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The Hamiltonian in a Box

Jia-Jun Wu （UCAS）

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bctp

Bethe Forum Multihadron Dynamics in a Box

Bonn, Germany, 10/09/2019

Outline

- Motivation
- Hamiltonian Effective Field Theory (HEFT)
- Eigenstate in the Lattice VS Resonance in the scattering
- The Angular momentum Mixing in the Hamiltonian
- The Multihadron system
- Summary

Resonance Properties: Mass , Width,
Pole position, Coupling, Structure

Experiments Observable
Differential Cross Sections

QCD theory

Resonance Properties: Mass , Width,
Pole position, Coupling, Structure

T matrix
Phase Shift,
inelasticity

Experiments Observable
Differential Cross Sections

Lattice Spectrum

Lattice QCD

QCD theory

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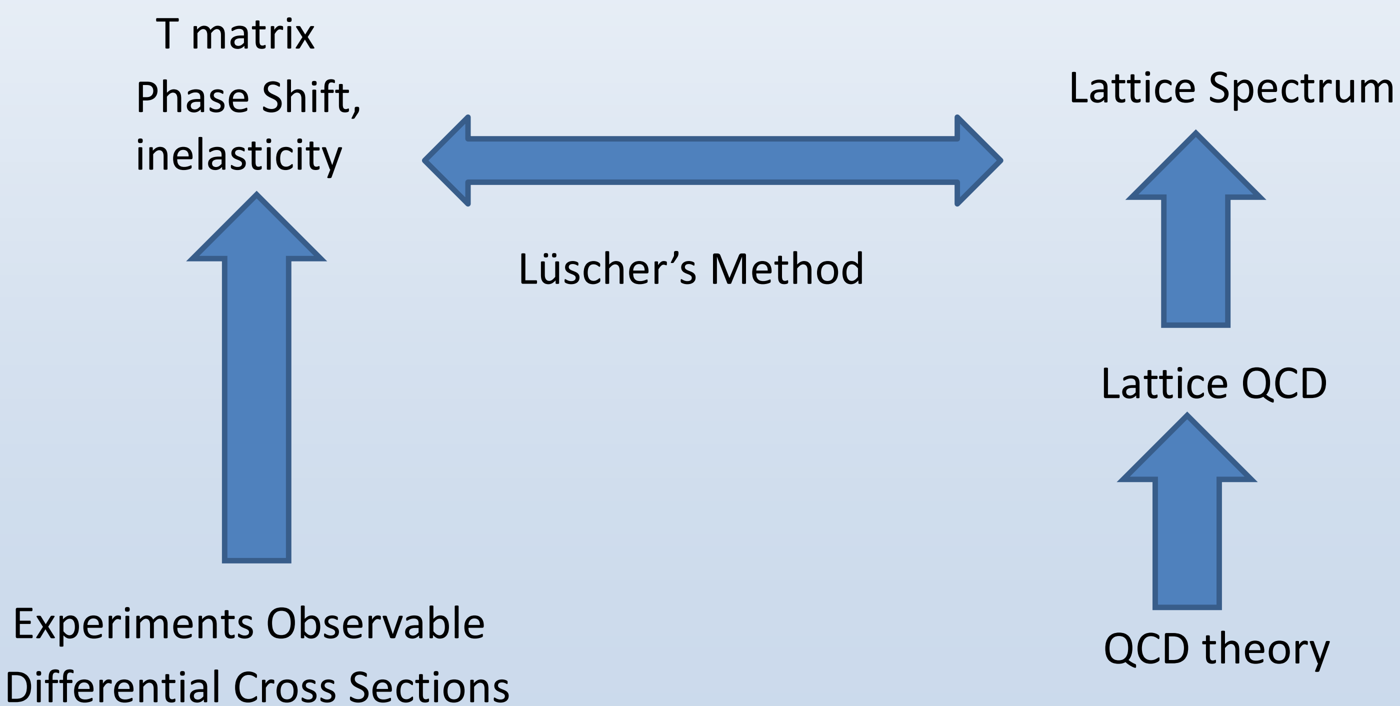
Resonance Properties: Mass , Width,
Pole position, Coupling, Structure



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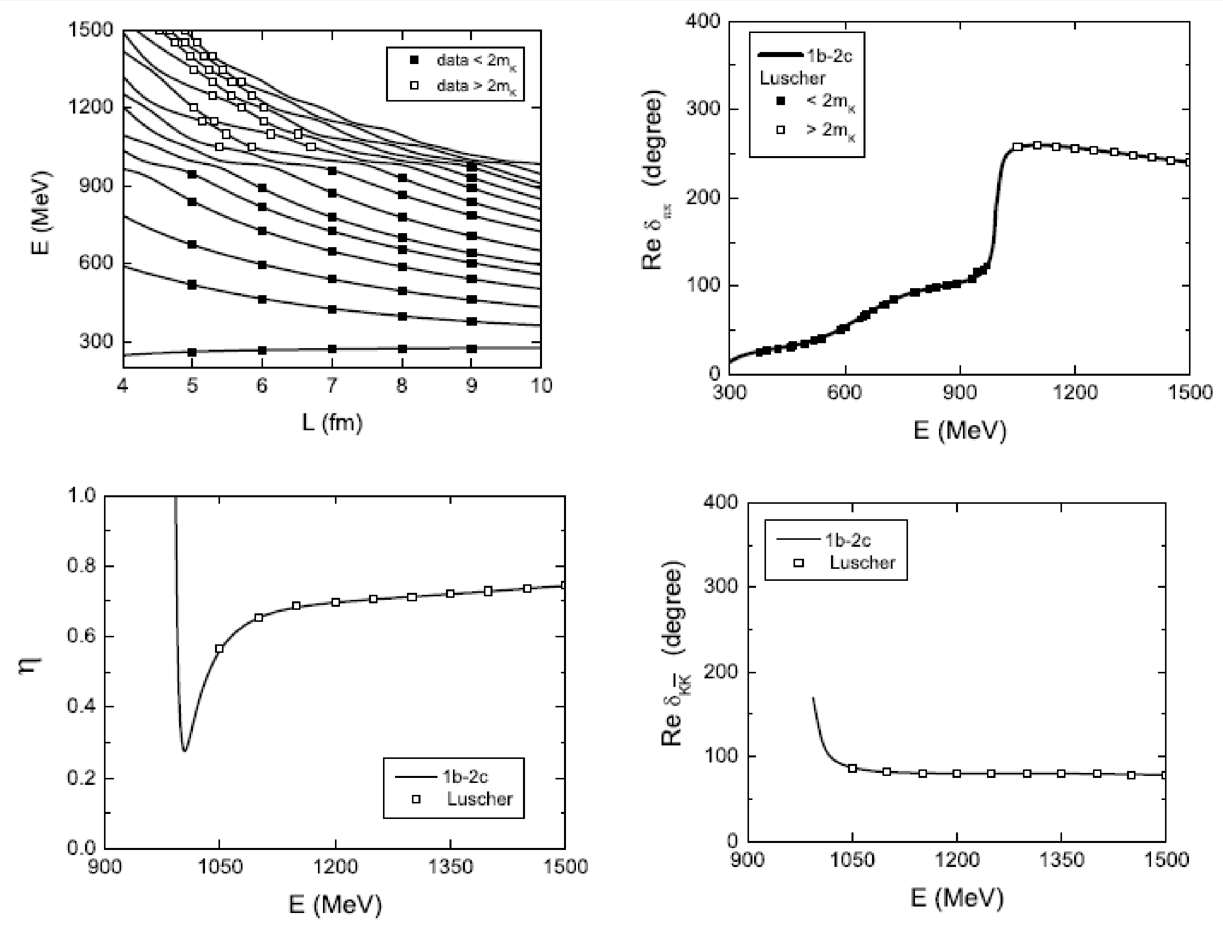
$\pi\pi \to \pi\pi$ & $\pi\pi \to \bar{K}K$ & $\bar{K}K \to \bar{K}K$



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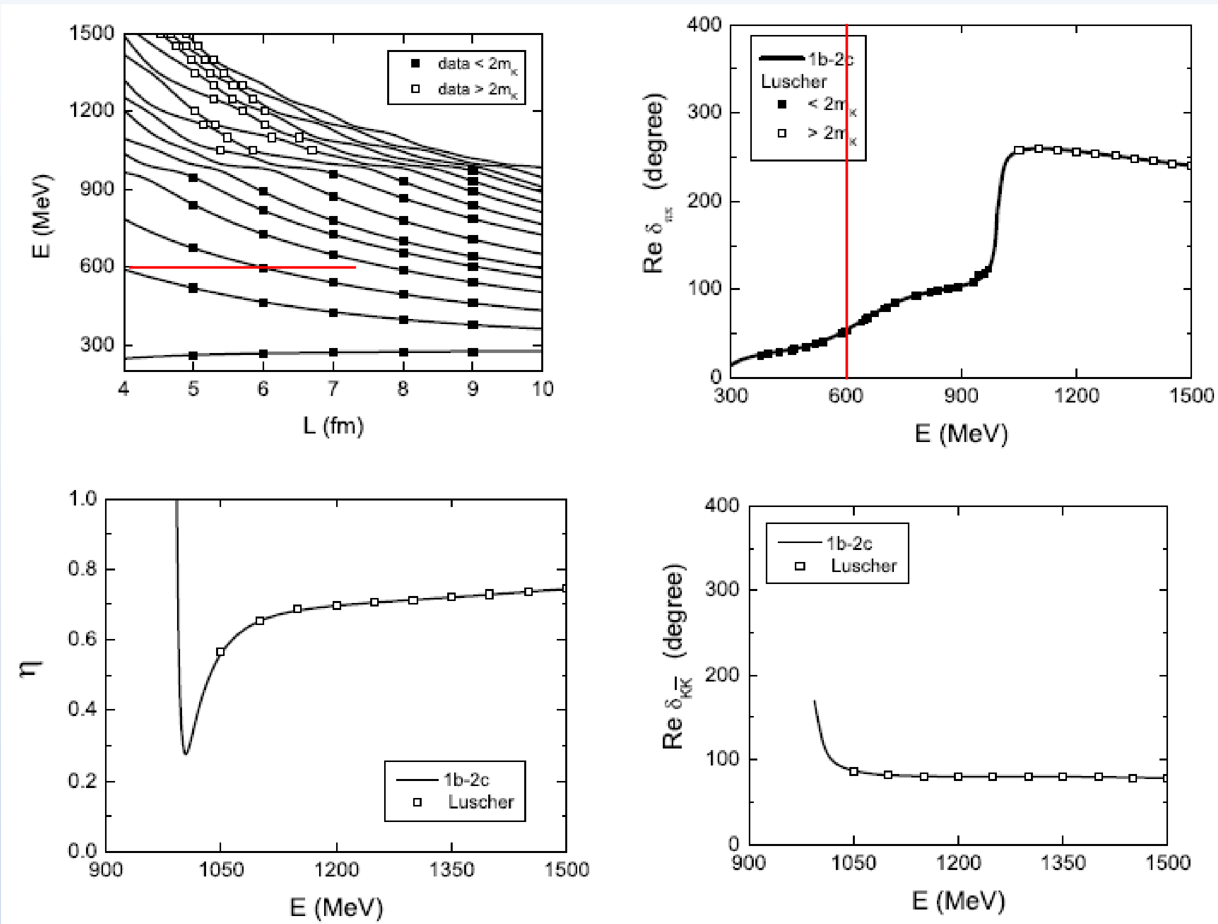


$\bar{K}K$ 阈下

L —— E —— $\delta_{\pi\pi}(E)$

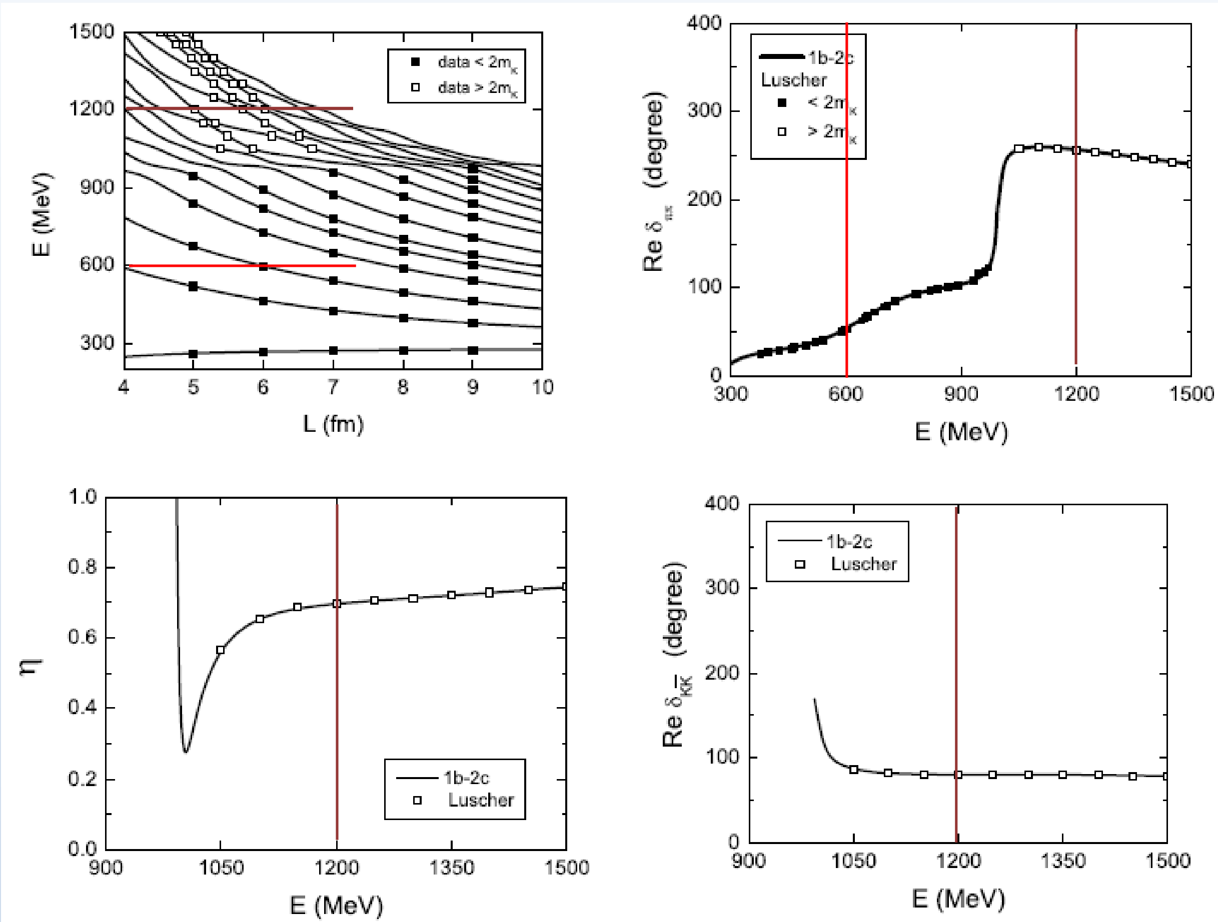
$$\delta_{\pi\pi}(k)=\Delta_{\pi\pi}(L) \bmod \pi$$

M. Luscher, NPB 354, 531 (1991).

$$\Delta_{\alpha}(L)=\tan^{-1}\left(\frac{q_{\alpha}\pi^{3/2}}{Z_{00}(1,q_{\alpha}^{2})}\right)$$

$$q_{\alpha}=\sqrt{\frac{E(L)^{2}}{4}-m_{\alpha}^{2}}$$

ππ –> ππ & ππ –> $\bar{K}$K & $\bar{K}$K–> $\bar{K}$K



$\bar{K}$K 阈下

L —— E —— $\delta_{\pi\pi}(E)$

$$\delta_{\pi\pi}(k)=\Delta_{\pi\pi}(L) \bmod \pi$$

M. Luscher, NPB 354, 531 (1991).

$\bar{K}$K 阈上

L_1, L_2, L_3 —— E

$\delta_{\pi\pi}(E), \quad \delta_{K\bar{K}}(E), \quad \eta(E)$

$$\Delta_\alpha(L)=\tan^{-1}\left(\frac{q_\alpha \pi^{3/2}}{Z_{00}(1,q_\alpha^2)}\right)$$

$$0= \cos\left(\Delta_{\pi\pi}(L)+\Delta_{K\bar{K}}(L)-\delta_{\pi\pi}(E)-\delta_{K\bar{K}}(E)\right)$$
$$-\eta\cos\left(\Delta_{\pi\pi}(L)-\Delta_{K\bar{K}}(L)-\delta_{\pi\pi}(E)+\delta_{K\bar{K}}(E)\right)$$

