Three-pion finite-volume spectrum at maximal isospin from lattice QCD

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Multihadron dynamics in a box
Bethe Center for Theoretical Physics, Bonn
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based on BH, A. Hanlon 1905.04277 [hep-lat]

Three-particle dynamics in QCD

three-particle dynamics relevant for many hadronic systems

- $\omega(782) \to 3\pi, \quad a_1(1260) \to 3\pi$
- $N(1440) \to N\pi\pi$
- $3N$ force for nuclear physics

weakly interacting $I = 3$ three-pion as a testbed

- signal-to-noise manageable
- formalism fully developed
- calculable in ChPT

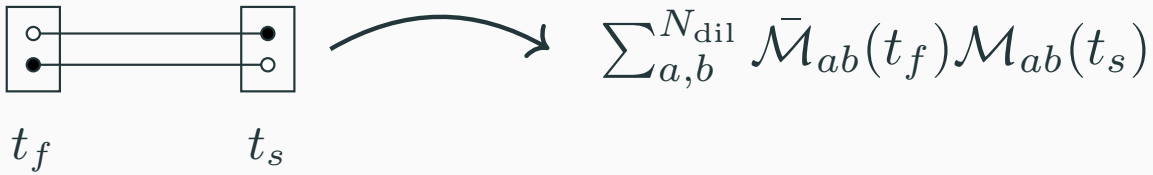
Lattice QCD spectroscopy

- 'large' set of irreducible operators

  (precise meaning of large depends on the system)

- *all-to-all propagators* furnished by stochastic LapH

[Peardon et al. 0905.2160
Morningstar et al. 1104.3870]

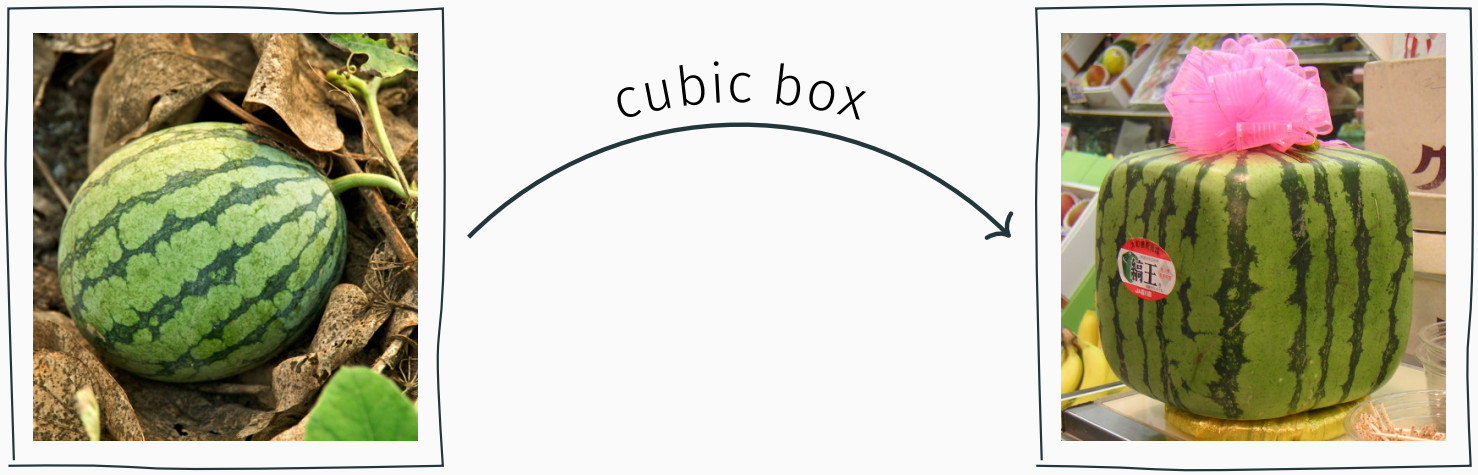


- reliable extraction of ground and excited states via solution of generalized eigenvalue problem

  (facilitated by Hermitian correlator matrices)

[Michael, Teasdale '83
Lüscher, Wolff '90
Blossier et al. 0902.1265]

Finite-volume symmetry



Three-pion interpolating operators

- coupling two pions with total momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$

  [Morningstar et al. 1303.6816
  see also Dudek, Edwards, Thomas 1203.6041]

