



UK Research  
and Innovation

# Finite-volume formalism and the t-channel cut

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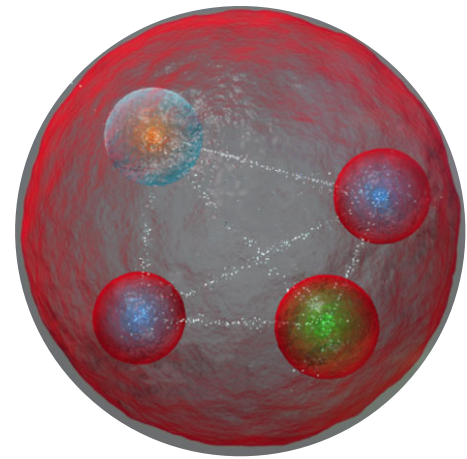
based on work with Andre Raposo



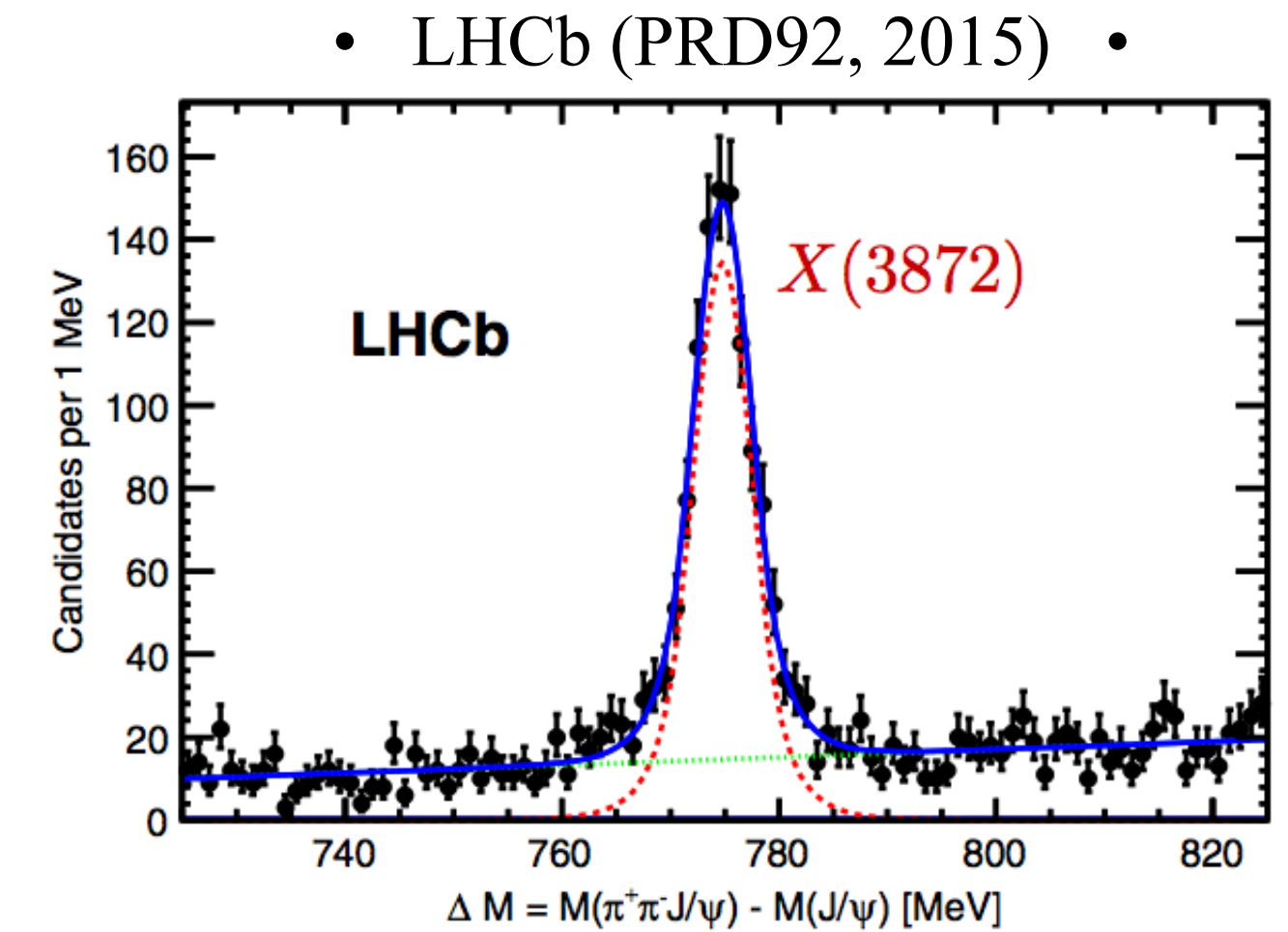
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# Multi-hadron observables

- Exotics, XYZs, tetra- and penta-quarks,  $H$  dibaryon



e.g.  $X(3872)$   
 $\sim |D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0}\rangle?$



- Electroweak, CP violation, resonant enhancement

CP violation in charm  $D \rightarrow \pi\pi, K\bar{K}$

$$\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$$

• LHCb (PRL, 2019) •

$f_0(1710)$  could enhance  $\Delta A_{CP}$   
 • Soni (2017) •

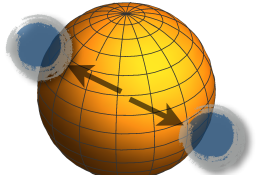
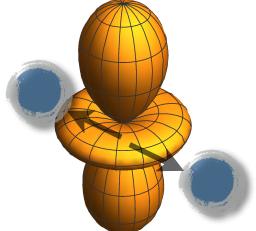
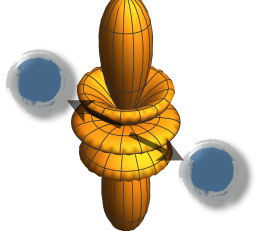
Resonant B decays  $B \rightarrow K^* \ell\ell \rightarrow K\pi \ell\ell$

$|X\rangle, |\rho\rangle, |K^*\rangle, |f_0\rangle \notin \text{QCD Fock space}$

# QCD Fock space

- At low-energies QCD = hadronic degrees of freedom  $\pi \sim \bar{u}d, K \sim \bar{s}u, p \sim uud$
- Overlaps of multi-hadron *asymptotic states*  $\rightarrow$  S matrix

$S(s) \equiv \langle \pi\pi, \text{out} |$

	$ \pi\pi, \text{in}\rangle$		
	$e^{2i\delta_0(s)}$	0	0
	0	$e^{2i\delta_1(s)}$	0
	0	0	$e^{2i\delta_2(s)}$

