Herbi-Fest I

From the early 1990's right up through today, these great papers were a gift to those of us trying to learn about supersymmetry, and not just the R-parity violating kind:

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North-Holland

NUCLEAR
PHYSICS B

R-PARITY VIOLATION AT HADRON COLLIDERS

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Using a new operator notation we present a systematic study of all possible R-parity violating signals at a hadron collider.

An Introduction to Explicit R-parity Violation

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I discuss the theoretical motivations for R-parity violation, review the experimental bounds and outline the main changes in collider phenomenology compared to conserved R-parity. I briefly comment on the effects of R-parity violation on cosmology.

I remember taking copious notes on the first one especially, while first trying to learn the subject shortly after it appeared. The tattered and coffee-stained copy is still on the bookshelf next to my desk.

Once upon a time, a few[†] years ago, three intrepid investigators (Herbi, Howie, and me) set out to create something wonderful: a book on supersymmetry!

Encouragement was plentiful:

“Vast royalty money, you’ll be crazy rich!”, they said.

“It will be easy and fun!” they said.

“What could go wrong?” they said.

But there were words of warning too: “Hurry, because you should try to beat the Tevatron Run 2 discovery,” they said.

I think we started planning in 1998, although the original emails seem to have been lost.

[†] 25, more or less

In 1998, the world was younger than today...

- A plucky new startup company called Google was launched, with the aspirational motto "don't be evil"
- World leaders: Bill Clinton, Tony Blair, Helmut Kohl, Boris Yeltsin
- In baseball, it was a horrible year:
  - New York Yankees finished 22 games ahead of the Boston Red Sox, win the World Series
  - Mark McGwire and Sammy Sosa both broke Roger Maris' home run record, using questionable medicinal practices
- Limits on supersymmetry from Fermilab Tevatron and LEP:
  $M_{\text{gluino}} > 190$ GeV,
  $M_{\text{squarks}} > 260$ GeV,
  $M_{\text{top squark}} > 70$ GeV,
  $M_{\text{charginos}} > 92$ GeV.

Two particularly delicious quote from emails from our contacts at Cambridge University Press, in July 1998:

"...we would be very interested indeed to discuss publication of your supersymmetry book. My own feeling is that it could not be **more timely** ..."

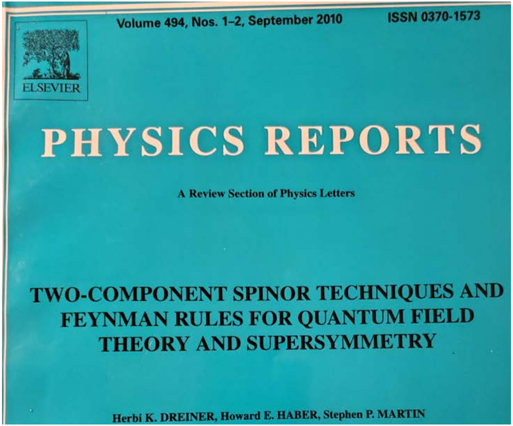
and in April 2000, after signing a contract:

"If it ever looks as though you won't be able to deliver by the **January 2002** deadline, please let me know."

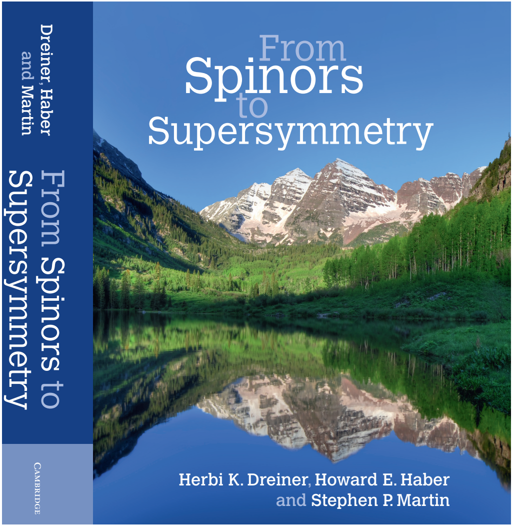
Emails similar to the last one followed once or twice a year for the next 22 years or so.

Why has it taken so long?

- deciding on a publisher (about a year?).
- unsuccessful attempts by me to convince Herbi and Howie that the $(-+++)$ metric would be a sign of sophistication and good taste.
- we were all doing Other Things, too.
- a short detour (8 years) to write a Physics Reports:



But, we are finally done!



1027 pages, pre-order now for delivery in July.

In my remaining time, I will talk about some work closely related to areas in which Herbi has made important contributions:
discrete symmetries, supersymmetry, and axions.

This is based on work in a paper arXiv:2106.14964 written with Prudhvi Bhattiprolu (Michigan).

Outline

1. Connection between the μ problem of supersymmetry and the strong CP problem of the Standard Model
2. Can solve both problems with the Kim-Nilles mechanism, get an axion
3. There are 4 distinct minimal versions ("Base models")
4. Reasons to look beyond the minimal Base models:
   - Why not?
   - Domain wall problem
   - Axion quality problem
5. Experimental consequences

Strong CP problem

QCD has a CP-violating term in the pure gluon sector:

$$\mathcal{L}_{\rm QCD} = \theta \frac{\alpha_S}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

as well as CP-violating phases in the 3×3 quark mass matrices M_u and M_d. Chiral phase rotations can eliminate one or the other, but the combination

$$\overline{\theta} \quad = \quad \theta + {\rm Arg}\left[{\rm Det}(M_u M_d)\right]$$

is invariant. The present limit on the neutron electron dipole moment gives

$$\overline{\theta} \quad < \quad 9 \times 10^{-11}.$$

Since CP violation in the electroweak sector is not small, this appears to be a serious fine-tuning problem.

