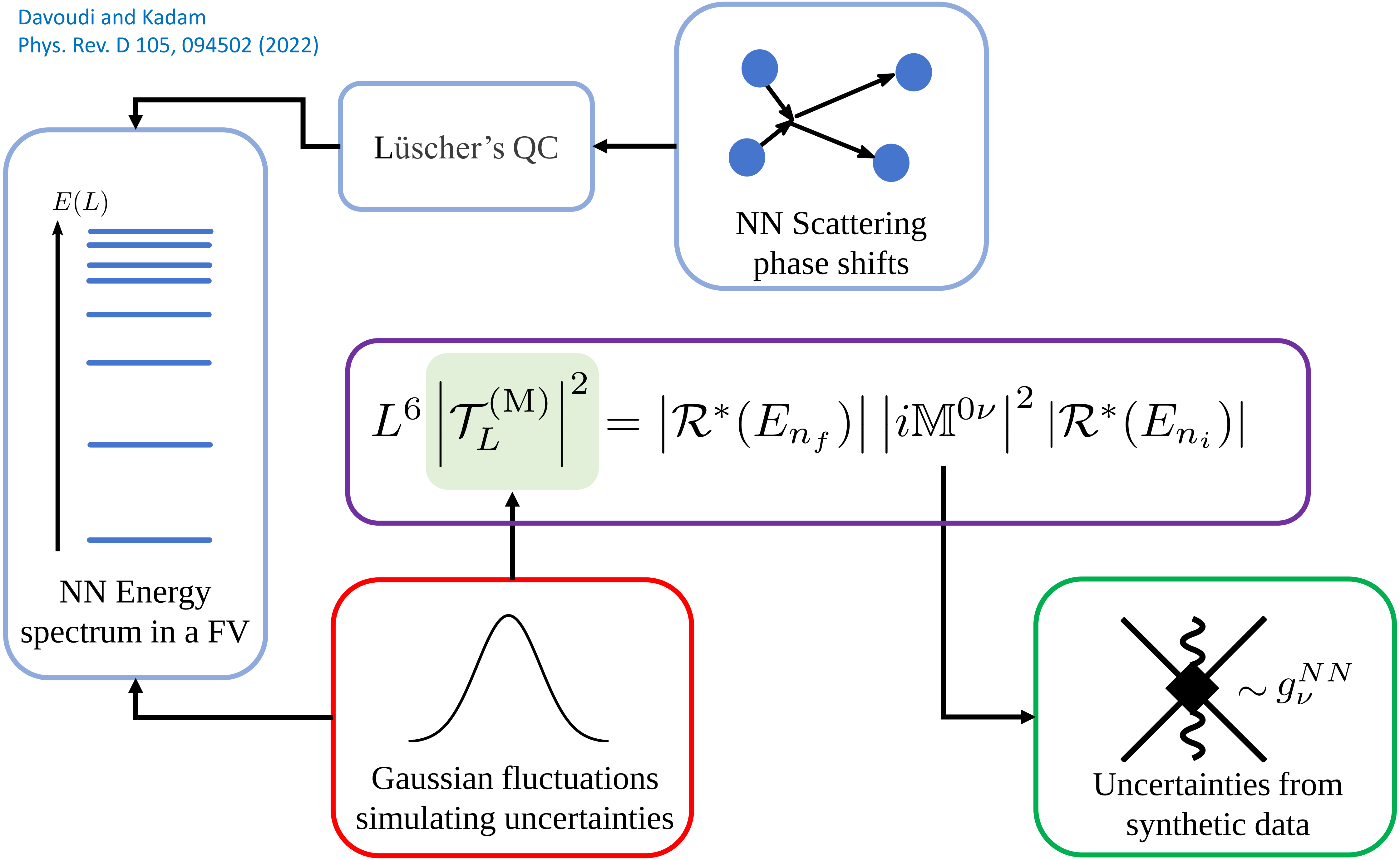
$E_{^1S_0} = -2.728,\ 19.043, \cdots$ MeV

$E_{*m} = -5.579,\ 13.688, \cdots$ MeV

$|\mathbf{P}_*| = 2\pi/L \approx 155$ MeV

$$i\mathcal{T}_L^{(\mathrm{E})} = i \int d\tau \, e^{E_1 \tau} G_L^{(\mathrm{E})}(\tau)$$

