Constraining matrix elements for BSM searches with dispersion relations

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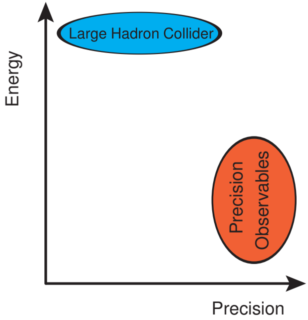
HISKP Colloquium

University of Bonn

How to search for physics beyond the Standard Model

- **Energy Frontier**
  - Direct searches in high-energy collisions
  - Reach limited (mainly) by energy of collider
- **Precision Frontier**
  - Indirect searches in precision observables
  - Reach limited by **precision of experiment and theory**
  - Can be sensitive to higher scales



Sakurai Prize 2023

2023 J. J. Sakurai Prize for Theoretical Particle Physics Recipient

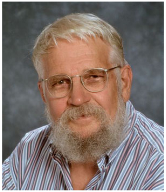
Heinrich Leutwyler
University of Bern

Citation:

"For fundamental contributions to the effective field theory of pions at low energies, and for proposing that the gluon is a color octet"

Background:

attached



Volume 47B, number 4 PHYSICS LETTERS 26 November 1973

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE ☆

H. FRITZSCH*, M. GELL-MANN and H. LEUTWYLER**
California Institute of Technology, Pasadena, Calif. 91109, USA

Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

In the discussion of hadrons, and especially of their electromagnetic and weak currents, a great deal of use has been made of a Lagrangian field theory model in which quark fields are coupled symmetrically to a neutral vector "gluon" field. Properties of the model are abstracted and assumed to be true for the real hadron system. In the last few years, theorists have abstracted not only properties true to each order of the coupling constant (such as the charge algebra $SU_3 \times SU_3$ and the manner in which its conservation is violated) but also properties that would be true to each order only if there were an effective cutoff in transverse momentum (for example, Bjorken scaling, V-A light cone algebra, extended V-A-S-T-P light cone algebra with finite quark bare masses, etc.).

We suppose that the hadron system can be described by a theory that resembles such a Lagrangian model. If we accept the stronger abstractions like exact asymptotic Bjorken scaling, we may have to assume that the propagation of gluons is somehow modified at high frequencies to give the transverse momentum cutoff. Likewise a modification at low frequencies may be necessary so as to confine the quarks and antiquarks behave as if they were composed of quarks and gluons.

We assume here the validity of quark statistics (equivalent to para-Fermi statistics of rank three, but with restriction of baryons to fermions and mesons to bosons). The quarks come in three "colors", but all physical states and interactions are supposed to be singlets with respect to the SU_3 of color. Thus, we do not accept theories in which quarks are real, observable particles; nor do we allow any scheme in which the color non-singlet degrees of freedom can be excited. Color is a perfect symmetry. (We should mention that even if there is a fourth "charmed" quark u′ in addition to the usual u, d, and s, there are still three colors and the principal conclusions set forth here are unaffected.)

For a long time, the quark-gluon field theory model used for abstraction was the one with the Lagrangian density

$$L = -\bar{q}\left[\gamma_\alpha(\partial_\alpha - igB_\alpha\lambda_\alpha) + M\right]q + L_B. \qquad (1)$$

Here M is the diagonal mechanical mass matrix of the quarks and L_B is the Lagrangian density of the free

Quantum chromodynamics (QCD)

- Protons and neutrons made of **quarks**
- Strong force mediated via **gluons**

Nobel Prize 2004

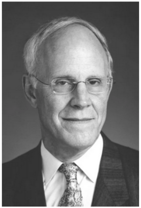


Photo from the Nobel Foundation archive.

David J. Gross

Prize share: 1/3

Photo from the Nobel Foundation archive.

