Fewbody hypernuclei





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- Motivation
- Chiral YN interactions and estimates of 3BF contributions
- SRG evolution of (hyper-)nuclear interactions
- Impact of an increased $\overline{E_{\Lambda}(^3_{\Lambda}H)}$ on hypernuclear binding
- Light S = -2 hypernuclei and Ξ hypernuclei
- Determination of CSB contact interactions and Λn scattering length
- Conclusions & Outlook

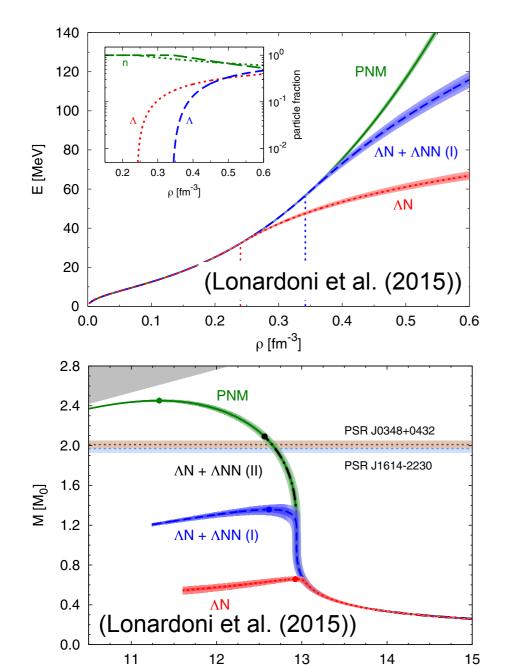
in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner

Hypernuclear interactions

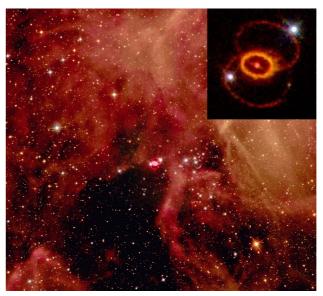


Why is understanding hypernuclear interactions interesting?

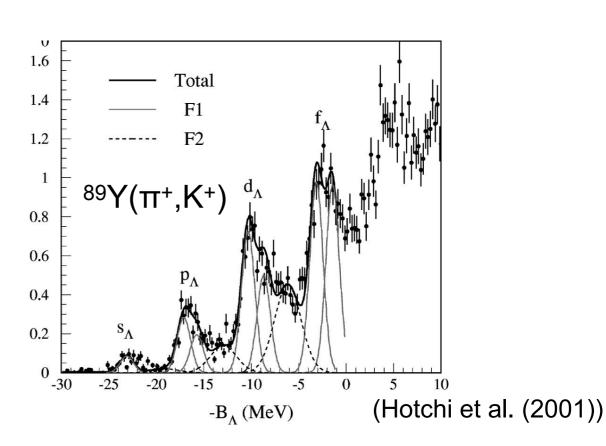
- "phenomenologically"
 - hyperon contribution to the EOS, neutron stars, supernovae
 - A as probe to nuclear structure



R [km]



(SN1987a, Wikipedia)



Hypernuclear interactions

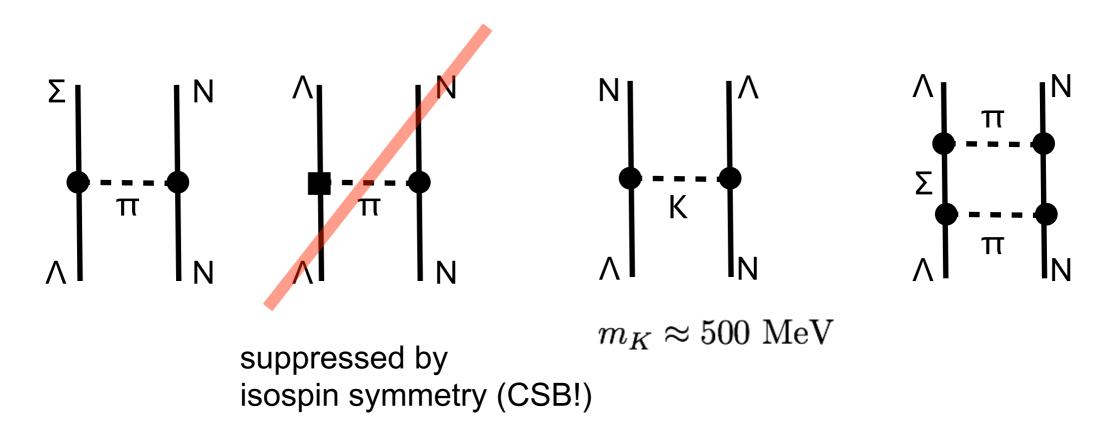


Why is understanding hypernuclear interactions interesting?

Hypernuclear interactions have interesting properties

For example

- Particle conversion process is sometimes long-range part of the interaction
- experimental access to explicit chiral symmetry breaking



Chiral NN & YN & YY interactions



(adapted from Epelbaum, 2008)

	BB force	3B force	4B force	
LO	X ++	—		5 (+1) NN/YN (YY) short range parameters
NLO	XXXX			23(+5) NN/YN (YY) short range parameters
N ² LO	 <	 - - - - 		

Additional constraints required (e.g. for YN only 35 data, but 23 parameters at NLO) data too scarce to uniquely determine the short range LECs!

Two realization for the YN interaction at NLO: NLO13 & NLO19 with different assumptions on the LECs (J. Haidenbauer et al., 2013 & 2019)

YY interaction at NLO (J. Haidenbauer et al., 2016 & 2019)

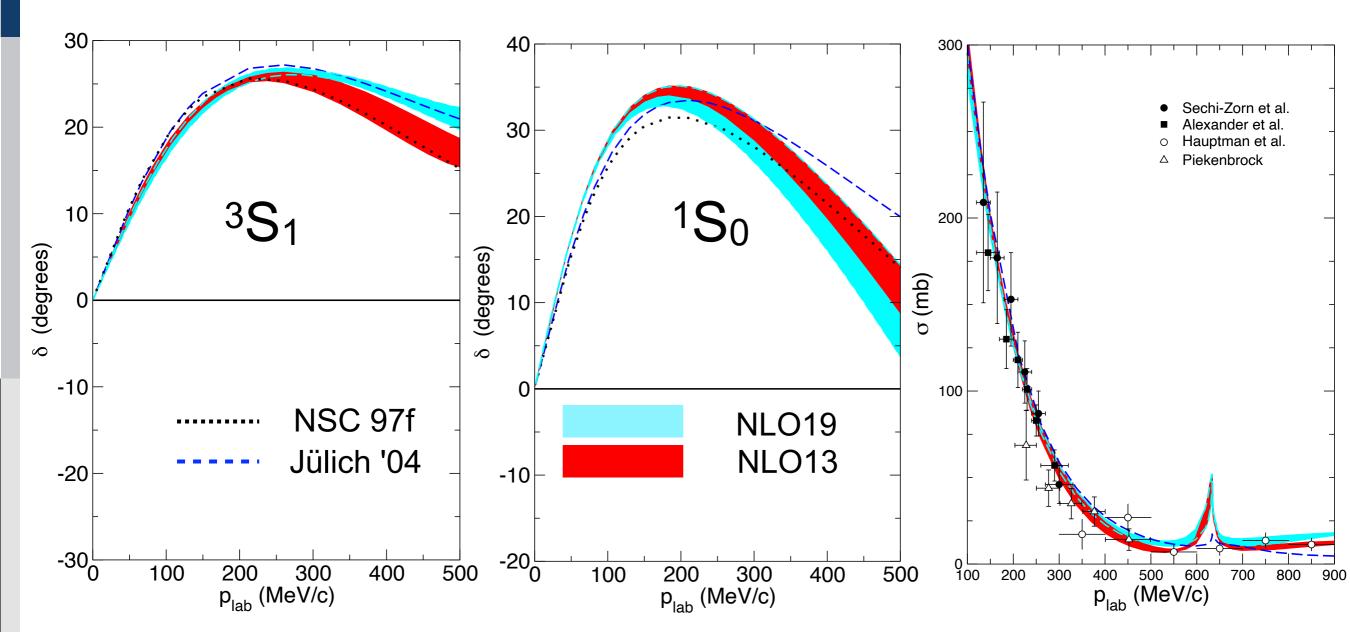
Chiral interactions include symmetries of QCD & retain flexibility to adjust to data Regulator required — cutoff is also used to estimate uncertainty

