Adventures from spinors to SUSY

Howard E. Haber
28 March 2023

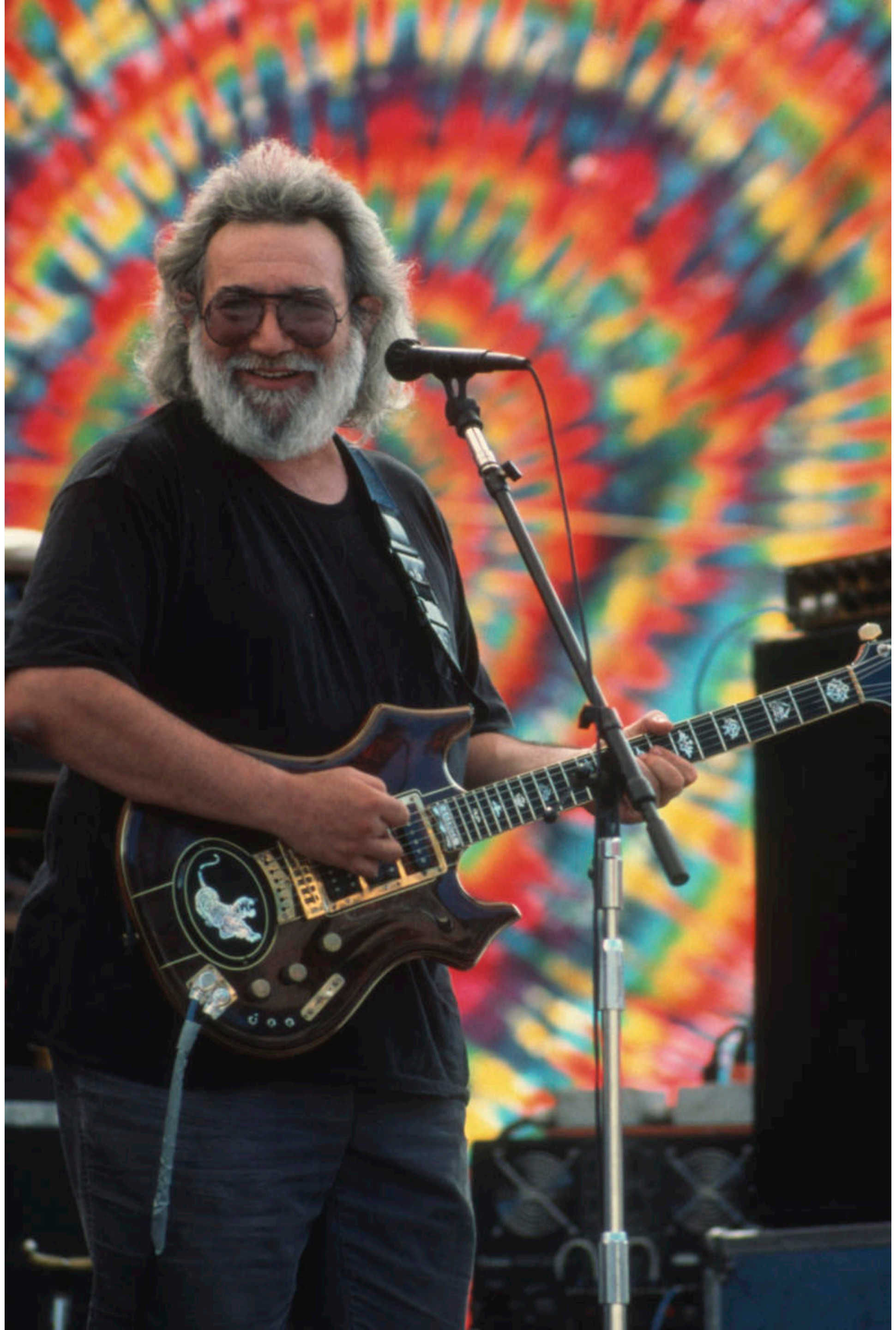
Herbi–Fest

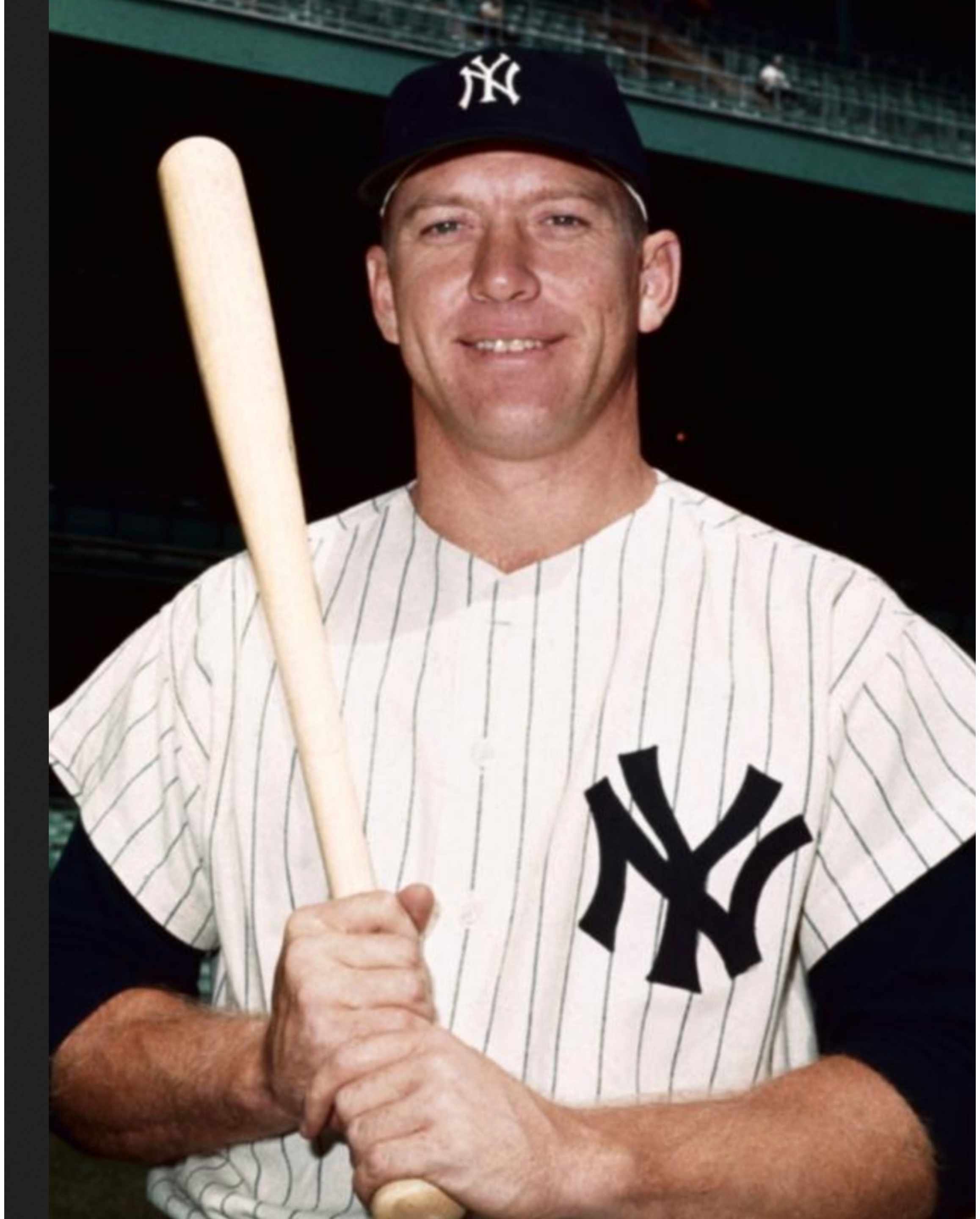
The early years: 1993—2008