  Clebsch-Gordan coefficients c

$$O^{(\mathbf{P},\Lambda)} = c^{(\Lambda)}_{\mathbf{p}_1,\mathbf{p}_2}\, \pi_{\mathbf{p}_1} \pi_{\mathbf{p}_2}$$

- three-particle operator by iterating two-particle Clebsch-Gordan coupling

  no subresonances, does not require construction from [Woss et al. 1904.04136]

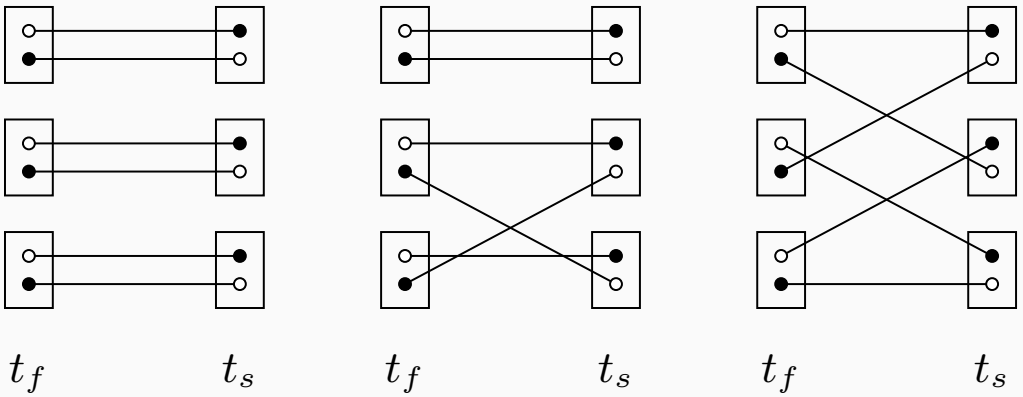
  keep track of duplicate operators

Example:

Λ	[-00][+0+][000]	[0-0][0++][000]	[00][000][00+]	[0+0][0-+][000]	[+00][-0+][000]
A_2^-		1			
B_2^-	1	1		1	1
	1	−1		−1	1

→ shared elementals

Contraction optimization



- contractions limiting factor as number of valence quark fields increases

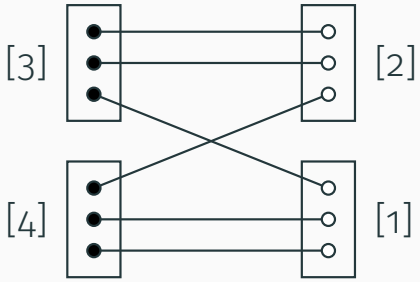
  [e.g. in recent Δ study Andersen, Bulava, BH, Morningstar 1710.01557]

- algorithm to systematically exploit redundancies

  duplicate diagrams from shared elementals

  $I = 2$ $\pi\pi$ diagrams as subexpressions

Example: two-baryon correlation function



- *in-diagram* optimization

  find the best contraction(s) in a diagram

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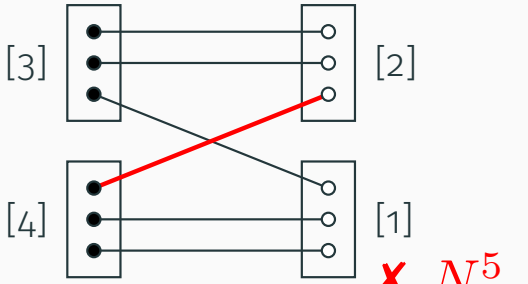

✗ N_{dil}^{5}

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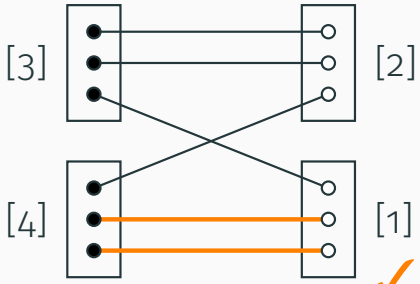