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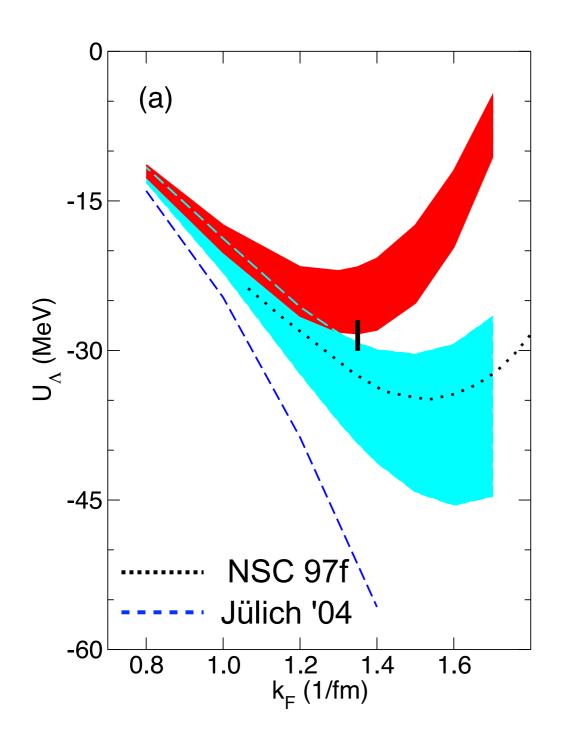
- Regularization required
 - Dependence on cutoff indicates uncertainty
- NLO13 and NLO19 interactions largely phase shift equivalent
 - differences indicate size of three-baryon interactions

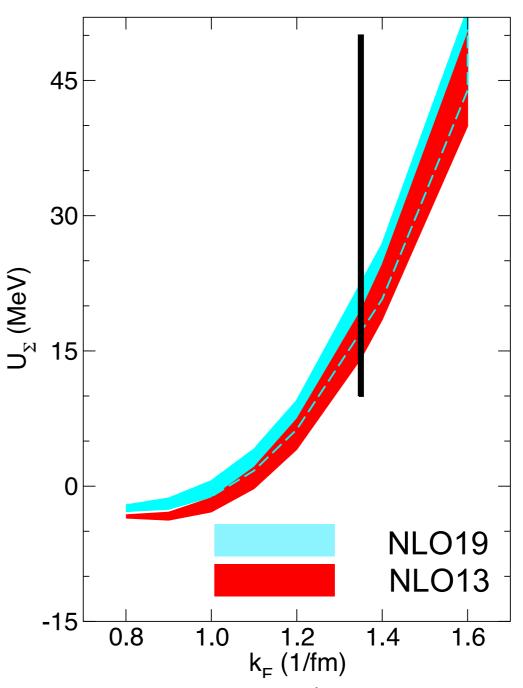


(Haidenbauer at al., 2019)

3BF contribution in nuclear matter?





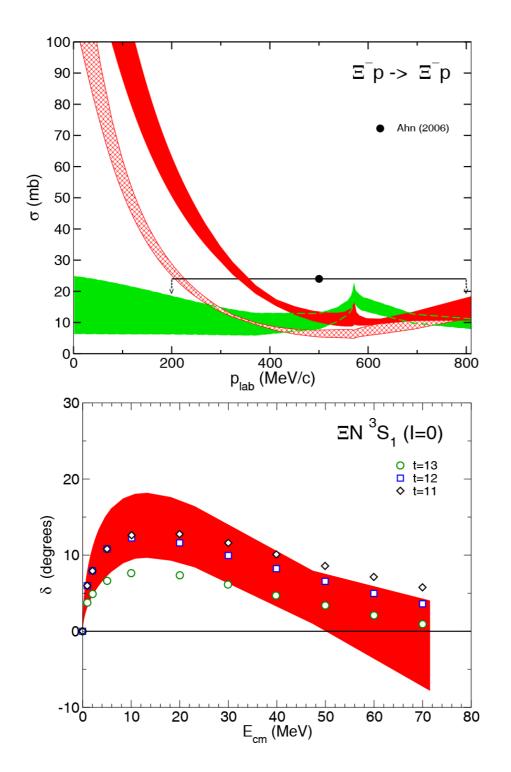


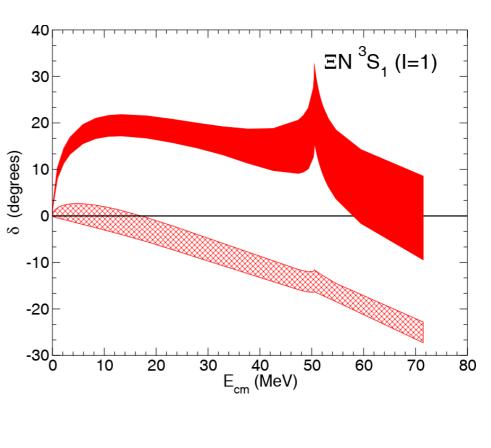
(Haidenbauer at al., 2020)

Indication that 3BF contribution is significant at saturation density Probably less important for light hypernuclei

YY interaction









(Haidenbauer at al., 2019)



adjusted to data & LQCD (HAL QCD) updated version consistent with Ξ -nuclei (only change in ΞN 3S_1)

Tools



Need reliable predictions for hypernuclei to further constrain interactions

Faddeev-Yakubovsky (FY) equations for A=3 and 4 (momentum space)

- long distance tails of wave functions can be well represented
- uses Jacobi coordinates separating off CM motion
- chiral interactions can be directly used
- hugh linear eigenvalue problem (dimension 109x109) even for A=4 systems
- is feasible only for A ≤ 4

(see AN, Glöckle, Kamada, 2002))

Jacobi-no core shell model (J-NCSM) for $A \ge 4$ (HO space)

- smaller dimensions allow to tackle p-shell nuclei
- exact antisymmetrization of wave functions can be prepared
- uses Jacobi coordinates separating off CM motion
- chiral interactions require similarity renormalization group (SRG) evolution
- long distance wave functions require large HO model spaces

(see Liebig et al., 2016; Le et al., 2020 & 2021)

Faddeev-Yakubovsky



Decompose wave function into five Yakubovsky components

$$\Psi = (1+P)(\psi_{1A} + \psi_{1B} + \psi_{2A} + \psi_{2B}) + (1-P_{12})(1+P)\psi_{1C}$$

and solve set of Yakubovsky equations

$$\psi_{1A} = G_0 t_{12} P(\psi_{1A} + \psi_{1B} + \psi_{2A}) + (1 + G_0 t_{12}) G_0 V_{123}^{(3)} \Psi$$

$$\psi_{1B} = G_0 t_{12} ((1 - P_{12})(1 - P_{23}) \psi_{1B} + P \psi_{2B})$$

$$\psi_{1C} = G_0 t_{14} (\psi_{1A} + \psi_{1B} + \psi_{2A} - P_{12} \psi_{1C} + P_{12} P_{23} \psi_{1C} + P_{13} P_{23} \psi_{2B})$$

$$\psi_{2A} = G_0 t_{12} ((P_{12} - 1) P_{13} \psi_{1C} + \psi_{2B})$$

$$\psi_{2B} = G_0 t_{34} (\psi_{1A} + \psi_{1B} + \psi_{2A})$$

$$(P = P_{12} P_{23} + P_{13} P_{23})$$



- each component expresses in natural kind of Jacobi coordinate
- improved convergence in terms of partial waves
- t-matrices enter the equations

We carefully checked convergence with respect to partial waves, stability with respect to mesh points, ..., accuracies of **20 keV** for binding energies are possible.