In the Minimal Supersymmetric Standard Model, the Lagrangian is determined by the gauge symmetries and the superpotential:

$$W = \mu H_u H_d + y_u H_u q\bar{u} + y_d H_d q\bar{d} + y_e H_d \ell\bar{e}$$

which contains two Higgs fields H_u and H_d, which both get VEVs.

There are two issues related to the μ mass term:

- μ is supersymmetric, but should be of the same order as the supersymmetry-breaking mass terms, presumably 1 TeV.
  Why? What is the connection? This is called **the μ problem**.
- In the formal limit $\mu \to 0$, there is a PQ symmetry, which must be either explicitly or spontaneously broken.

The Kim-Nilles idea: promote the constant μ to a product of gauge-singlet fields which get vacuum expectation values. For example:

$$W = \frac{\lambda_\mu}{M_{\rm Planck}} X Y H_u H_d + \frac{\lambda}{M_{\rm Planck}} X^3 Y.$$

Here λ_μ and λ are dimensionless constants. The scalar potential for X, Y including supersymmetry breaking terms:

$$\begin{aligned} V_{\rm soft} &= \frac{|\lambda|^2}{M_{\rm Planck}^2} \left(9|Y|^2 + |X|^2\right) |X|^4 \\ &\quad - \left(\frac{a_\mu}{M_{\rm Planck}} X Y H_u H_d + \frac{a}{M_{\rm Planck}} X^3 Y \right) + {\rm c.c.} - m_X^2 |X|^2 - m_Y^2 |Y|^2, \end{aligned}$$

with a_μ, a, m_X, and m_Y all of order the TeV scale. Minimum has:

$$\langle X \rangle \sim \langle Y \rangle \sim \sqrt{m_{\rm soft} M_{\rm Planck}},$$

where $m_{\rm soft} \sim \rm TeV$, and then

$$\mu = \frac{\lambda_\mu \langle X \rangle \langle Y \rangle}{M_{\rm Planck}} \sim {\rm TeV}.$$

This potential has a global Peccei-Quinn $U(1)$ symmetry, with charges:

	X	Y	H_u	H_d	q	ℓ	$\overline{u}$	$\overline{d}$	$\overline{e}$	$\overline{\nu}$
PQ	1	-3	-1	-1	1	1	0	0	0	0

This symmetry cannot be exact; it has a QCD anomaly.

When the scalar components of X, Y, H_u, and H_d get VEVs, the anomalous $U(1)$ symmetry is spontaneously broken, giving rise to a very light pseudo-Nambu-Goldstone boson, the axion. As usual in axion models, at the minimum of the potential, the physical CP-violating θ angle of QCD vanishes.

This model links, and simultaneously solves, the strong CP problem of the Standard Model and the μ problem of supersymmetry.

Peccei-Quinn anomalous global symmetry

The PQ symmetry current has an anomalous divergence,

$$\partial_\mu j^\mu_{\mathrm{PQ}} = \frac{\alpha_S}{4\pi} N G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} + \frac{\alpha}{4\pi} E F_{\mu\nu} \widetilde{F}^{\mu\nu},$$

where

$$\begin{aligned} N &= U(1)_{\mathrm{PQ}}\text{-}SU(3)_c\text{-}SU(3)_c \text{ anomaly } = \mathrm{Tr}\left[Q_f\, T(R_f)\right], \\ E &= U(1)_{\mathrm{PQ}}\text{-}U(1)_{EM}\text{-}U(1)_{EM} \text{ anomaly } = \mathrm{Tr}\left[Q_f q_f^2\right], \end{aligned}$$

with $T(R_f) = 1/2$ for a color triplet and 0 for a color singlet,
while $q_f =$ electromagnetic charge.

The axion mass from non-perturbative QCD effects is, numerically,

$$m_A = \left(\frac{10^{12}\ \mathrm{GeV}}{f_A}\right) 5.7\ \mu\mathrm{eV}.$$

If this is the only term in the potential expanded to quadratic order, then one also finds

$$\overline{\theta} \ = \ \frac{\langle A\rangle}{f_A} \ = \ 0,$$

so that the strong CP problem is solved.

Actually, there are 4 distinct implementations of the Kim-Nilles idea leading to a DFSZ axion. In a normalization where $H_u H_d$ has PQ charge -2:

Base model	Superpotential terms	PQ charges of (X, Y)
$\mathrm{B_I}$	$XYH_uH_d + X^3Y$	$(-1, 3)$
$\mathrm{B_{II}}$	$X^2H_uH_d + X^3Y$	$(1, -3)$
$\mathrm{B_{III}}$	$Y^2H_uH_d + X^3Y$	$(-\frac{1}{3}, 1)$
$\mathrm{B_{IV}}$	$X^2H_uH_d + X^2Y^2$	$(1, -1)$

$\mathrm{B_I}$ = Murayama, Suzuki, Yanagida, PLB291, 418, (1992).
$\mathrm{B_{II}}$ = Choi, Chun, Kim, hep-ph/9608222.
$\mathrm{B_{III}}$ and $\mathrm{B_{IV}}$ = SPM, hep-ph/0005116.