✓ N_{dil}^4

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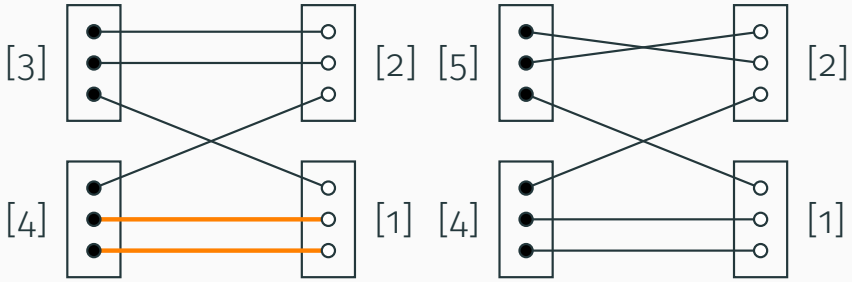

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  common subexpression elimination (CSE) in all diagrams

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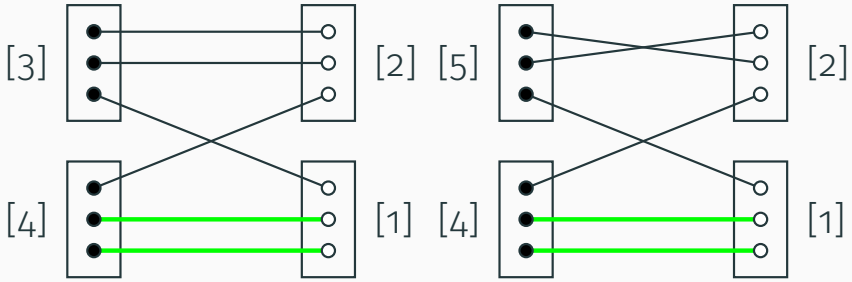

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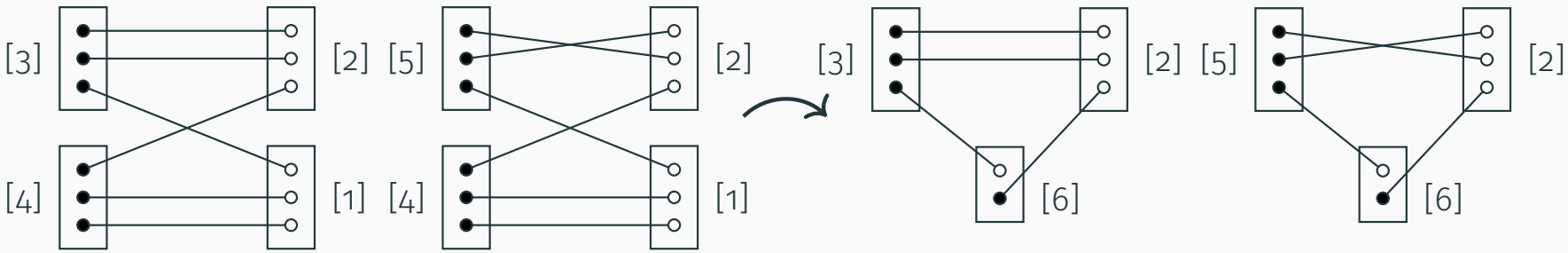