S.He,X.Feng,andC.Liu, JHEP 07(2005)011

$$q_\alpha=\sqrt{\frac{E(L)^2}{4}-m_\alpha^2}$$

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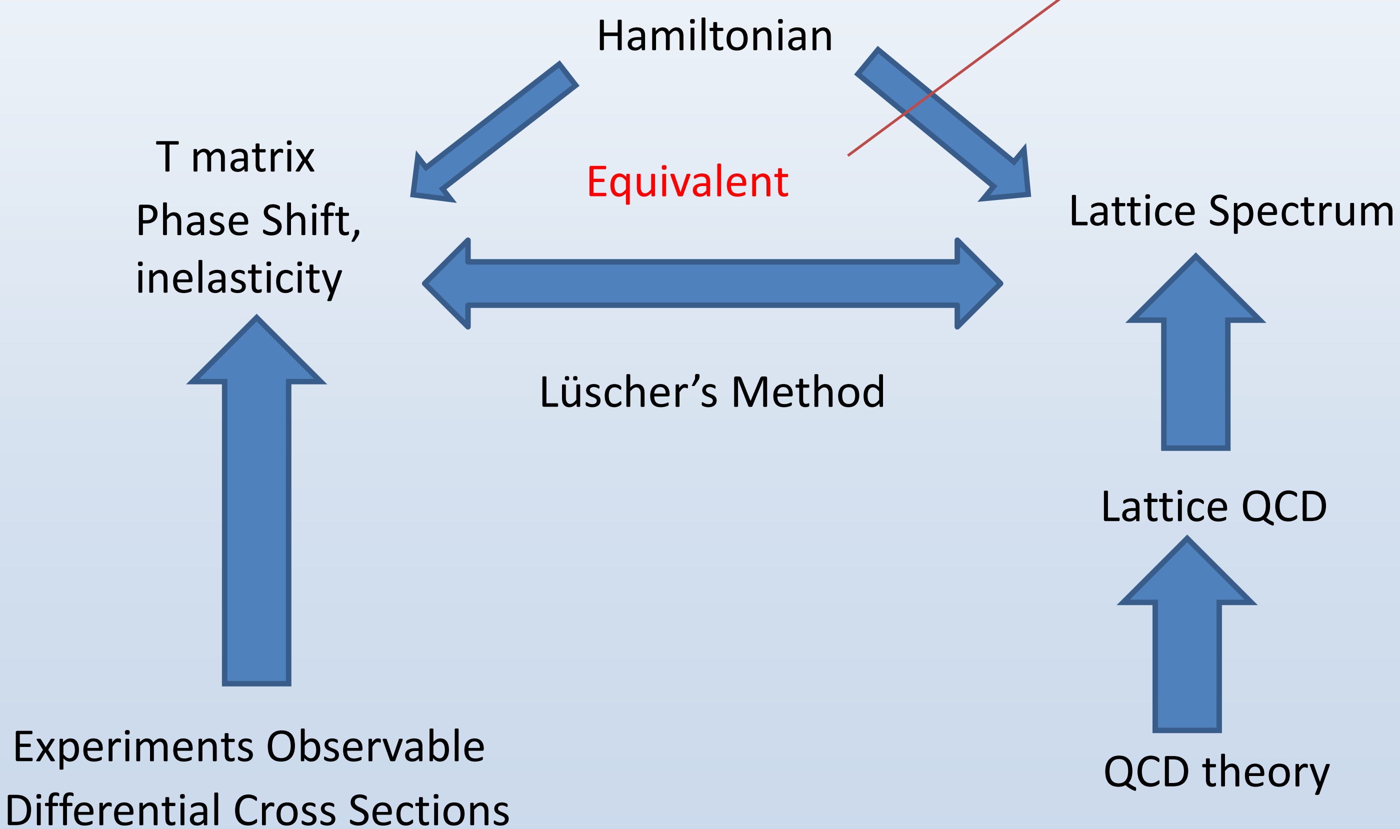
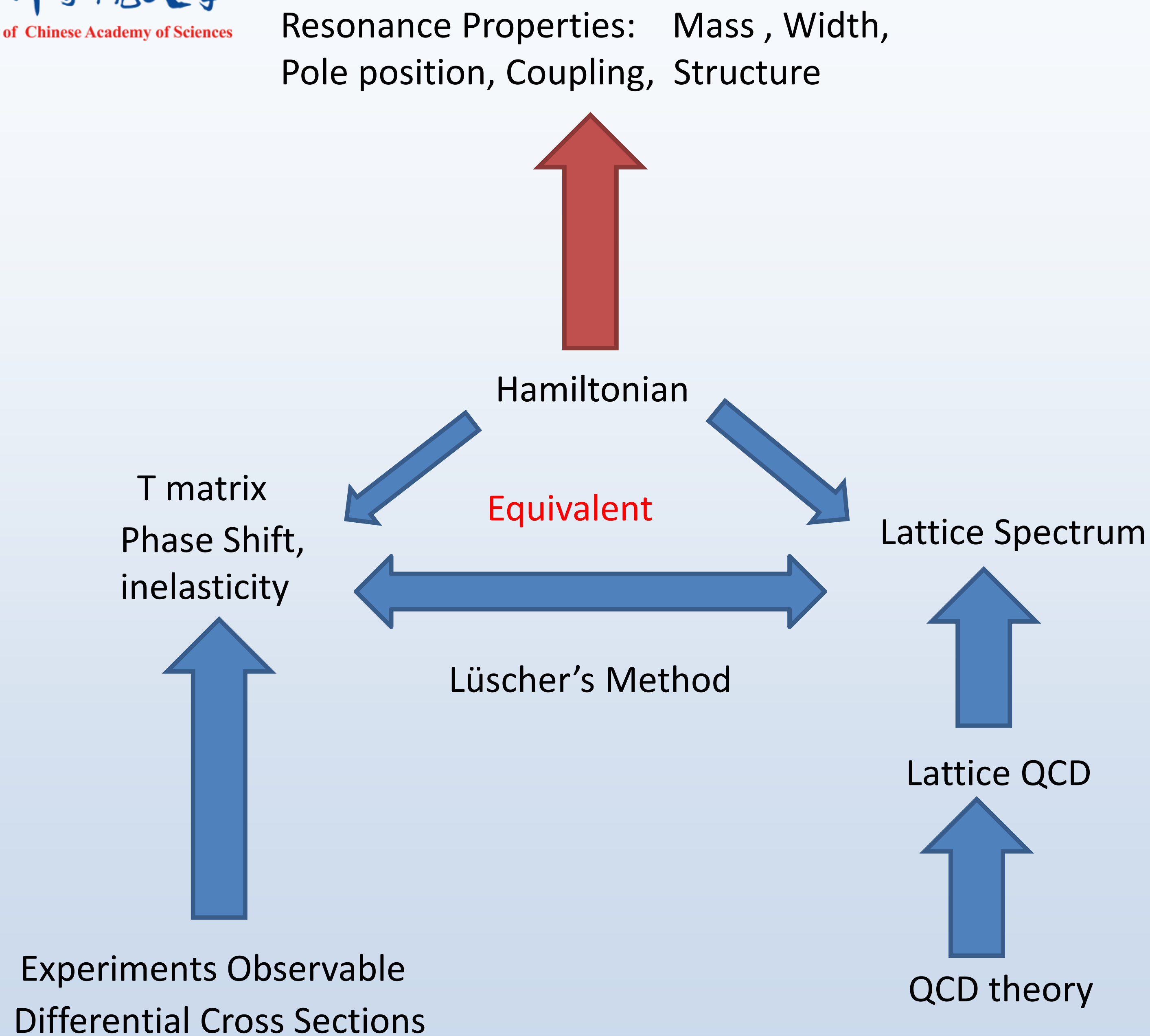
Resonance Properties: Mass , Width,
Pole position, Coupling, Structure

J. M. M. Hall etc. Phys. Rev. D 87, 094510
Jia-Jun Wu etc. Phys.Rev. C90 (2014) no.5, 055206
JPS Conf.Proc. 10 (2016) 062002




Resonance Properties: Mass , Width,
Pole position, Coupling, Structure
Hamiltonian
T matrix
Phase Shift,
inelasticity
Equivalent
Lattice Spectrum
Lüscher’s Method
Lattice QCD
Experiments Observable
Differential Cross Sections
QCD theory

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Hamiltonian EFT

Resonance Properties: Mass , Width,
Pole position, Coupling, structure

Hamiltonian

T matrix
Phase Shift,
inelasticity

Lattice Spectrum

Other Method

Resonance Properties: Mass , Width,
Pole position, Coupling, structure

Model (K matrix or others)

Lüscher Equation

T matrix
Phase Shift,
inelasticity

Lattice Spectrum

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Hamiltonian EFT

1. **Finite-volume matrix Hamiltonian model for a Δ→Nπ system**
   J.M.M. Hall, A.C.-P. Hsu, D.B. Leinweber, A.W.Thomas, R.D. Young
   Phys.Rev. D87 (2013) no.9, 094510
2. **Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD**
   Jia-Jun Wu, T.-S.H.Lee, A.W.Thomas, R.D. Young
   Phys.Rev. C90 (2014) no.5, 055206
3. **Hamiltonian effective field theory study of the N∗(1535) resonance in lattice QCD**
   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev.Lett. 116 (2016) no.8, 082004
4. **Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule**
   J.M.M. Hall, Waseem Kamleh, Derek B. Leinweber, Benjamin J. Menadue, Benjamin J. Owen, A.W.Thomas, R.D. Young
   Phys.Rev.Lett. 114 (2015) no.13, 132002
5. **Hamiltonian effective field theory study of the N∗(1440) resonance in lattice QCD**
   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.3, 034034
6. **Structure of the Λ(1405) from Hamiltonian effective field theory**
   Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.1, 014506
7. **Nucleon resonance structure in the finite volume of lattice QCD**
   Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas
   Phys.Rev. D95 (2017) no.11, 114507
8. **Structure of the Roper Resonance from Lattice QCD Constraints**
   Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W. Thomas
   Phys.Rev. D97 (2018) no.9, 094509

Hamiltonian EFT

Resonance Properties: Mass , Width,
Pole position, Coupling, structure

Hamiltonian

T matrix
Phase Shift,
inelasticity

Lattice Spectrum

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Hamiltonian EFT

$$H = H_0 + H_I$$

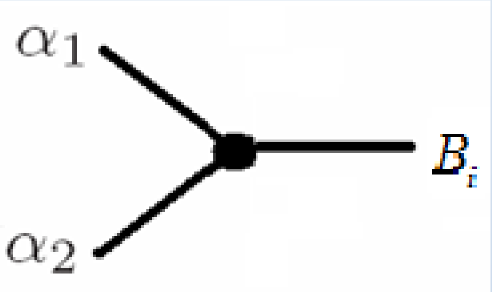
$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2} \right] \langle \alpha(k_\alpha)|$$

$|B_i\rangle$ bare state， bare mass m_i

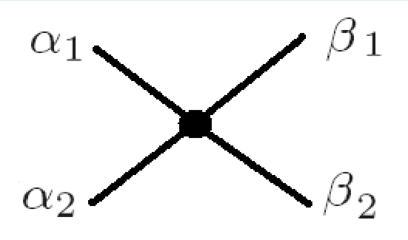
$|\alpha(k_\alpha)\rangle$ non-interaction channels

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$



$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



Hamiltonian EFT

Resonance Properties: Mass , Width,
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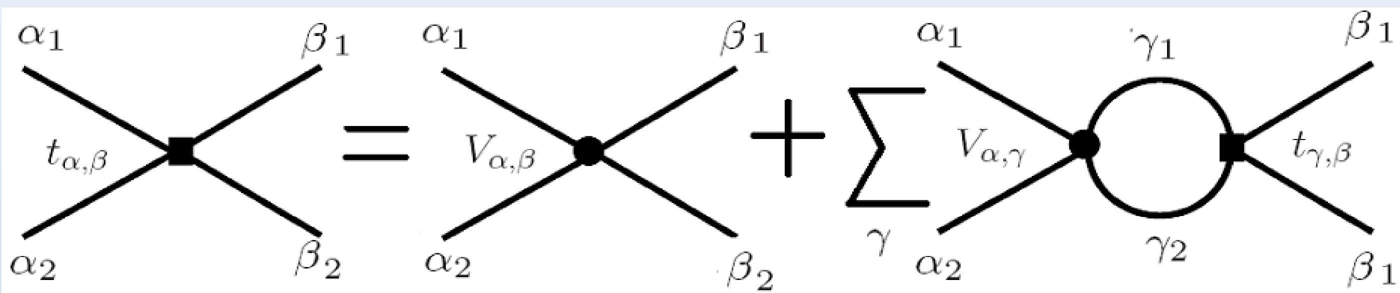
T matrix
Phase Shift,
inelasticity

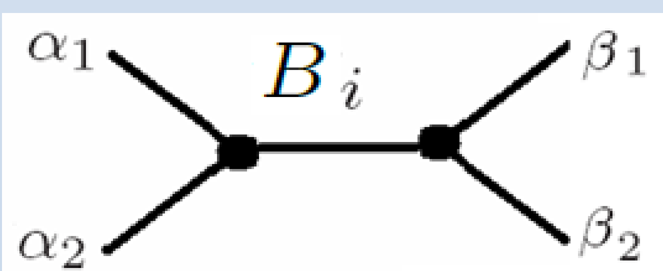
Lattice Spectrum

Connect to T and S matrix

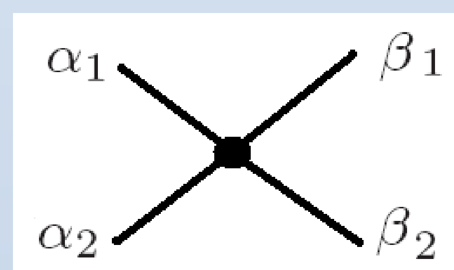
- T Matrix:

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\,\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$





$$g^*_{i,\alpha}\frac{1}{E-m_i}g_{i,\beta}$$



$$V_{\alpha,\beta}$$

- T Matrix:

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$

$$S_{\alpha,\beta}=1-i2\sqrt{\rho_\alpha}t_{\alpha,\beta}(k_{0\alpha},k_{0\beta},E)\sqrt{\rho_\beta}$$

$$\rho_\alpha=\frac{\pi k_{0\alpha}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha}=S_{\alpha,\alpha}$$

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Hamiltonian EFT

Resonance Properties: Mass , Width,
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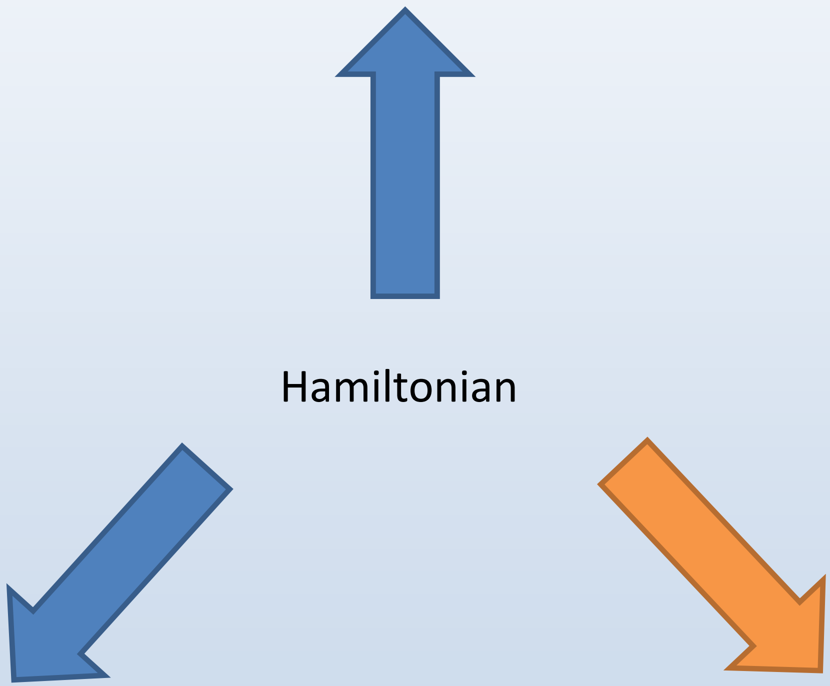


T matrix
Phase Shift,
inelasticity

Lattice Spectrum

Connect finite spectrum

- Hamiltonian with discrete momentum

Continuous: $\int \mathrm{d}\vec{k}$ and $\left|\alpha(\vec{k}_\alpha)\right\rangle$ and $\left\langle\beta(\vec{k}_\beta)\middle|\alpha(\vec{k}_\alpha)\right\rangle=\delta_{\alpha\beta}\delta(\vec{k}_\alpha-\vec{k}_\beta)$