(before the iPhone)


KEEP OFF LAWNS


Dynamic HTML
An Introduction to Quantum Field Theory



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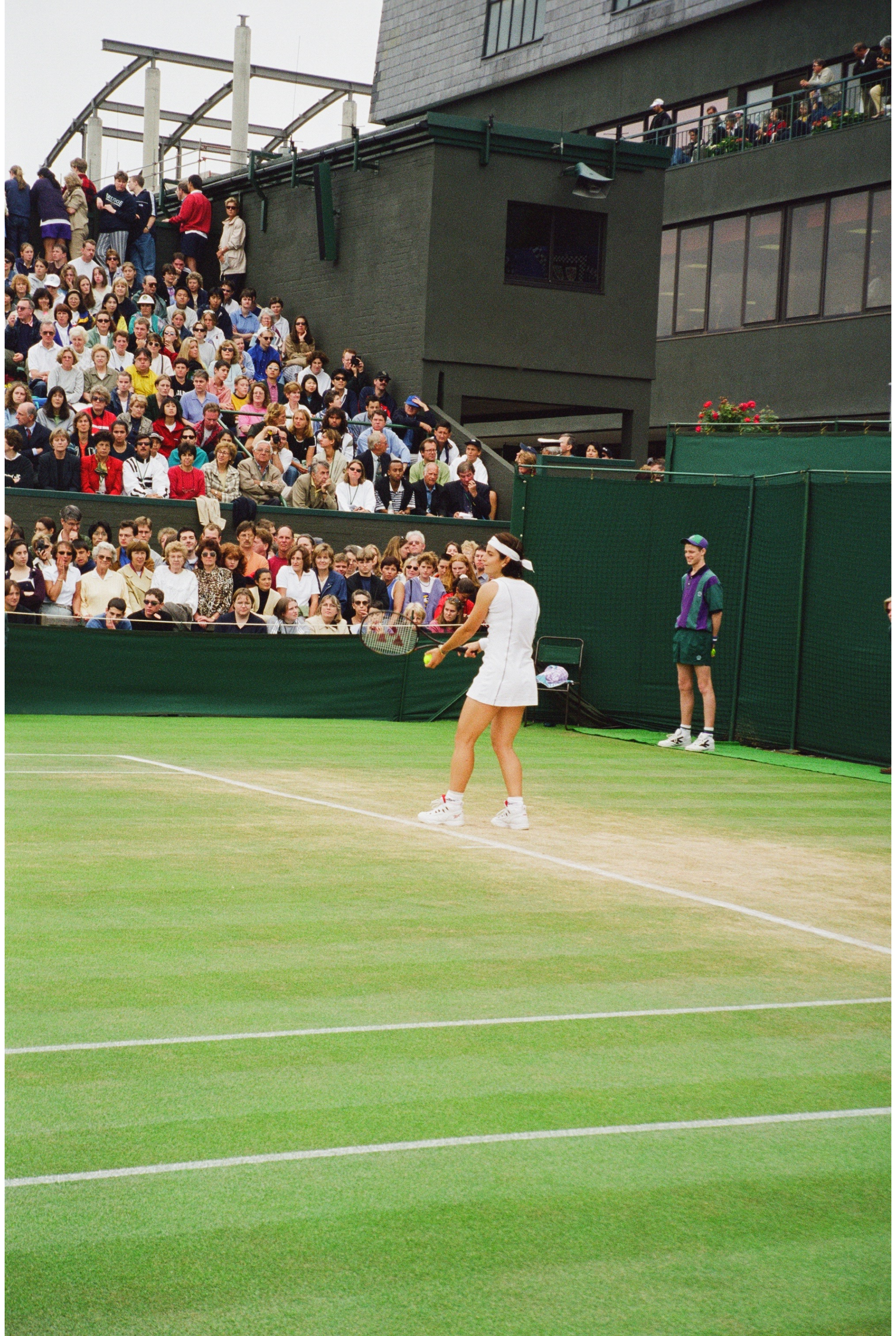


