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Anomaly-free gauged $U(1)'$ in local supersymmetry and baryon-number violation

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Abstract

The supersymmetric extension of the standard model suffers from a problem of baryon-number violation. Discrete (and global) symmetries introduced to protect the proton are unstable under gravitational effects. We add a gauged $U(1)_X$ to the standard model gauge group G_{SM} and require it to be anomaly-free. As new (chiral) superfields we only allow G_{SM}-singlets in order to maintain the good unification predictions. We find the most general set of solutions for the rational singlet charges. We embed our models in *local* supersymmetry and study the breaking of supersymmetry and $U(1)_X$ to determine M_X. We determine the full non-renormalizable and gauge invariant Lagrangian for the different solutions. We expect any effective theory to contain baryon- and lepton-number violating terms of dimension four suppressed by powers of M_X/M_{Pl}. The power is predicted by the $U(1)_X$ charges. We find consistency with the experimental bounds on the proton lifetime and on the neutrino masses. We also expect all supersymmetric models to have an unstable but longlived lightest supersymmetric particle. Consistency with underground experiments on upward going muons leads to stricter constraints than the proton decay experiments. These are barely satisfied.

1. Introduction

When incorporating supersymmetry into the Standard Model one immediately runs into a problem. The Standard Model conserves baryon- (B) and lepton-number (L) automatically and higher-dimensional $\Delta B, \Delta L \neq 0$ operators are suppressed by at least 4 powers of the scale of baryon- or lepton-number violation. However, in supersymmetry [1] the most general interactions involving the Standard Model (super-) fields and invariant under the Standard Model gauge group

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{1.1}$$

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Anomaly-free gauged R-symmetry in local supersymmetry

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Abstract

We discuss local R-symmetry as a potentially powerful new model building tool. We first review and clarify that a $U(1)$ R-symmetry can only be gauged in local and not in global supersymmetry. We determine the anomaly-cancellation conditions for the gauged R-symmetry. For the standard superpotential these equations have *no* solution, independently of how many Standard Model singlets are added to the model. There is also no solution when we increase the number of families and the number of pairs of Higgs doublets. When the Green-Schwarz mechanism is employed to cancel the anomalies, solutions only exist for a large number of singlets. We find many anomaly-free family-independent models with an extra $SU(3)_c$ octet chiral superfield. We consider in detail the conditions for an anomaly-free *family-dependent* $U(1)_R$ and find solutions with one, two, three and four extra singlets. Only with three and four extra singlets do we naturally obtain sfermion masses of the order of the weak scale. For these solutions we consider the spontaneous breaking of supersymmetry and the R-symmetry in the context of local supersymmetry. In general the $U(1)_R$ gauge group is broken at or close to the Planck scale. We consider the effects of the R-symmetry on baryon- and lepton-number violation in supersymmetry. There is no logical connection between a conserved R-symmetry and a conserved R-parity. For conserved R-symmetry we have models for all possibilities of conserved or broken R-parity. Most models predict dominant effects which could be observed at HERA.

1. Introduction

Supersymmetry combines fields of different spin into supermultiplets. It includes the special possibility of a symmetry which distinguishes between the fermionic and the bosonic component of a $N = 1$ supersymmetric superfield. Such symmetries are called R-symmetries and they are particular to supersymmetry. As such, they deserve special attention when considering the implications of supersymmetry. R-parity can be thought of as a discrete R-symmetry and has been widely discussed in the context of the

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Mimetic Dark Matter

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- Modifications of gravity in most cases involve adding new fields
- Modifications of GR are motivated in order to solve well known problems such as adding higher curvature terms, breaking diffeomorphism invariance, or in cosmology by adding an inflaton, curvaton, quintessence, or in Horndinski model adding a scalar field with heigher safe derivatives.

• Dark Matter

- Various new particles and interactions are proposed to account for missing mostly non-baryonic matter in the universe, such as supersymmetric neutralino, axions, WIMP, dilaton ...
- Lack of direct evidence for dark matter made some to propose modifications of GR at large scales.
- Most modifications of GR are made to solve one problem at a time.

