D π/K scattering and charm meson resonances from lattice QCD

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Bethe Forum on "*Multihadron Dynamics in a Box – A.D. 2022*"
Bonn, 15 – 19 August 2022

Lattice QCD spectroscopy

Finite-volume energy eigenstates

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle = \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

In each symmetry channel: matrix of correlators for **large bases of interpolating operators** with appropriate variety of structures. Use **distillation** to compute corrs.

$$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x) \left[\Gamma \overleftrightarrow{D} \overleftrightarrow{D} \dots \right] \psi(x) \qquad \sum_{\vec{p}_1, \vec{p}_2} C(\vec{P}, \vec{p}_1, \vec{p}_2) H(\vec{p}_1) H(\vec{p}_2)$$

$$\sum_{\vec{p}_1, \vec{p}_2, \vec{p}_3, \dots} C(\vec{P}, \vec{p}_1, \vec{p}_2, \vec{p}_3, \dots) H(\vec{p}_1) H(\vec{p}_2) H(\vec{p}_3) \dots$$

Lattice QCD spectroscopy

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$$\sum_{\vec{p}_1, \vec{p}_2, \vec{p}_3, \ldots} C(\vec{P}, \vec{p}_1, \vec{p}_2, \vec{p}_3, \ldots) H(\vec{p}_1) H(\vec{p}_2) H(\vec{p}_3) \ldots$$

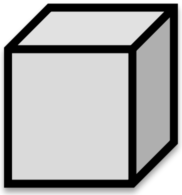
Variational method (generalised eigenvalue problem) → $\{E_n\}$

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)} \qquad \lambda^{(n)}(t) \sim e^{-E_n (t - t_0)}$$

$$v_i^{(n)} \to Z_i^{(n)} \equiv \langle 0 | \mathcal{O}_i | n \rangle \qquad \Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

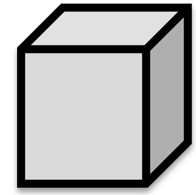
Scattering in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\rm cm}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Scattering in lattice QCD



Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\text{cm}}\}$ to **infinite-volume scattering *t*-matrix**.

$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[\mathbf{1} + i\,\boldsymbol{\rho}(E_{\text{cm}})\,\boldsymbol{t}(E_{\text{cm}})\left(\mathbf{1} + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\text{cm}}, L)\right)\right] = 0$$

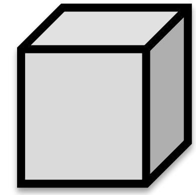
Infinite-volume scattering *t*-matrix

Effect of finite volume (including reduced sym.)

$$\rho_i(E_{\text{cm}}) = \frac{2k_i}{E_{\text{cm}}}$$

[Complication: reduced sym. of lattice vol. → 'mixing' of partial waves]

Scattering in lattice QCD



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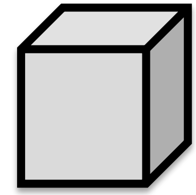

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Elastic scattering: one-to-one mapping $E_{cm} \leftrightarrow t(E_{cm})$

Analytically continue t in complex E_{cm} plane, look for poles.

[Complication: reduced sym. of lattice vol. → 'mixing' of partial waves]

Scattering in lattice QCD

$$\det\left[1 + i\,\rho(E_{\rm cm})\,\boldsymbol{t}(E_{\rm cm})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\rm cm}, L)\right)\right] = 0$$

Coupled channels (hadron-hadron and/or partial waves):

E.g. $\mathbf{t}(E_{\rm cm}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\rm cm}) & t_{\pi\pi\to K\bar{K}}(E_{\rm cm}) \\ t_{K\bar{K}\to\pi\pi}(E_{\rm cm}) & t_{K\bar{K}\to K\bar{K}}(E_{\rm cm}) \end{pmatrix}$

Given $\mathbf{t}(E_{cm})$: solutions → finite-volume spectrum $\{E_{cm}\}$
But we need: spectrum → $\mathbf{t}(E_{cm})$

Scattering in lattice QCD

$$\det\left[1 + i\,\rho(E_{\mathrm{cm}})\,\boldsymbol{t}(E_{\mathrm{cm}})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\mathrm{cm}}, L)\right)\right] = 0$$

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Given **t**(E_{cm}): solutions → finite-volume spectrum {E_{cm}}
But we need: spectrum → **t**(E_{cm})

Under-constrained (each E_{cm} constrains *t*-matrix at that E_{cm})
→ Parameterize E_{cm} dep. of *t*-matrix; fit {$E_{lattice}$} to {E_{param}}

Try different parameterizations, e.g. various *K*-matrix forms (unitarity) (also Breit Wigner, effective range expansion for elastic scattering).

$$t^{-1}_{ij} = \frac{1}{(2k_i)^{\ell_i}} K^{-1}_{ij} \frac{1}{(2k_j)^{\ell_j}} + I_{ij}$$

Scattering in lattice QCD

$$\det\left[1 + i\,\rho(E_{\text{cm}})\,\boldsymbol{t}(E_{\text{cm}})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\text{cm}}, L)\right)\right] = 0$$

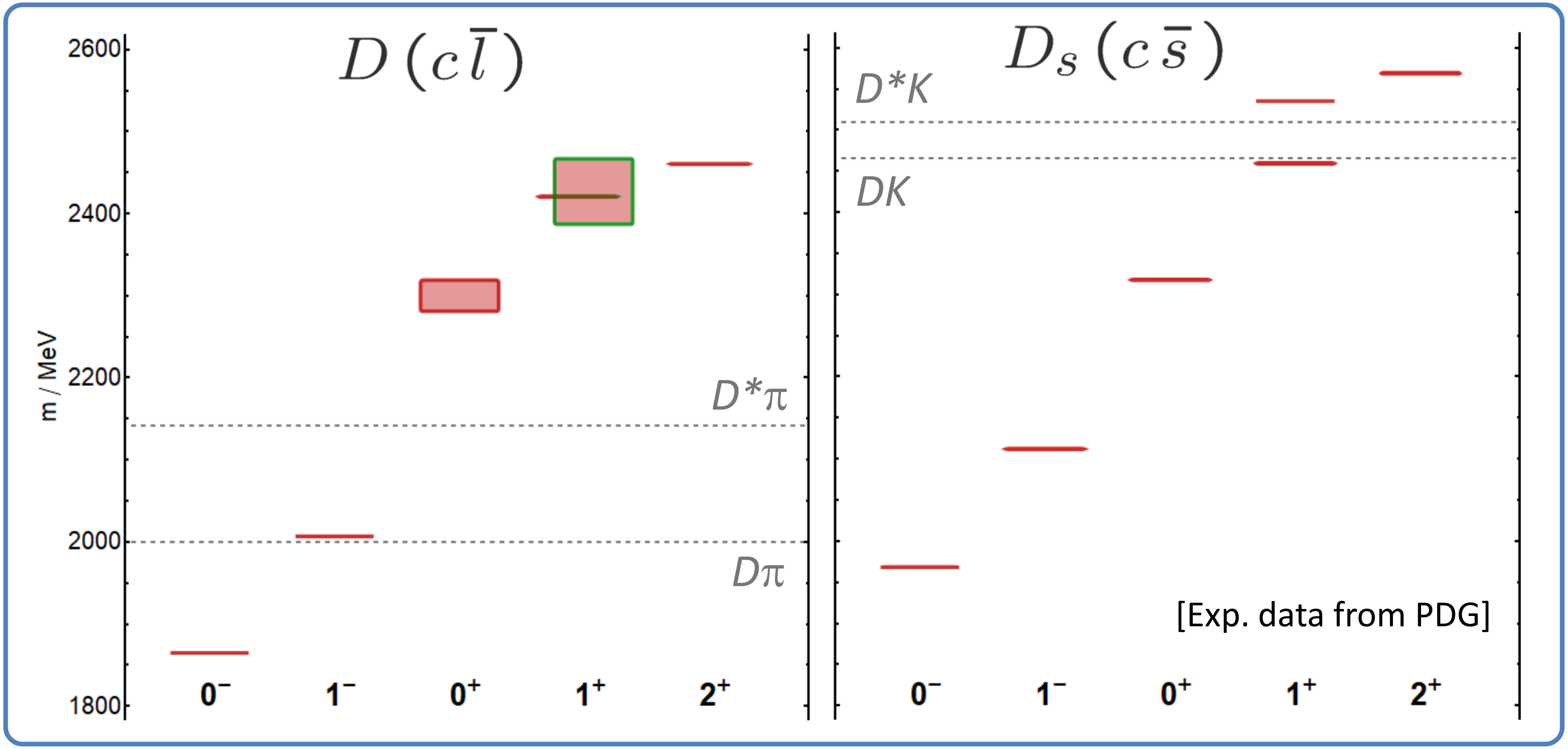
Require:

- Large set of E_{cm} in a range of channels:
  - various symmetry channels (irreps), and
  - overall non-zero momentum, different volumes, and/or twisted b.c.s
- Large enough spatial volume ($m_\pi L \gtrsim 4$)

This is for 2 hadron scattering – see other talks for >2 hadron scattering

Review in e.g. Briceño, Dudek, Young [Rev. Mod. Phys. 90, 025001 (2018)]

Charm (D) and charm-strange (D_s) mesons



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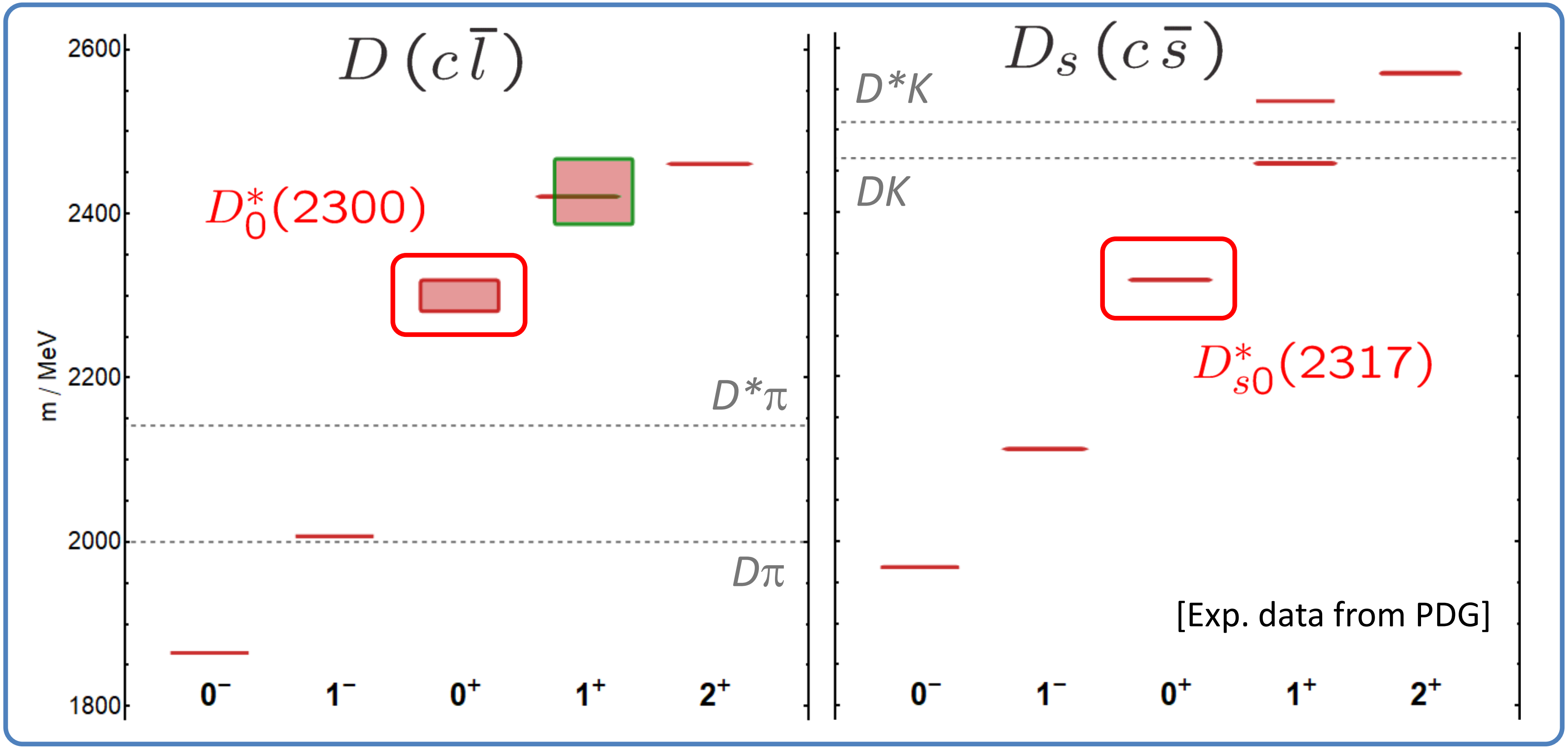

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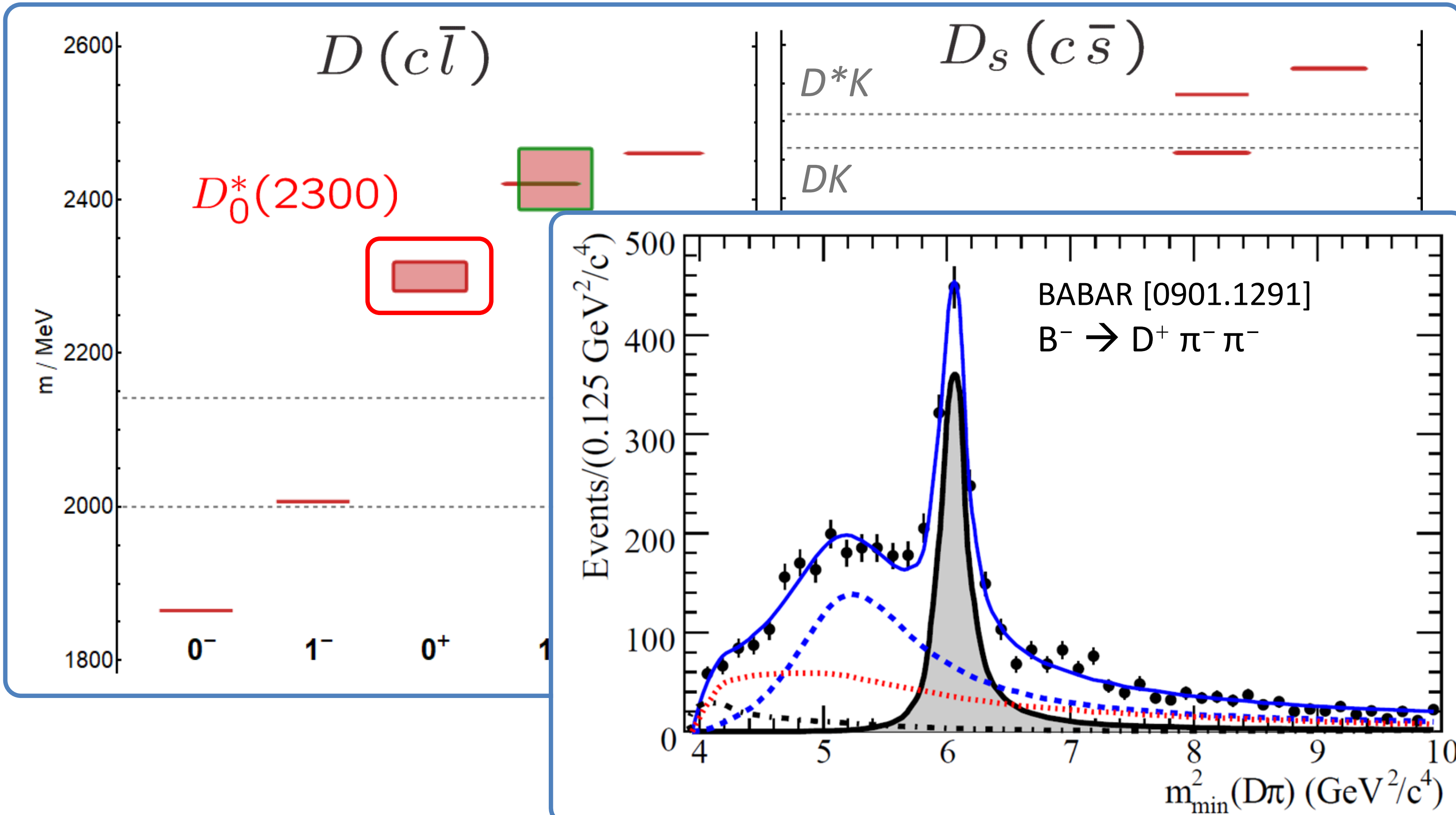

