Exploring Momentum Fraction in Hadrons through Lattice Quantum Chromodynamics Simulations

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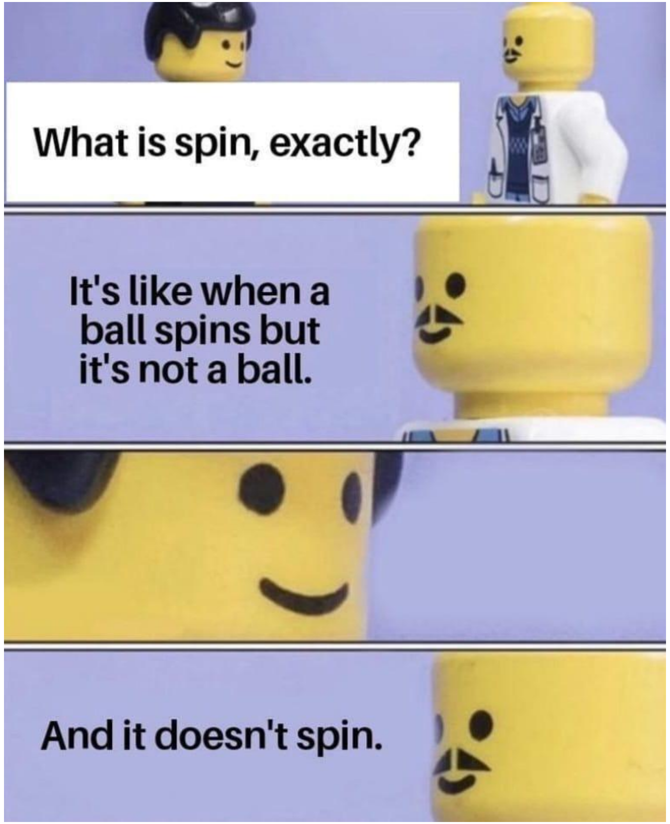
Spin


What is spin, exactly?
It's like when a ball spins but it's not a ball.
And it doesn't spin.

Why analyse the structure of hadrons?

- ➔ The European Muon Collaboration found that about half of the proton spin is carried by the valence quarks, this came to be known as the proton spin puzzle[1]
- ➔ This puzzle and the fact that the structure of protons is more accessible led them to be studied extensively
- ➔ The solution to this problem was given by the contribution of the sea quarks and the gluon
- ➔ To compute the proton spin, we need to evaluate the proton matrix elements of the euclidian energy momentum tensor
- ➔ This can be decompose into three generalized form factors

$$\langle N(p',s')|T^{\mu\nu;q,g}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^{q,g}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q,g}(q^2)\frac{i\sigma^{\{\mu\rho}q_\rho P^{\nu\}}}{2m_N} + C_{20}^{q,g}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m_N}\right]u_N(p,s)$$

- ➔ One of them, $A^{g,q}_{20}(q^2)$ in the zero momentum transfer limit, gives the average momentum fraction carried by quarks and gluon[2]

[1] J. Ashman et al. Nucl. Phys. B328, 1 (1989), [,351(1989)].
[2] X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249.

Why analyse the structure of pion and kaon?

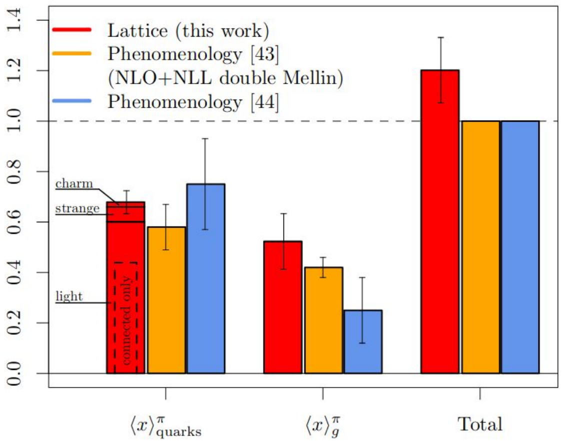
- ➔ On the other hand, particles as pion and kaon are more experimentally challenging
- ➔ There is not much work on the structure of these particles
- ➔ New experimental data will come from COMPASS++/Amber, related with quark and gluon dynamics[3]

Can we theoretically calculate the quark and gluon momentum fraction contributions to the mesons?