• Scale factor for metric

GR is not invariant under scale transformations of the metric:

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \qquad I = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R(g)$$

not-invariant

One gets in addition a kinetic term for $\Omega(x)$

Our proposal is to isolate scale factor in a covariant way:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu}\left(\tilde{g}^{\kappa\lambda}\partial_\kappa\phi\,\partial_\lambda\phi\right) \equiv \tilde{g}_{\mu\nu}\, S$$

$\tilde{g}_{\mu\nu}$ is an auxiliary metric

$g_{\mu\nu}$ is invariant under scale transformation $\tilde{g}_{\mu\nu} \to \Omega^2 \tilde{g}_{\mu\nu}$

Implications: $g^{\mu\nu} = \tilde{g}^{\mu\nu}\dfrac{1}{S} \Rightarrow g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = \tilde{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\,\dfrac{1}{S}$

$$= 1$$

$$\therefore \boxed{g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = 1}$$

metric $g^{\mu\nu}$ is constrained.

vary Action $-\frac{1}{2}\int d^4x \sqrt{-g(\tilde{g})}\, R(g(\tilde{g}))$

with respect to $\tilde{g}_{\mu\nu}$ and ϕ is equivalent to action

$$I = -\frac{1}{2}\int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, \lambda \left(g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - 1 \right)$$

λ = Lagrange multiplier

Equations of motion:

$$G_{\mu\nu} = T_{\mu\nu}^{matter} + \tilde{T}_{\mu\nu}$$

$$\tilde{T}_{\mu\nu} = 2\lambda\, \partial_\mu \phi\, \partial_\nu \phi = (G - T)\, \partial_\mu \phi\, \partial_\nu \phi$$

Scalar field equation: $\nabla^\nu\left((G-T)\,\partial_\mu\phi\,\partial_\nu\phi\right)=0$

Equation $g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi=1$ is the defining equation for ϕ being

synchronous time: $\phi=t \Rightarrow \frac{1}{\sqrt{\gamma}}\partial_0\left(\sqrt{\gamma}\,(G-T)\right)=0$

where $ds^2=dt^2-\gamma_{ij}(x,t)\,dx^i dx^j$,

Solution: $G-T=\frac{C(x)}{\sqrt{\gamma}}$, $C(x)$ = mimetic dust

If $\gamma_{ij}=\delta_{ij}\,a^2(t) \rightarrow G-T=\frac{C(x)}{a^3}$

Mimetic Gravity

- Mimetic gravity is a minimal modification of GR where with same degrees of freedom except for a longitudinal mode of graviton gets excited.

- It is possible to add to Einstein action any function $V(\phi) \xrightarrow{\text{synchronous gauge}} V(t)$ without breaking diffeomorphism invariance.

Equations of motion in this case become

$$\tilde{T}_{\mu\nu} = (G - T - 4V)\, \partial_\mu \phi\, \partial_\nu \phi$$

$$\nabla^\nu \left[(G - T - 4V) \right] = -V'(\phi)$$

With this it is possible to construct many interesting Cosmological models (In collaboration with A. Vikman)

Applications: Although idea is simple, the implications are surprisingly robust.

- Resolving Singularities

one of the main problems in GR is the appearance of singularities e.g. at initial time $t=0$ for Friedmann, Kasner or black hole solutions

The constraint on scalar field ϕ is invariant under constant shifts $\phi \to \phi + c$. To solve singularity problem we will use instead of potential $V(\phi)$, not invariant under constant shift

a function

$$f(K) \quad \text{where} \quad K = \Box\phi$$

$$K = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right)$$

In synchronous gauge $K = \frac{1}{\sqrt{\gamma}} \partial_0 \left(\sqrt{\gamma} \right)$ is the trace of intrinsic curvature.

$\gamma = \det \gamma_{ij}$

$$K_{\mu\nu} = \nabla_\mu \nabla_\nu \phi \rightarrow K = g^{\mu\nu} K_{\mu\nu}$$

Equations of motion, after adding a term to the action

$$\int d^4x \sqrt{-g}\, f(K)$$

Contributes to energy-momentum tensor

$$\tilde{T}_{\mu\nu} = 2\lambda\, \partial_\mu \phi\, \partial_\nu \phi + g_{\mu\nu} \left(K f' - f + g^{\rho\sigma} \partial_\rho f' \partial_\sigma \phi - \left(\partial_\mu f' \partial_\nu \phi + \partial_\nu f' \partial_\mu \phi \right) \right)$$