(see AN, Glöckle, Kamada (2002))



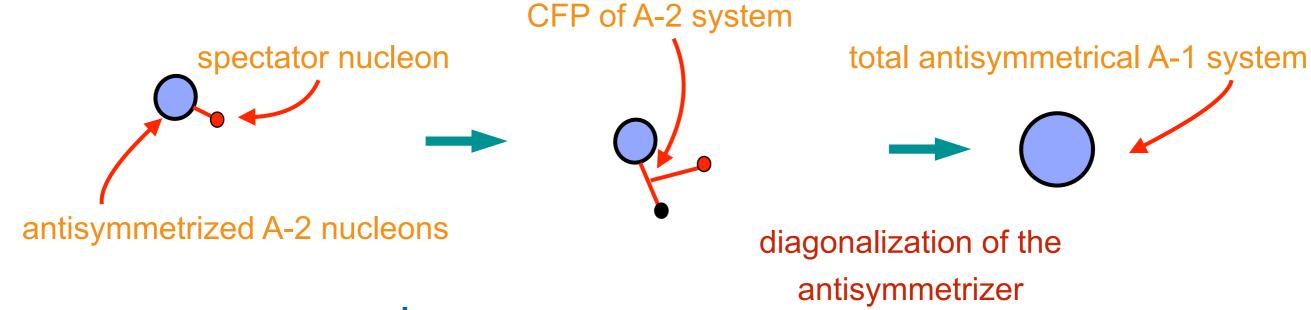
- uses Jacobi coordinates separating off the CM motion
- allows one to go beyond A=4
- efficient for soft interactions
- long distance tails of wave functions cannot be well represented
- requires soft interactions (effective NCSM, vlowk, SRG)

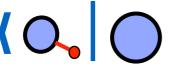
Basic idea: use HO states and soft interactions

- m-scheme uses single particle states (CM not separated)
 antisymmetrization for nucleons easily perform
 larger dimensions (see application to p-shell hypernuclei by Wirth et al. (2014,2016))
- Jacobi-NCSM uses relative coordinates
 antisymmetrization for nucleons difficult but possible for A ≤ 8 (cfp-coefficients)
 small dimensions (see also application to s-shell hypernuclei by Gazda et al. (2014))



First, generate antisymmetrized states for the A-1 nucleon system





The CFP coefficients () are obtained by diagonalization of the antisymmetrizer.

HO states guarantee:

- complete separation of antisymmetrized and other states
- independence of HO length/frequency

CFP coefficients will be openly accessible as **HDF5** data files (download server is in preparation (please ask me when interested!))

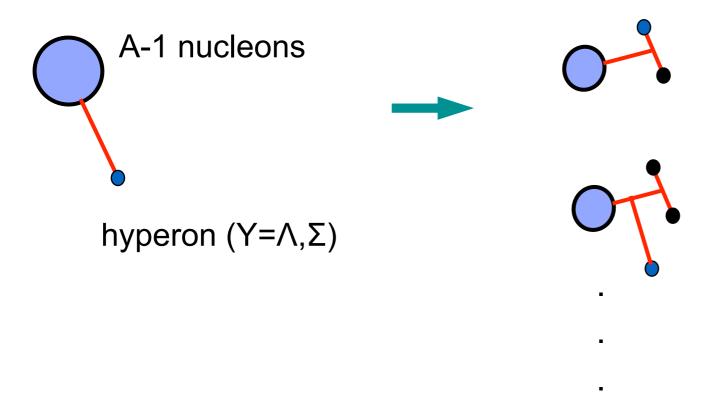
(Liebig, Meißner, AN (2016))

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Second, generate A-body hypernuclei state (no antisymmetrization required)

Third, rearrange baryons for the application of interactions, ...



Again HO states guarantee the independence of HO length/frequency.

Transition coefficients are also accessible as **HDF5** data files to anyone interested.

Leads to converged results for "soft" interactions.

(Liebig et al., 2016) (Le et al., 2020 & 2021)

SRG interactions



Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, ...)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

$$\frac{dH_s}{ds} = \left[\underbrace{\left[T,H(s)\right]},H(s)\right] \qquad H(s) = T+V(s)$$

$$\equiv^{\eta(s)} \text{ this choice of generator drives } \textit{V(s)} \text{ into a diagonal form in momentum space}$$

- V(s) will be phase equivalent to original interaction
- short range V(s) will change towards softer interactions
- Evolution can be restricted to 2-,3-, ... body level (approximation)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ is a measure of the width of the interaction in momentum space (Bogner et al., 2007)
- dependence of results on λ or s is a measure for missing terms

SRG interactions



The evolution naturally separated in 2- and 3-body,... parts.

$$\begin{split} \frac{dV_{ij}(s)}{ds} &= \left[\left[T_{ij}, V_{ij}(s) \right], T_{ij} + V_{ij}(s) \right] & \text{easily done} -- \text{still here the working horse} \\ \frac{dV_{ijk}(s)}{ds} &= \left[\left[T_{ij}, V_{jj}(s) \right], V_{ki}(s) + V_{jk}(s) + V_{ijk}(s) \right] + \left[\left[T_{jk}, V_{jk}(s) \right], V_{ki}(s) + V_{ijk}(s) \right] \\ &+ \left[\left[T_{ki}, V_{ki}(s) \right], V_{ij}(s) + V_{jk}(s) + V_{ijk}(s) \right] & \text{more involved} -- \text{doable} \\ &+ \left[\left[T_{ij} + T_k, V_{ijk}(s) \right], T_{ij} + T_k + V_{ij}(s) + V_{ij}(s) + V_{ki}(s) + V_{ijk}(s) \right] \end{split}$$

Fortunately the generation of 4-body SRG induced interactions seems to be not required

For 3N forces: induced interactions are of similar size a chiral 3N forces

For ΛNN : SRG induced-3BFs are large, probably much larger than chiral ones!

(see also Wirth et al. (2016))

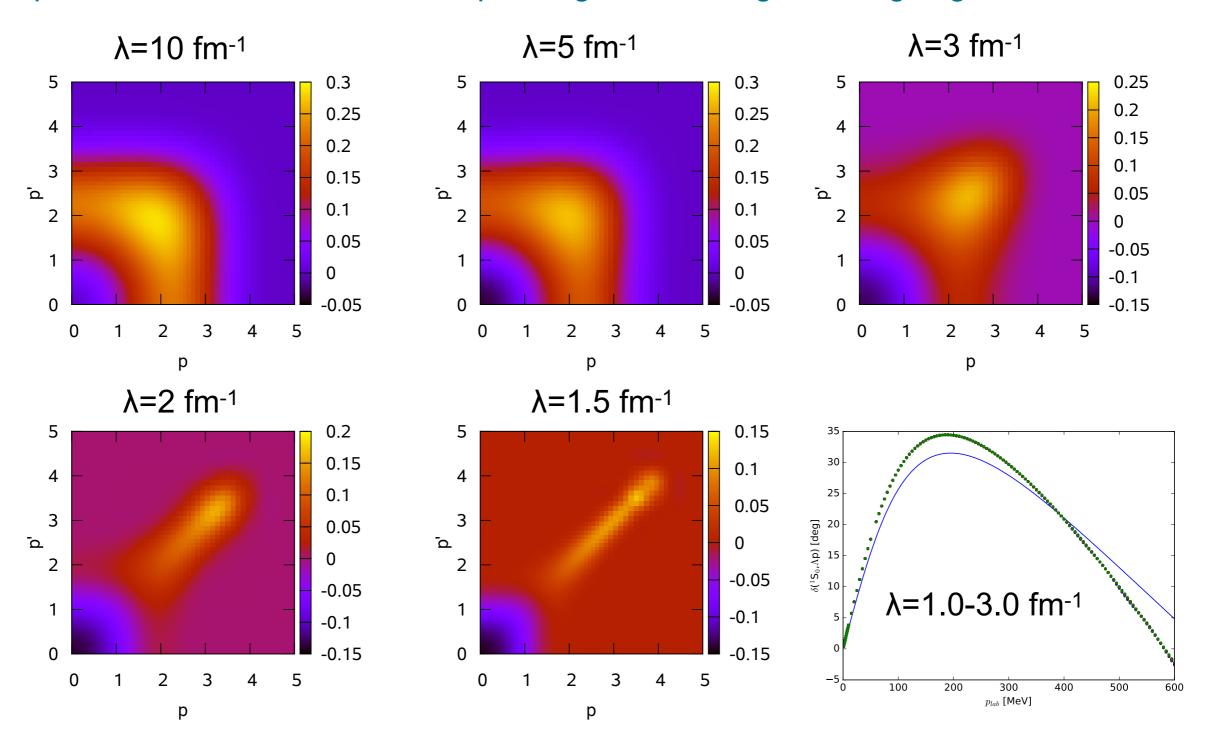
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SRG interactions (YN)