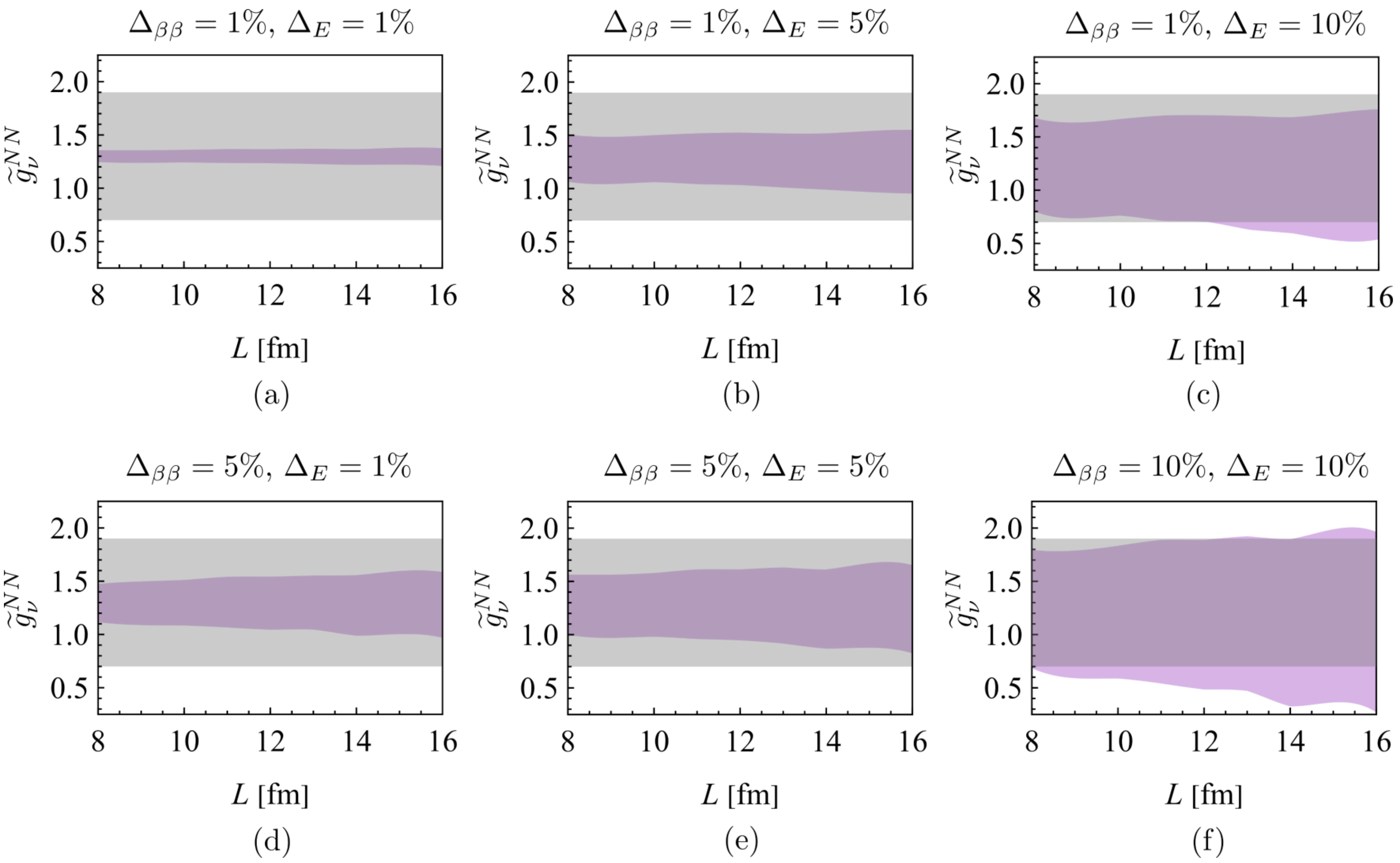
Davoudi and Kadam
Phys. Rev. D 105, 094502 (2022)



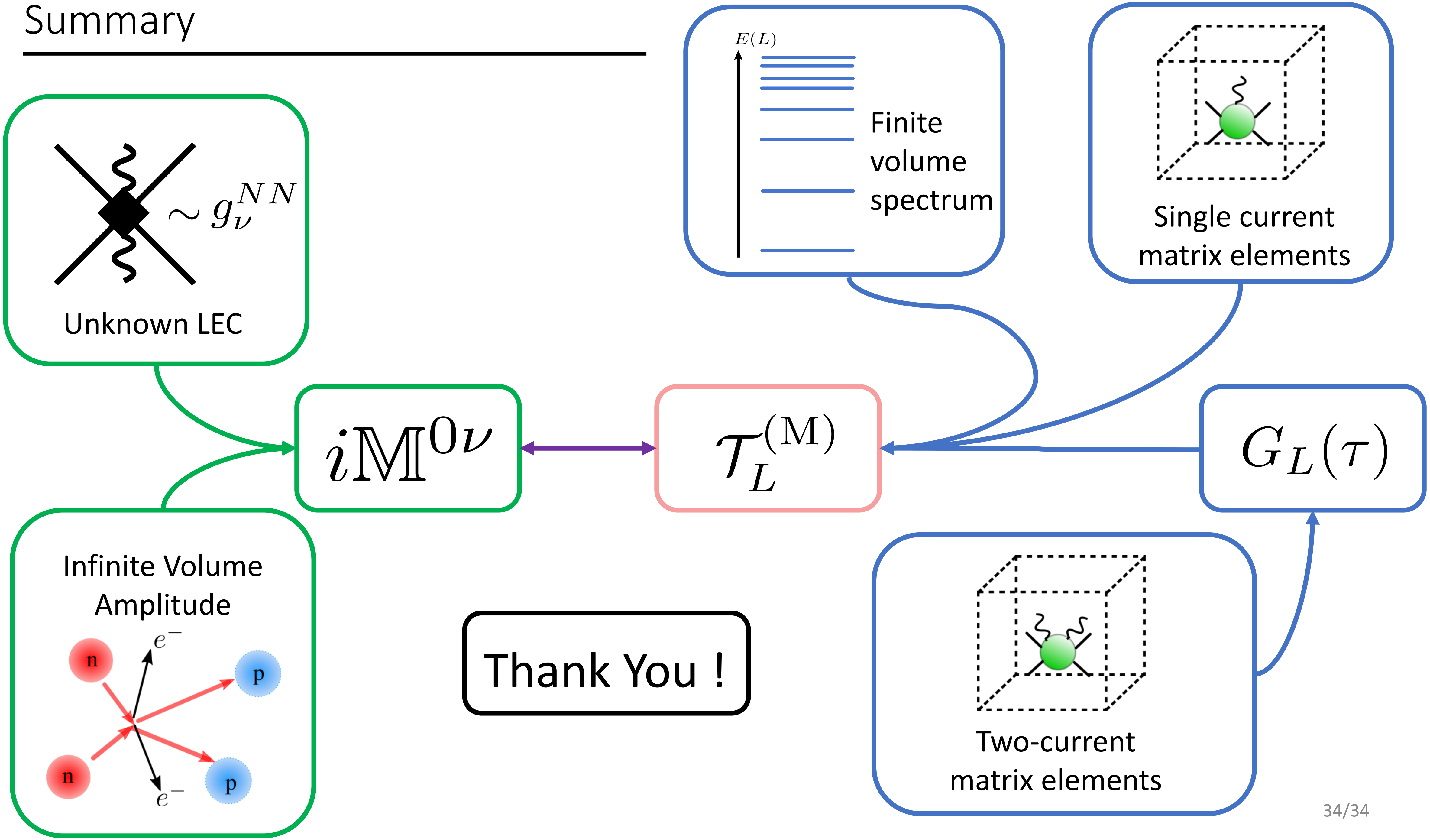
Davoudi and Kadam
Phys. Rev. D 105, 094502 (2022)