H. David Politzer

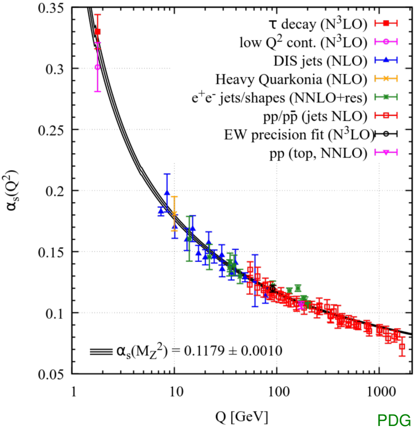
Prize share: 1/3

Photo from the Nobel Foundation archive.

Frank Wilczek

Prize share: 1/3

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction"

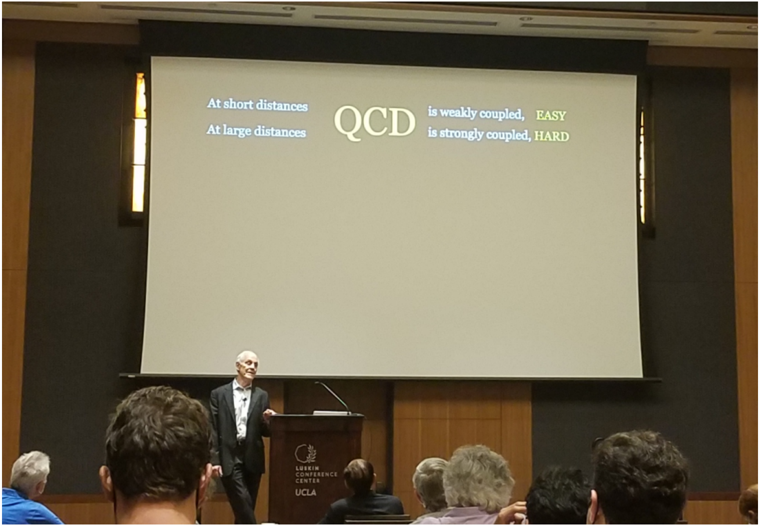


PDG 2019

Asymptotic freedom/confinement

Strong interactions become stronger at low energies, weaker at high energies. In consequence, quarks are confined into hadrons and never appear as free particles.

The challenge with strong interactions

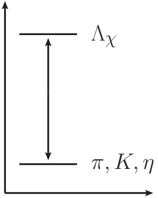


David Gross at Mani-Fest 2022

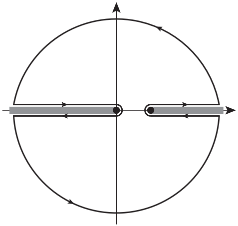
Model-independent, non-perturbative methods for low-energy QCD

1. **Effective field theories**: symmetries, separation of scales
   - Example: **chiral symmetry** of QCD 2023 Sakurai Prize citation: “For fundamental contributions to the effective field theory of pions at low energies”

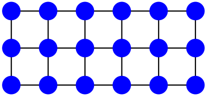
     $\hookrightarrow$ Chiral perturbation theory (ChPT)
   - Pions as pseudo Goldstone bosons
   - Expansion parameter: M_π/Λ_χ, $\Lambda_\chi \sim 1\,\text{GeV}$
   - Wide range of applications, from kaon decays to nuclear forces



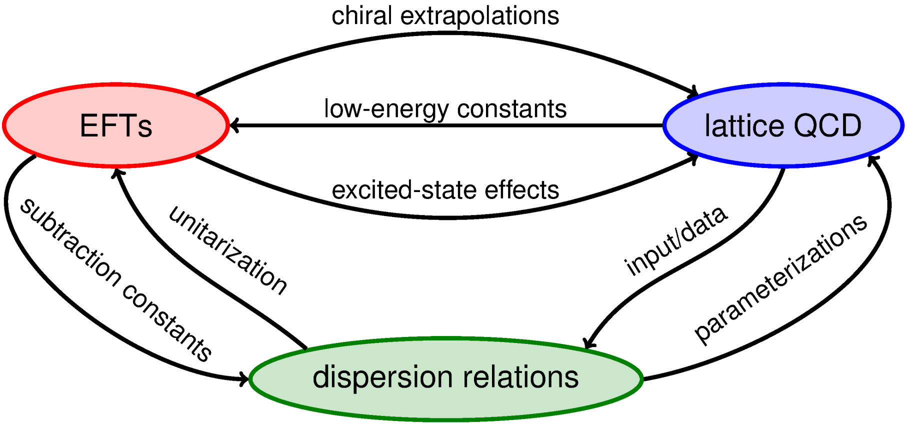
2. **Dispersion relations**: analyticity, unitarity, crossing symmetry
   - Example: optical theorem
   - Based on Cauchy’s theorem, analytic structure of amplitudes
   - Experimental input



