Resonances in coupled-channel scattering from Lattice QCD

David Wilson

based on work with the Hadron Spectrum Collaboration

hadspec

hadspec.org

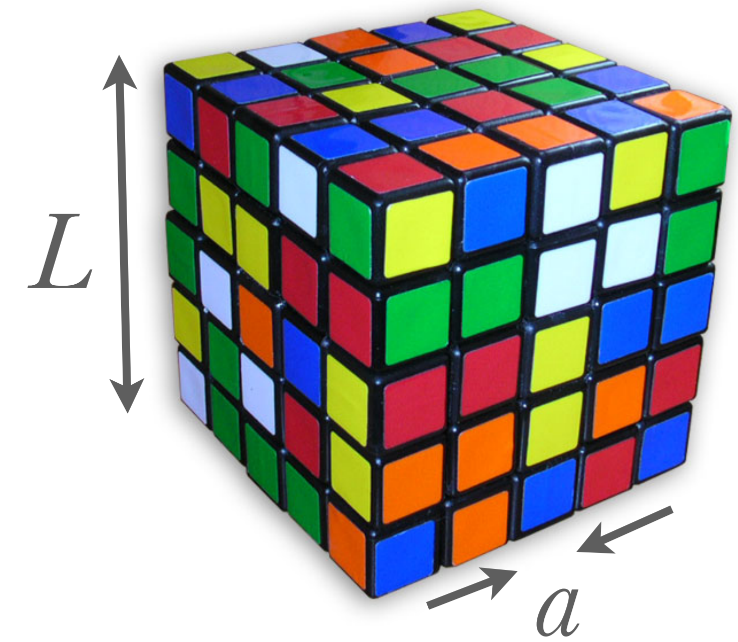
Bethe Forum
9-13 September 2019
Bonn

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

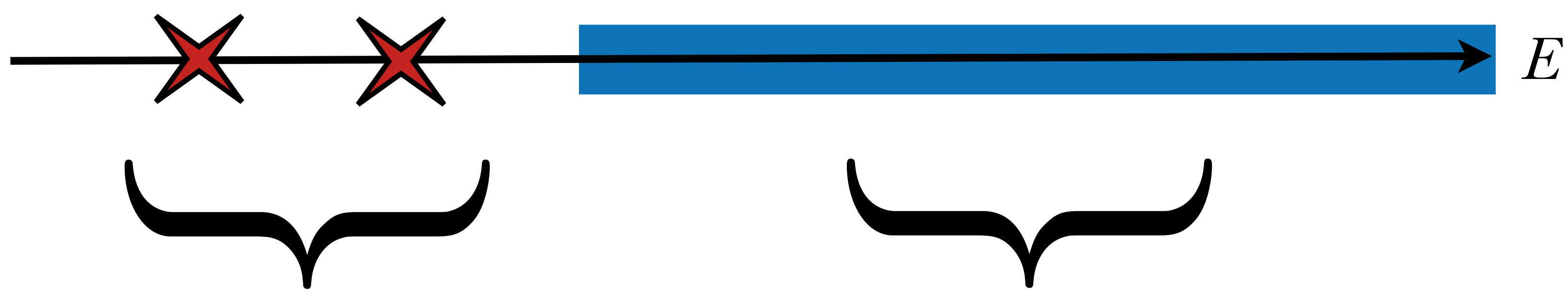
Science Foundation Ireland sfi For what's next

THE ROYAL SOCIETY

Scattering in a finite volume?



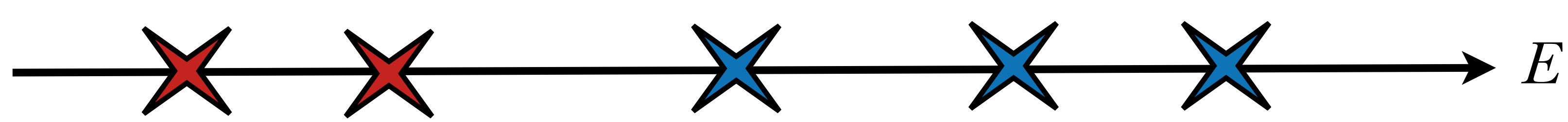
Infinite volume



Bound states

Meson-meson continuum

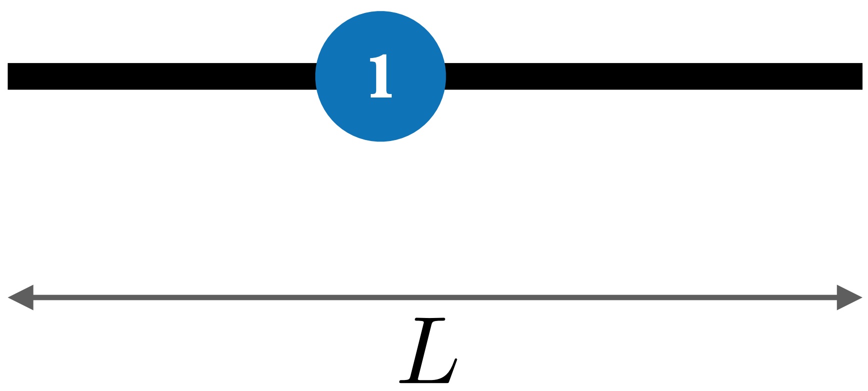
Finite volume

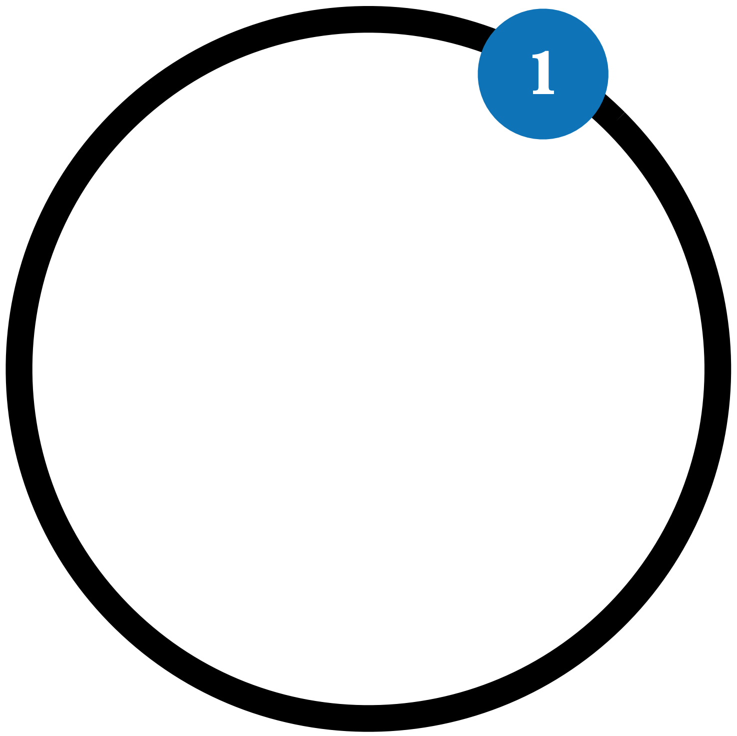


Momentum is quantised - no continuum

$$\vec{p} = \frac{2\pi}{L}\vec{n}$$

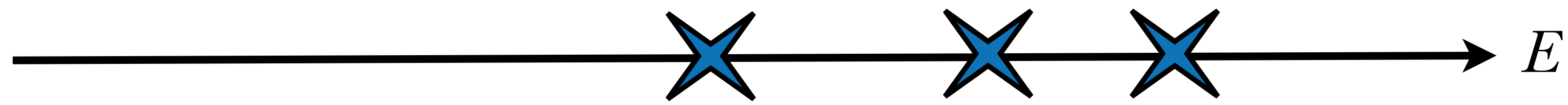
1-dimensional QM, periodic BC, single particle:



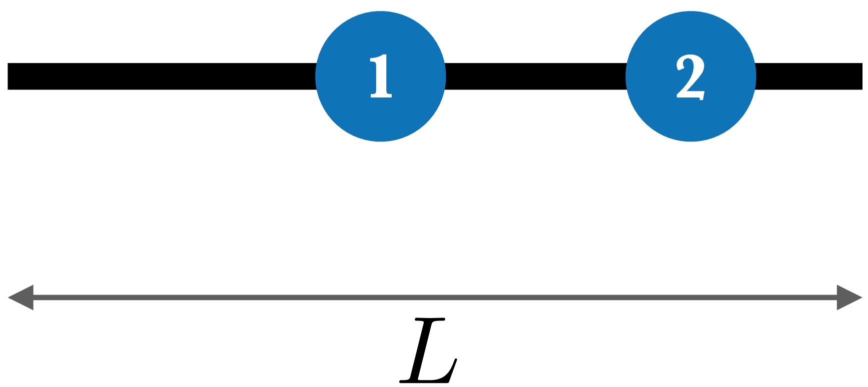


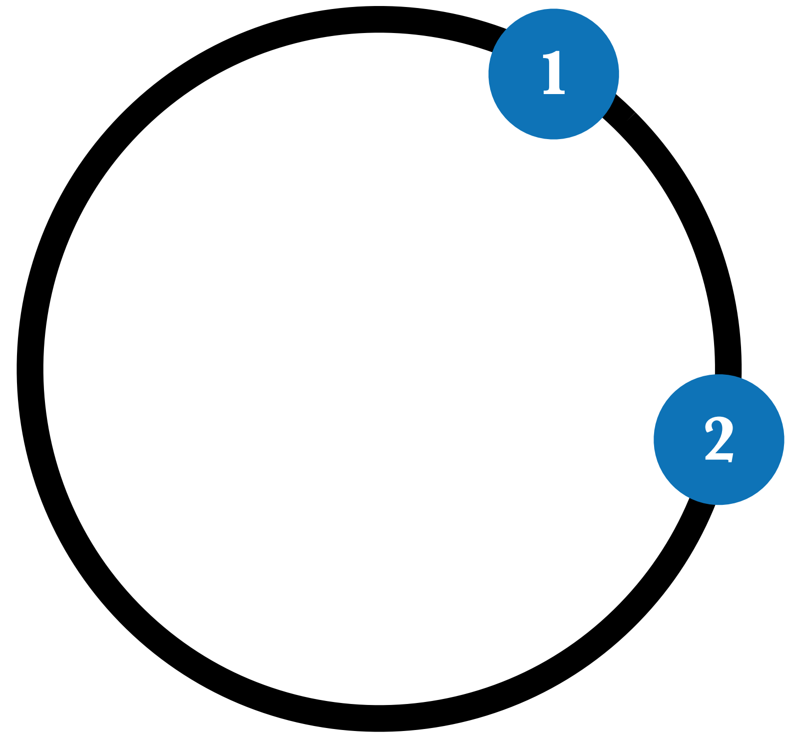
momentum is quantised: $p = \frac{2\pi n}{L}$

Lüscher et al



1-dimensional QM, periodic BC, two particles, no interactions



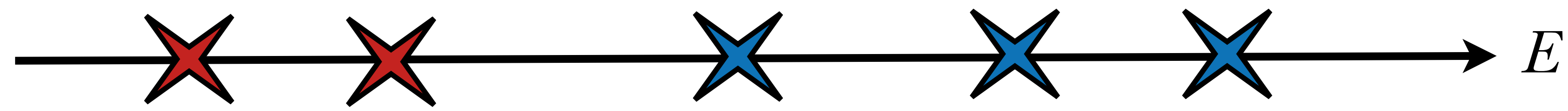


momentum is quantised: $p_i = \frac{2\pi n_i}{L}$

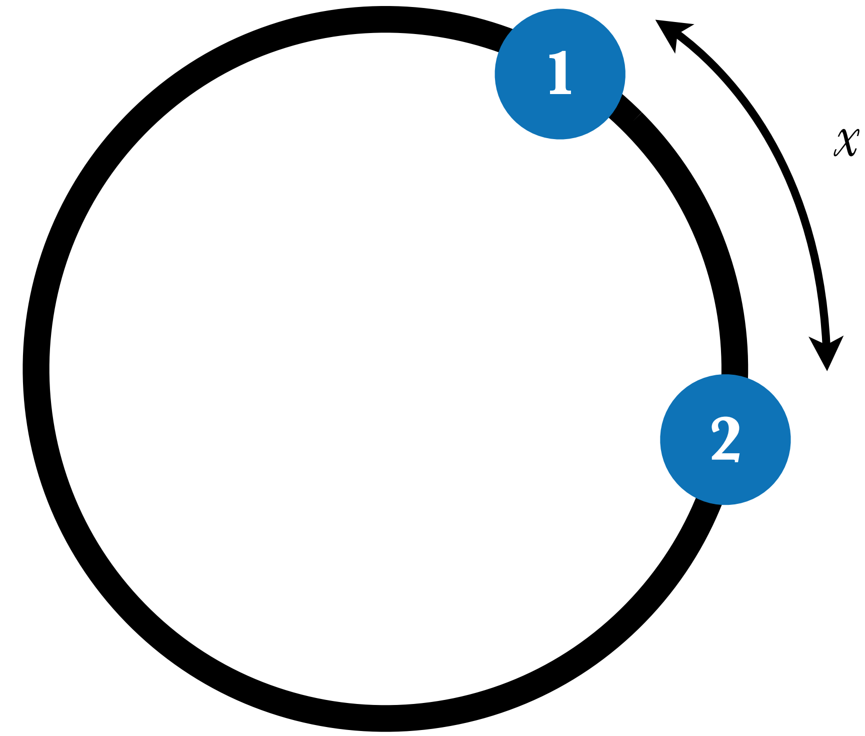
two particle energies are discrete:

$$E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}}$$

Lüscher et al



1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$



$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$

Phase shifts via Lüscher's method:
$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT
→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot\left(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

determinant condition:

- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations

- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \rightarrow \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad\qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

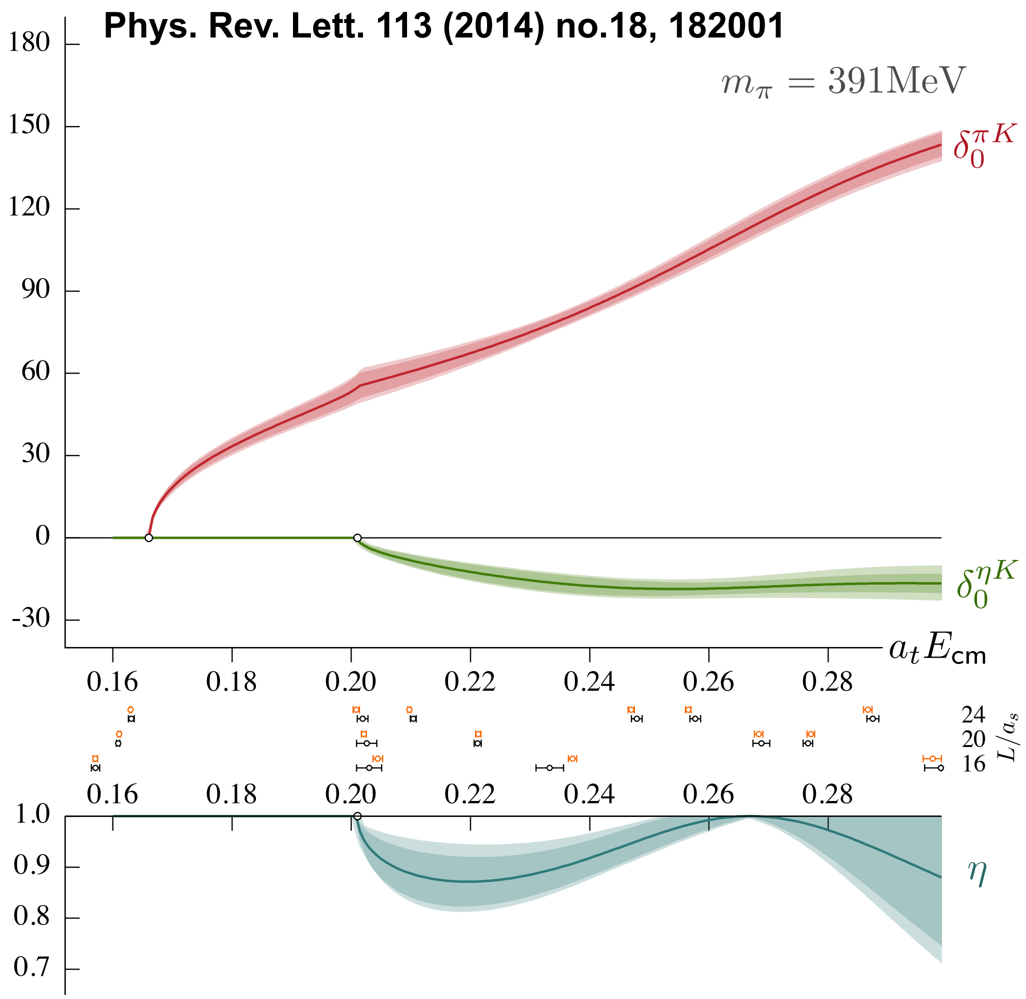
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

use a dispersion relation to generate a real part from iρ

- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut



operators used:

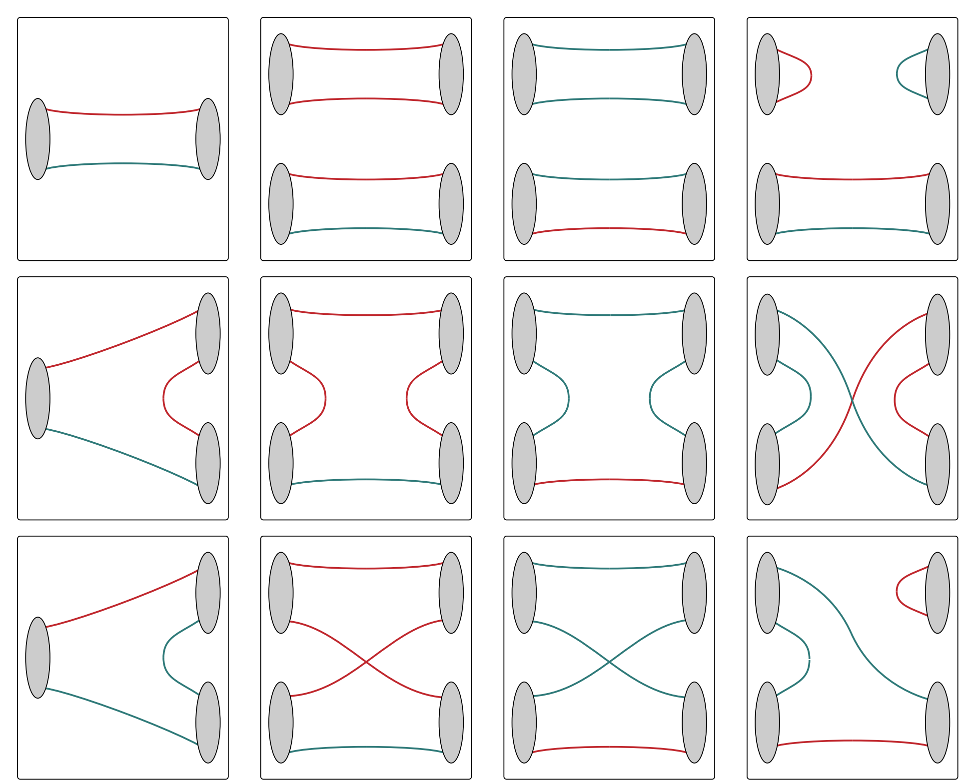
$$\bar{\psi}\Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D}\, \psi$$ local qq-like constructions

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$ two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$ uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009) compute a large correlation matrix & use GEVP to extract energies

many wick contractions, pi-K, eta-K & qq operators:



+ ...

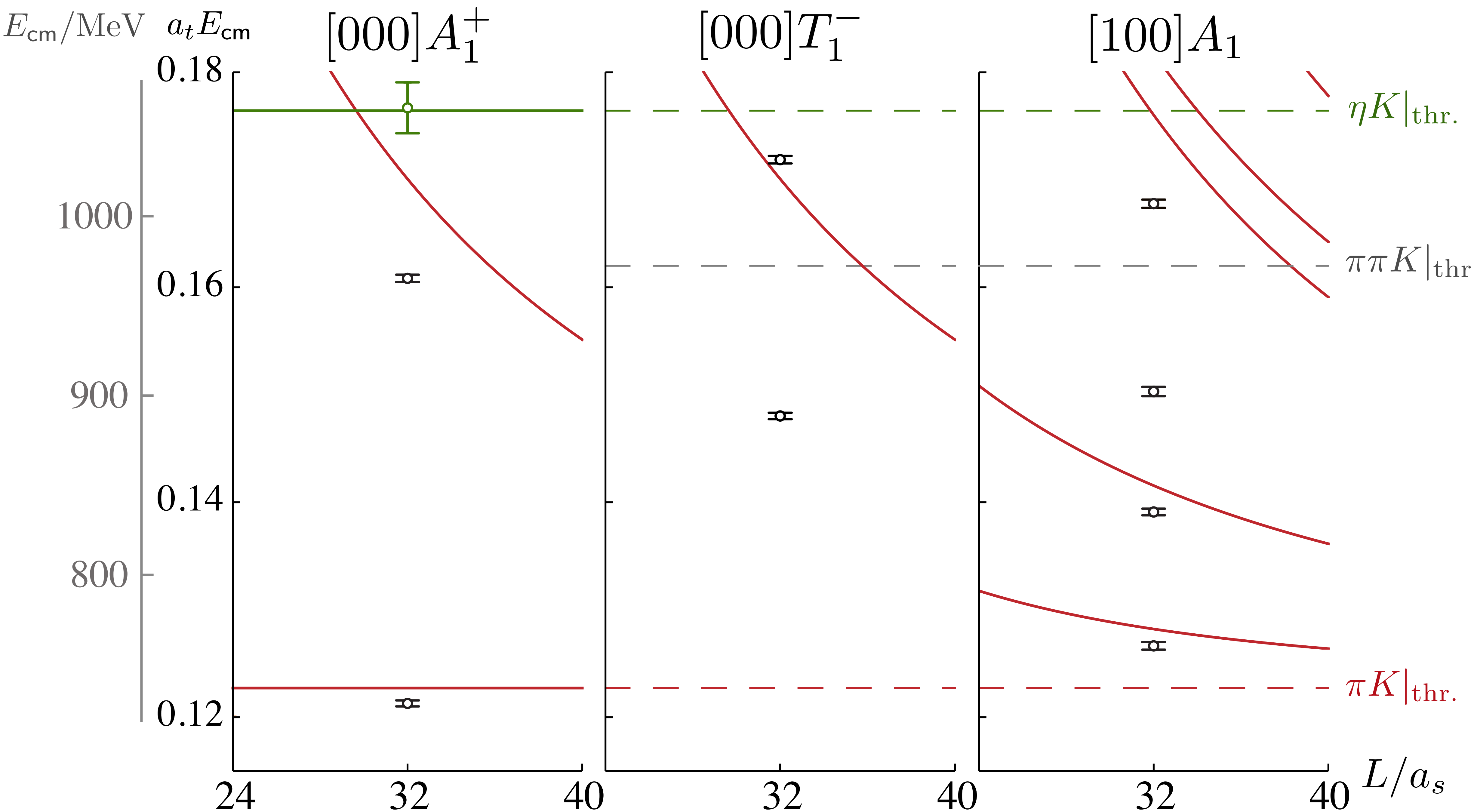
we investigate elastic pi-K scattering using two new lattices generated with L/a_s=24 and an existing lattice with L/a_s=32.

pion masses: 239, 284, 327 (& 391) MeV

anisotropic action (3.5 finer spacing in time) Wilson-Clover fermions

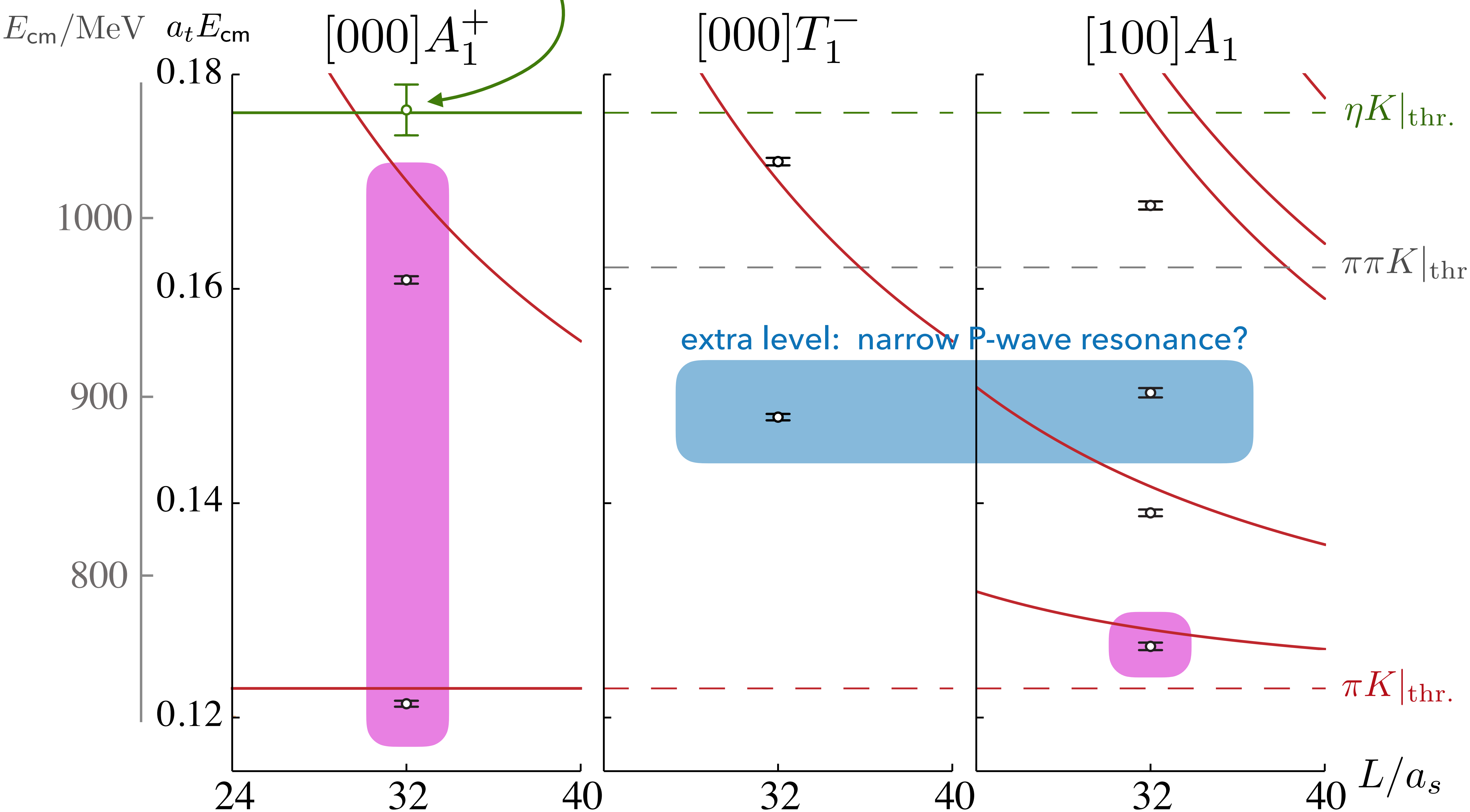
Phys. Rev. Lett. 123 (2019) no.4, 042002

$m_\pi = 239$ MeV



$m_\pi = 239\,\mathrm{MeV}$

elastic scattering only:
ignore levels around and above ηK



negative energy shifts: attraction in S-wave?