depends on  $s = E_{\text{cm}}^2$   
and angular variables

diagonal in angular momentum

$$\mathcal{M}_\ell(s) \propto e^{2i\delta_\ell(s)} - 1$$

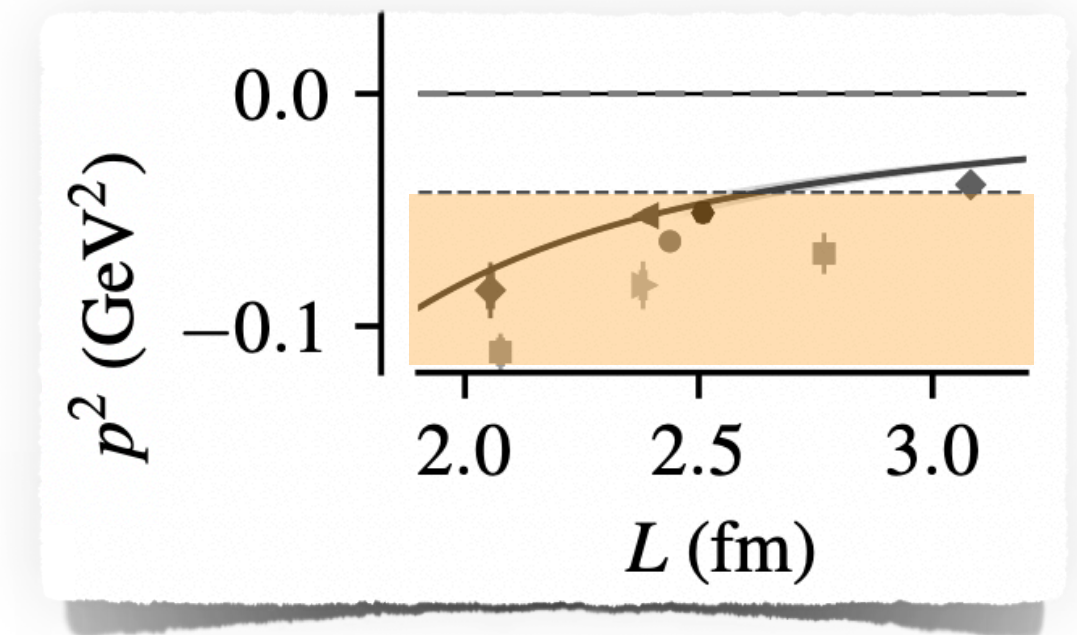
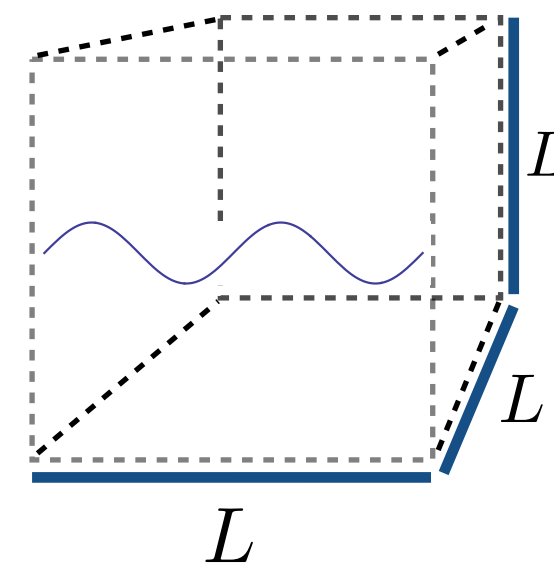
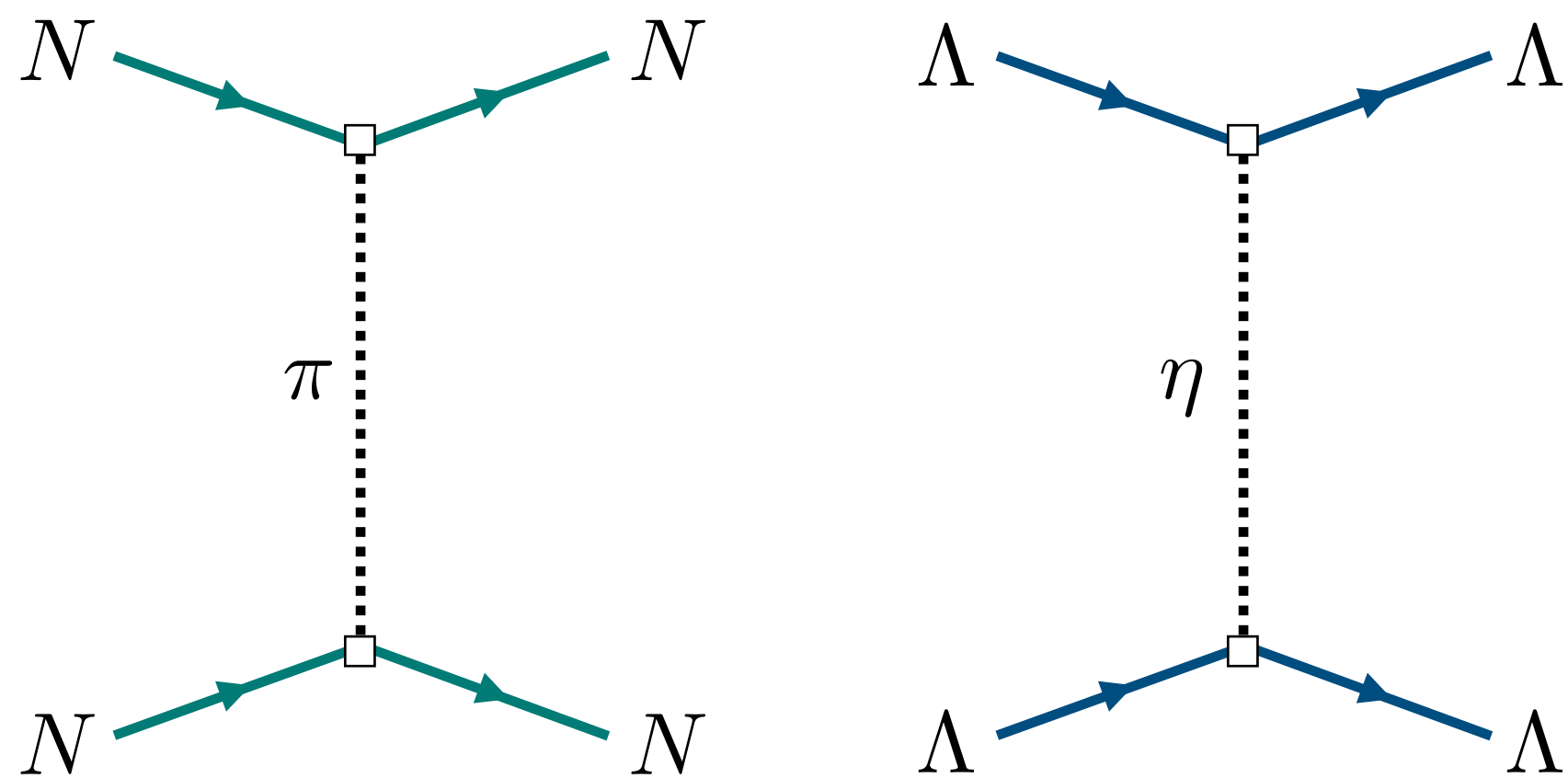
angular momentum also  
plays an important role in  
resonant analysis

- An enormous space of information

$|\pi\pi\pi\pi, \text{in}\rangle \quad |K\bar{K}, \text{in}\rangle \quad \dots$

# Motivation in this work...

- ❑ Lattice QCD has seen significant recent progress in multi-baryon scattering calculations (NPLQCD, CalLatt, Mainz)
- ❑ The extracted amplitudes have sub-threshold t-channel cuts (left-hand cuts) due to light meson exchanges
- ❑ Many calculations are extracting finite-volume energies on these cuts



Weakly bound H dibaryon from SU(3)-flavor-symmetric QCD  
Green, Hanlon, Junnarkar, Wittig, **PRL** (2021)

- ❑ The Lüscher scattering formalism (+extensions) is not applicable for these energies

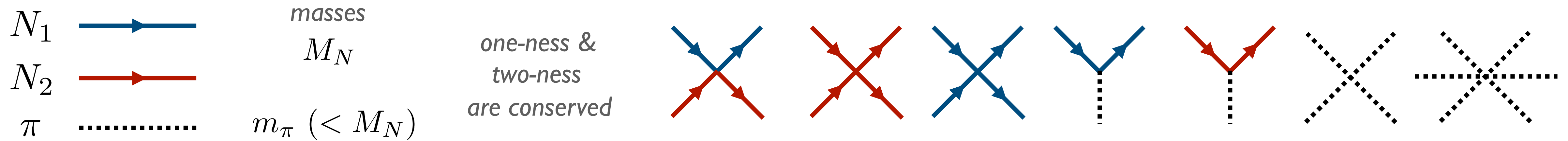
$$p \cot \delta(p)_{(\text{true})} \Big|_{\text{cut}} \in \mathbb{C}$$

$$p \cot \delta(p)_{(\text{f.v. extracted})} \Big|_{\text{cut}} \in \mathbb{R}$$

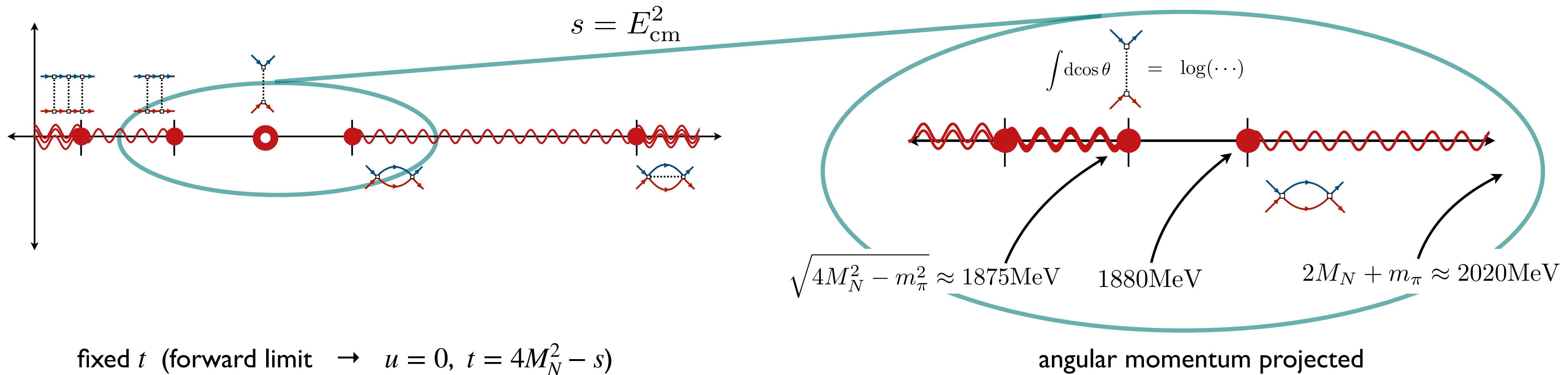
*We aim to provide a modification (extension) that resolves this issue*

# t-channel reminder

□ For concreteness, start with a theory of three scalars...