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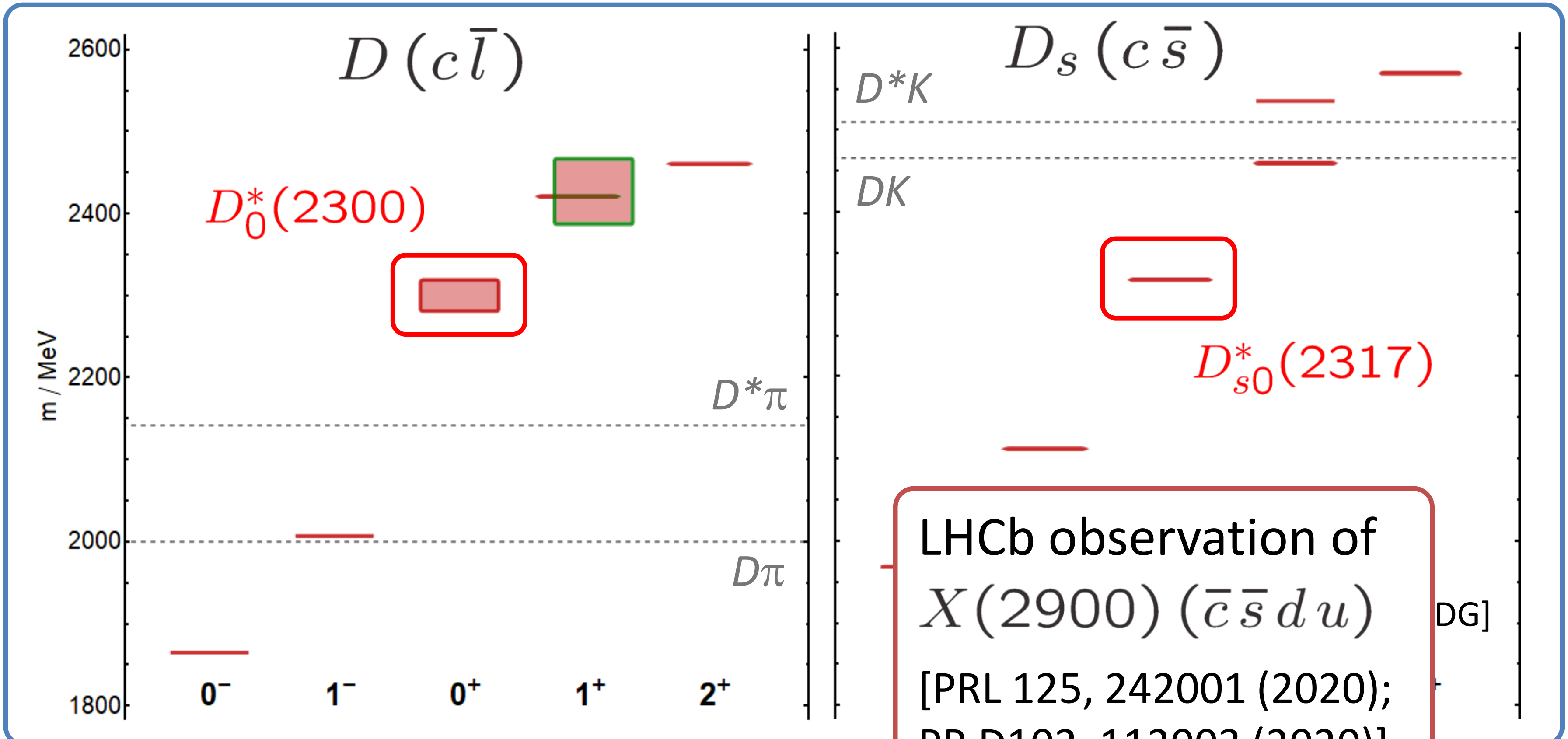

LHCb observation of
$X(2900)\;(\bar{c}\,\bar{s}\,d\,u)$
[PRL 125, 242001 (2020);
PR D102, 112003 (2020)]

Other calculations

Some other lattice QCD work on $D\,K$ and/or $D\,\pi$ scattering:

- Mohler *et al* [PR D87, 034501 (2013), 1208.4059];
- Liu *et al* [PR D87, 014508 (2013), 1208.4535];
- Mohler *et al* [PRL 111, 222001 (2013), 1308.3175];
- Lang *et al* [PR D90, 034510 (2014), 1403.8103];
- Bali *et al* (RQCD) [PR D96, 074501 (2017), 1706.01247];
- Alexandrou *et al* (ETM) [PR D101 034502 (2020), 1911.08435];
- Gregory *et al* [2106.15391]

Also:

- Martínez Torres *et al* [JHEP 05 (2015) 153, 1412.1706];
- Albaladejo *et al* [PL B767, 465 (2017), 1610.06727];
- Du *et al* [PR D98, 094018 (2018), 1712.07957];
- Guo *et al* [*PR D*98 014510 (2018), 1801.10122];
- Guo *et al* [EPJ C79, 13 (2019), 1811.05585]

DK (isospin=0)

[Cheung, CT, Wilson, Moir, Peardon, Ryan (HadSpec), JHEP 02 (2021) 100, arXiv:2008.06432]

Anisotropic lattices,
$a_s/a_t \approx 3.5$, $a_s \approx 0.12$ fm,
various volumes.

$N_f = 2+1$,
Wilson-clover fermions,
$m_\pi \approx 239$ MeV & 391 MeV.

Use many different fermion-bilinear

$$\sim \bar{\psi}\Gamma D \dots \psi$$

and *DK*, ... operators (built from 'optimised' *D* and *K* operators)

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

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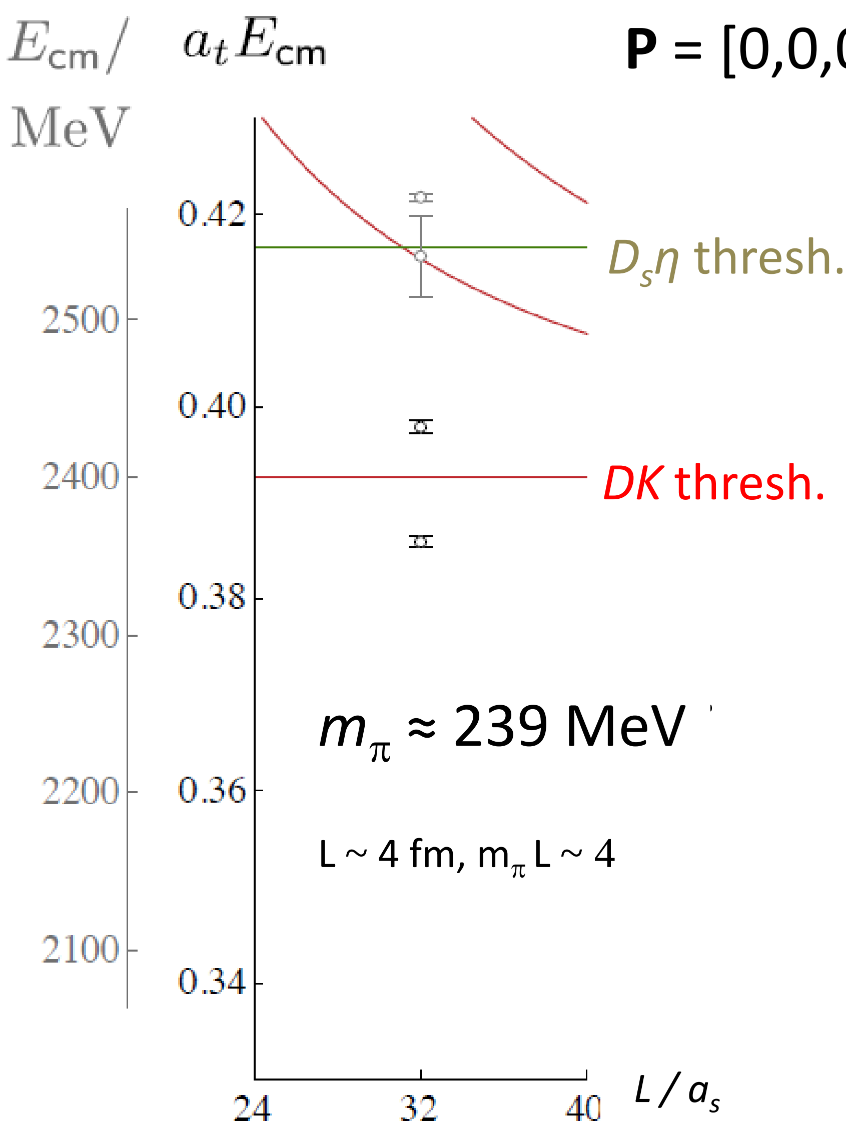
P = [0,0,0] A_1^+ ($J^P = 0^+, 4^+, \ldots$)



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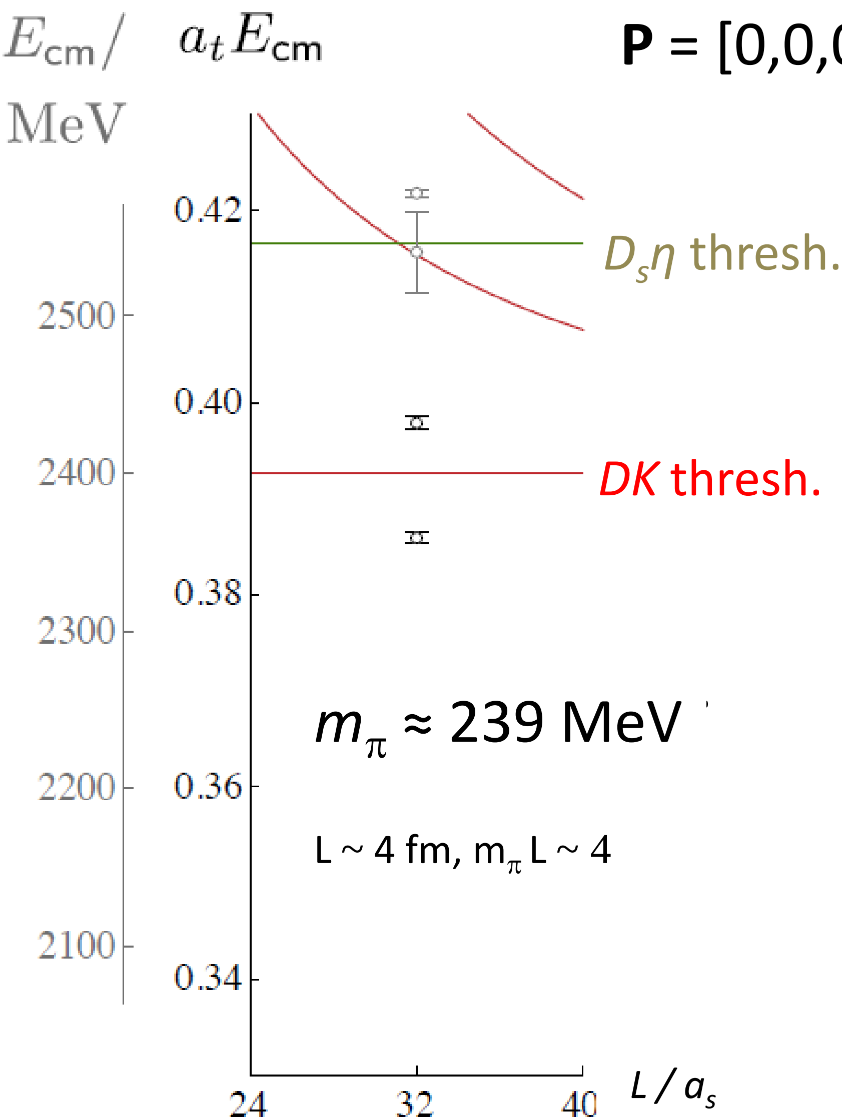
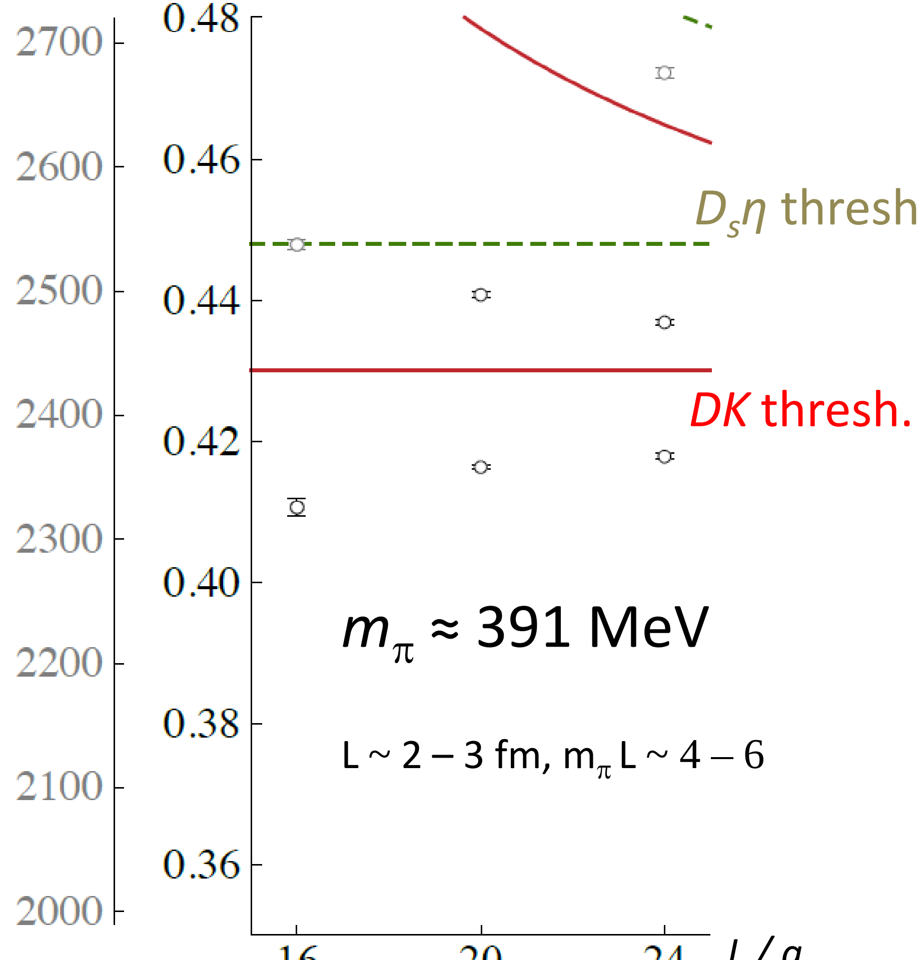
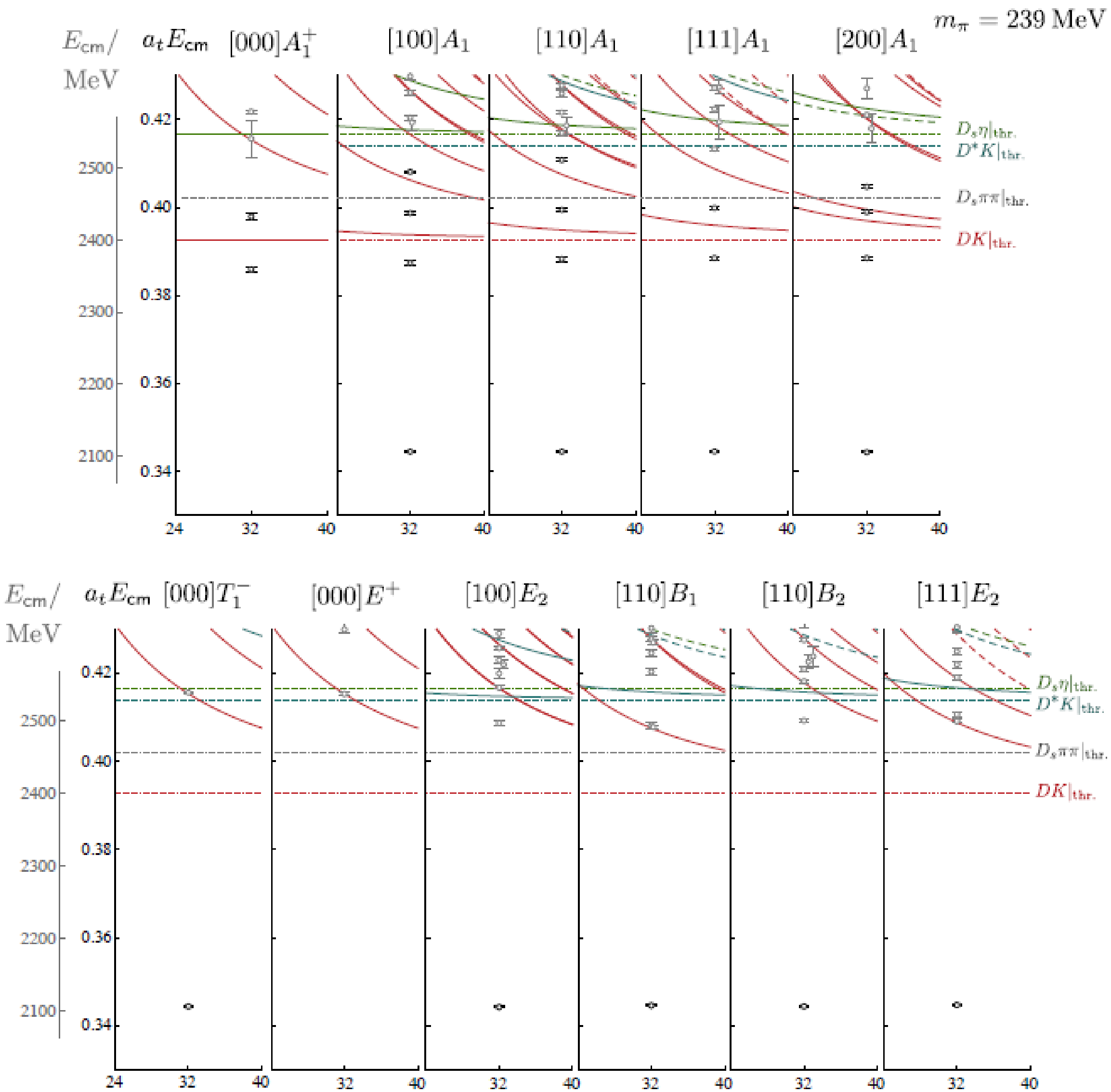




DK (isospin=0) – spectra

[JHEP 02 (2021) 100]

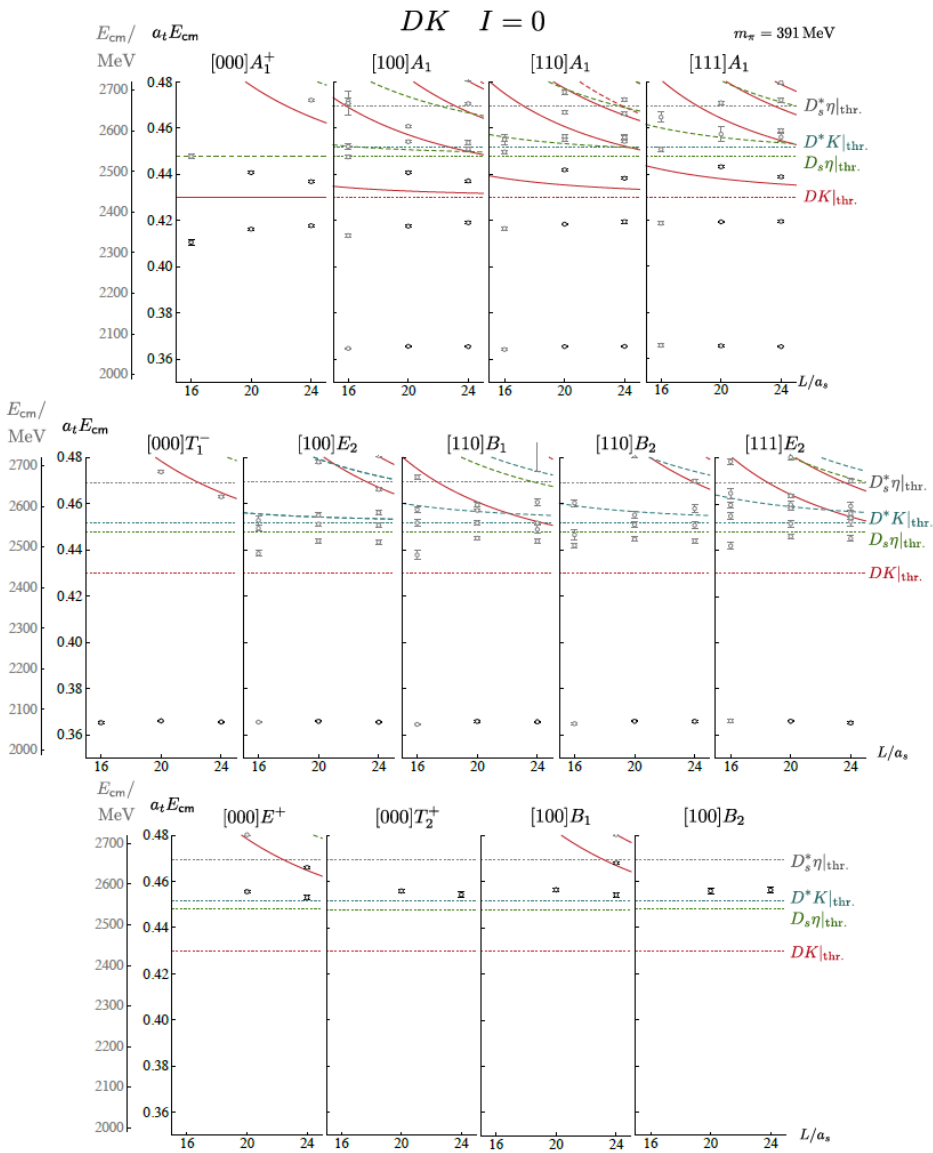


$m_\pi \approx 239$ MeV

Use 22 energy levels for ℓ = 0, 1

DK (isospin=0) – spectra

[JHEP 02 (2021) 100]