$\Delta_{\beta\beta}$: Uncertainty in four-point function

Δ_E : Uncertainty in NN energy eigenvalues



Summary



$E(L)$
Finite volume spectrum
Single current matrix elements
$\sim g_\nu^{NN}$
Unknown LEC
$i\mathbb{M}^{0\nu}$
$\mathcal{T}_L^{(\mathrm{M})}$
$G_L(\tau)$
Infinite Volume Amplitude
n
n
p
p
e^-
e^-
Thank You !
Two-current matrix elements

Backup

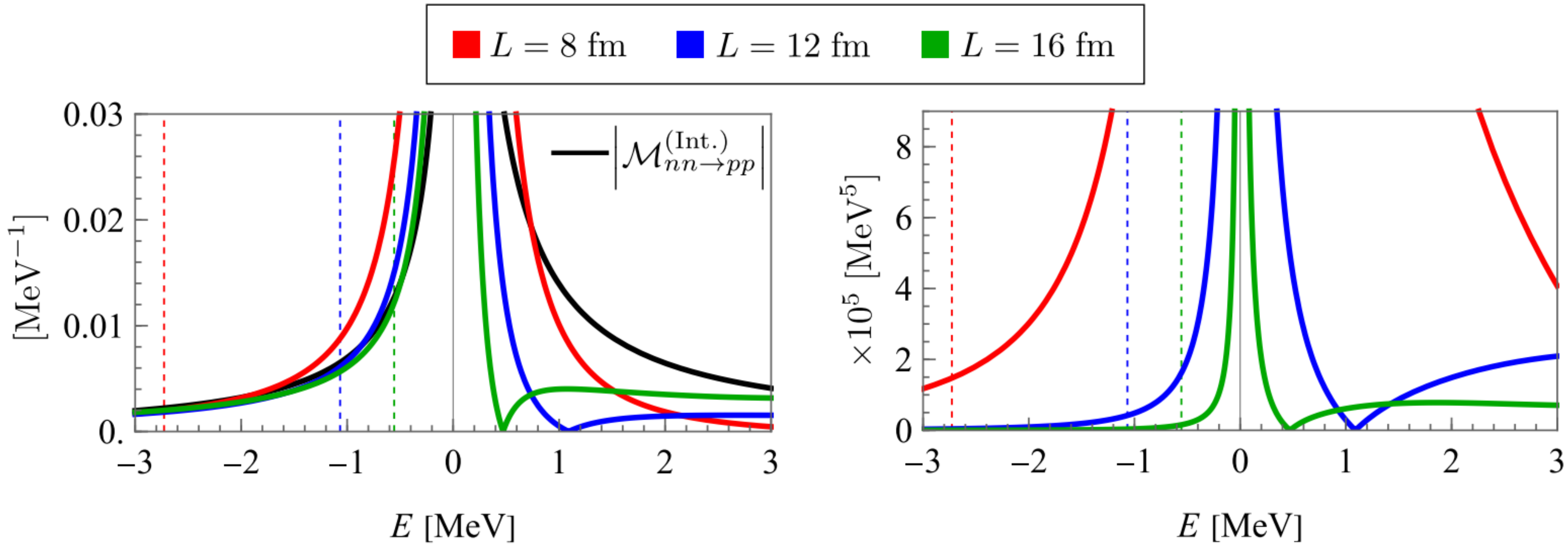


FIG. 7. $\left|\mathcal{M}^{0\nu,V(\text{Int.})}_{nn\to pp}\right|$ (left) and $\left|\mathcal{T}^{(M)}_{L}\right|$ (right) functions defined in Eqs. (30)-(28), with $L = 8$ fm (red), $L = 12$ fm (blue), and $L = 16$ fm (green) are plotted against the CM energy of the NN state, considering the kinematics $E_i = E_f \equiv E$. The effective neutrino mass $m_{\beta\beta}$ is set to 1 MeV. The dashed lines in both panels denote the ground-state energy eigenvalues in the corresponding volumes obtained from the quantization condition in Eq. (9) (as plotted in Fig. 2). Selected numerical values for the functions shown are provided in Appendix A.