To avoid singularity we implement ideas of limiting curvature

Choose: $f(K) = 1 + \frac{1}{2}K^2 - K \arcsin K - \sqrt{1-K^2}$

(we rescale $K \to \dfrac{K}{K_m}$ $\quad K_m$ limiting curvature)

Taylor expansion $f(K) = O(K^3)$ to agree with GR

For Friedman $\gamma_{ij} = a^2(t)\,\delta_{ij}$ $\qquad i,j = 1,2,3$

Einstein equations : $\frac{1}{12}\left(\frac{\dot\gamma}{\gamma}\right)^2 = \varepsilon\left(1 - \frac{\varepsilon}{\varepsilon_m}\right)$ $\qquad \varepsilon_m = 2K_m$

$$\gamma = a^6, \qquad \varepsilon = \frac{C}{\sqrt{\gamma}} + T_{00}$$

Solution $a(t) = \left(1 + \frac{3}{4}\varepsilon_m t^2\right)^{\frac{1}{3}}$ is an exact solution

$t < -\frac{1}{\sqrt{\varepsilon_m}} \rightarrow a(t) \sim t^{2/3}$ as in cold dominated universe

$-\frac{1}{\sqrt{\varepsilon_m}} < t < \frac{1}{\sqrt{\varepsilon_m}}$ we get a regular bounce

$t > \frac{1}{\sqrt{\varepsilon_m}}$ we get a dust dominated universe special curvature terms are regular during bounce and all curvature invariants remain bounded.

The multivalued function $f(R)$ satisfy matching conditions at branch points to insure regularity of cosmological evolution

- Non-singular Kasner universe

Kasner metric is : $ds^2 = dt^2 - t^{2p_1} dx^2 - t^{2p_2} dy^2 - t^{2p_3} dz^2$

$$p_1 + p_2 + p_3 = 1, \quad p_1^2 + p_2^2 + p_3^2 = 1$$

This is a homogeneous anisotropic solution of Einstein equations

$$R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = -\frac{16}{t^4} p_1 p_2 p_3 \qquad \text{singular at } t=0$$

In our case : $\left(\frac{\dot{\gamma}}{\gamma}\right)^2 = \frac{3\bar{J}^2}{2\gamma}\left(1 - \frac{\bar{J}^2}{8\varepsilon_m \gamma}\right)$, $\bar{J}^2 = J^i_j J^j_i = \text{const.}$

Exact solution $\gamma_{(i)} = \left(\frac{\bar{J}^2}{8\varepsilon_m}(1+3\varepsilon_m t^2)\right)^{\frac{1}{3}} \exp\left(2\sqrt{\frac{2}{3}}\frac{J_{(i)}}{\bar{J}} \sinh^{-1}\left(\sqrt{3\varepsilon_m}\, t\right)\right)$

$$\gamma_{(i)} \equiv \gamma_{ii}$$

If $\varepsilon_m t^2 \gg 1 \rightarrow \gamma_{(i)} \sim t^{2p_i^{\pm}}$; $p_i^{\pm} = \frac{1}{3} \pm \sqrt{\frac{2}{3}}\frac{J^i}{\bar{J}}$

reproduces familiar Kasner solution

We also get a non-singular Schwarzschild solution which is regular at $t=0$ but complicated behavior at event horizon.

A non-singular black-hole solution is obtained in the context of asymptotically free mimetic gravity (with T. Russ)

Consider the action

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left(-\mathcal{L} + \lambda\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - 1\right)\right)$$

$$\mathcal{L} = f(K)R + 2\Lambda(K) + (f(K)-1)\tilde{R} + h(\tilde{R})$$

where $\tilde{R} = 2\nabla^\mu\phi\,\nabla^\nu\phi\, G_{\mu\nu} - (\Box\phi)^2 + \nabla^\mu\nabla^\nu\phi\,\nabla_\mu\nabla_\nu\phi$

In synchronous gauge $\tilde{R} = {}^3R$ the curvature of the three-curvature slices : $-R = 2\dot{K} + K^2 + K_{ij}K^{ij} + {}^{3}R$

(only space derivatives)

Require: $f(K=0) = 1$, $K \to K_0$ $f(K_0) = \frac{1}{G(K_0)} \to \infty$

theory becomes asymptotically free.