Λp - Λp matrix element for the ${}^{1}S_{0}$ depending on incoming and outgoing momenta



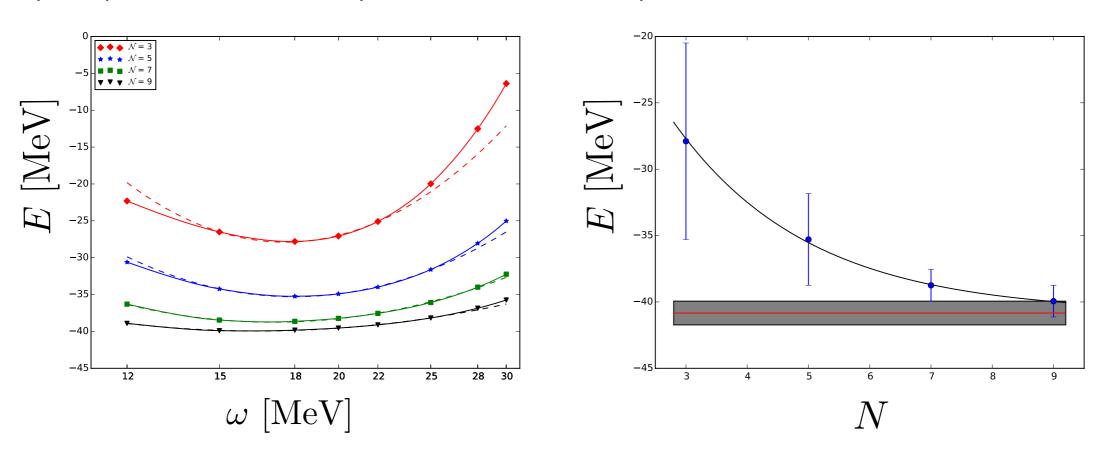
SC97f compared to SRG of EFT-NLO-600

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To get insight into the numerical accuracy, we set up a systematic analysis of the HO frequency dependence and dependence on maximal HO excitation taken into account.

 7 Li (1/2+) for Idaho-N3LO (SRG to $λ = 1.5 \text{ fm}^{-1}$)



$$E_b(\omega) = E_N + \kappa \left(\log(\omega) - \log(\omega_{opt})\right)^2$$
 $\longrightarrow E_N = E_{\infty} + A e^{-bN}$



(including the estimate of numerical accuracy)

(Liebig, Meißner, AN (2016))

A separation energies



- Hypernuclei —— primarily interested in dependence on YN interactions
- NN interactions are well constrained
- description of nuclear binding requires properly adjusted 3NFs
- for hypernuclei often of less importance if separation energies are considered

$$E_{\Lambda} = E(core) - E(hypernucleus)$$

YN: NLO19(650)

in MeV

		0+		1+	
NN interaction	E(3He)	E(⁴∧He)	E∧	E(⁴∧He)	E∧
N ⁴ LO(400)	-7.582	-9.139	1.556	-8.504	0.921
N ⁴ LO(450)	-7.424	-8.966	1.542	-8.340	0.916
N ⁴ LO(500)	-7.299	-8.808	1.509	-8.193	0.894
N ⁴ LO(550)	-7.209	-8.681	1.472	-8.079	0.870

Fortunately, YN interaction are relatively weak:

the nuclear core is only slightly affected by the presence of the hyperon

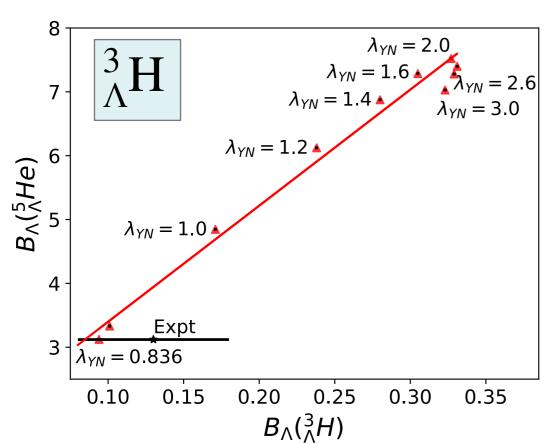
Correlation of separation energies

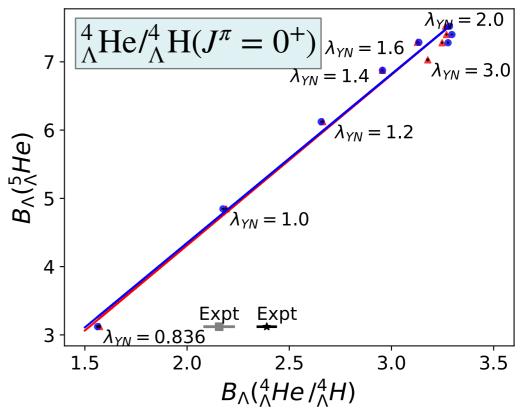


Separation energies of s-shell hypernuclei are strongly correlated (to

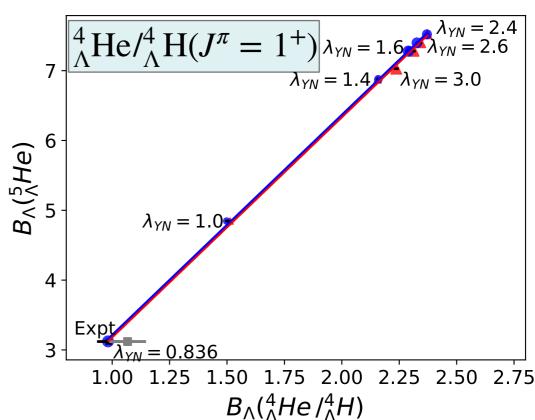


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- YN interaction: NLO19(600)
- strong overbinding for $\lambda \gtrsim 1.0 \text{ fm}^{-1}$
- but A=3 and A=5 consistently predicted for $\lambda \approx 0.836~\mathrm{fm^{-1}}$



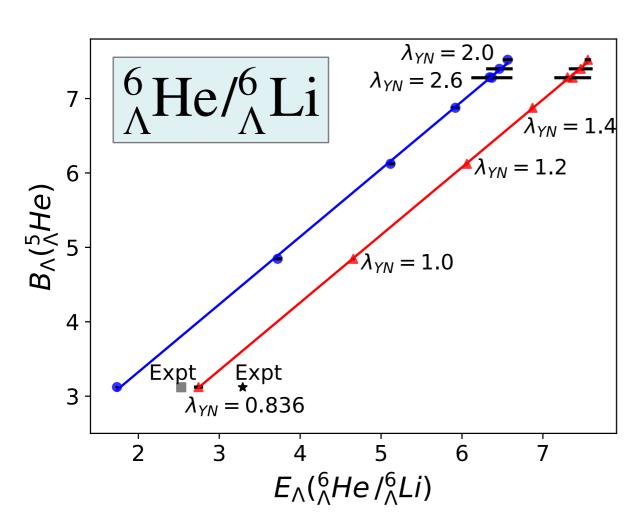
August 18th, 2022 $B_{\Lambda}({}^{4}_{\Lambda}He/{}^{4}_{\Lambda}H)$