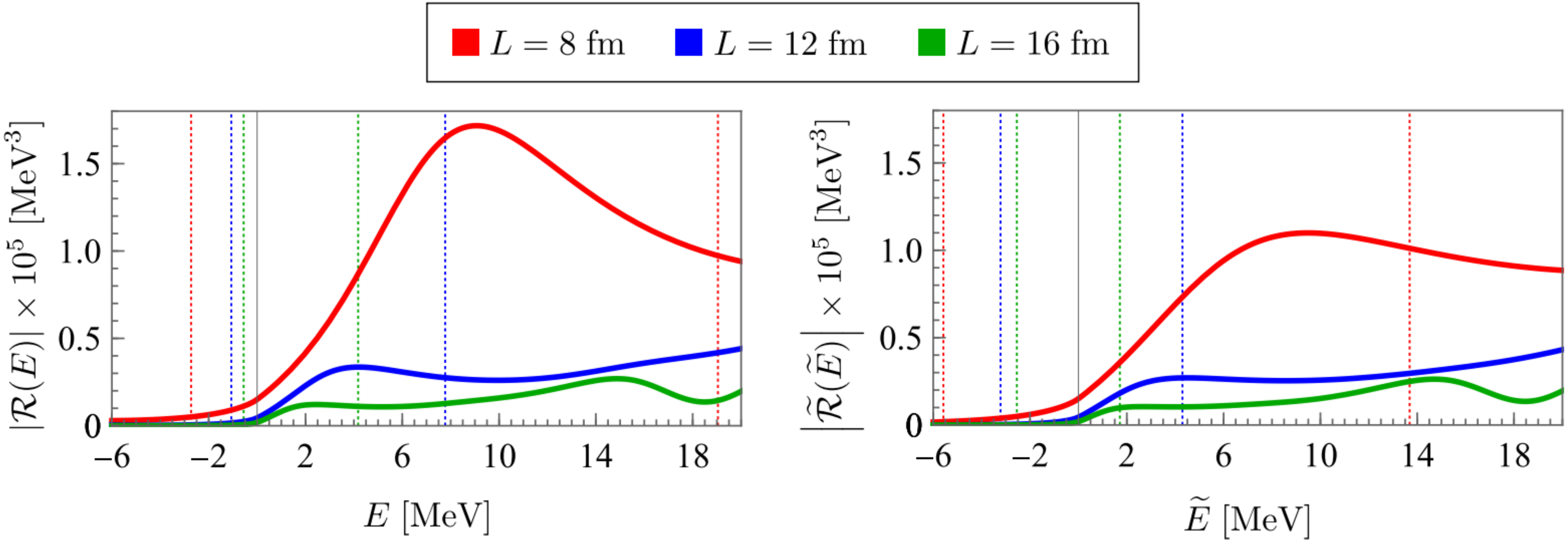


FIG. 3. The absolute values of the LL residue function in the ${}^1S_0$ (left) and ${}^3S_1$ (right) channels is plotted against the CM energy for three different volumes with $L = 8$ fm (red), $L = 12$ fm (blue), and $L = 16$ fm (green). Dashed lines indicate energy eigenvalues in the respective volumes. The numerical values of $|\mathcal{R}|$ and $|\widetilde{\mathcal{R}}|$ evaluated at the FV ground- and first excited-state energies in the corresponding volumes are provided in Appendix A.