$m_\pi \approx 391$ MeV

Use 34 energy levels for ℓ = 0, 1

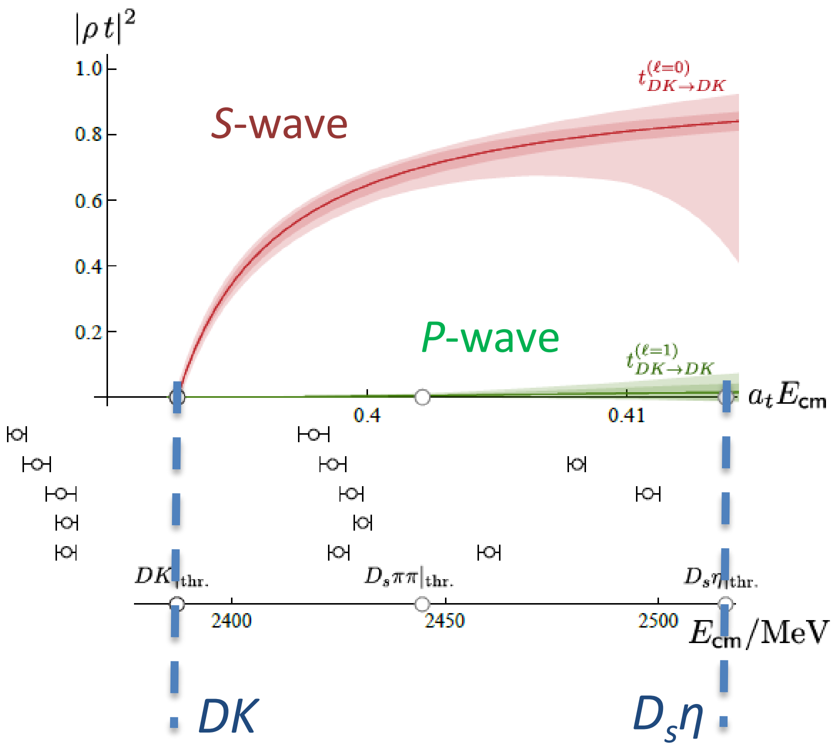
DK (isospin=0) – amplitudes

[JHEP 02 (2021) 100]

$m_\pi \approx 239$ MeV

(22 energy levels)

$\sim |\text{amp}|^2$



Elastic *DK* scattering in *S* and *P*-wave

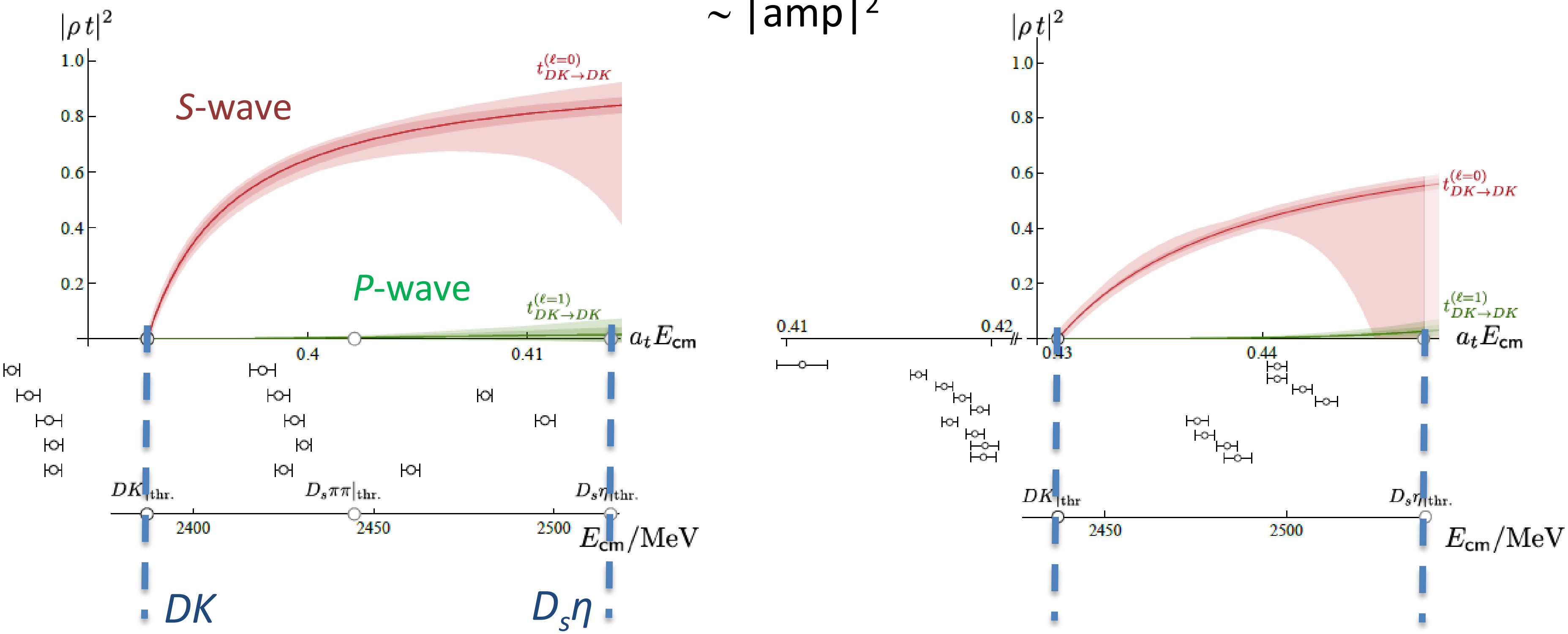
Sharp turn-on in *S*-wave at threshold

DK (isospin=0) – amplitudes

[JHEP 02 (2021) 100]

$m_\pi \approx 239$ MeV (22 energy levels)

$m_\pi \approx 391$ MeV (34 energy levels)

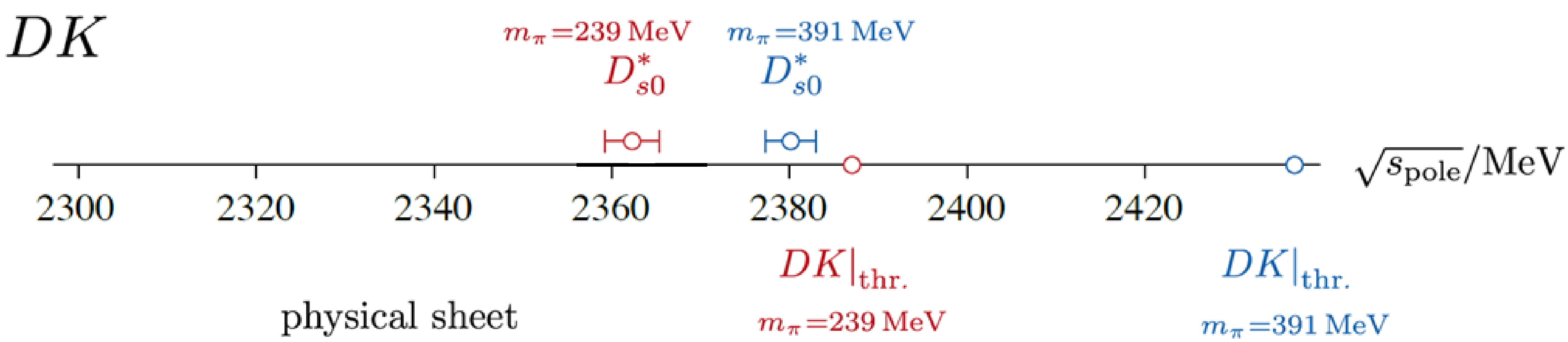


Elastic DK scattering in S and P-wave

Sharp turn-on in S-wave at threshold

DK (isospin=0) – S-wave poles

[JHEP 02 (2021) 100]



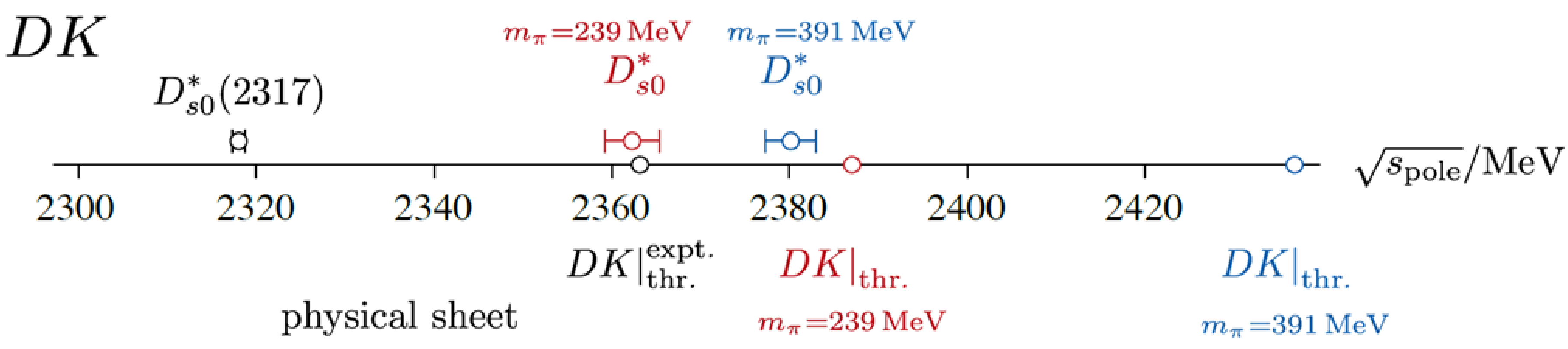
Bound-state pole strongly coupled to *S*-wave *DK*

ΔE = 25(3) MeV for $m_\pi \approx 239$ MeV

ΔE = 57(3) MeV for $m_\pi \approx 391$ MeV

DK (isospin=0) – S-wave poles

[JHEP 02 (2021) 100]



Bound-state pole strongly coupled to S-wave DK

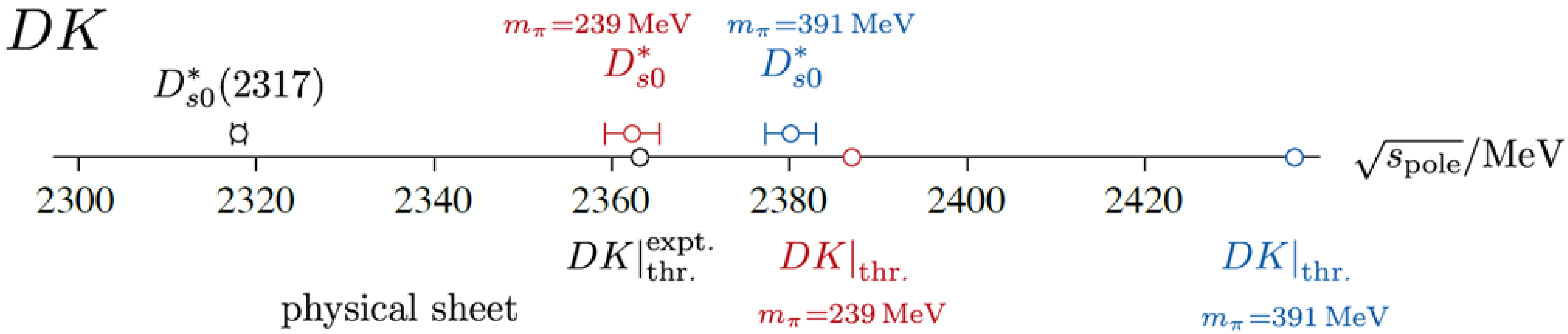
ΔE = 25(3) MeV for $m_\pi \approx 239$ MeV

ΔE = 57(3) MeV for $m_\pi \approx 391$ MeV

c.f. experiment ΔE ≈ 45 MeV (decays to $D_s \pi^0$)

DK (isospin=0) – *S*-wave poles

[JHEP 02 (2021) 100]



Bound-state pole strongly coupled to *S*-wave *DK*

ΔE = 25(3) MeV for $m_\pi \approx 239$ MeV $\quad Z \lesssim 0.11$

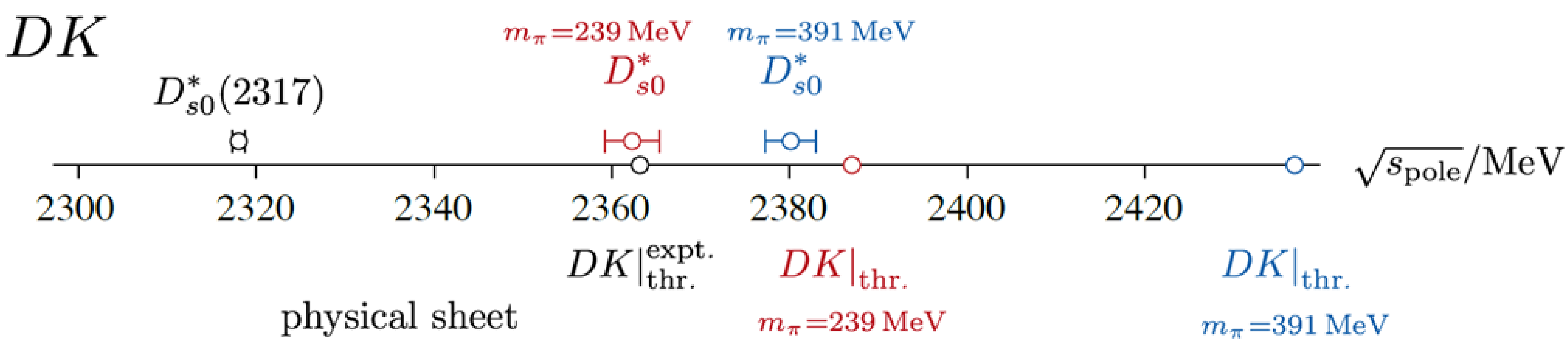
ΔE = 57(3) MeV for $m_\pi \approx 391$ MeV $\quad Z \approx 0.13(6)$

c.f. experiment ΔE ≈ 45 MeV (decays to $D_s \pi^0$)

Weinberg [PR 137, B672 (1965)] compositeness, $0 \leq Z \leq 1$
(assuming binding is sufficiently weak)

DK (isospin=0) – *S*-wave poles

[JHEP 02 (2021) 100]



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ΔE = 25(3) MeV for $m_\pi \approx 239$ MeV Z $\lesssim$ 0.11

ΔE = 57(3) MeV for $m_\pi \approx 391$ MeV Z ≈ 0.13(6)

c.f. experiment ΔE ≈ 45 MeV (decays to $D_s \pi^0$)

Also deeply bound state in *P*-wave, D_s^*, but doesn't strongly influence *DK* scattering at these energies

$D\bar{K}$ (isospin=0,1)

Exotic flavour $(\bar{l}\,\bar{l}\,c\,s)$

[JHEP 02 (2021) 100]

Use many operators,

$\sim D\bar{K}$

$D\bar{K}$ (isospin=0,1)

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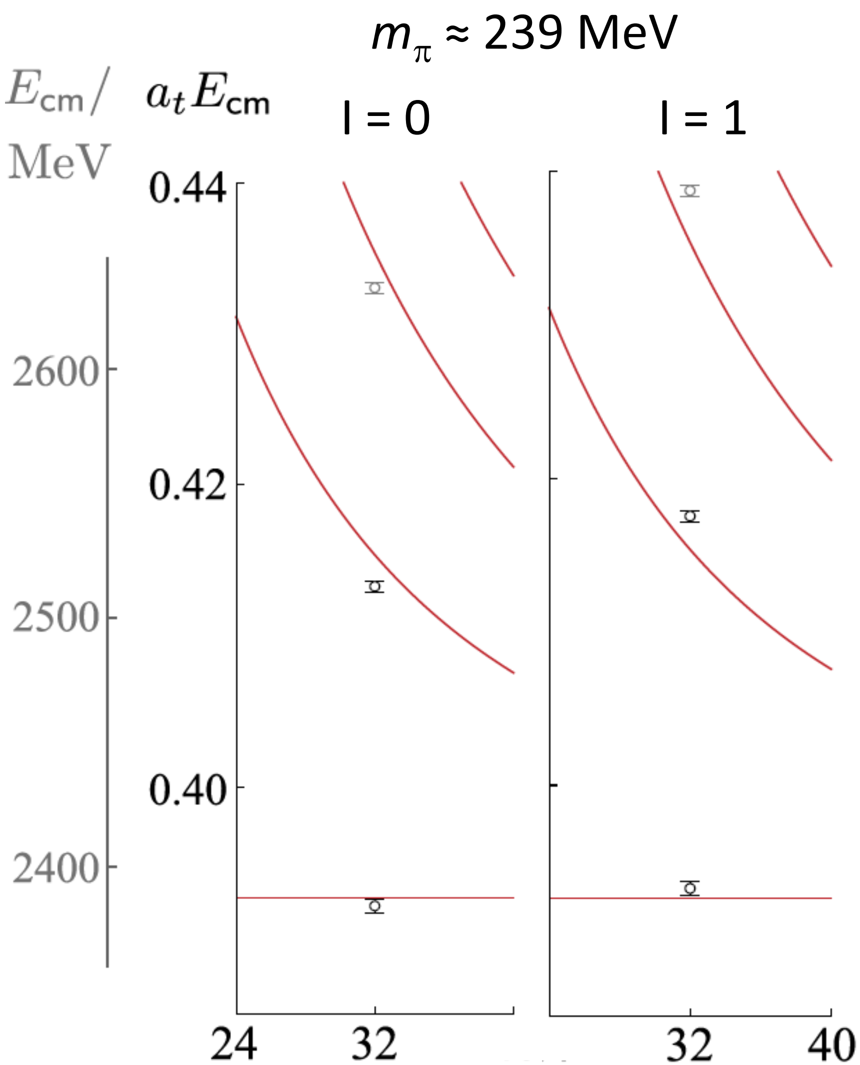
[JHEP 02 (2021) 100]

[0,0,0] $J^P = 0^+$, ...