S-wave amplitude:

$$t^{\ell=0} = \frac{K(s)}{1 + I(s)K(s)}, \qquad K(s) = \gamma_0 + \gamma_1 s$$

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \, \frac{\rho(s')}{(s' - s)(s' - s_0)}, \qquad \rho = 2k_{\mathsf{cm}}/\sqrt{s}$$

"Chew-Mandelstam" phase space
- once-subtracted dispersion relation
- produces real log from the imaginary part due to unitarity

- respects s-channel unitarity
- ignores left-hand cuts
- can produce subthreshold ("Adler"-like) zeros
- some forms of K(s) can produce physical sheet poles

which are foribben by analyticity - we check for these

on the real axis in the physical scattering region,
this is a lot like the familiar effective range parameterisation: $k_{\mathsf{cm}} \cot \delta_0 = \frac{1}{a} + \frac{1}{2} r k_{\mathsf{cm}}^2$

P-wave amplitude:

relativistic Breit-Wigner, coupling g_R, mass m_R

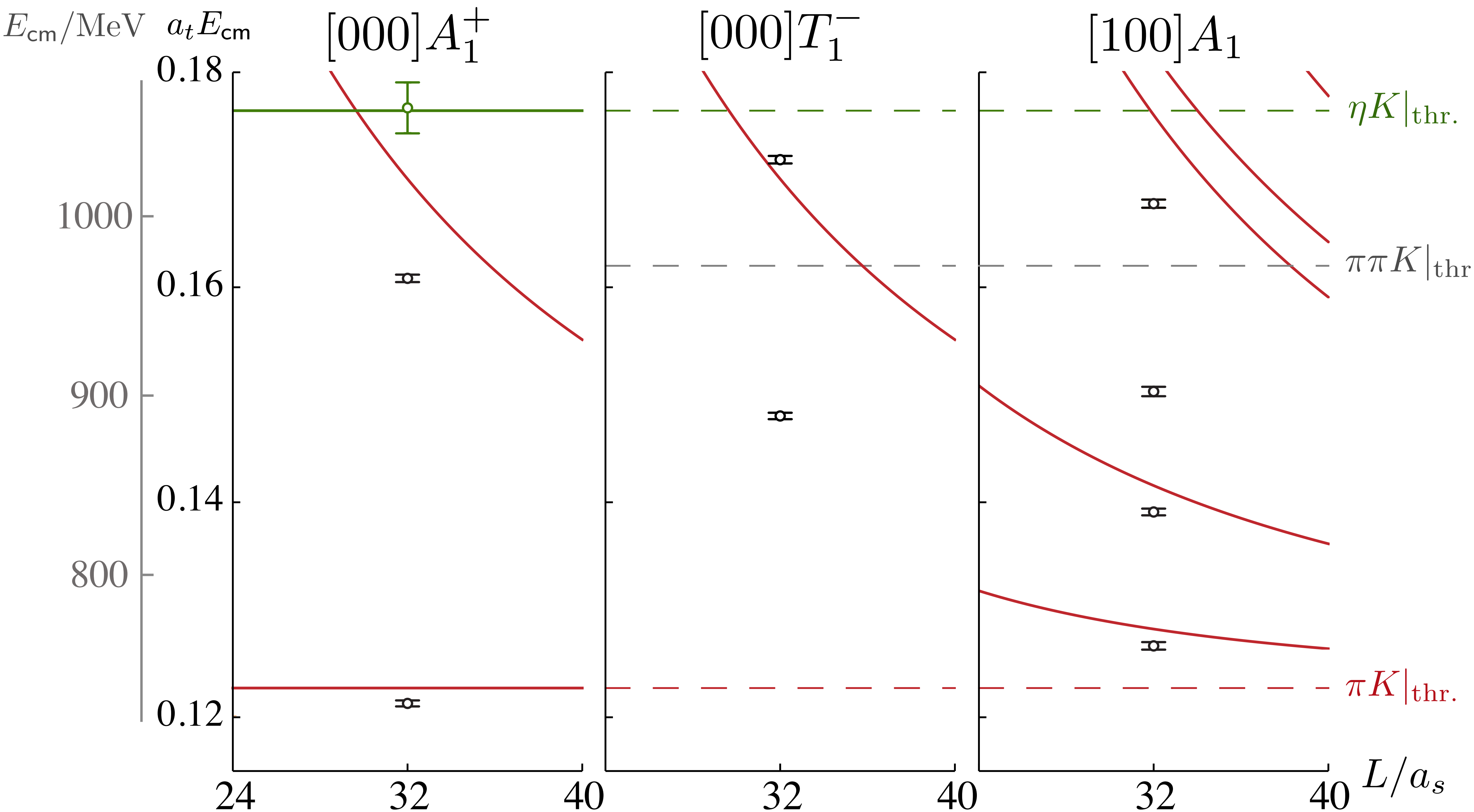
$$t^{(\ell=1)}(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma_{\ell=1}(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma_{\ell=1}(s)},$$

$$\Gamma_{\ell=1}(s) = \frac{g_R^2}{6\pi} \frac{k_{\mathsf{cm}}^3}{s}$$

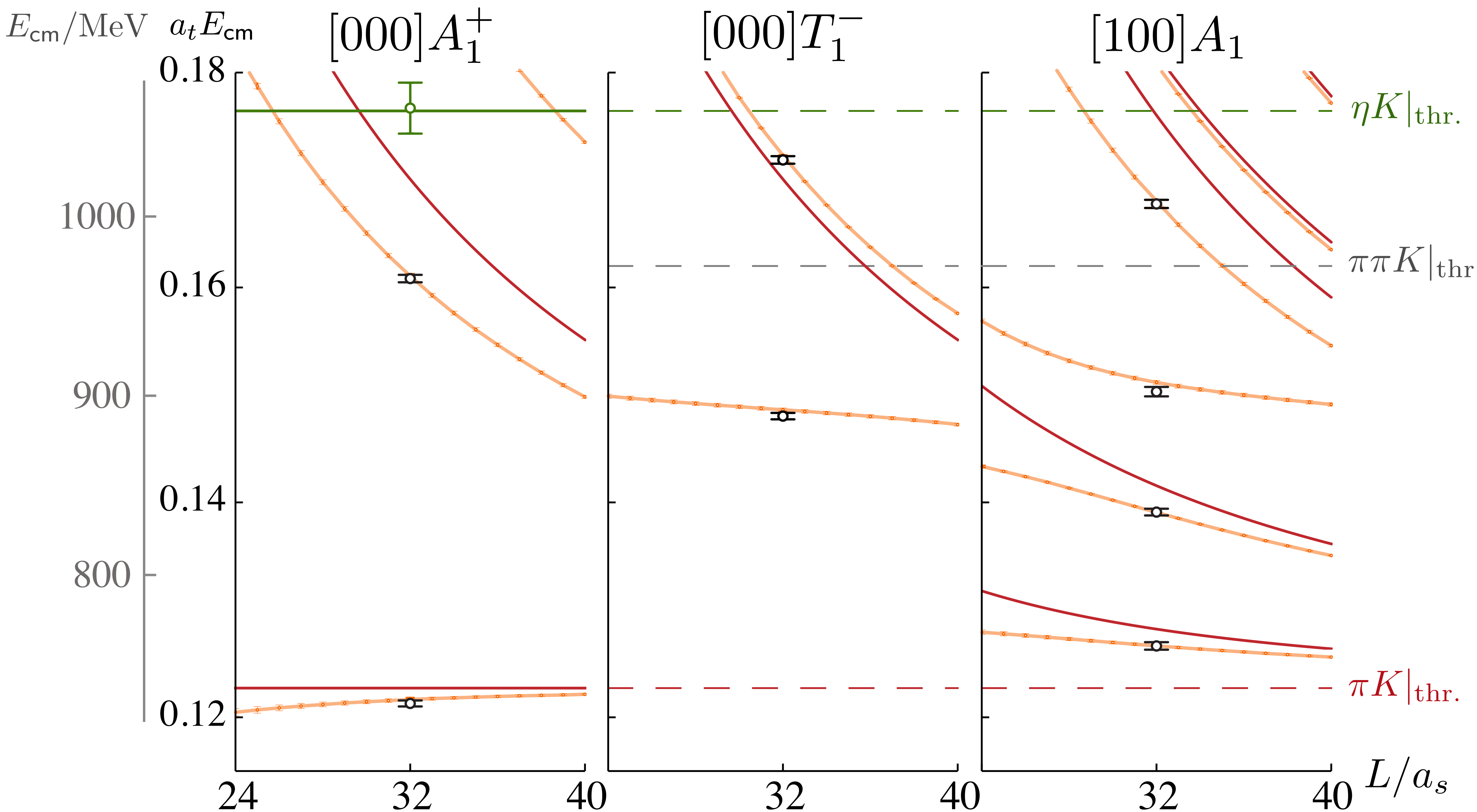
can describe narrow and broad resonance poles

