The nucleon-pion scattering lengths from lattice QCD at $m_\pi = 200$ MeV

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“Multihadron Dynamics in a Box - A.D. 2022”
Bethe Center for Theoretical Physics – U. of Bonn
Aug. 19th, 2022

Motivation

- ➔ $N\pi \to N\pi$ is a step toward:

$$N(1440) \to N\pi\pi \qquad\qquad N + A_\mu \to N\pi$$

- ➔ Scattering lengths confronted with ChPT: 'convergence' near the physical point?

- ➔ Direct lattice QCD determinations of scattering lengths constrain $\sigma_{\pi N}$

M. Hoferichter et al. PLB 760 (2016)

Difficulties with nucleon-pion scattering

(compared to meson-meson)

- Additional quark propagator in correlation functions

  → efficient algorithm for all-to-all: Stochastic LapH

  C. Morningstar, et al. PRD 83 (2011); M. Peardon, et al. PRD 80 (2009)

  → efficient contraction of hadron tensors

- Two partial waves for each non-zero J

  → exhaustive determination of B-matrix elements

  C. Morningstar, et al. NPB 910 (2016); M. Gockeler, et al. PRD 86 (2012)

- Worse signal-to-noise problem in baryon correlation functions

  → high-statistics on CLS ensemble D200 (open temporal bc's)

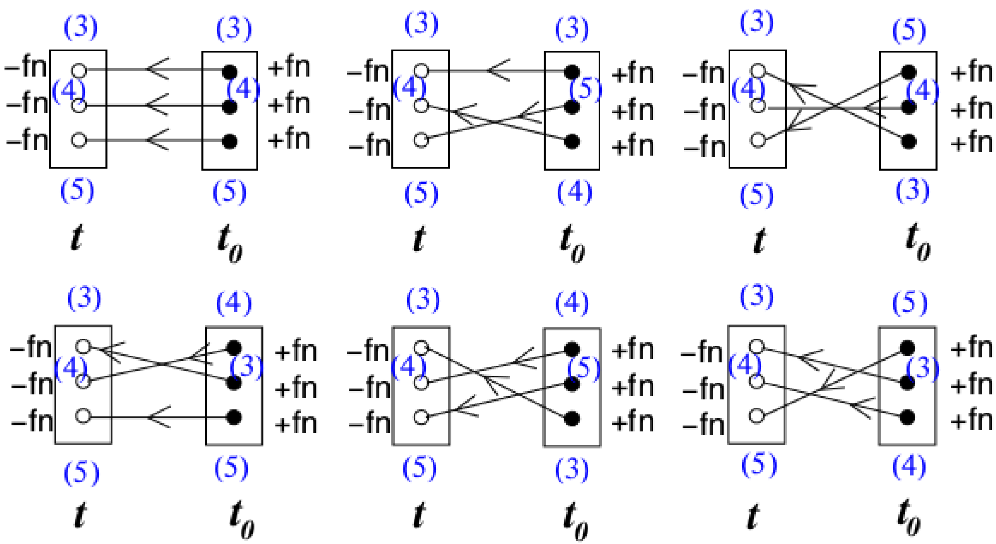
$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV},$$

$$N_{\text{meas}} = 2000,\ t_{\text{max}} = 25a,\ m_\pi t_{\text{bnd}} = 2.5$$

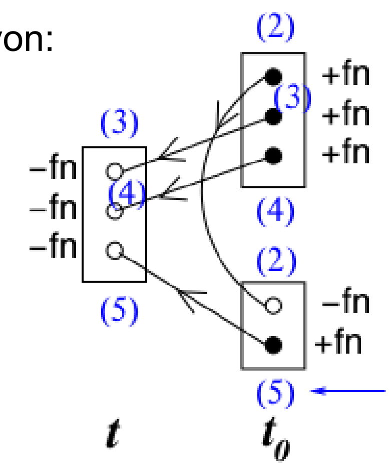
M. Bruno, et al. JHEP 02 (2015); JB and S. Schaefer NPB 874 (2013)

Correlation functions constructed by tensor contraction

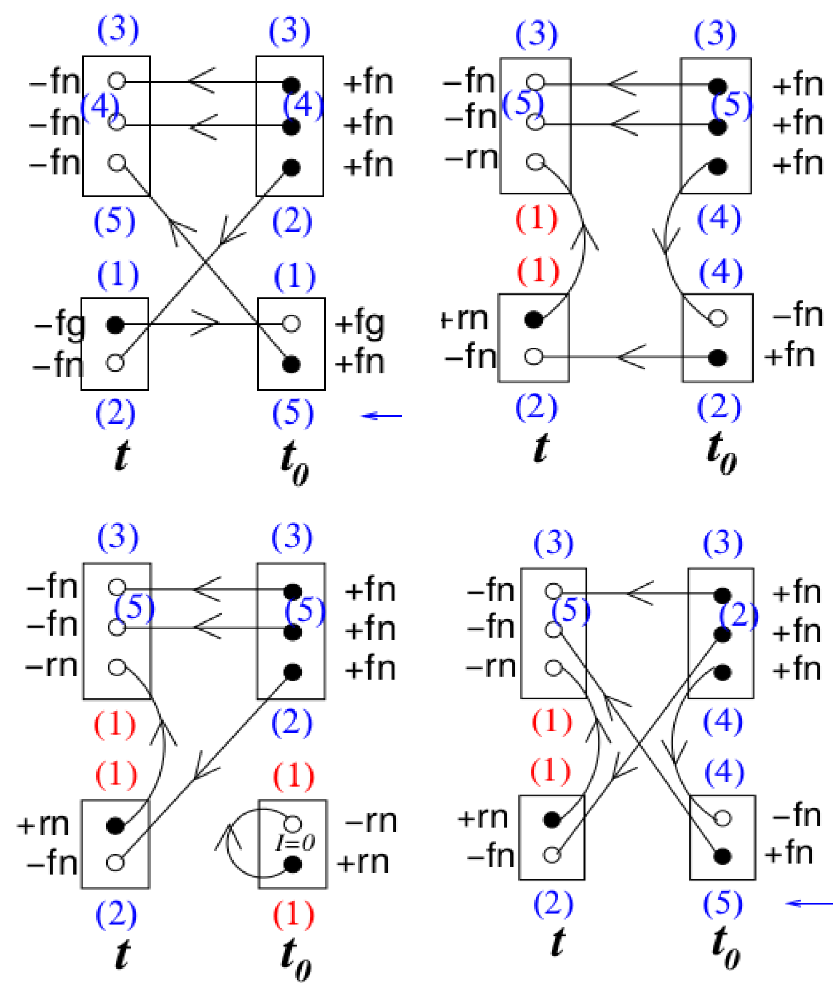
Single Baryon – Single Baryon:



Single Baryon – Meson+Baryon:



Meson+Baryon – Meson+Baryon:



Correlation functions constructed by tensor contraction

- Optimizations familiar to DFT simulations:
  - 'Path' optimization: find best contraction order
  - Common sub-expression elimination

B. Hörz, et al. PRC 103 (2021)

- Tensor construction/contractions now require leadership class computing:

  → large Frontera (TACC) allocation

- Part of a broad program to compute meson-baryon and baryon-baryon scattering amplitudes: Baryon Scattering Collaboration (BaSC)

C. Morningstar, S. Skinner (CMU); A. Nicholson (UNC); A. Walker-Loud, P. Vranas (LBL); A. Hanlon (BNL), B. Hörz (Intel), JB (DESY), F. Romero-Lopez (MIT), … additional members in Mainz, TCD, GSI/Darmstadt

Npi paper draft online: https://arxiv.org/pdf/2208.03867.pdf

Correlation functions constructed by tensor contraction

Isospin channel	D200 Number of Correlators
$I=0,\ S=0,\ NN$	8357
$I=0,\ S=-1,\ \Lambda, N\overline{K}, \Sigma\pi$	8143
$I=\frac{1}{2},\ S=0,\ N\pi$	696
$I=\frac{1}{2},\ S=-1,\ N\Lambda, N\Sigma$	17816
$I=1,\ S=0,\ NN$	7945
$I=\frac{3}{2},\ S=0,\ \Delta, N\pi$	3218
$I=\frac{3}{2},\ S=-1,\ N\Sigma$	23748
$I=0,\ S=-2,\ \Lambda\Lambda, N\Xi, \Sigma\Sigma$	16086
$I=2,\ S=-2,\ \Sigma\Sigma$	4589
Single hadrons (SH)	33

Quantization Condition

Below $N\pi\pi$ threshold (and away from left cut):

$$\det[K^{-1}(E_{\rm cm}^L) - B(L\mathbf{q}_{\rm cm})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531, ...