We call these "Base models", because we will consider extensions of them with vectorlike quark and lepton superfields.
For the Base models, the anomaly coefficients are

$$N = 3, \qquad E = 6.$$

Note an important difference from non-SUSY DFSZ models, where $E = 8$.
In the SUSY case, the Higgsinos contribute negatively to E.

Extensions of the Base Models

In the MSSM Base models, there is one pair of vectorlike[†] fields that get masses from the X, Y fields, namely the Higgs fields H_u, H_d.

Why should they be the only ones?

Consider extended models with pairs of vectorlike quarks and leptons, which can have small mixing with the ordinary fermions.

Current LHC limits on vectorlike quarks that decay promptly range from 1.2 to 1.5 TeV, depending on branching ratios to Standard Model states, for example $t' \to tZ$ or $t' \to tH$ or $t' \to bW$.

LHC limits on isosinglet vectorlike leptons are very weak; masses only constrained to be larger than about 150 GeV.

[†] "vectorlike" means left-handed and right-handed fermions transform the same way. Chiral superfields transform oppositely under the gauge group.

Vectorlike pairs of chiral superfields $\Phi + \overline{\Phi}$ that can be added to the base models, and their Standard Model gauge transformation properties.

Superfields	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$Q + \overline{Q}$	$(\mathbf{3}, \mathbf{2}, 1/6) + (\overline{\mathbf{3}}, \mathbf{2}, -1/6)$
$U + \overline{U}$	$(\mathbf{3}, \mathbf{1}, 2/3) + (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$
$D + \overline{D}$	$(\mathbf{3}, \mathbf{1}, -1/3) + (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$
$L + \overline{L}$	$(\mathbf{1}, \mathbf{2}, -1/2) + (\mathbf{1}, \mathbf{2}, 1/2)$
$E + \overline{E}$	$(\mathbf{1}, \mathbf{1}, -1) + (\mathbf{1}, \mathbf{1}, 1)$

The superpotential masses can be renormalizable, for example:

$$W = X\Phi\overline{\Phi} \qquad \text{(intermediate scale mass)}$$

or non-renormalizable, for example:

$$W = \frac{1}{M_{\rm Planck}} X^2\Phi\overline{\Phi} \qquad \text{(TeV scale mass)}$$

or a combination.

Net PQ charges of vectorlike superfields $\sum Q_{\Phi\overline{\Phi}} \equiv Q_{\Phi} + Q_{\overline{\Phi}}$:

Mass terms	$\mathrm{B_I}$	$\mathrm{B_{II}}$	$\mathrm{B_{III}}$	$\mathrm{B_{IV}}$	
$XY\,\Phi\overline{\Phi}$	-2	2	$-2/3$	0	
$X^2\Phi\overline{\Phi}$	2	-2	$2/3$	-2	$M_{\Phi\overline{\Phi}} \sim \mathrm{TeV}$
$Y^2\Phi\overline{\Phi}$	-6	6	-2	2	
$X\,\Phi\overline{\Phi}$	1	-1	$1/3$	-1	$M_{\Phi\overline{\Phi}} \sim f_A$
$Y\,\Phi\overline{\Phi}$	-3	3	-1	1	

PQ anomaly coefficients, and therefore couplings of axions to matter, are correlated to the sources, and scales, of the masses of extra matter particles.

There are clearly many non-minimal models of this type, with different PQ charges and therefore different axion properties, even if the vectorlike superfields are out of reach of future experiments.

Given the PQ charges of the extra fields, the PQ anomaly coefficients are:

$$
\begin{aligned}
N &= 3 + \sum Q_{Q\overline{Q}} + \frac{1}{2}\sum Q_{U\overline{U}} + \frac{1}{2}\sum Q_{D\overline{D}} \\
E &= 6 + \frac{5}{3}\sum Q_{Q\overline{Q}} + \frac{4}{3}\sum Q_{U\overline{U}} + \frac{1}{3}\sum Q_{D\overline{D}} + \sum Q_{L\overline{L}} + \sum Q_{E\overline{E}}
\end{aligned}
$$

In terms of these, the couplings of the axion to photons, electrons, neutrons, and protons are:

$$
\begin{aligned}
g_{A\gamma} &= \frac{\alpha}{2\pi f_A}(E/N - 1.92), \\
g_{Ae} &= \frac{m_e}{f_A}\frac{\sin^2\beta}{N}, \\
g_{An} &= \frac{m_n}{f_A}\left(-0.02 + [0.83 - 1.24\cos^2\beta]/N\right), \\
g_{Ap} &= \frac{m_p}{f_A}\left(-0.47 + [-0.437 + 1.302\cos^2\beta]/N\right)
\end{aligned}
$$

Note that the axion couplings to both EM field and to electrons are "accidentally" small for the Base Models with $E/N = 2$ and $N = 3$, but both can be much larger for extended models.