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SCIENCE AREA


ELLA

The modern era: 2009—2023

(after the iPhone)

Boston 2009 at the SUSY conference

Maroon Lake, CO
Summer 2009

Pine Creek Cookhouse
Summer 2009

Sabbatical in Santa Cruz with the Re-Entry softball team in May 2010

Giants vs. Red Sox
in May, 2010 with the
debut of Madison
Bumgarner

Herbi’s last week
on sabbatical in
Santa Cruz,
August 2010

Physics Reports

Physics Reports

48.2 CiteScore | 30.51 Impact Factor

Volume 494, Issues 1–2

Pages 1-196 (September 2010)

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Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry

Herbi K. Dreiner, Howard E. Haber, Stephen P. Martin

Pages 1-196

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Bamburg 2011
courtesy of the
Humboldt
foundation

Top of the Ute Trail
in Aspen, CO 2011

At Herbi’s house
October 2011

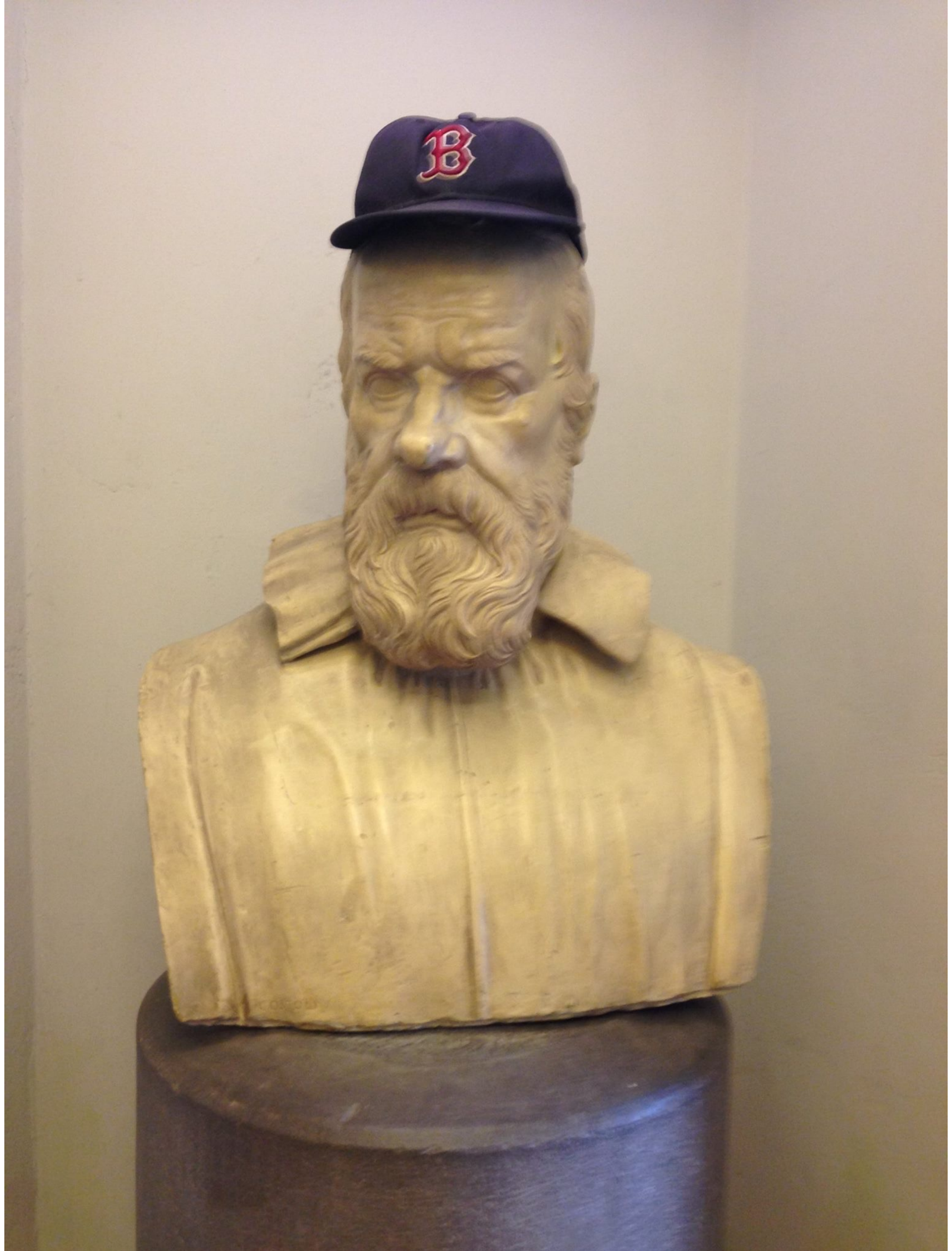
Berlin in June, 2012,
courtesy of the Humboldt
Foundation