- Block-diagonal in finite-volume irreps → additional ‘occurrence’ index $n_{\rm occ}$
- Total spin = ½ fixed → K-matrix is diagonal in J^P and $n_{\rm occ}$, but B dense.
- Truncate at $\ell_{\rm max} = 2$ for isospin I = 3/2 and $\ell_{\rm max} = 0$ for I = ½

Finite volume → Reduced symmetry

- Irreps where $(2J, \ell) = (3, 1)$ contributes for $\Delta(1232)$
- Irreps where $(1, 0)$ contributes for scattering lengths.

$\boldsymbol{d}$	Λ	dim.	contributing $(2J, \ell)^{n_{\rm occ}}$ for $\ell_{\rm max} = 2$
$(0,0,0)$	$G_{1\rm u}$	2	$(1,0)$
	$G_{1\rm g}$	2	$(1,1)$
	$H_{\rm g}$	4	$(3,1),\ (5,2)$
	$H_{\rm u}$	4	$(3,2),\ 5,2)$
	$G_{2\rm g}$	2	$(5,2)$
$(0,0,n)$	G_1	2	$(1,0),\ (1,1),\ (3,1),\ (3,2),\ (5,2)$
	G_2	2	$(3,1),\ (3,2),\ (5,2)^2$
$(0,n,n)$	G	2	$(1,0),\ (1,1),\ (3,1)^2,\ (3,2)^2,\ (5,2)^3$
(n,n,n)	G	2	$(1,0),\ (1,1),\ (3,1),\ (3,2),\ (5,2)^2$
	F_1	1	$(3,1),\ (3,2),\ (5,2)$
	F_2	1	$(3,1),\ (3,2),\ (5,2)$

Determination of finite-volume energies

C. Michael, I. Teasdale NPB 215 (1983)

- Solve $N_{\rm op} \times N_{\rm op}$ GEVP for a single $(t_0, t_{\rm d})$

$$C(t_{\rm d})\, v_n(t_0, t_{\rm d}) = \lambda_n(t_0, t_{\rm d}) C(t_0)\, v_n(t_0, t_{\rm d})$$

$$\Rightarrow D_n(t) = \tilde{C}_{nn}(t) = (v_n(t_0, t_{\rm d}), C(t) v_n(t_0, t_{\rm d}))$$

  - Pro: no ‘eigenvector pinning’
  - Con: no formal control over large-time asymptotics

- Need to ensure:
  - Results independent of $N_{\rm op}$ and $(t_0, t_{\rm d})$
  - $\sum_{m<n} \tilde{C}_{nm}(t)$ is small

Determination of finite-volume energies

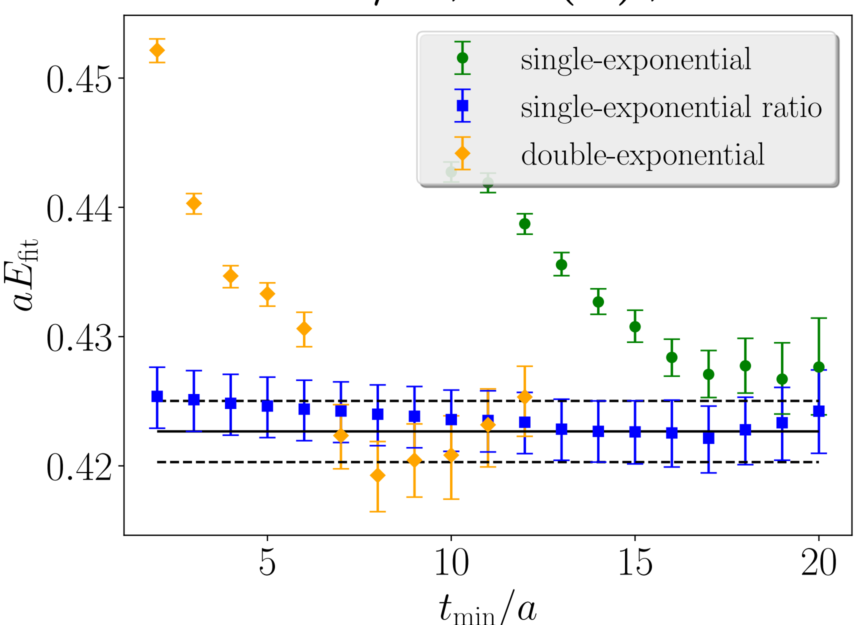
Fitting strategies:

- Single- (1-exp) and double-exponential (2-exp) fits to $D_n(t)$
- Single-exponential fits to the ratio (1-exp ratio):

$$R_n(t) = \frac{D_n(t)}{C_\pi(\boldsymbol{d}_\pi^2, t)\, C_{\mathrm{N}}(\boldsymbol{d}_{\mathrm{N}}^2, t)}$$

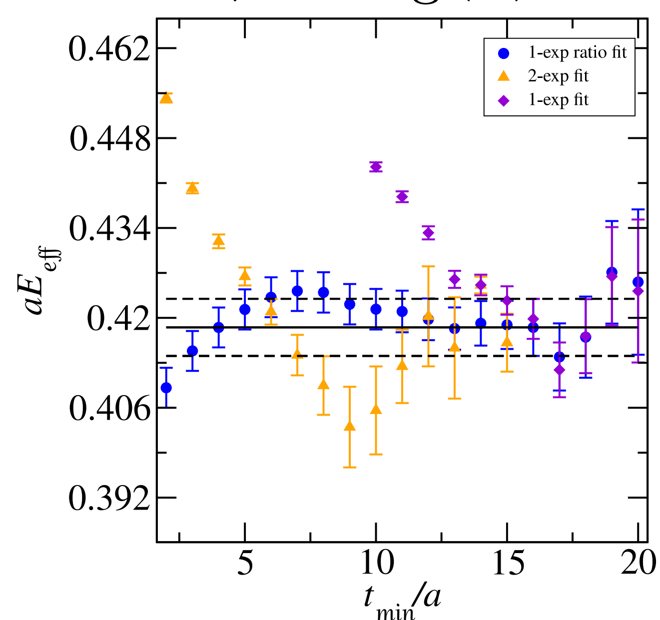
- Demand consistency btw. 1-exp ratio and 2-exp, also across GEVP's