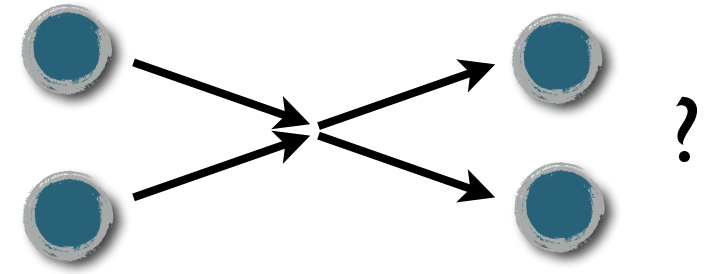


□ Analytic structure of  $\mathcal{M}(s)$ , the  $N_1 N_2 \rightarrow N_1 N_2$  amplitude



# s-channel cut (optical theorem)

□ For two-particle scattering energies, how do we know the analytic structure of  $\mathcal{M}(s) =$



□ The optical theorem tells us...

$$\rho(s)|\mathcal{M}_\ell(s)|^2 = \text{Im } \mathcal{M}_\ell(s)$$

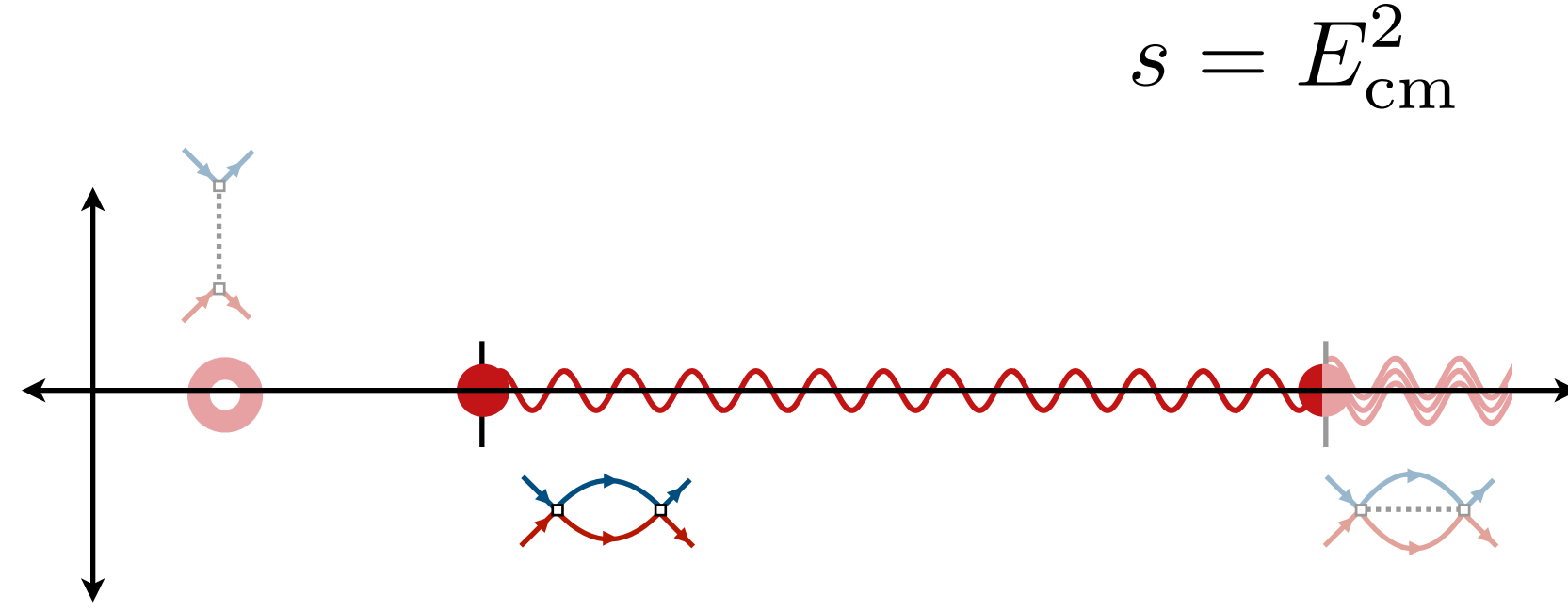
where  $\rho(s) = \frac{\sqrt{1 - 4M_N^2/s}}{32\pi}$  is the two-particle phase space

□ Unique solution is...

$$\mathcal{M}_\ell(s) = \frac{1}{\mathcal{K}_\ell(s)^{-1} - i\rho(s)}$$

K matrix (short distance)

phase-space cut (long distance)



Key message: *s-channel square-root branch cut from the optical theorem*

# s-channel cut (diagrammatic)

$$\mathcal{M}(s) = \text{[diagrams]} \dots$$

$$= \text{[Bethe-Salpeter kernel]} + \text{[chain of kernels]} + \dots$$

The first row shows a series of diagrams representing the s-channel cut, including a box diagram, a vertical chain, a loop, a shaded loop, and two ladder diagrams. The second row shows a series of diagrams representing the full amplitude, starting with a Bethe-Salpeter kernel and followed by chains of kernels connected by propagators, with  $i\epsilon$  labels on the propagators.

$$\mathcal{M}(s) = \text{[2-to-2 scattering diagram]}$$

A diagram showing two incoming particles (blue circles) scattering into two outgoing particles (blue circles) via a central interaction vertex.

propagating hadrons

= fully dressed

Bethe-Salpeter kernel

$$\text{[diagrams]} = \int [\text{real, analytic}]$$

$$\text{for } (2M_N)^2 < s < (2M_N + m_\pi)^2$$

*Framework for generic, EFT independent, all-orders diagrammatic relations*

# s-channel cut (diagrammatic)

$$\mathcal{M}(s) = \text{[diagrams]} \dots$$

$$= \text{[Bethe-Salpeter kernels]} + \dots$$

The diagrammatic equation shows the s-channel cut of the scattering amplitude  $\mathcal{M}(s)$ . The top row shows a series of diagrams representing different orders of perturbation theory, including tree-level contact terms, exchange diagrams, and loop diagrams. The bottom row shows the same amplitude as a sum of Bethe-Salpeter kernels, which are represented by circles with external lines. The kernels are connected by lines representing propagating hadrons, with some lines being fully dressed (indicated by a blue dot). The imaginary part of the amplitude is indicated by  $i\epsilon$  on the lines connecting the kernels.

$$\mathcal{M}(s) = \text{[diagram]}$$

The diagram shows a scattering process with two incoming particles (blue circles) and two outgoing particles (blue circles), connected by two lines representing propagating hadrons.

— — propagating hadrons

—•— = fully dressed

● Bethe-Salpeter kernel

$$\text{[diagrams]} = \int [\text{real, analytic}]$$

$$\text{for } (2M_N)^2 < s < (2M_N + m_\pi)^2$$

*Framework for generic, EFT independent, all-orders diagrammatic relations*



# s-channel cut (diagrammatic)

$$\mathcal{M}(s) = \text{[tree diagrams]} + \text{[loop diagrams]} + \dots$$

$$= \text{[BS kernel]} + \text{[dressed BS kernel]} + \text{[dressed BS kernel]} + \dots$$

