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Pion and Kaon Transverse Momentum-Dependent Parton Distribution Functions in Lattice QCD

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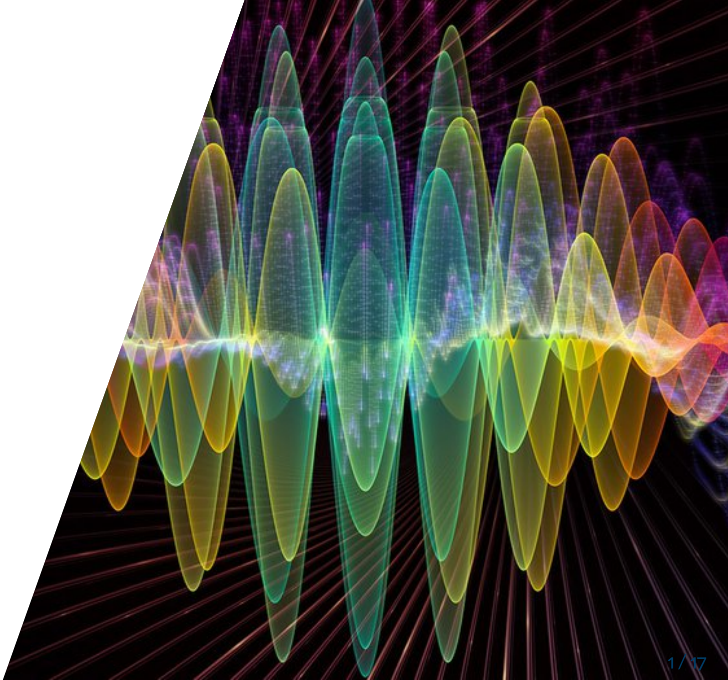

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Why Transverse Momentum-Dependent Parton Distribution Functions?

- Transverse momentum- dependent parton distribution functions (TMDPDFs) provide information on the structure of hadrons transversely to their boost direction,specifically the intrinsic transverse momentum of partons in hadrons.
- Less investigated so far compared to the collinear-PDFs.
- Lots of experimental efforts :
  - **COMPASS** experiment at CERN **[arXiv:hep-ex/0703049]**
  - **RHIC** at Brookhaven National Lab **[arXiv:2103.05419]**
  - **Electron-Ion Collider** being constructed at the Brookhaven National Laboratory
  - **SIDIS** from Jefferson Laboratory **[arXiv:1902.05022]**
  - **HERMES** at Desy **[arXiv:1407.2445]**
- Large Momentum Effective Field Theory (LaMET) framework has improved the computation of the physical light-cone PDFs, matching them from the Quasi-PDFs distribution calculated on the lattice

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Transverse Momentum-Dependent Parton Distribution Functions

The physical Transverse Momentum-Dependent Parton Distribution Functions (TMDPDFs) can be obtained in the LaMET framework through a matching of the Quasi-TMDPDFs $\tilde{f}^{TMD}$:

$$f^{\mathrm{TMD}}(x,b,\mu,\zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln(\zeta_z/\zeta) K(b,\mu)} \tilde{f}^{\mathrm{TMD}}(x,P_z,b;\mu) S_r{}^{1/2}(b,\mu) + \\ + \mathcal{O}\left(\Lambda^2_{\mathrm{QCD}}/\zeta_z, M^2/(P_z)^2, 1/(b^2\zeta_z)\right)$$

[arXiv:1801.05930]

- $\tilde{f}^{\mathrm{TMD}}$ Rapidity-Independent Quasi-Transverse Momentum-Dependent Parton Distribution Function (Quasi-TMDPDF)

Matching Coefficients

- S_r Rapidity-Independent Reduced Soft Function
- K Rapidity-Dependent Collins-Soper kernel
- H Perturbative Matching Kernel

Definitions of Quasi-TMDPDFs, Quasi-Beam Functions and Staple-Shaped Wilson Line

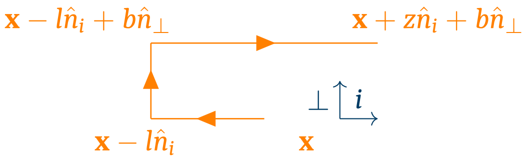
Quasi-Transverse Momentum-Dependent Parton Distribution Functions