$I = 1/2,\ G(3), \mathrm{lvl}\, 1$



$(t_{\mathrm{min,N}} = 10a)$

$I = 3/2,\ H_{\mathrm{g}}(0), \mathrm{lvl}\, 0$



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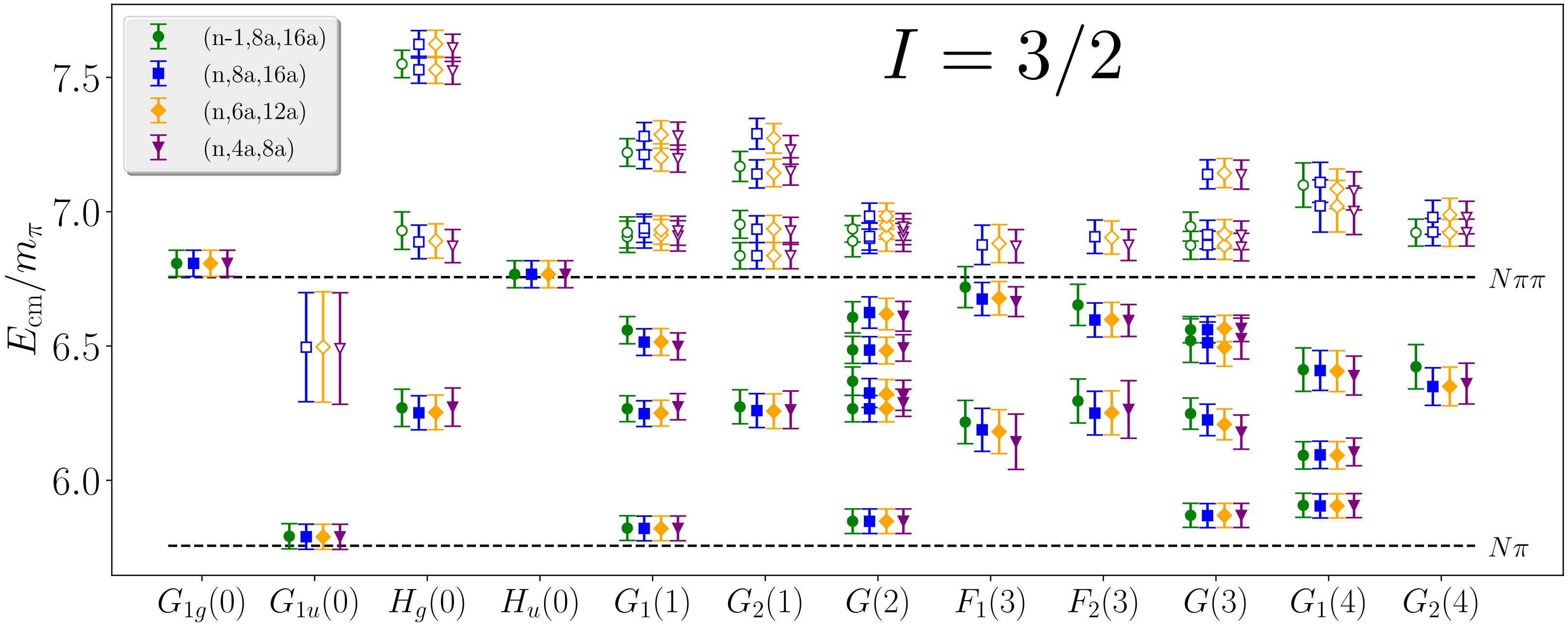

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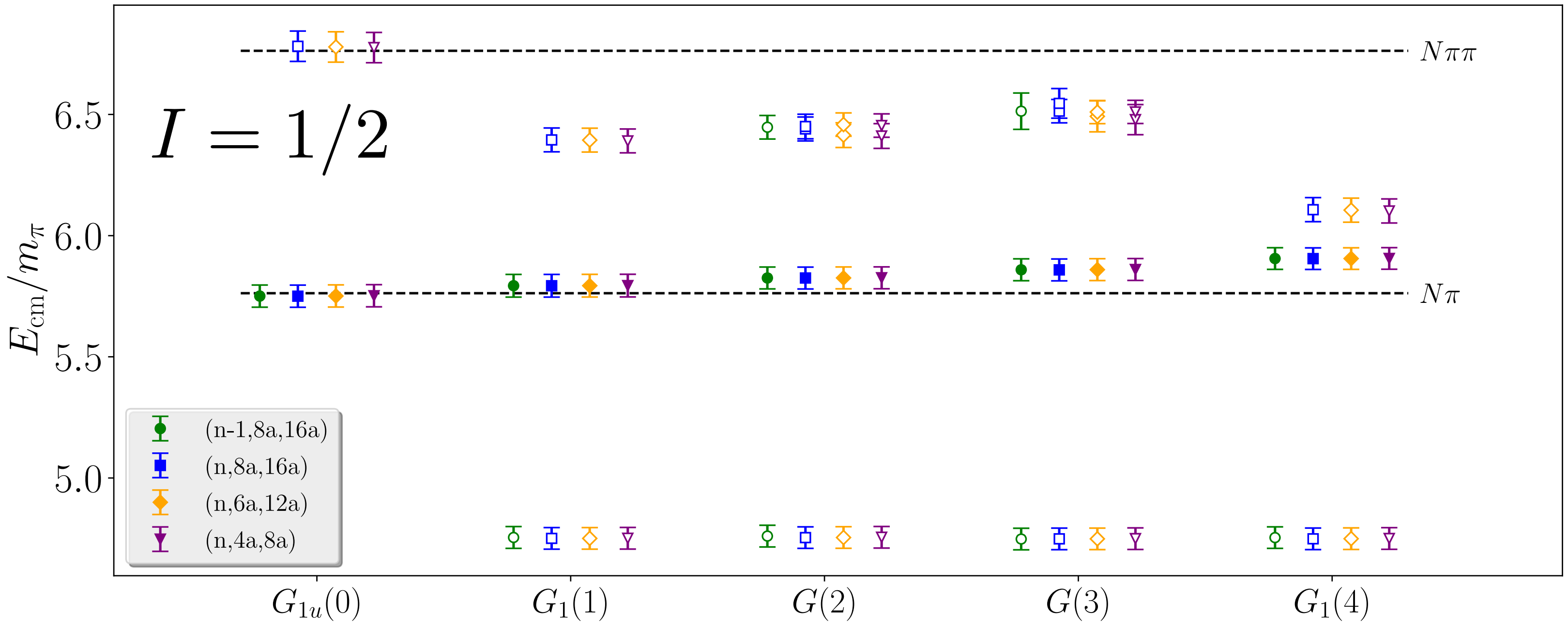
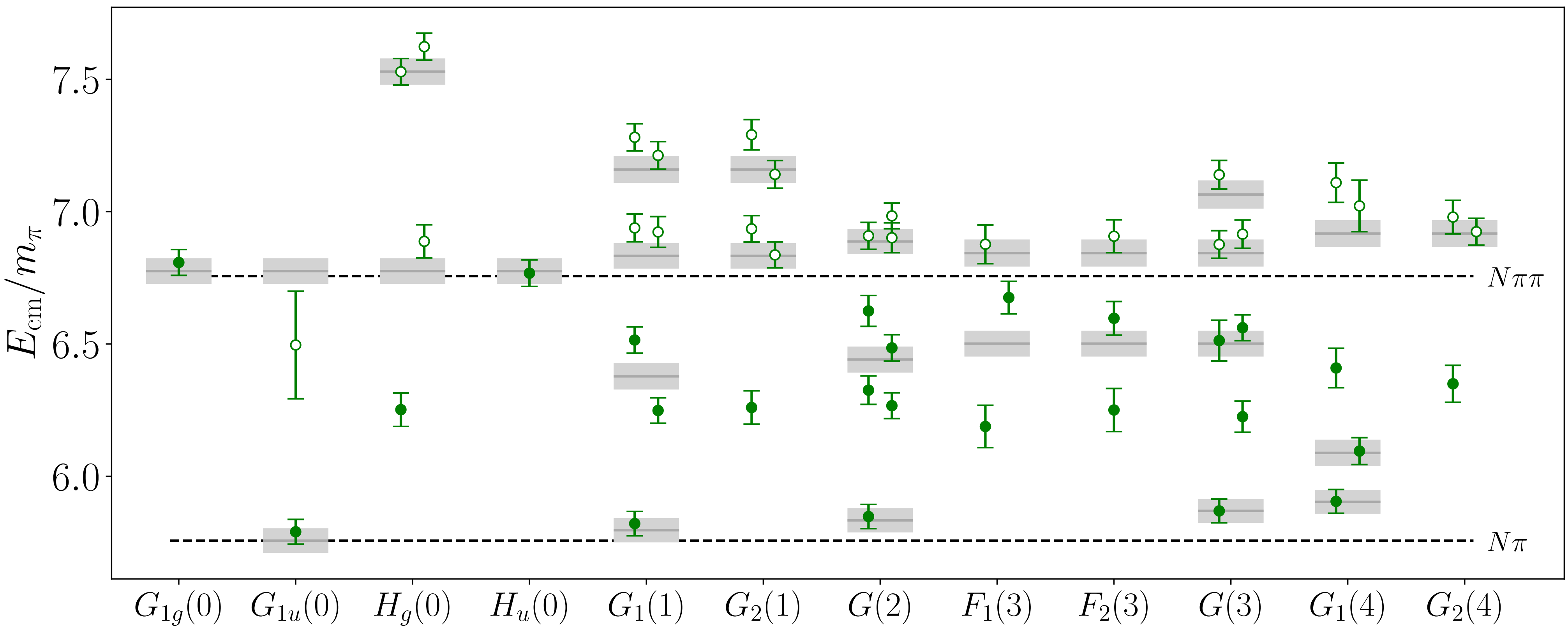
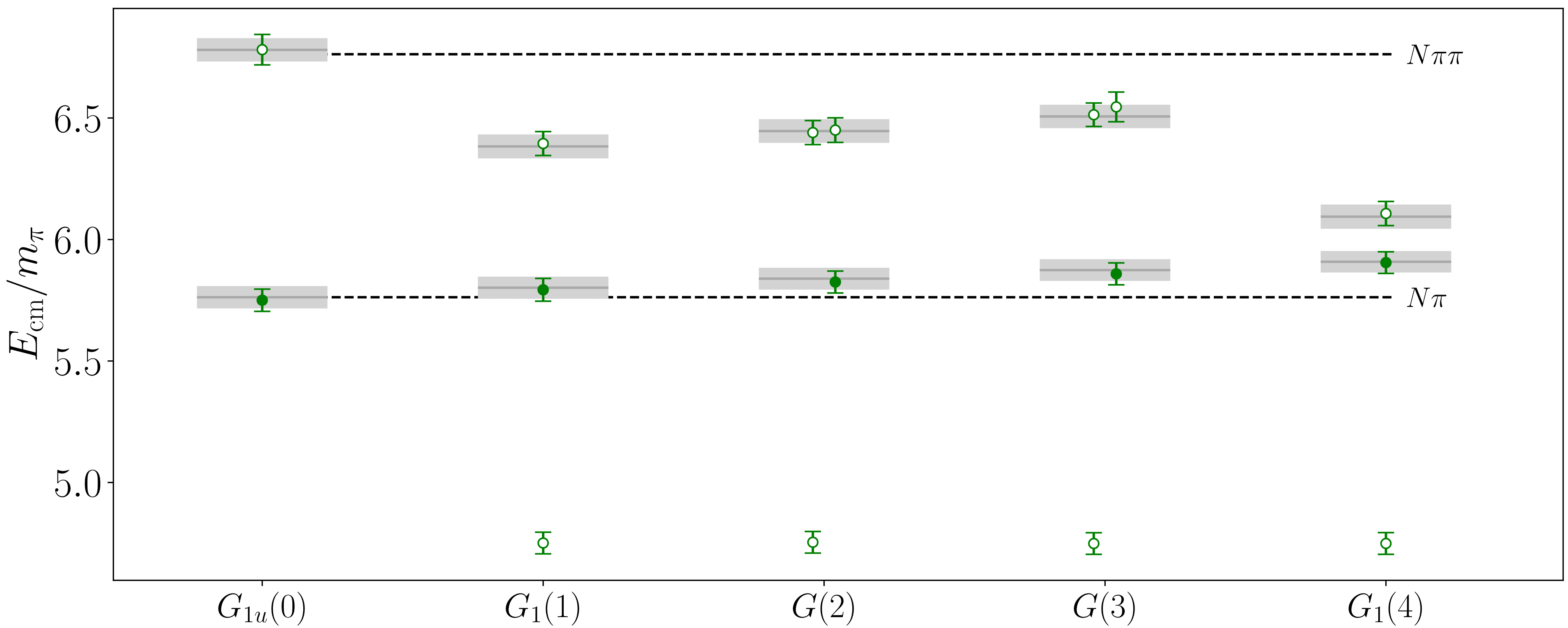