3. **Lattice QCD**: Monte-Carlo simulations
   - Solve QCD on a lattice



Interplay among methods



Searching for physics beyond the SM at the Precision Frontier

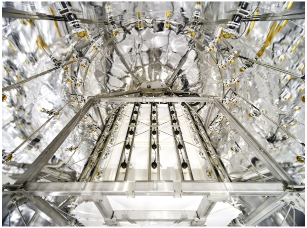
- **Low-energy precision observables**
  - **Dipole moments** ($\ell = e, \mu, \tau$; $n, \dots$)

    $\hookrightarrow$ Fermilab, J-PARC, PSI, ...
  - **Rare decays** (B mesons, kaons, pions)

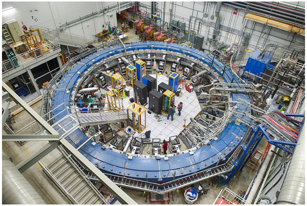
    $\hookrightarrow$ Belle II, LHCb, NA62, PIONEER, ...
  - **Atomic nuclei as BSM laboratories**
    - Direct detection searches for **dark matter**

      $\hookrightarrow$ XENONnT, LZ, PANDAX, DARWIN, ...
    - **Lepton flavor violation**: $\mu \to e$ conversion in nuclei

      $\hookrightarrow$ Mu2e, COMET
    - Neutrino oscillation experiments: DUNE, HyperK
  - Many, many more



XENONnT detector



Fermilab E989 storage ring

Experimental program at Precision Frontier

Need to understand **complex systems**

to isolate/interpret a potential BSM signal

Example: anomalous magnetic moment of the muon

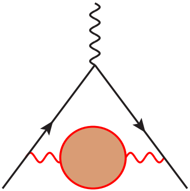
- **Anomalous magnetic moment of the muon**

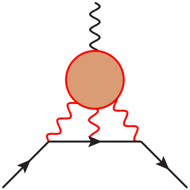
$$a_\mu^{\text{exp}} = 116{,}592{,}061(41) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = 116{,}591{,}810(43) \times 10^{-11}$$
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11} [4.2\sigma]$$

- Fermilab experiment: $\Delta a_\mu^{\text{exp}} = 16 \times 10^{-11}$

  $\hookrightarrow$ needs to be matched by theory

- Uncertainty in a_μ^{SM} dominated by **strong interactions**
  - **Hadronic vacuum polarization**
  - **Hadronic light-by-light scattering**





Experimental program at Precision Frontier

Control over **SM prediction** crucial for discovery potential (and interpretation of limits)

Complex analysis

Principle of maximal analyticity

Scattering amplitudes and form factors are represented by a complex function that exhibits no further singularities except for those required by general principles such as unitarity and crossing symmetry.

- **Unitarity**: “right-hand cut”

  $\hookrightarrow$ when particles can go on-shell

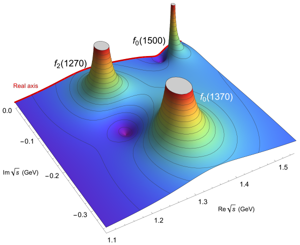
- **Crossing symmetry**: “left-hand cut”

  $\hookrightarrow$ crossed channels

- **Bound states/resonances**

  $\hookrightarrow$ poles on real axis/on second Riemann sheet

- Other singularities: partial-wave expansion, anomalous thresholds, . . .