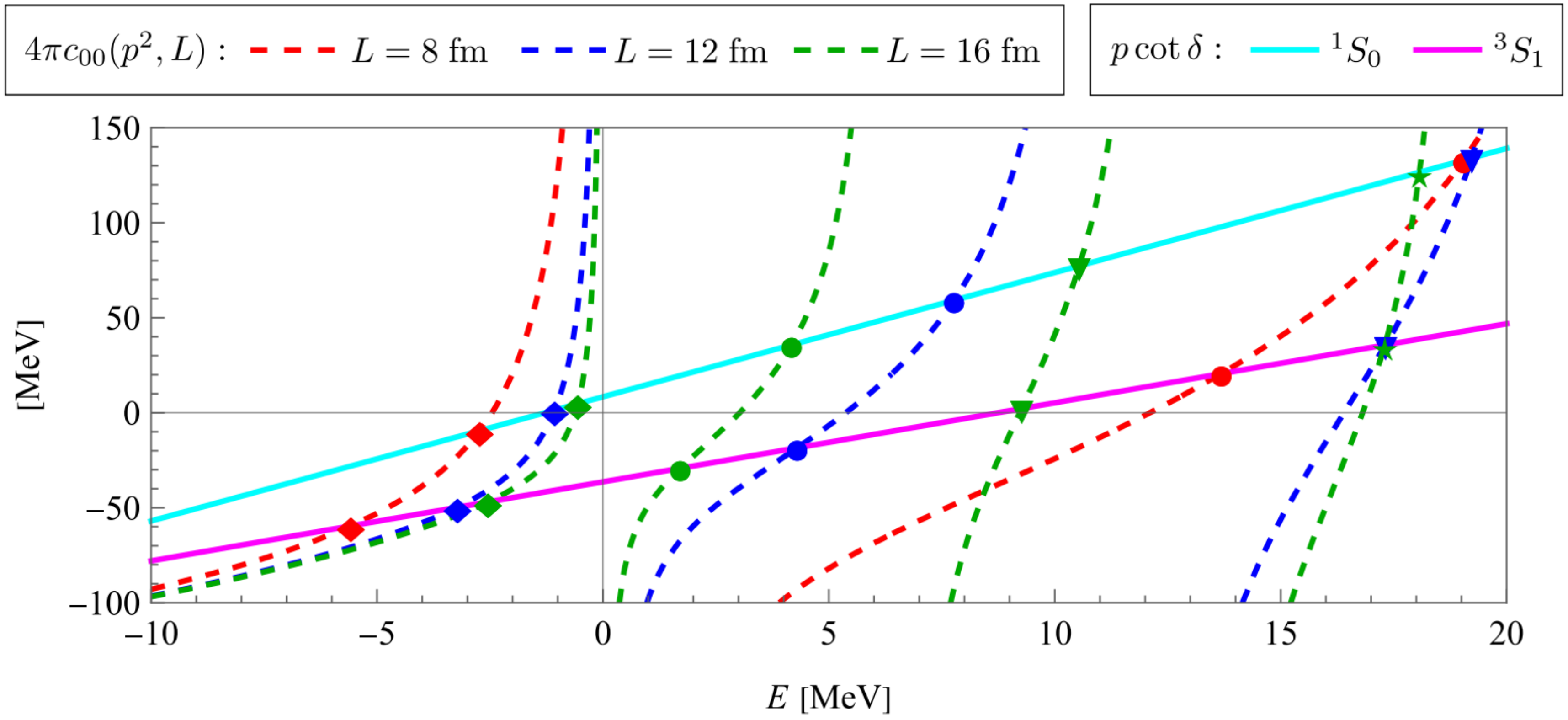


FIG. 2. The effective-range function (solid lines) and Lüscher's function (dotted lines) in Eq. (9) are plotted independently against the CM energy of NN systems. Equation (4) is used for the effective-range function with the effective-range expansion parameters given in Eq. (14) for the two channels, $^1S_0$ (cyan) and $^3S_1$ (magenta). The function $4\pi c_{00}(p^2, L)$ is plotted for three different volumes with $L = 8$ fm (red), $L = 12$ fm (blue) and $L = 16$ fm (green). The diamonds, circles, triangles, and stars denote, respectively, the location of energy eigenvalues of the ground, first, second, and third excited states in each volume, and satisfy the quantization condition in Eq. (9) (and its counterpart for the $^3S_1$ channel). The numerical values associated with this figure are provided in Appendix A.

$0\nu\beta\beta$ Decay

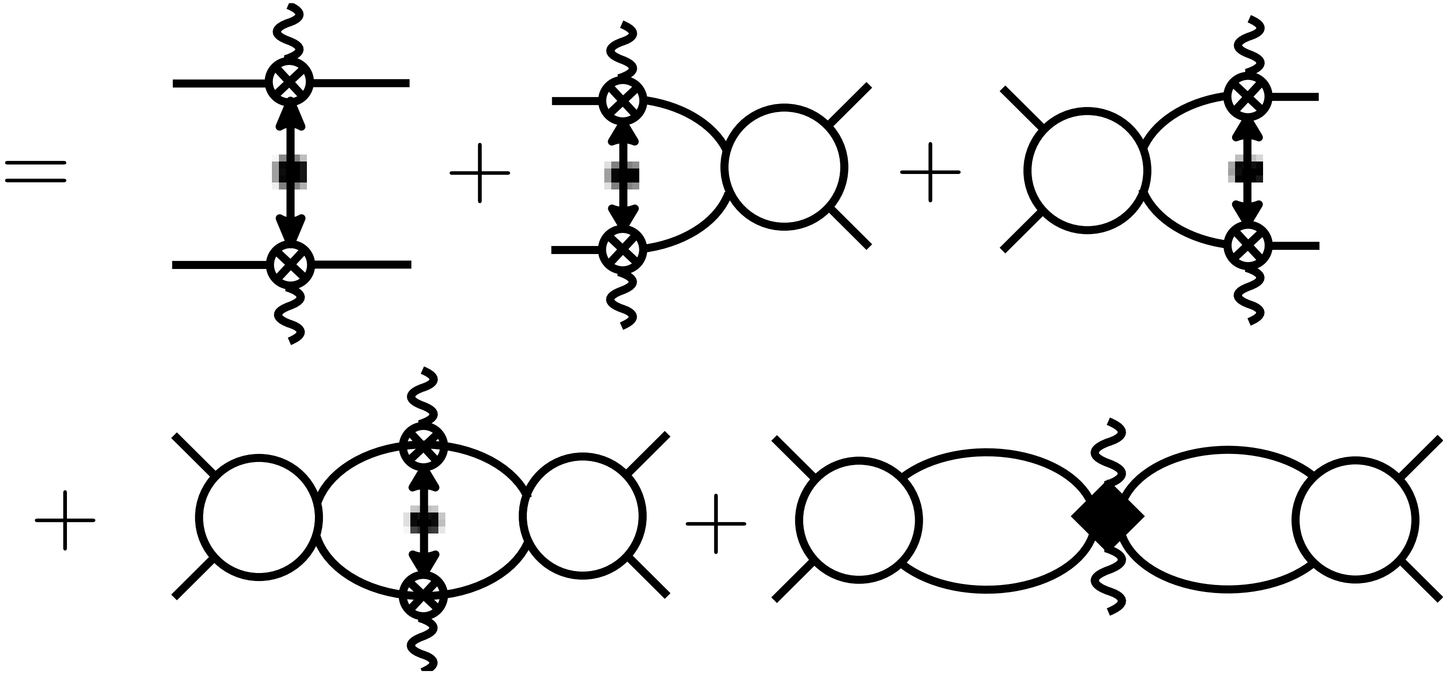
Infinite Volume

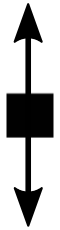
At LO in Pionless EFT

Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, Piarulli, Van Kolck and Wiringa Phys. Rev. C 100, 055504 (2019)