I=3/2 finite-volume spectrum



Solid points included in the fit: within 1-sigma of inelastic threshold.

I=1/2 finite-volume spectrum



Solid points included in the fit: lowest energies in all irreps with leading *s*-wave

Fits to lattice energies

Amplitude parametrizations:

- resonant *p*-wave:

$$\frac{q_{\rm cm}^3}{m_\pi^3}\cot\delta^{3/2}_{3/2^+} = \frac{6\pi\sqrt{s}}{m_\pi^3 g_{\rm BW}^2}(m_\Delta^2 - s)$$

- all other waves:

$$\frac{q_{\rm cm}^{2\ell+1}}{m_\pi^{2\ell+1}}\cot\delta^{I}_{J^P} = \frac{\sqrt{s}}{m_\pi A^I_{J^P}}$$

- S-waves and resonant p-wave are important, only mild sensitivity to other waves.

Fits to lattice energies

Two fitting strategies to determine parameters $\{p_n\}$

- Determinant residual (DR): C. Morningstar et al., Nucl. Phys. B924 (2017)

$$\chi^2\left(\{p_n\}\right) = \sum_{ij} \det_i\left(\{p_n\}, \frac{E_{\rm cm}}{m_\pi}, m_\pi L, \frac{m_{\rm N}}{m_\pi}\right) C_{ij}^{-1} \times$$

$$\det_j\left(\{p_n\}, \frac{E_{\rm cm}}{m_\pi}, m_\pi L, \frac{m_{\rm N}}{m_\pi}\right)$$

- Spectrum method (SP): P. Guo et al., Phys. Rev. D 88 (2013)

$$\chi^2\left(\{p_n\}\right) = \sum_{ij}\left(\frac{q^2_{{\rm cm},i}}{m_\pi^2} - \frac{q^{2,{\rm QC}}_{{\rm cm},i}}{m_\pi^2}\left(\{p_n\}\right)\right) C_{ij}^{-1}\left(\frac{q^2_{{\rm cm},j}}{m_\pi^2} - \frac{q^{2,{\rm QC}}_{{\rm cm},j}}{m_\pi^2}\left(\{p_n\}\right)\right)$$

Fits to lattice energies

Two fitting strategies: to determine parameters $\{p_n\}$

- Determinant residual (DR):
  - No root finding or identification of roots with levels
  - Covariance recalculated always
  - Residuals/covariance not precisely determined

- Spectrum method (SP):
  - Root finding is tricky
  - Covariance independent of $\{p_n\}$, more precise
  - Terms for $m_\pi L, \dfrac{m_{\mathrm{N}}}{m_\pi}$ have little effect