After the PQ symmetry breaking, a discrete subgroup

$$Z_{N_{\rm DW}} \quad = \quad e^{i2\pi k/N_{\rm DW}}, \qquad k = 0, 1, \ldots, N_{\rm DW} - 1$$

is left unbroken. $N_{\rm DW}$ = "Domain Wall number" = the number of inequivalent degenerate minima of the axion potential.

No domain wall problem if:

- PQ symmetry broken before inflation, so that observable universe is a single patch that initially had a single common value of θ,
- $Z_{N_{\rm DW}}$ embedded in a continuous gauge symmetry, so $N_{\rm DE}$ "different" vacua are actually the same, or
- $N_{\rm DW} = 1$

The integer $N_{\rm DW}$ depends on the PQ-QCD-QCD anomaly coefficient N, and the charges of the scalars that get VEVs.

In the Base Models,

$$N_{\rm DW} \quad = \quad \begin{cases} 6 \text{ in Base Models } \mathrm{B_I}, \mathrm{B_{II}}, \text{ and } \mathrm{B_{IV}}, \\ 18 \text{ in Base Model } \mathrm{B_{III}}. \end{cases}$$

To get $N_{\rm DW} = 1$, we must have instead:

$$N \quad = \quad \begin{cases} \pm\frac{1}{2} \text{ in extensions of } \mathrm{B_I}, \mathrm{B_{II}}, \text{ and } \mathrm{B_{IV}}, \\ \pm\frac{1}{6} \text{ in extensions of } \mathrm{B_{III}}. \end{cases}$$

This in turn requires an **odd number** of extra vectorlike quarks at the intermediate scale.
Couplings to matter scale like $1/N$, so enhanced by a factor of 6 or 18.

Examples of extended models with $N_{DW} = 1$:

Model extension	Base	Mass terms	$1/N$	$3E/N$
$\mathbf{10}+\overline{\mathbf{10}}$ at $M_{\rm int}$	$\mathrm{B_I}$	$YQ\overline{Q} + XU\overline{U}$	2	20, −4
$\mathbf{5}+\overline{\mathbf{5}}$ at TeV, $\mathbf{5}+\overline{\mathbf{5}}$ at $M_{\rm int}$	$\mathrm{B_I}$	$XYD\overline{D} + YD'\overline{D}'$	2	44, 20, −4, −28
		$Y^2D\overline{D} + XD'\overline{D}'$	2	44, 20, −4, −28
$\mathbf{5}+\overline{\mathbf{5}}$ at TeV, $\mathbf{10}+\overline{\mathbf{10}}$ at $M_{\rm int}$	$\mathrm{B_I}$	$YQ\overline{Q} + YU\overline{U} + X^2D\overline{D}$	−2	68, 44, 20, −4
		$YQ\overline{Q} + XU\overline{U} + XYD\overline{D}$	−2	44, 20, −4, −28
		$XQ\overline{Q} + YU\overline{U} + Y^2D\overline{D}$	−2	44, 20, −4, −28
	$\mathrm{B_{II}}$	$XQ\overline{Q} + XU\overline{U} + X^2D\overline{D}$	2	68, 44, 20, −4
	$\mathrm{B_{IV}}$	$XQ\overline{Q} + XU\overline{U} + X^2D\overline{D}$	2	32, 20, 8, −4
$\mathbf{10}+\overline{\mathbf{10}}$ at TeV, $\mathbf{5}+\overline{\mathbf{5}}$ at $M_{\rm int}$	$\mathrm{B_I}$	$XYQ\overline{Q} + XYU\overline{U} + XD\overline{D}$	2	20, −4, −28, −52
		$XYQ\overline{Q} + X^2U\overline{U} + YD\overline{D}$	2	44, 20, −4, −28
		$X^2Q\overline{Q} + Y^2U\overline{U} + YD\overline{D}$	2	20, −4, −28, −52
	$\mathrm{B_{II}}$	$X^2Q\overline{Q} + X^2U\overline{U} + XD\overline{D}$	−2	20, −4, −28, −52
	$\mathrm{B_{III}}$	$Y^2Q\overline{Q} + XYU\overline{U} + YD\overline{D}$	6	44, 20, −4, −28
		$Y^2Q\overline{Q} + Y^2U\overline{U} + XD\overline{D}$	6	20, −4, −28, −52
	$\mathrm{B_{IV}}$	$X^2Q\overline{Q} + XYU\overline{U} + XD\overline{D}$	2	32, 20, 8, −4
		$X^2Q\overline{Q} + X^2U\overline{U} + XD\overline{D}$	−2	20, 8, −4, −16
		$X^2Q\overline{Q} + X^2U\overline{U} + YD\overline{D}$	2	20, 8, −4, −16
$\mathbf{10}+\overline{\mathbf{10}}$ at TeV, $\mathbf{10}+\overline{\mathbf{10}}$ at $M_{\rm int}$	$\mathrm{B_I}$	$XYQ\overline{Q} + XYU\overline{U} + XQ'\overline{Q}' + YU'\overline{U}'$	−2	68, 44, 20, −4
		$XYQ\overline{Q} + X^2U\overline{U} + YQ'\overline{Q}' + XU'\overline{U}'$	−2	44, 20, −4, −28
	$\mathrm{B_{II}}$	$X^2Q\overline{Q} + XYU\overline{U} + XQ'\overline{Q}' + XU'\overline{U}'$	2	68, 44, 20, −4
	$\mathrm{B_{III}}$	$XYQ\overline{Q} + Y^2U\overline{U} + YQ'\overline{Q}' + YU'\overline{U}'$	−6	68, 44, 20, −4
		$Y^2Q\overline{Q} + XYU\overline{U} + YQ'\overline{Q}' + XU'\overline{U}'$	−6	44, 20, −4, −28
	$\mathrm{B_{IV}}$	$XYQ\overline{Q} + X^2U\overline{U} + XQ'\overline{Q}' + XU'\overline{U}'$	2	20, 8, −4, −16
		$X^2Q\overline{Q} + XYU\overline{U} + XQ'\overline{Q}' + XU'\overline{U}'$	−2	20, 8, −4, −16
		$X^2Q\overline{Q} + XYU\overline{U} + XQ'\overline{Q}' + YU'\overline{U}'$	2	32, 20, 8, −4