Cirigliano, Dekens, de Vries, Graesses, Mereghetti, Pastore and van Klock Phys. Rev. Lett. 120, 202001

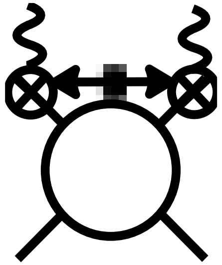
$$i\mathcal{M}_{0\nu} =$$





$$\sim \frac{m_{\beta\beta}}{|\mathbf{q}|^2}$$

Static Neutrino Potential



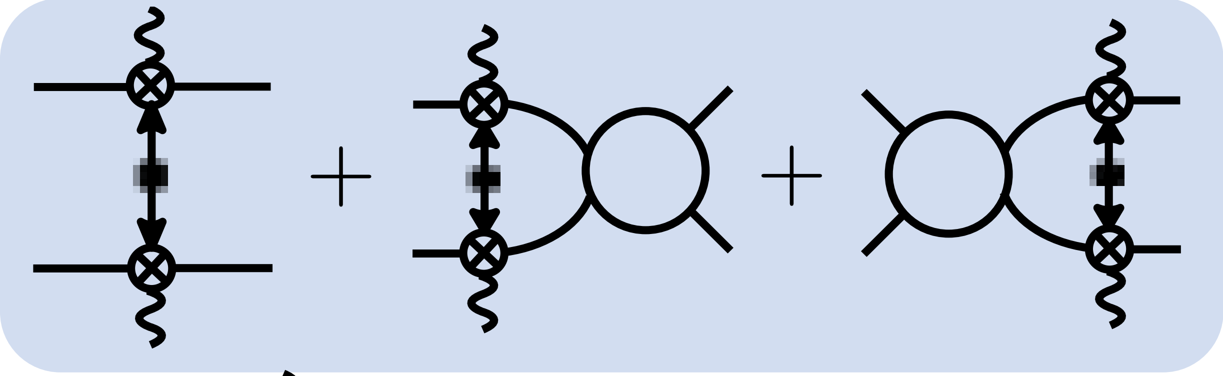
Radiative Neutrinos
Higher Order

At LO in Pionless EFT

Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, Piarulli, Van Kolck and Wiringa Phys. Rev. C 100, 055504 (2019)

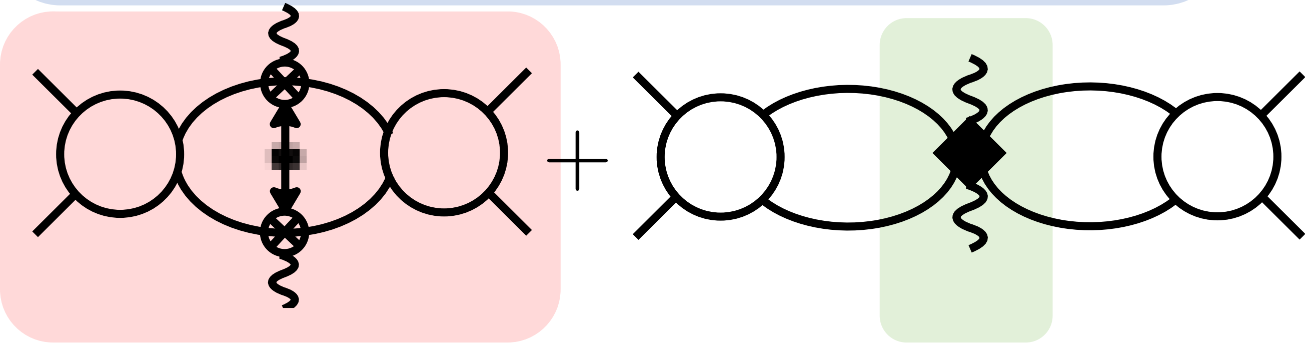
Cirigliano, Dekens, de Vries, Graesses, Mereghetti, Pastore and van Klock Phys. Rev. Lett. 120, 202001

$i\mathcal{M}_{0\nu} =$



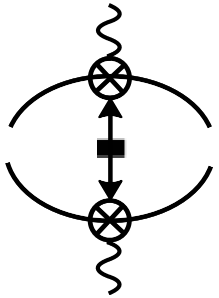
Won't contribute to FV correlation function

Divergent $+$

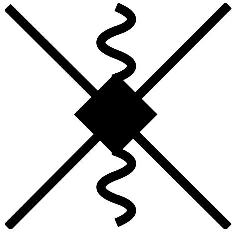


Scale dependent LEC

$I_{0\nu}^{\infty}(E_i, E_f) \sim$



New LEC →



$\sim g_\nu^{NN}$

$$\mu \frac{d}{d\mu}\left(\frac{g_\nu^{NN}}{C_0^2}\right) = \frac{1+3g_A^2}{2}\,\frac{M_N^2}{16\pi^2}$$

TABLE I. Lower bounds achievable for $m_{\beta\beta}$ by some $0\nu\beta\beta$ experiments, depending on their reached sensitivities (upper group) or sensitivity goals (lower group). The different results correspond to the different quenching of g_A, according to the definitions in Eq. (9). The 1σ uncertainties on $m_{\beta\beta}$ are calculated by assuming uncertainties both on the matrix elements and phase space factors, according to [1] and [8], respectively.