$\langle x \rangle_g^X$ $\langle x \rangle_q^X$ $X \in K, \pi$

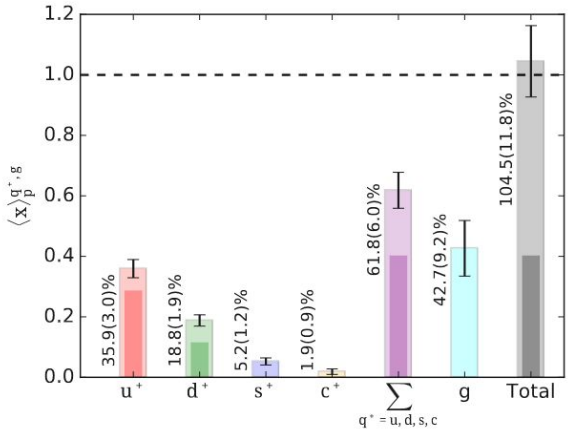
[3] B. Adams et al., “Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER),” (2018)

State of the art



- [43] Authors of Ref. [44] communicating their results at 2 GeV in the MS scheme.
- [44] P.C. Barry et al, arXiv:2108.05822 [hep-ph].

C. Alexandrou et al. Phys. Rev. Lett. 127, 252001



C. Alexandrou, et al. Phys. Rev. D 101, 094513

- There are studies about the composition of pion and proton
- Around 60% of the momentum fraction come from the quarks
- The rest comes from the gluon which is only disconnected
- This calculations are done using data from cB211.072.64[4]

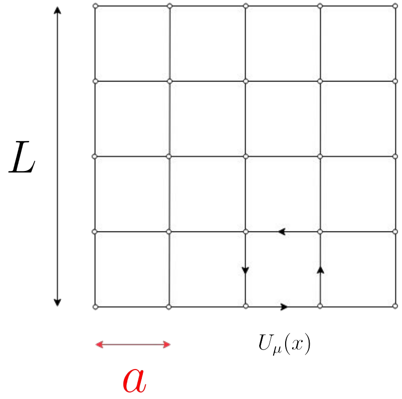
[4]C. Alexandrou et al. Phys. Rev. D 98, 054518

What is a LQCD ensemble?

- The expectation value of an operator is given by

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D[U] D[\bar{\psi}, \psi] \mathcal{O} e^{-S[U,\bar{\psi},\psi]} \qquad Z = \int D[U] D[\bar{\psi}, \psi] e^{-S[U,\bar{\psi},\psi]}$$

- The numerical evaluation of the integral can be perform by sampling the integrand in points of the lattice with certain probability
- There are different algorithms to choose the sampling and the acceptance of the field configuration such as Markov chains, Hybrid Monte Carlo, etc…[5, 6]



- The resulting set of configurations is called ensemble
- Each ensemble has different fixed parameters as lattice spacing, masses, number of quark flavors, etc…

[5]M. Lusher, Computational Strategies in Lattice QCD, 2009, arXiv:1002.4232 [hep-lat]
[6]Bhavna Prasad talk

What is novel in what we are doing?

- We use three ensembles with different lattice spacing, in order to take the continuum limit

- In this work beyond the pion we include the kaon to study the effects of having a strange quark.

$Ensemble$	V/a^4	$L[fm]$	$a[fm]$
cB211.072.64	$64^3 \times 128$	5.1	0.07957
cC211.060.80	$80^3 \times 160$	5.44	0.06821
cD211.054.96	$96^3 \times 192$	5.47	0.05692

- The momentum fraction is calculated using three $N_f = 2+1+1$ twisted mass fermions ensembles

- We use a weighted average based on the Akaike Information Criterion[7]

$$P_i = \frac{1}{Z} e^{-\frac{1}{2}\chi_i^2 + N_{dof}}$$

$$Z = \sum_i e^{-\frac{1}{2}\chi_i^2 + N_{dof}}$$

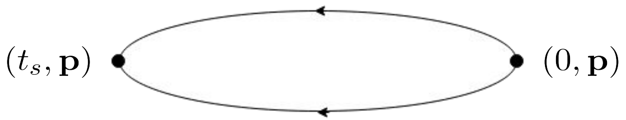
[7]William I. Jay and Ethan T. Neil, Phys. Rev. D 103, 114502