Since the global Peccei-Quinn symmetry cannot be exact, expect it to be violated by some superpotential term of dimension p,

$$W = \frac{1}{M_{\rm Planck}^{p-3}} X^j Y^{p-j}$$

and corresponding supersymmetry-breaking terms of the same form. In terms of the axion field, both give corrections to the potential linear in A, resulting in

$$V \;=\; \frac{1}{2} m_A^2 A^2 - \delta \frac{f_A^{p+1}}{M_{\rm Planck}^{p-2}} A$$

where δ is dimensionless. Since this term is linear in the axion field, it will develop a non-zero VEV, and the effective strong CP angle will be non-zero:

$$|\theta_{\rm eff}| \;=\; \frac{\langle A \rangle}{f_A} \;=\; \delta \frac{f_A^{p+2}}{\mathcal{M}_{\rm QCD}^4 M_{\rm Planck}^{p-2}}$$

This is the **axion quality problem**.

Because it will be important below, let me emphasize:

Define $p =$ "protection power" $=$ the lowest mass dimension of superpotential terms that violates the PQ symmetry

Larger $p \leftrightarrow$ "higher quality" axion

The neutron electric dipole moment limit requires that the axion quality measure p must satisfy:

$$p + 2 \quad > \quad \frac{88 + \log_{10}(\delta)}{9.4 - \log_{10}(f_A/10^9\,\mathrm{GeV})}.$$

If $\delta \approx 1$, then:

$$\begin{aligned}
p = 8 \quad &\rightarrow \quad f_A \lesssim 4 \times 10^9 \text{ GeV} \\
p = 9 \quad &\rightarrow \quad f_A \lesssim 3 \times 10^{10} \text{ GeV} \\
p = 10 \quad &\rightarrow \quad f_A \lesssim 1 \times 10^{11} \text{ GeV} \\
p = 11 \quad &\rightarrow \quad f_A \lesssim 4 \times 10^{11} \text{ GeV} \\
p = 12 \quad &\rightarrow \quad f_A \lesssim 1 \times 10^{12} \text{ GeV}
\end{aligned}$$

However, δ could be smaller than 1. (Recall that the electron Yukawa coupling is 2.8×10^{-6}.) So perhaps even $p = 7$ could be consistent with $f_A = 10^9$ GeV.

Z_n discrete symmetries to suppress Peccei-Quinn symmetry violation

Each chiral superfield Φ with charge z_Φ (mod n) transforms as

$$\Phi \quad \to \quad e^{2\pi i z_\Phi / n}\Phi,$$

and gauginos may transform as

$$\lambda \quad \to \quad e^{2\pi i R/n}$$

In the special case $R \neq 0$, the discrete symmetry is an R-symmetry.
For a given axion quality exponent p, how large must n be?

$$\begin{array}{rcl}
p = 7 & \to & n \geq 12 \\
p = 8 & \to & n = 12 \text{ or } \geq 16 \\
p = 9 & \to & n = 15 \text{ or } \geq 18 \\
p = 10 & \to & n = 16 \text{ or } \geq 18 \\
p = 11 & \to & n \geq 21 \\
p = 12 & \to & n = 20 \text{ or } \geq 24
\end{array}$$

Found by brute force: try all possible charges for each n.

The discrete Z_n symmetry also should be anomaly free.

To ensure this, can require $Z_n \subset U(1)$ anomaly free, possibly with charged heavy fermions. (Ibañez and Ross, Banks and Dine)

For $G = SU(3)_c$ and $SU(2)_L$ and $U(1)_Y$, the $Z_n \times G \times G$ anomalies are:

$$
\begin{aligned}
A_3 &= n_g(2z_q + z_{\overline{u}} + z_{\overline{d}} - 4R) + 6R + 2\Delta_{Q\overline{Q}} + \Delta_{U\overline{U}} + \Delta_{D\overline{D}} \\
A_2 &= n_g(3z_q + z_\ell - 4R) + z_{H_u} + z_{H_d} + 2R + 3\Delta_{Q\overline{Q}} + \Delta_{L\overline{L}} \\
A_1 &= n_g(z_q + 3z_\ell + 8z_{\overline{u}} + 2z_{\overline{d}} + 6z_{\overline{e}} - 20R) + 3z_{H_u} + 3z_{H_d} - 6R \\
&\quad + \Delta_{Q\overline{Q}} + 3\Delta_{L\overline{L}} + 8\Delta_{U\overline{U}} + 2\Delta_{D\overline{D}} + 6\Delta_{E\overline{E}}
\end{aligned}
$$

We chose a normalization for A_1 so that it is an integer.

