

THREE-PION FINITE-VOLUME SPECTRUM AT MAXIMAL ISOSPIN FROM LATTICE QCD

Ben Hörz (Lawrence Berkeley National Lab)

Multihadron dynamics in a box

Bethe Center for Theoretical Physics, Bonn

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based on BH, A. Hanlon 1905.04277 [hep-lat]

THREE-PARTICLE DYNAMICS IN QCD

three-particle dynamics relevant for many hadronic systems

- $\omega(782) \rightarrow 3\pi, \quad a_1(1260) \rightarrow 3\pi$
- $N(1440) \rightarrow N\pi\pi$
- $3N$ force for nuclear physics

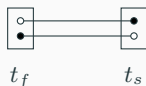
weakly interacting $I = 3$ **three-pion** as a **testbed**

- signal-to-noise manageable
- formalism fully developed
- calculable in ChPT

- ‘large’ set of irreducible operators
(precise meaning of large depends on the system)

- *all-to-all propagators* furnished by stochastic LapH

[Peardon et al. 0905.2160
Morningstar et al. 1104.3870]



The diagram shows two square boxes representing source and sink operators. The left box is labeled t_f and contains two points: an open circle at the top and a solid black dot at the bottom. The right box is labeled t_s and contains two points: a solid black dot at the top and an open circle at the bottom. Two horizontal lines connect the top points and the bottom points of the two boxes. A curved arrow points from this diagram to the right.

$$\sum_{a,b}^{N_{\text{dil}}} \bar{\mathcal{M}}_{ab}(t_f) \mathcal{M}_{ab}(t_s)$$

- reliable extraction of ground and excited states via solution of generalized eigenvalue problem
(facilitated by Hermitian correlator matrices)

[Michael, Teasdale '83
Lüscher, Wolff '90
Blossier et al. 0902.1265]

FINITE-VOLUME SYMMETRY



cubic box



THREE-PION INTERPOLATING OPERATORS

- coupling two pions with total momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$

[Morningstar et al. 1303.6816

see also Dudek, Edwards, Thomas 1203.6041]

Clebsch-Gordan coefficients c

$$O^{(\mathbf{P}, \Lambda)} = c_{\mathbf{p}_1, \mathbf{p}_2}^{(\Lambda)} \pi_{\mathbf{p}_1} \pi_{\mathbf{p}_2}$$

- three-particle operator by iterating two-particle Clebsch-Gordan coupling

no subresonances, does not require construction from

[Woss et al. 1904.04136]

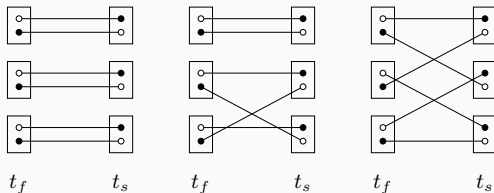
keep track of duplicate operators

Example:

Λ	$[-00][+0+][000]$	$[0-0][0++][000]$	$[000][000][00+]$	$[0+0][0-+][000]$	$[+00][-0+][000]$
A_2^-			1		
B_2^-	1	1		1	1
	1	-1		-1	1

} → shared elementals

CONTRACTION OPTIMIZATION



- contractions limiting factor as number of valence quark fields increases

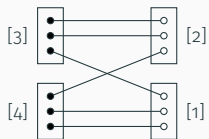
[e.g. in recent Δ study Andersen, Bulava, BH, Morningstar 1710.01557]

- algorithm to systematically exploit redundancies

duplicate diagrams from shared elementals

$I = 2 \pi\pi$ diagrams as subexpressions

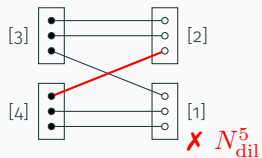
EXAMPLE: TWO-BARYON CORRELATION FUNCTION



- *in-diagram* optimization

find the best contraction(s) in a diagram

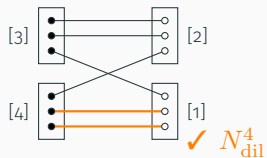
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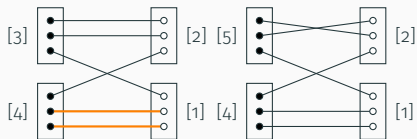
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- *in-diagram* optimization