Three- and two- point functions

- Two-point function

$$C^X(t_s, \mathbf{p}) = \langle X(t_s, \mathbf{p}) X(0, \mathbf{p}) \rangle$$

$$X \in K, \pi$$

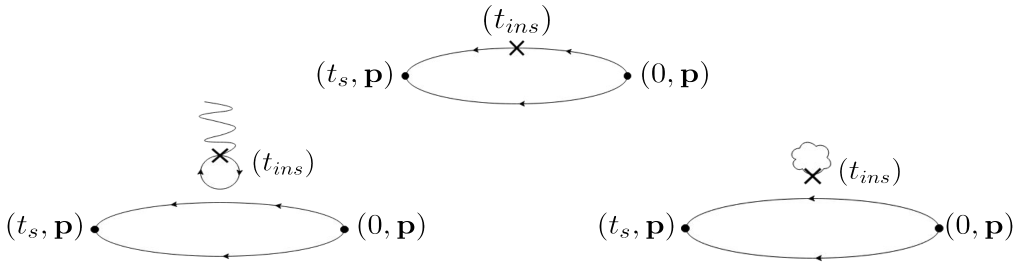


- Three-point function

$$C^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p}) = \langle X(t_s, \mathbf{p}) T^f_{\mu\nu}(t_{ins}) X(0, \mathbf{p}) \rangle$$

$$T^f_{\mu\nu} = \text{Euclidian energy momentum tensor}$$

$$f \in u, d, s, c \text{ and } g$$

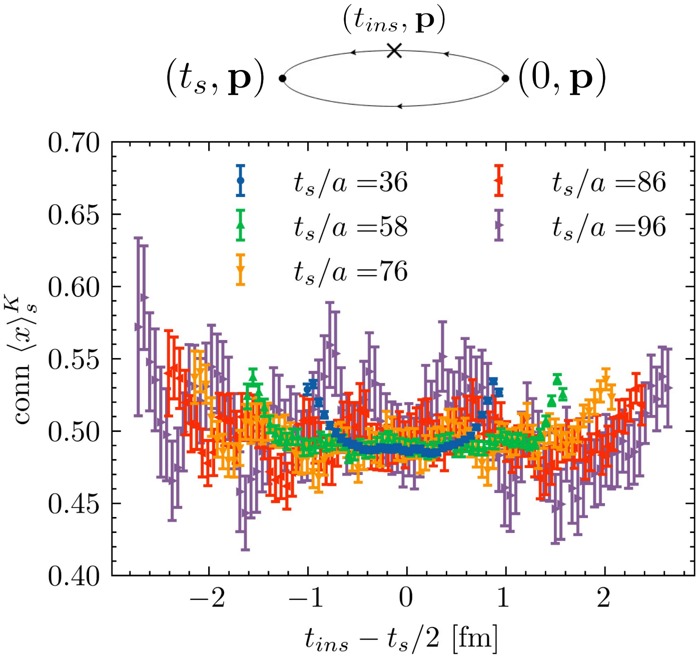


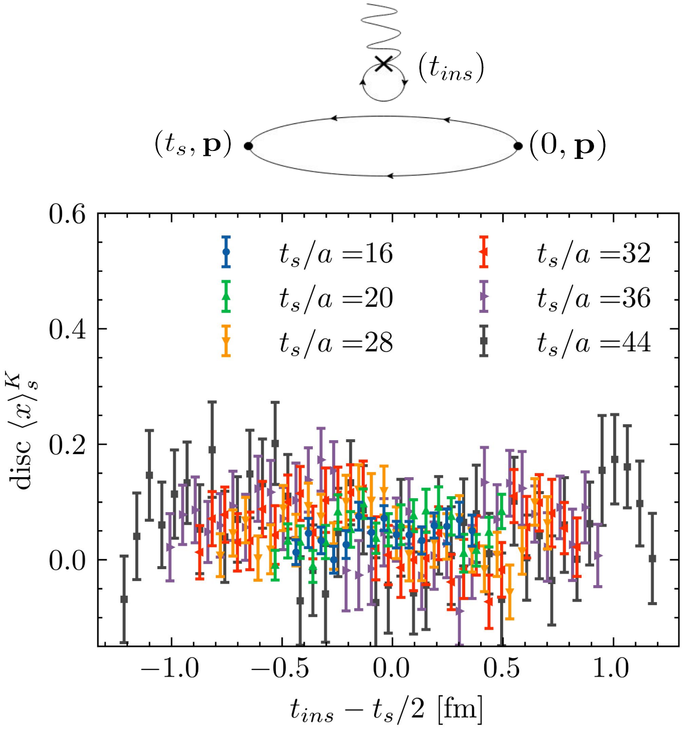
- In order to extract the matrix elements, we can calculate the ratio between the three- and two-point function. In the large enough time limit is proportional to the average momentum fraction