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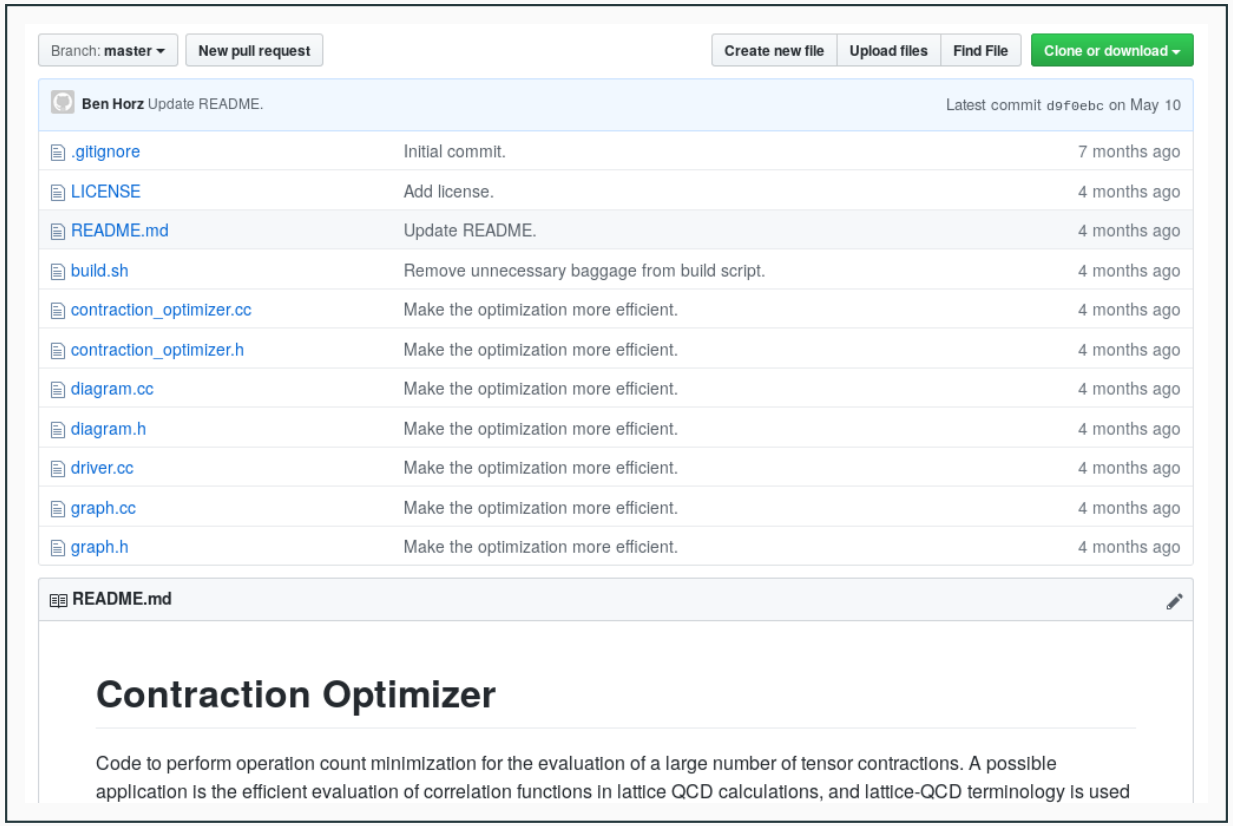
  common subexpression elimination (CSE) in all diagrams

- **Current work:** all 2π & 3π correlators (DC: diagram consolidation)

	N^2_{dil}	N^3_{dil}
w/o CSE, w/o DC	36,860,400	44,042,400
w/o CSE, w/ DC	10,035,600	11,810,400
w/ CSE	2,789,370	761,093

15x

contraction_optimizer



- implementation in C++ publicly available
  `https://github.com/laphnn/contraction_optimizer`
- treats arbitrary collections of tensor contractions
- Python interface partially implemented

Ensemble details

	$m_\pi\,[\mathrm{MeV}]$	L/a	$m_\pi L$	$a\,[\mathrm{fm}]$	N_{cfg}	dilution	N_η
D200	200	64	4.16	0.064	1100	(TF,SF,LI16)	6

$N_{\mathrm{dil}} = 64$

- $N_{\mathrm{f}} = 2 + 1$ CLS ensemble with nonperturbatively improved Wilson fermions

[Bruno et al. 1411.3982
Bruno, Korzec, Schaefer 1608.08900]