Exact black hole solution: $\Lambda = \frac{2}{3} K^2 (f-1)$

$$f = \frac{1}{1 - \frac{K^2}{K_0^2}}$$

$$\frac{1}{2}(1+w) K_0 t = \frac{K_0}{K} - \operatorname{atanh} \frac{K}{K_0} - \sqrt{2} \arctan\left(\sqrt{2} \frac{K}{K_0}\right)$$

$$\rho \propto a^{-3(1+w)}$$

For $K^2 \ll K_0^2 \longrightarrow$ Friedmann

Mimetic Horava Gravity & Renormalizability

Examining components of Riemann & Ricci tensors in synchronous gauge:

$$R^0{}_{ijk} = \nabla_j K_{ki} - \nabla_k K_{ji}$$

$$R^0{}_0 = -\underline{\partial_0 K} - K_i{}^j K_j{}^i$$

$$R^0{}_{i0j} = \underline{\partial_0 K_{ij}} - K_{il} K^l{}_j$$

$$R^0{}_i = \nabla_j K_i{}^j - \nabla_i K$$

$$R^i{}_{0jk} = \nabla_j K^i{}_k - \partial_k K^i{}_j$$

$$R_i{}^j = -{}^3R_i{}^j - \partial_0 K_i{}^j - K K_i{}^j$$

$$R^l{}_{ijk} = {}^3R^l{}_{ijk} + K_i{}^l K_{lk} - K_k{}^l K_{lj}$$

$$R = -2\,\underline{\partial_0 K} - K^2 - K_i{}^j K_j{}^i - {}^3R$$

$$R^l{}_{i0k} = \underline{\partial_0 \Gamma^l_{ik}} - \nabla_k K_i{}^l$$

$f(K)$ in synchronous gauge $\sim (\partial_0 \gamma)^n$ does not change graviton propagator

The derivative $n_\mu = \partial_\mu \phi$ provides, in synchronous gauge, a normal to project along time direction. : $n_0 = 1$, $n_i = 0$

Define projection operator $P_\mu{}^\nu = \delta_\mu^\nu - n_\mu n^\nu$; $n^\nu = g^{\nu\rho} n_\rho$

Can project any 4-dimensional tensor to space direction:

$$P_\mu{}^\rho P_\rho{}^\nu = P_\mu{}^\nu , \qquad \bar{V}_\mu = P_\mu{}^\nu V_\nu \longrightarrow \bar{V}_0 = 0, \quad \bar{V}_i = V_i$$

$\therefore$ $n_\mu = \partial_\mu \phi$ can be used to construct Horava type gravity without breaking 4-dimensional diffeomorphism invariance

We can include higher space-derivatives but restricting to only second order time derivatives improving renormalizability while avoiding non-unitarity.

Example: $\bar{R}^{\sigma}{}_{\rho\mu\nu} = P^{\sigma}_{\delta} P^{\gamma}_{\rho} P^{\alpha}_{\mu} P^{\beta}_{\nu} R^{\delta}{}_{\gamma\alpha\beta} + 2\, \nabla_{[\mu|}\phi \nabla^{\sigma}\phi \nabla_{\rho} \nabla_{|\nu]}\phi$

coincides with ${}^{3}R^{l}{}_{ij\mu}$ in synchronous gauge

$$\bar{R}_{\mu\nu\kappa\lambda} = R_{\mu\nu\kappa\lambda} - \nabla^{\rho}\phi \nabla^{\sigma}\phi \left[\left(\partial_{\mu}\phi\, \partial_{\kappa}\phi\, R_{\rho\nu\sigma\lambda} - \mu \leftrightarrow \nu \right) - \kappa \leftrightarrow \lambda \right]$$
$$+ 2 \left(\nabla_{\mu}\nabla_{\kappa}\phi\, \nabla_{\nu}\nabla_{\lambda}\phi - \nabla_{\mu}\nabla_{\lambda}\phi\, \nabla_{\nu}\nabla_{\kappa}\phi \right)$$

Then $\tilde{R}_{0ijk} = 0$, $\tilde{R}_{0i0j} = 0$, $\tilde{R}_{lijk} = -{}^3R_{lijk} + (K_{lj}K_{ik} - K_{lk}K_{ij})$

has no second time derivatives

e.g. $\displaystyle\int_M d^4x \sqrt{-g}\,\tilde{R} = \int_M d^4x \sqrt{\gamma}\,({}^3R + K^2 - K_i{}^j K_j{}^i)$

$\displaystyle = \int_M d^4x \sqrt{-g}\, R - 2\int_{\partial M} d^3x \sqrt{\gamma}\, K$

Ghost free mimetic massive gravity

Formulation of gravity with mimetic field ϕ allows to solve problem of finding a consistent ghost free massive gravity.