$$R^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p}) = \frac{C^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p})}{C^X(t_s, \mathbf{p})} \xrightarrow[t_s \to \infty]{t_s - t_{ins} \to \infty} \langle x \rangle^X_f$$

How does the ratio $R^{f,X}_{\mu\nu}$ look like?

$$\bar{T}^q_{\mu\nu} = -\frac{(i)^{\kappa_{\mu\nu}}}{4}\,\bar{q}\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu - \delta_{\mu\nu}\frac{1}{2}\gamma_\rho \overleftrightarrow{D}_\rho\right) q$$

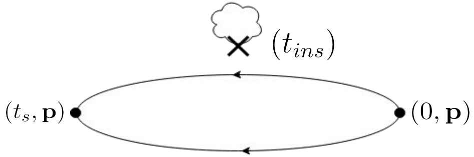


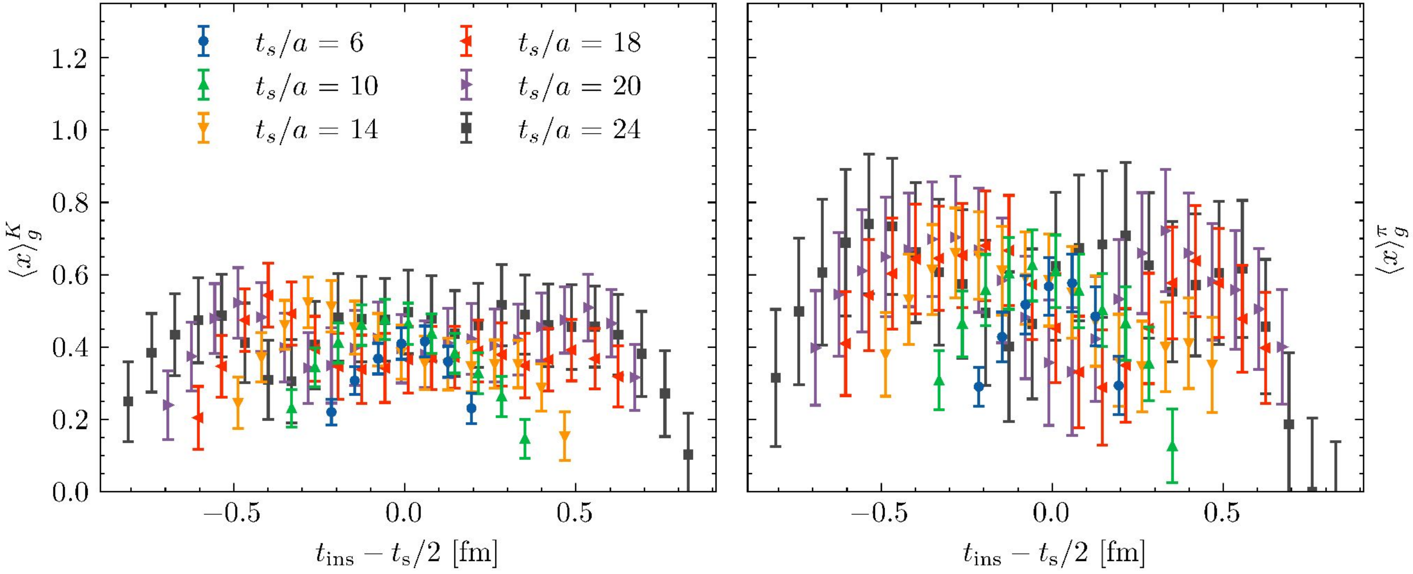


Ratio for quark connected and disconnected contributions, for different values of t_s

And for the gluon?