On his way to
Maroon Lake near
Aspen, CO in 2012

Hard at work on the book outside of Paradise bakery in Aspen, Co in 2012

Another Ph.D. granted to one of Herbi's students in September 2012.

Santa Cruz visit
in March 2015



Florence, summer of 2015

Super Bowl Sunday
2018 in Santa Cruz

Followed by a triumphant visit to Canada

Christmas Market in Bonn, December 2018

Munich workshop in summer of 2019

Updating Simon Capelin of Cambridge University Press on progress on the book (while attending the 2019 Cambridge Folk Festival)

Herbo’s last visit to Santa Cruz in January, 2020

My last visit to Bonn before the pandemic

My last evening
in Bonn on
March 8, 2020.

"The new book by Dreiner, Haber, and Martin is a must have for folks who are interested in beyond the Standard Model phenomenology. It contains innumerable lessons for performing quantum field theory calculations both at the conceptual and technical level, by way of many concrete examples within the Standard Model and its supersymmetric extension. I expect this will become a go-to reference for everyone from graduate students to seasoned researchers."
Prof. Tim Cohen, CERN/EPFL and the University of Oregon

"The book gives a self-contained description of the Standard Model of particle physics and its supersymmetric extension. It is well suited for students, as well as experienced researchers in the field. Its unique feature is the comprehensive description of quantum field theory and its application to particle physics in the framework of two-component (Weyl) spinors … The book will be of enormous help to all those that try to teach and try to learn the subject."
Prof. Hans-Peter Nilles, Universität Bonn

"This is a massive, definitive text on phenomenological supersymmetry in quantum field theory by three giants of the field. The book develops two-component spinor formalism and its practical use in amplitude computations with many phenomenological examples up to one loop order. Supersymmetric extensions of the Standard Model are also covered and many other gems besides."
Prof. Ben Allanach, University of Cambridge

Supersymmetry is an extension of the successful Standard Model of particle physics; it relies on the principle that fermions and bosons are related by a symmetry, leading to an elegant predictive structure for quantum field theory. This textbook provides a comprehensive and pedagogical introduction to supersymmetry and other aspects of particle physics at the high-energy frontier. Aimed at graduate students and researchers, it also discusses concepts of physics beyond the Standard Model, including extended Higgs sectors, grand unification, and the origin of neutrino masses.