Quasi-TMDPDFs in the LaMET framework can be written in terms of Quasi-Beam Functions $h_{0,\Gamma}$, as follows

$$\tilde{f}^{\mathrm{TMD}}{}_{\Gamma}\left(x, P_z, b; \mu\right) \equiv \lim_{l \to \infty} \int \frac{dz}{2\pi} e^{-ixzP_z} \mathcal{Z}_{\Gamma\Gamma'}\left(\mu, b, z, l\right) \frac{2P_z}{N_\Gamma} h_{0,\Gamma'}\left(z, P_z, b, l\right)$$

$$h_{0,\Gamma}\left(z, P_z, b, l\right) = \langle N(P_z)|\bar{\psi}\left(0\right)\Gamma\mathcal{W}\left(b, z, l\right)\psi\left(b\hat{n}_\perp + z\hat{n}_i\right)|N(P_z)\rangle$$

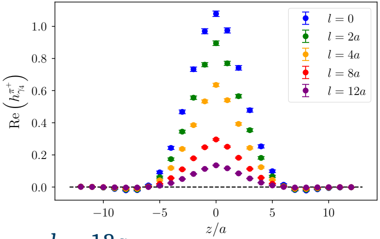
with $|N\left(P_z\right)\rangle$ being the hadronic state with boost P_z, ψ the quark field and $\mathcal{W}$ the staple-shaped Wilson line.



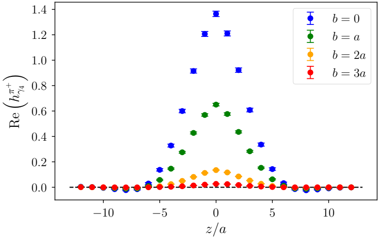
Divergences Introduced by the Staple-Shaped Wilson Line Operator

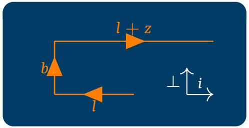
Quasi-Transverse Momentum-Dependent Parton Distribution Functions

$b = 2a$



$l = 12a$





Lattice Characteristics

Ensemble name	$L^3 \times T$	a (fm)	$a\mu_l$	m_π (MeV)	$a\mu_s$	m_K (MeV)
cA211.53.24	$24^3 \times 48$	0.093	0.0053	350	0.0221	554

- $\mathcal{W}$ **adds linear divergences** depending on the total length of the staple **and logarithmic divergences** coming from the cusps of the staple.
- **In LaMET also pinch-pole singularities** coming from the gluon exchange between the transverse segments of the staple **at the limit of** $l \to \infty$.

Form Factor

Reduced Soft Function

- Mesonic Form Factor :

$$F_{\Gamma}\left(b, P_z\right) = \langle \pi\left(P'_z\right) \mid \left(\bar{d}\Gamma d\right)\left(b\hat{n}_{\perp}\right)\left(\bar{u}\Gamma u\right)(0)|\pi\left(P_z\right)\rangle \,, \qquad \textbf{[arXiv:1910.11415]}$$

with $P_z = (P_0, 0, 0, P)$ and $P'_z = (P_0, 0, 0, -P)$.

- In the LaMET framework,factorized into:

$$S_r(b, \mu) = \frac{F_{\Gamma}\left(b, P_z, \mu\right)}{N_{\Gamma} J_1}\left(1 - \frac{\alpha_s}{4\pi} C_F\left(h_0^{\Gamma} + 2\pi^2 + \frac{J_2}{J_1} + h_1^{\Gamma}\frac{J_3}{J_1}\right)\right) + \mathcal{O}\left(\alpha_s^2\right)$$

h_0^{Γ},h_1^{Γ} and C_F coefficients and J_1,J_2 and J_3 integrals of expressions involving the quasi-TMDWF Φ.

- In the limit of $P_z \to \infty$ and at the leading order w.r.t. α_s : **[arXiv:2106.13027v2]**]

$$S_r(b, \mu) = \frac{F_{\Gamma}\left(b, P_z, \mu\right)}{N_{\Gamma}|\tilde{\phi}\left(0, b, P_z, \mu\right)|^2} + \mathcal{O}\left(\alpha_s\right)$$

We will use this last expression for now.

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Quasi-Transverse Momentum-Dependent Wave Function and Wilson Loop