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[JHEP 02 (2021) 100]

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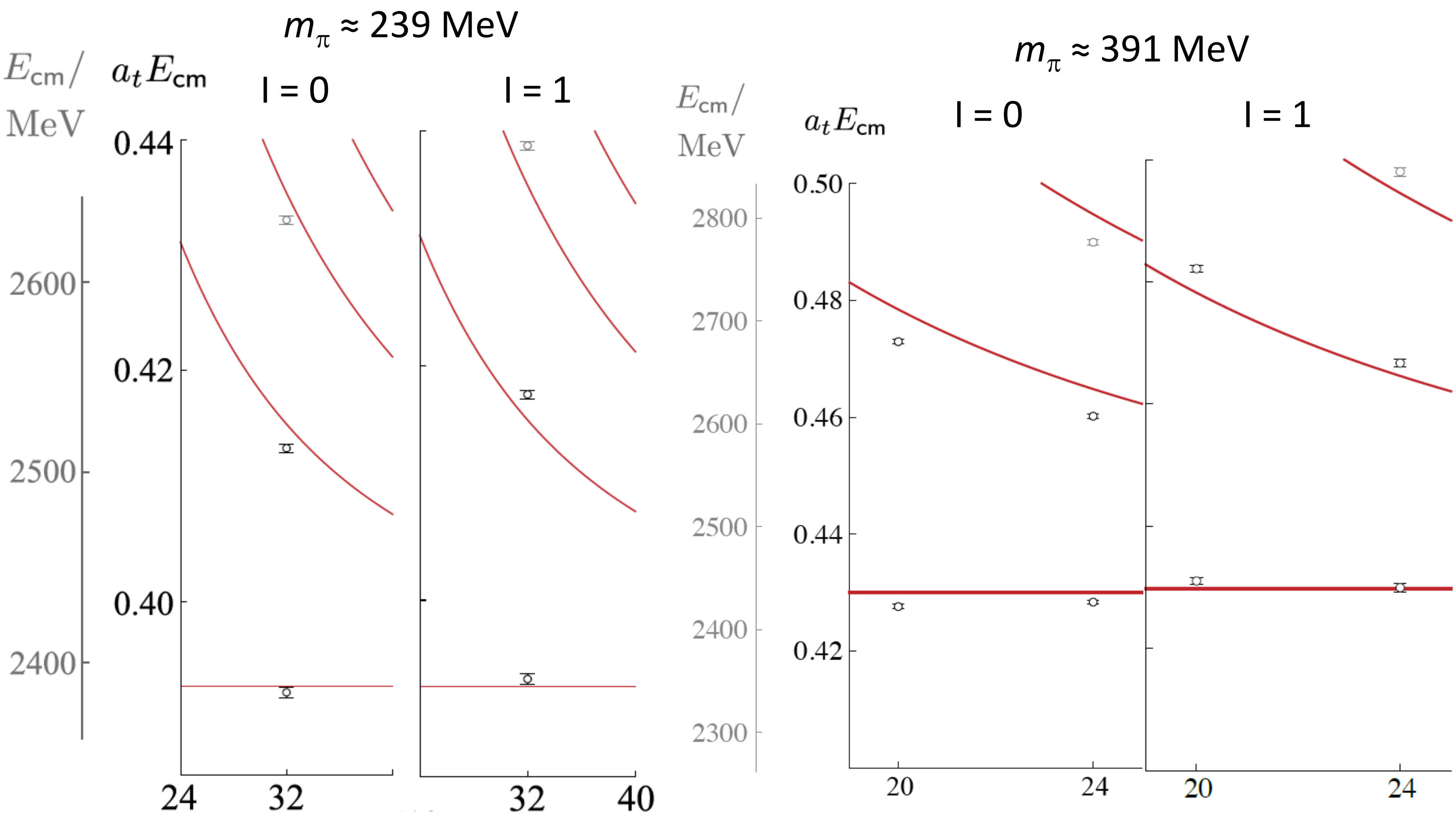

$D\bar{K}$ (isospin=0,1)

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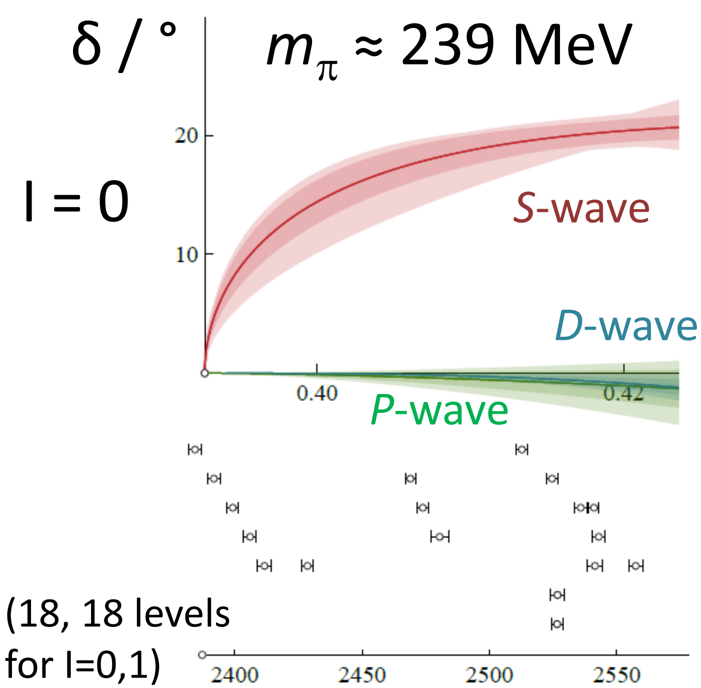
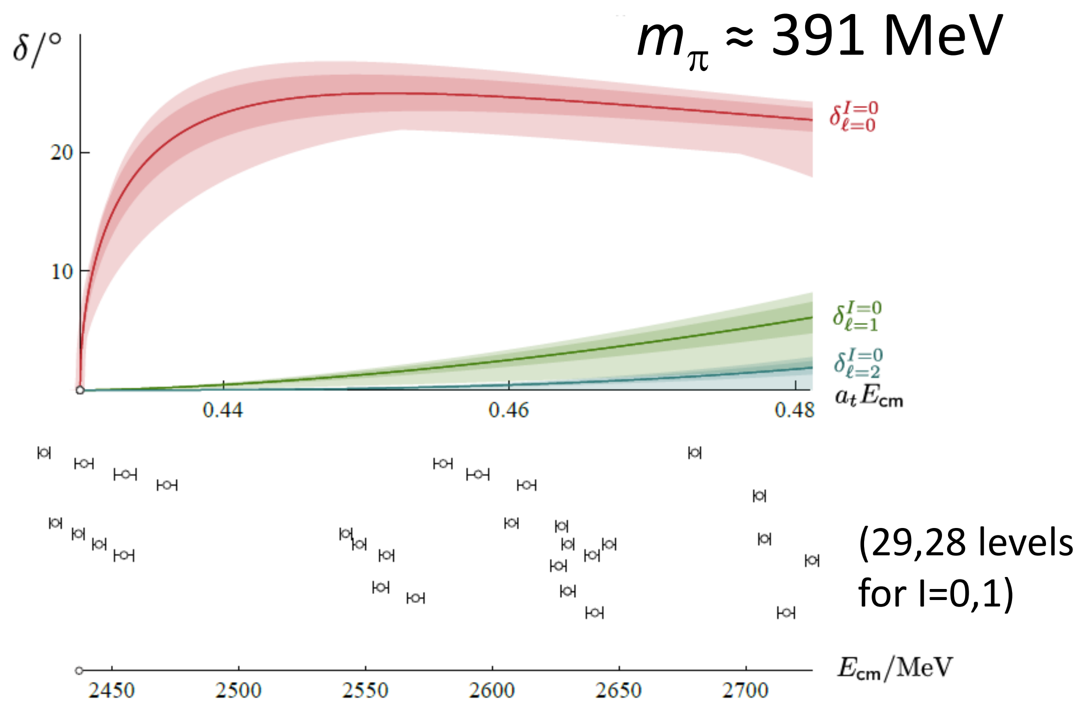
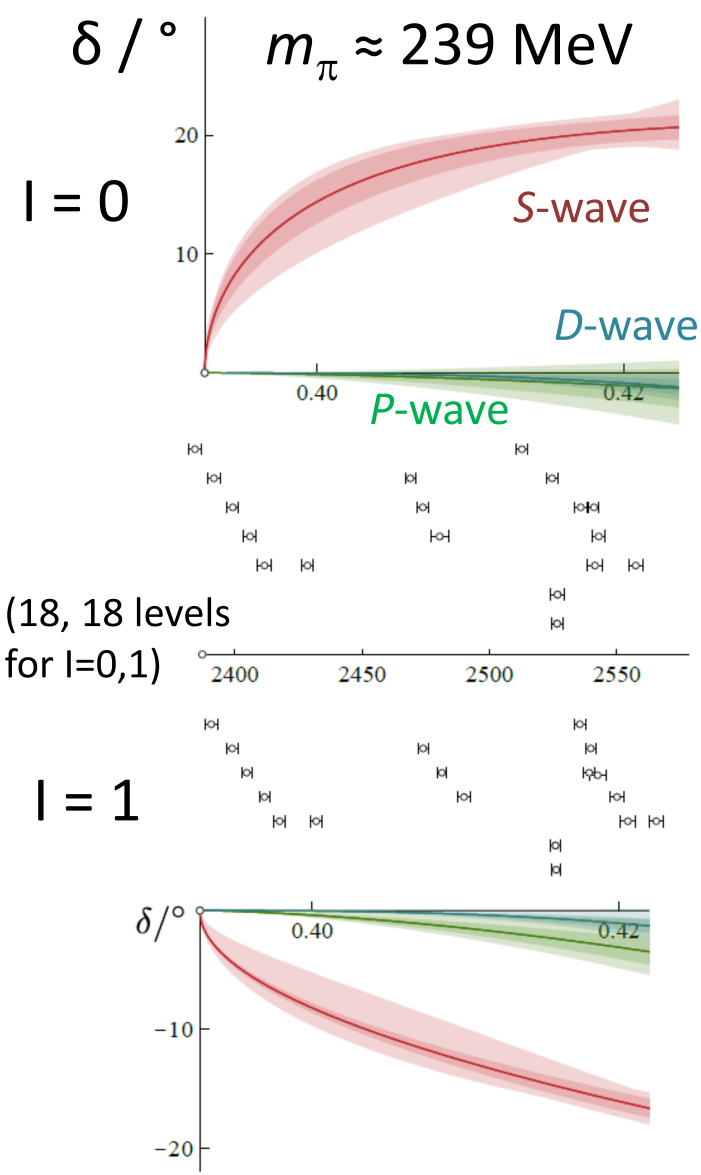


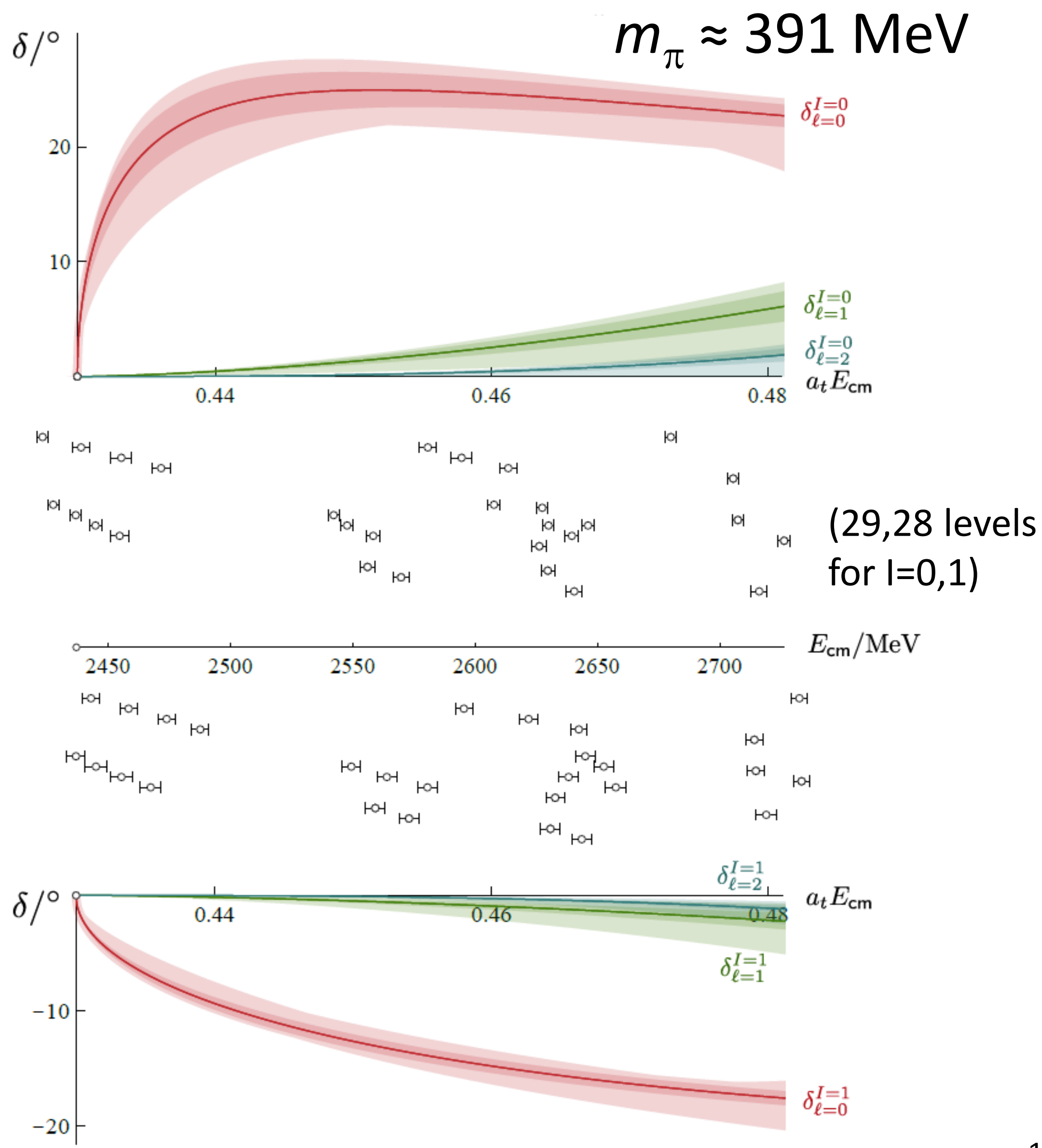


$D\bar{K}$ (isospin=0,1)

Exotic flavour $(\bar{l}\bar{l}cs)$

[JHEP 02 (2021) 100]



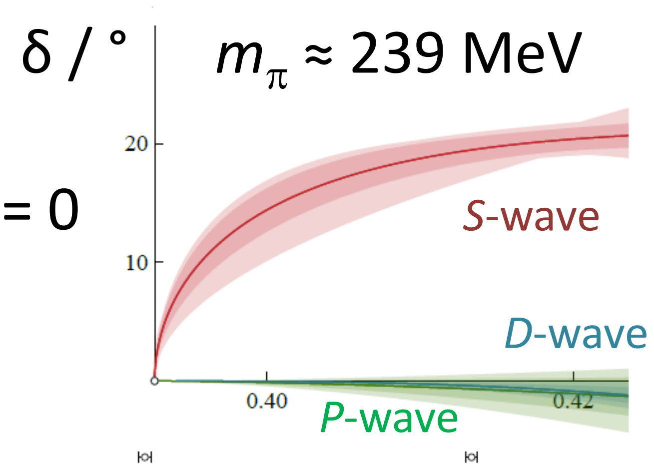


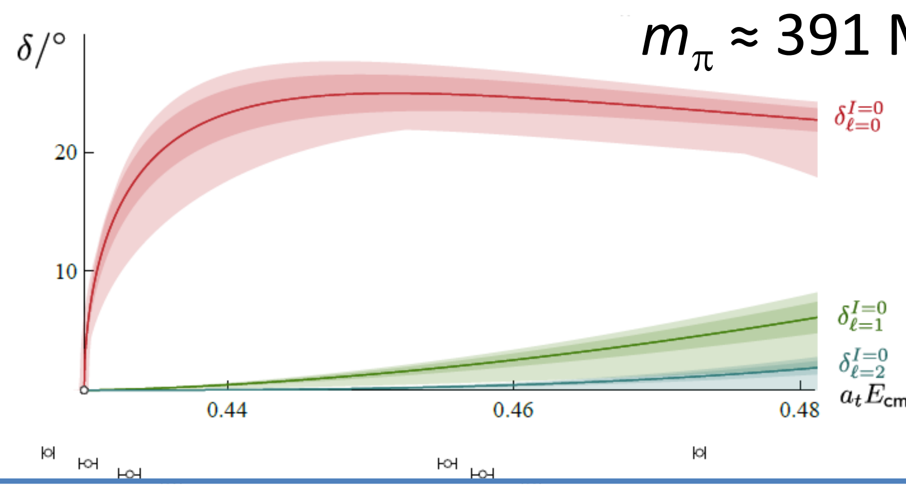
$D\bar{K}$ (isospin=0,1)