Cover image: Sierralara/RooM/Getty Images

Designed by EMC Design Ltd

CAMBRIDGE UNIVERSITY PRESS

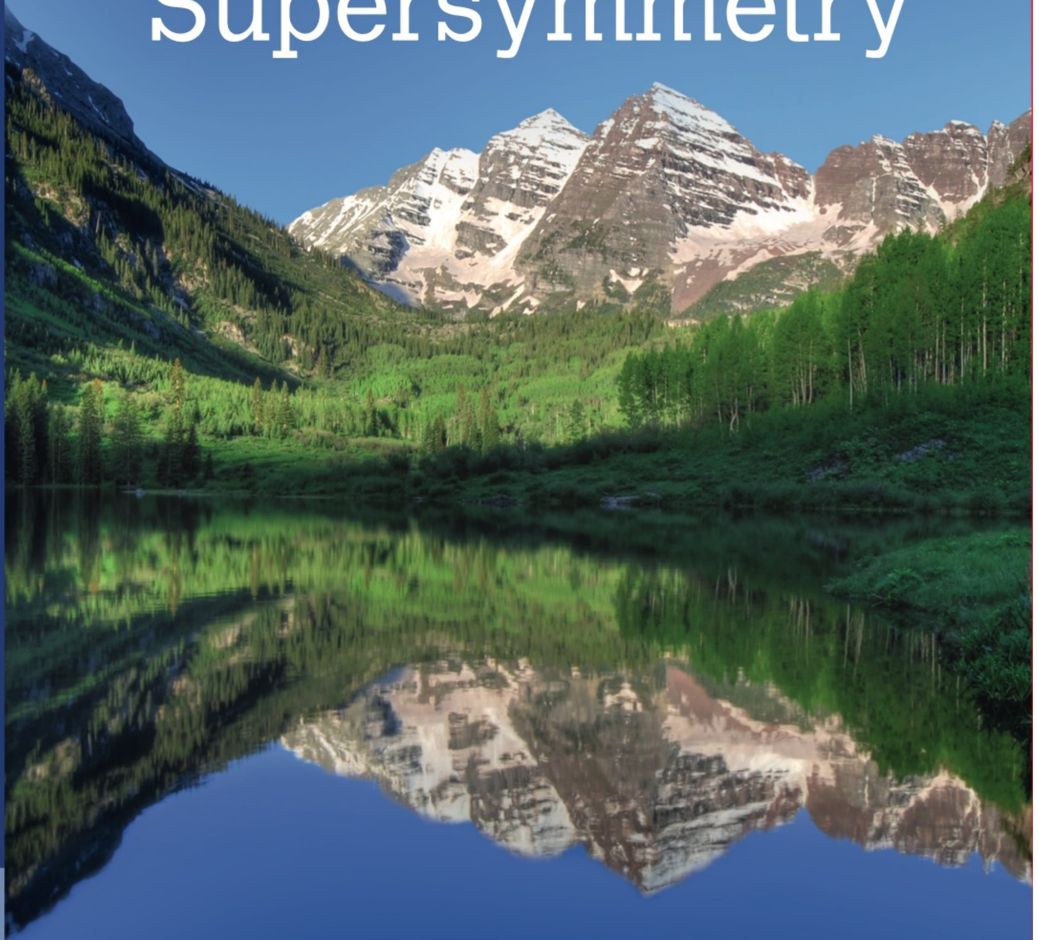
ISBN 978-0-521-80088-4

Dreiner, Haber, and Martin

From Spinors to Supersymmetry

CAMBRIDGE

From Spinors to Supersymmetry



Herbi K. Dreiner, Howard E. Haber, and Stephen P. Martin

Puzzling over a famous result of Weisskopf

In QED, the (unrenormalized) inverse propagator to all orders is given by

$$S^{-1}(p) = \not{p}\left(1 - \Sigma(p^2)\right) - m - \Sigma_D(p^2)\,,$$

where $-i\left[\not{p}\,\Sigma(p^2) + \Sigma_D(p^2)\right]$ is the sum of all 1PI diagrams contributing to the electron two-point function. The pole mass, denoted by m_p, corresponds to a zero of $S^{-1}(p)$. Thus setting $\not{p} = m$ and $p^2 = m^2$, where m is the bare mass, it follows that at one-loop order,

$$m_p = m + m\Sigma(m^2) + \Sigma_D(m^2)\,.$$

The electron mass counterterm is defined by $\delta m \equiv m_p - m$. In a modern calculation, one obtains the gauge invariant one-loop result in QED,

$$\delta m = m\Sigma(m^2) + \Sigma_D(m^2) = \frac{\alpha m}{2\pi}\left[B_0(m^2; 0, m^2) - (1-\epsilon)B_1(m^2; 0, m^2)\right],$$

where $\epsilon \equiv 2 - \frac{1}{2}d$ and

$$B_0(p^2; m_a^2, m_b^2) = -16\pi^2 i\mu^{2\epsilon}\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2 - m_a^2 + i\varepsilon)[(q+p)^2 - m_b^2 + i\varepsilon]},$$

$$p^\mu B_1(p^2; m_a^2, m_b^2) = -16\pi^2 i\mu^{2\epsilon}\int\frac{d^d q}{(2\pi)^d}\frac{q^\mu}{(q^2 - m_a^2 + i\varepsilon)[(q+p)^2 - m_b^2 + i\varepsilon]},$$

are Passarino-Veltman loop functions and μ is an arbitrary mass scale.

If δm is evaluated in $d = 4$ spacetime dimensions with an ultraviolet cutoff Λ, then one can derive a result first obtained by Weisskopf in 1934 (thanks to a subsequent erratum),[1]

$$\delta m = \frac{3\alpha m}{2\pi} \ln\left(\frac{\Lambda}{m}\right) + \text{finite terms.}$$

Weisskopf's breakthrough was to realize that potentially linear and quadratic divergences canceled exactly, a result we understand today as being a consequence of chiral symmetry in the limit of $m \to 0$. Thus, the hierarchy problem of QED was resolved, only to reappear in the Standard Model in the computation of the mass counterterms for the W, Z and Higgs boson.

[1] This is Exercise 7.1 of DHM.

The self-energy of the electron

V. WEISSKOPF

Zeitschrift für Physik, 89: 27–39 (1934). Received 13 March 1934.

The self-energy of the electron is derived in a closer formal connection with classical radiation theory, and the self-energy of an electron is calculated when the negative energy states are occupied, corresponding to the conception of positive and negative electrons in the Dirac 'hole' theory. As expected, the self-energy also diverges in this theory, and specifically to the same extent as in ordinary single-electron theory.