$\downarrow$

Discrete: $\sum_{\mathrm{i}}\left(2\pi/\mathrm{L}\right)^3$ and $\left(2\pi/\mathrm{L}\right)^{-3/2}\left|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\right\rangle_\alpha$ and ${}_\beta\left\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}\middle|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\right\rangle_\alpha=\delta_{\alpha\beta}\delta_{\mathrm{ij}}$

$$H_0=\sum_{i=1,n}\left|B_i\right\rangle m_i\left\langle B_i\right|+\sum_{\alpha,\mathrm{i}}\left|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\right\rangle_\alpha\left[\sqrt{m_{\alpha_B}^2+k_\alpha^2}+\sqrt{m_{\alpha_M}^2+k_\alpha^2}\right]_\alpha\left\langle\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\right|$$

$$H_I=\sum_{\mathrm{j}}\left(2\pi/\mathrm{L}\right)^{3/2}\sum_\alpha\sum_{i=1,n}\left[\left|\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}\right\rangle_\alpha g_{i,\alpha}^{+}\left\langle B_i\right|+\left|B_i\right\rangle g_{i,\alpha}\,{}_\alpha\left\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}\right|\right]$$

$$+\sum_{\mathrm{i,j}}\left(2\pi/\mathrm{L}\right)^3\sum_{\alpha,\beta}\left|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\right\rangle_\alpha v_{\alpha,\beta}\,{}_\beta\left\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}\right|$$

- Hamiltonian Matrix for discrete momentum

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Eigen-Value $\longleftrightarrow$ Lattice Spectrum

$$(H_0 + H_I)\,|\Psi> = E\,|\Psi>$$

Eigen-Vector

- For example： πN scattering in the Δ resonance energy region

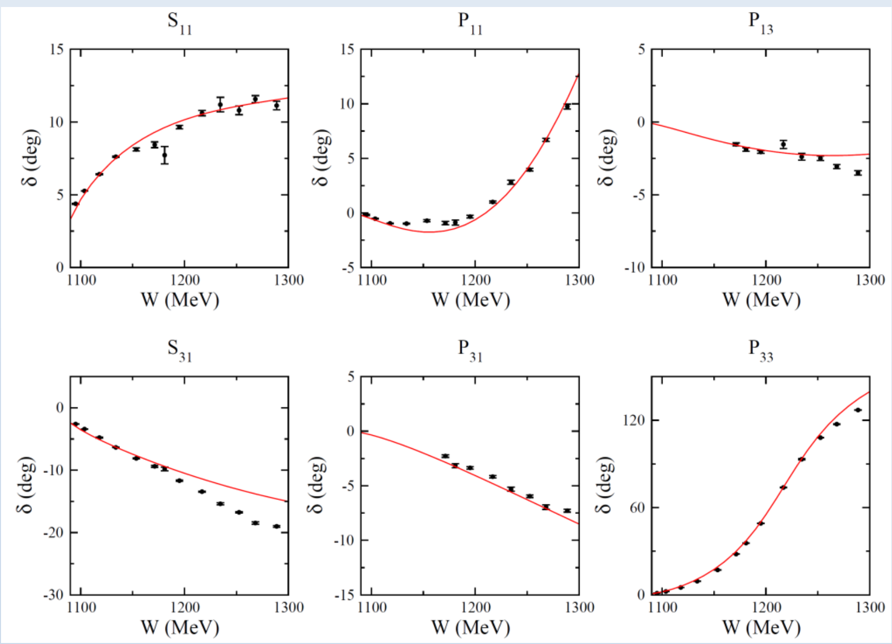
$$H_0=\begin{pmatrix} m_1 & 0 & 0 & \cdots \\ 0 & \sqrt{k_0^2+m_N^2}+\sqrt{k_0^2+m_\pi^2} & 0 & \cdots \\ 0 & 0 & \sqrt{k_1^2+m_N^2}+\sqrt{k_1^2+m_\pi^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

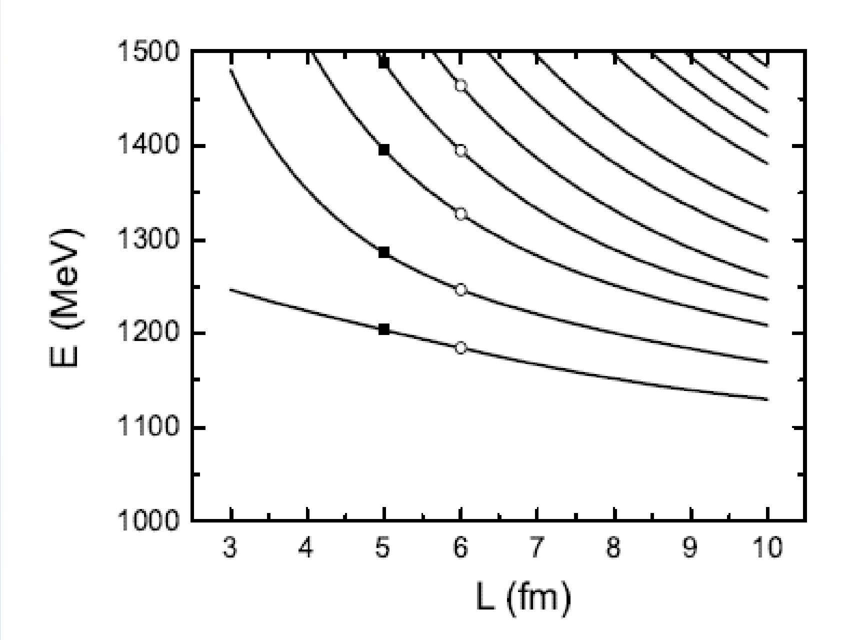
$$H_I=\begin{pmatrix} m_0 & g_{\pi N}^{fin}(k_0) & g_{\pi N}^{fin}(k_1) & \cdots \\ g_{\pi N}^{fin}(k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_1) & \cdots \\ g_{\pi N}^{fin}(k_1) & v_{\pi N,\pi N}^{fin}(k_1,k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$g_{\pi\pi}^{fin}(k_n)=\left(\frac{2\pi}{L}\right)^{\frac{3}{2}} g_{\pi N}(k_n) \quad v_{\pi\pi,\pi\pi}^{fin}(k_n,k_m)=\left(\frac{2\pi}{L}\right)^{3} v_{\pi N,\pi N}(k_n,k_m)$$

$$g_{\pi N}(k_n) \;\; v_{\pi N,\pi N}(k_n,k_m)$$

From Sato-Lee model





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Why equivalent with Lüscher Equation ?

$$k_{on}cot(\delta_E) = \frac{4}{\pi E}\frac{1}{g^2_{\pi N}(k_{on})}\left[\mathrm{P}\int k^2 dk - \left(\frac{2\pi}{L}\right)^3 \frac{1}{4\pi}\sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}}\right]\frac{g^2_{\pi N}(k_n)}{E_i - \sqrt{m_N^2+k_n^2} - \sqrt{m_\pi^2+k_n^2}}.$$

$$= \frac{2}{\pi}\left[\mathrm{P}\int k^2 dk - \left(\frac{2\pi}{L}\right)^3 \frac{1}{4\pi}\sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}}\right]\frac{1}{k_n^2 - k_{on}^2} \longrightarrow \frac{2}{\sqrt{\pi}L}Z_{00}(1;(\frac{kL}{2\pi})^2)$$

Zeta Function as for Luscher

$$+\frac{4}{\pi E}\left[\mathrm{P}\int k^2 dk - \left(\frac{2\pi}{L}\right)^3 \frac{1}{4\pi}\sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}}\right]\times$$

$$\left[\frac{g^2_{\pi N}(k_n)/g^2_{\pi N}(k_{on}) - 1}{E_i - \sqrt{m_N^2+k_n^2} - \sqrt{m_\pi^2+k_n^2}} + \dots\right] \longrightarrow$$

No Singularity

Would be suppressed by e^{-Lm}

HEFT

Resonance Properties: Mass , Width, Pole position, Coupling

Complex Continuous
Momentum Space

Hamiltonian

Real Continuous
Momentum Space

Real Discrete
Momentum Space

T matrix
Phase Shift,
inelasticity

Lattice Spectrum

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What are the eigenstates in the FV ?

$$(H_0+H_I)\,|\Psi> = E\,|\Psi>$$

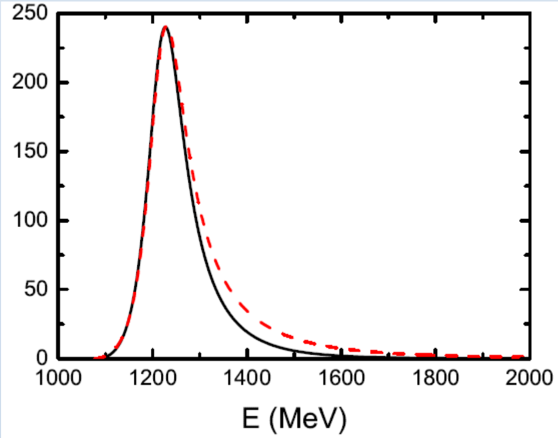
$$|\Psi_E\rangle = C_0|B> + \sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}} C_E(\vec{k}_n)|\alpha(\vec{k}_n)\rangle$$

What is the relationship between these eigenstates and resonance ?

Infinite-Volume

The eigenstate of Hamiltonian is final scattering state, which are continuum,

$$|\langle B|\Psi^{(+)}_{E,i}\rangle|^2 = \frac{\bar{\Gamma}_{\pi N}(k_{\pi N};E)}{E-m_0-\Sigma(E)}$$



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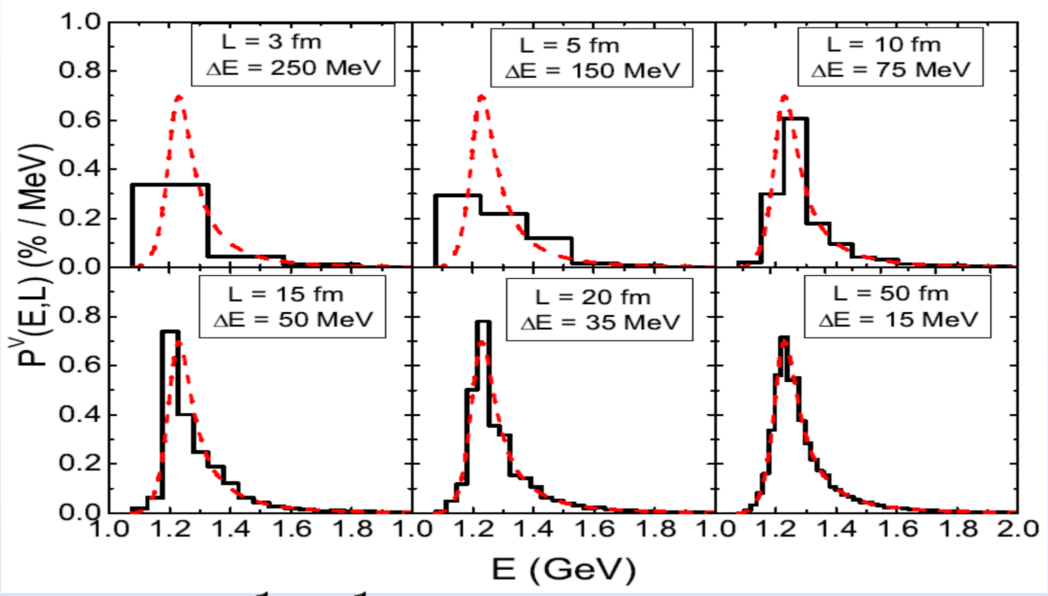
Eigenstate VS Resonance

Sato-Lee Model for $\Delta(1232)$ region

Jia-jun Wu etc.
Phys.Rev. D95 (2017) no.11, 114507



Finite-Volume
Black Solid line

$$P^V(E_k^{ave}, L) = \frac{1}{Z^V}\frac{1}{\Delta E}\sum_{E_k^{ave}-\frac{\Delta E}{2}\le E_\alpha \le E_k^{ave}+\frac{\Delta E}{2}} |\langle B|\Psi^V_{E_\alpha}\rangle|^2$$

Infinite-Volume
Red dashed line

$$P(E) = \frac{1}{Z}\sum_{i=1,n_c} \pi k_i E_{M_i}(k_i) E_{B_i}(k_i) |\langle B|\Psi^{(+)}_{E,i}\rangle|^2$$

LQCD -> N* resonance -> coupled-channel data

Since $P^V(E)$-> P(E) as volume size increase, P(E) which can be readily calculated using ANL-Osaka Hamiltonian can already bridge

Structure of Baryon Resonance

- 3 quark
- 5 quark
- Meson Cloud + 3 quark core
- Meson-Baryon Molecule
- Dynamical generated state

… …

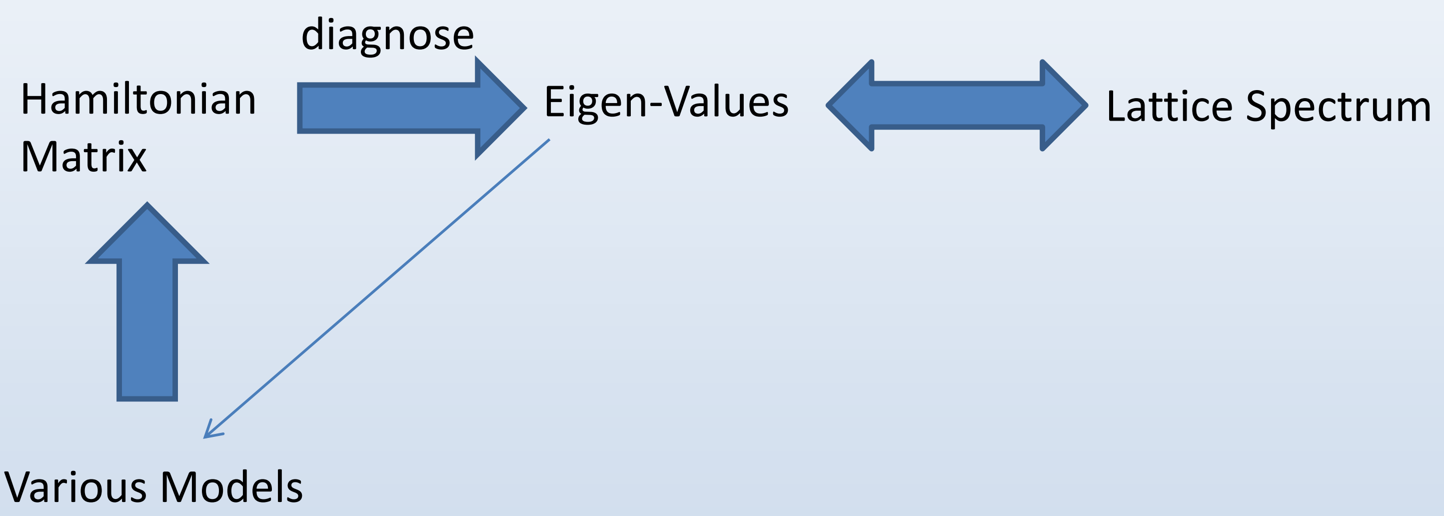
Question: How to distinguish them ???

Fitting from the limited data from Experiments and Lattice.

Model with various parameters are always powerful !!!