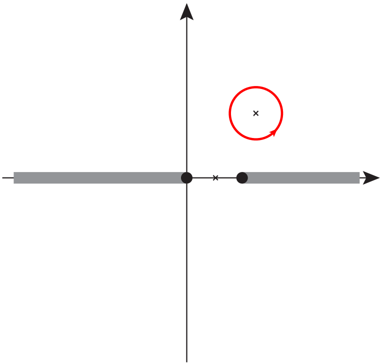


Peláez, Rodas, Ruiz de Elvira 2022

From Cauchy's theorem to dispersion relations

- **Cauchy's theorem**

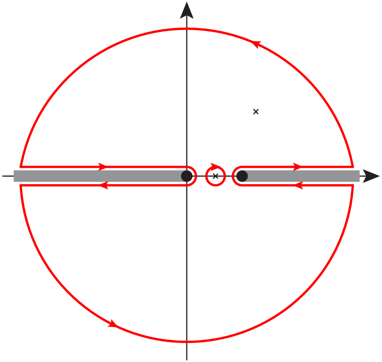
$$f(s) = \frac{1}{2\pi i} \int_{\partial\Omega} \frac{ds'\, f(s')}{s' - s}$$



From Cauchy's theorem to dispersion relations

- **Cauchy's theorem**

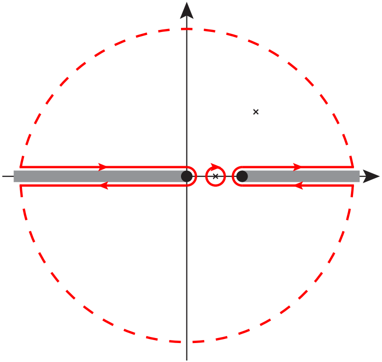
$$f(s) = \frac{1}{2\pi i}\int_{\partial\Omega}\frac{ds'\, f(s')}{s'-s}$$



- **Dispersion relation**

$$f(s) = \frac{g}{s - M^2} + \frac{1}{\pi} \int_{\text{cuts}} \frac{\mathrm{d}s' \, \mathrm{Im}\, f(s')}{s' - s}$$

$\hookrightarrow$ **analyticity**



From Cauchy's theorem to dispersion relations

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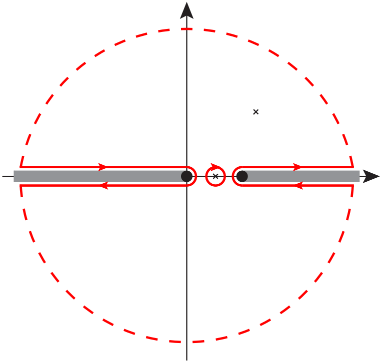
$$f(s) = \frac{g}{s - M^2} + \frac{1}{\pi}\int_{\text{cuts}} \frac{ds' \, \text{Im}\, f(s')}{s' - s}$$

$\hookrightarrow$ **analyticity**

- Subtractions

$$f(s) = \frac{g}{s - M^2} + \underbrace{C}_{f(0)+\frac{g}{M^2}} + \frac{s}{\pi}\int_{\text{cuts}} \frac{ds' \, \text{Im}\, f(s')}{s'(s' - s)}$$



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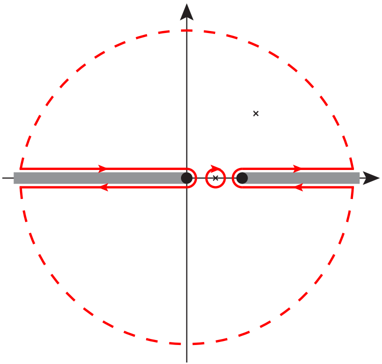
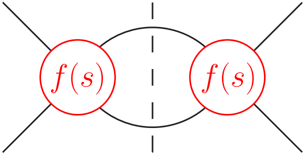


- Imaginary part from **Cutkosky rules**

$\hookrightarrow$ forward direction: **optical theorem**

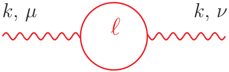
- **Unitarity** for partial waves: $\text{Im}\,f(s) = \rho(s)|f(s)|^2$

- Residue g **reaction-independent**



A concrete example: vacuum polarization by leptons

- Vacuum polarization of the photon by a **lepton loop**:



$$= -i\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)\Pi(k^2)$$

$$\Pi(k^2) - \Pi(0) = \frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\log\frac{m_\ell^2 - x(1-x)k^2}{m_\ell^2}$$