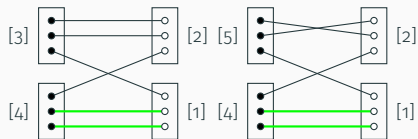
find the best contraction(s) in a diagram

EXAMPLE: TWO-BARYON CORRELATION FUNCTION



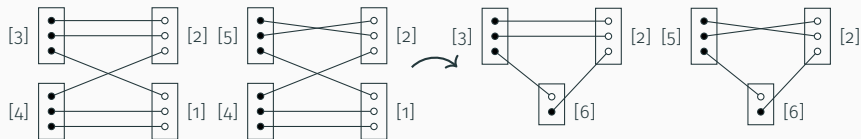
- *in-diagram* optimization
find the best contraction(s) in a diagram
- *between-diagram* optimization
common subexpression elimination (CSE) in all diagrams

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common subexpression elimination (CSE) in all diagrams

- **Current work:** all 2π & 3π correlators (DC: diagram consolidation)

	N_{di1}^2	N_{di1}^3	
w/o CSE, w/o DC	36,860,400	44,042,400	} 15x
w/o CSE, w/ DC	10,035,600	11,810,400	
w/ CSE	2,789,370	761,093	

contraction_optimizer

The screenshot shows the GitHub repository page for 'contraction_optimizer'. At the top, it indicates the current branch is 'master' and there is a 'New pull request' button. On the right, there are buttons for 'Create new file', 'Upload files', 'Find File', and 'Clone or download'. Below this, a commit history table is displayed, showing the most recent commit by Ben Horz updating the README. The table lists various files and their commit messages, such as 'Initial commi.', 'Add license.', and 'Make the optimization more efficient.' for multiple files including 'gltignore', 'LICENSE', 'README.md', 'build.sh', 'contraction_optimizer.cc', 'contraction_optimizer.h', 'diagram.cc', 'diagram.h', 'driver.cc', 'graph.cc', and 'graph.h'. Below the commit history, the README.md file is open, showing the title 'Contraction Optimizer' and a brief description: 'Code to perform operation count minimization for the evaluation of a large number of tensor contractions. A possible application is the efficient evaluation of correlation functions in lattice QCD calculations, and lattice-QCD terminology is used'.

File	Commit Message	Time
gltignore	Initial commi.	7 months ago
LICENSE	Add license.	4 months ago
README.md	Update README.	4 months ago
build.sh	Remove unnecessary baggage from build script.	4 months ago
contraction_optimizer.cc	Make the optimization more efficient.	4 months ago
contraction_optimizer.h	Make the optimization more efficient.	4 months ago
diagram.cc	Make the optimization more efficient.	4 months ago
diagram.h	Make the optimization more efficient.	4 months ago
driver.cc	Make the optimization more efficient.	4 months ago
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Contraction Optimizer

Code to perform operation count minimization for the evaluation of a large number of tensor contractions. A possible application is the efficient evaluation of correlation functions in lattice QCD calculations, and lattice-QCD terminology is used

- implementation in C++ publicly available
https://github.com/laphnn/contraction_optimizer
- treats arbitrary collections of tensor contractions
- Python interface partially implemented

ENSEMBLE DETAILS

	m_π [MeV]	L/a	$m_\pi L$	a [fm]	N_{cfg}	dilution	N_η
D200	200	64	4.16	0.064	1100	(TF,SF,LI16)	6

$N_{\text{dil}} = 64$

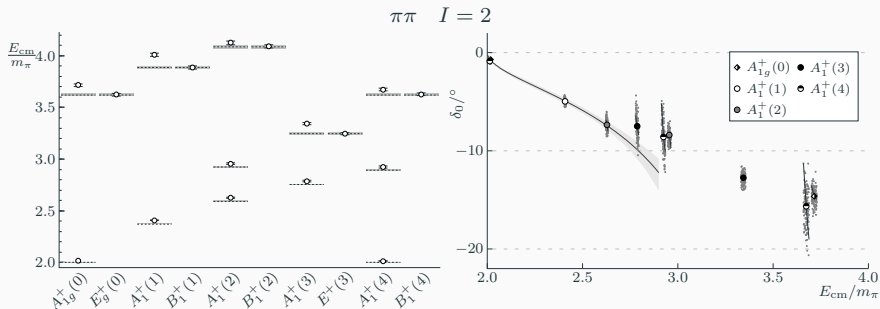
- $N_f = 2 + 1$ CLS ensemble with nonperturbatively improved Wilson fermions