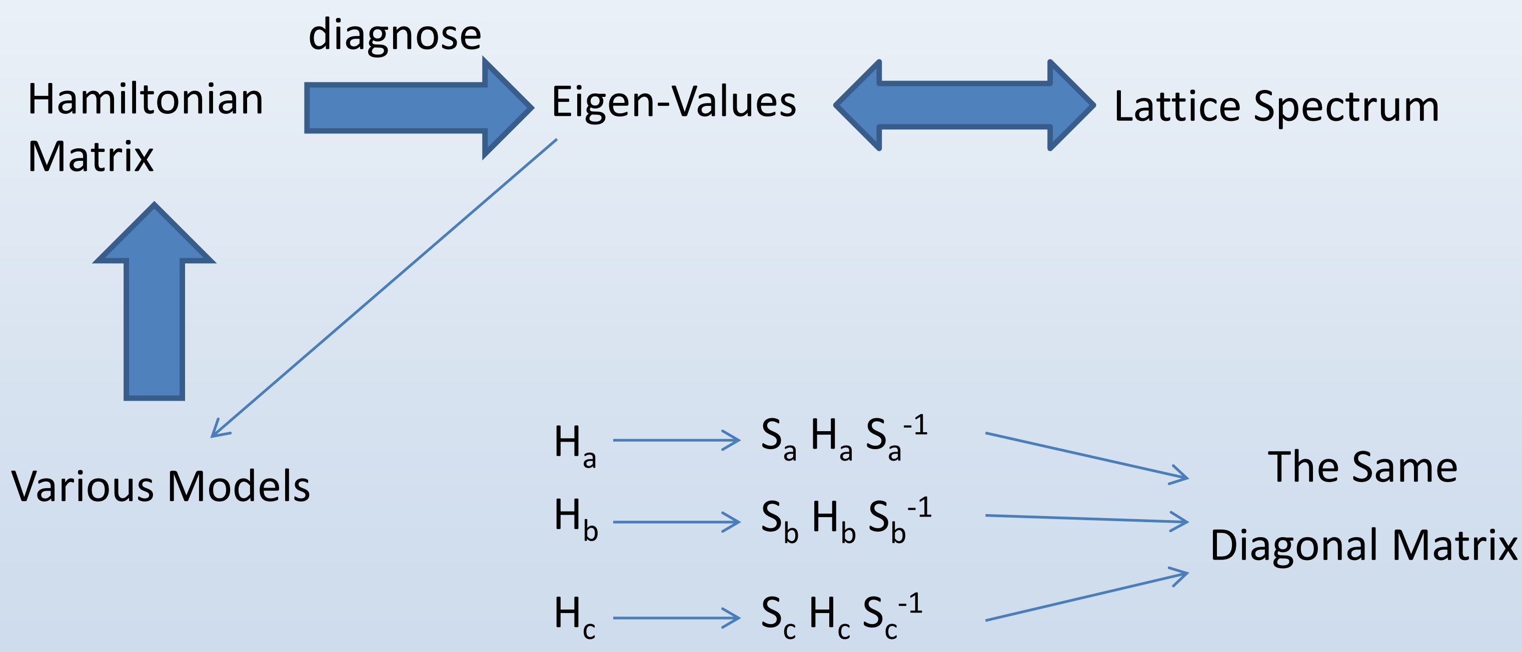
How to distinguish Models？？？

- From our Finite-Volume Hamiltonian matrix:



How to distinguish Models？？？

- From our Finite-Volume Hamiltonian matrix:



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How to distinguish Models？？？

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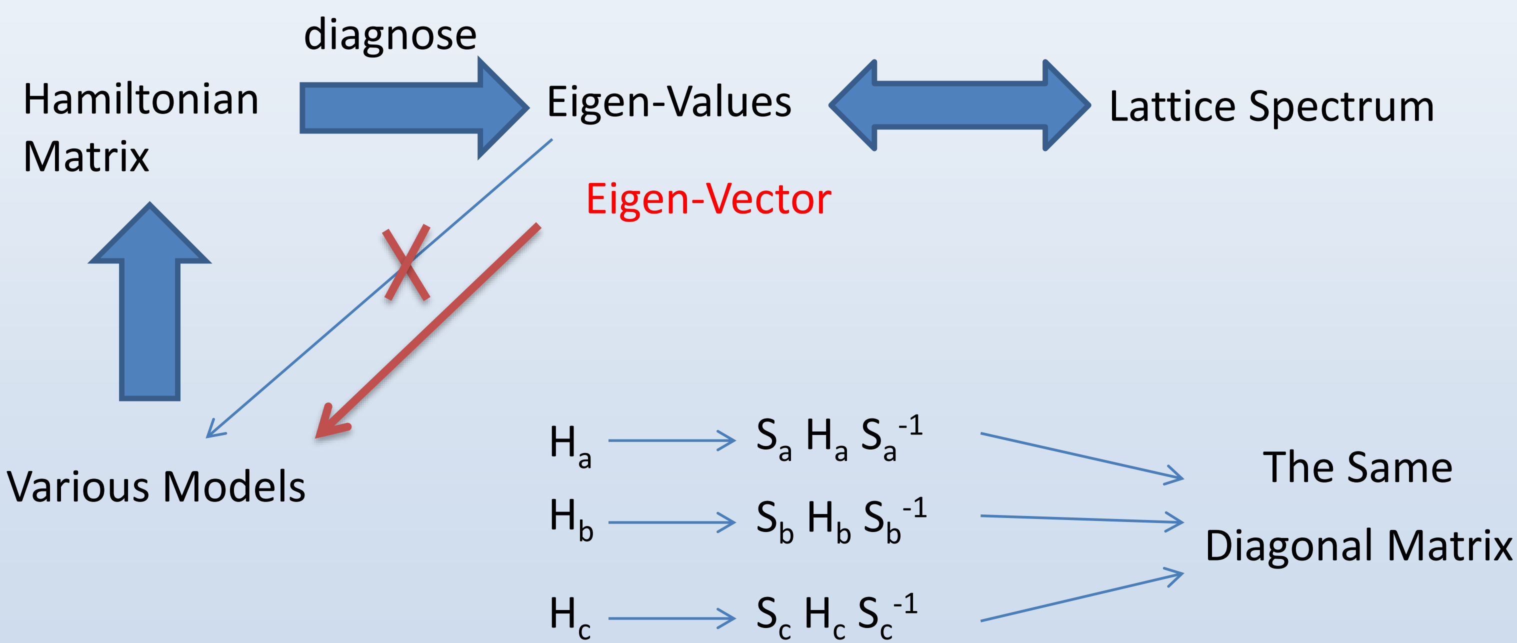


How to distinguish Models？？？

- From our Finite-Volume Hamiltonian matrix:

Hamiltonian Matrix $\xrightarrow{\text{diagnose}}$ Eigen-Values $\Longleftrightarrow$ Lattice Spectrum

Eigen-Vector

Various Models

$$\left\langle \alpha \middle| \hat{O} \middle| \alpha \right\rangle \quad \text{VS} \quad \left\langle \Psi \middle| \hat{O} \middle| \Psi \right\rangle$$

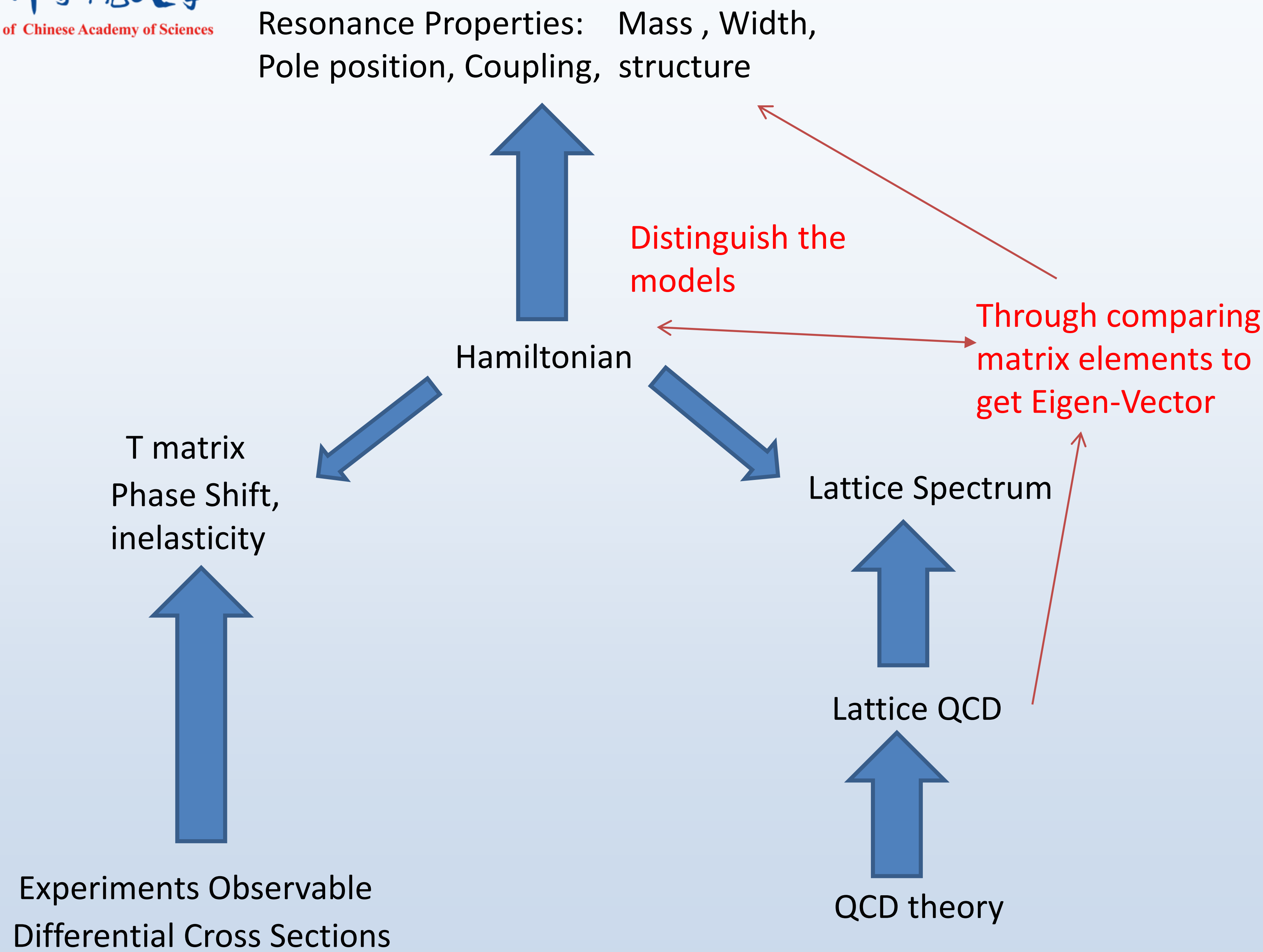
Theory or Model　　Lattice

However, the Eigen-Vector can not be measured from experiment or lattice. Actually, the eigenvector is the interpretation of the eigenstate, which is based on the knowledge of ourselves.

In the HEFT, we take bare state and non-interaction hadrons states as the basic states. It means we fix a frame here. Then the eigenvector can be calculated by following ways.

|α>: bare state or non-interaction states

|Ψ>: eigenstate of interaction Hamiltonian


Resonance Properties: Mass , Width,
Pole position, Coupling, structure
Distinguish the models
Through comparing matrix elements to get Eigen-Vector
Hamiltonian
T matrix
Phase Shift, inelasticity
Lattice Spectrum
Lattice QCD
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Unfortunately, current Lattice techniques can not measure all Eigen-states and Eigen-vectors for Baryon Resonances.
At present, for the Baryon spectrum, pure 3 quark local operators are mostly used, while rarely some calculations use Meson-Baryon non-local operators.

C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

For the 3 quark operators:

- Overlap with the meson baryon scattering states is suppressed by factor of thousand relative to the nucleon.
- Therefore, the 3 quark operator excites the bare state of the Hamiltonian model.
- Thus, the Eigen-states seen on the lattice should have large bare state components.

HEFT

1. Fitting Experimental Data to fix the parameters.
2. Using these fitted parameters to generate the Finite-Volume Hamiltonian matrix. And for high pion mass, another parameter for the mass slope is needed.
3. Calculate the eigenvalue and eigenvector of this matrix to compare with Lattice data.

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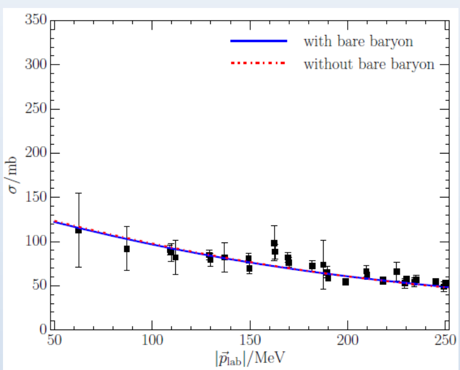
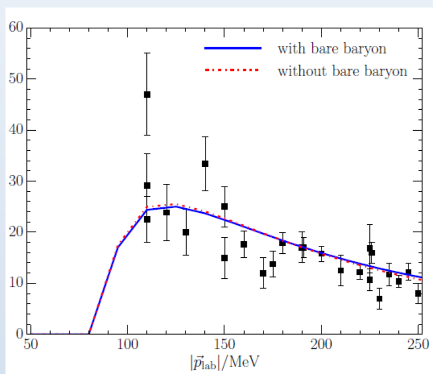
$\Lambda^*(1405)$

Zhan-wei Liu etc. Phys.Rev. D95 (2017) no.1, 014506

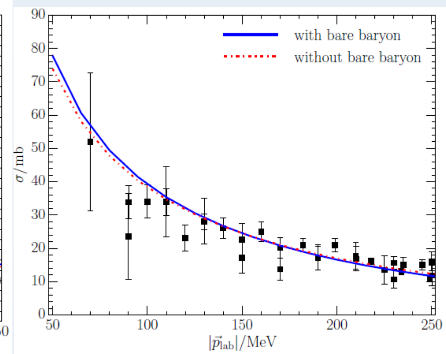
- I=0, $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$ and $\bar{K}\Xi$
- I=1, $\pi\Sigma$, $\bar{K}N$, $\pi\Lambda$



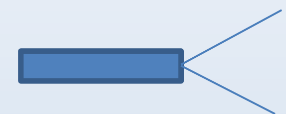
(a) $K^-p \to K^-p$



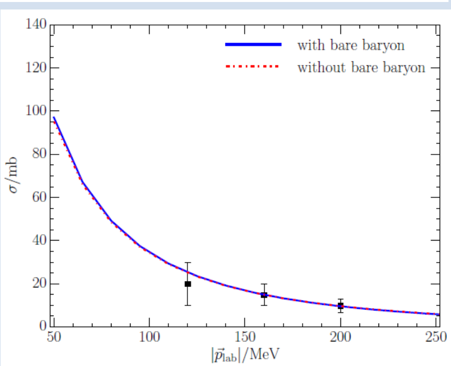
(b) $K^-p \to \bar{K}^0 n$



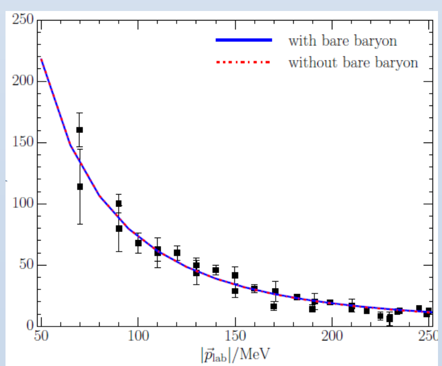
(c) $K^-p \to \pi^-\Sigma^+$



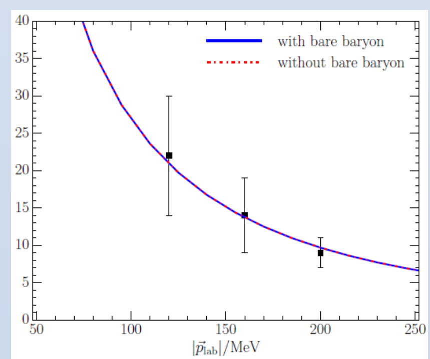
$$\frac{\sqrt{3}g^I_{\alpha,B_0}}{2\pi f}\sqrt{\omega_\pi(k)}u(k)$$



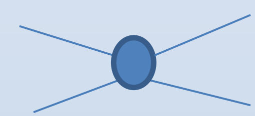
(d) $K^-p \to \pi^0\Sigma^0$



(e) $K^-p \to \pi^+\Sigma^-$



(f) $K^-p \to \pi^0\Lambda$

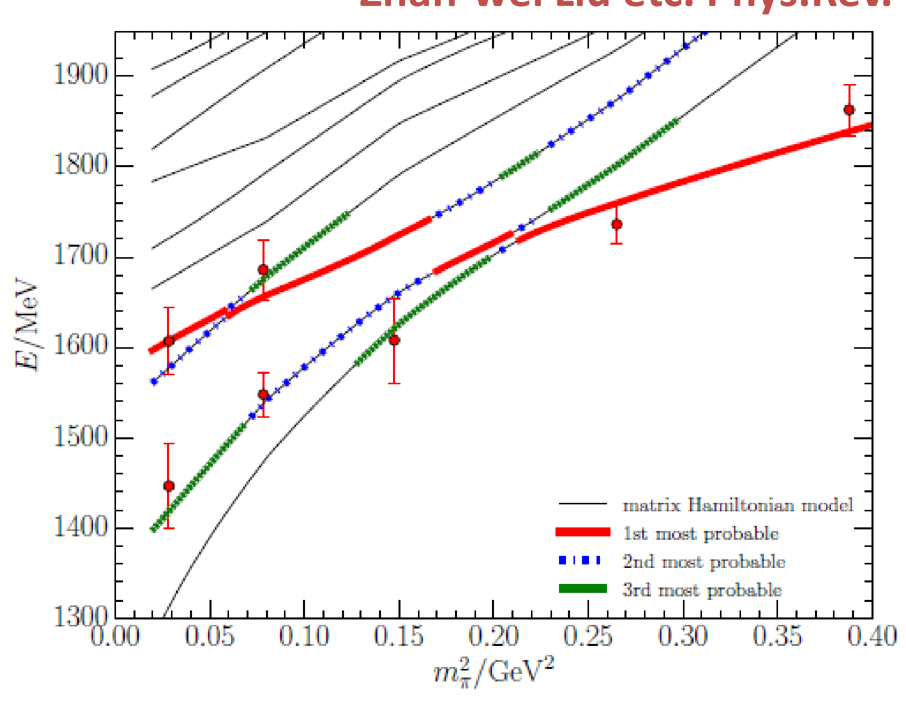


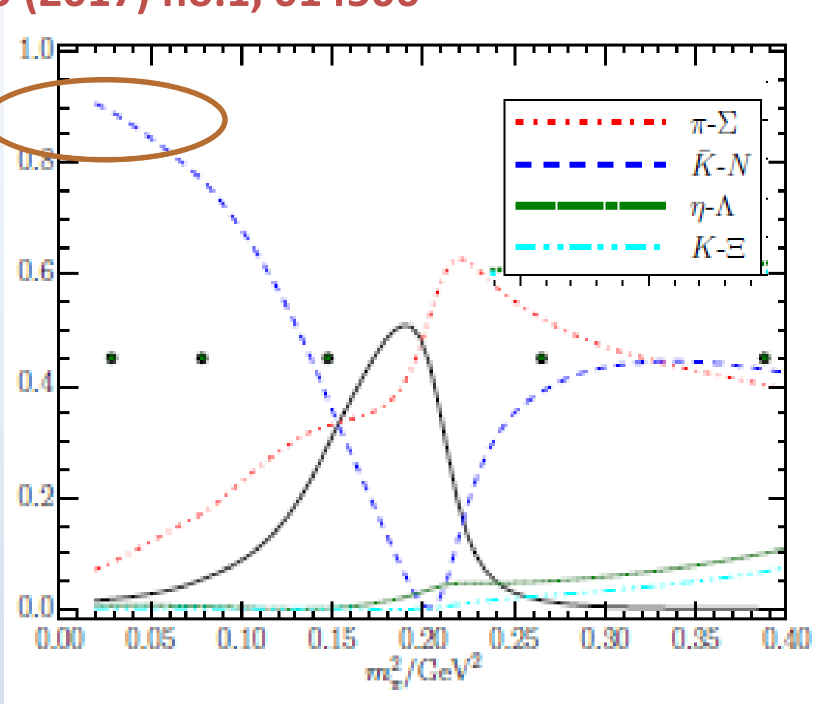
$$g^I_{\alpha,\beta}\frac{[\omega_{\alpha_M}(k)+\omega_{\beta_M}(k')]\,u(k)u(k')}{8\pi^2 f^2\sqrt{2\omega_{\alpha_M}(k)}\sqrt{2\omega_{\beta_M}(k')}}$$