- Logarithm develops a cut once $m_\ell^2 - x(1-x)k^2$ can become negative
  $\hookrightarrow k^2 > 4m_\ell^2$ with discontinuity

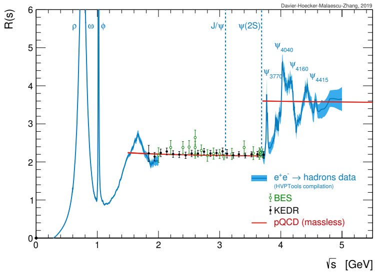
$$\text{disc}\,\Pi(k^2) = 2i\text{Im}\,\Pi(k^2) = -2i\theta(k^2 - 4m_\ell^2)\frac{\alpha}{3}\sqrt{1-\frac{4m_\ell^2}{k^2}}\left(1+\frac{2m_\ell^2}{k^2}\right)$$

- Dispersion relation

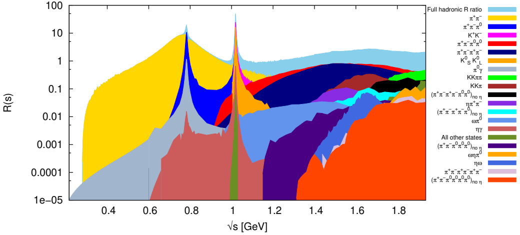
$$\Pi(k^2) = \Pi(0) + \frac{k^2}{\pi}\int_{4m_\ell^2}^\infty ds\frac{\text{Im}\,\Pi(s)}{s(s-k^2)}$$

- Physical value on real axis defined by $k^2 \to k^2 + i\epsilon$

A concrete example: vacuum polarization by hadrons



Davier, Hoecker, Malaescu, Zhang 2019



Keshavarzi, Nomura, Teubner 2018

- For quarks at low energy perturbation theory breaks down
- But: know the imaginary from **unitarity**

$$\operatorname{Im}\Pi(s) = -\frac{s}{4\pi\alpha}\sigma_{\text{tot}}\big(e^+e^- \to \text{hadrons}\big) = -\frac{\alpha}{3}R_{\text{had}}(s)$$

Master formula for HVP contribution to a_μ

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)$$

General strategy

- Hadronic vacuum polarization is the **ideal case**

  $\hookrightarrow$ only one Lorentz structure, one kinematic variable, automatic subtraction

- Otherwise, typical steps include derivation of
  - **Lorentz decomposition** e.g. Bardeen, Tung 1968, Tarrach 1975
  - **Unitarity relation** e.g. Watson 1954
  - **Solution of dispersion relation** e.g. Omnès 1958
- Rest of the talk: **Rescattering corrections to proton decay matrix elements**
  - Compact, but non-trivial example for dispersive constraints
  - Interplay with lattice QCD and ChPT

Searches for proton decay

- **Best limit on proton decay** from SuperKamiokande

$$\mathrm{Br}[p \to \pi^0 e^+] > 2.4 \times 10^{34}\mathrm{yr}$$

- Many other exclusive channels, but also inclusive limits, e.g., $p \to e^+ X$
- Future experiments:
  - HyperKamiokande: another order of magnitude in $p \to \pi^0 e^+$
  - DUNE, JUNO: competitive limits for kaon modes
- Why measure all these different modes?

Channel	Limit [10^{30} y]
$p \to \pi^0 e^+$	2.4×10^4
$p \to \pi^0 \mu^+$	1.6×10^4
$p \to \pi^+ \bar{\nu}$	3.9×10^2
$p \to K^0 e^+$	1.0×10^3
$p \to K^0 \mu^+$	3.6×10^3
$p \to K^+ \bar{\nu}$	5.9×10^3
$p \to \eta e^+$	1.0×10^4
$p \to \eta \mu^+$	4.7×10^3
$n \to \pi^- e^+$	5.3×10^3
$n \to \pi^- \mu^+$	3.5×10^3
$n \to \pi^0 \bar{\nu}$	1.1×10^3
$n \to K^0 \bar{\nu}$	1.3×10^2
$n \to \eta \bar{\nu}$	1.6×10^2
$p, n \to e^+ X$	0.6
$p, n \to \mu^+ X$	12