minimise g_R, m_R along with S-wave parameters until spectra are well-described

$m_\pi = 239\,\mathrm{MeV}$

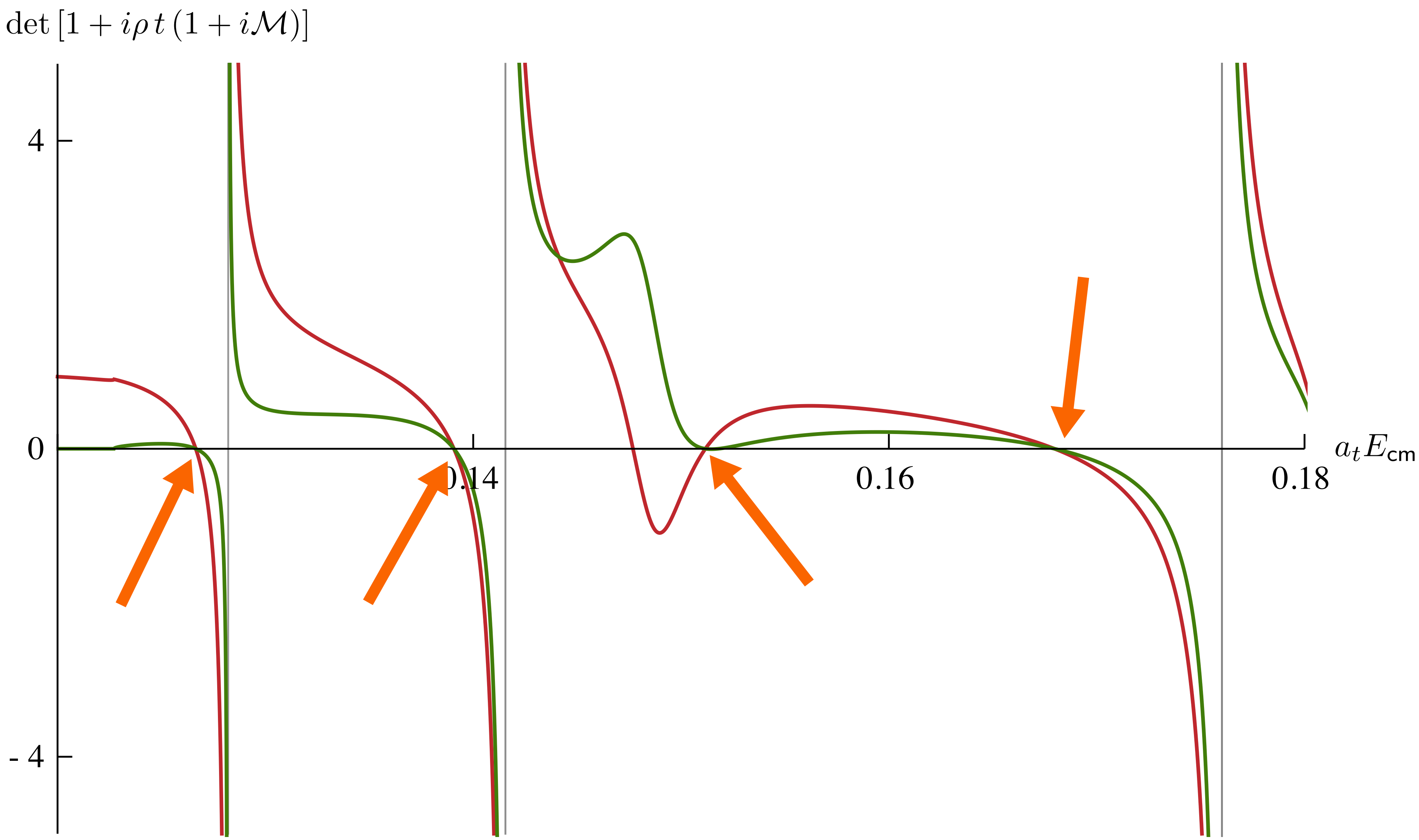


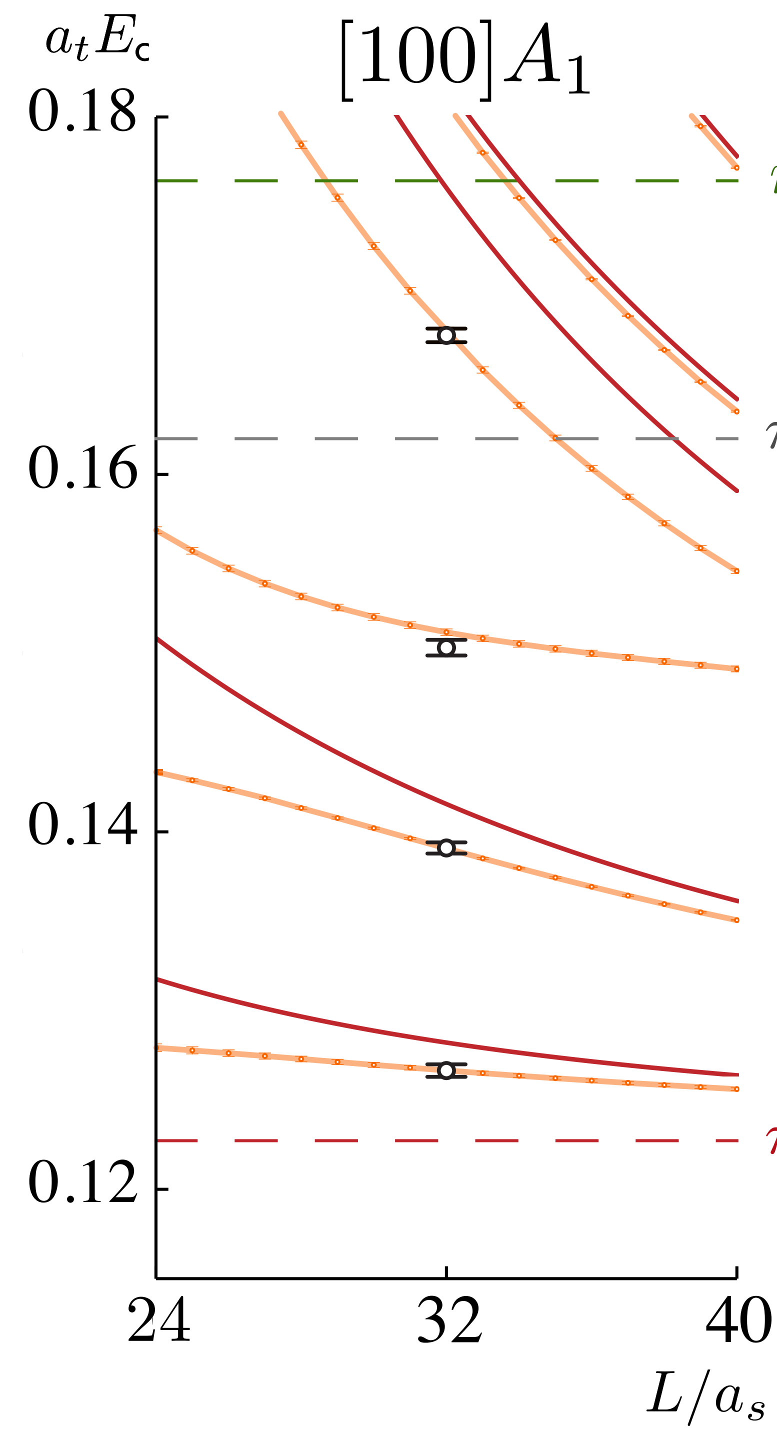
$m_\pi = 239\,\mathrm{MeV}$



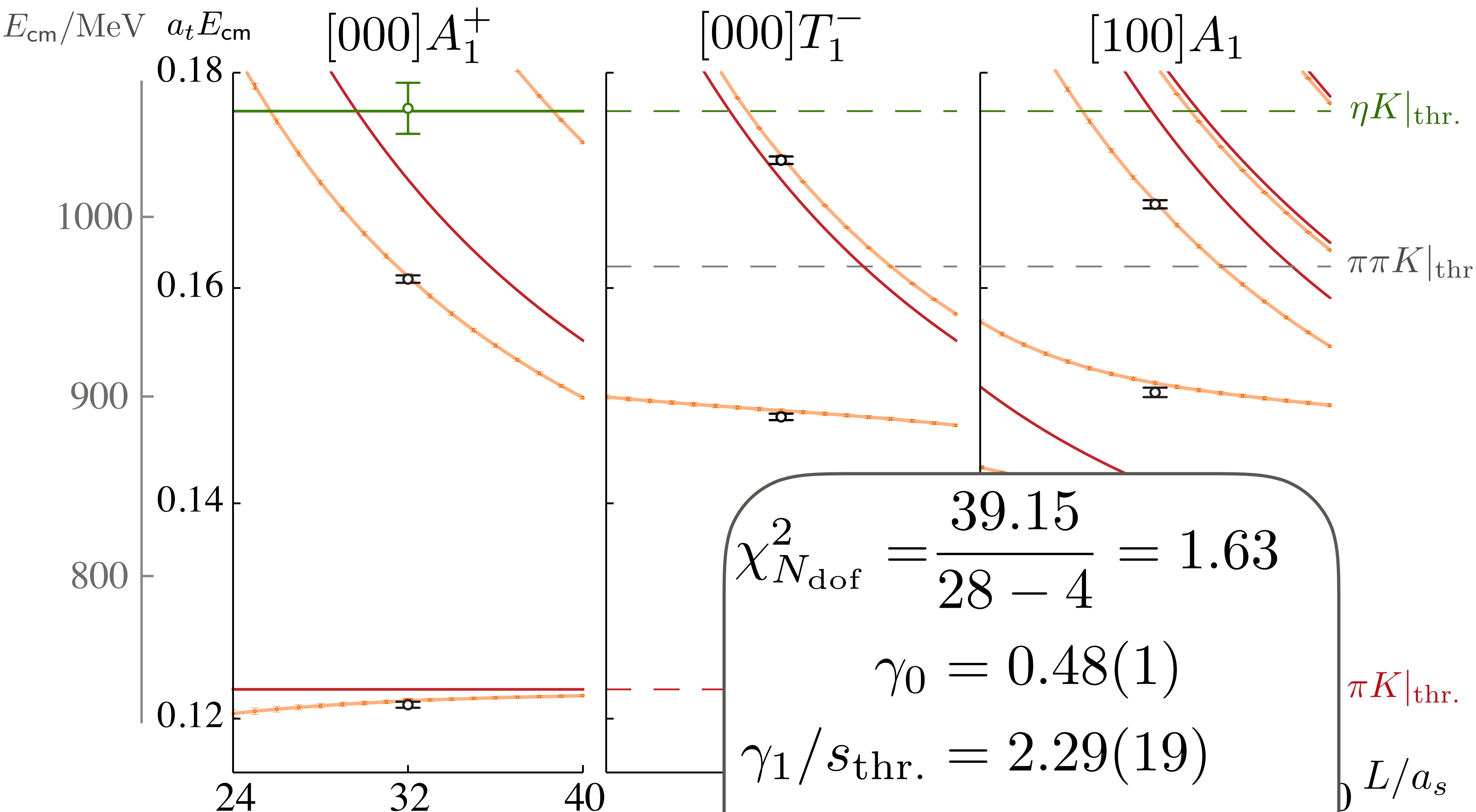
spectrum fitting

$m_\pi = 239\,\mathrm{MeV}$





$m_\pi = 239\,\mathrm{MeV}$



$$\chi^2_{N_{\mathrm{dof}}} = \frac{39.15}{28-4} = 1.63$$

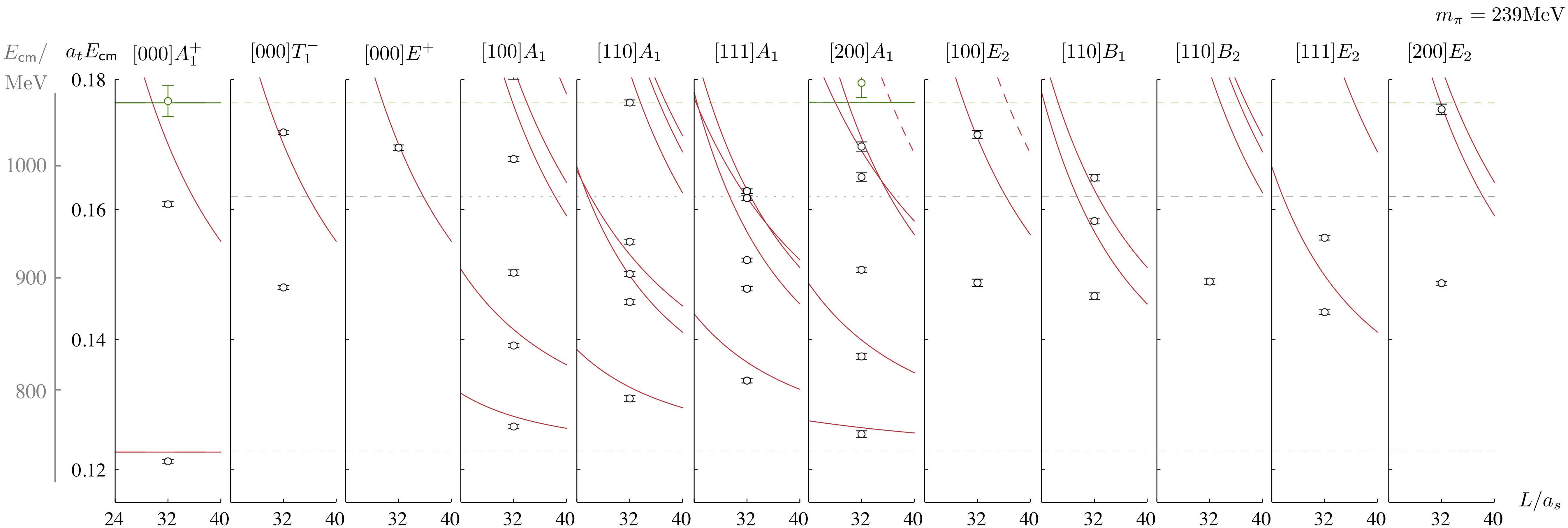
$$\gamma_0 = 0.48(1)$$

$$\gamma_1/s_{\mathrm{thr.}} = 2.29(19)$$

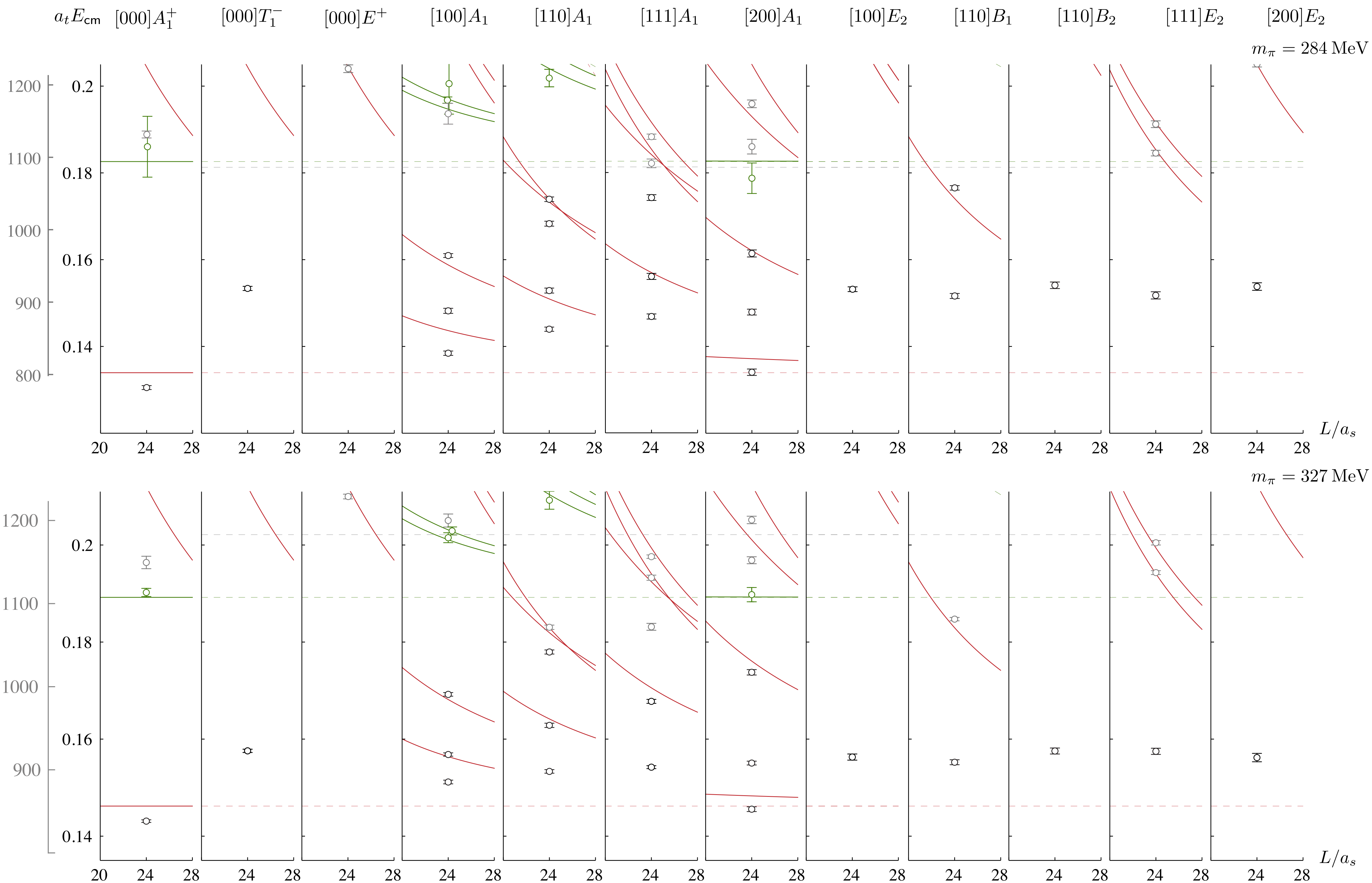
$$m_R = 904(1)\ \mathrm{MeV}$$

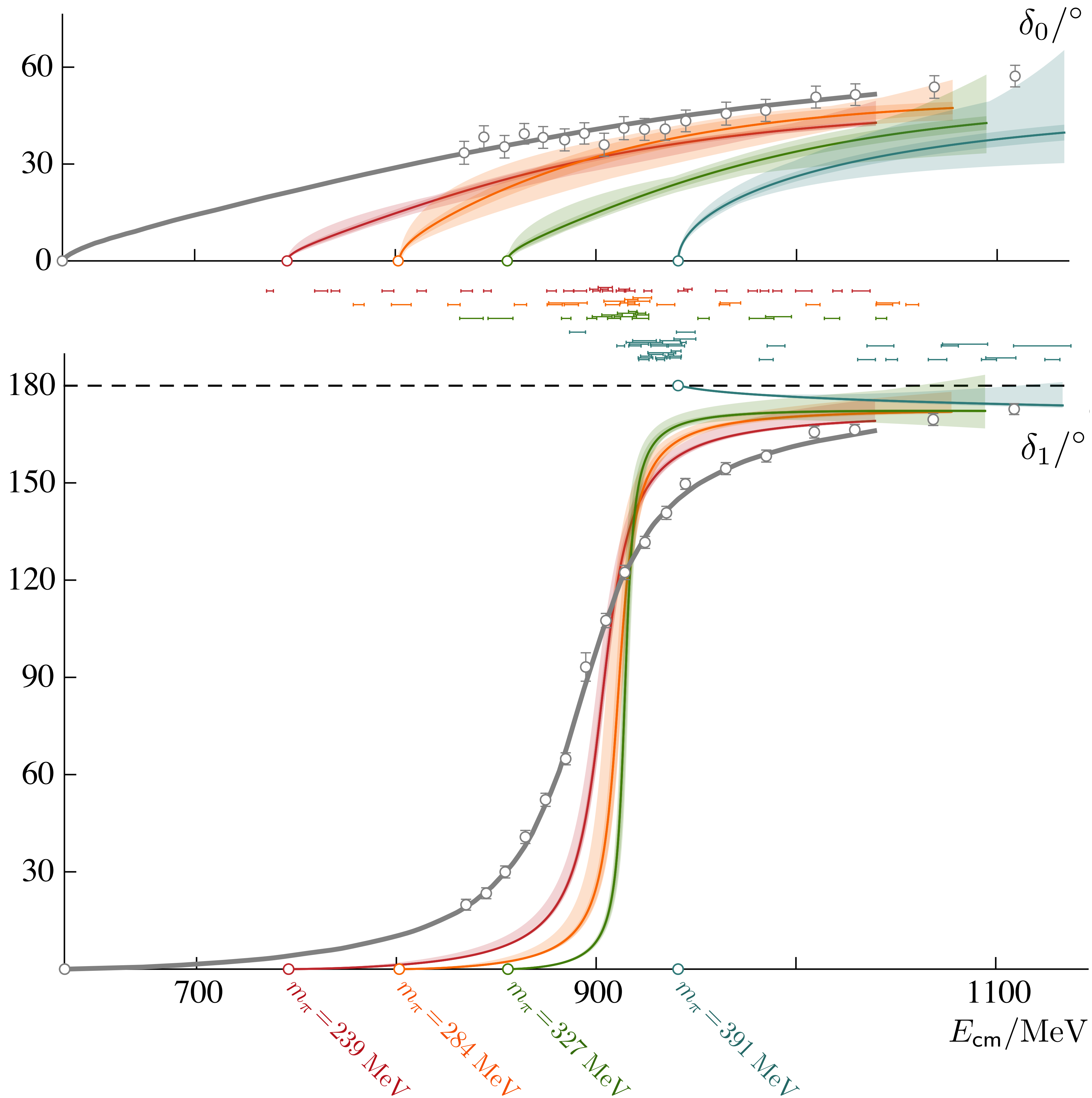
$$g_R = 4.81(7)$$

4 light quark masses
constant (approximately physical) strange quark mass (different to the previous talk)