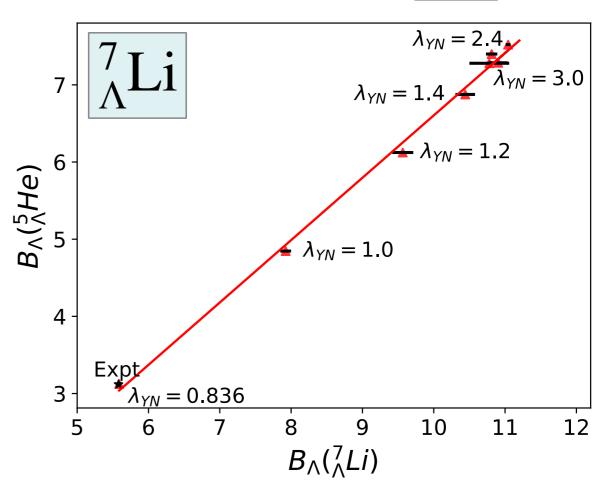
p-shell hypernuclei



Separation energies of p-shell hypernuclei are also correlated (to







- YN interaction: NLO19(600)
- $^{7}_{\Lambda} \text{Li}$ astonishingly well reproduced at "magic" $\lambda \approx 0.836 \text{ fm}^{-1}$

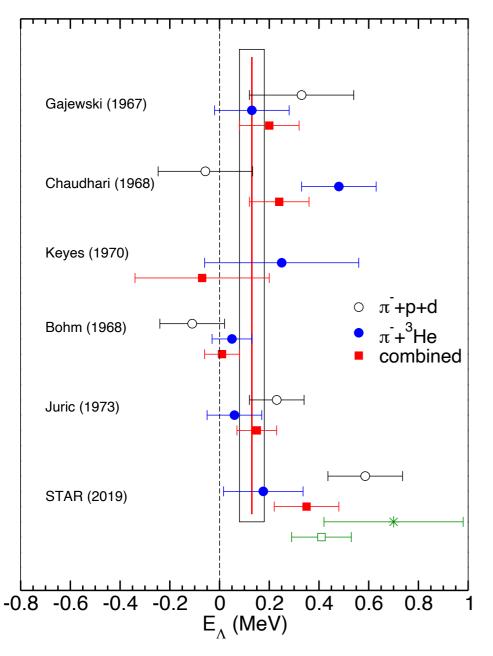
A=6 in our calculations not particle stable

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Increased binding of ${}^3_{\Lambda}H$



- $^3_\Lambda {
 m H}$ is used to determine relative strength of $^3{
 m S}_1/\,^1{
 m S}_0$ interactions or relative size of $a_3^{\Lambda N}$ and $a_1^{\Lambda N}$
- Jurič et al. (1973!)
 new: Adam et al. (2019)



$$E_{\Lambda} \begin{pmatrix} 3 \text{ H} \end{pmatrix} = (130 \pm 50) \text{ keV}$$

 $E_{\Lambda} \begin{pmatrix} 3 \text{ H} \end{pmatrix} = (410 \pm 120) \text{ keV}$

- separation energies of light hypernuclei have often been obtained many years ago from emulsion data
- systematic uncertainties?
- different experiments contradict each other

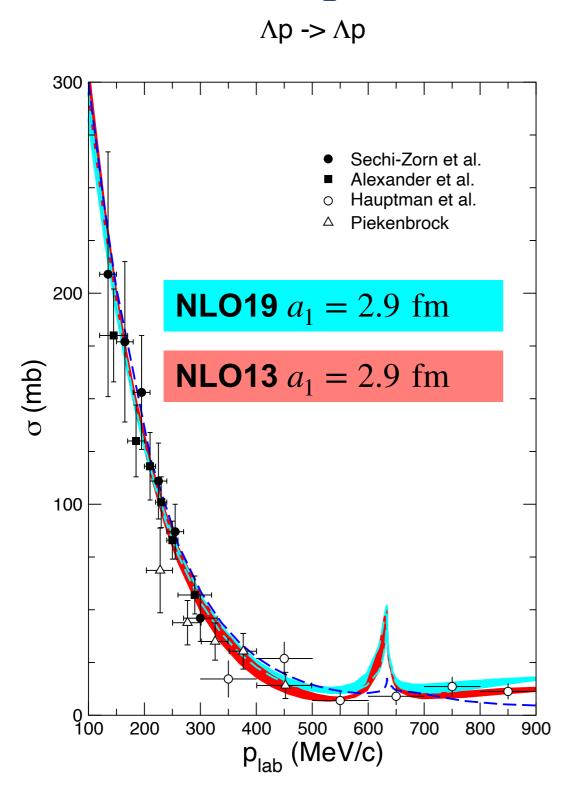
What would be the impact on hypernuclear binding in general if $^3_\Lambda H$ the binding increases?

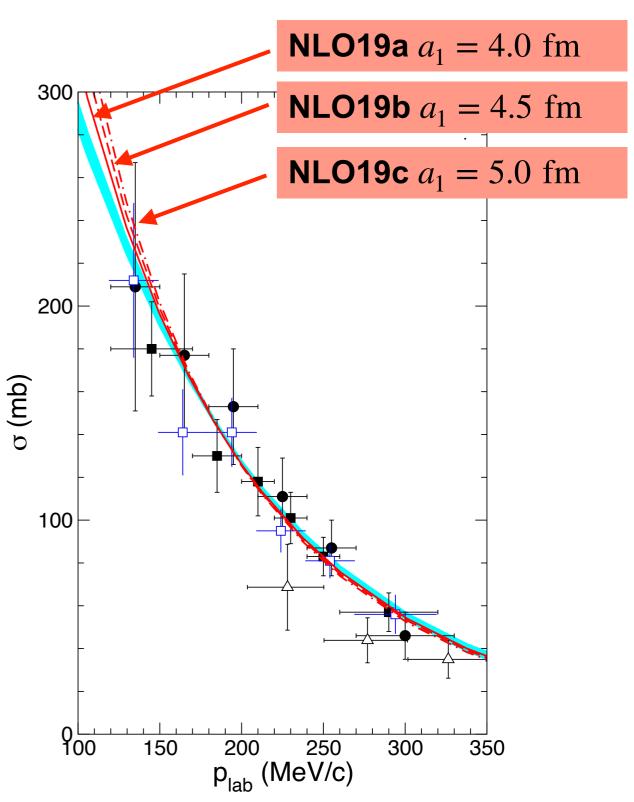
Equivalent to: What happens if $a_1^{\Lambda N}$ increases?