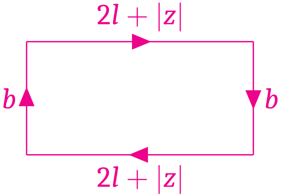
Reduced Soft Function

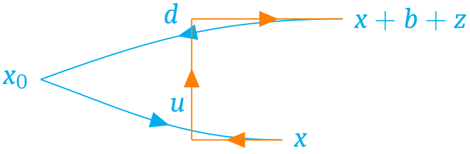
The Quasi-Transverse Momentum Dependent Wave-Function Φ (Quasi-TMDWF) is the Fourier transformation of the coordinate-space Quasi-TMDWF $\tilde{\phi}$:

$$\Phi(x, b, P_z, \mu) = \int_{-\infty}^{+\infty} dx \frac{P_z}{2\pi} e^{ixzp_z} \tilde{\phi}(z, b, P_z, \mu)$$

where

$$\tilde{\phi}(z, b, P_z, \mu) = \lim_{l \to \infty} \frac{\phi(z, b, l, P_z, \mu)}{\sqrt{Z_E(b, 2l + |z|, \mu)}}, \quad \phi(z, b, l, P_z, \mu) = \langle 0 | \bar{\psi}(0) \Gamma \mathcal{W}(b, z, l) \psi(b\hat{n}_\perp + z\hat{n}_i) | N(P_z) \rangle$$





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Collins-Soper Kernel

- It is the only quantity that regulates the rapidity-dependence of the physical TMDPDF.
- At the leading order w.r.t. $\mathcal{O}(\alpha_s)$ and within the assumption of $P_z \to \infty$:

$$K(b,\mu) = \lim_{l\to\infty} \frac{1}{\ln\left(\frac{P_{z1}}{P_{z2}}\right)} \ln\left|\frac{\phi(0,b,l,P_{z1},\mu)\,\phi(0,0,l,P_{z2},\mu)}{\phi(0,b,l,P_{z2},\mu)\,\phi(0,0,l,P_{z1},\mu)}\right| + \mathcal{O}(\alpha_s)$$

- In soon future works we plan to investigate this quantity considering the finite momentum case and up to $\mathcal{O}\left(\alpha_s^2\right)$.

RI/MOM scheme vs RI-short

Renormalization Procedures

RI/MOM

- Renormalization factors calculated in RI/MOM as usual, **only difference in the vertex function** defined as

$$G_{0,\Gamma}(b,z,l,\mu) = \sum_{\mathbf{x}_f,\mathbf{x}} e^{-i\mu\cdot(x_f-x_0)} \langle u(x_f)\bar{u}(x)\Gamma\mathcal{W}\,(b,z,l)\,d(x+b\hat{n}_\perp+z\hat{n}_i)\bar{d}(x_0)\rangle.$$

RI-short

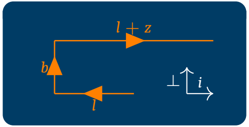
- New subtracted vertex function G_Γ as

$$G_\Gamma(b,z,\mu) = \lim_{l\to\infty} \frac{G_{0,\Gamma}(b,z,l,\mu)}{\sqrt{Z_E(b,2l+|z|,\mu)}},$$

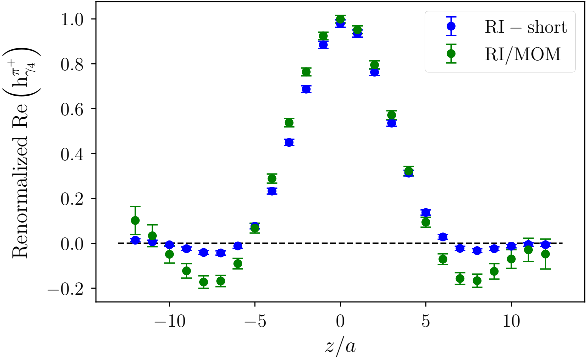
- **The square root of the vacuum expectation value of the Wilson loop Z_E cancels the divergences** on b and l, as well as the linear divergence on z.
- **We can now fix the scales of the renormalization factors to a short distance region** where perturbation theory is valid, since in the LaMET framework we work at high-momenta regimes.
- The renormalization procedure then follows the usual RI/MOM, except for the fact that also the matrix elements need to be subtracted by Z_E.

RI-short Outperforms RI/MOM at Large Distances

Renormalization Procedures



$b = 2a$, for RI-short $b_0 = 1a$ and $z_0 = 0$



- At large values of z ,RI/MOM scheme has large errors.
- This is the sign of a remaining linear divergence
- RI-short handles the remaining divergences

Short Distance Ratio Scheme

Renormalization Procedures

- It uses the subtracted bare matrix elements **[arXiv:2205.13402]**

$$h_\Gamma(z, P_z, b) = \lim_{l\to\infty} \frac{h_{0,\Gamma}(z, P_z, b, l)}{\sqrt{Z_E(b, 2l + |z|, \mu)}} .$$

- Only remaining divergences are the UV divergences associated with the quark field and its end-points connecting to the gauge links.
- Since remaining divergences are multiplicative, we can take the following ratio

$$h_\Gamma{}^{SDR}(z, z_0, P_z, b, b_0, \mu) = Z^{SDR}(z_0, b_0, \mu)\, h_\Gamma(z, P_z, b) ,$$