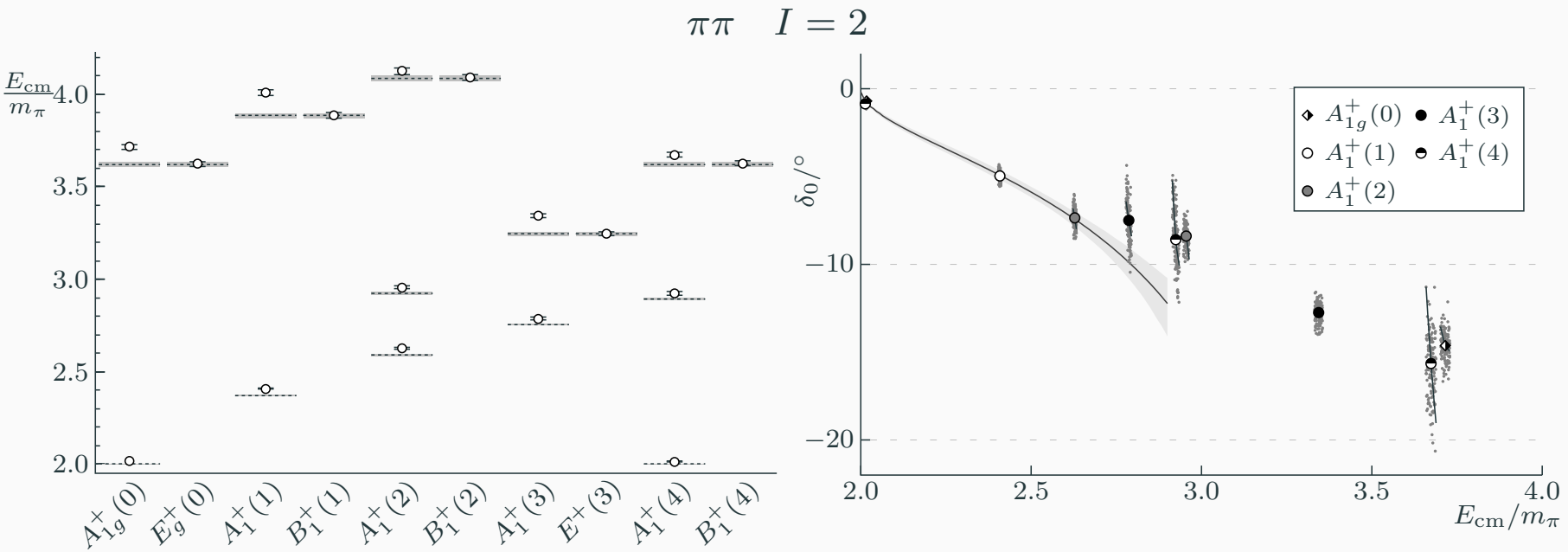
- open temporal boundary conditions

  interpolators always seperated from boundary by at least $m_\pi t_s = 2.2$

- used previously for $I = 1$ $\pi\pi$ scattering in $g - 2$ HVP calculation

[Gérardin et al. 1904.03120]

$I = 2$ TWO-PION SPECTRUM AND SCATTERING



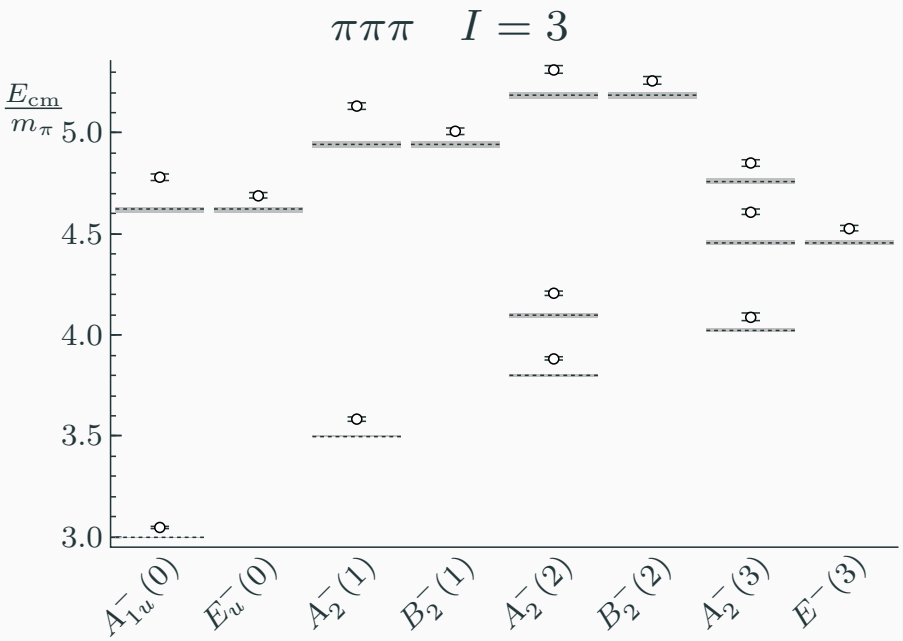
effective range expansion for $q^2/m_\pi^2 < 1$

$$q \cot \delta_0 = -\frac{1}{a_0} + \frac{r_0}{2} q^2$$

$$m_\pi a_0 = 0.1019(88)$$

$$m_\pi r_0 = 9.0(2.4)$$

$I = 3$ THREE-PION SPECTRUM



- precision at few-permille level
- significant energy shifts in all irreps

different energy shifts in irreps with degenerate noninteracting levels

might suggest sensitivity to different combinations of low-energy scattering parameters