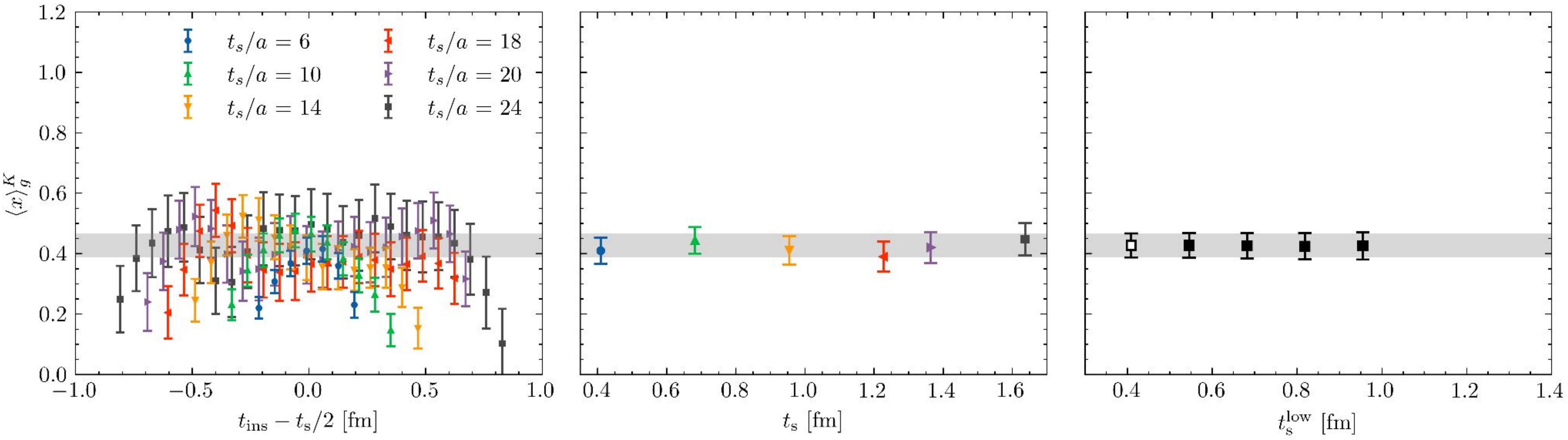
$$\bar{T}^{g}_{\mu\nu} = (i)^{\kappa_{\mu\nu}} \left(F_{\mu\rho} F_{\nu\rho} + F_{\nu\rho} F_{\mu\rho} - \delta_{\mu\nu} \frac{1}{2} F_{\rho\sigma} F_{\rho\sigma} \right)$$





Ratio for gluon disconnected contributions. As before, plotted for different values of t_s.

Plateau fit



Plateau fit. In the left panel is shown the bare ratio. In the middle panel, is shown the plateau fit per t_s . In the right panel is shown the fit per t_s^{low} . The gray band is the value after the weighted average. The white marker is the value with the highest probability.

Renormalization procedure

Re-normalize the results, including mixing

$$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g + Z_{gq} \langle x \rangle_q$$

$$\langle x \rangle_q^R = Z_{qq} \langle x \rangle_q + Z_{qg} \langle x \rangle_g$$