Experiment	Isotope	$t^{1/2}$(90% C.L.)(10^{25} yr)	Lower bound for $m_{\beta\beta}$(eV)		
			g_{nucleon}	g_{quark}	$g_{\text{phen.}}$
IGEX [9]	^{76}Ge	1.57	0.31 ± 0.03	0.49 ± 0.05	1.44 ± 0.16
HEIDELBERG-MOSCOW [10]	^{76}Ge	1.9	0.28 ± 0.03	0.44 ± 0.05	1.31 ± 0.14
GERDA-I [11]	^{76}Ge	2.1	0.26 ± 0.03	0.42 ± 0.05	1.25 ± 0.14
KamLAND-Zen-I [12]	^{136}Xe	1.9	0.18 ± 0.02	0.29 ± 0.03	1.06 ± 0.12
KamLAND-Zen-II [13]	^{136}Xe	1.3	0.22 ± 0.02	0.35 ± 0.04	1.28 ± 0.14
EXO-200 [14]	^{136}Xe	1.1	0.24 ± 0.03	0.38 ± 0.04	1.39 ± 0.15
Combined Ge [11]	^{76}Ge	3.0	0.22 ± 0.02	0.35 ± 0.04	1.05 ± 0.11
Combined Xe	^{136}Xe	2.6	0.15 ± 0.02	0.25 ± 0.03	0.91 ± 0.10
Combined Ge + Xe	^{76}Ge/^{136}Xe		0.15 ± 0.01	0.24 ± 0.02	0.81 ± 0.07
CUORE [15]	^{130}Te	9.5	0.07 ± 0.01	0.11 ± 0.01	0.39 ± 0.04
GERDA-II [16]	^{76}Ge	15	0.10 ± 0.01	0.16 ± 0.02	0.47 ± 0.05
SuperNEMO [17]	^{82}Se	10	0.07 ± 0.01	0.12 ± 0.01	0.36 ± 0.04

TABLE II. Sensitivity and exposure necessary to discriminate between $\mathcal{NH}$ and $\mathcal{IH}$: the goal is $m_{\beta\beta} = 8$ meV. The two cases refer to the unquenched value of $g_A = g_{\text{nucleon}}$ (mega) and $g_A = g_{\text{phen.}}$ (ultimate). The calculations are performed assuming *zero background* experiments with 100% detection efficiency and no fiducial volume cuts. The last column shows the maximum value of the product $B \cdot \Delta$ in order to actually comply with the zero background condition.

Experiment	Isotope	$t^{1/2}$(yr)	Exposure (estimate) $M \cdot T$ (ton · yr)	$B \cdot \Delta_{(\text{zero bkg})}$(counts/kg/yr)
Mega Te	^{130}Te	6.8×10^{27}	2.1	4.7×10^{-4}
Mega Ge	^{76}Ge	2.3×10^{28}	4.1	2.4×10^{-4}
Mega Xe	^{136}Xe	9.7×10^{27}	3.2	3.2×10^{-4}
Ultimate Te	^{130}Te	2.3×10^{29}	71	1.4×10^{-5}
Ultimate Ge	^{76}Ge	5.1×10^{29}	93	1.1×10^{-5}
Ultimate Xe	^{136}Xe	3.3×10^{29}	109	9.2×10^{-6}

Table 1. Limits on neutrinoless DBDs $T_{1/2}^{0\nu-\mathrm{exp}}$ (claim for evidence is denoted in [42]). $Q_{\beta\beta}$: Q-value for the $0^+ \to 0^+$ ground-state transition. $G^{0\nu}$: kinematical (phase space volume) factor ($g_A = 1.25$ and $R = 1.2$ fm $A^{1/3}$). $\langle m_\nu \rangle$: the upper limit on the effective Majorana neutrino mass, deduced from $T_{1/2}^{0\nu-\mathrm{exp}}$ by assuming the ISM [236] ($g_A^{\mathrm{eff}} = 1.25$, UCOM src), the EDF [131] ($g_A^{\mathrm{eff}} = 1.25$, UCOM src), the (R)QRPA ($1.00 \leqslant g_A^{\mathrm{eff}} \leqslant 1.25$, the modern self-consistent treatment of src), and the IBM-2 [130] ($1.00 \leqslant g_A^{\mathrm{eff}} \leqslant 1.25$, Miller–Spencer src), NMEs (see section 10). src means short-range correlations.