this analysis: m_π~239, 284 & 327 MeV



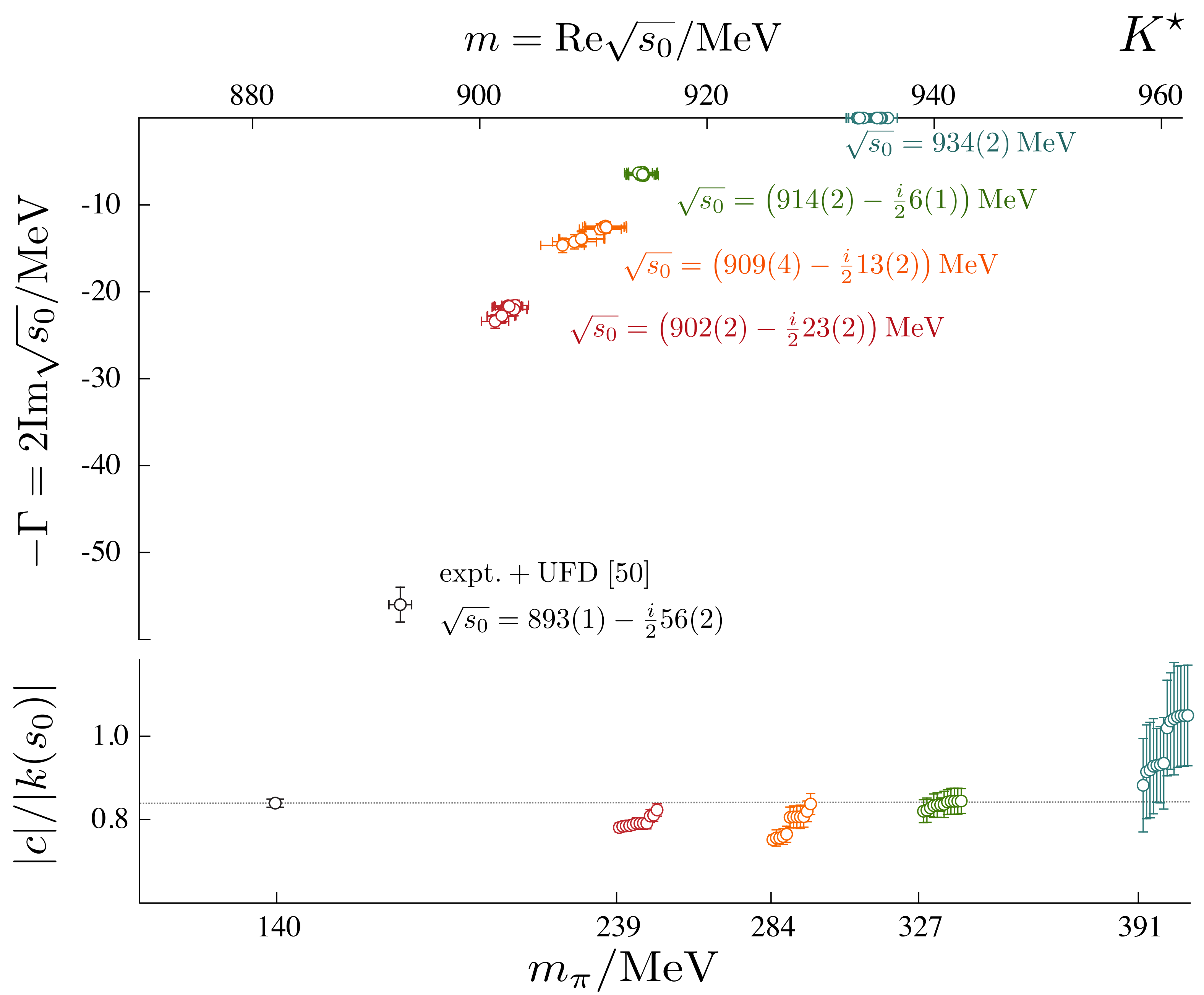


$\delta_0/°$
60
30
0
180
150
120
90
60
30
$\delta_1/°$
700
900
1100
$E_{\mathsf{cm}}/\mathrm{MeV}$
$m_\pi = 239$ MeV
$m_\pi = 284$ MeV
$m_\pi = 327$ MeV
$m_\pi = 391$ MeV

scattering-amplitude poles ⇔ spectroscopic content

$$t \sim \frac{c^2}{s_0 - s}$$

P-wave is easy:

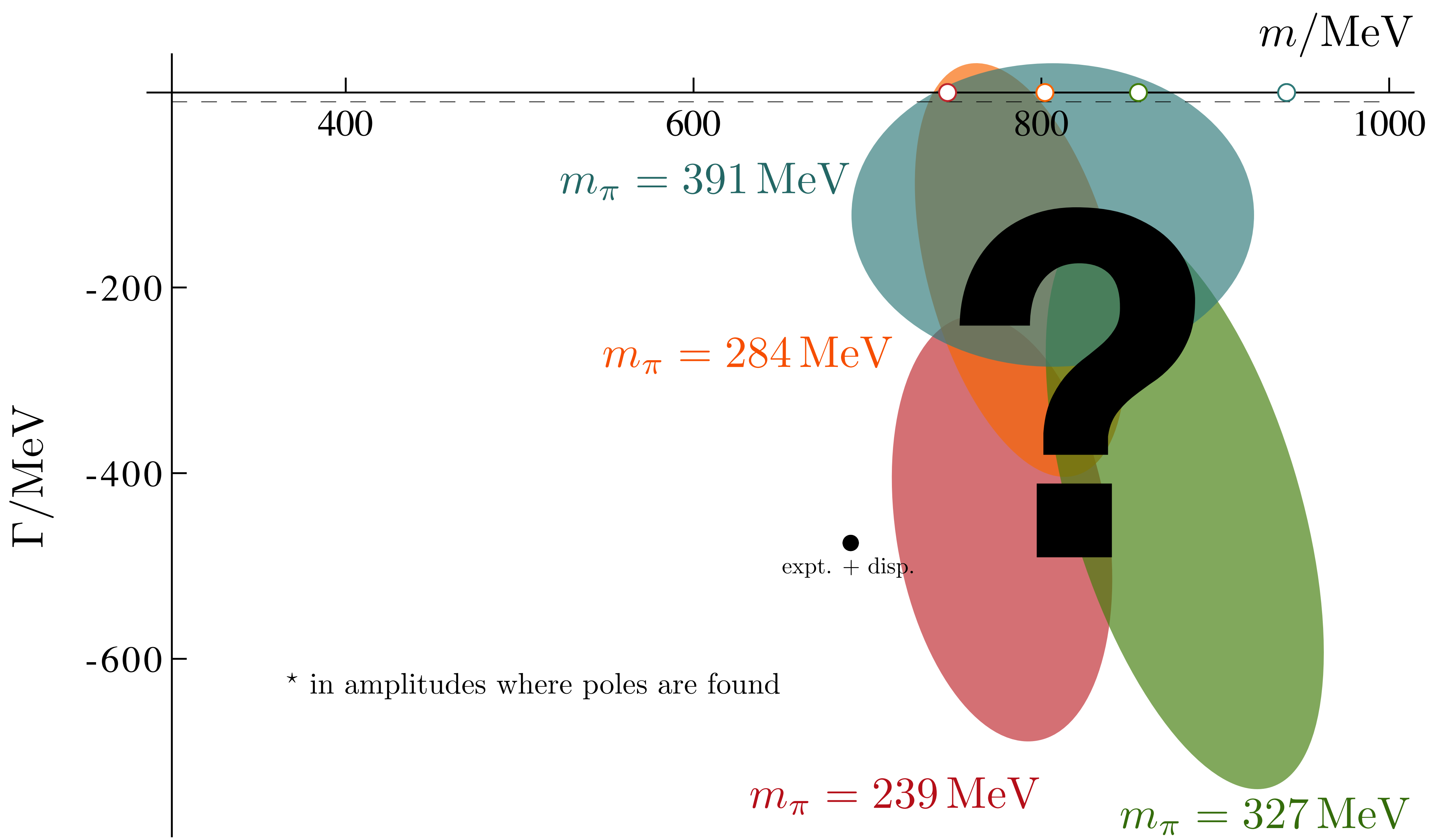


scattering-amplitude poles ⇔ spectroscopic content

$$t \sim \frac{c^2}{s_0 - s}$$

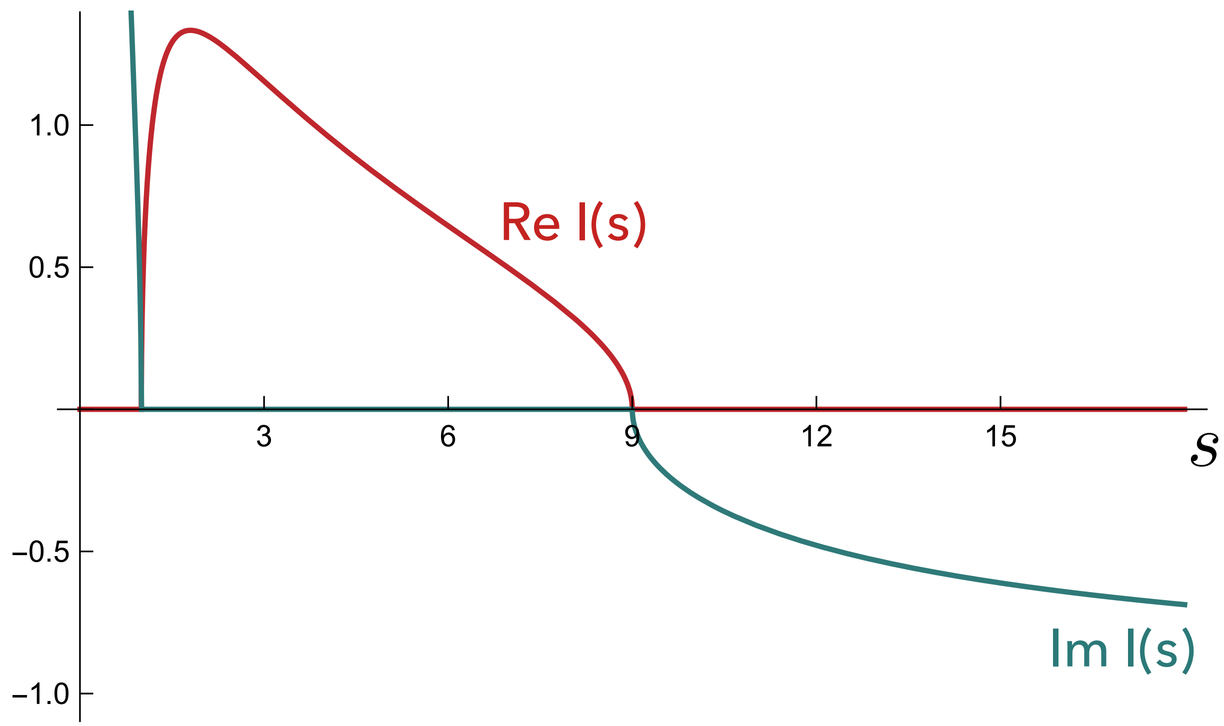
S-wave is inconclusive

- many amplitudes have poles, but not all
- broad spread in position possible at all 4 masses
- uncertainties from different parameterisations often don't overlap
- some amplitudes have no nearby poles: cannot claim a robust determination