- Introduce 4-scalar fields ϕ^A $\quad A = 0, 1, 2, 3$
- Consider induced metric $H^{AB} = g^{\mu\nu} \partial_\mu \phi^A \partial_\nu \phi^B$
- Expand $H^{AB} = \eta^{AB} + \bar{h}^{AB}$ and add mass term

Fierz-Pauli $\int d^4x \sqrt{-g} \, \frac{m^2}{8} \left(\bar{h}^2 - \bar{h}^A_B \bar{h}^B_A \right)$ + higher orders

This action exhibits a ghost state that could be stabilized for the combination $S^A{}_B = \sqrt{\delta^A{}_B + \bar{h}^A{}_B} - \delta^A{}_B$ by using $S^A{}_B$ instead of $\bar{h}^A{}_B$

ghost mode gets excited for time dependent background

Three scalar fields out of the 4 ϕ^A are absorbed by graviton to become massive with 5 degrees of freedom. The fourth scalar is a potential ghost mode problem solved by identifying the mimetic field $\phi = \phi^0$

$$I = \int d^4x \sqrt{-g}\left(-\frac{1}{2}R + \frac{m^2}{8}\left(\frac{1}{2}\bar{h}^2 - \bar{h}^A_B\,\bar{h}^B_A\right) + \lambda\left(g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0 - 1\right)\right)$$

non-Fierz pauli mass term

$g_{\mu\nu}$ will absorb only 3-degrees of freedom to become massive

- Mimetic inflation avoiding self-reproduction

(work with M. Khaldich also)

In slow roll inflation with scalar potentials there exists a regime of self reproduction if inflation begins at Planck scale and one must resort to fine tuning of initial conditions.

problem could be solved by coupling inflaton to mimetic fields

Consider the action

inflaton

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2}R + \lambda\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - 1\right) + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - C(K)V(\varphi)\right]$$

For simplicity consider the special case

$$C(K) = 1 + \frac{K}{m} \qquad m << 1 \quad \text{(in Planck units)}$$

$$V(\varphi) = \frac{1}{2}\frac{m^2\varphi^2}{1+\varphi^2}\left(1 + m\varphi^4\right)$$

For $\varphi < 1 \longrightarrow V(\varphi)$ describes a massive scalar field

$\varphi > 1 \longrightarrow V(\varphi) \simeq \frac{1}{2}m^2\left(1 - \frac{1}{\varphi^2} + m\varphi^4\right)$

In this approximation equations of motion become

$$K^2 = \frac{3}{2}m^2\left(1-\frac{1}{\varphi^2}+m\varphi^4\right)+\frac{3}{2}\dot{\varphi}^2 \qquad *$$

$$\ddot{\varphi} + K\dot{\varphi} + \left(1+\frac{K}{m}\right)\left(\frac{m^2}{\varphi^3}+2m^3\varphi^3\right) = 0$$

In the slow roll approximation we show that

$$1 < \varphi < m^{-\frac{1}{2}}, \quad \dot{\varphi}^2 < V$$ and inflation is

dominantly driven by potential, and condition for self-reproduction

is not fullfilled: decrease of classical background field during typical Hubble time $\delta\varphi_{cl} \simeq \dot{\varphi}\, t_H$ is always larger than amplitude of quantum fluctuations in Hubble scale so that the scalar field decreases in total

At $\varphi \simeq m^{-\frac{1}{2}}$ the energy density $\varepsilon \simeq m$

At $\varphi > m^{-\frac{1}{2}}$ kinetic term in equation $*$ is dominant

slow roll condition is still satisfied giving inflationary stage

Conclusions

- Mimetic modification of GR has unexpected far reaching conditions
- Eliminates need to introduce arbitrary new scalar fields to solve multitudes of problems
- It offers a simple solution to problem of dark matter in the form of dust.

- provides a natural way to 3+1 splitting of space-time using vector $n_\mu = \partial_\mu \phi$

- Can add higher powers of intrinsic curvature without changing propagator causing non-unitarity

- Can resolve singularities of space-time yielding smooth Friedmann-Kasner & black hole solutions at $t=0$

It appears that there is unlimited number of applications