(Le et al., 2020)

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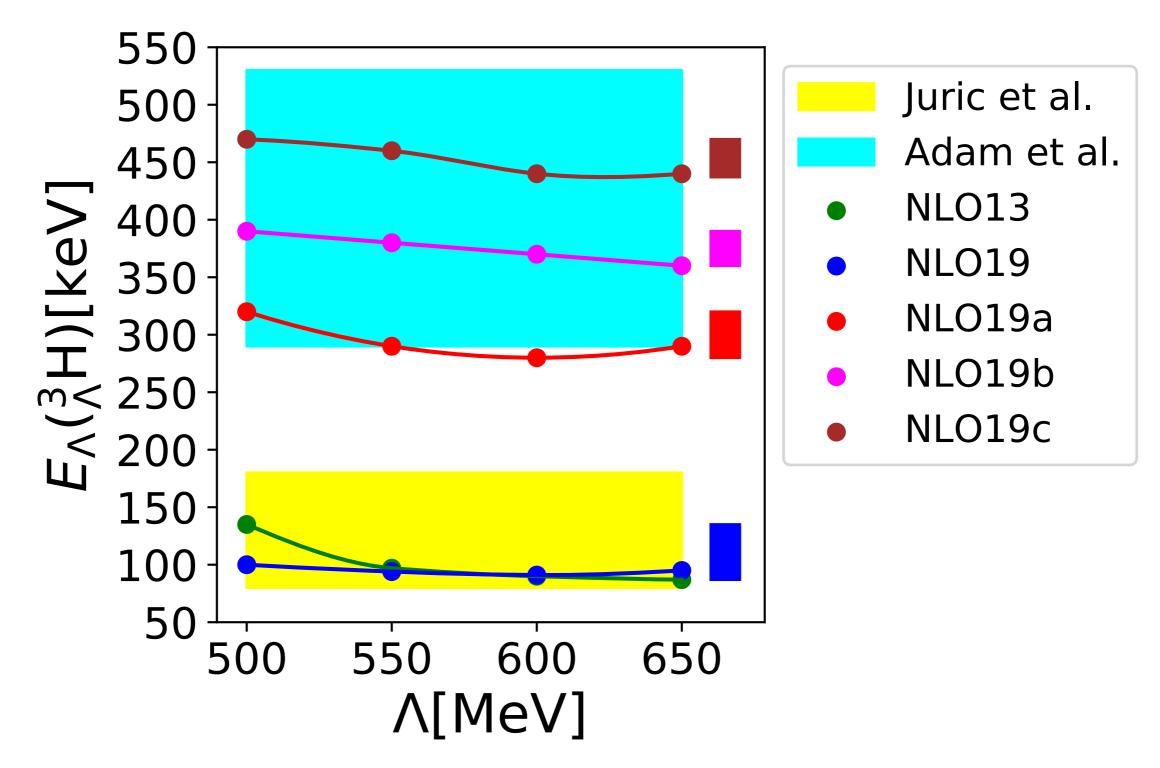




... can be done without changing the nice description of available YN data



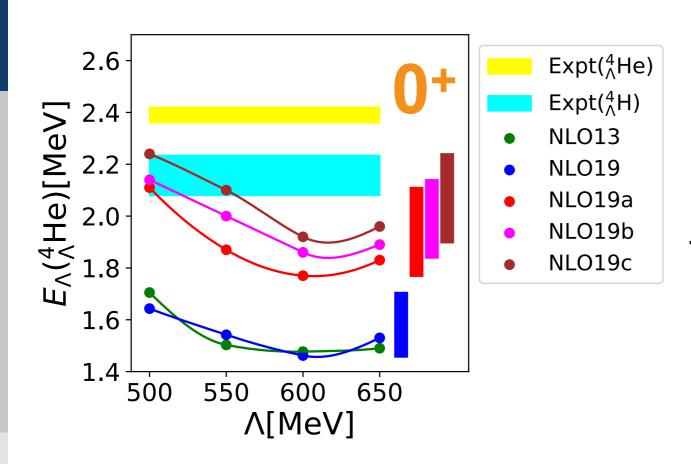


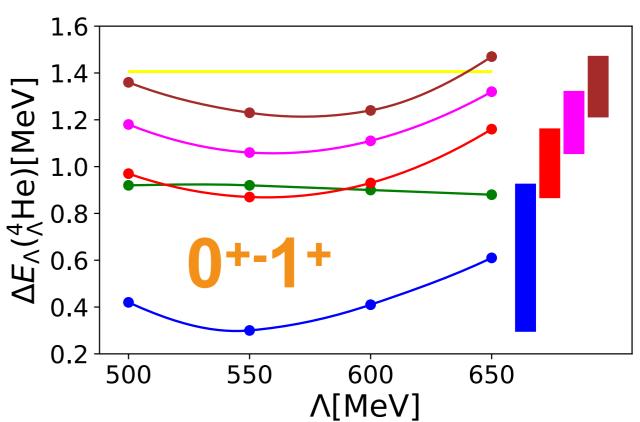


... changes the hypertriton binding energy as expected





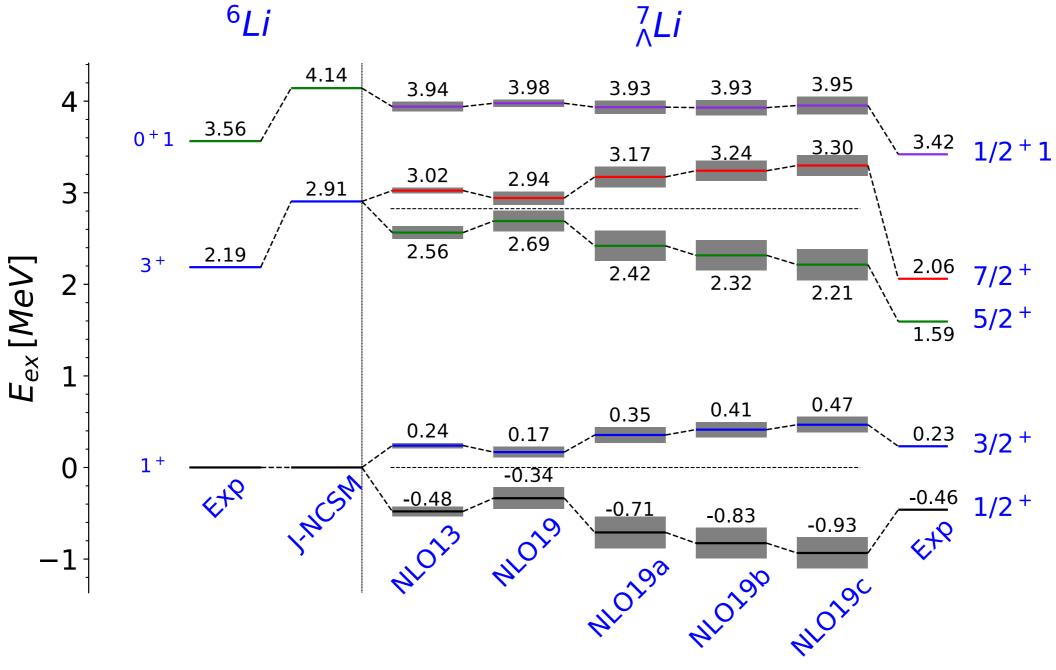




... improves the description of $^4_\Lambda He$







... does not distort the description of $^7_\Lambda Li$



In summary: an increase of the hypertriton binding energy is not excluded by binding energies of other light hypernuclei!

S=-2 hypernuclei — $_{\Lambda\Lambda}^{6}{\rm He}$

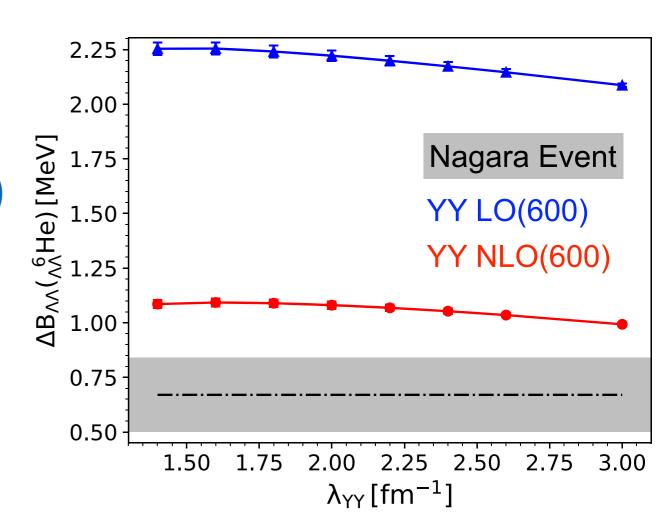


• $\Lambda\Lambda$ excess binding energy

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$$

$$= 2E \begin{pmatrix} A-1 \\ \Lambda \end{pmatrix} - E \begin{pmatrix} A \\ \Lambda \end{pmatrix} - E \begin{pmatrix} A-2 \\ \Lambda \end{pmatrix}$$