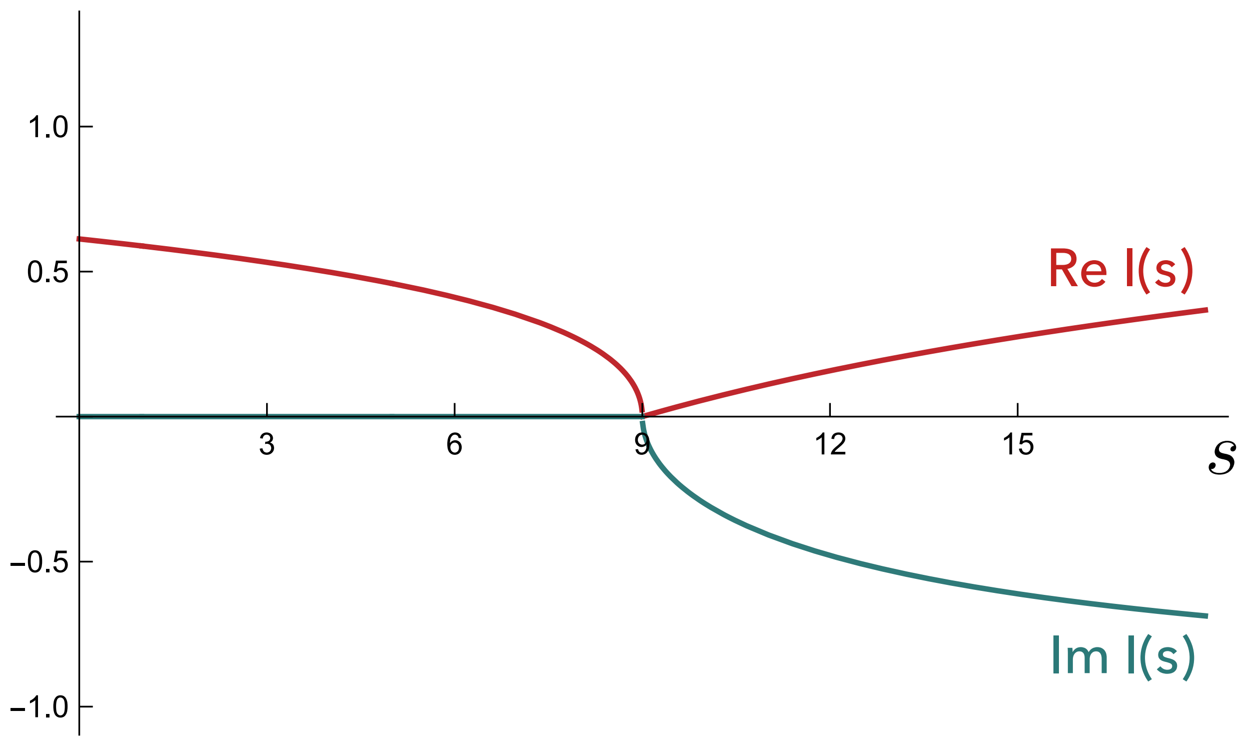
"simple" phase space

unequal masses: $m_1=1$, $m_2=2$



$$I(s) = -i\rho(s)$$
$$= -\frac{i}{s}\left(s-(m_1+m_2)^2\right)^{\frac{1}{2}}\left(s-(m_2-m_1)^2\right)^{\frac{1}{2}}$$

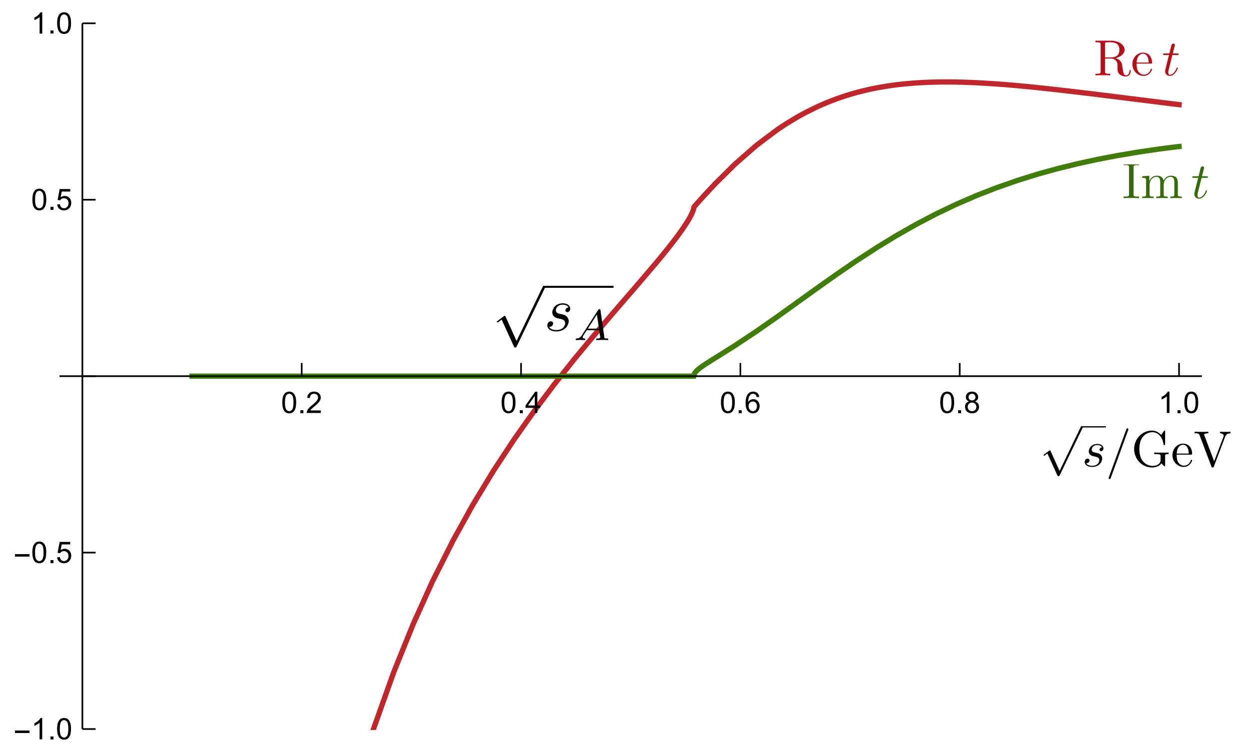
"Chew-Mandelstam" phase space



$$I(s) = I(s_0) - \frac{s-s_0}{\pi}\int_{s_0}^{\infty} ds' \, \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

$$\mathrm{Im} I(s) = -\rho(s)$$

a zero in t below pi-K threshold



from fit using 239 MeV spectra

amplitude has CM phase space, linear K(s), zero arises from minimisation

often argued as an essential feature
arises in chiral perturbation theory, at leading order:

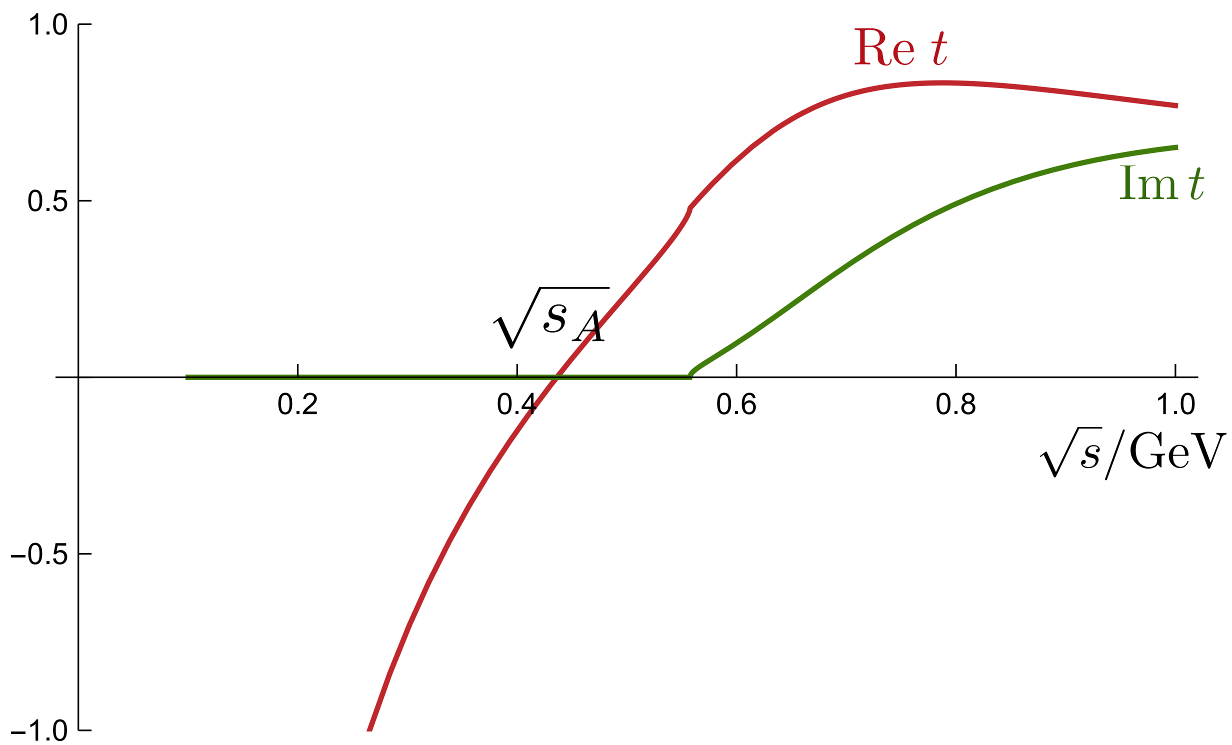
$$s_A = m_K^2 + m_\pi^2 \pm 2\left(4m_\pi^4 - 7m_K^2 m_\pi^2 + 4m_K^4\right)^{\frac{1}{2}}$$

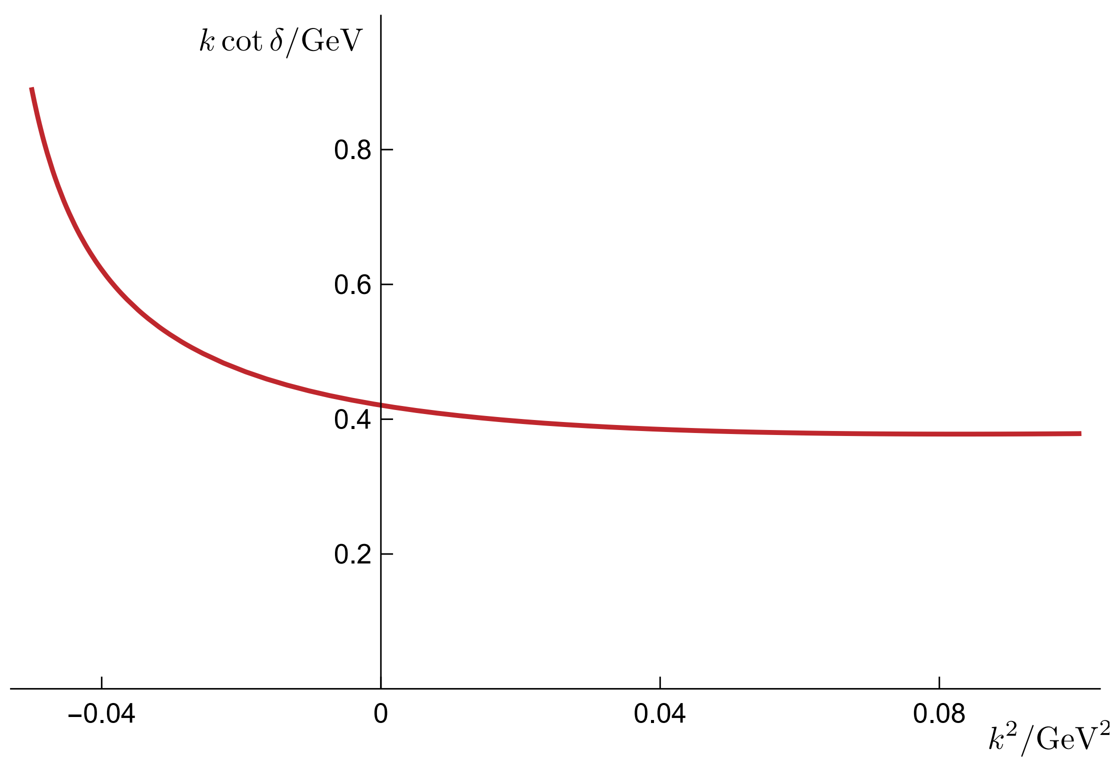
~(480 MeV)^2 at all mass points considered