Fits to lattice energies

- I = 3/2

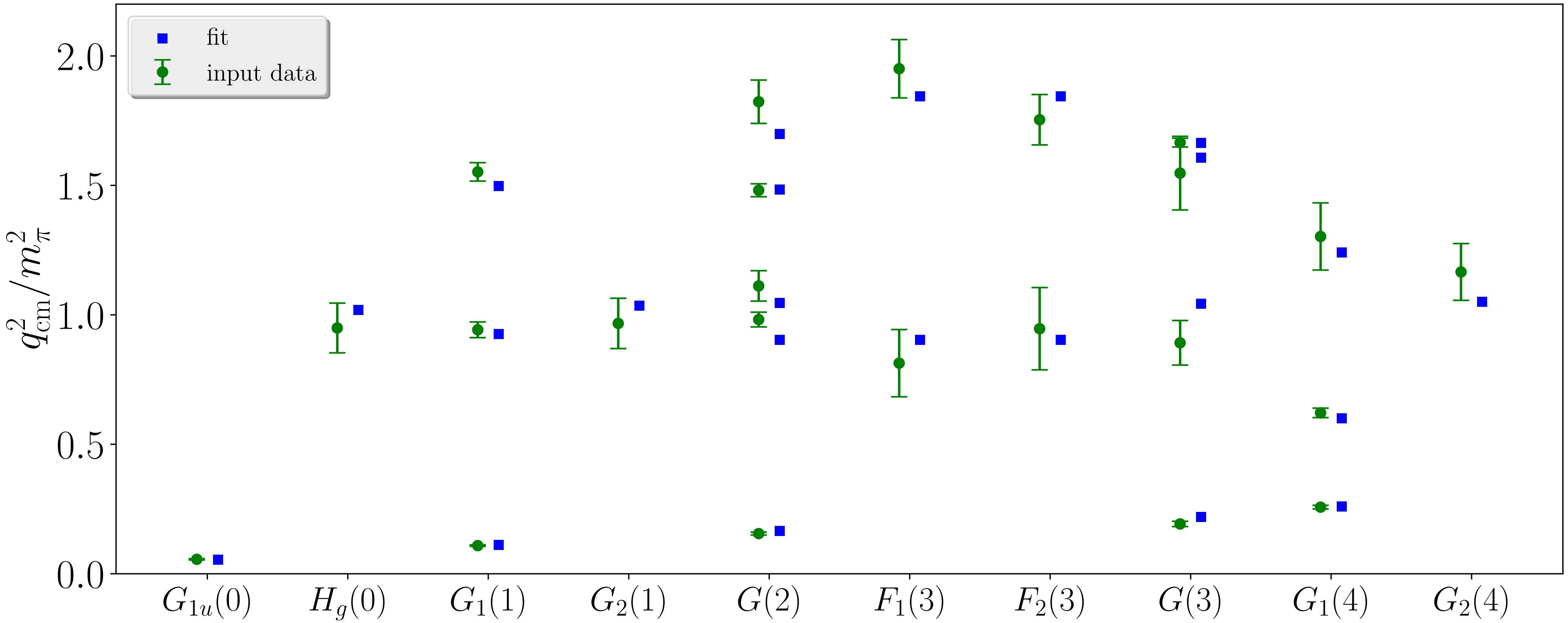
Fit	N_{pw}	$A_{1/2^-}$		M_Δ/M_π	$A_{1/2^+}$	$A_{3/2^-}$	$A_{5/2^-}$	χ^2	dofs
SP	2	-1.56(4)	13.8(6)	6.281(16)	—	—	—	44.38	$23-3$
DR	2	-1.57(5)	14.4(5)	6.257(36)	—	—	—	14.91	$23-3$
SP	5	-1.53(4)	14.7(7)	6.290(18)	-0.19(6)	-0.46(12)	0.37(10)	30.17	$25-6$

- I = ½

Fit	N_{pw}	$A_{1/2^-}$	χ^2	dofs
SP	1	0.82(12)	1.68	$5-1$
DR	1	0.92(22)	1.72	$5-1$
SP	1	0.82(13)	0.79	$4-1$