Exotic flavour $(\bar{l}\,\bar{l}\,c\,s)$

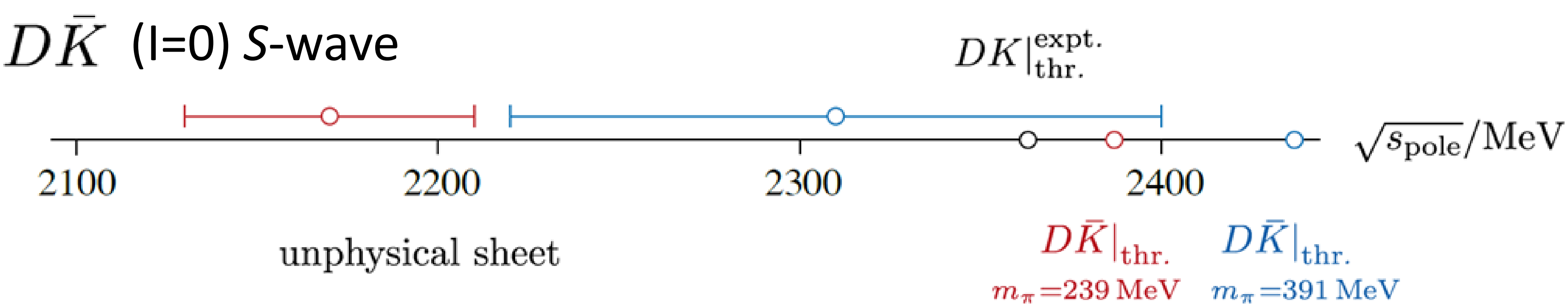
[JHEP 02 (2021) 100]

I = 0

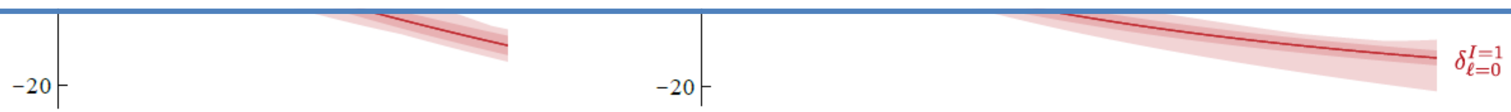




$D\bar{K}$ (I=0) *S*-wave



Suggestion of a **virtual bound-state** pole (**exotic flavour**)



Dπ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 10 (2016) 011]

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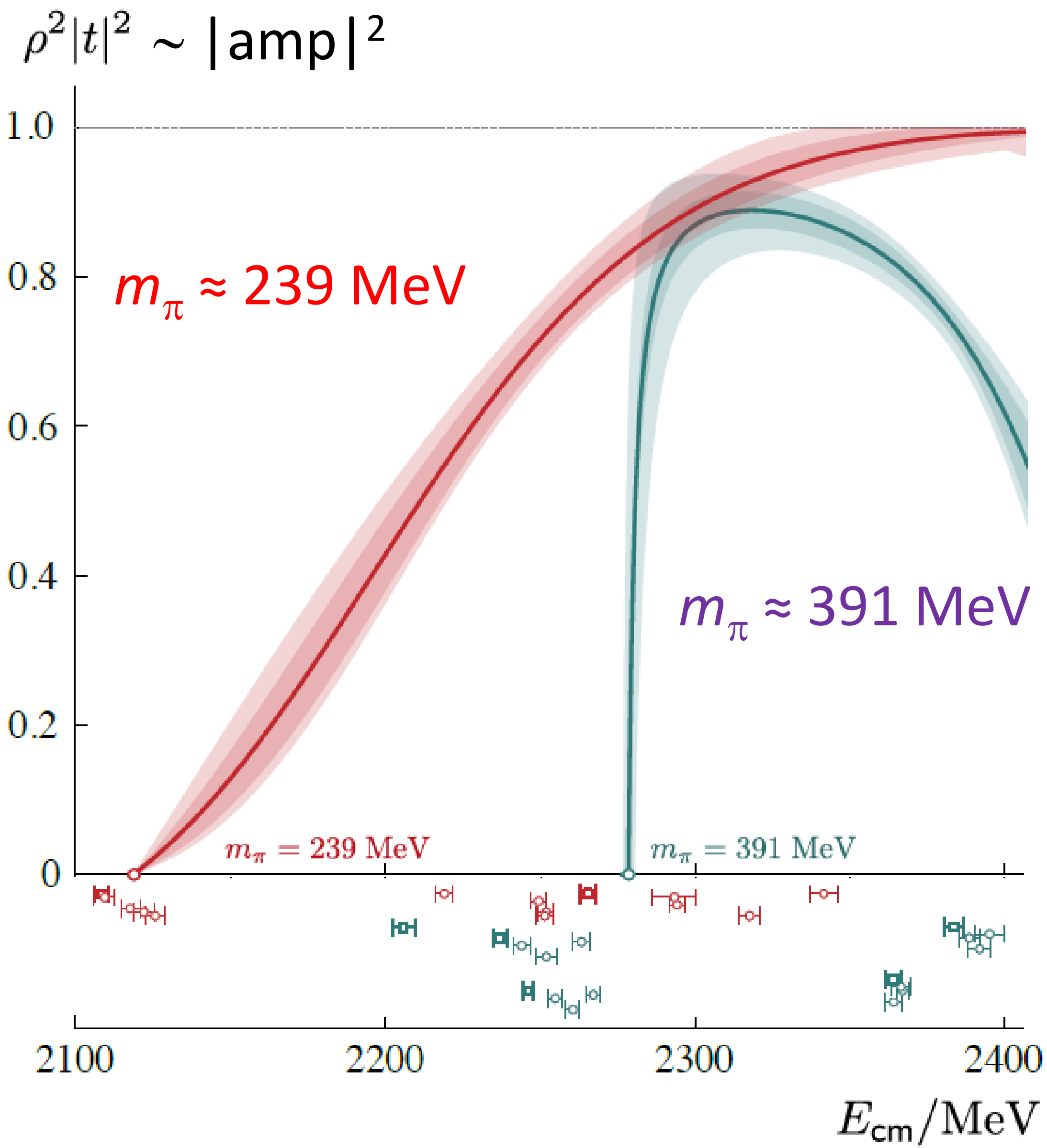

$m_\pi \approx 239$ MeV
29 energy levels
(1 volume)

$m_\pi \approx 391$ MeV
47 energy levels
(3 volumes)

Dπ (isospin=1/2) – S-wave

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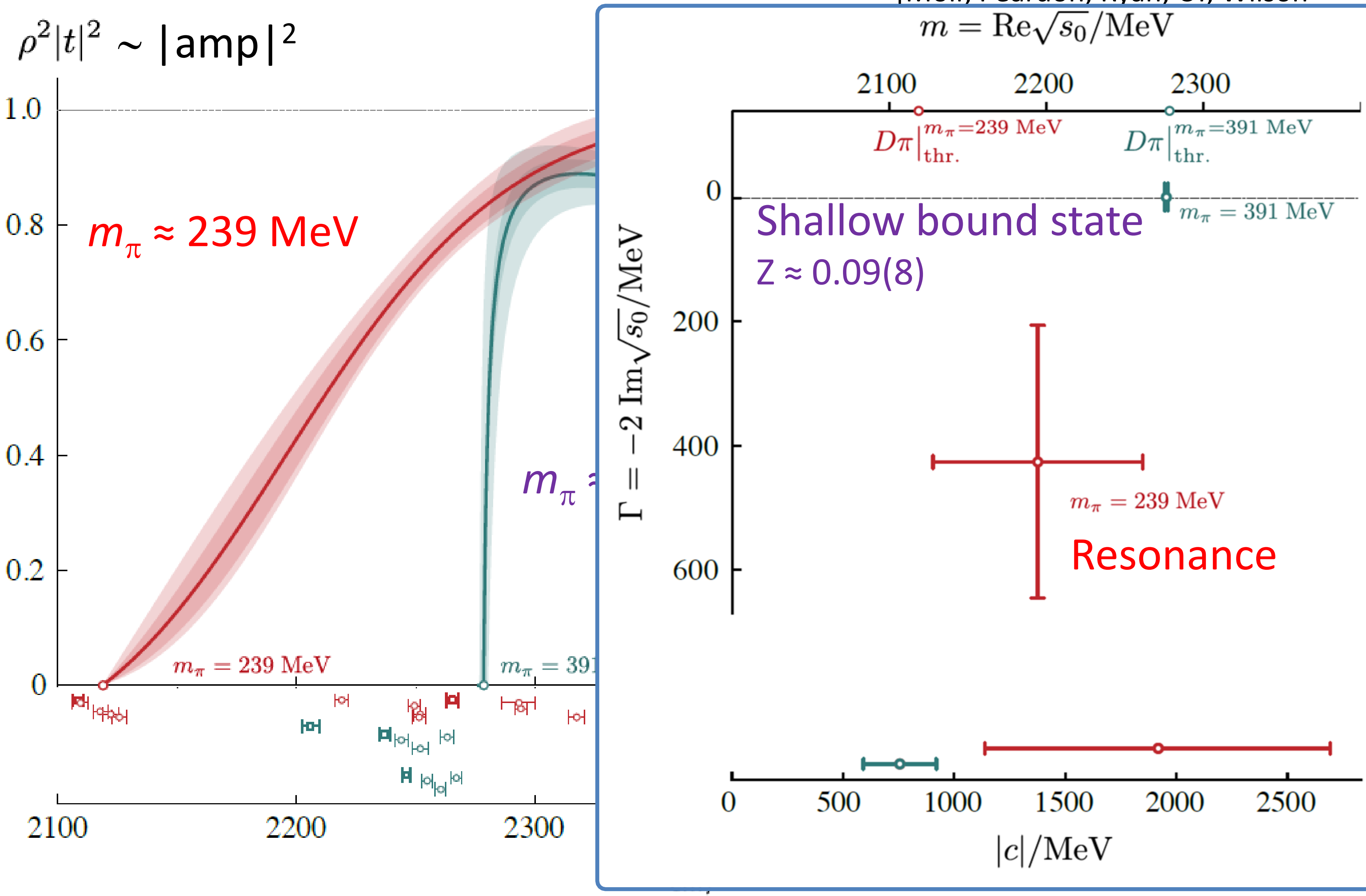

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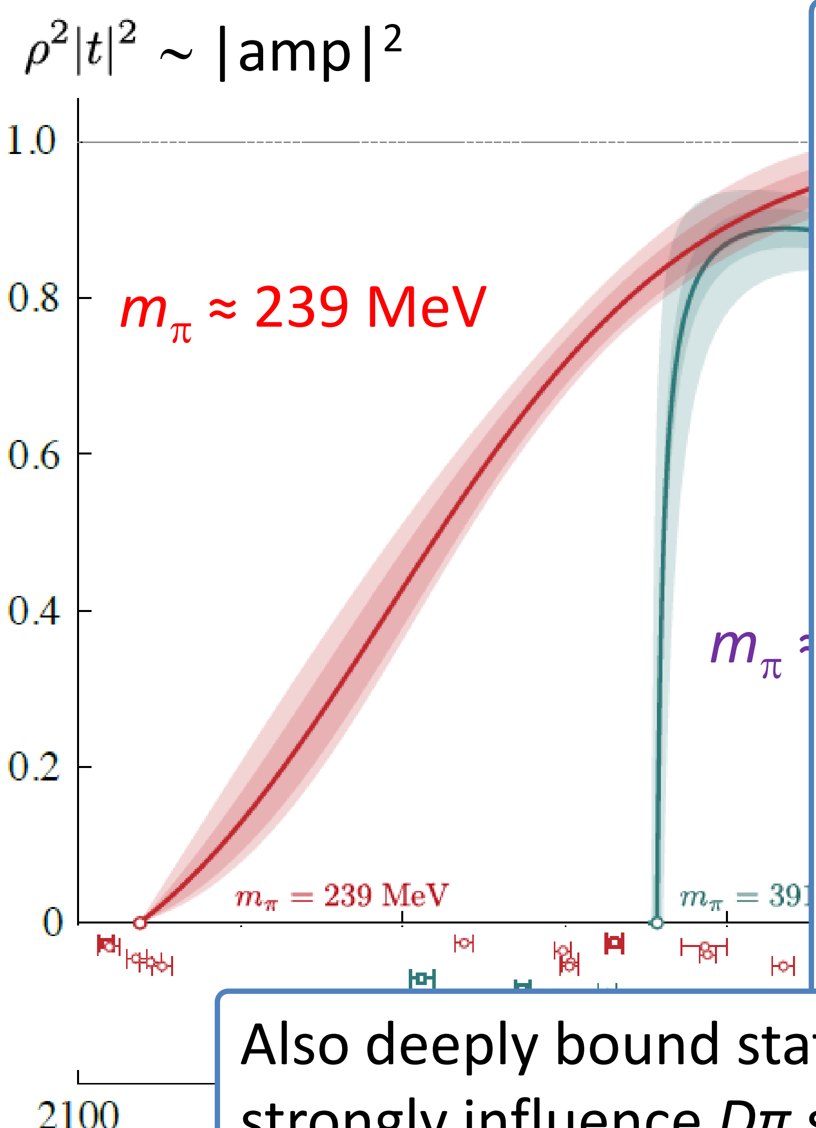
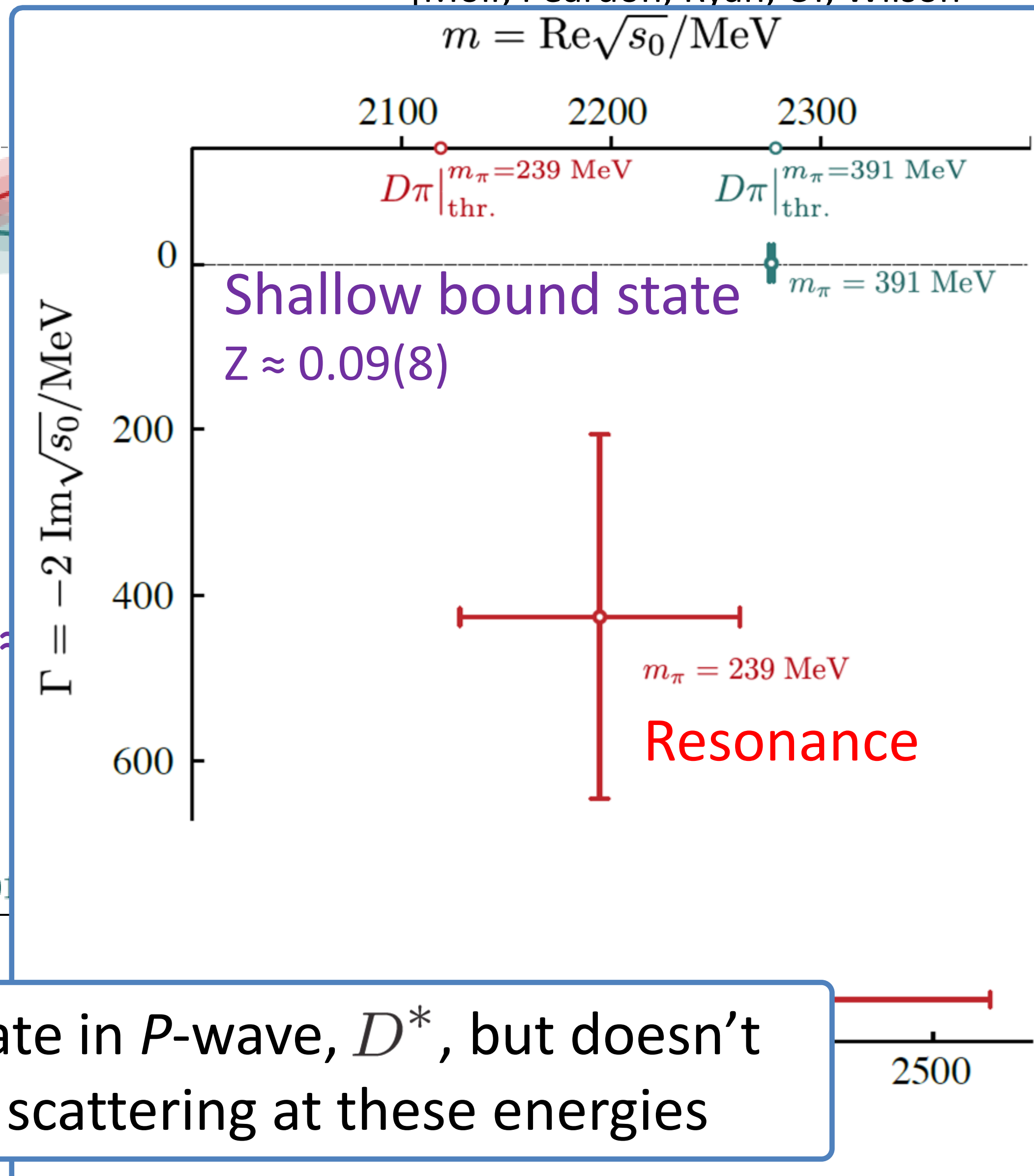
[Moir, Peardon, Ryan, CT, Wilson