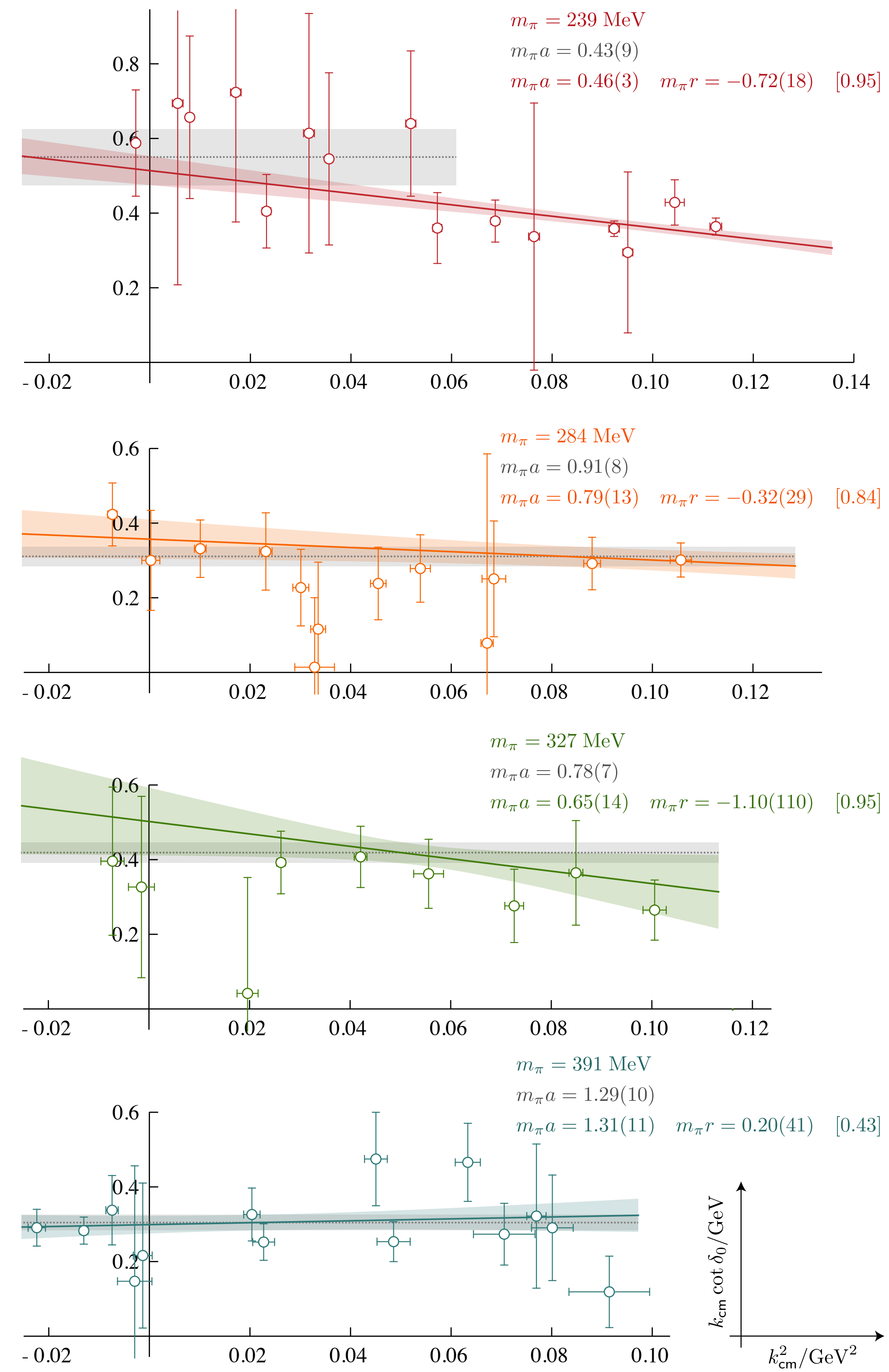
SU(3) ChiPT, I=1/2, S-wave projection

strong correlation with extracted pole position

adler zeros

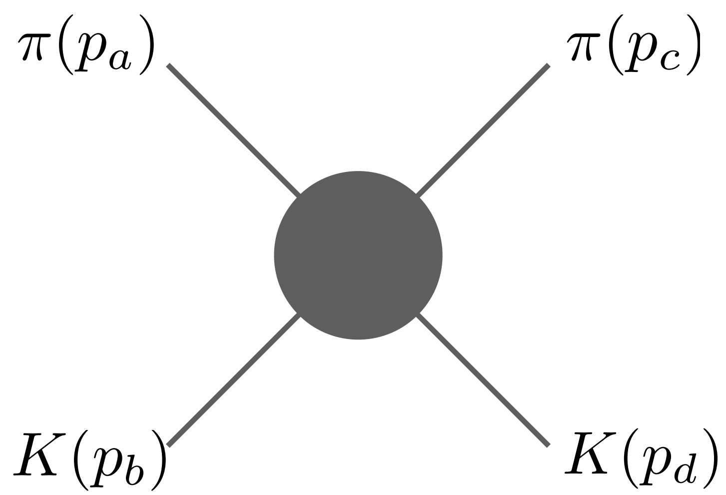


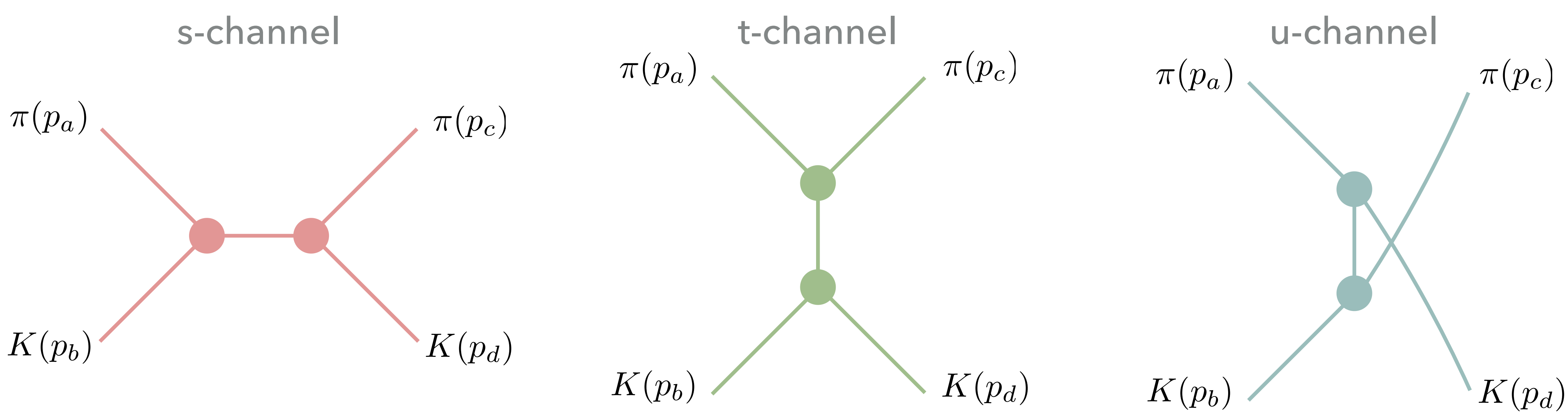




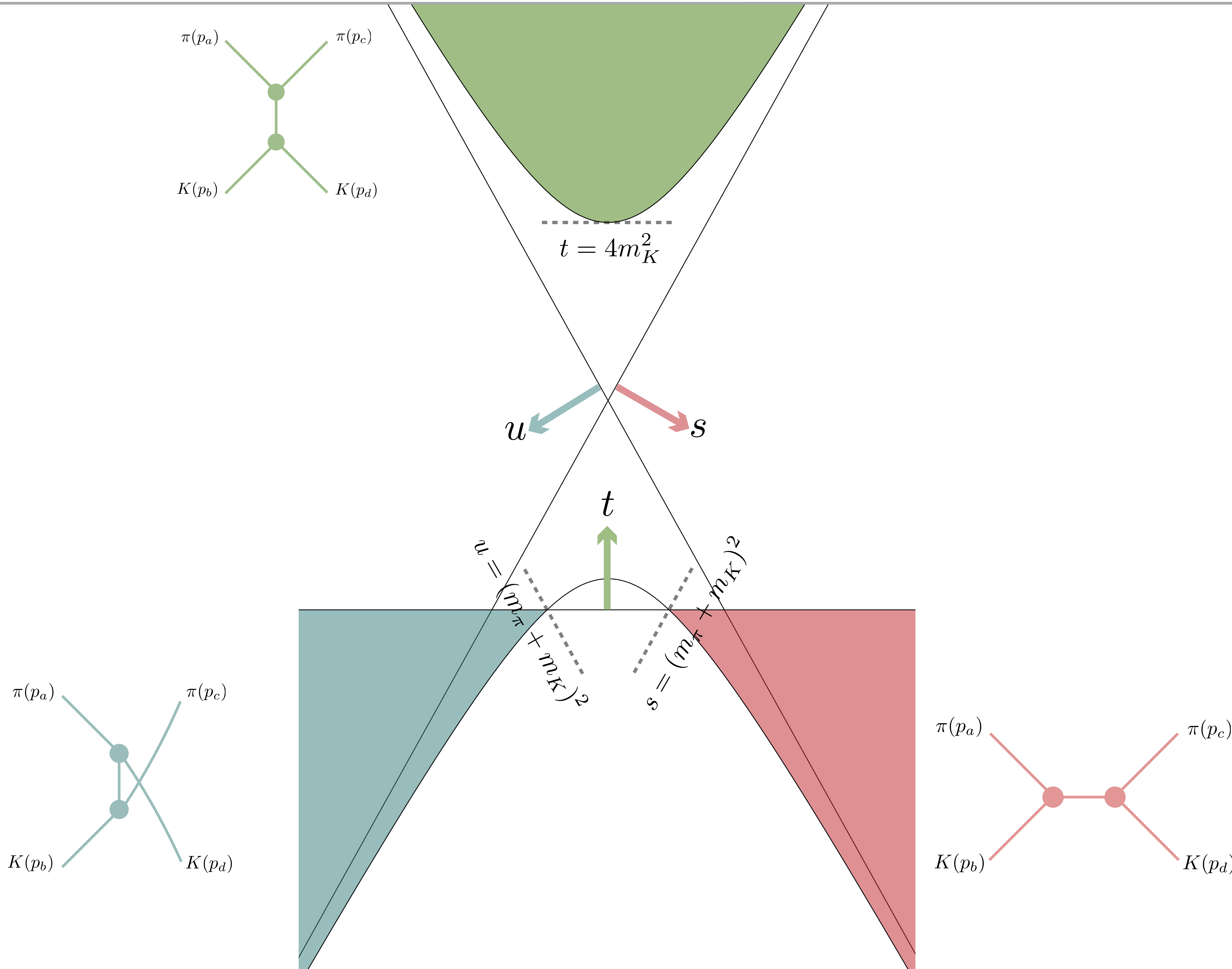
zero in t below threshold
⇒ pole in $k\cot\delta$ at negative k^2