$i\epsilon$        $i\epsilon$        $i\epsilon$

$$\mathcal{M}(s) = \text{[2-to-2 scattering diagram]}$$

propagating hadrons

= fully dressed

Bethe-Salpeter kernel

$$\text{[tree diagrams]} = \int [\text{real, analytic}]$$

$$\text{for } (2M_N)^2 < s < (2M_N + m_\pi)^2$$

*Framework for generic, EFT independent, all-orders diagrammatic relations*

# s-channel cut (diagrammatic)

$$\mathcal{M}(s) = \text{tree} + \text{Y} + \text{loop} + \text{cut} + \text{box} + \text{box} + \text{loop} + \dots$$

$$= \text{BS kernel} + \text{BS kernel} \text{---} \text{BS kernel} + \text{BS kernel} \text{---} \text{BS kernel} \text{---} \text{BS kernel} + \dots$$

$$\text{BS kernel} \text{---} \text{BS kernel} = \text{Re} \left[ \text{BS kernel} \text{---} \text{BS kernel} \right] + i \text{Im} \left[ \text{BS kernel} \text{---} \text{BS kernel} \right]$$

$$i \text{Im} \left[ \text{BS kernel} \text{---} \text{BS kernel} \right] = -i \text{Im} \int_{\mathbf{k}} \frac{B(\mathbf{k})^2}{(2\omega_{\mathbf{k}})[(P-k)^2 - M_N^2 + i\epsilon]} = i\pi \int_{\mathbf{k}} \frac{\delta((P-k)^2 - m^2)}{2\omega_{\mathbf{k}}} B_{\ell m}(|\mathbf{k}_{\text{cm}}|)^2$$

$$= B_{\ell m}(s) i\rho(s) B_{\ell m}(s) = \text{BS kernel} \text{---} \text{cut} \text{---} \text{BS kernel}$$

$i\rho(s) \propto \sqrt{s - 4M_N^2}$

$$\mathcal{M}(s) = \text{tree} + \text{Y} + \text{loop} + \dots$$

— — — propagating hadrons

—•— = fully dressed

● Bethe-Salpeter kernel

$$\text{tree} + \text{Y} + \text{loop} = \int [\text{real, analytic}]$$

$$\text{for } (2M_N)^2 < s < (2M_N + m_\pi)^2$$

Framework for generic, EFT independent, all-orders diagrammatic relations

$$\mathbf{k}_{\text{cm}}^2 \stackrel{!}{=} E_{\text{cm}}^2/4 - M_N^2 = p(E_{\text{cm}})^2$$

imaginary part is set exactly on-shell

cut is trivial in angular momentum

# s-channel cut (diagrammatic)

$$\mathcal{M}(s) = \text{diagram}_1 + \text{diagram}_2 + \text{diagram}_3 + \dots$$

$$\text{diagram}_2 = \text{diagram}_2^{\text{p.v.}} + \text{diagram}_2^{\text{cut}}$$

$i\rho(s) \propto \sqrt{s - 4M_N^2}$

defines the K matrix  $\mathcal{K}(s)$

$$= \left[ \text{diagram}_1 + \text{diagram}_2^{\text{p.v.}} + \text{diagram}_3^{\text{p.v.}} + \dots \right] + \left[ \text{diagram}_1 + \text{diagram}_2^{\text{p.v.}} + \dots \right] \text{diagram}_2^{\text{cut}} \left[ \text{diagram}_1 + \text{diagram}_2^{\text{p.v.}} + \dots \right]$$

$i\rho(s) \propto \sqrt{s - 4M_N^2}$

$$= \mathcal{K}(s) + \mathcal{K}(s)i\rho(s)\mathcal{K}(s) + \dots = \frac{1}{\mathcal{K}(s)^{-1} - i\rho(s)}$$

$$\mathcal{M}(s) = \text{diagram}$$

— (blue) — propagating hadrons

— (blue with dot) — fully dressed

● (grey) Bethe-Salpeter kernel

$$\text{diagram} = \int [\text{real, analytic}]$$

$$\text{for } (2M_N)^2 < s < (2M_N + m_\pi)^2$$

Framework for generic, EFT independent, all-orders diagrammatic relations

$$k_{\text{cm}}^2 \stackrel{!}{=} E_{\text{cm}}^2/4 - M_N^2 = p(E_{\text{cm}})^2$$

imaginary part is set exactly on-shell

cut is trivial in angular momentum

# The importance of on-shell-ness

- s-channel cut arises from the imaginary part of two-particle loops

continuum of two-particle scattering states

loops can go on-shell, carry physical energy

pole is hit, need  $i\epsilon$

Dirac delta enforces physical energy (on-shell)

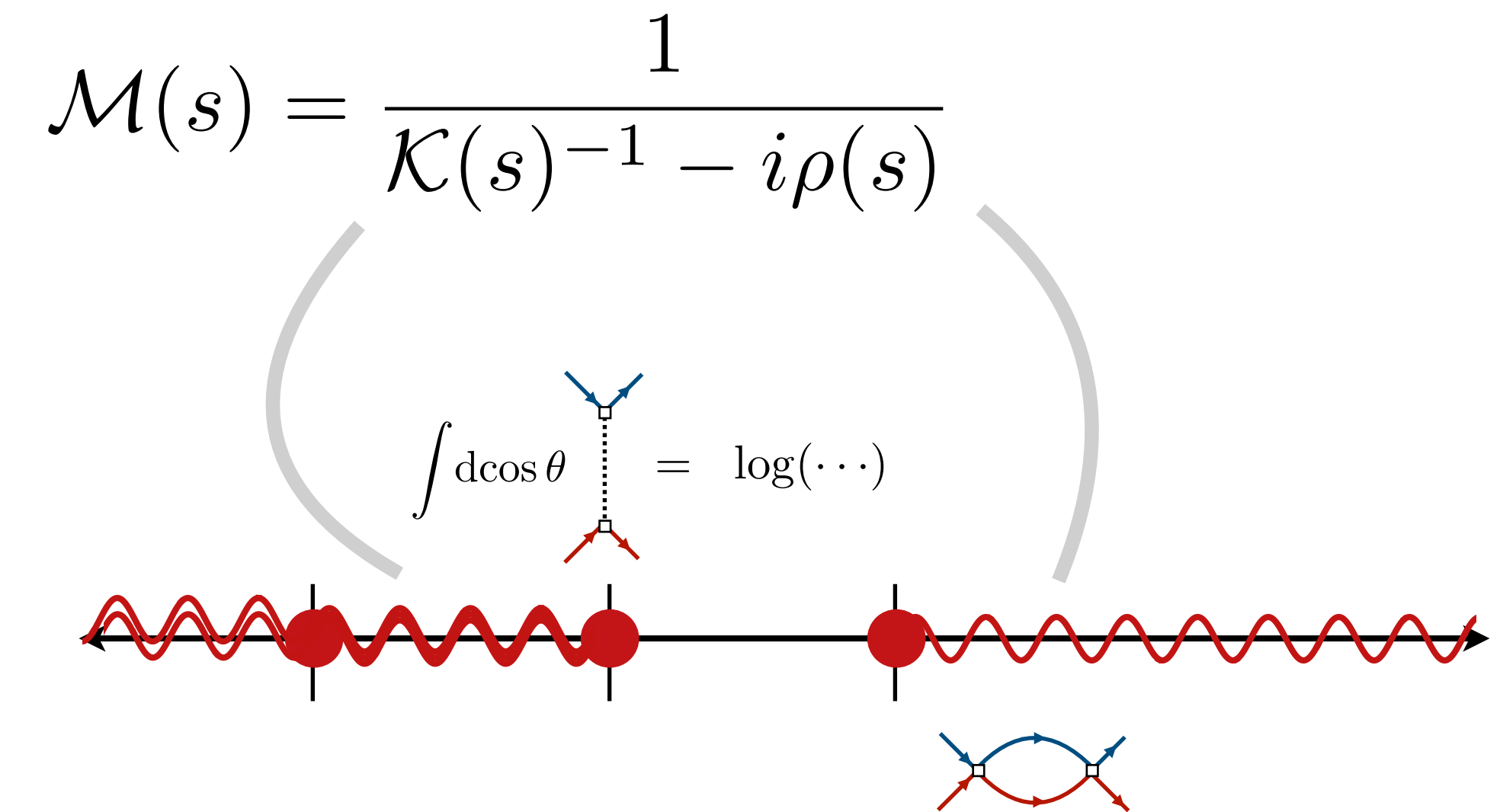
- Above threshold, K-matrix is set identically on-shell

- Sub-threshold continuation follows from...