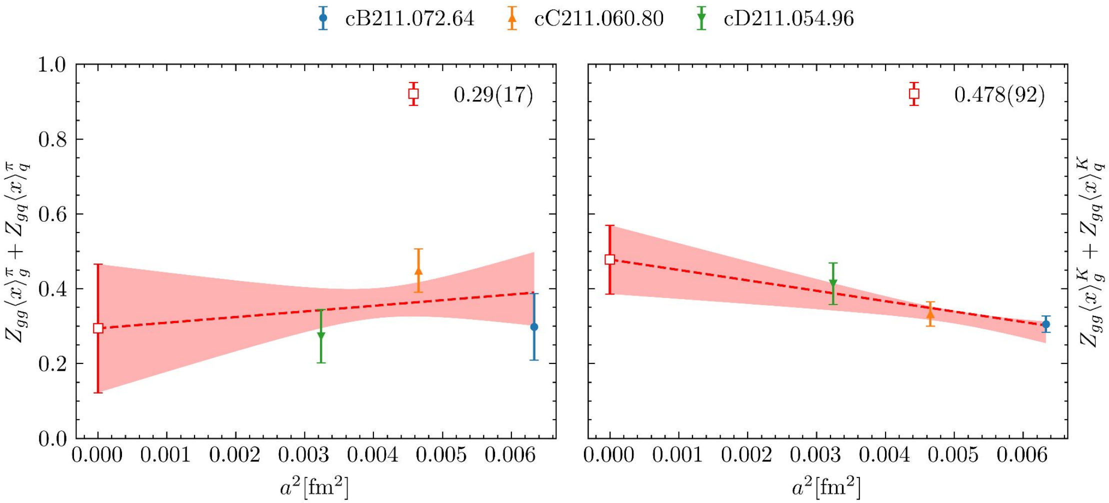
Test the momentum sum rule

$$\langle x \rangle_g^R + \langle x \rangle_q^R = 1$$

Continuum limit

- We can take the continuum limit as a linear fit in the square of the lattice spacing

$$f(a^2) = c_0 + c_1 a^2$$

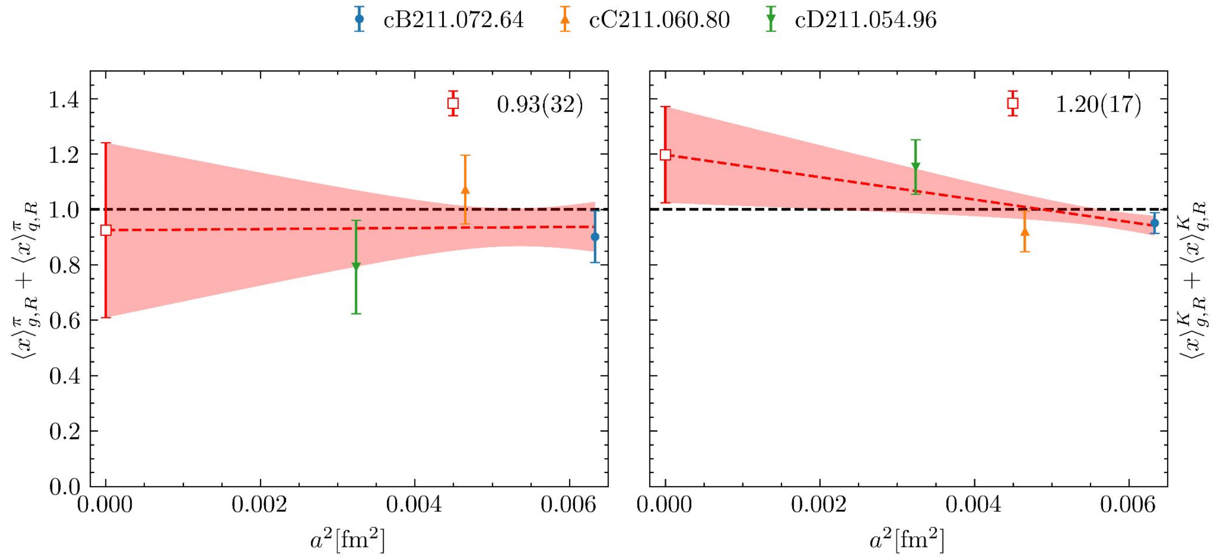


Green, yellow and blue markers are the values per ensemble. Red marker with open symbol is the continuum limit value. The red band is the error propagation of the fit. The dotted line is the value of the fitted function.

Sum rule

The renormalized contributions of gluon and quarks should obey the sum rule

$$\langle x \rangle_g^R + \langle x \rangle_q^R = 1?$$



Continuum limit for the sum of all components of the mesons. The results are compatible within one sigma for pion and two sigmas for kaon.