Standard Model EFT

Standard Model effective field theory

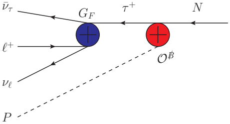
$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda_{\text{BSM}}} \sum_k C_k^{(5)} \mathcal{O}_k^{(5)} + \frac{1}{\Lambda_{\text{BSM}}^2} \sum_k C_k^{(6)} \mathcal{O}_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda_{\text{BSM}}^3}\right)$$

- Impose SM symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y$ on higher-dimensional operators Buchmüller, Wyler 1986, Grzadkowski et al. 2010
  - Dim 5: one operator structure, **Weinberg operator** $\mathcal{O}_k^{(5)} = (\tilde{\phi}^\dagger L_p)^T C(\tilde{\phi}^\dagger L_r)$
  - Dim 6: $\underbrace{15}_{\text{bosonic}} + \underbrace{19}_{\text{two-fermion}} + \underbrace{25}_{\text{four-fermion}} = 59$ B-conserving operator structures
- ***B*-violating operator structures** Wilczek 1979, Weinberg 1979

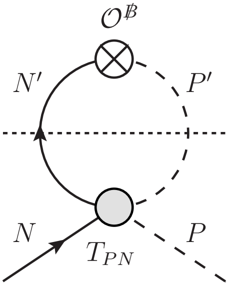
$$\begin{aligned}
Q_{duq} &= \varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(q_s^{\gamma j}\right)^T C L_t^k\right]\\
Q_{qqu} &= \varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[\left(q_p^{\alpha j}\right)^T C q_r^{\beta k}\right]\left[\left(u_s^\gamma\right)^T C e_t\right]\\
Q_{qqq} &= \varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[\left(q_p^{\alpha j}\right)^T C q_r^{\beta k}\right]\left[\left(q_s^{\gamma m}\right)^T C L_t^n\right]\\
Q_{duu} &= \varepsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(u_s^\gamma\right)^T C e_t\right]
\end{aligned}$$

- How can we constrain the $C_k^{(6)}$?

Matrix elements for nucleon decay



- In all cases: need hadronization of quark-level operators
  $\hookrightarrow$ **matrix elements**
- For most operators dominant limits from two-body decays, e.g., $p \to \pi^0 e^+$
  $\hookrightarrow$ **kinematics fixed**
- Exception: operators with τ require off-shell processes such as $p \to \pi^0 \ell^+ \nu_\ell \bar{\nu}_\tau$ because **decay into τ kinematically forbidden**
- Momentum dependence of the form factors from dispersion relations $\Rightarrow$ **pion–nucleon rescattering**



Matrix elements for nucleon decay: normalization

X_i	$W_0^{X_iL}(0)$	$W_1^{X_iL}(0)$	$W_0^{X_iR}(0)$	$W_1^{X_iR}(0)$
U_1	0.151(31)	−0.134(18)	−0.159(35)	0.169(37)
S_1	0.043(4)	0.028(7)	0.085(12)	−0.026(4)
S_2	0.028(4)	−0.049(7)	−0.040(6)	0.053(7)
S_3	0.101(11)	−0.075(13)	−0.109(19)	0.080(17)
S_4	−0.072(8)	0.024(6)	−0.044(5)	−0.026(6)
S_{1+2+4}	0.000(0)	0.000(0)	0.000(0)	0.000(0)
S_{2-3-4}	0.000(0)	0.000(0)	0.112(15)	0.000(12)

Yoo et al. 2022

- Normalizations from lattice QCD

$$\langle\pi^0|\left[\bar{u}^c P_A d\right] u_B|p\rangle = \frac{1}{\sqrt{2}}\langle\pi^+|\left[\bar{u}^c P_A d\right] d_B|p\rangle \equiv \frac{1}{\sqrt{2}} U_1^{AB}$$

$$\langle K^0|\left[\bar{u}^c P_A s\right] u_B|p\rangle \equiv S_1^{AB} \quad \langle K^+|\left[\bar{u}^c P_A s\right] d_B|p\rangle \equiv S_2^{AB} \quad \langle K^+|\left[\bar{u}^c P_A d\right] s_B|p\rangle \equiv S_3^{AB} \quad \langle K^+|\left[\bar{d}^c P_A s\right] u_B|p\rangle \equiv S_4^{AB}$$