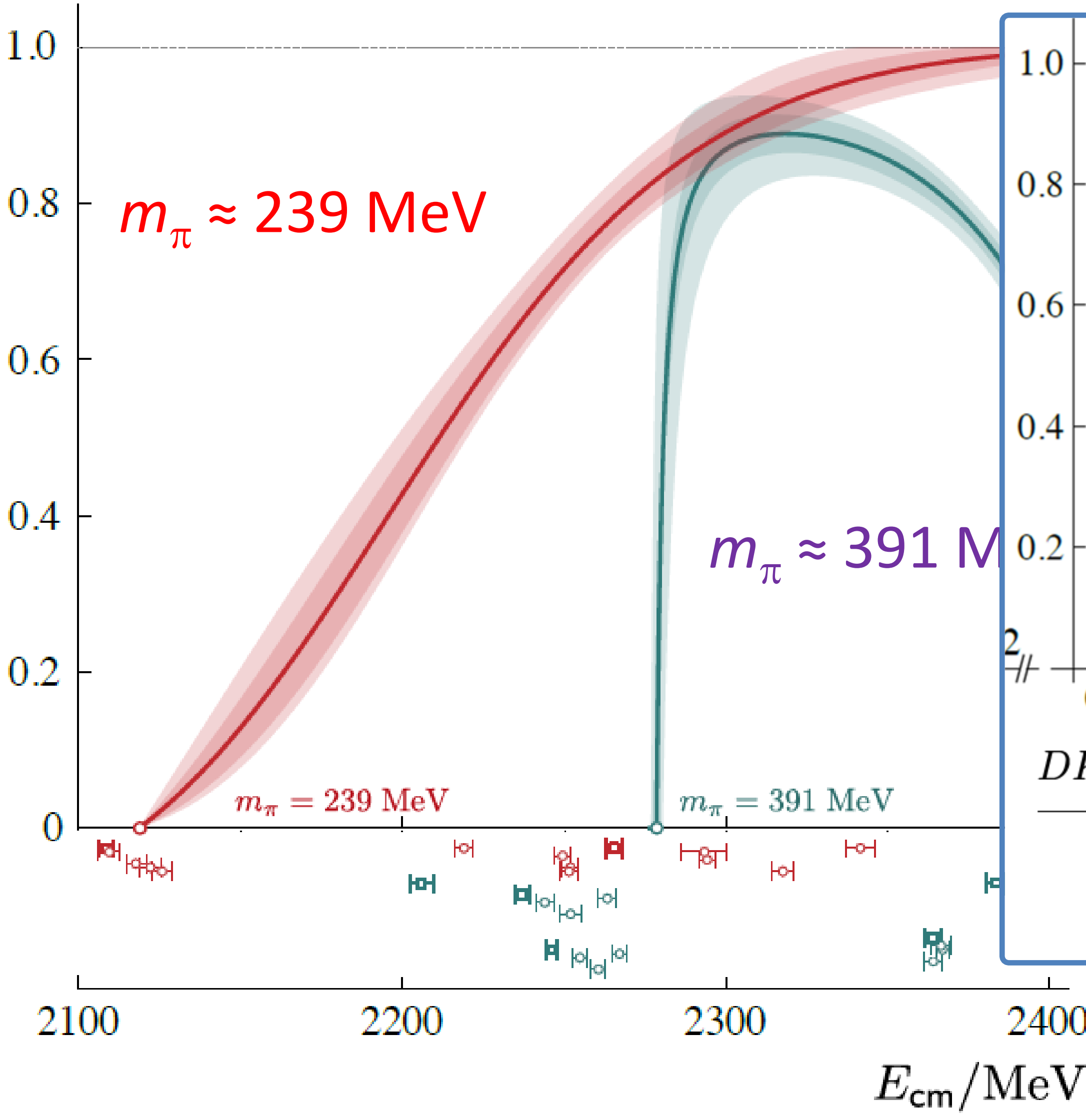
Also deeply bound state in *P*-wave, D^*, but doesn't strongly influence *Dπ* scattering at these energies

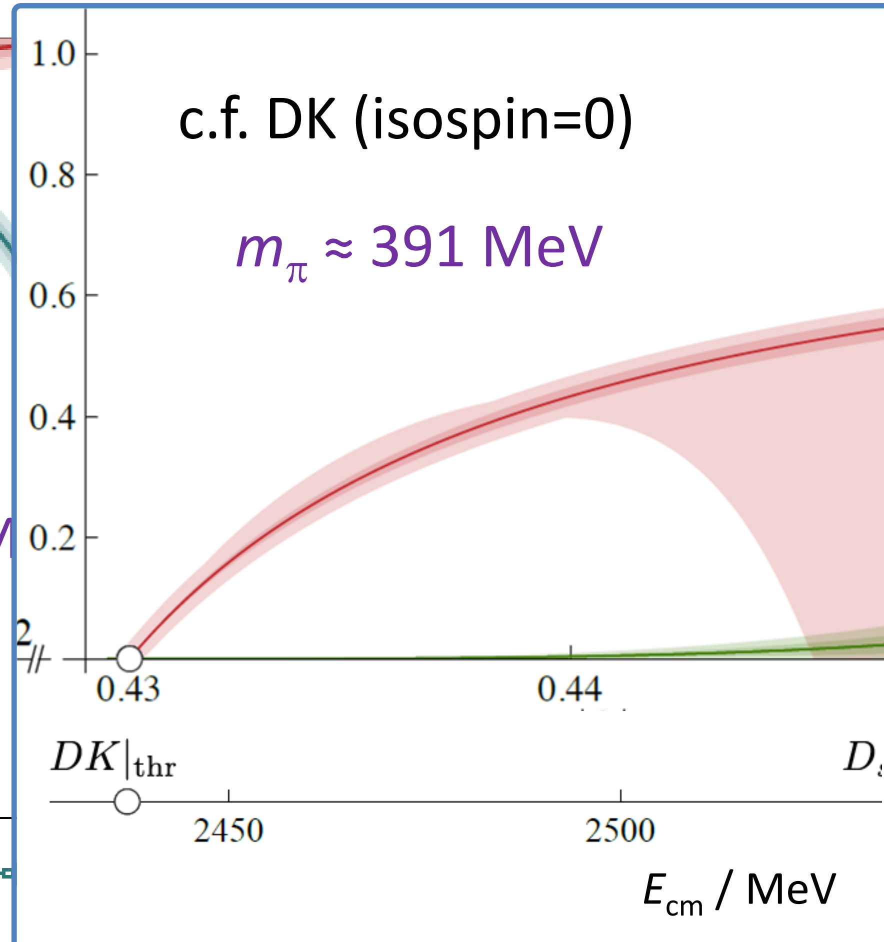
Dπ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 10 (2016) 011]

$\rho^2 |t|^2 \sim |\text{amp}|^2$

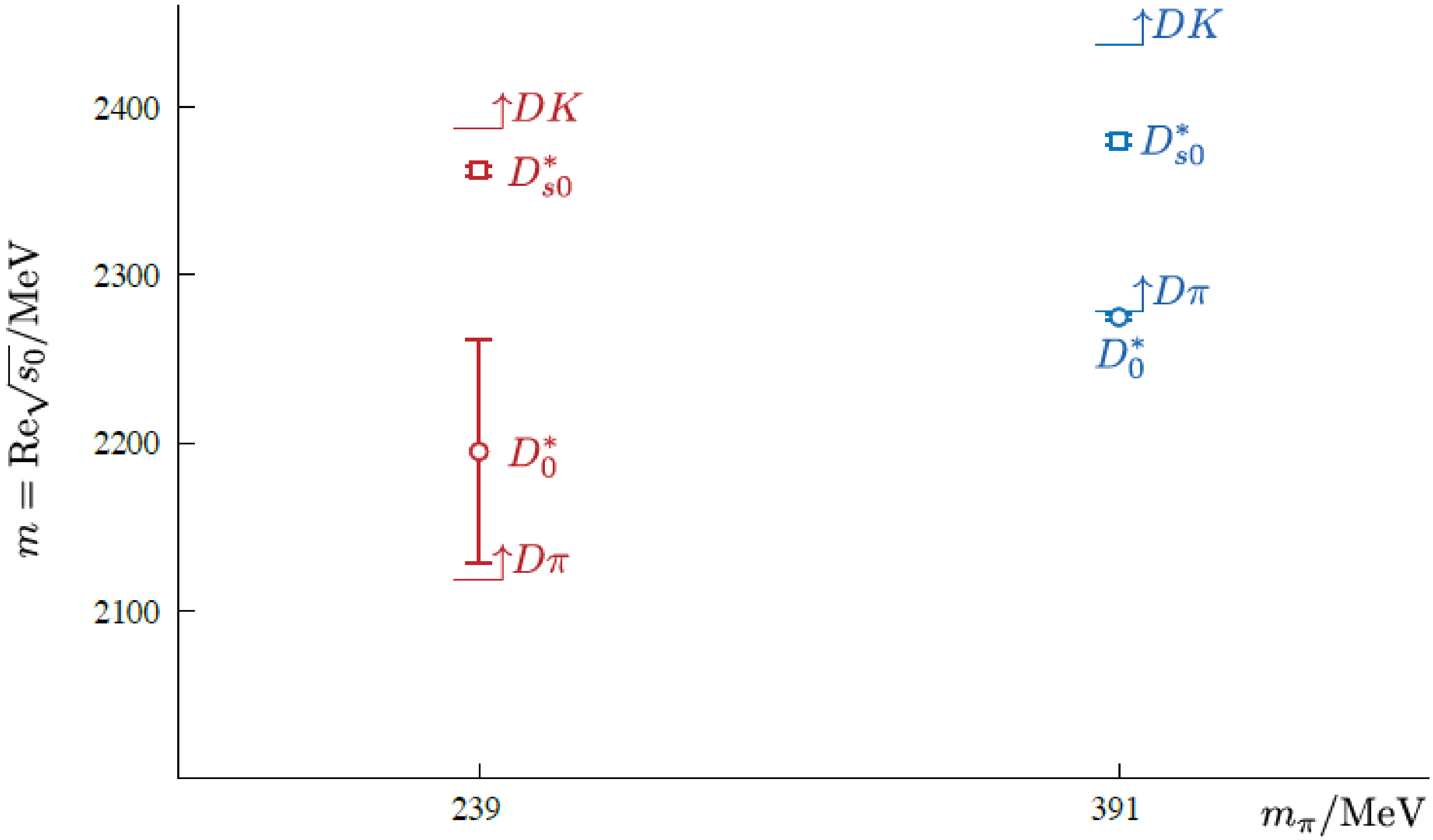




47 energy levels
(3 volumes)

DK and Dπ – S-wave poles

[JHEP 07 (2021) 123, JHEP 02 (2021) 100, JHEP 10 (2016), 011]



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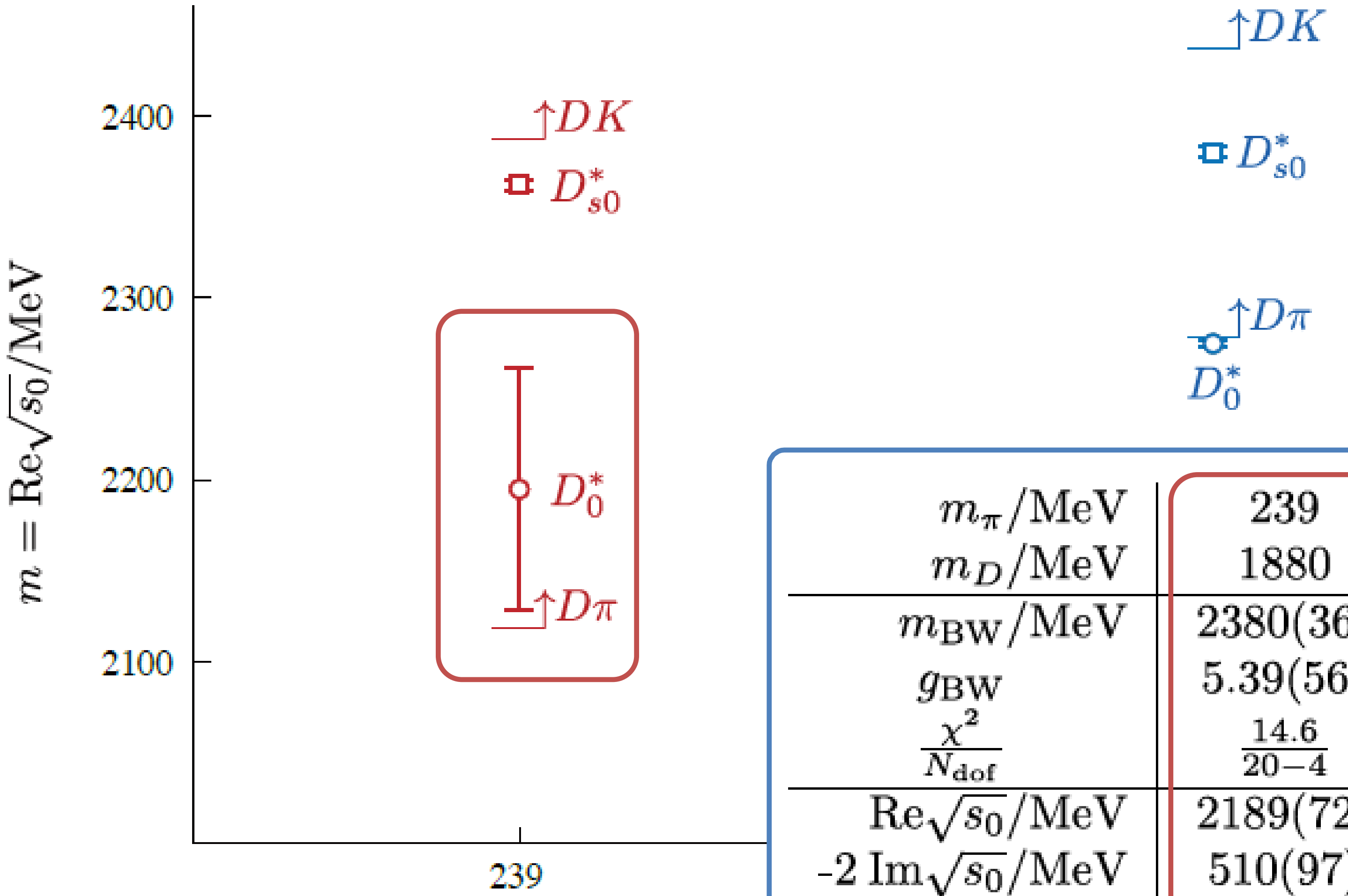
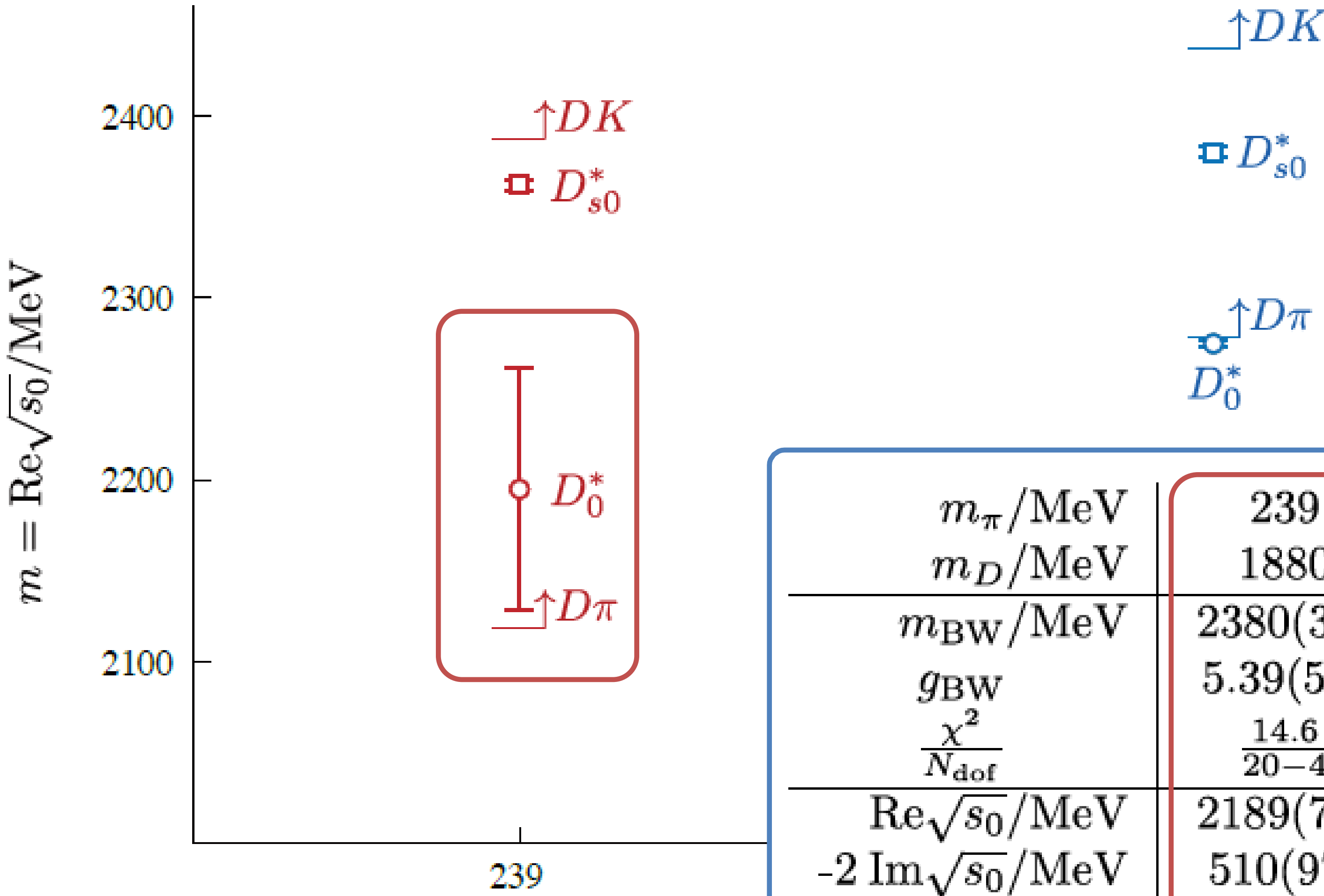


m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\mathrm{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\mathrm{Re}\sqrt{s_0}$/MeV	2189(72)	2275(1)
$-2\,\mathrm{Im}\sqrt{s_0}$/MeV	510(97)	-
$\lvert c\rvert$/MeV	2391(411)	826(133)

DK and Dπ – S-wave poles

[JHEP 07 (2021) 123, JHEP 02 (2021) 100, JHEP 10 (2016), 011]



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D_0^* pole position may be lower than currently reported exp. mass. (See also Du *et al*, PRL 126, 192001 (2021), 2012.04599)

SU(3) flavour symmetry

[JHEP 02 (2021) 100]

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SU(3) multiplets:

$D_{(s)}$ $\overline{\mathbf{3}}$ Light/strange meson $\mathbf{8}$ or $\mathbf{1}$

$$\overline{\mathbf{3}} \otimes \mathbf{8} \to \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \quad \overline{\mathbf{3}} \otimes \mathbf{1} \to \overline{\mathbf{3}}$$

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[JHEP 02 (2021) 100]

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$(I=0)$ DK-$D_s\eta$: $\overline{\mathbf{3}} \oplus \overline{\mathbf{15}}$

$(I=\frac{1}{2})$ $D\pi$-$D\eta$-$D_s\bar{K}$: $\overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=1)$ DK-$D_s\pi$: $\mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=0)$ $D\bar{K}$: $\mathbf{6}$

$(I=\frac{1}{2})$ D_sK, $(I=1)$ $D\bar{K}$, $(I=\frac{3}{2})$ $D\pi$: $\overline{\mathbf{15}}$

SU(3) flavour symmetry

[JHEP 02 (2021) 100]