Weinberg – Tomozawa Term

$\Lambda^*(1405)$

Zhan-wei Liu etc. Phys.Rev. D95 (2017) no.1, 014506





the $\Lambda^*(1405)$ is predominantly a

molecular $\bar{\mathrm{K}}$ N bound State,

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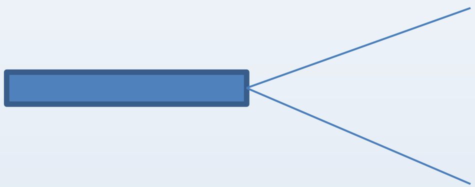
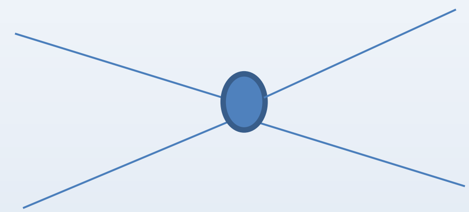
N*(1535)

Zhan-wei Liu etc. Phys.Rev.Lett. 116 (2016) no.8, 082004

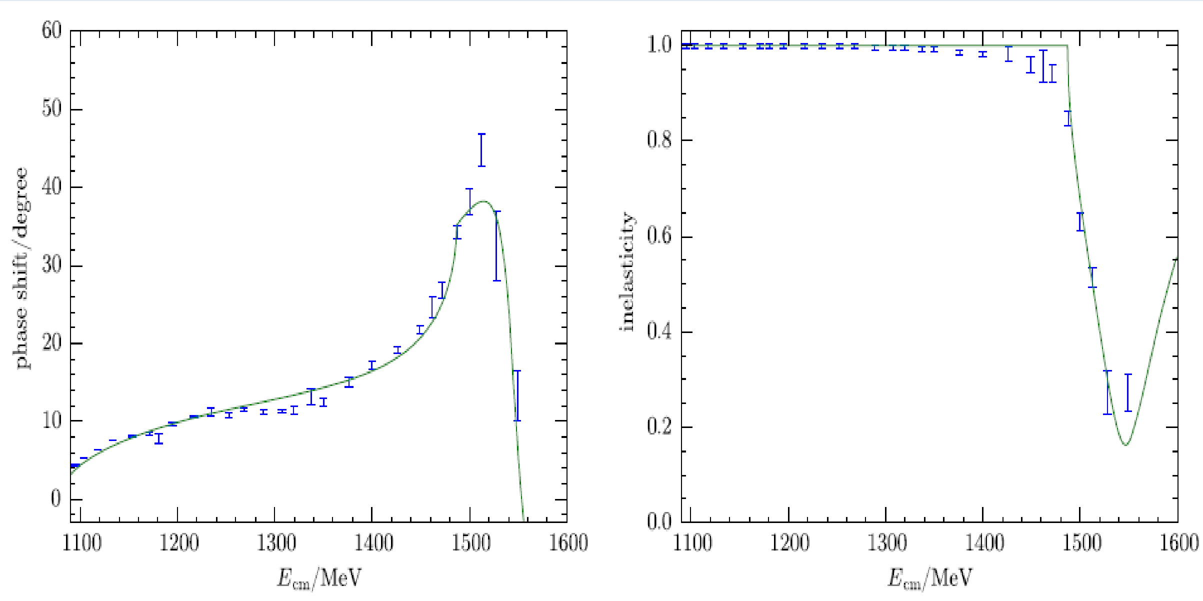
- 2 Channels: πN and ηN





$$G^2_{iN}(k) = \left(3g^2_{N_0^* iN}/4\pi^2 f^2\right)\omega_i(k)u^2(k)$$

$$\frac{3g^S_{\pi N}\tilde{u}(k)\tilde{u}(k')}{4\pi^2 f^2}$$



$$g^S_{\pi N} = -0.0608 \pm 0.0004$$
$$m_0 = 1601 \pm 14\ \text{MeV}$$
$$g_{N_0^*\pi N} = 0.186 \pm 0.006$$
$$g_{N_0^*\eta N} = 0.185 \pm 0.017$$

$$\chi^2_{\rm DOF} = 6.8$$

$$1531 \pm 29 - i88 \pm 2\ \text{MeV}$$

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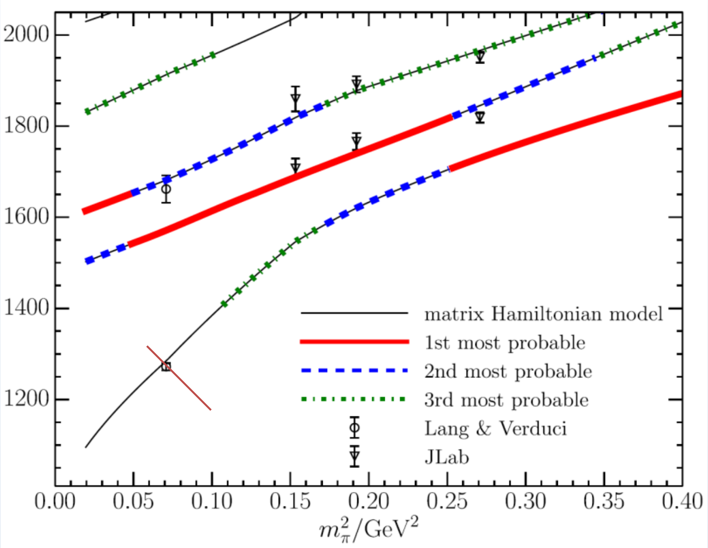
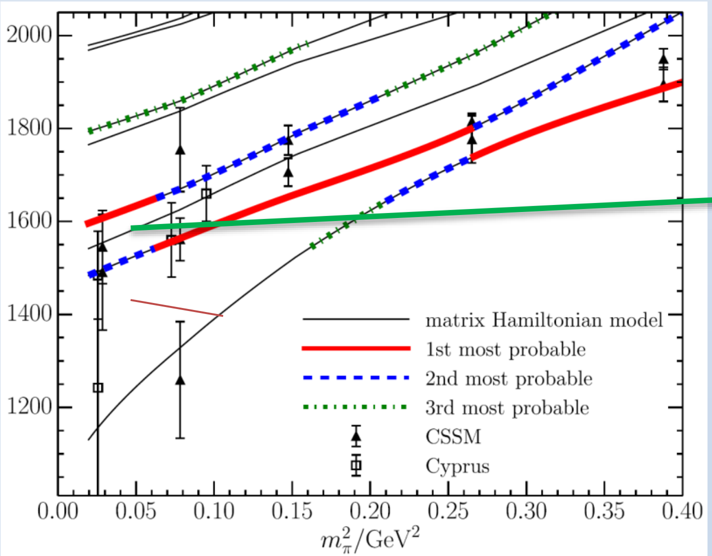
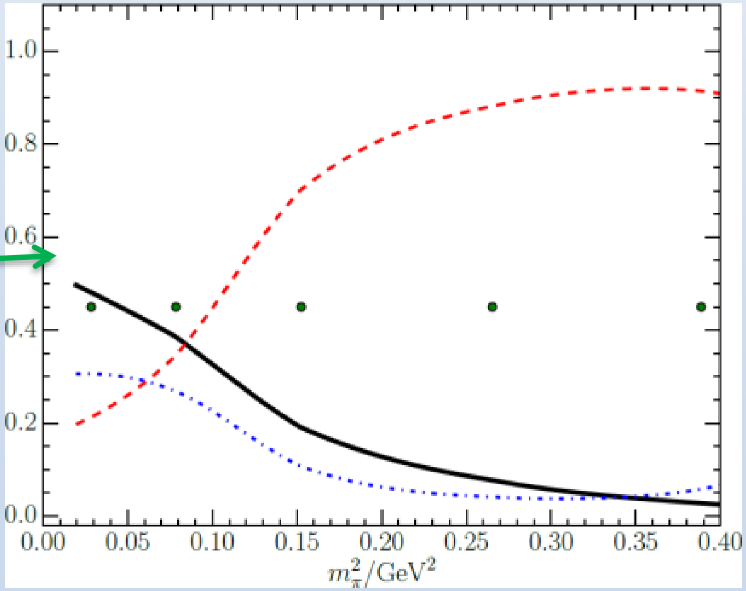
N*(1535)

Zhan-wei Liu etc. Phys.Rev.Lett. 116 (2016) no.8, 082004



The main components (at least 50%) of N*(1535) is from the 3 quark core.





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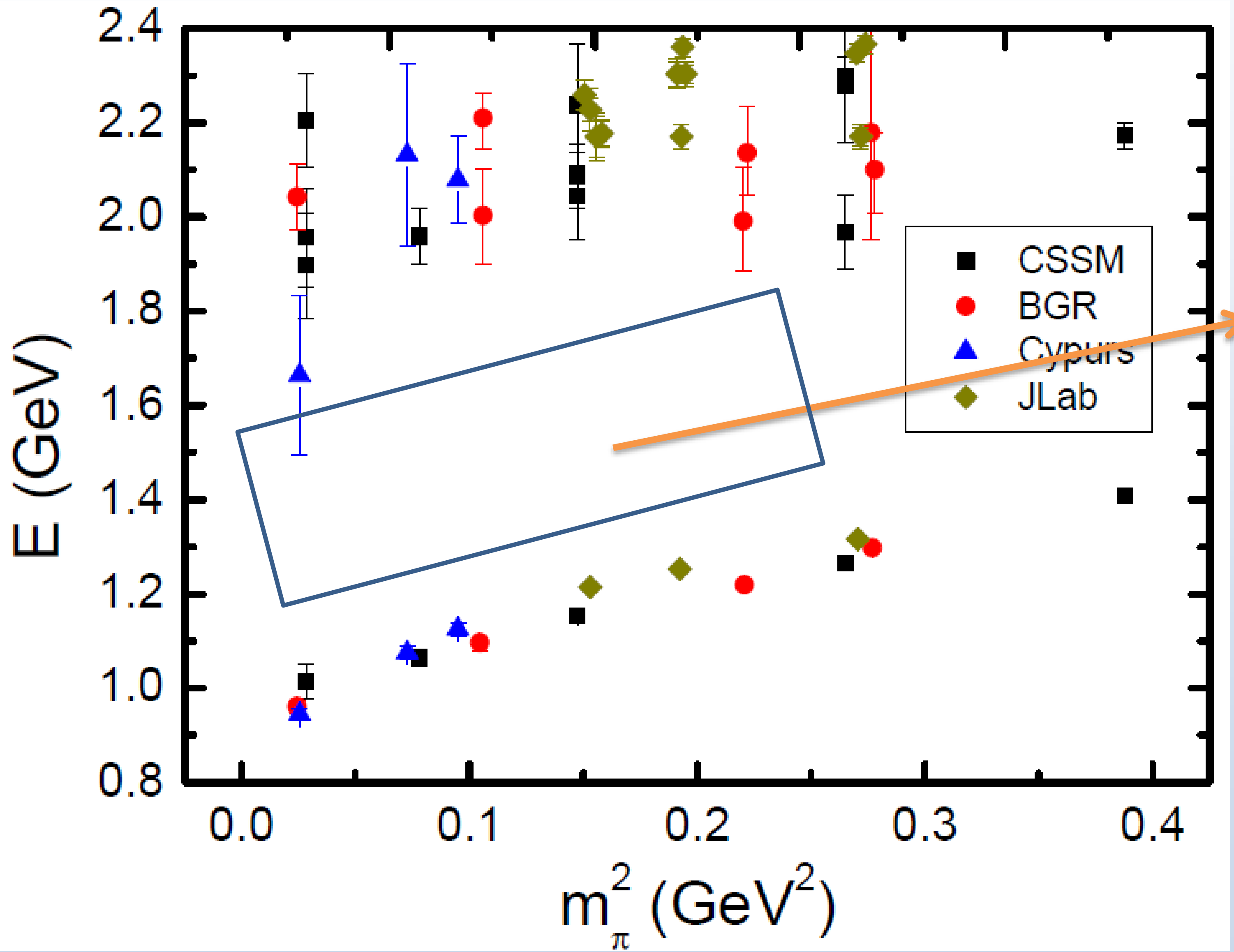
N*(1440)



No State extracted from 3 quark operator

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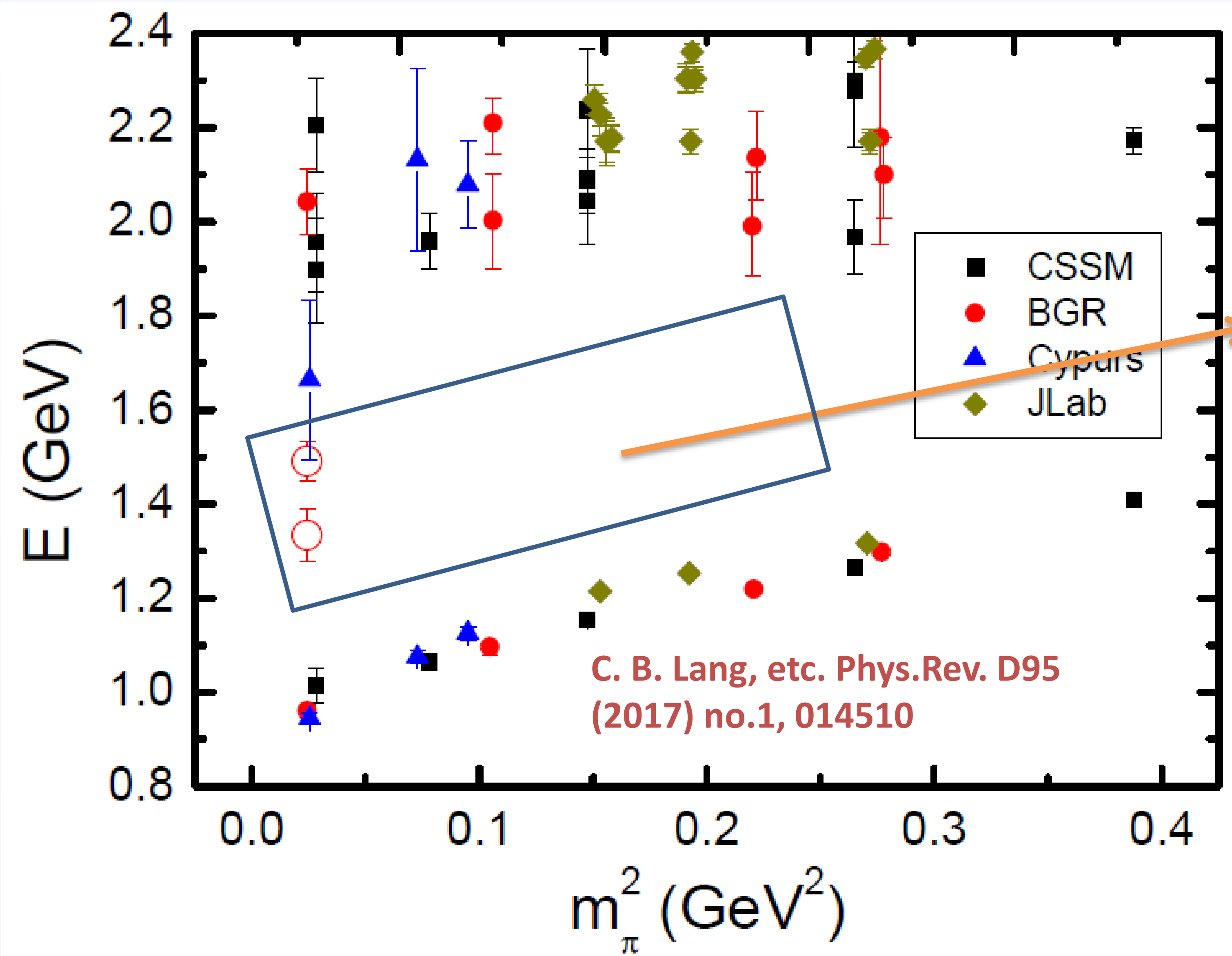
N*(1440)



