Naturalness from a String Perspective and the emergence of the Higgs field

Steve Abel (IPPP + Cern)

When I say something new it is mainly based on work w/ Keith Dienes and Luca Nutricati

Oxford '92



Oxford '92



Life:

Herbi was a brash American who was also somehow trying to escape the German draft and string theory.

As an American he also calls himself 'erbi.

Oxford was a fantastic time for me and all of us

He does not wear ties (in Oxford ties are sometimes required by law)

When in Oxford we took a honeymoon road trip to Ioannina and around Greece in a renault clio.

We both went to RAL

Only one to Durham

Many thanks to Herbi (and Boris Johnson) we are both German





Dipole moments of the electron, neutrino and neutron in the MSSM without R-parity symmetry

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ABSTRACT: We show that in the MSSM without R-parity symmetry there are *no* new contributions to electron and neutron electric dipole moments (EDMs) at one-loop induced by the R-parity violating Yukawa couplings. Non-zero EDMs for the electron and neutron first arise at the two-loop level. As an example we estimate the contribution of a two-loop graph which induces electron EDMs. On the other hand, we show that the (Majorana) neutrino electric and magnetic transition moments are non zero even at the one-loop level. Constraints on the R-parity violating couplings are derived from the existing bounds on the neutrino dipole moments.

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Testing locality at colliders via Bell's inequality?

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We consider a measurement of correlated spins at LEP and show that it does *not* constitute a *general* test of local-realistic theories via Bell's inequality. The central point of the argument is that such tests, where the spins of two particles are inferred from a scattering distribution, can be described by a local hidden variable theory. We conclude that with present experimental techniques it is not possible to test locality via Bell's inequality at a collider experiment. Finally we suggest an improved fixed-target experiment as a viable test of Bell's inequality.

Is it possible to test Bell's inequality via e.g. $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$

Establish spin by pion momentum $e^+e^- \rightarrow Z^0 \rightarrow \tau^+ \tau^- \rightarrow \pi^+ \bar{\nu}_{\tau} \pi^- \nu_{\tau}$

Differential cross-section $P_{QM}(\cos \theta_{\pi\pi}) = \frac{(d\sigma/d \cos \theta_{\pi\pi})(e^+e^- \rightarrow \pi^+\pi^- \nu_\tau \bar{\nu}_\tau)}{\sigma(e^+e^- \rightarrow \pi^+\pi^- \nu_\tau \bar{\nu}_\tau)}$ = $\frac{1}{2}(1-\frac{1}{3}\cos \theta_{\pi\pi})$

Insert into Bell's inequality gives $9 - \cos \theta_{\hat{a}\hat{c}} \ge |\cos \theta_{\hat{a}\hat{b}} - \cos \theta_{\hat{b}\hat{c}}|$

Always satisfied => this process is not a test of Bell. (Ultimately because the angles are written in terms of commuting variables.)

Also true of e.g. $t\bar{t}$ systems at LHC and e.g. $H \rightarrow ZZ$ (Casas et al; Barr et al)

In the paper we proposed a fixed target experiment that *would* be a consistent test of Bell's inequality ...

The Papers:



Science Slams - how do they work?

Herbi Dreiner gives the lowdown on the Dortmund champions



Motivation for rest:

BSM driven for a long time (at least since 1992) by (un)Naturalness we see in QFT

Guided whole supersymmetry story: But nowadays "where is SUSY? Nowhere?" (Georg)

It is worth looking around for another paradigm of naturalness!



Motivation for rest:

The end result of this study is the following circle of ideas for naturalness beyond SUSY



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The weird acausal timeline:

2021	CW potential in strings
2010	Angelantonj and friends
1995	Misaligned supersymmetry (Dienes)
1991	Kutasov/Seiberg
1988	Gauge thresholds (Kaplunovsky)
1984	Superstrings
1981	Zagier
1973	Coleman-Weinberg potential
1939/40	Rankin/Selberg

1. Coleman Weinberg done stringily

Let's start our story by examining the one-loop CW effective potential in field theory (and similar amplitudes where we don't care about the external momenta):



giving ...

$$\Lambda(\phi) = \sum_{n} \int \frac{d^4k}{(2\pi)^4} (-1)^F \log(k^2 + M_n^2)$$

where masses can be functions of the Higgs ϕ and we are forced to put in a cut-off.