Riemann sheet specification:  $i\rho(s) \rightarrow \pm |\rho(s)|$

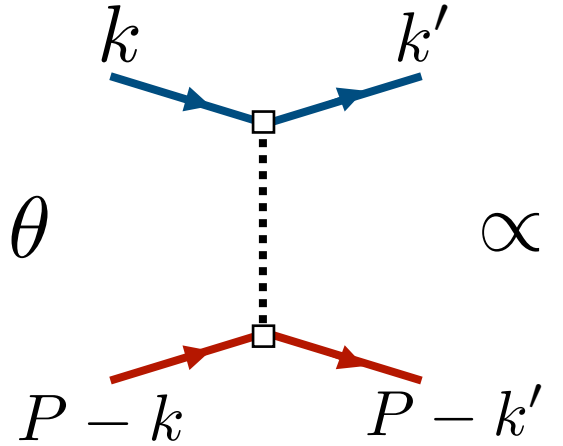
near-threshold analyticity of  $K(s)$

- t-channel appears further down but is intrinsically tied to the continuation of an **on-shell** diagram



# Show me the cut!

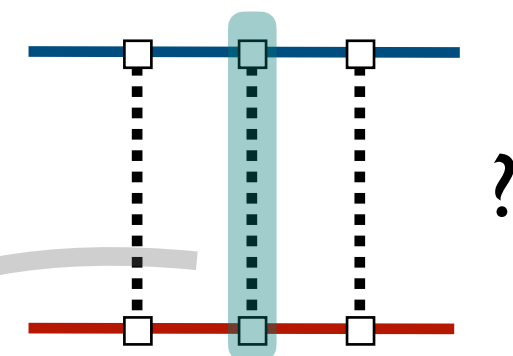
□ Evaluating the simple exchange diagram gives



$$\int d\cos\theta \propto \int d\cos\theta \left[ \frac{1}{-(\mathbf{k}_{\text{cm}} - \mathbf{k}'_{\text{cm}})^2 + m_\pi^2 - i\epsilon} \right] = \frac{1}{k_{\text{cm}}^2} \log \left[ \frac{4\mathbf{k}_{\text{cm}}^2 + m_\pi^2 - i\epsilon}{m_\pi^2} \right] \quad (\text{analytic for } k_{\text{cm}}^2 > -m_\pi^2/4)$$

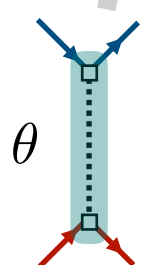
$\xrightarrow{\text{on-shell kinematics}}$ 
 $\frac{1}{s/4 - M_N^2} \log \left[ \frac{s - 4M_N^2 + m_\pi^2 - i\epsilon}{m_\pi^2} \right] \quad (\text{analytic for } s > 4M_N^2 - m_\pi^2)$

□ What about diagrams like

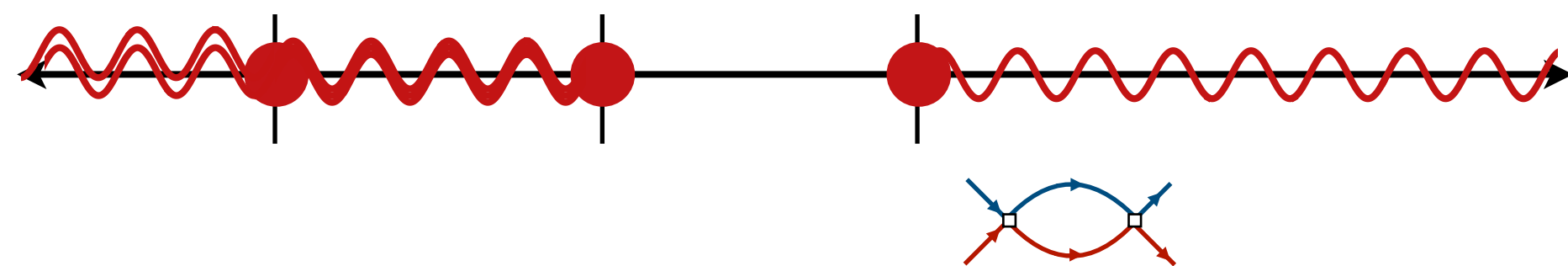


Naive statement: Internal loops are summed over real  $k \rightarrow$  never get cuts from middle exchange

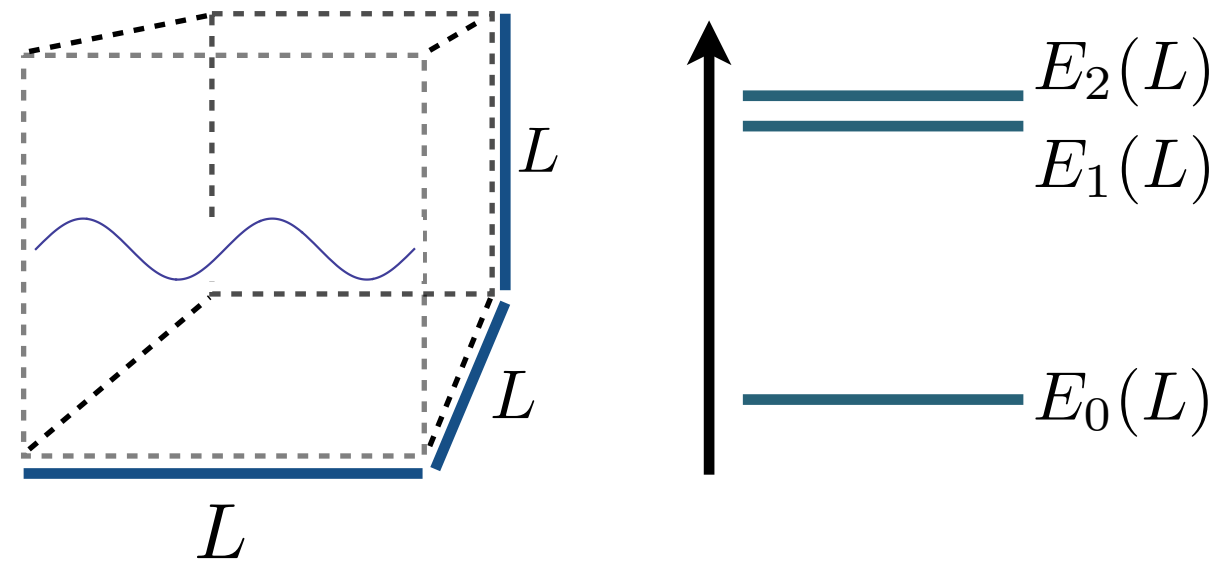
**True statement:** In the  $\rho(s)$  decomposition, the middle exchange is set on-shell  $\rightarrow$  contributes to the leading K-matrix cut



$$\int d\cos\theta = \log(\dots)$$

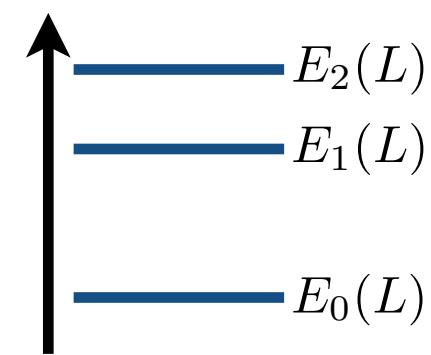


# The finite-volume

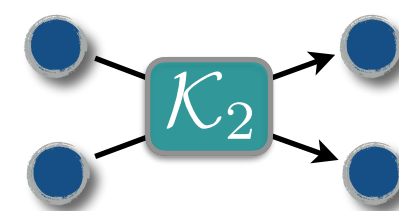


- ❑ Finite, **cubic**, periodic spatial volume (extent  $L$ )
- ❑ **Discrete** momenta and energies
- ❑  $L$  is large enough to neglect  $e^{-m_\pi L}$
- ❑ Finite  $T$  and non-zero lattice spacing assumed negligible

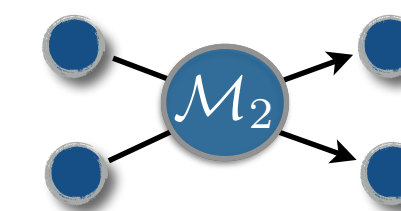
Quantisation condition relates energies to K-matrices, and thereby amplitudes



finite volume



unitarity



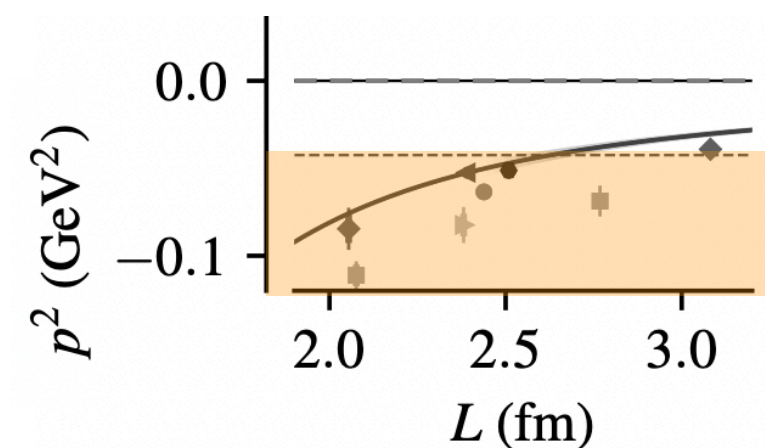
$$\det[\mathcal{K}^{-1}(s) + F(P, L)] = 0$$

$$F(P, L) \equiv \text{Matrix of known geometric functions}$$

• Lüscher (1989) • many others •

$F(P, L)$  is real below threshold

→ only solvable if  $K(s)$  is real below threshold



$$\frac{1}{s/4 - M_N^2} \log \left[ \frac{s - 4M_N^2 + m_\pi^2 - i\epsilon}{m_\pi^2} \right]$$