- Two form factors: $X_i^{AB} = P_B\left[W_0^{X_i^{AB}}(s) + \frac{\not{q}}{m_N} W_1^{X_i^{AB}}(s)\right]u_N(p)$, write $X_{iA} \equiv X_i^{AL}$
- Found two new relations:

$$S_{1A} + S_{2A} + S_{4A} = 0 \text{ (isospin)} \qquad S_{2L} - S_{3L} - S_{4L} = 0 \text{ (Fierz)}$$

Matrix elements for nucleon decay: unitarity and ChPT

- For which scalar functions should one write dispersion relations?
  - Need to **avoid kinematic singularities and zeros**: $W_0(s)$, $W_1(s)$
  - Would like **simple unitary relations**: $W_\pm(s) = W_0(s) \pm \frac{\sqrt{s}}{m_N} W_1(s)$ because

$$\text{Im}\, W_+(s) = W_+(s) e^{-i\delta_{0+}(s)} \sin\delta_{0+}(s) \qquad \text{Im}\, W_-(s) = W_-(s) e^{-i\delta_{1-}(s)} \sin\delta_{1-}(s)$$

    with πN phase shifts $\delta_{\ell\pm}$, $j = \ell \pm 1/2$
  - Further constraint from **baryon-pole diagrams** (from ChPT Aoki et al. 2000)

$$W_0^{\text{ChPT}}(s) = W_0(0)\left[1 - \frac{m_B}{m_N}\frac{W_1(0)}{W_0(0)}\frac{s}{m_B^2 - s}\right]$$

$$W_1^{\text{ChPT}}(s) = W_1(0)\frac{m_B^2}{m_B^2 - s}$$

  - Suppressed isospin label $I = \{1/2, 1, 0\} \Rightarrow m_B = \{m_N, m_\Sigma, m_\Lambda\}$

$\hookrightarrow$ how can we combine all of that?

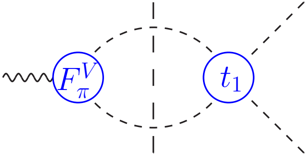
Omnès solution for pion vector form factor

- Electromagnetic form factor of the pion

$$\langle \pi^{\pm}(p')|j^{\mu}_{\text{em}}(0)|\pi^{\pm}(p)\rangle = \pm(p+p')^{\mu} F^V_\pi(s) \qquad s=(p'-p)^2$$

- **Unitarity**

$$\text{Im}\, F^V_\pi(s) = \theta\big(s-4M_\pi^2\big) F^V_\pi(s) e^{-i\delta_1^1(s)} \sin\delta_1^1(s)$$



$\hookrightarrow$ **final-state theorem**: phase of F^V_π equals $\pi\pi$ P-wave phase δ_1^1 Watson 1954

- Solution in terms of Omnès function Omnès 1958

$$F^V_\pi(s) = P(s)\Omega_1^1(s) \qquad \Omega_1^1(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

$\hookrightarrow$ can be predicted in terms of elastic phase shift!

- Function $P(s)$ free of 2π cut, but may still have other singularities (3π, 4π, …)

Matrix elements for nucleon decay: solution strategy

- Would be tempted to write

$$W_+(s) = P_+(s)\Omega_{0+}(s) \qquad \Omega_{0+}(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} ds' \frac{\delta_{0+}(s')}{s'(s'-s)}\right\}$$

$$W_-(s) = P_-(s)\Omega_{1-}(s) \qquad \Omega_{1-}(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} ds' \frac{\delta_{1-}(s')}{s'(s'-s)}\right\}$$

with $s_{\text{th}} = (m_N + M_P)^2$, $P = \pi, K$

- But: $W_\pm(s) = W_0(s) \pm \frac{\sqrt{s}}{m_N} W_1(s)$ has **kinematic singularity** $\sim \sqrt{s}$