- NN, YN and YY interactions contribute
- use NN and YN that describe nuclei and single Λ hypernuclei
- small λ_{YY} dependence
- LO overbinds YY
- NLO predicts binding fairly well



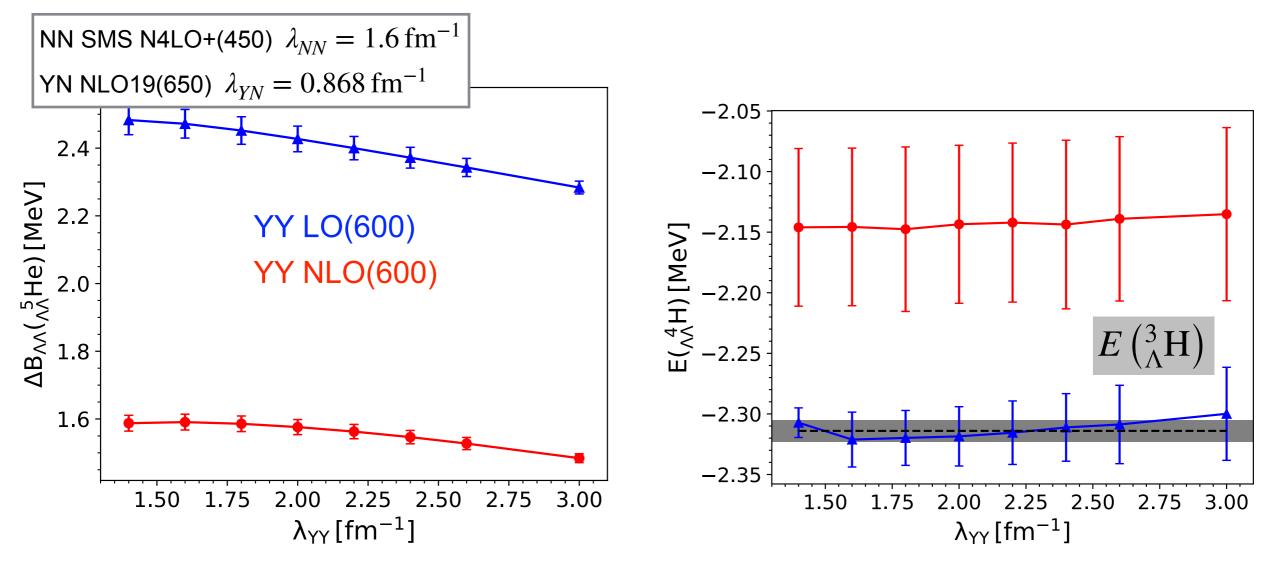
NN SMS N4LO+(450) $\lambda_{NN} = 1.6 \, \mathrm{fm^{-1}}$ YN NLO19(650) $\lambda_{YN} = 0.868 \, \mathrm{fm^{-1}}$

(Le et al., 2021)

Can an S = -2 bound state for A = 4.5 be expected?

S=-2 hypernuclei $-\frac{5}{\Lambda\Lambda}$ He & $\frac{4}{\Lambda\Lambda}$ H





- A = 5: $\Lambda\Lambda$ excess binding energy & A = 4: binding energy
- A = 5: LO & NLO predicts bound state
- A = 4: NLO unbound, LO at threshold to binding (see also Contessi et al., 2019)
- excess energy larger for A=5 than for A=6 (in contrast to Filikhin et al., 2002!)

S = -2 bound state for A = 5 can be expected,

for A = 4 less likely but not ruled out!

E hypernuclei



- experimentally accessible: Ξ^- capture process (experimental data for ${}^{15}_{\Xi}C$ and ${}^{12}_{\Xi}Be$)
- $\Xi N \Lambda\Lambda$ conversion channel open: possibly short life times/difficult calculations
- HAL QCD & chiral YY interactions indicate suppression $\Xi N \Lambda\Lambda$ transition
- ΞN interaction relevant: Ξ is often the second hyperon to appear in neutron matter

Identify possibly interesting states:

calculations based on chiral interactions neglecting $\Xi N - \Lambda\Lambda$ transitions

(keeping $\Xi N - \Lambda \Sigma, \Sigma \Sigma$) states are bound states

finetuning of $^{11}S_0$ interaction to correct for missing $\Lambda\Lambda$ channel neglect YN interaction to avoid transitions to $\Lambda\Lambda$ perturbative width estimates indicate small widths \checkmark

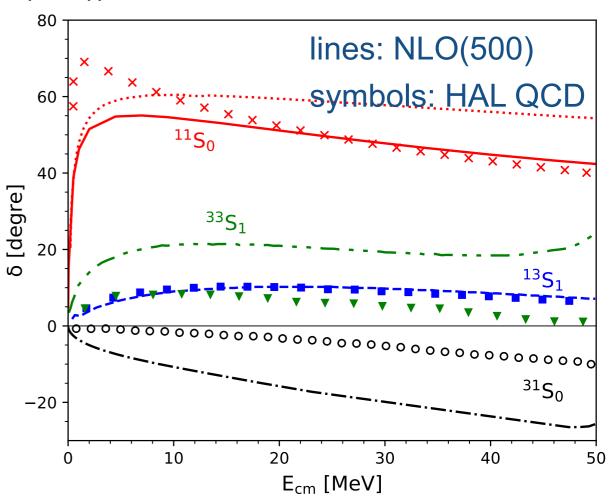
Here: look at ${}^{7}_{\Xi}H$ (exp. expected), ${}^{5}_{\Xi}H$, ${}^{4}_{\Xi}H$ and ${}^{4}_{\Xi}n$ explore possible bound states and their widths

E hypernuclei



Ξ separation energies (NLO(500) and SMS N⁴LO⁺(450))

	B_{Ξ} [MeV]	Γ [MeV]
$_{\Xi}^{4}\mathrm{H}(1^{+},0)$	0.48 ± 0.01	0.74
$_{\Xi}^{4}$ n(0 ⁺ , 1)	0.71 ± 0.08	0.2
$_{\Xi}^{4}$ n(1 ⁺ , 1)	0.64 ± 0.11	0.01
$_{\Xi}^{4}H(0^{+},0)$	_	_
${}_{\Xi}^{5}\mathrm{H}({}_{2}^{1}{}^{+},{}_{2}^{1})$	2.16 ± 0.10	0.19
$_{\Xi}^{7}\mathrm{H}(\frac{1}{2}^{+},\frac{3}{2})$	3.50 ± 0.39	0.2



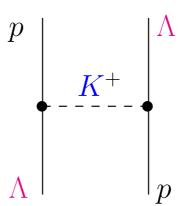
	$V^{S=-2}$			
	$^{11}S_0$	$^{31}S_0$	$^{13}S_{1}$	$^{33}S_{1}$
${}^{4}_{\Xi}\mathrm{H}(1^{+},0)$	- 1.95	0.02	- 0.7	- 2.31
$_{\Xi}^{4}$ n(0 ⁺ , 1)	-0.6	0.25	-0.004	-0.74
$_{\Xi}^{4}$ n(1 ⁺ , 1)	-0.02	0.16	-0.13	-1.14
$_{\Xi}^{4} H(0^{+}, 0)$	-0.002	0.08	-0.01	-0.006
${}_{\Xi}^{5}\mathrm{H}(1/2^{+},1/2)$	-0.96	0.94	-0.58	-3.63
$_{\Xi}^{7} H(1/2^{+}, 3/2)$	- 1.23	1.79	-0.79	-6.74