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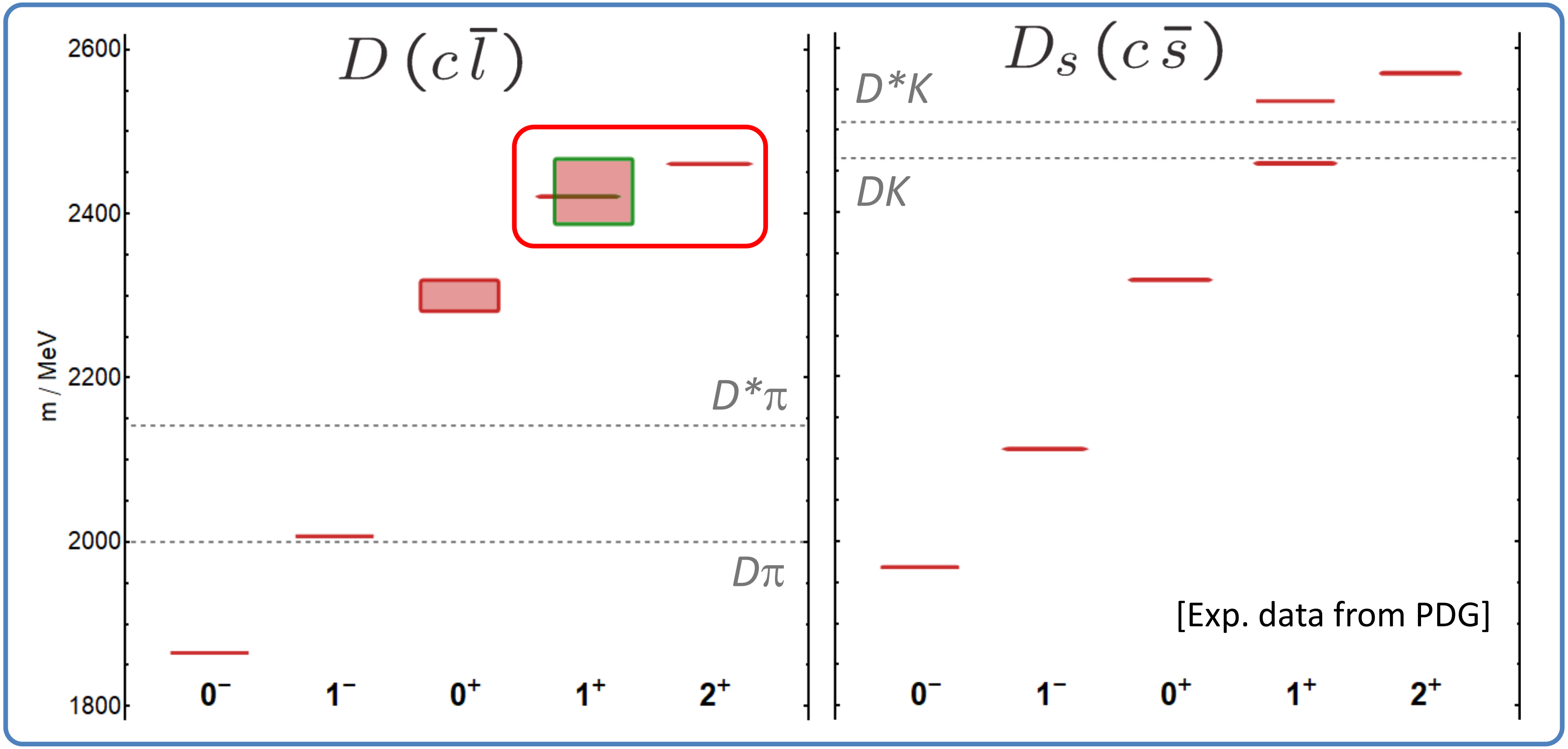
$(I=\frac{1}{2})\ D_sK$, $(I=1)\ D\bar{K}$, $(I=\frac{3}{2})\ D\pi$: $\overline{\mathbf{15}}$

S-wave results [broken SU(3)] suggest:

- $\overline{\mathbf{3}}$ resonance/bound state
- $\mathbf{6}$ virtual bound state
- $\overline{\mathbf{15}}$ weak repulsion

[See also PR D87, 014508 (2013) (1208.4535); PL B767, 465 (2017) (1610.06727); PR D98, 094018 (2018) (1712.07957); *PR D98* 014510 (2018) (1801.10122); *EPJ C79, 13 (2019)* (1811.05585); arXiv:2106.15391]

Charm (D) and charm-strange (D_s) mesons



D* π (isospin=1/2)

[**Lang**, Wilson (HadSpec), arXiv:2205.05026]

Scattering involving non-zero spin hadrons [see also Woss, CT, Dudek, Edwards, Wilson, arXiv:1802.05580 (JHEP)]

J = ℓ ⊗ S and different partial waves with the same J^P can mix dynamically,

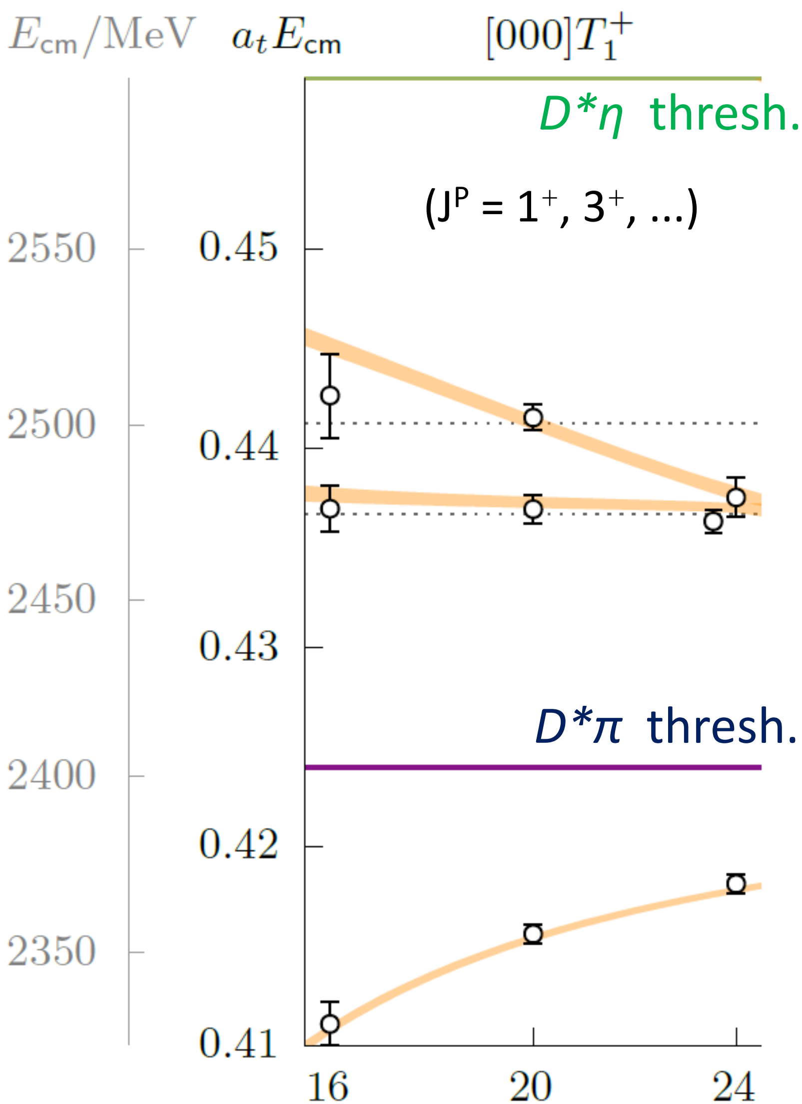
e.g. $J^P = 1^+$ ($^{2S+1}\ell_J = {}^3S_1, {}^3D_1$) $\quad \mathbf{t} = \begin{bmatrix} t({}^3S_1|{}^3S_1) & t({}^3S_1|{}^3D_1) \\ t({}^3S_1|{}^3D_1) & t({}^3D_1|{}^3D_1) \end{bmatrix}$

Finite-volume lattice QCD: reduced sym → additional 'mixing'

D* π (isospin=1/2)

$m_\pi \approx 391$ MeV

[arXiv:2205.05026]



Use many different fermion-bilinear

$$\sim \bar{\psi}\Gamma D \ldots \psi$$

and $D^*\pi$, ... operators

D* π (isospin=1/2)

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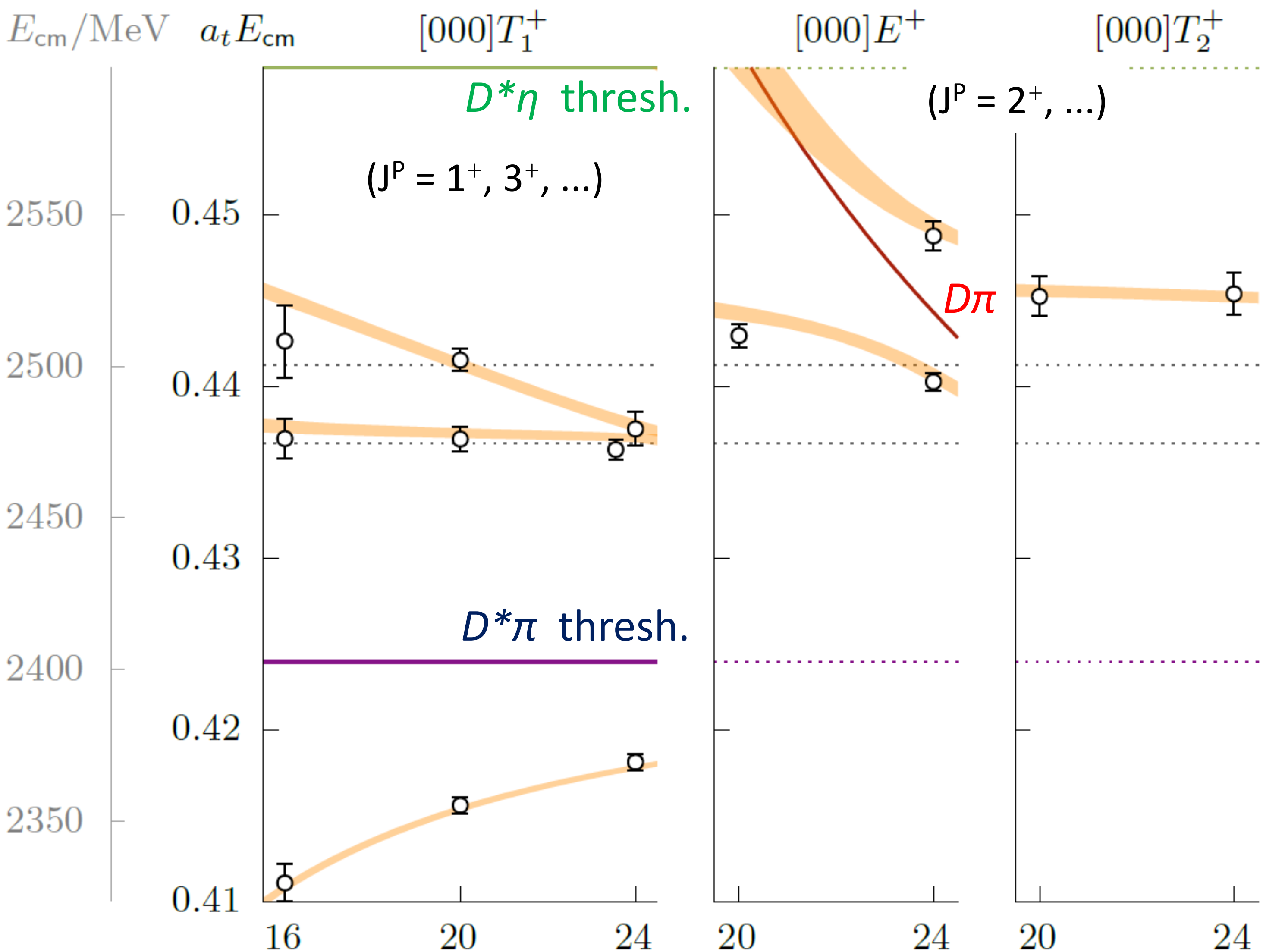
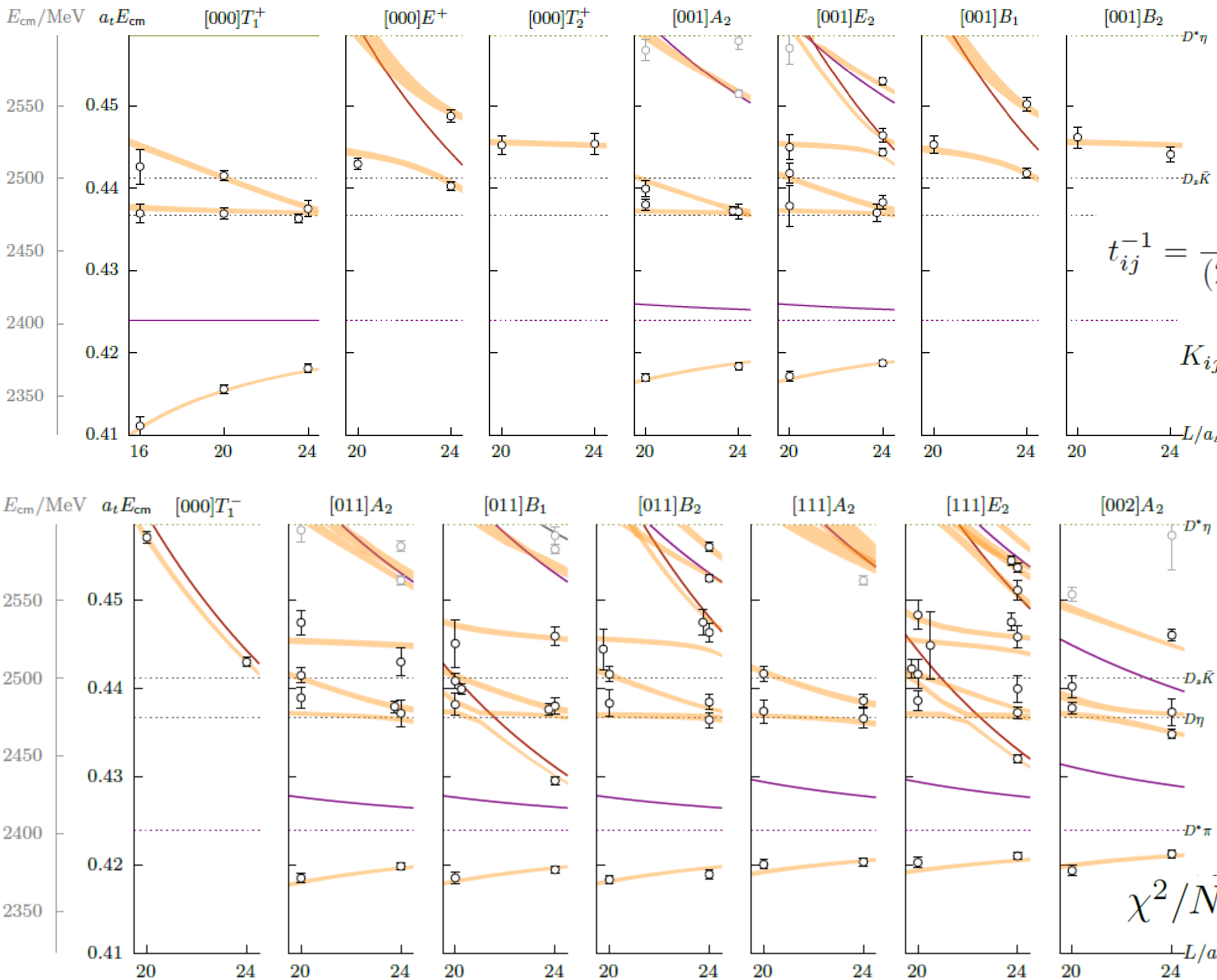
[arXiv:2205.05026]



D* π (isospin=1/2)

(94 energy levels to constrain $J^P = 1^+, 2^+, 0^-, 1^-, 2^-$)

[arXiv:2205.05026]



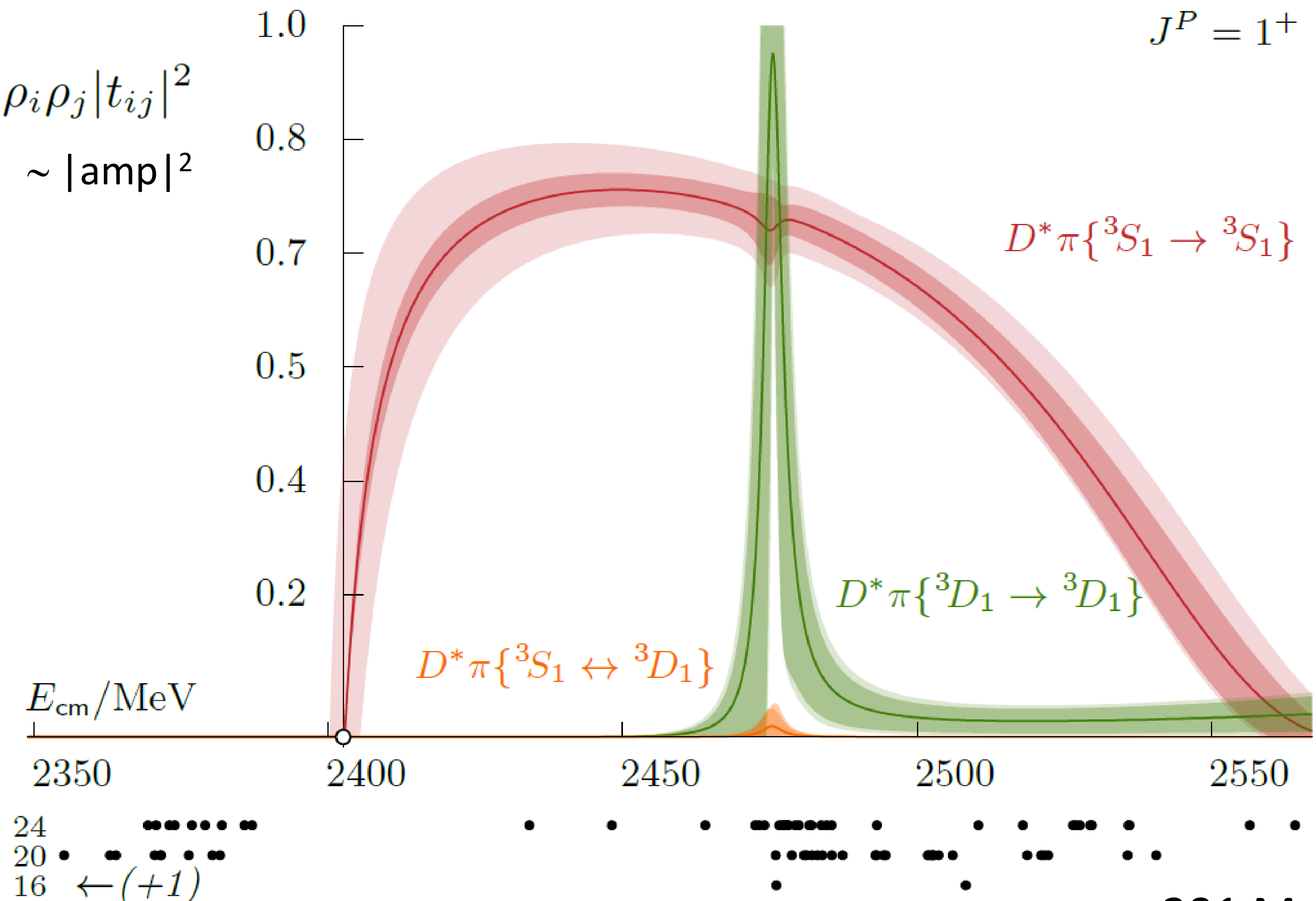
$$t_{ij}^{-1} = \frac{1}{(2k_i)^{\ell_i}} K_{ij}^{-1} \frac{1}{(2k_j)^{\ell_j}} + I_{ij}$$

$$K_{ij} = \sum_{p=1}^{2} \frac{g_{p,i}\, g_{p,j}}{m_p^2 - s} + \gamma_{ij}$$

$$\chi^2/N_{\rm dof} = \frac{95.0}{94-15} = 1.20$$

D* π (isospin=1/2)