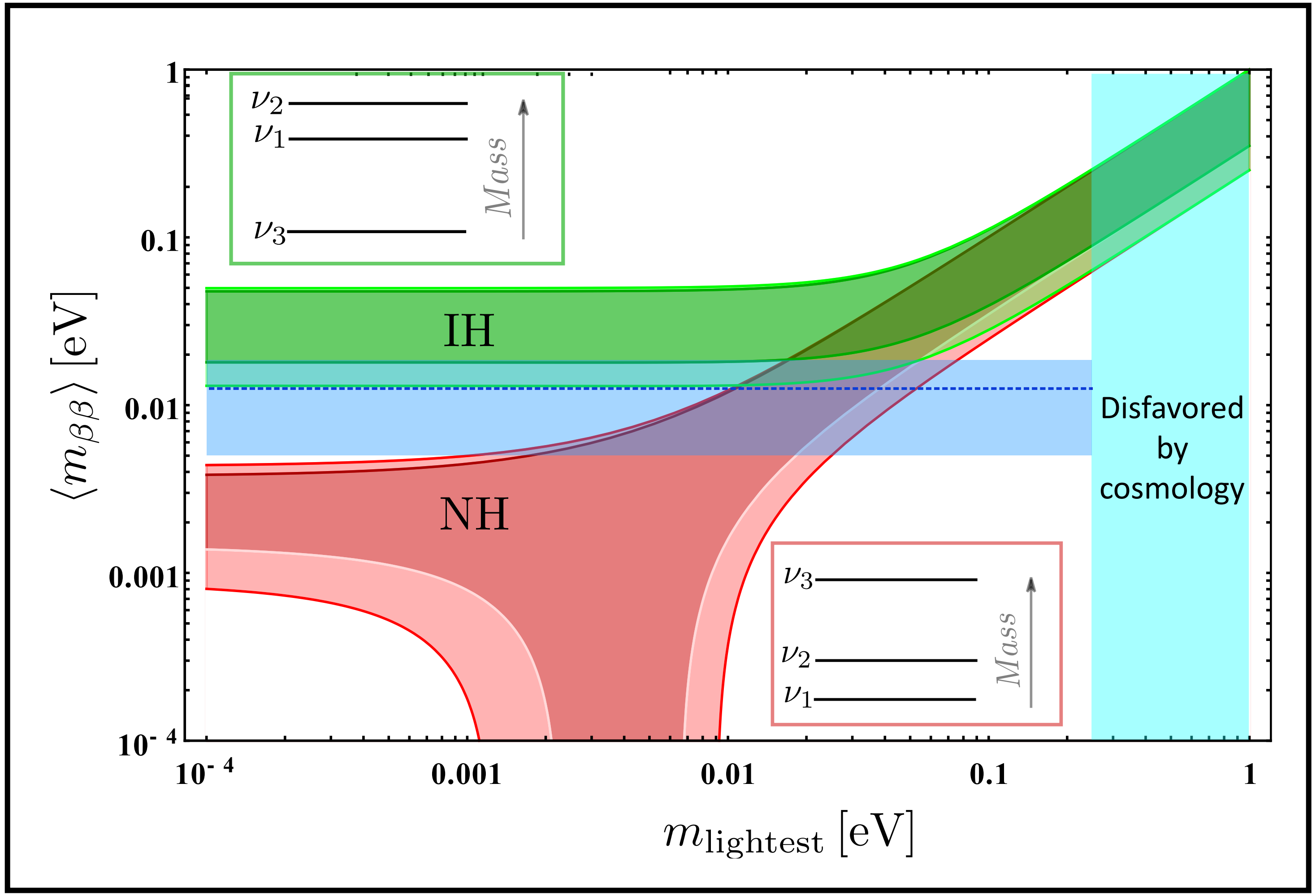
Isotope	A (%)	$Q_{\beta\beta}$ (MeV)	$G^{0\nu}$ (10^{-14} y)	$T_{1/2}^{0\nu-\mathrm{exp}}$ (10^{24} y)	NME	$\lvert\langle m_\nu\rangle\rvert$ eV (eV)	Future experiments
^{48}Ca	0.19	4.276	7.15	0.014 [237]	ISM	19.1	CANDLES
					EDF	7.0	
^{76}Ge	7.8	2.039	0.71	19 [36, 227, 228]	ISM, EDF	0.51, 0.31	GERDA
					(R)QRPA	(0.20, 0.32)	
					EDF	(0.26, 0.35)	
	7.8	2.039	0.71	22 [42]	ISM, EDF	0.47, 0.29	—
					(R)QRPA	(0.18, 0.30)	
					EDF	(0.24, 0.32)	
	7.8	2.039	0.71	16 [229, 230]	ISM, EDF	0.55, 0.34	MAJORANA
					(R)QRPA	(0.22, 0.35)	
					EDF	(0.28, 0.38)	
^{82}Se	9.2	2.992	3.11	0.36 [38, 234, 235]	ISM, EDF	1.88, 1.17	SuperNEMO
					(R)QRPA	(0.76, 1.28)	MOON
					EDF	(1.12, 1.49)	
^{100}Mo	9.6	3.034	5.03	1.0 [38, 234]	EDF	0.46	MOON
					(R)QRPA	(0.38, 0.73)	AMoRE
					EDF	(0.62, 1.06)	
^{116}Cd	7.5	2.804	5.44	0.17 [238]	EDF	1.15	COBRA
					(R)QRPA	(1.20, 2.16)	$CdWO_4$
^{130}Te	34.5	2.529	4.89	3.0 [231, 232, 239]	ISM, EDF	0.52, 0.27	CUORE
					(R)QRPA	(0.25, 0.43)	
					EDF	(0.33, 0.46)	
^{136}Xe	8.9	2.467	5.13	5.7 [40]	ISM, EDF	0.44, 0.23	EXO, NEXT
					(R)QRPA	(0.17, 0.30)	KamLAND-Zen
^{150}Nd	5.6	3.368	23.2	0.018 [38, 240]	EDF	4.68	SuperNEMO
					(R)QRPA	(2.13, 2.88)	SNO+ DCBA

Including Uncertainties in NMEs for $\langle m_{\beta\beta} \rangle = 0.05$ eV

Avignone, Elliott and Engel
Reviews of Modern Physics, 80 (2008)

Dell'Oro, Marcocci, Viel and Vissani
Advances in High Energy Physics (2016)

Vergados, Ejiri and Simkovic,
Rep. Prog. Phys. 75 106301 (2012)



Need to reduce uncertainties in $0\nu\beta\beta$ decay NMEs !!!