Anomaly-free condition:

$$
\frac{A_3 + m_3 n}{k_3} = \frac{A_2 + m_2 n}{k_2} = \frac{A_1 + m_1 n}{5k_1} = \rho_{GS}
$$

where m_1, m_2, m_3 are integers, and k_1, k_2, k_3 are Kac-Moody levels, and ρ_{GS} = Green-Schwarz mechanism constant.

Possible stronger assumptions on anomaly cancellation:

- If $k_3 = k_2 = 1$, then $A_3 = A_2 \pmod{n}$.
- If $k_1 = 1$ also, then $A_1 = 5A_3 \pmod{n}$.
- If no Green-Schwarz mechanism, then $A_3 = A_2 = A_1 = 0 \pmod{n}$.

More constraints:

- Assume Z_n charges are generation-independent
- Require that MSSM superpotential terms are allowed:

$$z_{\overline{u}} = -z_{H_u} - z_q + 2R, \qquad z_{\overline{d}} = -z_{H_d} - z_q + 2R$$

$$z_{\overline{e}} = -z_{H_d} - z_\ell + 2R$$

- Require defining Base Model superpotential terms are allowed:
    - $z_{H_d} = -z_{H_u} - z_X - z_Y + 2R$ for $\mathrm{B_I}$ and extensions
    - $z_{H_d} = -z_{H_u} - 2z_X + 2R$ for $\mathrm{B_{II}}$ and $\mathrm{B_{IV}}$ and extensions
    - $z_{H_d} = -z_{H_u} - 2z_Y + 2R$ for $\mathrm{B_{III}}$ and extensions

Consider Base Models with non-R discrete symmetries that only allow $p \geq 7$ Peccei-Quinn violating operators, and $A_2 = A_3 \pmod{n}$.

All such discrete symmetries can be classified: 5 infinite families for each of B_{I}, B_{II}, B_{IV}, and 15 infinite families for $\mathrm{B}_{\mathrm{III}}$.

But, if we further require $A_1 = 5A_3 \pmod{n}$, then only two survive!

- Z_{36} for $\mathrm{B}_{\mathrm{III}}$ that allows only $p \geq 12$,
- Z_{36} for B_{IV} that allows only $p \geq 8$.

If we further require $\rho_{GS} = 0$ (no Green-Schwarz mechanism at work), then **none** survive with $R = 0$ (gauginos uncharged under the discrete symmetry).

If we consider discrete R-symmetries, and/or extensions of the base models, then there are **many** possibilities. . .

A small subset of the examples with Z_n^R that have complete anomaly cancellation $A_1 = A_2 = A_3 = 0 \pmod{n}$ with $\rho_{\rm GS} = 0$:

Base	Extension	n	R	p	X	Y	H_u	N	$3E$
$\mathrm{B_I}$	none	54	3	8	39	-3	1	3	18
	$X^2 D\overline{D} + XYL\overline{L}$	22	1	8	9	-3	9	4	14
	$XYD\overline{D} + Y^2 L\overline{L}$	24	1	8	7	5	7	2	-2
$\mathrm{B_{II}}$	none	54	3	8	9	33	7	3	18
	$X^2 D\overline{D} + Y^2 L\overline{L}$	20	2	8	5	9	14	2	34
	$Y^2 D\overline{D} + X^2 L\overline{L}$	32	1	10	3	-7	5	6	18
	$Y^2 D\overline{D} + XYL\overline{L}$	108	6	20	11	87	22	6	30
	$Y^2 Q\overline{Q} + X^2 U\overline{U} + X^2 E\overline{E}$	24	1	10	5	11	5	8	34
	$Y^2 Q\overline{Q} + X^2 U\overline{U} + Y^2 E\overline{E}$	56	1	18	11	25	-3	8	58
$\mathrm{B_{III}}$	none	54	3	10	5	-9	1	3	18
	$XYD\overline{D} + XYL\overline{L}$	20	2	8	5	9	6	$\frac{8}{3}$	$\frac{46}{3}$
	$X^2 D\overline{D} + X^2 L\overline{L}$	24	1	8	9	-1	1	$\frac{10}{3}$	$\frac{62}{3}$
	$X^2 Q\overline{Q} + Y^2 U\overline{U} + X^2 E\overline{E}$	24	1	10	5	11	5	$\frac{8}{3}$	$\frac{46}{3}$
	$X^2 Q\overline{Q} + Y^2 U\overline{U} + Y^2 E\overline{E}$	56	1	18	11	25	-3	$\frac{8}{3}$	$\frac{22}{3}$
$\mathrm{B_{IV}}$	none	12	1	7	-4	5	1	3	18
		54	3	8	-9	39	1	3	18
		108	3	10	90	21	7	3	18
	$Y^2 D\overline{D} + X^2 L\overline{L}$	12	2	8	5	9	2	4	14
	$X^2 D\overline{D} + Y^2 L\overline{L}$	16	1	8	-4	5	3	2	22
	$XYQ\overline{Q} + Y^2 U\overline{U} + Y^2 E\overline{E}$	24	4	10	5	-1	2	4	32
	$Y^2 Q\overline{Q} + X^2 U\overline{U} + X^2 E\overline{E}$	28	1	11	11	4	11	4	14
	$Y^2 Q\overline{Q} + XYU\overline{U} + X^2 E\overline{E}$	60	2	16	7	-5	2	5	22

Discrete symmetries for extensions of the Base models

My pledge to you: no more big annoying tables that you won't read or remember.