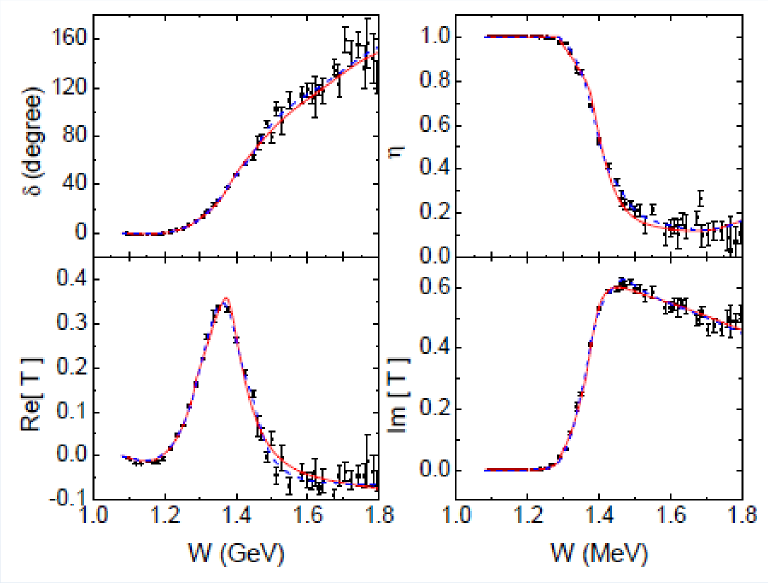
No State extracted from 3 quark operator

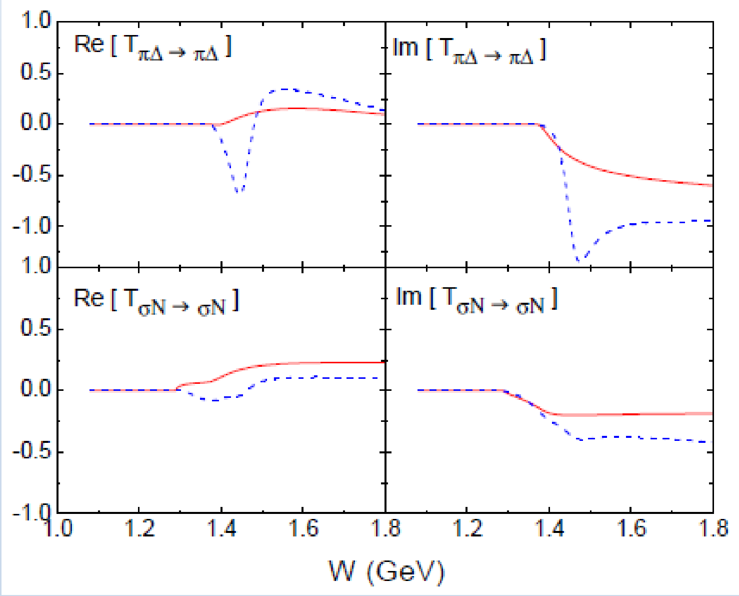
N*(1440)

Jia-jun Wu etc. arXiv: 1703.10715

Include 3 channels: πN, $\pi\Delta$ and σN

Parameter	I	II
$g^S_{\pi N,\pi N}$	1.156	0.634
$g^S_{\pi N,\pi\Delta}$	−0.662	−0.378
$g^S_{\pi N,\sigma N}$	−0.415	−1.738
$g^S_{\pi\Delta,\pi\Delta}$	−0.438	−0.581
$g^S_{\pi\Delta,\sigma N}$	1.332	0.964
$g^S_{\sigma N,\sigma N}$	10.000	10.000
m^0_B/GeV	2.000	1.7000
$g_{B_0\pi N}$	0.268	0.954
$g_{B_0\pi\Delta}$	1.544	−0.118
$g_{B_0\sigma N}$	—	−2.892
$\Lambda_{\pi N}$/GeV	0.5953	0.6302
$\Lambda_{\pi\Delta}$/GeV	1.5000	1.4318
$\Lambda_{\sigma N}$/GeV	1.5000	1.4533
Pole (MeV) (uuu)	$2012.28 - 42.09\,i$	$1355.57 - 70.81\,i$
Pole (MeV) (upu)	$1392.92 - 167.13\,i$	$1362.33 - 100.53\,i$





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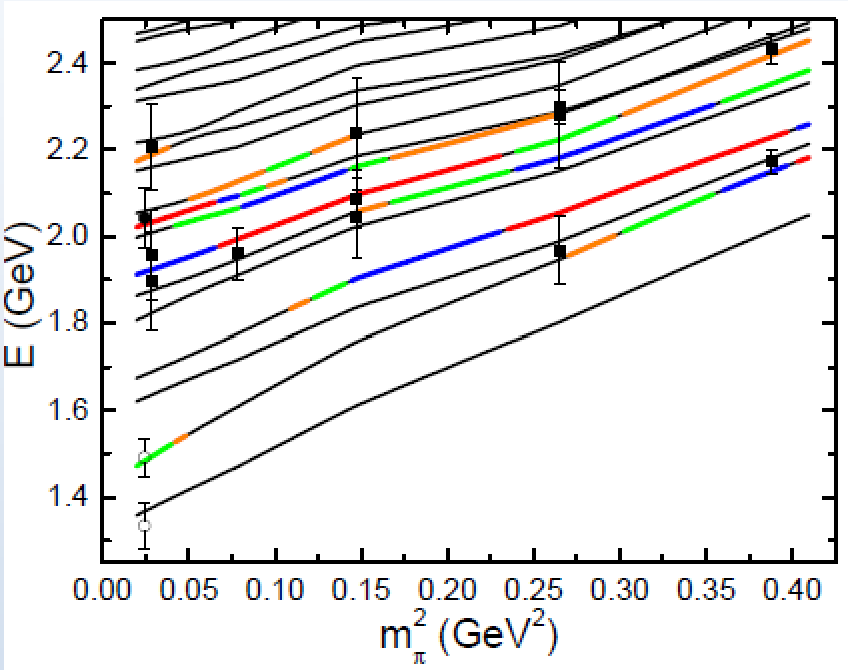
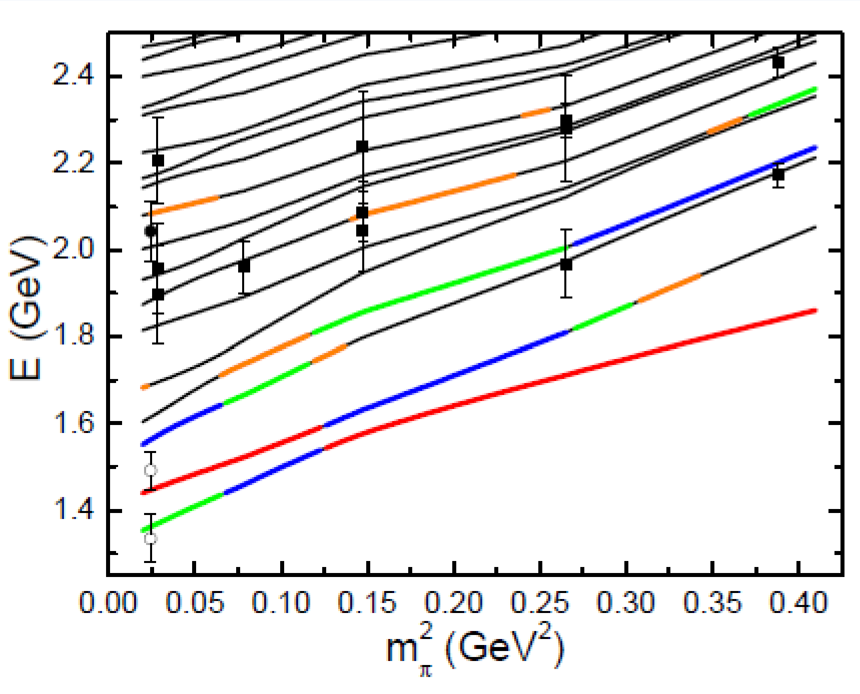
N*(1440)

Jia-jun Wu etc. arXiv: 1703.10715

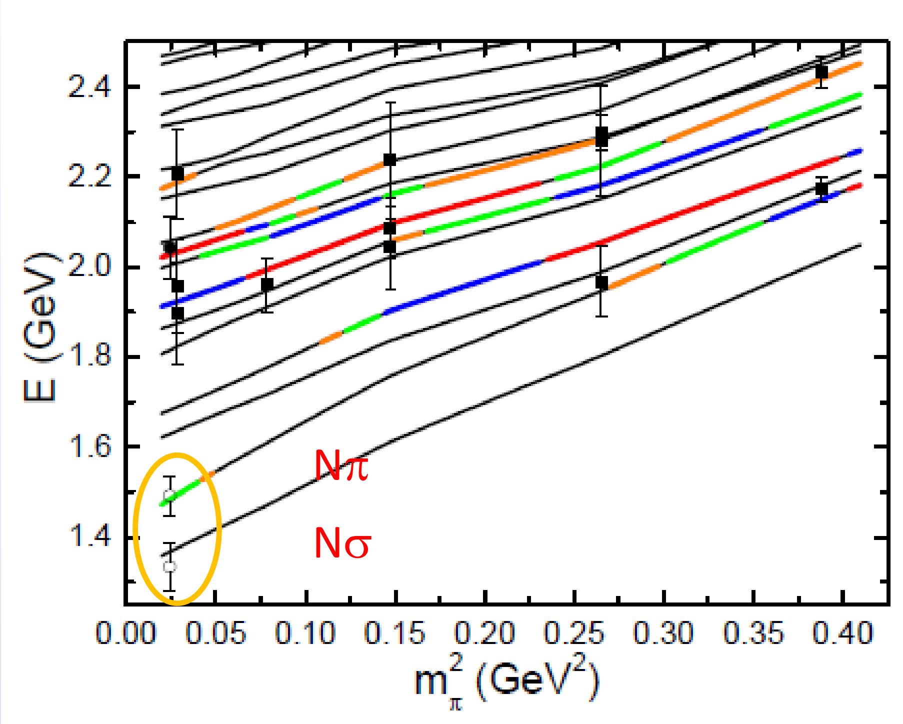


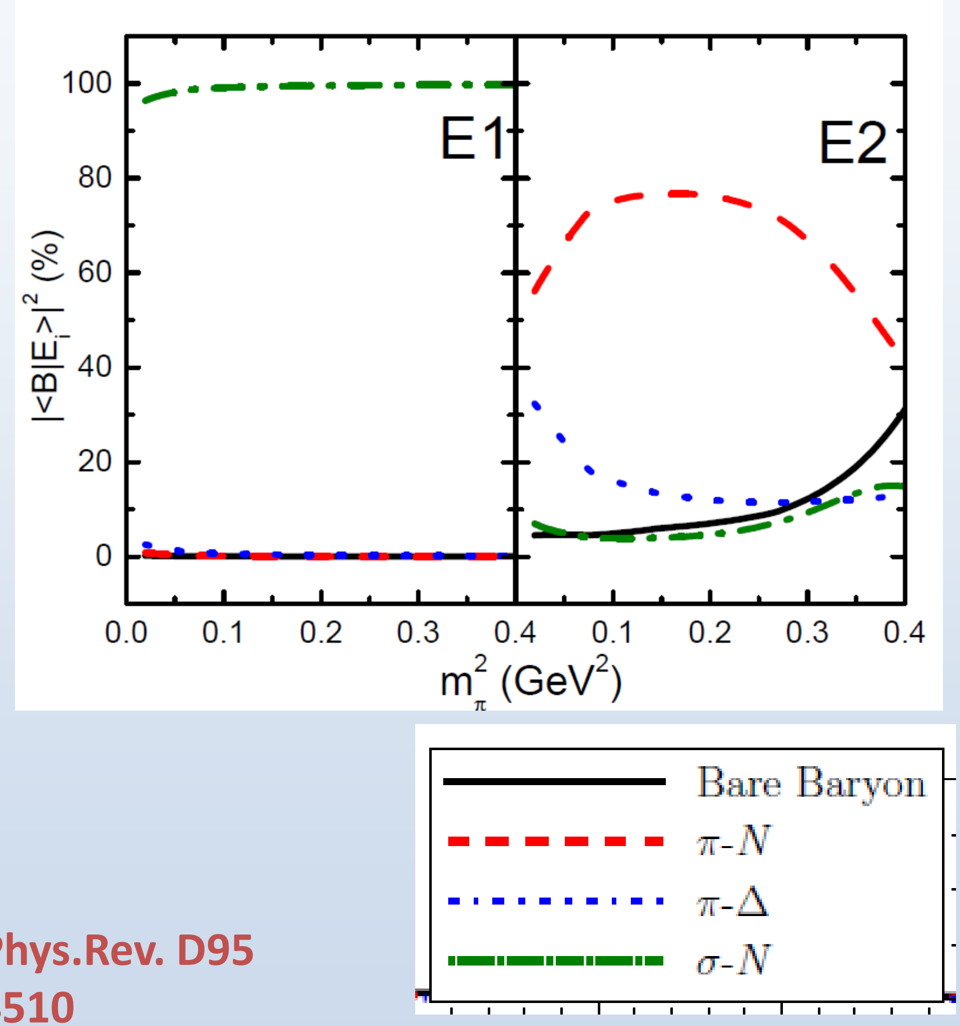
The first scenario with a bare state for P11 around the pole at 2.0 GeV can fit both Lattice data and experimental data well, it indicates that N*(1440) seems a re-scattering state, and first radial excitation of nucleon should be around 2.0 GeV.



The Second scenario with a bare state for N*(1440) fit the experimental data well. But the largest possibility for bare state does not touch the lattice point. Thus, it fails to explain Lattice data.

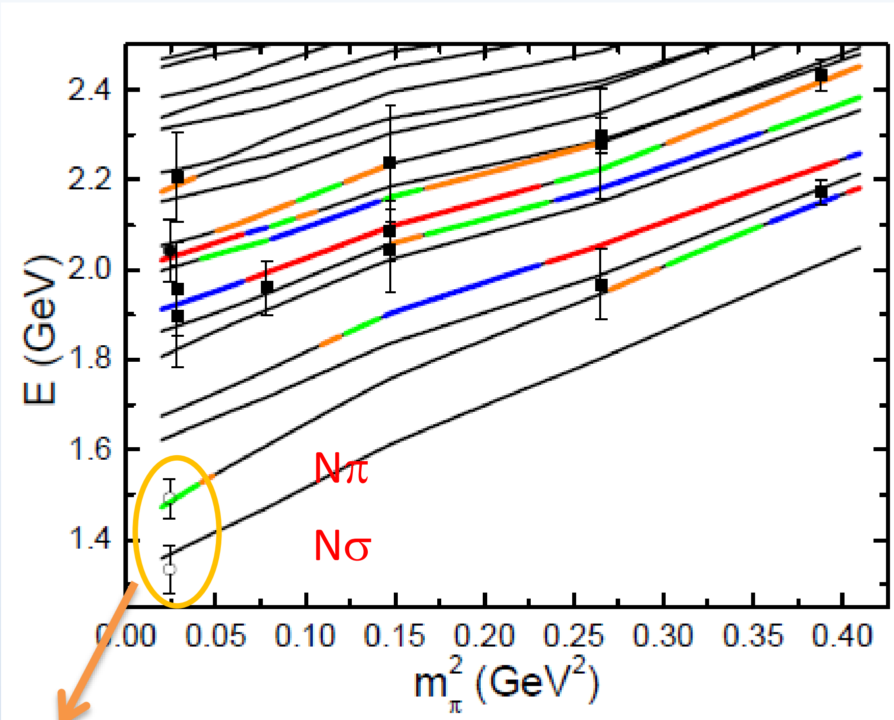
N*(1440)

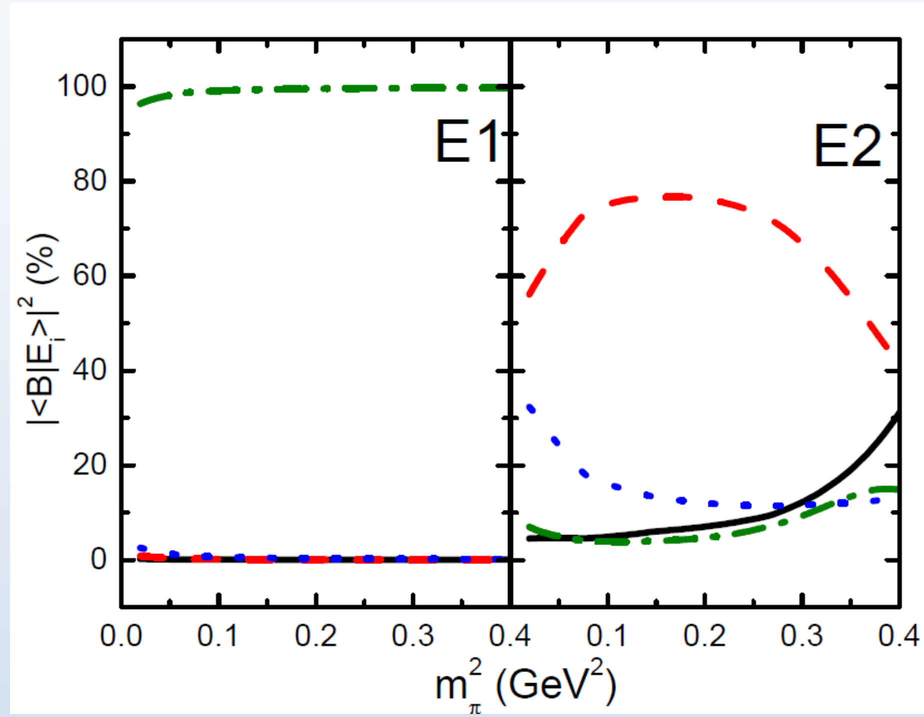


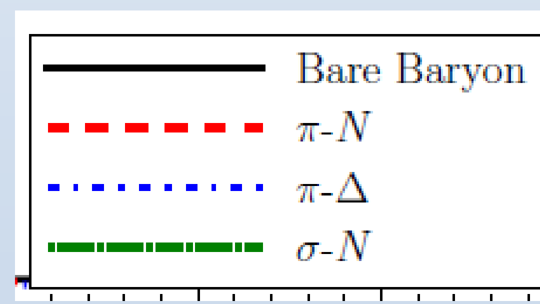


C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

N*(1440)







It is not a FIT !!!