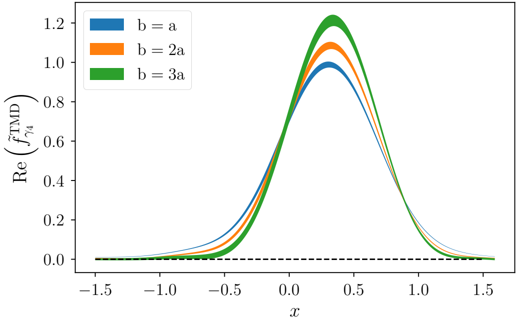
where,

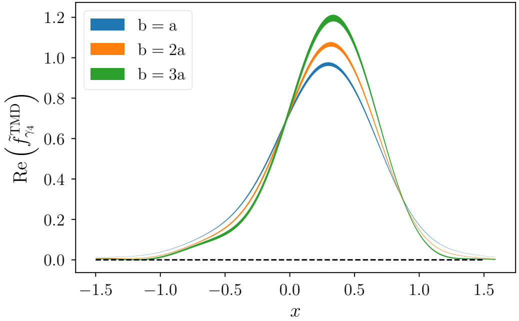
$$Z^{SDR}(z_0, b_0, \mu_0) = \frac{1}{h_\Gamma(z = z_0, P_z = 0, b = b_0)} .$$

- Since the remaining divergences are independent of the length of the Wilson line, one is free to choose z_0 and b_0, as long as they lie in the short distance perturbative region.

$\overline{\text{MS}}$ Renormalized Quasi-TMDPDFs

Results

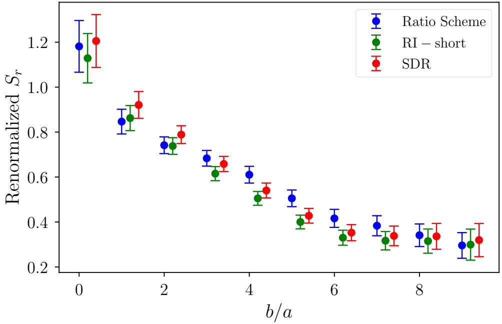




- On the left graph we show the pion case, whereas on the right we show the kaon case.
- $|\vec{P_z}| \approx 1.67$ GeV in order to ensure LaMET framework.
- Converted into the $\overline{\text{MS}}$ scheme from the RI-short non-perturbative scheme.
- Calculated with Discrete Fourier Transform method.

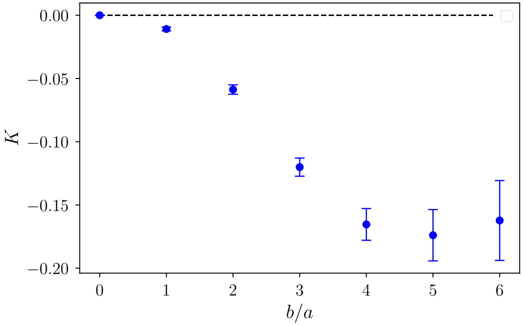
Reduced Soft Function Non-Perturbatively Renormalized with Different Methods

Results



- To check validity of our new techniques, we compare them with the Ratio scheme non-perturbative renormalization procedure, used by Y.Li et al. in **[arXiv:2106.13027v2]**
- S_r calculated at $|\vec{P_z}| \approx 2.78$ GeV to ensure LaMET framework

Collins-Soper Kernel Results



- Results calulcated at momenta $|\vec{P_z}| \approx 1.67$ GeV and $|\vec{P_z}| \approx 2.78$ GeV.
- Signal-to-noise ratio decreases very fast with b. Needed much bigger statistics for $b \geq 5a$.

Conclusions

- Good signal-to-noise ratio results for the Quasi-TMDPDFs.
- RI-short and SDR procedures have shown to handle better residual divergences compared to the standard RI/MOM scheme.
- Our results for the Reduced Soft Function are compatible with the ones presented by Y.Li at al. in [arXiv:2106.13027v2].

In our next work we want to :

- Use the Backus-Gilbert method for calculating the Quasi-TMDPDFs.
- Reduced Soft Function calculated with finite momentum assumption and second order of expansion in α_s.
- Perform the matching to the physical point with the Collins-Soper Kernel calculated in a more rigorous way, considering corrections up to $\mathcal{O}(\alpha_s^2)$ and not taking the limit $P_z \to \infty$.
- Get a first peek into final physical TMDPDFs once we calculate the Hard Kernel.
- Investigate the physical point.

Thanks For Your Attention!

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