English translation provided in Arthur I. Miller, *Early Quantum Electrodynamics: a source book* (Cambridge University Press, 1994).

Correction to the paper: The self-energy of the electron

Zeitschrift für Physik, 90: 817–18 (1934). Received 20 July 1934.

On [p. 166] of the paper cited above, there is a computational error which has seriously garbled the results of the calculation for the electrodynamic self-energy of the electron according to the Dirac hole theory. I am greatly indebted to Mr Furry (University of California, Berkeley) for kindly pointing this out to me.

The degree of divergence of the self-energy in the hole theory is *not*, as asserted in [the preceding paper], just as great as in the Dirac one-electron theory, but the divergence is only logarithmic. The expression for the electrostatic and electrodynamic parts of the self-energy E of an electron with momentum p now correctly reads, in the notations used in [the preceding paper]:

$$E = E^{\mathrm{S}} + E^{\mathrm{D}},$$

$$E^{\mathrm{S}} = \frac{e^2}{h(m^2c^2 + p^2)^{1/2}}(2m^2c^2 + p^2)\int_{k_0}^{\infty}\frac{\mathrm{d}k}{k} + \text{finite terms},$$

$$E^{\mathrm{D}} = \frac{e^2}{h(m^2c^2 + p^2)^{1/2}}(m^2c^2 - \tfrac{4}{3}p^2)\int_{k_0}^{\infty}\frac{\mathrm{d}k}{k} + \text{finite terms}.$$

1939: Scalar fields portend an energy scale associated with new phenomena that is close at hand.

JULY 1, 1939 PHYSICAL REVIEW VOLUME 56

On the Self-Energy and the Electromagnetic Field of the Electron

V. F. WEISSKOPF
University of Rochester, Rochester, New York

(Received April 12, 1939)

The charge distribution, the electromagnetic field and the self-energy of an electron are investigated. It is found that, as a result of Dirac's positron theory, the charge and the magnetic dipole of the electron are extended over a finite region; the contributions of the spin and of the fluctuations of the radiation field to the self-energy are analyzed, and the reasons that the self-energy is only logarithmically infinite in positron theory are given. It is proved that the latter result holds to every approximation in an expansion of the self-energy in powers of e^2/hc. The self-energy of charged particles obeying Bose statistics is found to be quadratically divergent. Some evidence is given that the "critical length" of positron theory is as small as $h/(mc)\cdot\exp(-hc/e^2)$.

The situation is, however, entirely different for a particle with Bose statistics. Even the Coulombian part of the self-energy diverges to a first approximation as $W_{st} \sim e^2 h/(mca^2)$ and requires a much larger critical length that is $a = (hc/e^2)^{-\frac{1}{2}} \cdot h/(mc)$, to keep it of the order of magnitude of mc^2. This may indicate that a theory of particles obeying Bose statistics must involve new features at this critical length, or at energies corresponding to this length; whereas a theory of particles obeying the exclusion principle is probably consistent down to much smaller lengths or up to much higher energies.

If one employs dimensional regularization to evaluate the Passarino-Veltman loop functions, then one obtains

$$\delta m = \frac{3\alpha m}{4\pi}\left[\frac{1}{\epsilon} - \ln\left(\frac{m^2}{Q^2}\right) + \frac{4}{3} + \mathcal{O}(\epsilon)\right],$$

where the regularization scale Q is defined by $Q^2 \equiv 4\pi e^{-\gamma}\mu^2$, and γ is Euler's constant.

The puzzle: The mass shift δm is defined in an on-mass shell (OS) renormalization scheme. How can δm possibly depend on the regularization scale Q which is arbitrary? Indeed, one had better find that

$$\frac{\partial}{\partial Q^2}\delta m = 0 .$$

Returning to

$$\delta m = \frac{3\alpha m}{4\pi}\left[\frac{1}{\epsilon} - \ln\left(\frac{m^2}{Q^2}\right) + \frac{4}{3} + \mathcal{O}(\epsilon)\right] ,$$

note that m is the pole mass (which is physical). On the other hand, we have not yet formally defined α. One should view $\alpha = \alpha(Q)$, with implicit Q dependence. One could formally provide a physical definition of $\alpha \simeq 1/137$ (e.g., the Thomson limit of QED). However, surely

$$\alpha(Q) = \alpha + \mathcal{O}(\alpha^2),$$

and the problem of the Q dependence of δm remains.

Defining α in QED—a closer look

In QED, the unrenormalized photon self-energy function has the form

$$\Pi_{\mu\nu}(p) = \left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\Pi(p^2)\,,$$

where

$$\begin{aligned}\Pi(p^2) &= -\frac{2\alpha}{\pi}\left[B_1(p^2;m^2,m^2) + B_{21}(p^2;m^2,m^2)\right]\\ &= \frac{2\alpha}{\pi}\left\{\frac{1}{6\epsilon} - \int_0^1 x(1-x)\ln\left(\frac{m^2 - p^2x(1-x)}{Q^2}\right)\,dx + \mathcal{O}(\epsilon)\right\},\end{aligned}$$

$\Pi(p^2)$, α, and m should be understood to be bare quantities (prior to renormalization).