C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

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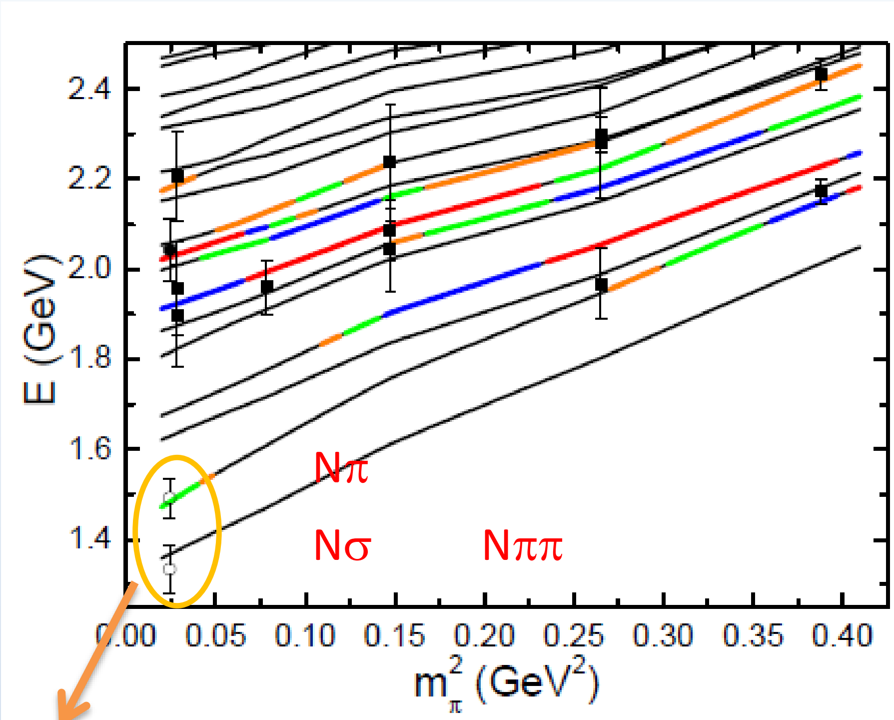
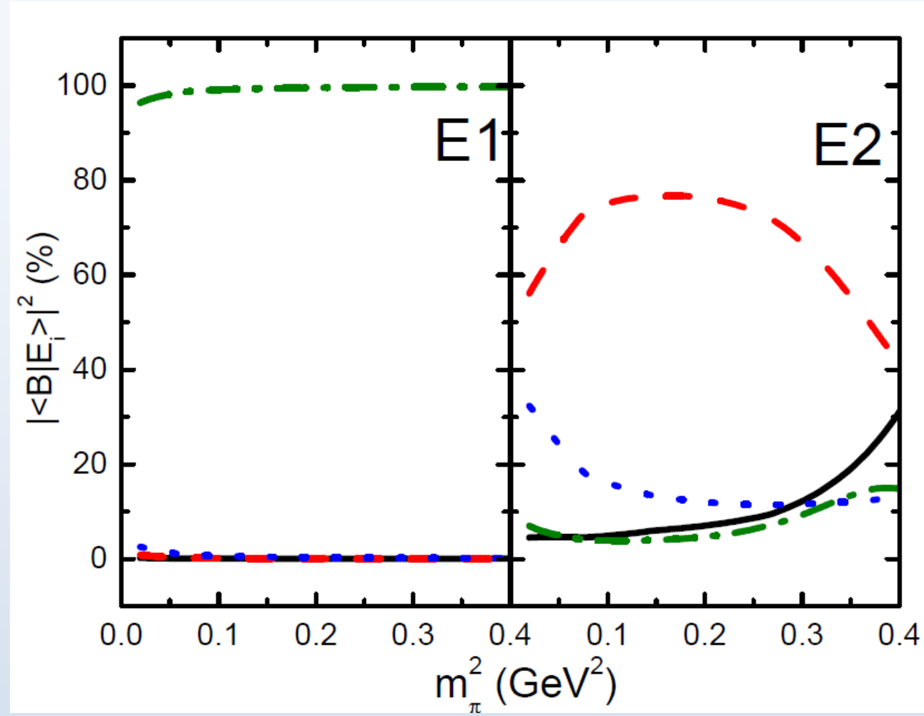
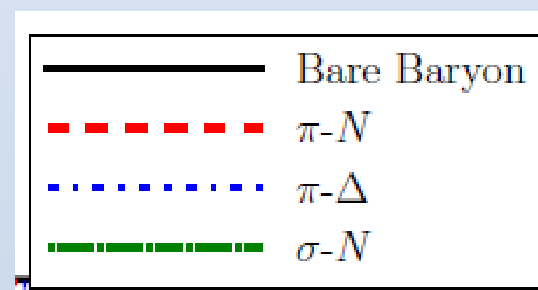
N*(1440)



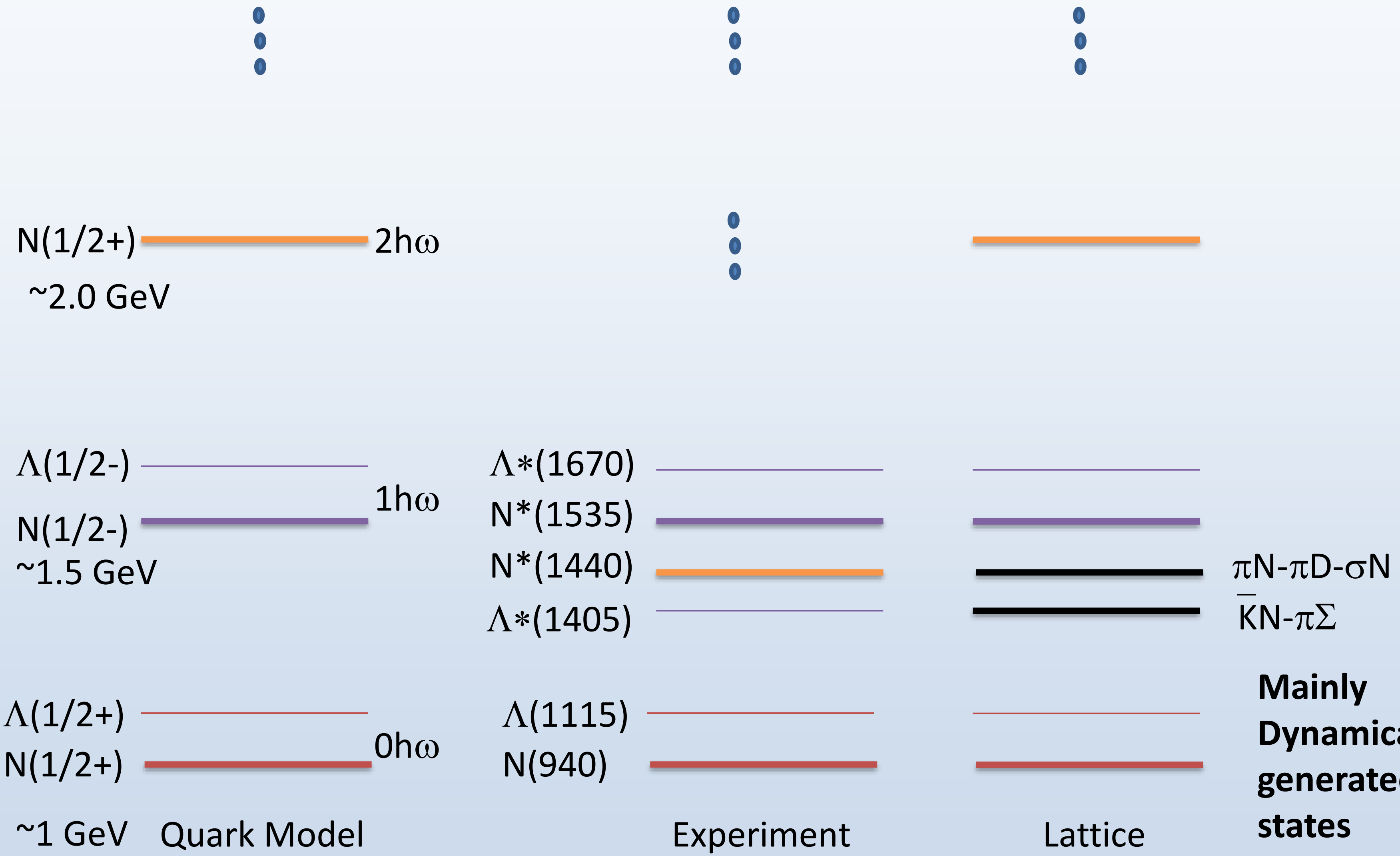




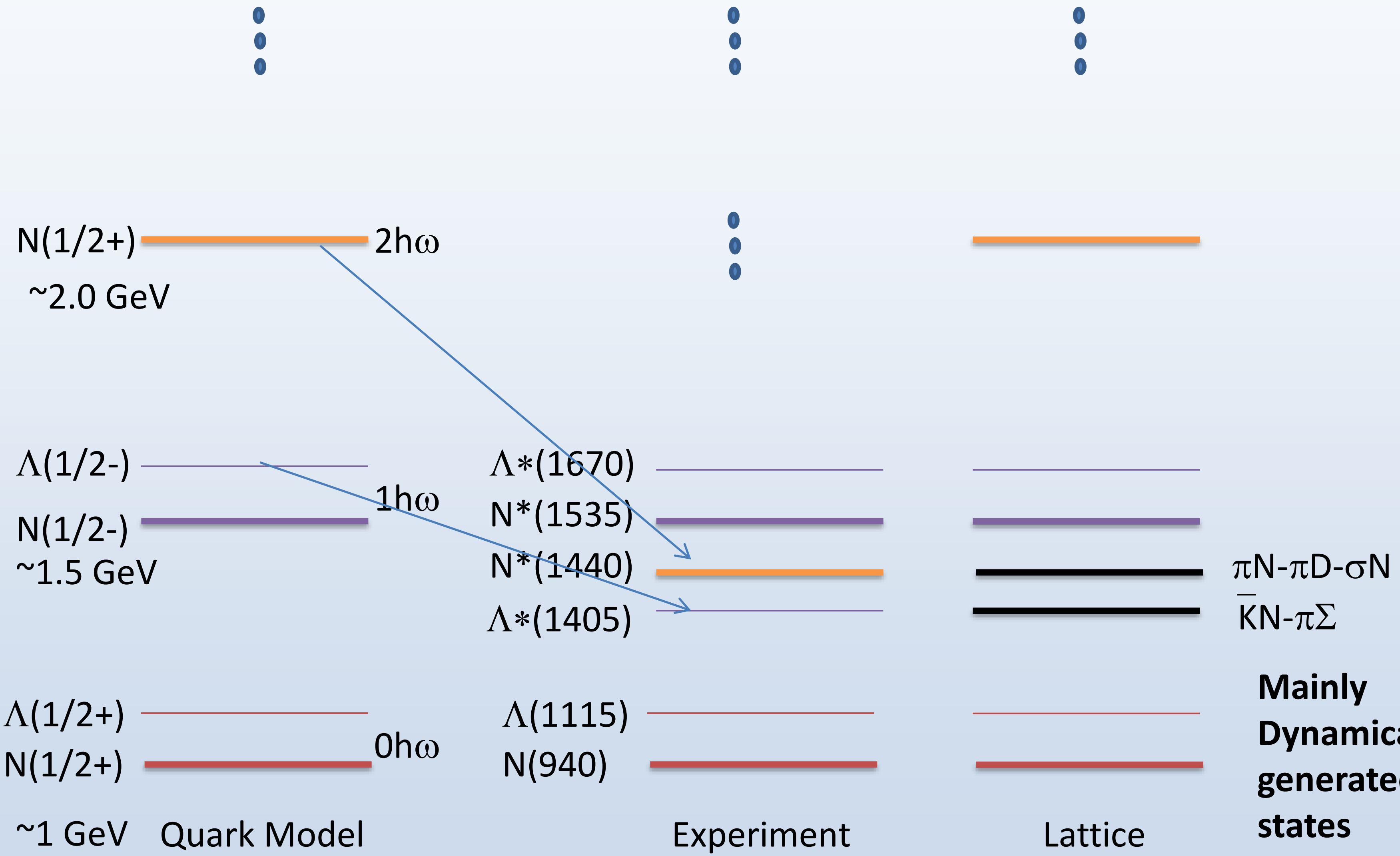
It is not a FIT !!!

C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

We need more data and detailed study, for the contribution from Nππ three body.



Mainly Dynamical generated states Not in the Quark Model

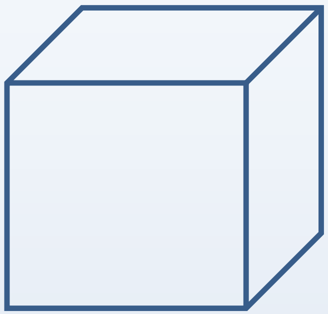


Mainly Dynamical generated states Not in the Quark Model

Outline

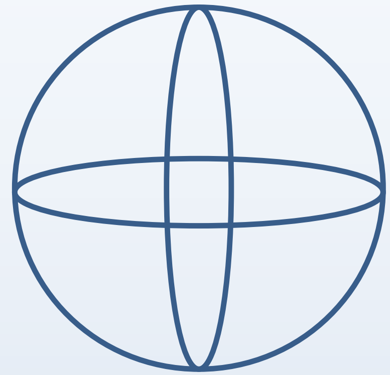
- Motivation
- Hamiltonian Effective Field Theory (HEFT)
- Eigenstate in the Lattice VS Resonance in the scattering
- The Angular momentum Mixing in the Hamiltonian
- The Multihadron system
- Summary and Outlook

Finite volume VS Infinite volume



O_h

$A_1^{\pm}, A_2^{\pm}, E^{\pm},$
$T_1^{\pm}, T_2^{\pm}$



O(3)

$L^p = 0^+, 1^-, 2^+, \ldots\ldots$

O(3)	**O_h**
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$

O_h	**O(3)**
A_1^+	$0^+, 4^+, \ldots.$
E^+	$2^+, 4^+, \ldots.$
T_1^+	$4^+, \ldots.$
T_2^+	$2^+, 4^+, \ldots.$
A_2^-	$3^-, 7^-, 9-, \ldots.$
T_1^-	$1^-, 3^-, \ldots.$
T_2^-	$3^-, \ldots.$

Finite volume VS Infinite volume

$$\hat{H}_L = \hat{H}_{0L} + \sum_{\mathbf{n}',\mathbf{n}\in\mathbb{Z}^3} V_L(\frac{2\pi\mathbf{n}'}{L}, \frac{2\pi\mathbf{n}}{L}) |\mathbf{n}'\rangle \langle\mathbf{n}|$$

$$\sum_{\mathbf{n}\in\mathbb{Z}^3} \rightarrow \sum_{\mathbf{n}\in\mathbb{Z}^3}^{|\mathbf{n}|^2\leq N_{\mathrm{cut}}}$$

For $N_{\mathrm{cut}} = 600$, we have 61565 states.