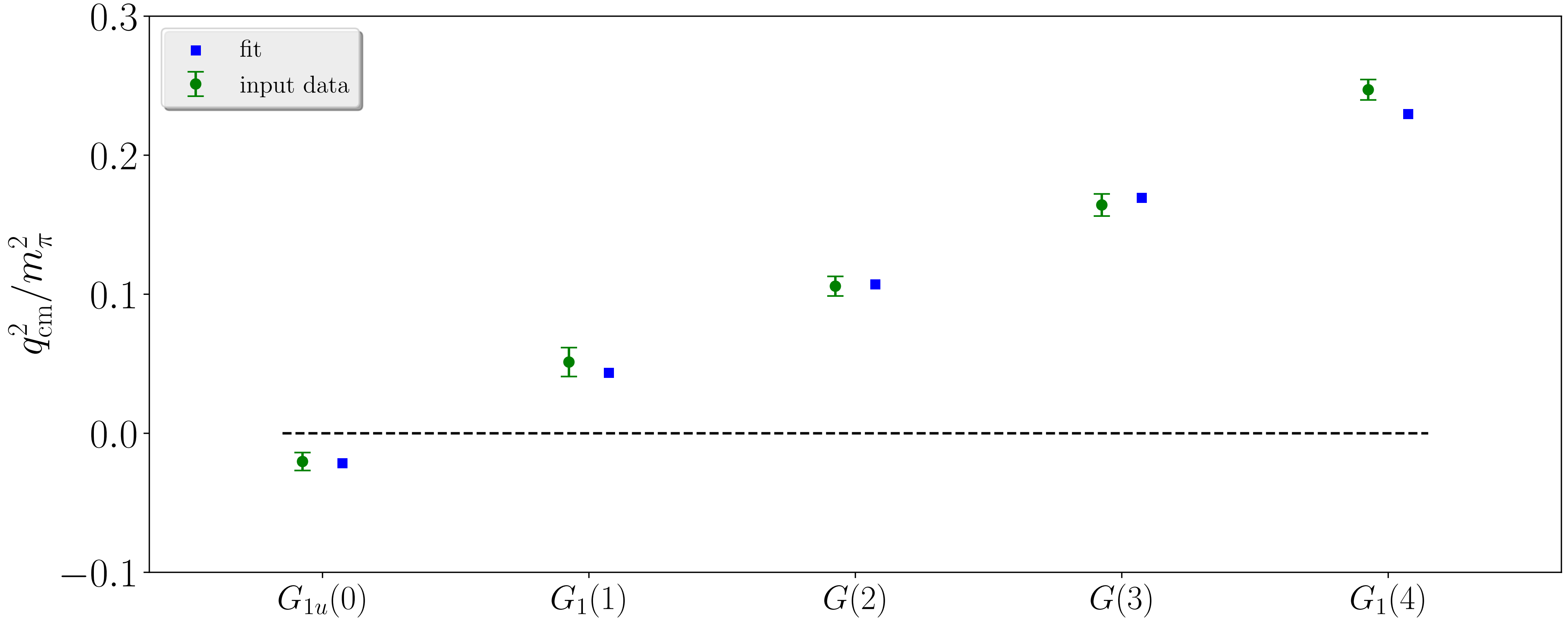
Fits to lattice energies

I = 3/2: all 5 partial waves, SP



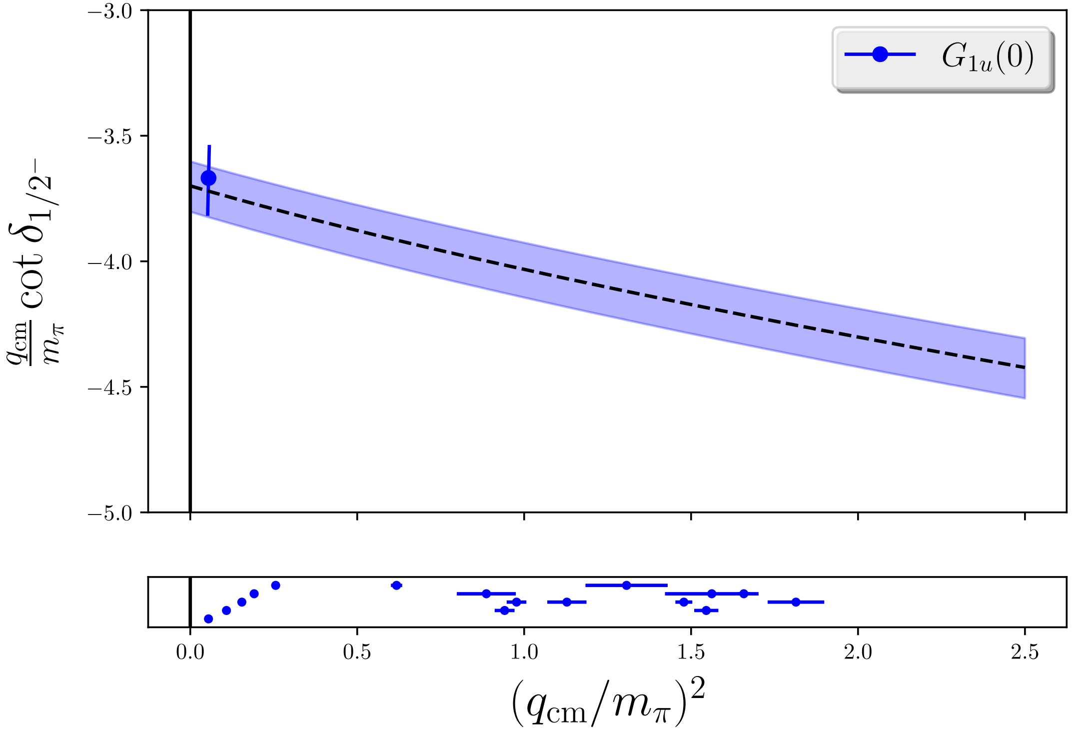
Fits to lattice energies

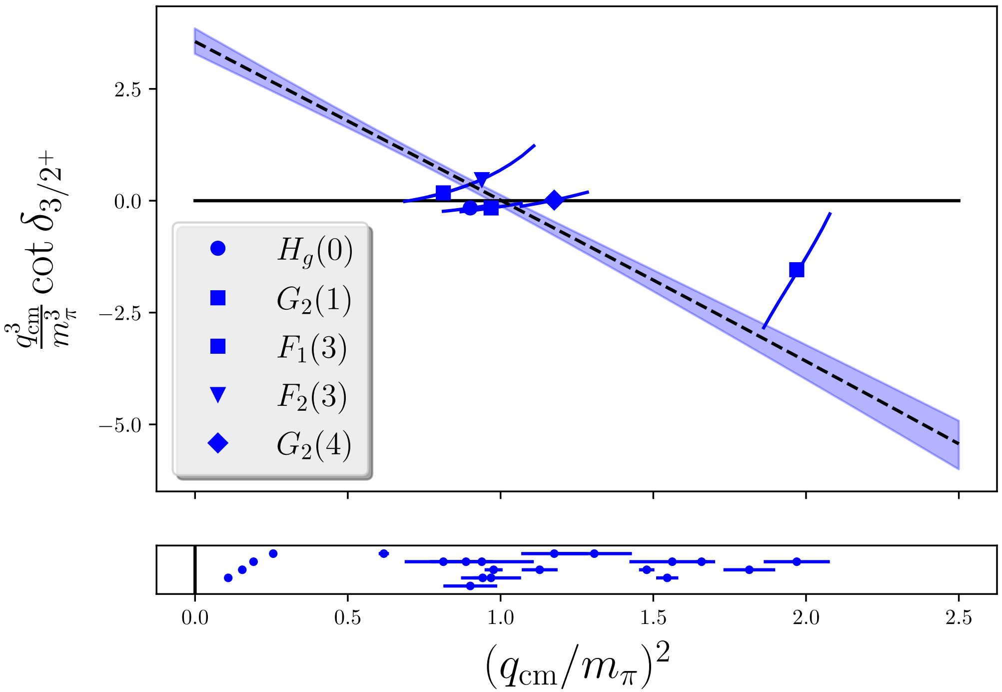
I = 1/2: 1 partial wave (SP)



Fits to lattice energies

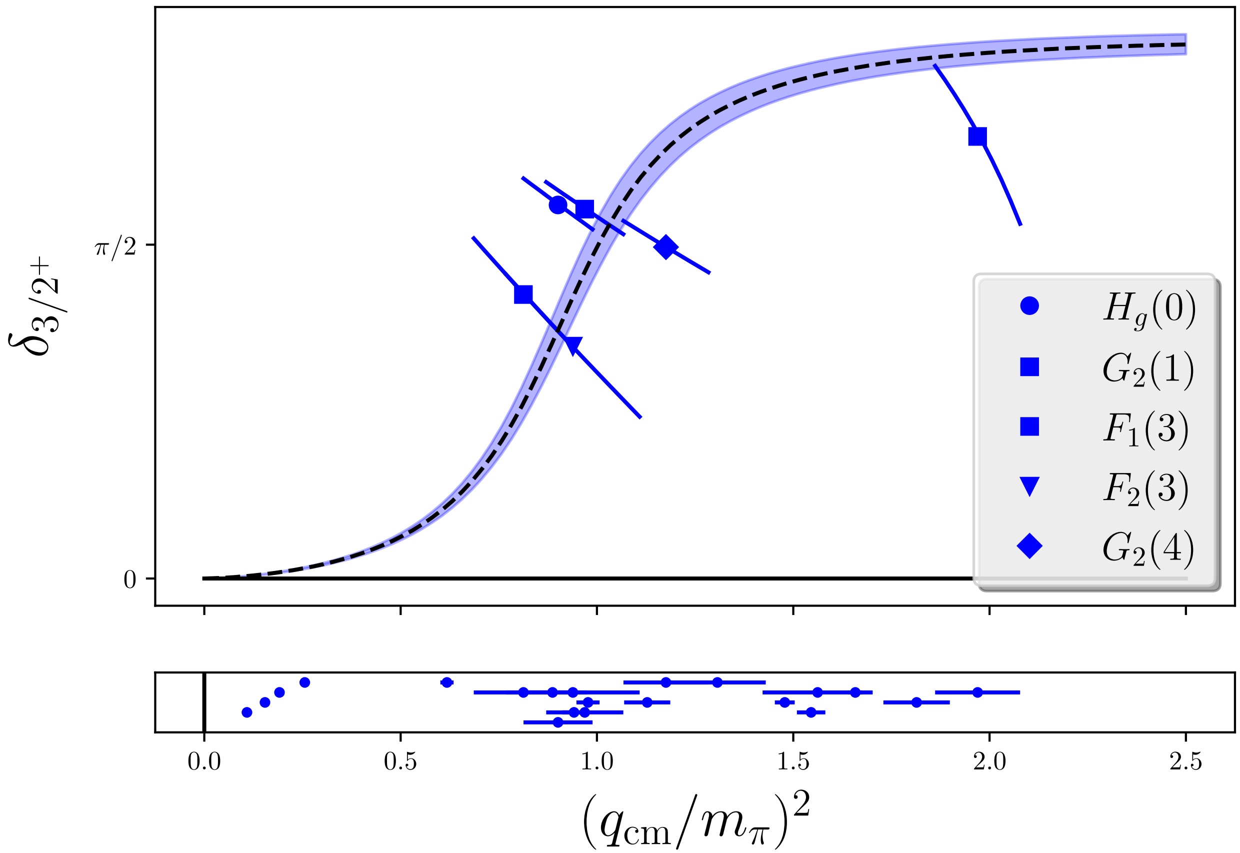
I = 3/2:





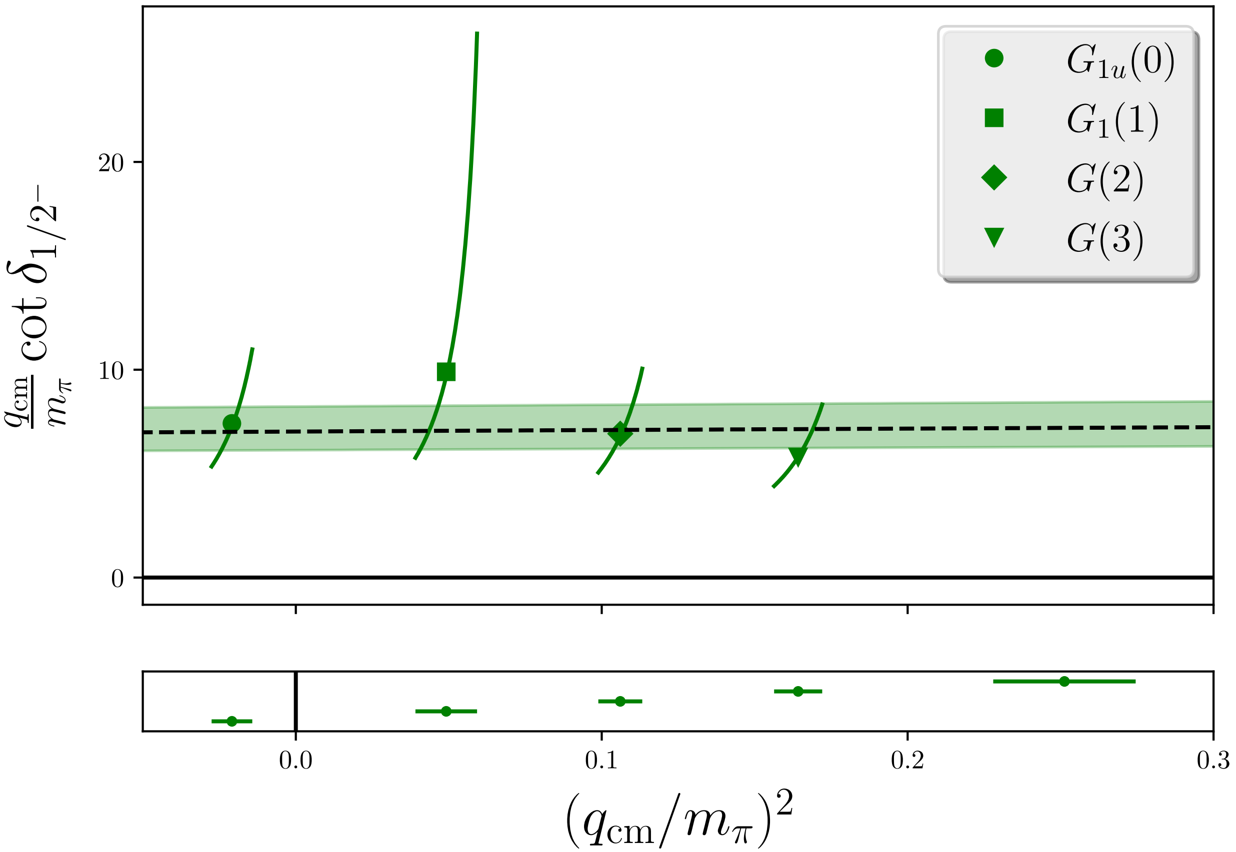
Fits to lattice energies

I = 3/2:



Fits to lattice energies

I = 1/2:



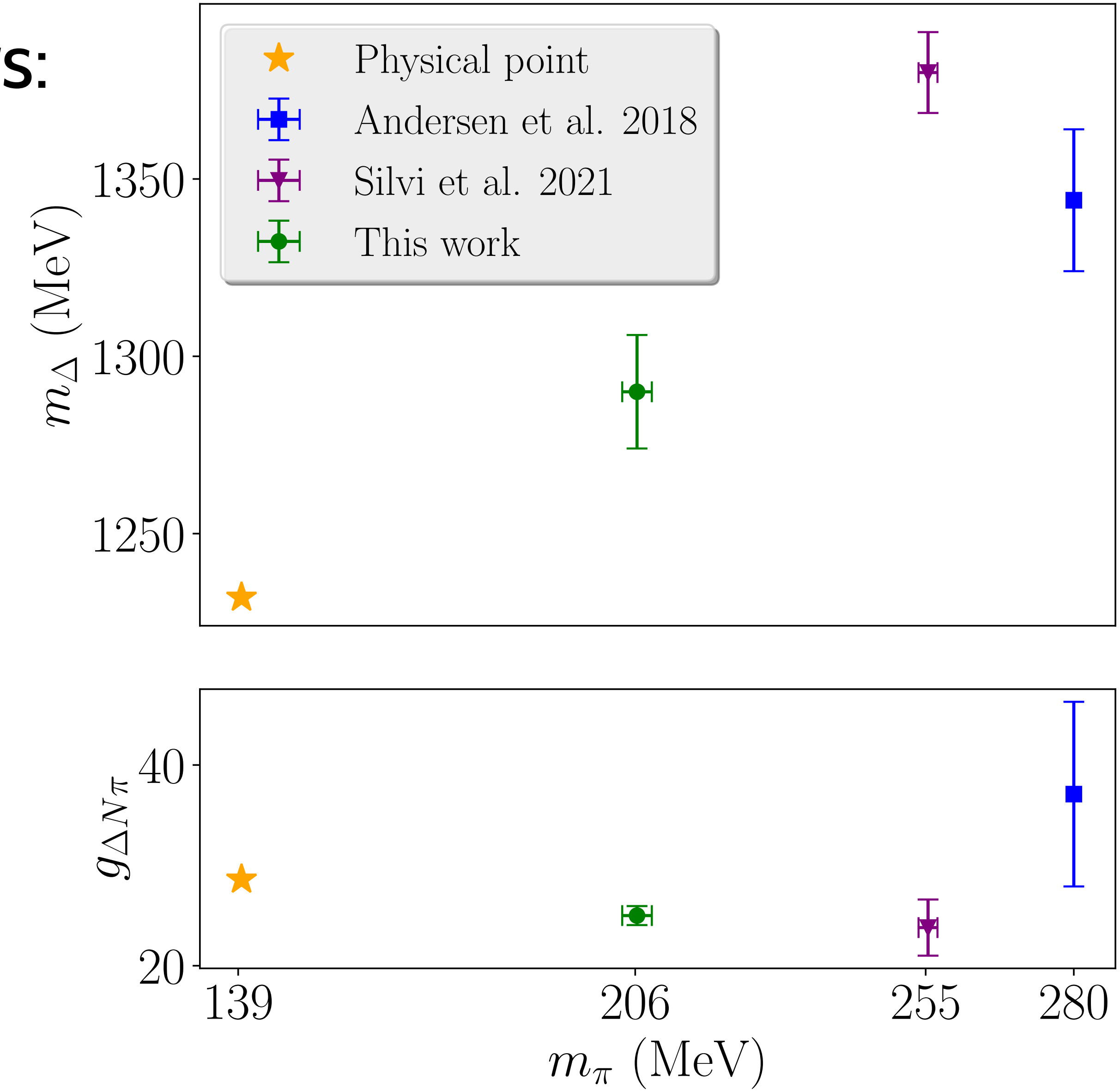
Comparison with Existing Results

Delta(1232) parameters:

➔ Additional (unpublished):

D. Mohler, PoS LATTICE2012 (2012)
V. Verducci, *PhD Thesis* (2014)
F. Pittler et al., PoS LATTICE2021 (2022)

➔ Physical point from PDG '20



Comparison with Existing Results

Scattering lengths:

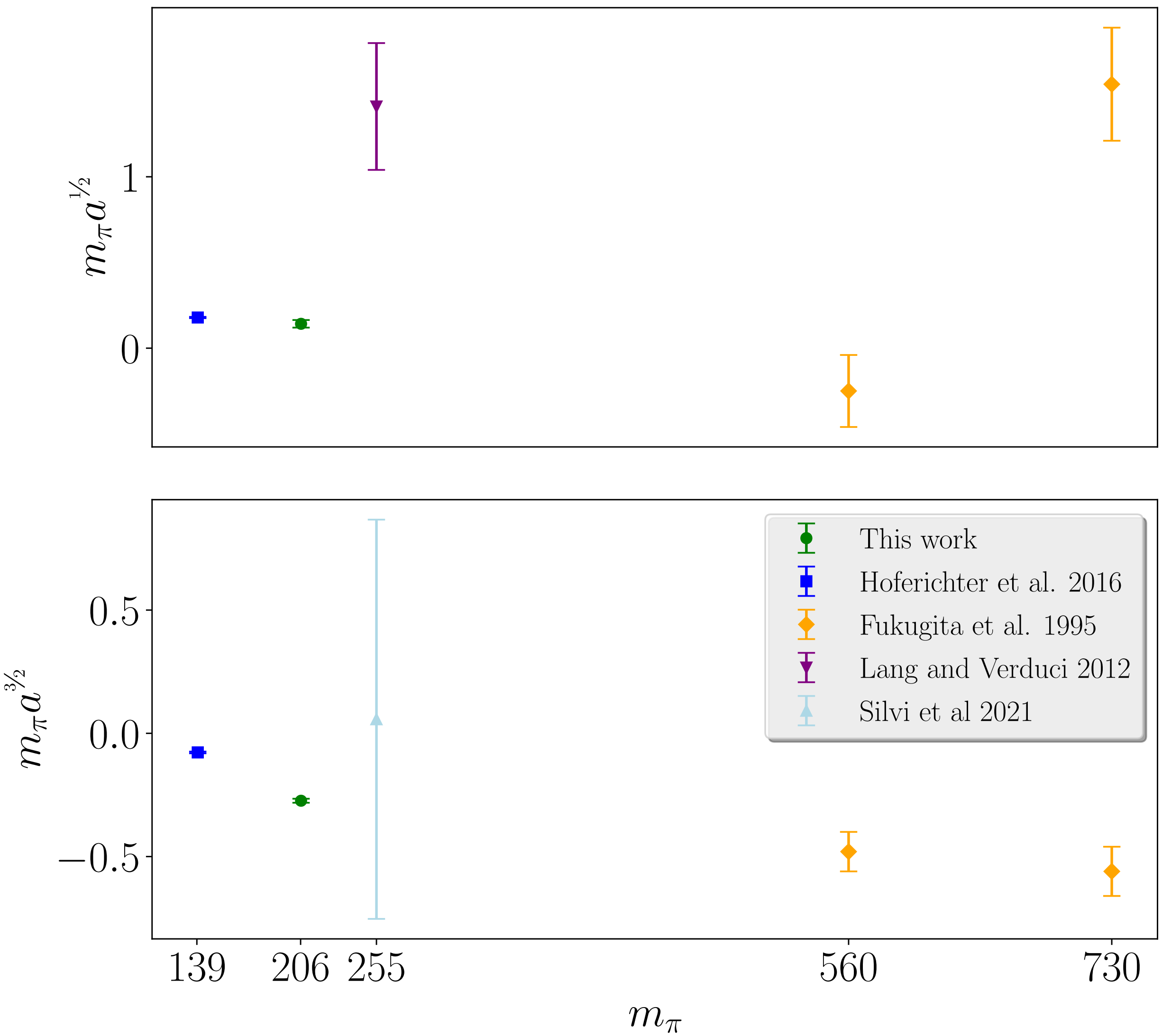
➔ This work:

$$m_\pi a_0^{3/2} = -0.2735(81)$$
$$m_\pi a_0^{1/2} = 0.142(22)$$

➔ Pheno (isospin limit):

$$m_\pi a_0^{3/2} = -0.0775(35)$$
$$m_\pi a_0^{1/2} = 0.1788(38)$$

M. Hoferichter et al. PLB 760 (2016)

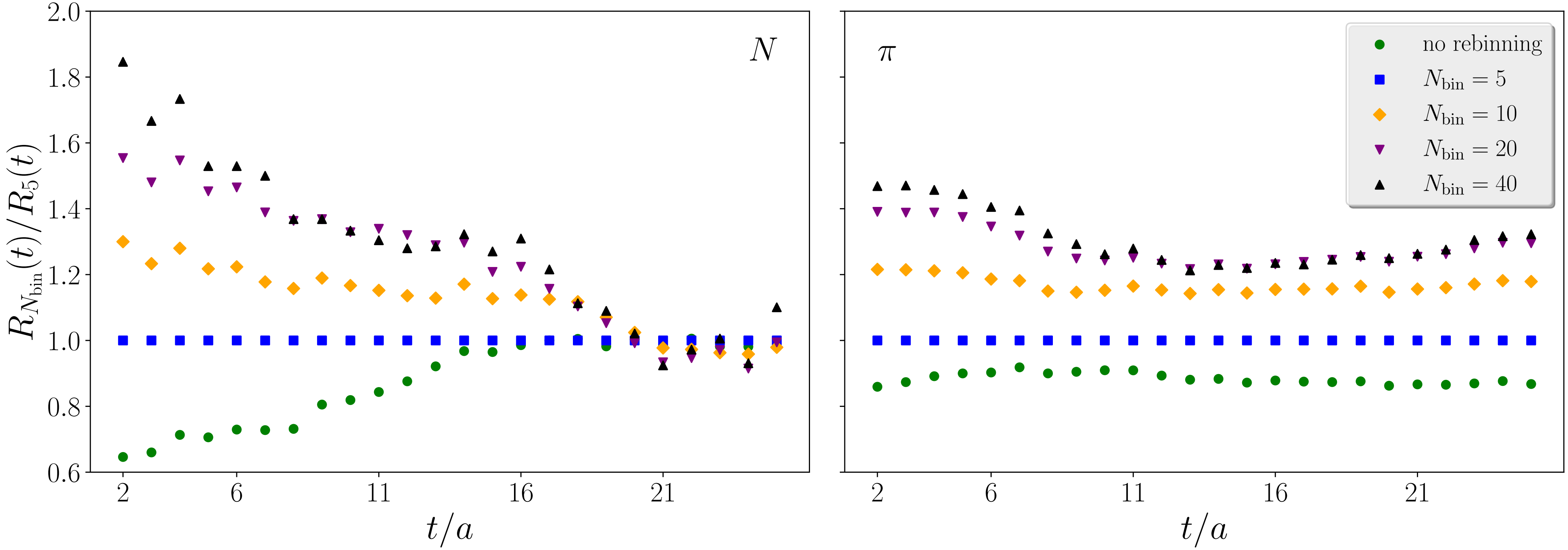


Conclusions

- Npi scattering amplitudes near/at the physical point are difficult, but within reach.
- Usual systematics need to be addressed: residual finite-volume effects, continuum limit
- Energy resolution of Delta(1232) limited by $m_\pi L,$ larger volumes needed
- (Preliminary) interface with ChPT: NLO can describe scattering lengths, but with an LEC different than pheno.
- Stay Tuned for more from the Baryon Scattering Collaboration (BaSC)!

Autocorrelations

- Relative errors on $C_\pi(t)$ and $C_\mathrm{N}(t)$



Ratio Fit Comparison

- For I=3/2, G1u(0), lvl 0 compare ratio fits to simultaneous fit:

$$C_{\pi \mathrm{N}}(t) = A\mathrm{e}^{-\Delta E\, t}\left\{1 + B_{\pi}\mathrm{e}^{-\Delta E_{\pi}\, t}\right\} \times \left\{1 + B_{\mathrm{N}}\mathrm{e}^{-\Delta E_{\mathrm{N}}\, t}\right\}$$

$$C_{\pi,\mathrm{N}}(t) = A_{\pi,\mathrm{N}}\mathrm{e}^{-m_{\pi,\mathrm{N}} t}\left\{1 + B_{\pi,\mathrm{N}}\mathrm{e}^{-\Delta E_{\pi,\mathrm{N}} t}\right\}$$