# Back to the derivation

$$\mathcal{M}_L(P) = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

matrix of known geometric functions

$$\text{diagram 2} = \text{diagram 2a} + \text{diagram 2b}$$

$-F(P, L)$

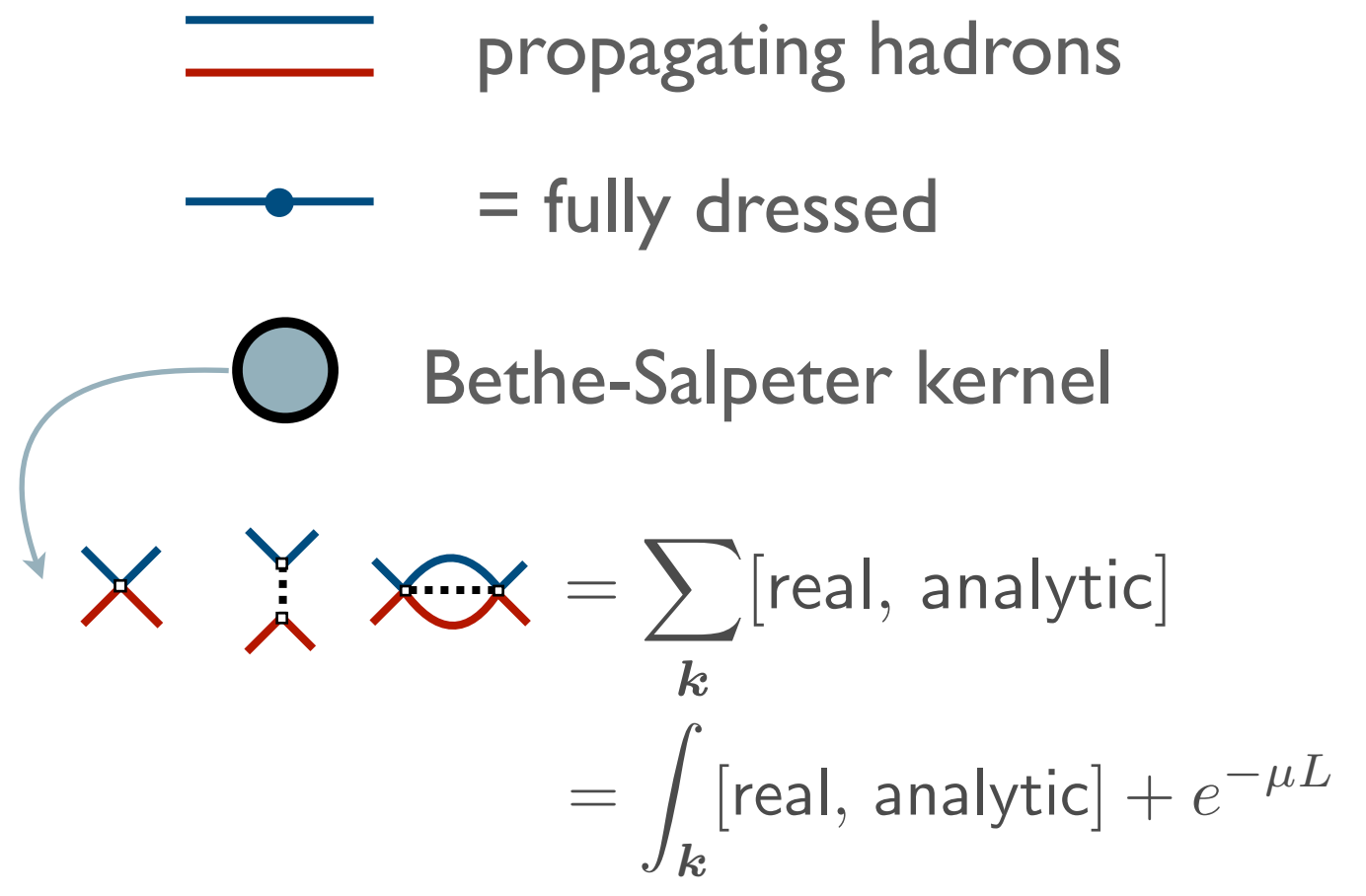
defines the K matrix  $\mathcal{K}(s)$

$$= \left[ \text{diagram 1} + \text{diagram 2a} + \text{diagram 2b} + \dots \right] + \left[ \text{diagram 2a} + \text{diagram 2b} + \dots \right] \text{diagram 2b} \left[ \text{diagram 1} + \text{diagram 2a} + \text{diagram 2b} + \dots \right]$$

$-F(P, L)$

$$= \mathcal{K}(s) - \mathcal{K}(s)F(P, L)\mathcal{K}(s) + \dots = \frac{1}{\mathcal{K}(s)^{-1} + F(P, L)}$$

$\mathcal{M}_L(P)$  = finite-volume correlator  
poles are finite-volume energies



for  $(2M_N)^2 < s < (2M_N + m_\pi)^2$

Framework for generic, EFT independent, all-orders diagrammatic relations

Key details:  
matrices on angular momentum  $\otimes$  channels  
 $\mathcal{K}(s)$  populated with physical (on-shell) partial waves

# Origins / issues of on-shell projection

$$i\rho(s) \propto \sqrt{s - 4M_N^2} = i \operatorname{Im} \left[ \frac{1}{i\epsilon} \right]$$

Exactly set on the mass shell by a Dirac delta function  
 ... a result that leads to the correct sub-threshold analytic continuation

$$-F(P, L) = \frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \text{p.v.}$$

Set on the mass shell by the relation:

$$\left[ \frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right] \frac{1}{2\omega_{\mathbf{k}}} \frac{\mathcal{S}(\mathbf{k}_{\text{cm}}^2) - \mathcal{S}(p(E_{\text{cm}})^2)}{\mathbf{k}_{\text{cm}}^2 - p(E_{\text{cm}})^2} = e^{-\mu L}$$

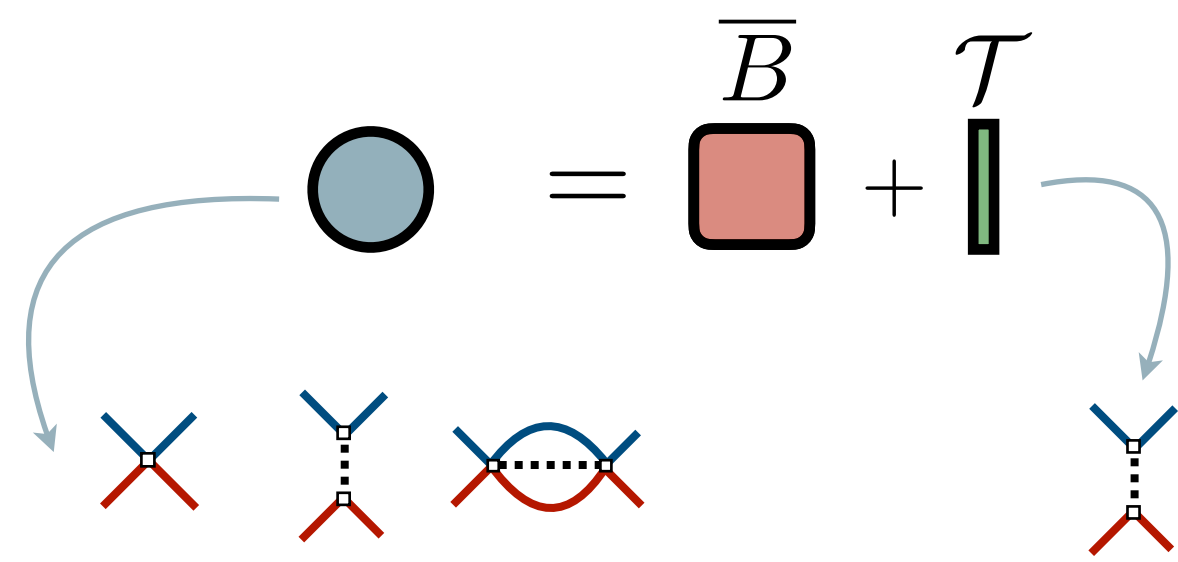
... a result that breaks on the cut

$$\frac{1}{k_{\text{cm}}^2} \log \left[ \frac{4k_{\text{cm}}^2 + m_{\pi}^2 - i\epsilon}{m_{\pi}^2} \right]$$