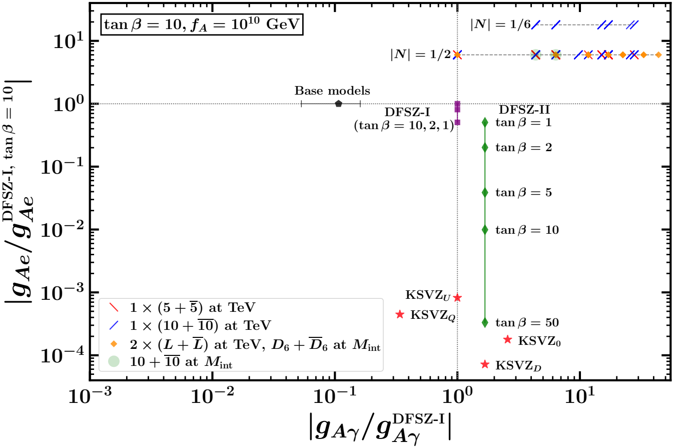
You can find the big annoying tables in our paper.

For every (N, E), there exists (an infinite number of) discrete symmetries Z_n to protect the PQ symmetry, although this may require rather large n.

From a low-energy point of view, we cannot hope to determine the discrete symmetry anyway! In the foreseeable future, our only experimental handles are:

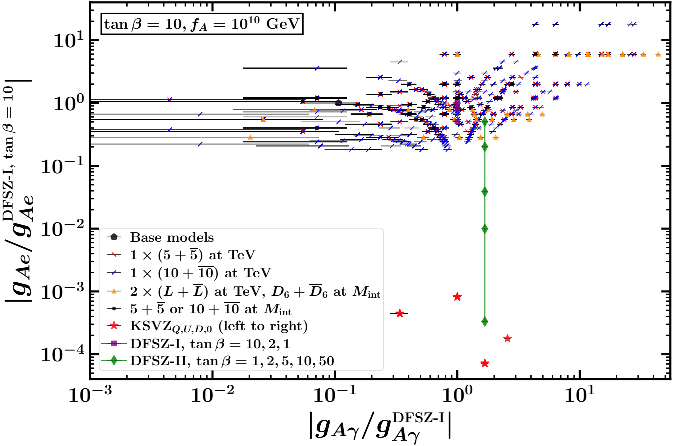
- If an axion is discovered, the anomaly coefficients (N, E).
- Possible extra vectorlike quarks or leptons at the TeV scale

Couplings for various axion models, normalized to the non-SUSY DFSZ-I model with large $\tan\beta$:

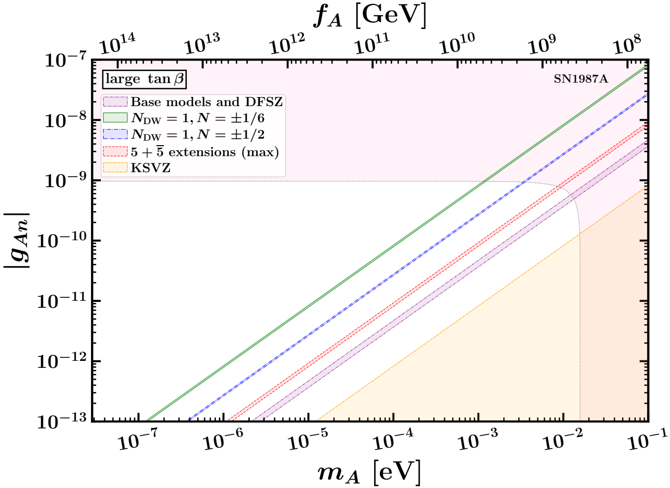


- Base models have accidentally small coupling to EM field.
- SUSY models with $N_{DW} = 1$ have enhanced couplings to both electrons and EM fields.

Same, but a more complete survey of SUSY models with axions associated with the Kim-Nilles mechanism:



- Coupling of axion to EM fields can be accidentally small due to cancellation (with large uncertainties)
- Coupling to electron can be suppressed by up to a factor of about 5 compared to DFSZ-I models