(For $L = 3$ fm, $N_{\mathrm{cut}} \sim 10$ GeV)

In the early works, for pure s- and p- waves,

$61565 \rightarrow$ about 600 .

O(3)	O_h
0^+	A_1^+
1^-	T_1^-

- Original basis $|\mathbf{n}\rangle$: $\sum_{N=0}^{N_{\mathrm{cut}}=600} C_3(N) \sim 60,000$
- Basis $|N;l,m\rangle$ with $l_{\mathrm{cut}} = 4$: $\sum_{N=0}^{600} 25 \sim 600 \times 25$
- Basis $|N,l;\Gamma = \mathbf{A}_1^+, f, \alpha\rangle$: $\sum_{N=0}^{600} 2 \sim 600 \times 2$
- Orthonormalization needs the inner products——P-Matrix

$$\hat{H}_L = \hat{H}_{0L} + \sum_{N',N} \sum_{\Gamma,F',F} v_{\Gamma,F',F}(k_{N'}, k_N) \sum_{\alpha} |N';\Gamma,F',\alpha\rangle \langle N;\Gamma,F,\alpha|$$

$|\mathbf{n}\rangle$

$\sum_{\hat{\mathbf{n}}} Y_{lm}(\hat{\mathbf{n}})\,|\mathbf{n}\rangle$

$|N; l, m\rangle$

$\sum_{m} [C_l]_{\Gamma,f,\alpha;m}\,|N; l, m\rangle$

$|N, l; \Gamma, f, \alpha\rangle$

Orthonormalization

$|N; \Gamma, F, \alpha\rangle$

$$[P_N]_{l',m';l,m} := \langle N; l', m' | N; l, m\rangle = \sum_{\hat{\mathbf{n}}} Y^*_{l'm'}(\hat{\mathbf{n}}) Y_{lm}(\hat{\mathbf{n}})$$

$$\begin{pmatrix} 25\times 25 \\ \text{for } l_{\text{cut}} = 4 \end{pmatrix} \xrightarrow{[C]^\dagger [P][C]} \begin{pmatrix} \boxed{\mathbf{A}_1^+} & & \\ & \boxed{} & \\ & & \ddots \end{pmatrix}$$

2×2

O(3)	O_h
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$

$$[P_{N;\Gamma,\alpha}]_{l',f';l,f} := \langle N, l'; \Gamma, f', \alpha | N, l; \Gamma, f, \alpha\rangle$$
$$= \sum_{m',m} [C_{l'}]^*_{\Gamma,f',\alpha;m'} [P_N]_{l',m';l,m} [C_l]_{\Gamma,f,\alpha;m}$$

Combination Coefficients: $[G_{l;\Gamma}]_{N',F';N,F} = \sum_f \langle N'; \Gamma, F', \alpha | N'; l, \Gamma, f, \alpha\rangle \langle N; l, \Gamma, f, \alpha | N; \Gamma, F, \alpha\rangle$

Example of Isospin-2 Scattering

- $l_{\text{cut}} = 4$, only s-, d- and g-waves are present

- Separable potential model:

$$v_l(p,k) = f_l(p) G_l f_l(k)$$

$$f_l(k) \sim \frac{(d_l \times k)^l}{(1 + (d_l \times k)^2)^{l/2+2}}$$

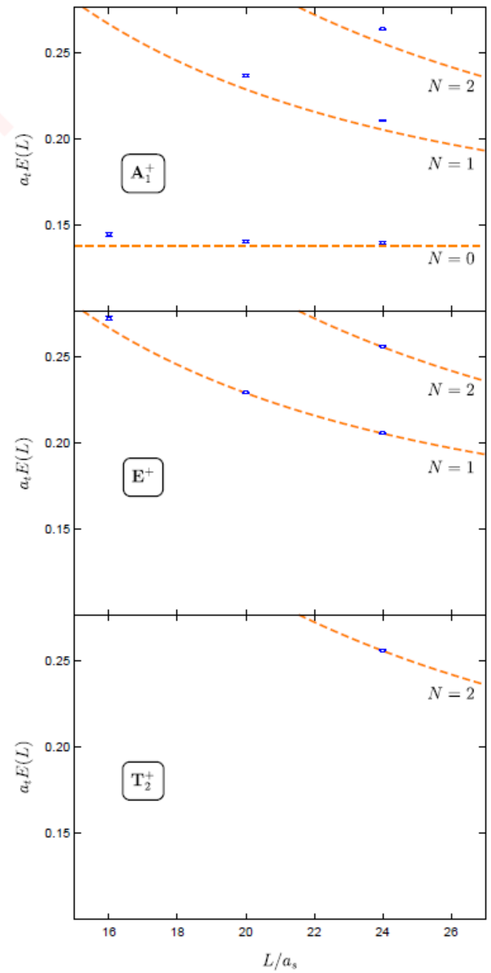
- 6 parameters: $G_0, G_2, G_4, d_0, d_2, d_4$

- Dimensions of Hamiltonians ($N_{\text{cut}} = 600$):

$$\mathbf{A}_1^+ : 923 \quad \mathbf{E}^+ : 965 \quad \mathbf{T}_2^+ : 963$$

- The fitted data: 11 energy levels

PRD 86, 034031 (2012)
Jozef J. Dudek et al.



PRELIMINARY

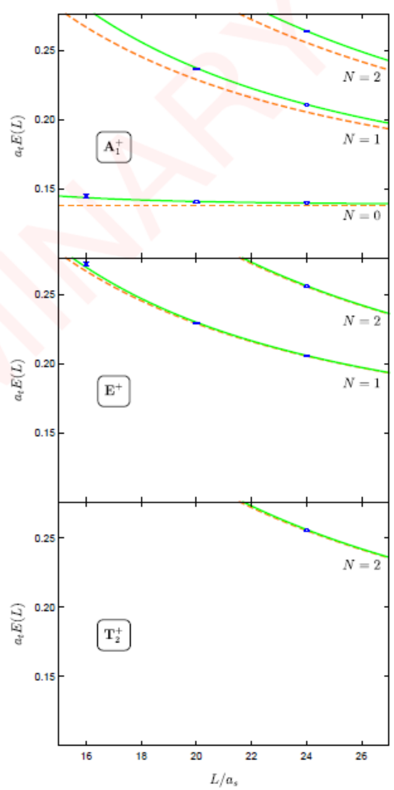
Example of Isospin-2 Scattering

Components of eigenstates

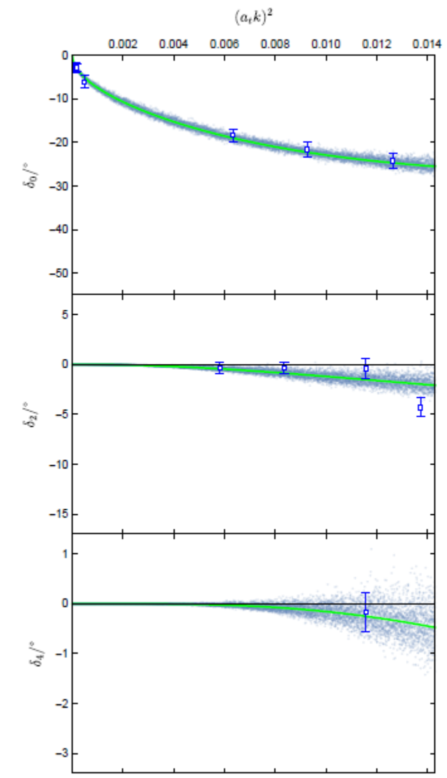
A_1^+	$N=0$	$N=1$	$N=2$	$N=3$	$N=4$	$\cdots$
1st	99.7	0.2	0.0	0.0	0.0	$\cdots$
2nd	0.1	97.4	1.9	0.2	0.0	$\cdots$
3rd	0.0	1.5	94.5	2.8	0.3	$\cdots$

- Fitting $\rightarrow$ Parameters $\rightarrow$ $\hat{H}$ and $\hat{H}_L$
- $\hat{H} \rightarrow \delta_l(E)$
- $\hat{H}_L \rightarrow E_n(\Gamma, L)$
- $\hat{H}_L \rightarrow$ Eigenstates

Volume dependent spectra



Phase shifts with errors



PRELIMINARY

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Outline

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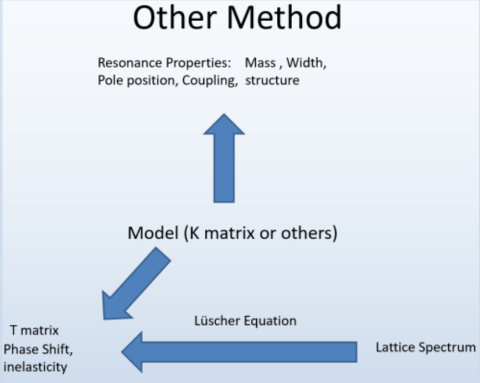
Story from …

Akaki (阿卡基. 儒塞斯基) asked :

What is really benefit compare to other Process ?



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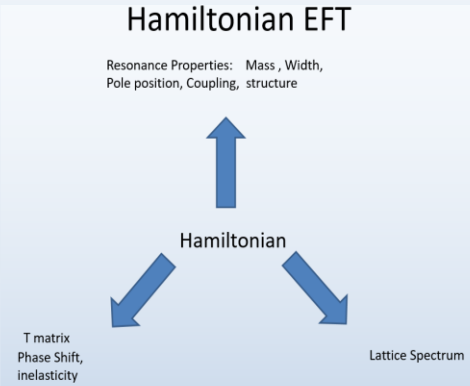
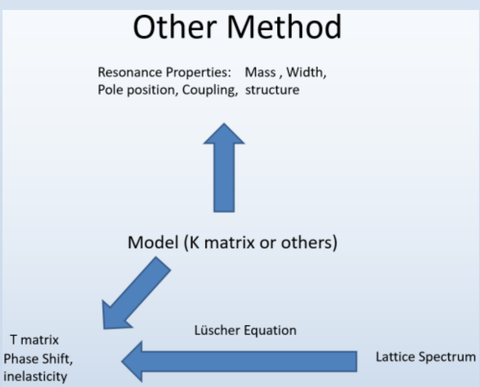


I try to answer:

1. It is a different formalism
2. EigenVector
3. Different parameters method

……



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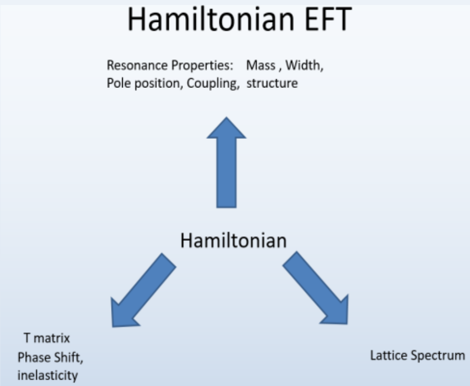
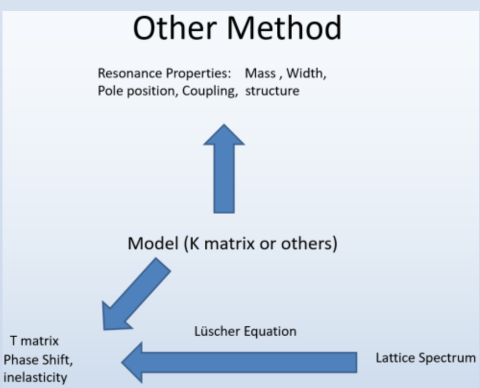


I try to answer:

1. It is a different formalism Exponential Suppressed Correction
2. EigenVector Model dependent
3. Different parameters method Also can be applied in other Method

……



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Story from …

Akaki (阿卡基. 儒塞斯基) asked :

What is really benefit compare to other Process ?

I try to answer:

1. It is a different formalism Exponential Suppressed Correction
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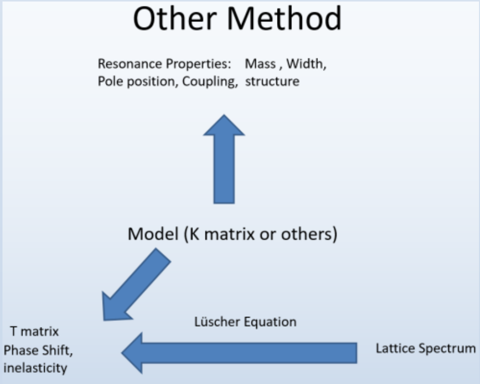

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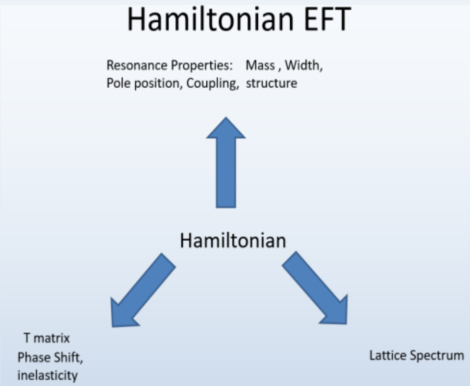

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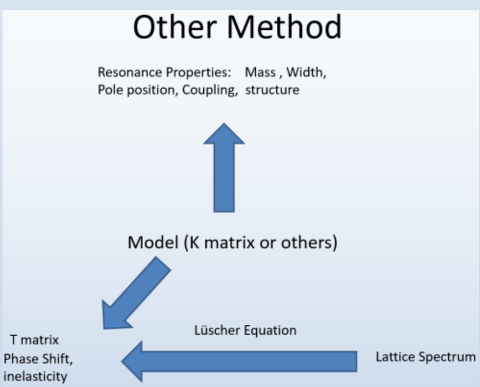

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But, now I found

Bethe Forum Multihadron Dynamics in a Box

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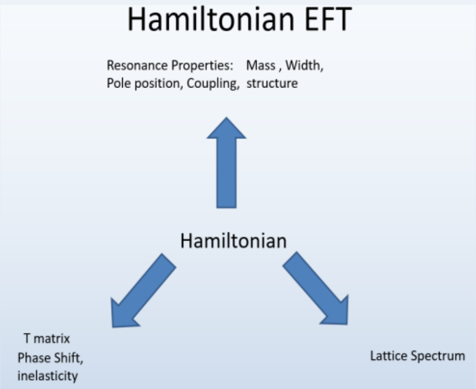

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Exponential Suppressed Correction

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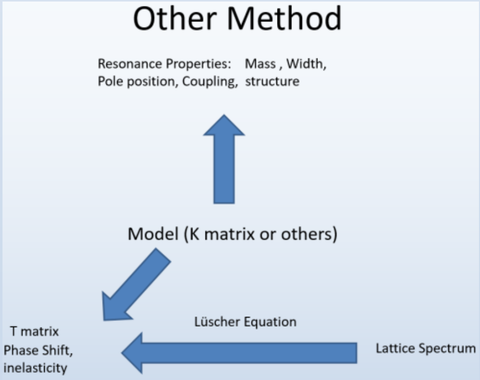

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Hamiltonian Approach can be extended to multi-hadron case

But, now I found

Bethe Forum Multihadron Dynamics in a Box

The Multihadron system

H=

	$\mid B>$	$\mid \alpha_{ij}\beta_k>$	$\mid \beta_1\,\beta_2\,\beta_3>$
$\mid B>$	diagonal	$B \to \alpha_{ij}\beta_k$ decay	0
$\mid \alpha_{ij}\beta_k>$		$\alpha_{ij}\beta_k \to \alpha'_{ij}\beta'_k$ Rest Frame two body	$\alpha_{ij} \to \beta_i\beta_j$ Boost Frame decay
$\mid \beta_1\,\beta_2\,\beta_3>$			$\beta_1\beta_2\beta_3 \to \beta_1\beta_2\beta_3$ Three bodies contact Term

The Multihadron system

$H=$

	$\|B\rangle$	$\|\alpha_{ij}\beta_k\rangle$	$\|\beta_1\beta_2\beta_3\rangle$
$\|B\rangle$	diagonal √	$B \to \alpha_{ij}\beta_k$ decay √	0 √
$\|\alpha_{ij}\beta_k\rangle$		$\alpha_{ij}\beta_k \to \alpha'_{ij}\beta'_k$ √ Rest Frame two body	$\alpha_{ij} \to \beta_i\beta_j$? Boost Frame decay
$\|\beta_1\beta_2\beta_3\rangle$			$\beta_1\beta_2\beta_3 \to \beta_1\beta_2\beta_3$? Three bodies contact Term

Long way, Heavy work ……

Summary

- HEFT can connect experimental data, Lattice data and effective model.
- HEFT studies Hamiltonian in the different Momentum spaces, "Complex / Real", "Continuous / Discrete" .
- Eigenstates in the finite volume are the discrete states of continuous final scattering states in the infinite volume.
- Eigenvector can be used to explore the internal structure of resonance based on basic states setting.
- The angular momentum mixing can be studied in the HEFT frame work.

THANKS VERY MUCH

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