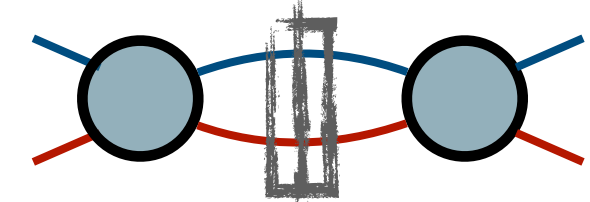
$$\frac{1}{s/4 - M_N^2} \log \left[ \frac{s - 4M_N^2 + m_{\pi}^2 - i\epsilon}{m_{\pi}^2} \right]$$

## Resolution:

separate the *t*-channel exchange from the Bethe Salpeter kernel



change the details of the cut to keep *T* off-shell



modify index space to reach a new quantisation condition

$$\ell, m \longrightarrow |\mathbf{k}_{\text{cm}}|, \ell, m$$

$E_n(L)$  can now constrain modified *K* matrix + the cut

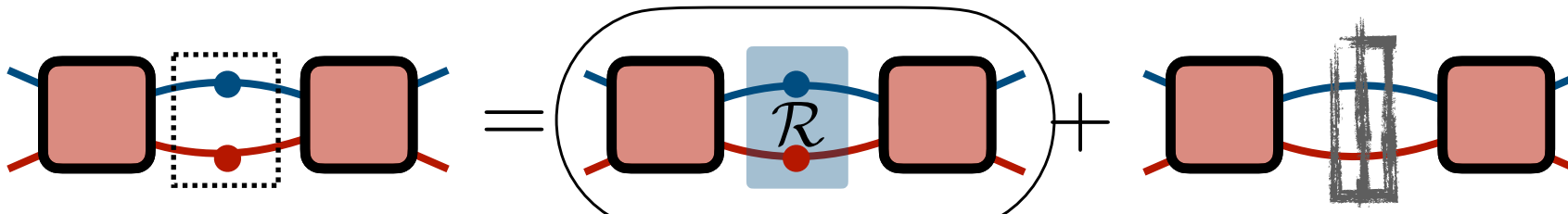
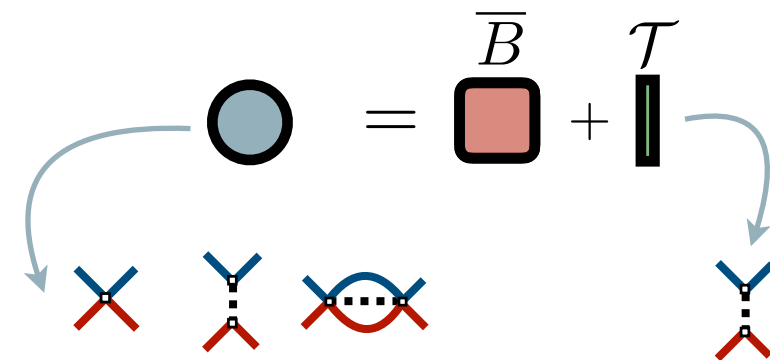
$$\bar{\mathcal{K}}(s) \quad g^2 \log[\dots]$$

Known integral equations relate this to the standard *K* (or the amplitude)



# Some cutting details

$$\mathcal{M}_L(P) = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]} + \dots$$



$$\frac{1}{L^3} \sum_{\mathbf{k}} \frac{1}{2\omega_{\mathbf{k}}} \frac{\mathcal{A}(\mathbf{k}_{\text{cm}}^2) \mathcal{B}(\mathbf{k}_{\text{cm}}^2)}{k_{\text{cm}}^2 - p(E_{\text{cm}})^2}$$

$$\frac{1}{L^3} \sum_{\mathbf{k}} \frac{1}{2\omega_{\mathbf{k}}} \frac{\mathcal{A}(\mathbf{k}_{\text{cm}}^2) \mathcal{B}(p(E_{\text{cm}})^2)}{k_{\text{cm}}^2 - p(E_{\text{cm}})^2} H(\mathbf{k}_{\text{cm}})$$

smooth cutoff  $H$

abbreviating slightly we write...

$$\sum \frac{\mathcal{A}_{\text{off}} \mathcal{B}_{\text{off}}}{k_{\text{cm}}^2 - p^2}$$

$$\sum \frac{\mathcal{A}_{\text{off}} \mathcal{B}_{\text{on}}}{k_{\text{cm}}^2 - p^2} (1 - H) + \sum \frac{\mathcal{A}_{\text{off}} \mathcal{B}_{\text{off}} - \mathcal{A}_{\text{off}} \mathcal{B}_{\text{on}}}{k_{\text{cm}}^2 - p^2}$$

$$\sum \frac{\mathcal{A}_{\text{off}} \mathcal{B}_{\text{on}}}{k_{\text{cm}}^2 - p^2} H$$

□ We are still angular-momentum projecting everywhere (*not really the issue*)

□ Sum is promoted to an additional index... can always be collapsed later

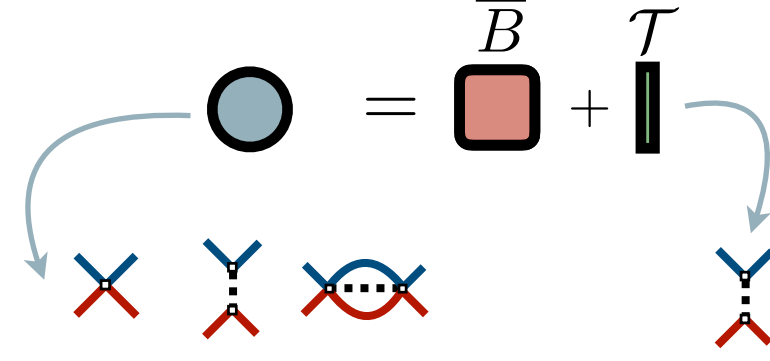
$$-S(P, L)_{|\mathbf{k}'_{\text{cm}}|, \ell', m', |\mathbf{k}_{\text{cm}}|, \ell, m}$$

□ Only the known log function is evaluated off-shell

# First sum the safe blobs

$$\mathcal{M}_L(P) = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]} + \dots$$

$$\text{[diagram 2]} = \text{[diagram 2a]} + \text{[diagram 2b]}$$



defines the modified K matrix  $\bar{\mathcal{K}}(s)$

$$= \left[ \text{[diagram 1]} + \text{[diagram 2a]} + \dots \right] + \left[ \text{[diagram 2b]} + \text{[diagram 3]} + \dots \right] \text{[vertical bar]} \left[ \text{[diagram 2a]} + \text{[diagram 2b]} + \dots \right] + \dots$$

$$-S(P, L)_{|\mathbf{k}'_{cm}|, \ell', m', |\mathbf{k}_{cm}|, \ell, m}$$

$$= \bar{\mathcal{K}}(s) - \bar{\mathcal{K}}(s)S(P, L)\bar{\mathcal{K}}(s) + \dots = \frac{1}{\bar{\mathcal{K}}(s)^{-1} + S(P, L)}$$

Key details:

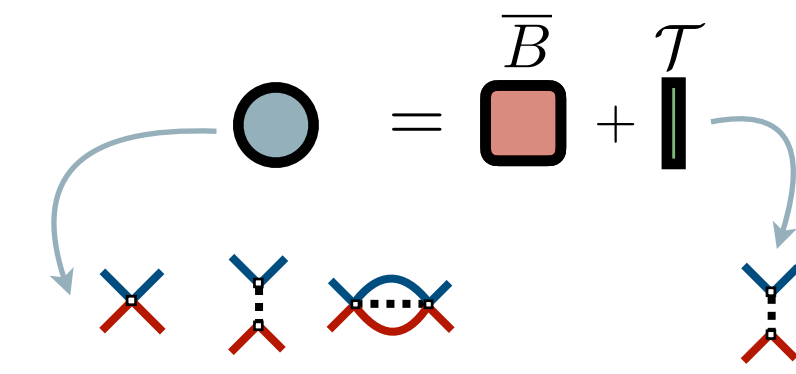
- matrices on angular momentum  $\otimes |\mathbf{k}_{cm}|$
- $\bar{\mathcal{K}}(s)$  on shell but missing parts of diagrams
- but now have significant redefinition freedom