Summary and future work

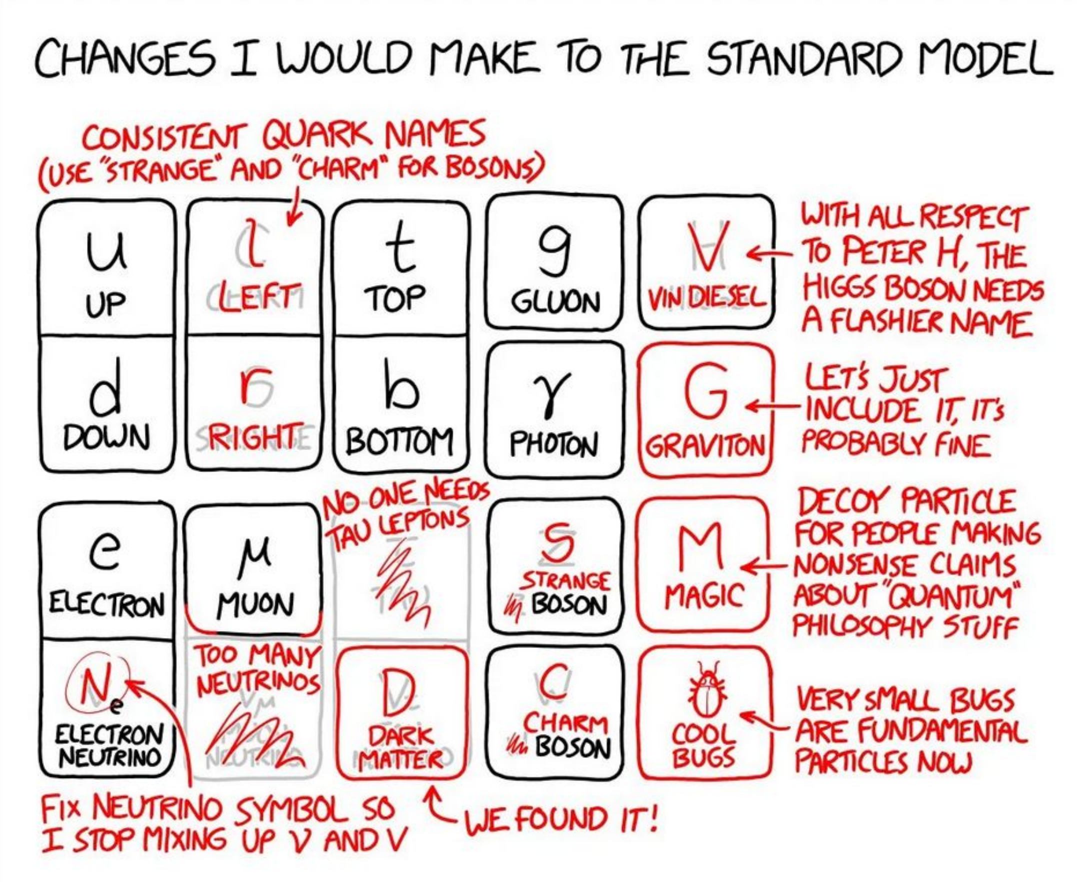
Preliminary

	$\langle x \rangle_g^{\pi,R}$	$\langle x \rangle_q^{\pi,R}$	$\langle x \rangle_g^{K,R}$	$\langle x \rangle_q^{K,R}$
cB211.072.64	0.298(89)	0.604(29)	0.305(22)	0.646(29)
cC211.060.80	0.448(58)	0.62(11)	0.332(32)	0.589(66)
cD211.054.96	0.273(71)	0.52(15)	0.414(56)	0.740(82)

Summary of every contribution from quarks and gluon for three ensembles.

- This work agrees with previous results
- Sum rule is satisfied
- Increase statistics for each ensemble
- Add one more ensemble for the continuum limit
- To do a similar analysis for nucleon

Thank you!


CHANGES I WOULD MAKE TO THE STANDARD MODEL
CONSISTENT QUARK NAMES (USE "STRANGE" AND "CHARM" FOR BOSONS)
u UP
l LEFT
t TOP
g GLUON
V VIN DIESEL
WITH ALL RESPECT TO PETER H, THE HIGGS BOSON NEEDS A FLASHIER NAME
d DOWN
r RIGHT
b BOTTOM
γ PHOTON
G GRAVITON
LET'S JUST INCLUDE IT, IT'S PROBABLY FINE
e ELECTRON
μ MUON
NO ONE NEEDS TAU LEPTONS
S STRANGE BOSON
M MAGIC
DECOY PARTICLE FOR PEOPLE MAKING NONSENSE CLAIMS ABOUT "QUANTUM" PHILOSOPHY STUFF
N ELECTRON NEUTRINO
TOO MANY NEUTRINOS
D DARK MATTER
C CHARM BOSON
COOL BUGS
VERY SMALL BUGS ARE FUNDAMENTAL PARTICLES NOW
FIX NEUTRINO SYMBOL SO I STOP MIXING UP ν AND V
WE FOUND IT!