Can make a *toy* of what string theory does using the *Schwinger worldline formalism:* Define parameter *t* which in some sense is the total "length" of the worldline around the one-loop bubble. Total contribution is the *integral* of amplitude over all possible *t*.



$$\begin{split} \Lambda &= \sum_{n} \int \frac{d^4k}{(2\pi)^4} (-1)^F \log(k^2 + M_n^2) \\ &= \sum_{n} \int \frac{d^4k}{(2\pi)^4} \int \frac{dt}{t} (-1)^F e^{-t(k^2 + M_n^2)} \\ &= \sum_{n} \int_{M_{UV}^{-2}}^{\infty} \frac{dt}{t^3} (-1)^F e^{-t M_n^2} \end{split}$$

Can identify a "particle partition function" as a weighted sum over the spectral density:

$$Z(t) = \operatorname{Str}\left[t^{-2}e^{-tM^2}\right]$$

Performing the integral of Z(t) indeed gives the usual CW effective potential:

$$\Lambda(\phi) = \frac{1}{2}M_{UV}^4 + \frac{M_{UV}^2}{32\pi^2} \operatorname{Str} M^2 - \frac{1}{64\pi^2} \operatorname{Str} M^4 \log\left(c\frac{M^2}{M_{UV}^2}\right)$$

From which we can infer the running Higgs mass-squared from the double derivative:

$$m_{\phi}^2 = \frac{M_{\rm UV}^2}{32\pi^2} \operatorname{Str} \partial_{\phi}^2 M^2 - \operatorname{Str} \partial_{\phi}^2 \left[\frac{M^4}{64\pi^2} \log\left(c\frac{M^2}{M_{\rm UV}^2}\right) \right]$$

This is the origin of the unfortunate naturalness problem associated with the Higgs mass. It is associated with the quadratic UV divergence in the EFT, and led to all sorts of speculation (e.g. *Veltman* condition). CW themselves said this should be "*zero at the origin of field space*".

Thus Schwinger gives us an alternative worldline picture of the integral (which remember depends purely on the mass-spectrum):



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$$\Lambda = \int_0^\infty \frac{d\tau_2}{\tau_2} \int_{-1/2}^{1/2} d\tau_1 \, Z(\pi \alpha' \tau_2)$$

With an eye towards an eventual connection to string theory, let's define a dimensionless Schwinger parameter $\tau_2 = t/\pi \alpha'$

Also let's introduce a dummy variable and enlarge our region of integration with it:



Thus far this is all field theory. But now suppose our theory had an exact symmetry under $\tau_2 \rightarrow 1/\tau_2$

This symmetry is clearly not fieldtheoretic! But let's pursue it anyway.

- What effects would this have?
- How could we interpret this?



The symmetry $\tau_2 \rightarrow 1/\tau_2$ is a redundancy in the description and we should remove it (and a spurious factor of 2) by folding along the axis of symmetry:

What does this folding imply for UV vs IR?

- There is no longer a notion of increasingly UV or IR "directions" → all directionality is lost. "Non-orientable"
- The two divergences (UV and IR) have been folded on top of each other
- Thus, *there is only one divergence*.
 You can call it UV or IR according to your choice/convention → meaningless distinction!



What happens if we try to implement this symmetry perturbatively?

• Strip is already invariant
$$\Lambda = \int_{S} \frac{d\tau_1 d\tau_2}{\tau_2} Z(\pi \alpha' \tau_2)$$

• Measure is already invariant

• Thus all that remains is to make partition function invariant in such a theory!

$$Z(\pi \alpha' \tau_2) = Z(\pi \alpha' / \tau_2)$$

• But this symmetry is very hard to arrange in a particle theory because recall all we have to play with in Z is the spectrum of masses. Seems to require an infinite tower of states. For example

$$Z = \frac{1}{\tau_2^2} \sum_{\vec{n} \in \mathbb{Z}^8} e^{-\pi \vec{n}^2 \tau_2} = \tau_2^2 \sum_{\vec{n} \in \mathbb{Z}^8} e^{-\pi \vec{n}^2 / \tau_2}$$

by Poisson resummation, but this implies an insanely tuned spectrum $M_{\vec{n}}^2 = \vec{n}^2 / \alpha'$ What string theory does is arrange such a tuning and symmetry.

2. Modular invariance

The miracle of string theory can be put down to such insane tuning being inherent *because* the finiteness of the theory is guaranteed (just like in our toy example) by a symmetry, namely modular invariance. Let us revisit the cosmological constant ...

Closed string theory instead maps out a torus:







How should we do such integrals in a UV complete way? As in the toy example the partition function must be invariant under the symmetry, and this severely constrains the theory:

$$Z(\tau) = Z(\tau + 1) = Z(-1/\tau)$$

Note: Performing such calculations now becomes an issue in *string phenomenology* (*i.e.*, the question is how one should approach extracting "low-energy" phenomenological predictions from string theory)...

Herbi knows very well the traditional approach (although he tries to hide it) -

- Start with a suitable configurations that obey the symmetry ("string model")
- Enumerate the massless states that arise in such models
- Construct a field-theoretic Lagrangian that describes the dynamics of these states
- Analyze this Lagrangian using all of the regular tools of QFT without further regard for the origins of these states within string theory

While this approach may well be sufficient for certain purposes, it *generically* fails precisely we are asking about terms with positive mass dimension, so it has nothing to say about naturalness.

Can we get a general formulation like the Coleman Weinberg potential?

Thanks to modular invariance there's a way to write such integrals generically, as a supertrace over the infinite tower of *physical (level matched) masses*, without ever specifying the theory. Much more Coleman-Weinberg like: cosmological constant even looks similar to the field theory *quadratically divergent* piece:

$$\Lambda = \frac{1}{24} \mathcal{M}^2 \mathrm{STr} M^2$$

- Dienes, Misaligned SUSY, 1994
- Kutasov, Seiberg, 1994
- Dienes, Moshe, Myers 1995

But note this definitely is *not* a normal field theory object — this supertrace is over the *infinite* string tower of physical states!! e.g. in non-supersymmetric models ...