The quantity $\Pi(p^2)$ enters the expression for the exact propagator,

$$\langle 0|\, T A_\mu(x) A_\nu(y)\, |0\rangle_{\mathrm{FT}} = \frac{-i}{p^2[1+\Pi(p^2)]}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \frac{i\xi p^\mu p^\nu}{p^4}\,,$$

The renormalized fields and parameters in some renormalization scheme (denoted by the subscript R) are given by

$$A_B^\mu = Z_3^{1/2} A_R^\mu\,, \qquad \alpha_B = Z_\alpha \mu^{2\epsilon}\alpha_R\,, \qquad m_p = m_B + \delta m\,.$$

where the subscript B indicates a bare parameter and m_p is the (physical) pole mass. In fact, $Z_\alpha = Z_3^{-1}$, which is a consequence of the QED Ward identity, $Z_1 = Z_2$.[1]

[1] Recall that Z_1 and Z_2 are the renormalization constants of the $ee\gamma$ vertex and the electron field, respectively, and $Z_\alpha^{1/2} \equiv Z_1 Z_2^{-1} Z_3^{-1/2}$.

It then follows that

$$\frac{\alpha_B}{1+\Pi_B(p^2)} = \frac{\alpha_R}{1+\Pi_R(p^2)} \, .$$

Consider two different renormalization schemes for defining α. In the $\overline{\mathrm{MS}}$ renormalization scheme at one loop, one simply subtracts off the term proportional to ϵ^{-1}. That is,

$$\Pi_{\overline{\mathrm{MS}}}(p^2) \equiv \Pi(p^2) - \frac{\alpha}{3\pi\epsilon} = -\frac{2\alpha}{\pi}\int_0^1 x(1-x)\ln\left(\frac{m^2 - p^2 x(1-x)}{Q^2}\right) dx \, .$$

Alternatively, in the on-shell (OS) renormalization scheme where $\Pi_{\mathrm{OS}}(0) = 0$,

$$\Pi_{\mathrm{OS}}(p^2) \equiv \Pi(p^2) - \Pi(0) = -\frac{2\alpha}{\pi}\int_0^1 x(1-x)\ln\left(\frac{m^2 - p^2 x(1-x)}{m^2}\right) dx$$

In this renormalization scheme, $\alpha_{\mathrm{OS}} \simeq 1/137$.

Plugging in the $\overline{\mathrm{MS}}$ and OS scheme results into

$$\frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{1+\Pi_{\overline{\mathrm{MS}}}(p^2)} = \frac{\alpha_{\mathrm{OS}}}{1+\Pi_{\mathrm{OS}}(p^2)} .$$

one can derive the one-loop relation,

$$\alpha_{\overline{\mathrm{MS}}}(Q) = \alpha_{\mathrm{OS}}\left\{1-\frac{\alpha_{\mathrm{OS}}}{3\pi}\ln\left(\frac{m^2}{Q^2}\right)+\mathcal{O}(\alpha_{\mathrm{OS}}^2)\right\} .$$

Note that, in the one-loop approximation, $\alpha_{\mathrm{OS}} = \alpha_{\overline{\mathrm{MS}}}(Q=m)$.

To reiterate,

$$\alpha(Q) = \alpha + \mathcal{O}(\alpha^2),$$

and the problem of the Q dependence of δm remains.

Back to Basics—deriving the RGEs

In renormalization by minimal subtraction, coupling constants are redefined in order to remove all ultraviolet divergence poles in ϵ from expressions for amplitudes and masses. This means that each Lagrangian parameter X corresponding to an N-field coupling is written as an expansion in the number of loops ℓ, containing counterterms $c^X_{\ell,n}$ with only (hence "minimal") poles in ϵ:

$$X_B = \mu^{\epsilon\rho_X}\left(X + \sum_{\ell=1}^{\infty}\frac{1}{(16\pi^2)^\ell}\sum_{n=1}^{\ell}\frac{c^X_{\ell,n}}{\epsilon^n}\right),$$

where the X_B cannot depend on our choice of Q, the X are the corresponding $\overline{\mathrm{MS}}$ parameters (which do depend on Q, and are finite as $\epsilon \to 0$), and $\rho_X = N - 2$.

The counterterm coefficients $c^X_{\ell,n}$ are polynomials in the $\overline{\mathrm{MS}}$ parameters (collectively called Y below), with no explicit dependence on Q and satisfy the following identity:[3]

$$\left(-\rho_X + \sum_Y \rho_Y Y \frac{\partial}{\partial Y}\right) c^X_{\ell,n} = 2\ell c^X_{\ell,n}\,,$$

where the sums over Y (which can include X itself) are taken over all $\overline{\mathrm{MS}}$ parameters that appear in the polynomials $c^X_{\ell,n}$. The counterterms are chosen in such a way that all observable quantities, when written in terms of the $\overline{\mathrm{MS}}$ parameters, do not contain any poles in ϵ.

---

[3] This is Exercise 11.5 of DHM.