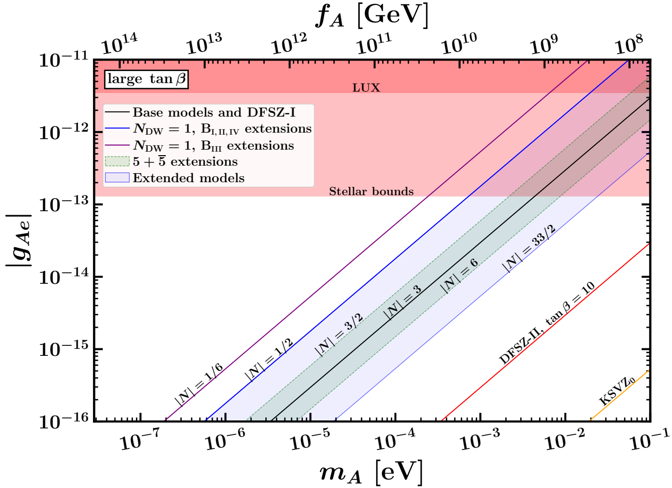


The SN 1987a bound on g_{Ap} and g_{An} amounts to (Carenza et al, 1906.11844):

$$g_{An}^2 + 0.61 g_{Ap}^2 + 0.53 g_{An} g_{Ap} \quad \lesssim \quad 8.26 \times 10^{-19},$$

which, in our models, becomes

$$\frac{f_A}{10^9 \text{ GeV}} > \sqrt{0.15 + 0.66/N^2}.$$

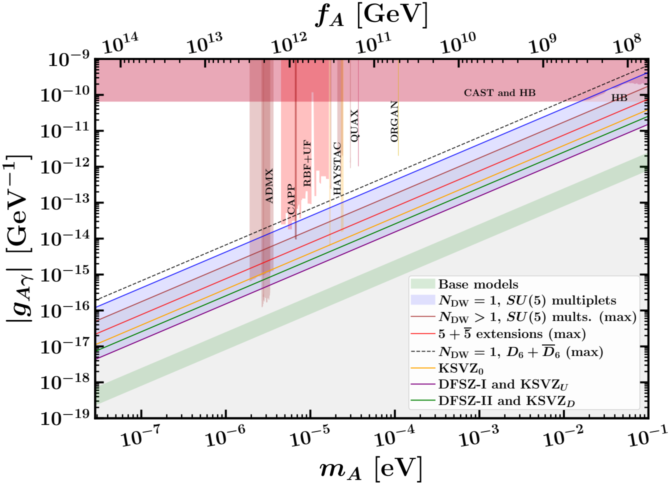


The strongest "Stellar bounds" come from observed brightness of stars at the tip of the red giant branch:

$$|g_{Ae}| < 1.3 \times 10^{-13},$$

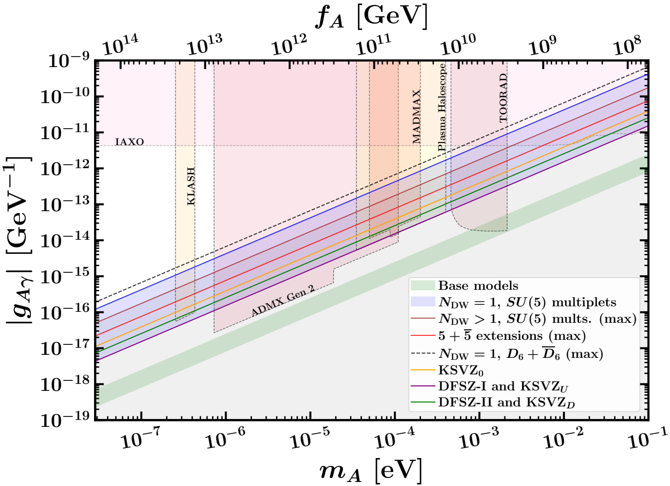
which for our models translates into

$$f_A > \frac{\sin^2 \beta}{|N|} 3.9 \times 10^9 \text{ GeV}.$$



By coincidence, get very similar constraints $|g_{A\gamma}| < 6.5 \times 10^{-11}$ GeV^{-1} from two completely different sources

- Evolution of Horizontal Branch (HB) stars
- CAST Helioscope = CERN Axion Solar Telescope, detects solar axion conversions to X-rays using a very strong magnetic field.



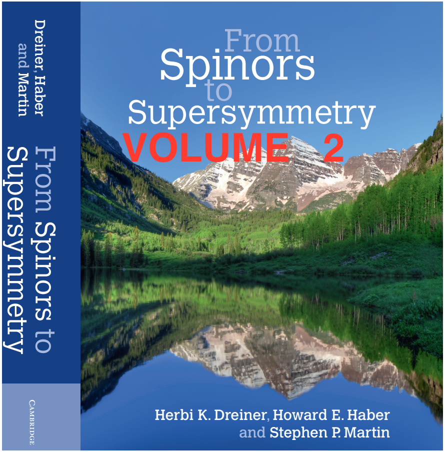
Future constraints from helioscope IAXO (successor to CAST) searches for solar axions, and searches for dark matter axions (ADMX and other haloscopes, ...).

Note Base Models mostly *not* covered, due to Higgsino contributions to EM anomaly, but extensions with $N_{\rm DM}=1$ are covered, if they are the dark matter.

Outlook

- The minimal supersymmetric DFSZ axion following from the Kim-Nilles mechanism has suppressed couplings to EM fields
- Non-minimal axion models with extra vectorlike quarks and leptons can have enhanced couplings by more than 2 orders of magnitude, especially if the domain wall problem is solved
- Prospects for direct detection are:
    - not encouraging for the SUSY Base Models
    - encouraging for the extended models, at least if the axion is the dark matter
- Existence proofs of anomaly-free discrete symmetries that solve the axion quality problem.

Our project for the next 25 years:



This will be the post-discovery edition.
Happy 60th, Herbi!