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Easiest way to find these expressions in terms of the physical states: Rankin-Selberg-Zagier

- In number theory: Rankin, Selberg (1939,40), Zagier (1981)
- In string theory: Angelantonj, Florakis, Pioline (2011)

The method of projecting to the physical states basically unfolds the origami:



Note the important difference from the usual picture. There is now clearly no single "IR cusp". All cusps contribute equally to the integral:



- All cusps equivalent under modular transformations, no "ultra UV" anywhere.
- As in our toy example, also not even an obvious IR/UV orientation

The fact that this infinite supertrace is finite can be put down to the fact that the spectral density behaves as follows as $\tau_2 \rightarrow 0$:

$$g(\tau_2) \sim \tau_2^{-1} \operatorname{Str}\left(e^{-\pi \tau_2 \alpha' M^2}\right) \longrightarrow c_0$$

In other words *Str(1)=0 despite no SUSY!* This explains why there is no term that is quartic in the string scale. (Nothing to do with SUSY).

The supertrace relation Str(1)=0 is just one example of a *magic cancellation* that runs across the entire string spectrum, suppressing divergences and/or ensuring the finiteness of string amplitudes relative to naive QFT expectations.

There are others: it turns out the Str(1)=0 relation is just the tip of the iceberg!

3. The Higgs mass?

Let's turn to the Higgs mass. How can we use this technology to express its emergent potential?

First assume that the partition function is a function of the higgs. Then begin with the naive expression and modular complete it:





Generally the result is logarithmically divergent if massless states. And in any case we need to find a way to regulate the theory at some IR scale *n* in order to extract a physical "running" potential even if no massless states

In other words, *I cannot choose a QFT* because my choice of whether the neutrino is light enough at a scale *n* to be called massless and be subtracted from the integral is completely arbitrary and would break modular invariance.

Instead we should see how an EFT *emerges* from string theory renormalisation by defining it with a modular invariant "Wilsonian" regulator instead:

$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^2} \widehat{\mathcal{G}}(\mu, \tau, \overline{\tau}) F(\tau, \overline{\tau})$$



Required properties of Wilsonian regulator, $\widehat{\mathbf{G}}$?



• c) Remember, in Rankin-Selberg all the cusps are equivalent IR cusps, so they should all be equally crushed ...

$$\tau_2^* \equiv 1/\tau_2^* \implies \widehat{\mathcal{G}}(\mu, \tau, \overline{\tau}) = \widehat{\mathcal{G}}(M_s^2/\mu, \tau, \overline{\tau})$$



• We adapted and modified a regulator of Kiritsis and Kounnas

The result is a smooth modular invariant stringy Coleman-Weinberg potential

Infinite sum of Bessel functions that has the following magical behaviour ...

$$\widehat{m}_{\phi}^2 = \frac{\xi}{4\pi^2} \frac{\widehat{\Lambda}(\mu)}{\mathcal{M}^2} + \partial_{\phi}^2 \widehat{\Lambda}(\mu)$$

$$\widehat{\Lambda}(\mu,\phi) = \frac{1}{24} \mathcal{M}^2 \operatorname{Str} M^2 - c' \operatorname{Str}_{M \gtrsim \mu} M^2 \mu^2 \quad - \operatorname{Str}_{0 \leq M \leq \mu} \left[\frac{M^4}{64\pi^2} \log\left(c\frac{M^2}{\mu^2}\right) + c'' \mu^4 \right]$$

 $c = 2e^{2\gamma + 1/2}, c' = 1/(96\pi^2), \text{ and } c'' = 7c'/10$

So Higgs mass begins at a UV value we can calculate, has RG running, maybe GUT breaking, EW and QCD phase transition, yada yada yada. But then it must eventually wind up at the exact same value in the IR. And everything is finite. Like this...



$$\lim_{\mu \to 0} \hat{m}_{\phi}^2(\mu) = \frac{\xi}{4\pi^2} \frac{\Lambda}{\mathcal{M}^2} + \frac{1}{24} \mathcal{M}^2 \mathrm{Str} \partial_{\phi}^2 M^2$$

4. Implications for Naturalness?

This picture solves the *technical* hierarchy problem — (Cartoon stolen from John March-Russell)



So what would a new paradigm for a natural theory look like?

$$SM + \sum_{\text{scalar tower } i} |\lambda|^2 |s_i|^2 |\phi|^2 + \sum_{\text{formion tower } i} \lambda' f_{L,i}^{\dagger} \phi f_{R,i}$$

scalar tower i

fermion tower i

where

$$\operatorname{Str}_{SM}|\lambda_{SM}|^2 + \operatorname{tr}|\lambda|^2 - \operatorname{tr}|\lambda'|^2 = 0$$

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Happy 60th Herbi ...!