[arXiv:2205.05026]



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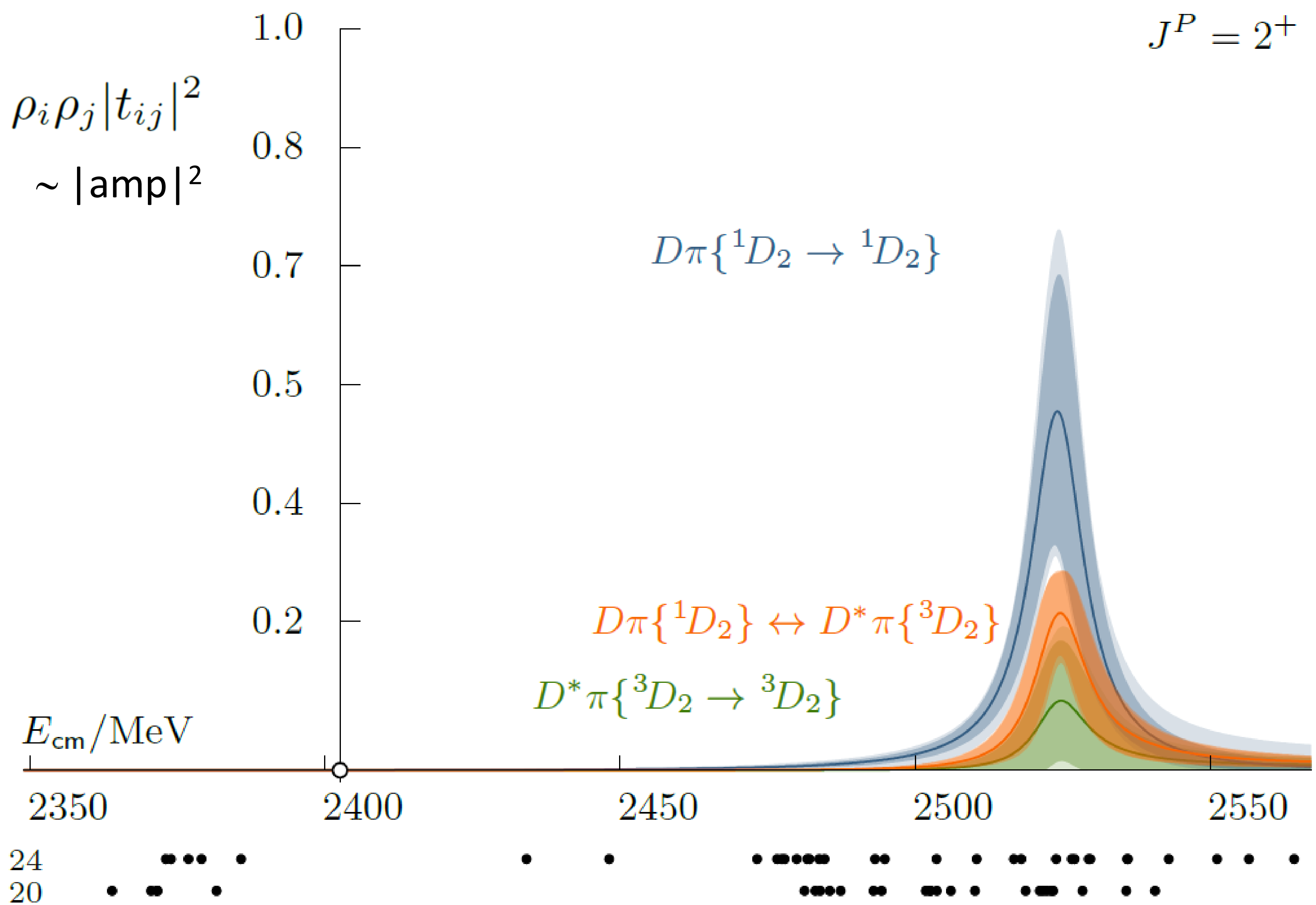

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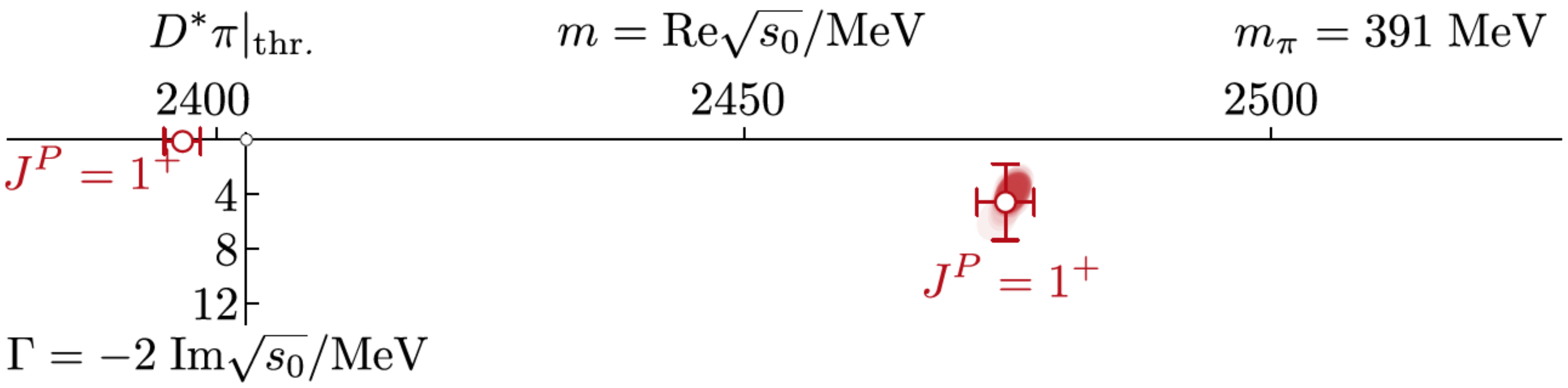
D* π (isospin=1/2) – poles

[arXiv:2205.05026]



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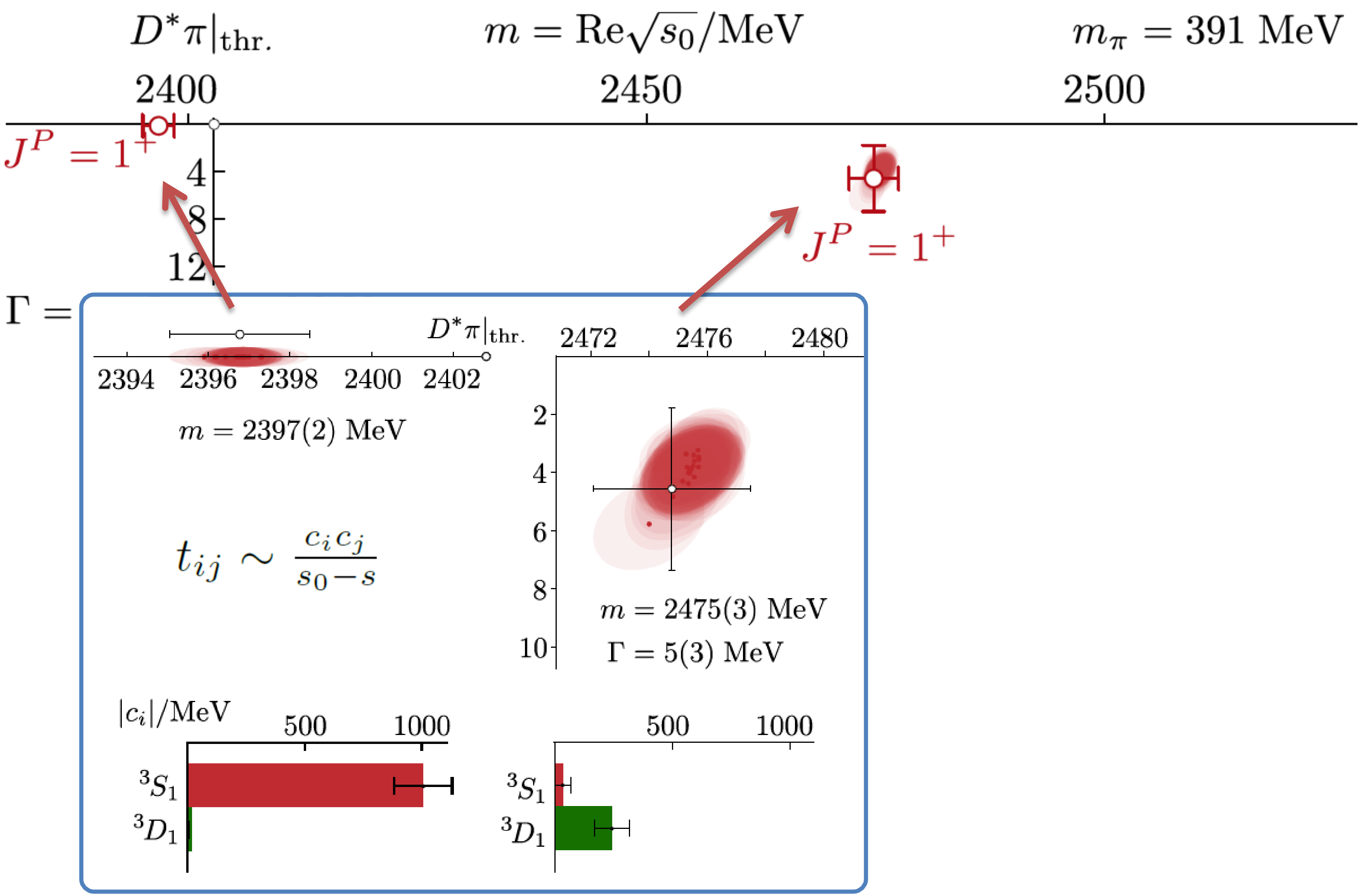

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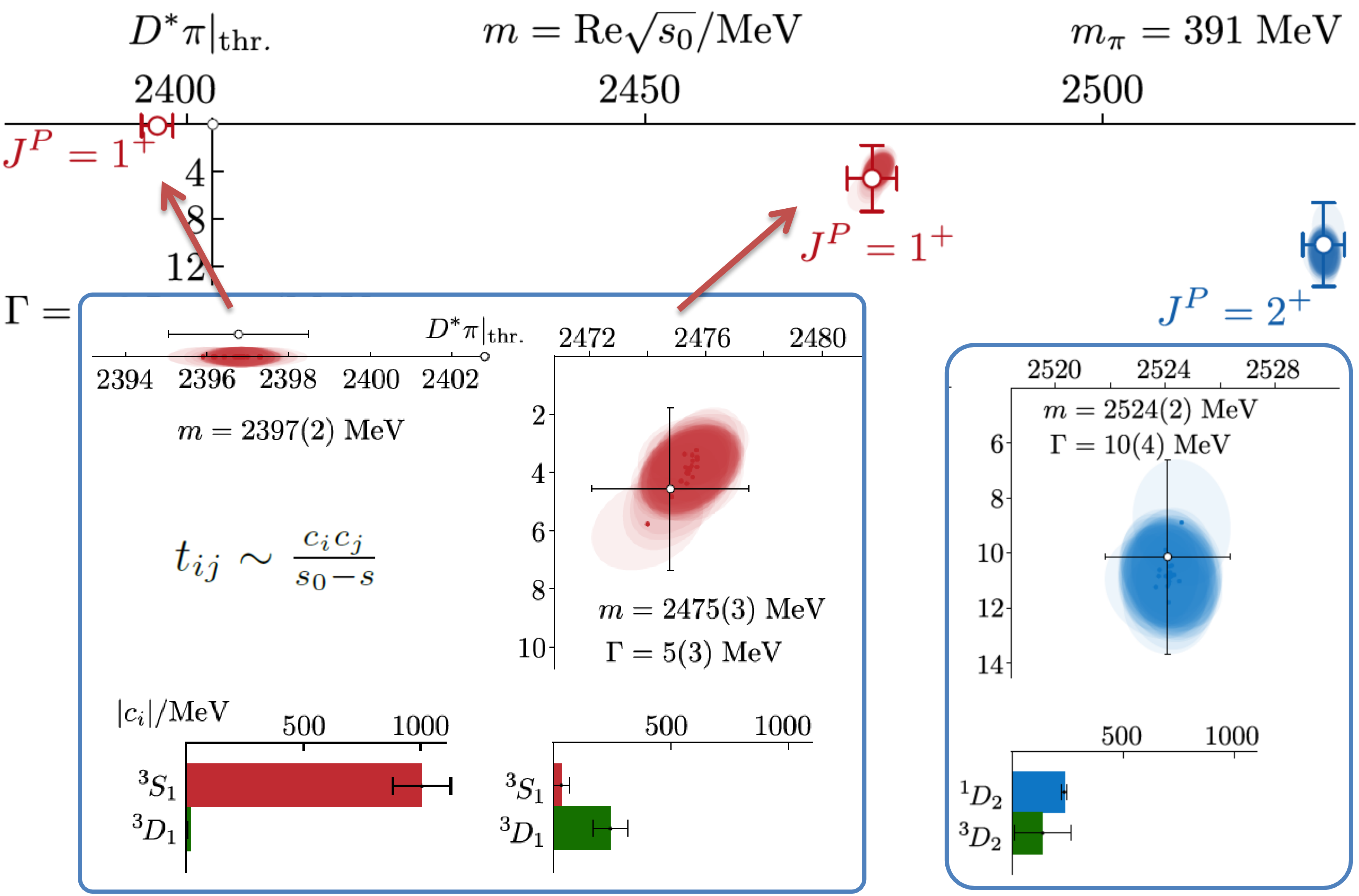
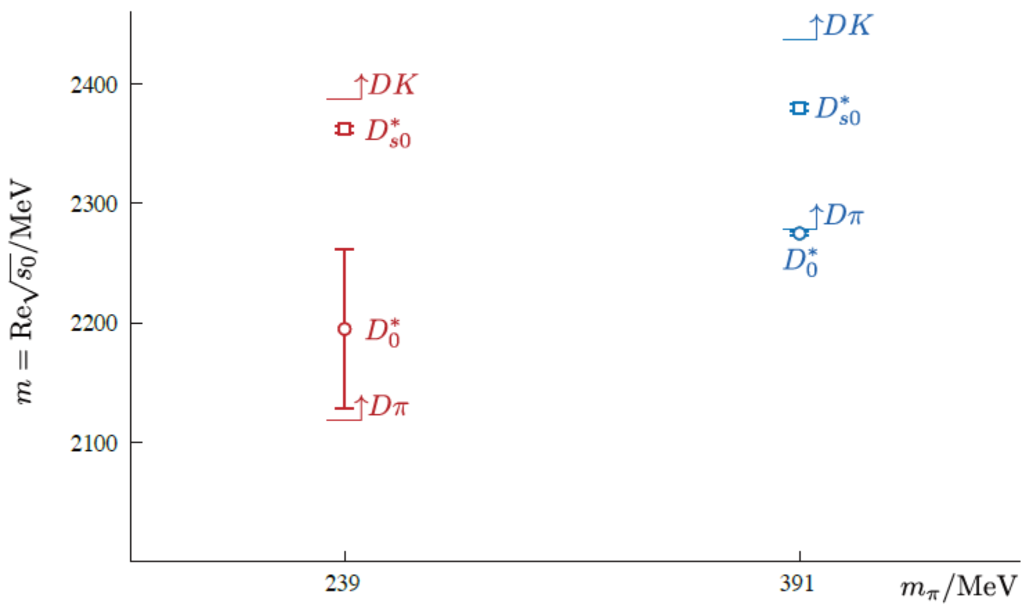


Summary

- Map out energy-dependence of scattering amps using lattice QCD.

- S-wave scattering of psuedoscalars ($J^P=0^+$)
  - Isospin-0 *DK:* bound state
  - Isospin-1/2 *Dπ*: bound state/resonance
  - Exotic-flavour isospin-0 $D\bar{K}$: suggestion of virtual bound state



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- Lighter (or heavier) light quarks? With SU(3) flavour sym?
- Further up in energy, inelastic scattering (3-hadron scattering)

Acknowledgements

Hadron Spectrum Collaboration

[*www.hadspec.org*]

Jefferson Lab and surroundings, USA:

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ODU: Raúl Briceño[1], Andrew Jackura ([1] and Jefferson Lab)

Trinity College Dublin, Ireland: Michael Peardon, Sinéad Ryan, *Nicolas Lang*

UK: University of Cambridge: CT, David Wilson, *Daniel Yeo, James Delaney*

Edinburgh: Max Hansen; Southampton: Bipasha Chakraborty

Tata Institute, India: Nilmani Mathur

Ljubljana, Slovenia: Luka Leskovec