# Completing the story...

$$\mathcal{M}_L(P) = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots$$

$$\text{Diagram 2} = \text{Diagram 2a} + \text{Diagram 2b}$$

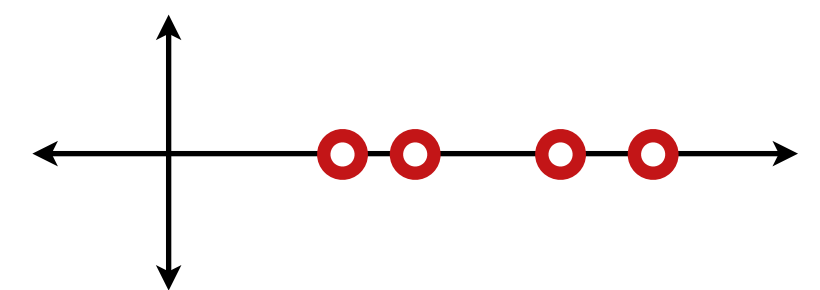
$-S(P, L)_{|\mathbf{k}'_{\text{cm}}|, \ell', m', |\mathbf{k}_{\text{cm}}|, \ell, m}$



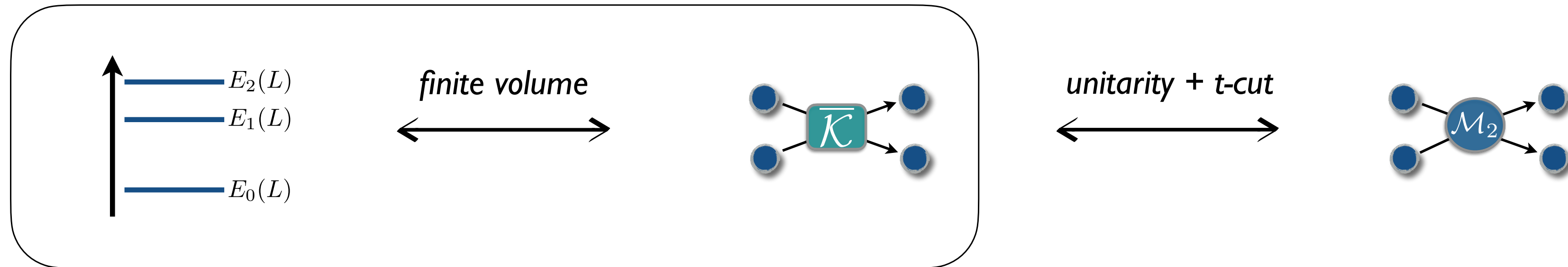
$$= \left[ \text{Diagram 1} + \text{Diagram 2a} + \text{Diagram 3a} + \dots \right] + \left[ \text{Diagram 1} + \text{Diagram 2a} + \dots \right] \text{Diagram 2b} \left[ \text{Diagram 1} + \text{Diagram 2a} + \dots \right]$$

$-S(P, L)_{|\mathbf{k}'_{\text{cm}}|, \ell', m', |\mathbf{k}_{\text{cm}}|, \ell, m}$

$$= [\bar{\mathcal{K}}(s) + g^2 \mathcal{T}] - [\bar{\mathcal{K}}(s) + g^2 \mathcal{T}] S(P, L) [\bar{\mathcal{K}}(s) + g^2 \mathcal{T}] + \dots = \frac{1}{[\bar{\mathcal{K}}(s) + g^2 \mathcal{T}]^{-1} + S(P, L)}$$



# Quantisation condition



$S(P, L) =$  Matrix of known geometric functions

$\mathcal{T} =$  Matrix of known off-shell logs

$$\det_{|\mathbf{k}_{\text{cm}}| \ell m} \left[ [\bar{\mathcal{K}}(s) + g^2 \mathcal{T}]^{-1} + S(P, L) \right] = 0$$

$$\mathcal{T}_{|\mathbf{k}_{\text{cm}}|00, |\mathbf{k}'_{\text{cm}}|00} = -\frac{1}{|\mathbf{k}_{\text{cm}}| |\mathbf{k}'_{\text{cm}}|} \log \left[ \frac{2\omega_{\mathbf{k}_{\text{cm}}} \omega_{\mathbf{k}'_{\text{cm}}} - 2|\mathbf{k}_{\text{cm}}| |\mathbf{k}'_{\text{cm}}| - 2M_N^2 + m_\pi^2 - i\epsilon}{2\omega_{\mathbf{k}_{\text{cm}}} \omega_{\mathbf{k}'_{\text{cm}}} + 2|\mathbf{k}_{\text{cm}}| |\mathbf{k}'_{\text{cm}}| - 2M_N^2 + m_\pi^2 - i\epsilon} \right]$$

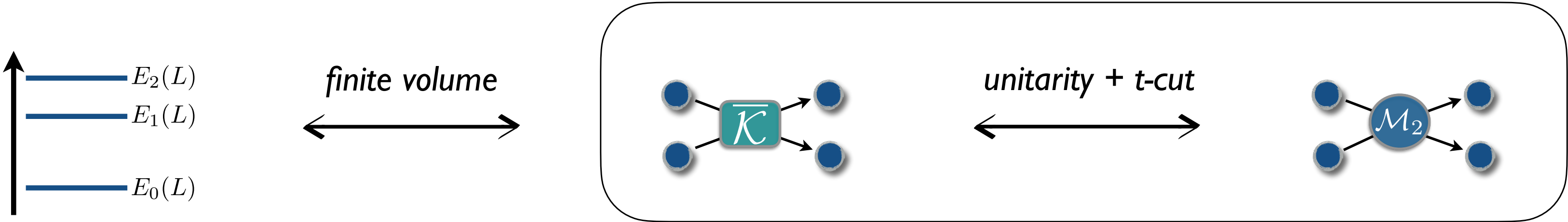
New matrix space, truncated by cutoff function

Holds up to  $e^{-m_\pi L}$

*Inspired in part three-particle work...*

*Blanton, Briceño, Döring, Draper, Mai, Meißner, Müller, Hammer, MTH, Pang, Romero-López, Rusetsky, Sharpe*

# Integral equations



- Exactly in the spirit of the three-particle approach • MTH, Sharpe (2015) • Agadjanov *et.al*, (2016) (Optical potential) •

- Define a finite-volume amplitude with the correct limit: 
$$\mathcal{M}_L(P) = \frac{1}{[\overline{\mathcal{K}}(s) + g^2 \mathcal{T}]^{-1} + S(P, L)}$$

- Formally take an infinite-volume limit to derive an integral equation

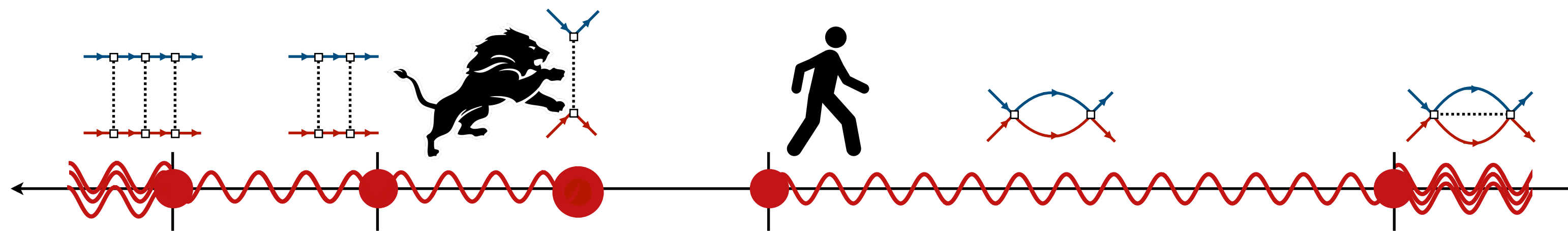
$$\mathcal{M}(s, t) = \lim_{\text{os}} \lim_{\epsilon \rightarrow 0} \lim_{L \rightarrow \infty} \mathcal{M}_L(P + i\epsilon)_{|\mathbf{k}'_{\text{cm}}| \ell m, |\mathbf{k}_{\text{cm}}| \ell m}$$

Note: *derivation strategy = logically separate from evaluation strategy*

# Summary and outlook

- ❑ t-channel cut is an *identically on-shell* + angular-momentum projection + *infinite-volume* effect
- ❑ Using *on-shell* + angular-momentum projected + *infinite-volume* Bethe-Salpeter kernels gives incorrect finite- $L$  description of the correlator
- ❑ A modified derivation solves this at the expense of a new quantisation condition (and a new  $K$ -matrix)
- ❑ Integral equations can be used to relate the latter to the standard quantity, with the cut included
- ❑ Publication forthcoming
- ❑ Much to explore... finite- $L$  effects near (but not on) the cut, realistic systems, dispersive applications

*Thanks to Jeremy Green, other Mainzers, CallLatt, Raúl Briceño for discussions!*



*Thanks for listening! Questions?....*