Since bare quantities cannot depend on the arbitrary choice of renormalization scale Q, it follows that $Q\,dX_B/dQ = 0$, which yields the renormalization group equation (RGE). That is,

$$Q\frac{dX}{dQ}+\epsilon\rho_X\left(X+\sum_{\ell=1}^{\infty}\frac{1}{(16\pi^2)^\ell}\sum_{n=1}^{\ell}\frac{c^X_{\ell,n}}{\epsilon^n}\right)+\sum_{\ell=1}^{\infty}\frac{1}{(16\pi^2)^\ell}\sum_{n=1}^{\ell}\frac{1}{\epsilon^n}\sum_Y Q\frac{dY}{dQ}\frac{\partial c^X_{\ell,n}}{\partial Y}=0.$$

Matching powers of ϵ in the above expansions and noting that X is finite as $\epsilon\to 0$, it follows that QdX/dQ contributes only to the terms of the ϵ expansions with ϵ^1 and ϵ^0. Hence,

$$Q\frac{dX}{dQ}=-\epsilon\rho_X X+\sum_{\ell=1}^{\infty}\frac{1}{(16\pi^2)^\ell}\left(-\rho_X+\sum_Y\rho_Y Y\frac{\partial}{\partial Y}\right)c^X_{\ell,1},$$

where we have self-consistently used $QdY/dQ=-\epsilon\rho_Y Y+\cdots$ to obtain the last term above.

The beta functions are defined to be the ϵ-independent parts of QdX/dQ,

$$\beta_X \equiv Q\frac{dX}{dQ}\bigg|_{\epsilon=0} = Q\frac{dX}{dQ} + \epsilon\rho_X X\,.$$

More explicitly,

$$\beta_X = \sum_{\ell=1}^{\infty} \frac{1}{(16\pi^2)^\ell}\left(-\rho_X + \sum_Y \rho_Y Y\frac{\partial}{\partial Y}\right) c^X_{\ell,1}.$$

Note that QdX/dQ, unlike β_X, crucially contains a "zero-loop" term, $-\epsilon\rho_X X$ if $\rho_X \neq 0$.

Example: The electromagnetic coupling in QED in the $\overline{\text{MS}}$ scheme satisfies:

$$Q\frac{d\alpha}{dQ} = -2\epsilon\alpha + \frac{2\alpha^2}{3\pi} + \mathcal{O}(\alpha^3)\,.$$

Solving this equation to one-loop accuracy,

$$\alpha_{\overline{\text{MS}}}(Q) = \alpha_{\text{OS}}\left\{1 - \epsilon\ln\left(\frac{Q^2}{m^2}\right) + \mathcal{O}(\epsilon^2)\right\} + \mathcal{O}\left(\alpha_{\text{OS}}^2\right),$$

thereby confirming that an $\mathcal{O}(\epsilon)$ term has been missed in the previous derivation of $\alpha_{\overline{\text{MS}}}(Q)$.

Returning again to

$$\delta m = \frac{3\alpha_{\overline{\mathrm{MS}}}(Q)m}{4\pi}\left[\frac{1}{\epsilon} - \ln\left(\frac{m^2}{Q^2}\right) + \frac{4}{3} + \mathcal{O}(\epsilon)\right] ,$$

where we put $\alpha = \alpha_{\overline{\mathrm{MS}}}(Q)$ in our previous expression, we can now re-express the result in terms of α_{OS}, thereby obtaining[3]

$$\delta m = \frac{3m\alpha_{\mathrm{OS}}}{4\pi}\left[\frac{1}{\epsilon} + \frac{4}{3} + \mathcal{O}(\epsilon)\right] .$$

Indeed, in terms of on-shell parameters, δm is explicitly independent of the $\overline{\mathrm{MS}}$ renormalization scale Q, as originally expected.

---

[3]This is Exercise 19.3 of DHM.

Equivalently, we can return to

$$\delta m = \frac{3\alpha m}{4\pi}\left[\frac{1}{\epsilon} - \ln\left(\frac{m^2}{Q^2}\right) + \frac{4}{3} + \mathcal{O}(\epsilon)\right] ,$$

and

$$Q\frac{d\alpha}{dQ} = -2\epsilon\alpha + \frac{2\alpha^2}{3\pi} + \mathcal{O}(\alpha^3) .$$

Then,

$$\frac{d}{dQ}\delta m = \frac{3m}{4\pi}\left[\frac{1}{\epsilon} - \ln\left(\frac{m^2}{Q^2}\right) + \frac{4}{3} + \mathcal{O}(\epsilon)\right]\frac{d\alpha}{dQ} + \frac{3\alpha m}{2\pi Q}$$
$$= -\frac{3\alpha m}{2\pi Q} + \frac{3\alpha m}{2\pi Q} + \mathcal{O}(\epsilon\alpha) + \mathcal{O}(\alpha^2) = 0.$$

at one-loop accuracy in the $\epsilon \to 0$ limit.[4]

[4] This is Exercsie 19.2 of DHM.

Ten years ago, Herbi came to Santa Cruz to help me celebrate my 60^{th} birthday. He also had a milestone birthdays to celebrate as well. With much joy for our many years of friendship and collaboration, I am most happy to return the favor!

Happy 60^{th} birthday, Herbi !!