[Bruno et al. 1411.3982
Bruno, Korzec, Schaefer 1608.08900]

- open temporal boundary conditions
interpolators always separated from boundary by at least $m_\pi t_s = 2.2$
- used previously for $I = 1$ $\pi\pi$ scattering in $g - 2$ HVP calculation

[Géardin et al. 1904.03120]

$I = 2$ TWO-PION SPECTRUM AND SCATTERING



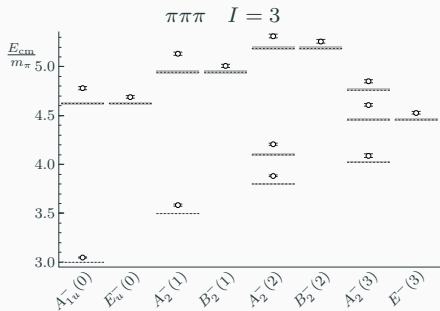
effective range expansion for $q^2/m_\pi^2 < 1$

$$q \cot \delta_0 = -\frac{1}{a_0} + \frac{r_0}{2} q^2$$

$$m_\pi a_0 = 0.1019(88)$$

$$m_\pi r_0 = 9.0(2.4)$$

$I = 3$ THREE-PION SPECTRUM



- precision at few-permille level
- significant energy shifts in all irreps

different energy shifts in irreps with degenerate noninteracting levels

might suggest sensitivity to different combinations of low-energy scattering parameters

THREE-PARTICLE $1/L$ EXPANSION

ground-state energy in $A_{1u}^-(0)$ irrep [Beane, Detmold, Savage 0707.1670; Hansen, Sharpe 1602.00324]

$$\Delta E_3 = \frac{12\pi a_0}{m_\pi L^3} \left\{ 1 - \left(\frac{a_0}{\pi L}\right) \mathcal{I} + \left(\frac{a_0}{\pi L}\right)^2 (\mathcal{I}^2 + \mathcal{J}) + \frac{3\pi a_0}{m_\pi^2 L^3} + \frac{64\pi^2 a_0^2 \mathcal{C}_3}{m_\pi L^3} + \frac{6\pi r_0 a_0^2}{L^3} \right. \\ \left. + \left(\frac{a_0}{\pi L}\right)^3 [c_L \log(N_{\text{cut}}) - \mathcal{I}^3 + \mathcal{I}\mathcal{J} + 15\mathcal{K} + \mathcal{C}_F + \mathcal{C}_4 + \mathcal{C}_5] \right\} - \frac{\mathcal{M}_{3,\text{th}}}{48m_\pi^3 L^6} + O(L^{-7})$$

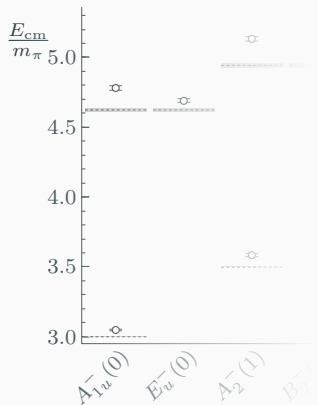
- LO energy shift three times larger than for two pions
our data: $\Delta E_3/\Delta E_2 = 2.78(21)$
- subtracted three-particle threshold amplitude at $O(L^{-6})$

$$\frac{m_\pi^2 \mathcal{M}_{3,\text{th}}}{48(m_\pi L)^6} = 0.0113(43)$$

(scheme-dependent) nonzero value

relies on correct treatment of two-particle scattering below threshold

A FIRST EXCITED STATE



- $1/L$ expression available for **first excited state** in $A_{1u}^-(0)$

[Pang et al. 1902.01111]

predicts $E_3'/m_{\pi} = 5.41(8)$

- no consistent description of spectrum

effect of r_0 and $M_{3,th}$ numerically small

- full FV formalism required to describe data

FV unitarity predicts $E_3'/m_{\pi} \approx 4.75$

[Mai, Döring 1807.04746; priv. comm.]

- extracted $I = 3$ three-pion spectrum across several irreps
all data public to facilitate analysis by other groups
- $1/L$ expansion does not seem to describe at-rest spectrum well
predictions based on full formalism promising
- technical improvement: optimized correlation function construction
code publicly available and generally applicable