$\hookrightarrow$ $P_\pm(s)$ have to inherit this cut

- Idea: $W_0(s) = W_+(s) + W_-(s)$ again free of kinematic singularities

$\hookrightarrow$ singularities have to cancel between $P_\pm(s)$

- Ansatz:

$$W_0(s) = W_0(0)\Big[(1-\alpha)\Omega_{0+}(s) + \alpha\frac{m_B^2}{m_B^2 - s}\Omega_{1-}(s)\Big]$$

since baryon pole sits in $W_-(s)$

- ChPT fixes $\alpha = -\frac{m_B}{m_N}\frac{W_1(0)}{W_0(0)}$

Matrix elements for nucleon decay: solution strategy II

- Second idea:

$$W_+(s)W_-(s) = \big[W_0(s)\big]^2 - \frac{s}{m_N^2}\big[W_1(s)\big]^2$$

  is again free of kinematic singularities and has an Omnès solution with $\delta_{0+} + \delta_{1-}$

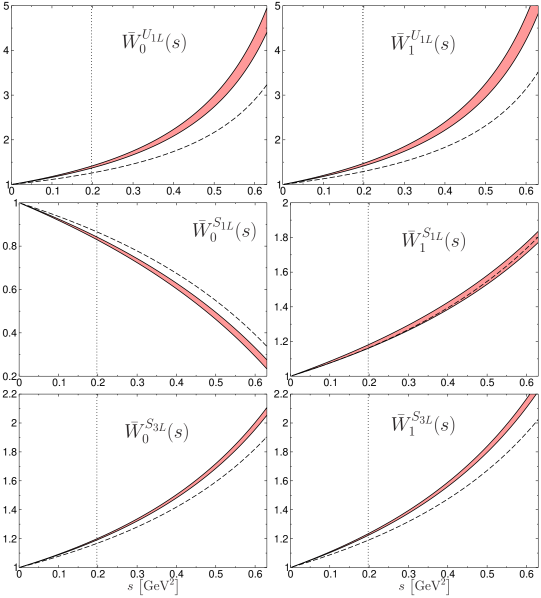
- Second ansatz:

$$W_+(s)W_-(s) = \big[W_0(0)\big]^2\Omega_{0+}(s)\Omega_{1-}(s)\frac{m_B^2}{m_B^2 - s}(1+\beta s)$$

$$\beta = (1-2\alpha)\bigg[\dot{\Omega}_{0+} - \dot{\Omega}_{1-} - \frac{1}{m_B^2}\bigg] - \frac{\big[W_1(0)\big]^2}{m_N^2\big[W_0(0)\big]^2} \qquad \dot{\Omega}_{\ell\pm} = \frac{d\Omega_{\ell\pm}(s)}{ds}\bigg|_{s=0}$$

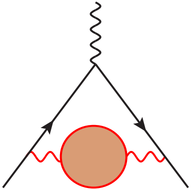
$\hookrightarrow$ implements **normalization, unitarity, and chiral constraints**

Matrix elements for nucleon decay: results

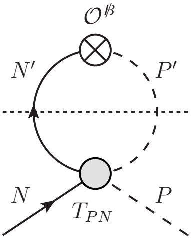


- Typical limits:
  - Two-body decays:
    $|C_i| \lesssim (10^{-15}/\,\text{GeV})^2$
  - Four-body decays:
    $|C_i| \lesssim (10^{-10}/\,\text{GeV})^2$
    $\hookrightarrow$ phase space and G_F
- Closes flat directions for τ operators
- Important input for global analysis of B-violating sector of SMEFT

Conclusions



- Matrix elements for BSM searches from **dispersion relations**
  - Convert quark-level operators to observables
  - Non-perturbative methods required
  - Analyticity and unitarity implemented via dispersion relations
- Rescattering corrections to **proton-decay matrix elements**
  - Unitarity corrections from meson–nucleon rescattering
  - Momentum dependence of form factors
  - Application to τ-mediated nucleon decay



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