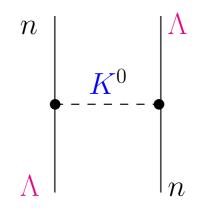
CSB contributions to YN interactions





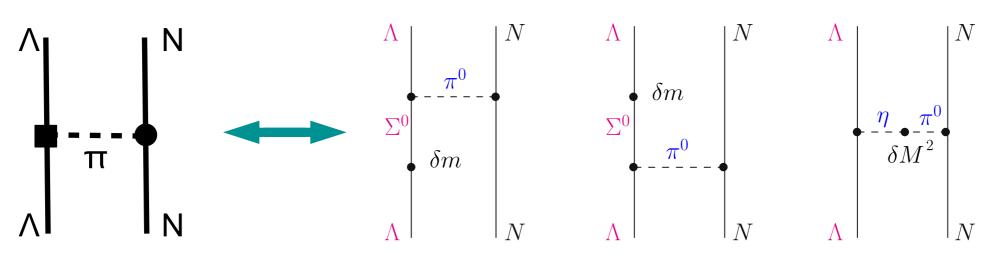
- formally leading contributions:
 Goldstone boson mass difference
 - very small due to the small relative difference of kaon masses





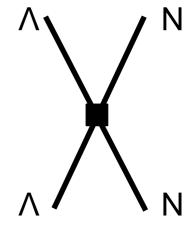
- subleading but most important
 - effective CSB $\Lambda\Lambda\pi$ coupling constant (Dalitz, van Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2 \frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_{\Lambda}} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_{\eta}^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106) f_{\Lambda\Sigma\pi}$$



- so far less considered, but equally important
 - CSB contact interactions (for singlet and triplet)

Aim: use A=4 hypernuclei to determine the two unknown CSB LECs and predict Λn scattering



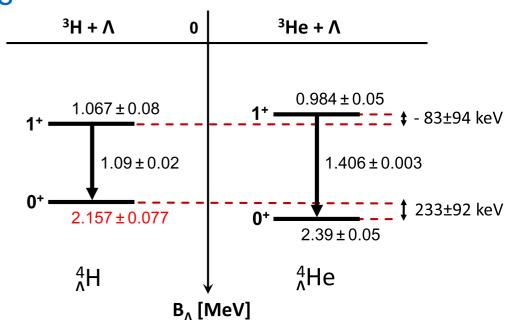
Fit of contact interactions



 Adjust the two CSB contact interactions to one main scenario (CSB1, shown here) and two more for testing (CSB2,CSB3) sensitivities

 Problem: large experimental uncertainty of experiment

- here only fit to central values to test
- theoretical uncertainties



Size of LECs as expected by power counting

(Schulz et al., 2016; Yamamoto, 2015)

$$\frac{m_d - m_u}{m_u + m_d} \left(\frac{M_{\pi}}{\Lambda}\right)^2 C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4 \,\text{GeV} \propto 6 \cdot 10^{-3} \cdot 10^4 \,\text{GeV}$$

Λ	NLO13		NLO19		
	C_s^{CSB}	C_t^{CSB}	C_s^{CSB}	C_t^{CSB}	
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}	
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}	
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}	
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}	

The values of the LECs are in 10^4 GeV⁻²

CSB contributions in $^4_{\Lambda}{\rm He}$

JÜLICH Forschungszentrum

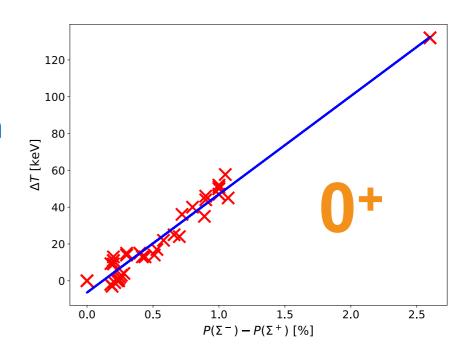
- perturbative calculations of CSB
- breakdown in kinetic energy, YN and NN interaction
- kinetic energy less important for chiral interactions

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interaction	$\langle T \rangle_{\rm CSB}$	$\langle V_{YN} \rangle_{\mathrm{CSB}}$	V_{NN}^{CSB}	$\Delta E_{\Lambda}^{pert}$	ΔE_{Λ}
NLO13(500)	44	200	16	261	265
NLO13(550)	46	191	20	257	261
NLO13(600)	44	187	20	252	256
NLO13(650)	38	189	18	245	249
NLO19(500)	14	224	5	243	249
NLO19(550)	14	226	7	247	252
NLO19(600)	22	204	12	238	243
NLO19(650)	26	207	12	245	250



interaction	$\langle T \rangle_{\rm CSB}$	$\langle V_{YN} \rangle_{\mathrm{CSB}}$	V_{NN}^{CSB}	$\Delta E_{\Lambda}^{pert}$	ΔE_{Λ}
NLO13(500)	5	-90	15	-71	-66
NLO13(550)	5	-86	18	-63	-56
NLO13(600)	4	-83	19	-59	-53
NLO13(650)	3	-80	17	-59	-55
NLO19(500)	1	-84	3	-80	-75
NLO19(550)	2	-81	2	-77	-72
NLO19(600)	4	-82	6	-71	-67
NLO19(650)	4	-79	9	-66	-69



Prediction for Λn scattering



• assuming the current experimental situation for ${}^4_{\Lambda} H / {}^4_{\Lambda} He$

• without CSB: $a_s^{\Lambda n} \approx 2.9 \ fm$ with CSB1: $a_s^{\Lambda n} \approx 3.3 \ fm$

• improved description of Λp data

almost independent of cutoff & NLO variant

CSB of triplet is smaller than of singlet

for "CSB1": currently best experimental values

	$a_s^{\Lambda p}$	$a_t^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda n}$	$\chi^2(\Lambda p)$	$\chi^2(\Sigma N)$	$\chi^2(\text{total})$
NLO13(500)	-2.604	-1.647	-3.267	-1.561	4.47	12.13	16.60
NLO13(550)	-2.586	-1.551	-3.291	-1.469	3.46	12.03	15.49
NLO13(600)	-2.588	-1.573	-3.291	-1.487	3.43	12.38	15.81
NLO13(650)	-2.592	-1.538	-3.271	-1.452	3.70	12.57	16.27
NLO19(500)	-2.649	-1.580	-3.202	-1.467	3.51	14.69	18.20
NLO19(550)	-2.640	-1.524	-3.205	-1.407	3.23	14.19	17.42
NLO19(600)	-2.632	-1.473	-3.227	-1.362	3.45	12.68	16.13
NLO19(650)	-2.620	-1.464	-3.225	-1.365	3.28	12.76	16.04

An accurate prediction for the Λn interaction is possible using hypernuclei!

Fitting to older values 350/240 keV splitting (**CSB2**) results in significantly different scattering length: $a_s^{\Lambda n} < a_s^{\Lambda p} \approx 3.8 \pm 0.2 \, fm$ (refit necessary!)

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Conclusions & Outlook



- YN & YY interactions are interesting
 - EOS and hyperon puzzle
 - · access to explicit chiral symmetry breaking
- YN & YY interactions not well understood
 - conversion processes often drive long range part of interaction
 - scarce YN, almost no YY data (more accurate lattice data soon?)
- Hypernuclei provide important constraints
 - 1S_0 ΛN scattering length & ${}^3_{\Lambda} H$ & impact on other hypernuclei
 - ${}^1S_0 \Lambda\Lambda$ scattering length & ${}^6_{\Lambda\Lambda}$ He & predictions for A=4,5
 - Light Ξ-hypernuclei exist and provide information on the ΞN interaction
 - CSB of ΛN scattering & $^4_{\Lambda}{\rm He}$ / $^4_{\Lambda}{\rm H}$
- J-NCSM
 - reliable predictions are possible for ranges of interactions for S=-1 and -2
- next steps
 - **need SRG induced 3BFs** to validate choice of λ_{YN}
 - estimates of chiral 3BFs are needed (implementing Petschauer et al., (2016))
 - study CSB of p-shell hypernuclei
 - study dependence of S=-2 results on chiral orders and regulators.