Three-particle $1/L$ expansion

ground-state energy in $A_{1u}^-(0)$ irrep [Beane, Detmold, Savage 0707.1670; Hansen, Sharpe 1602.00324]

$$\Delta E_3 = \frac{12\pi a_0}{m_\pi L^3}\left\{1 - \left(\frac{a_0}{\pi L}\right)\mathcal{I} + \left(\frac{a_0}{\pi L}\right)^2(\mathcal{I}^2 + \mathcal{J}) + \frac{3\pi a_0}{m_\pi^2 L^3} + \frac{64\pi^2 a_0^2 \mathcal{C}_3}{m_\pi L^3} + \frac{6\pi r_0 a_0^2}{L^3}\right.$$
$$\left. + \left(\frac{a_0}{\pi L}\right)^3 [c_L \log(N_{\text{cut}}) - \mathcal{I}^3 + \mathcal{I}\mathcal{J} + 15\mathcal{K} + \mathcal{C}_F + \mathcal{C}_4 + \mathcal{C}_5]\right\} - \frac{\mathcal{M}_{3,\text{th}}}{48 m_\pi^3 L^6} + O(L^{-7})$$

- LO energy shift three times larger than for two pions

  our data: $\Delta E_3 / \Delta E_2 = 2.78(21)$

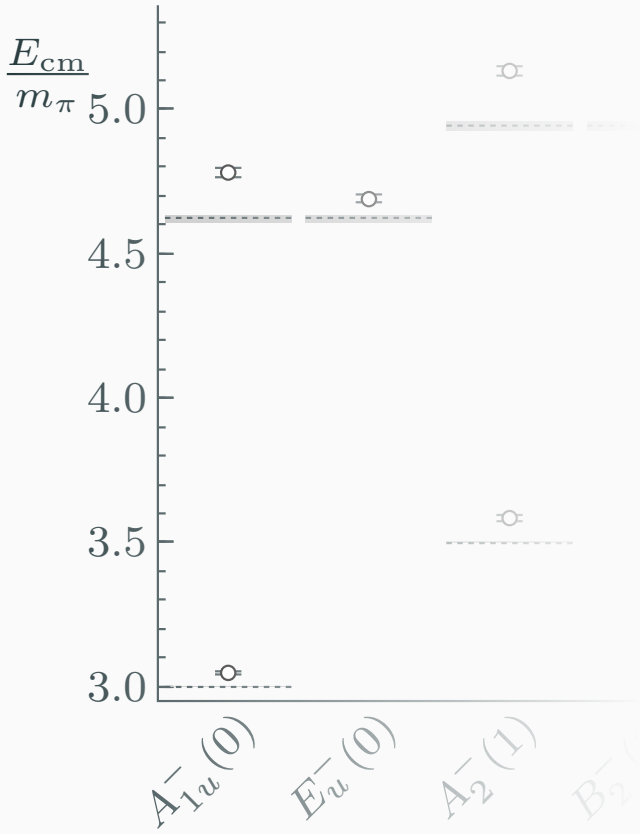
- subtracted three-particle threshold amplitude at $O(L^{-6})$

  $\frac{m_\pi^2 \mathcal{M}_{3,\text{th}}}{48(m_\pi L)^6} = 0.0113(43)$

  (scheme-dependent) nonzero value

  relies on correct treatment of two-particle scattering below threshold

A first excited state



- $1/L$ expression available for first excited state in $A_{1u}^-(0)$ [Pang et al. 1902.01111]

  predicts $E_3'/m_\pi = 5.41(8)$

- no consistent description of spectrum

  effect of r_0 and $M_{3,\mathrm{th}}$ numerically small

- full FV formalism required to describe data

  FV unitarity predicts $E_3'/m_\pi \approx 4.75$

  [Mai, Döring 1807.04746; priv. comm.]

Summary & Outlook

- extracted $I = 3$ three-pion spectrum across several irreps

  all data public to facilitate analysis by other groups

- $1/L$ expansion does not seem to describe at-rest spectrum well

  predictions based on full formalism promising

- technical improvement: optimized correlation function construction

  code publicly available and generally applicable