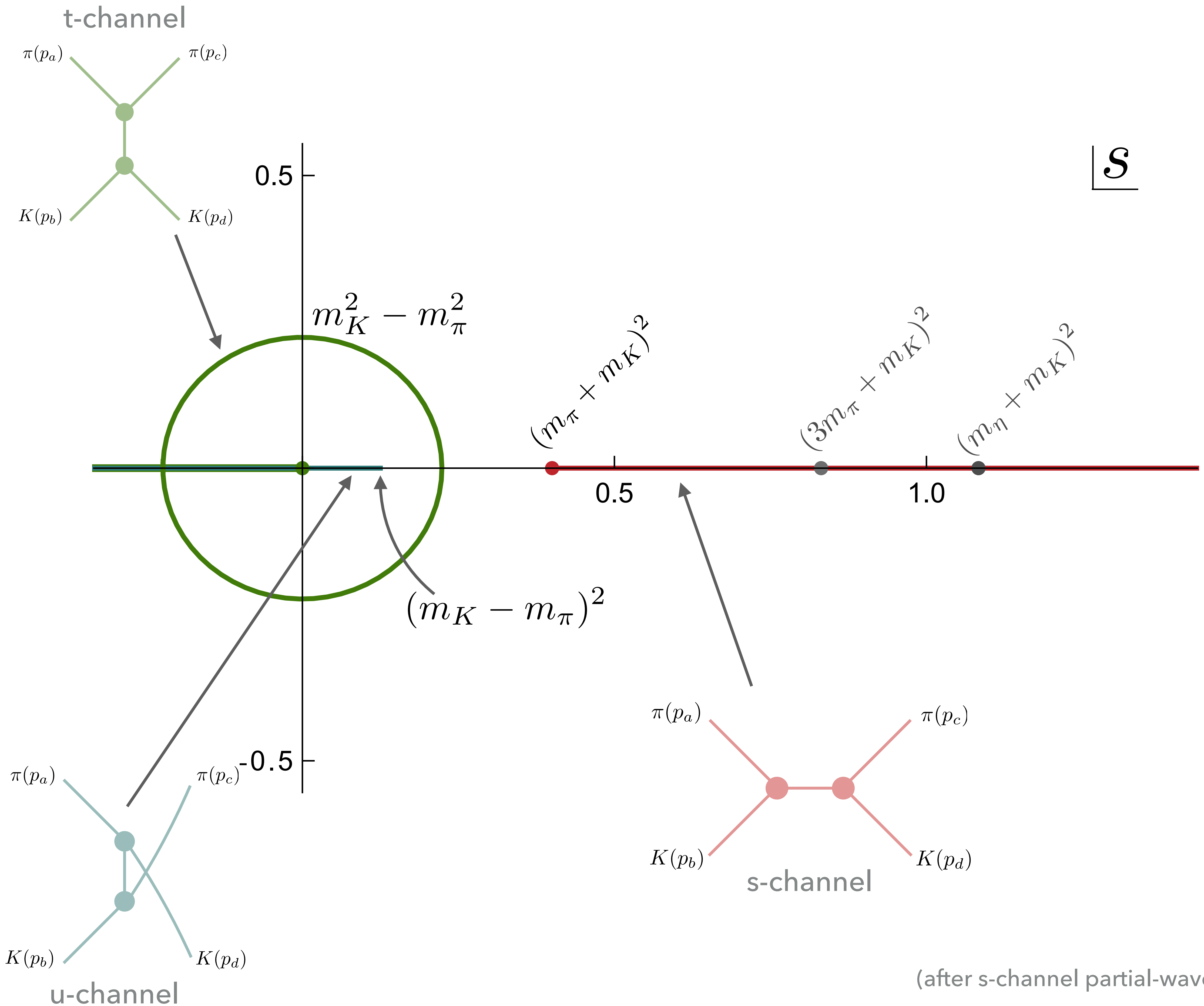
no clear sign from lattice spectra


$\pi(p_a)$
$\pi(p_c)$
$K(p_b)$
$K(p_d)$

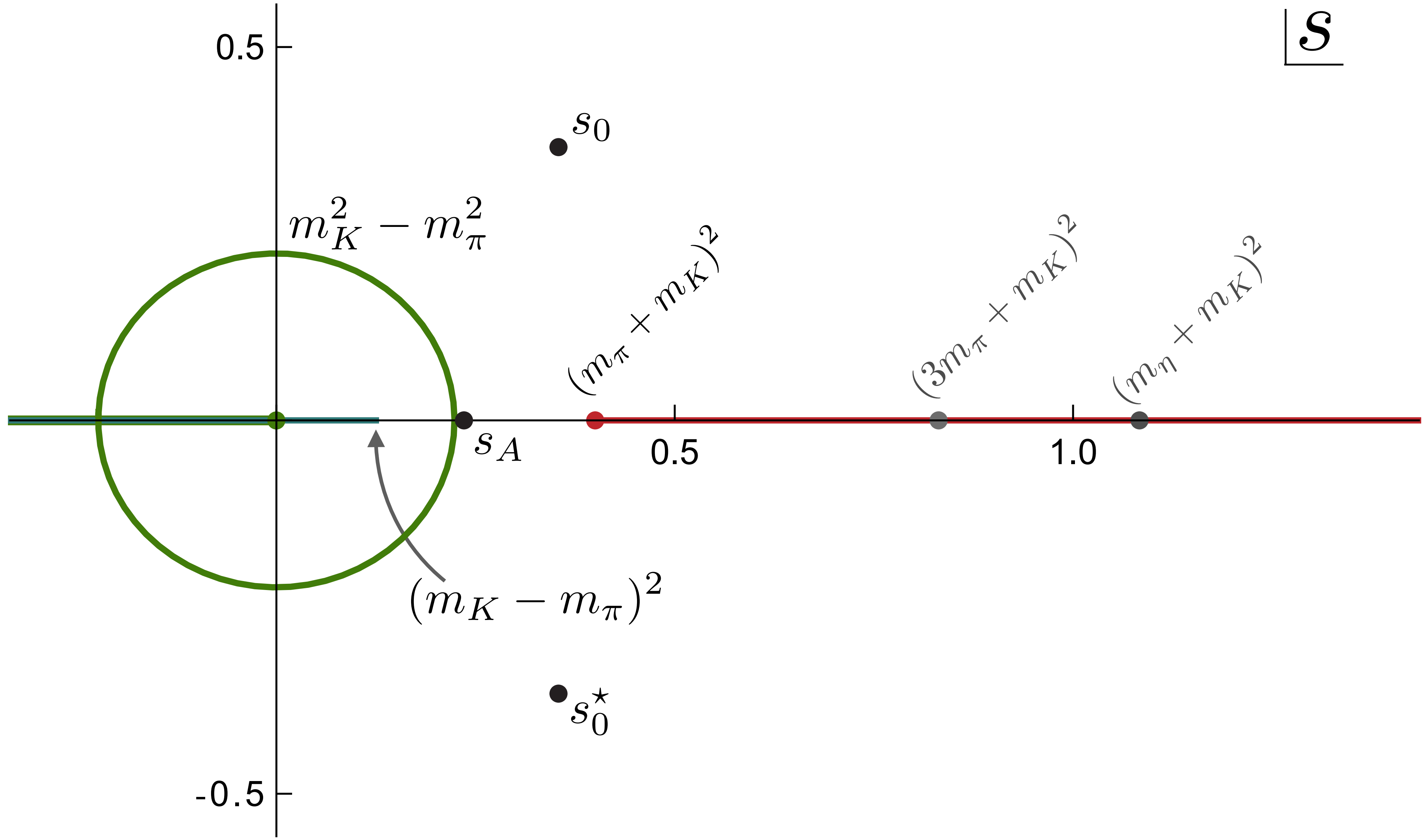

s-channel
t-channel
u-channel
$\pi(p_a)$
$\pi(p_c)$
$K(p_b)$
$K(p_d)$
$\pi(p_a)$
$\pi(p_c)$
$K(p_b)$
$K(p_d)$
$\pi(p_a)$
$\pi(p_c)$
$K(p_b)$
$K(p_d)$

cross-channel scattering

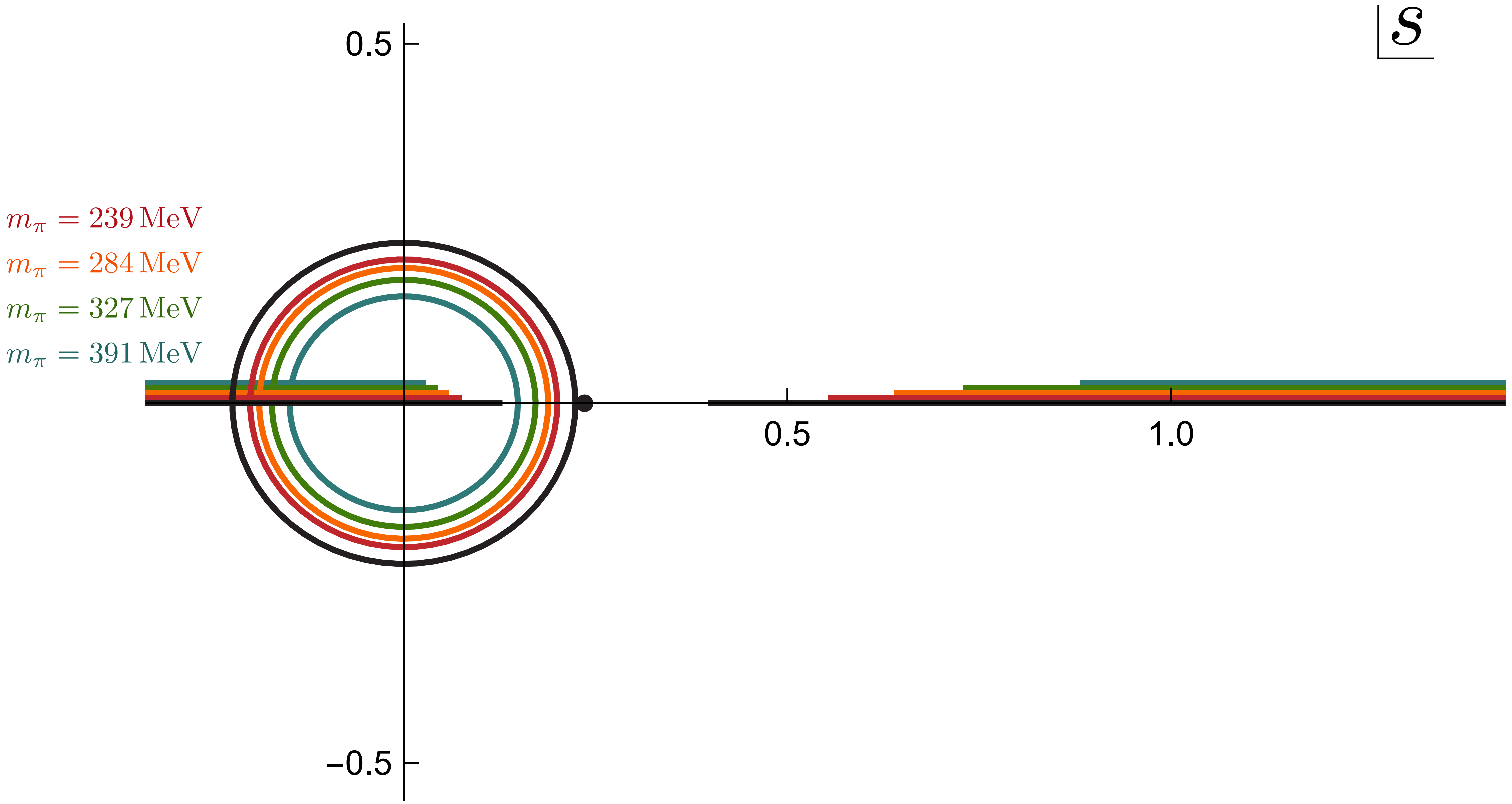




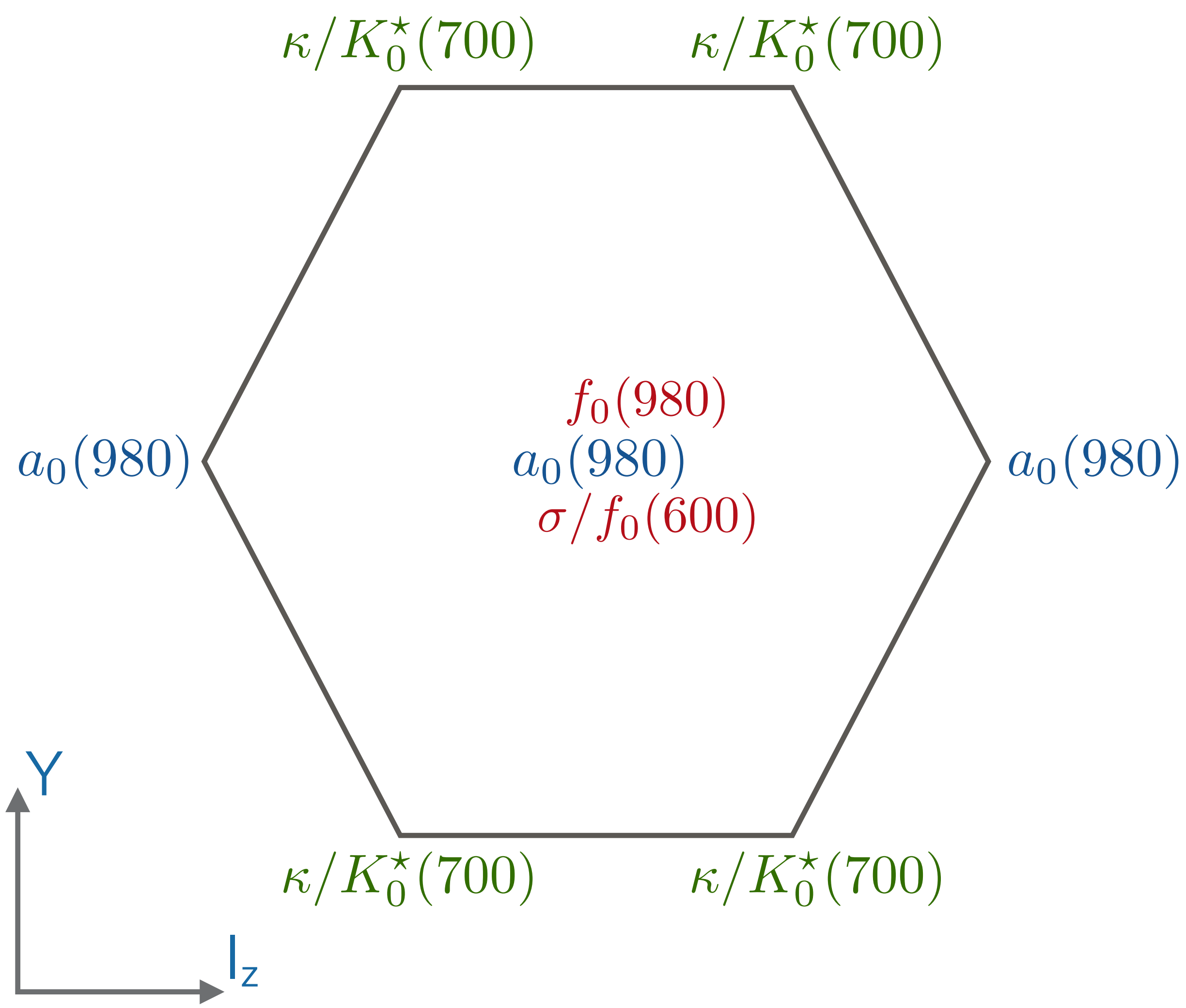
(after s-channel partial-wave projection)



hard to argue for zero at s_A without including left cuts

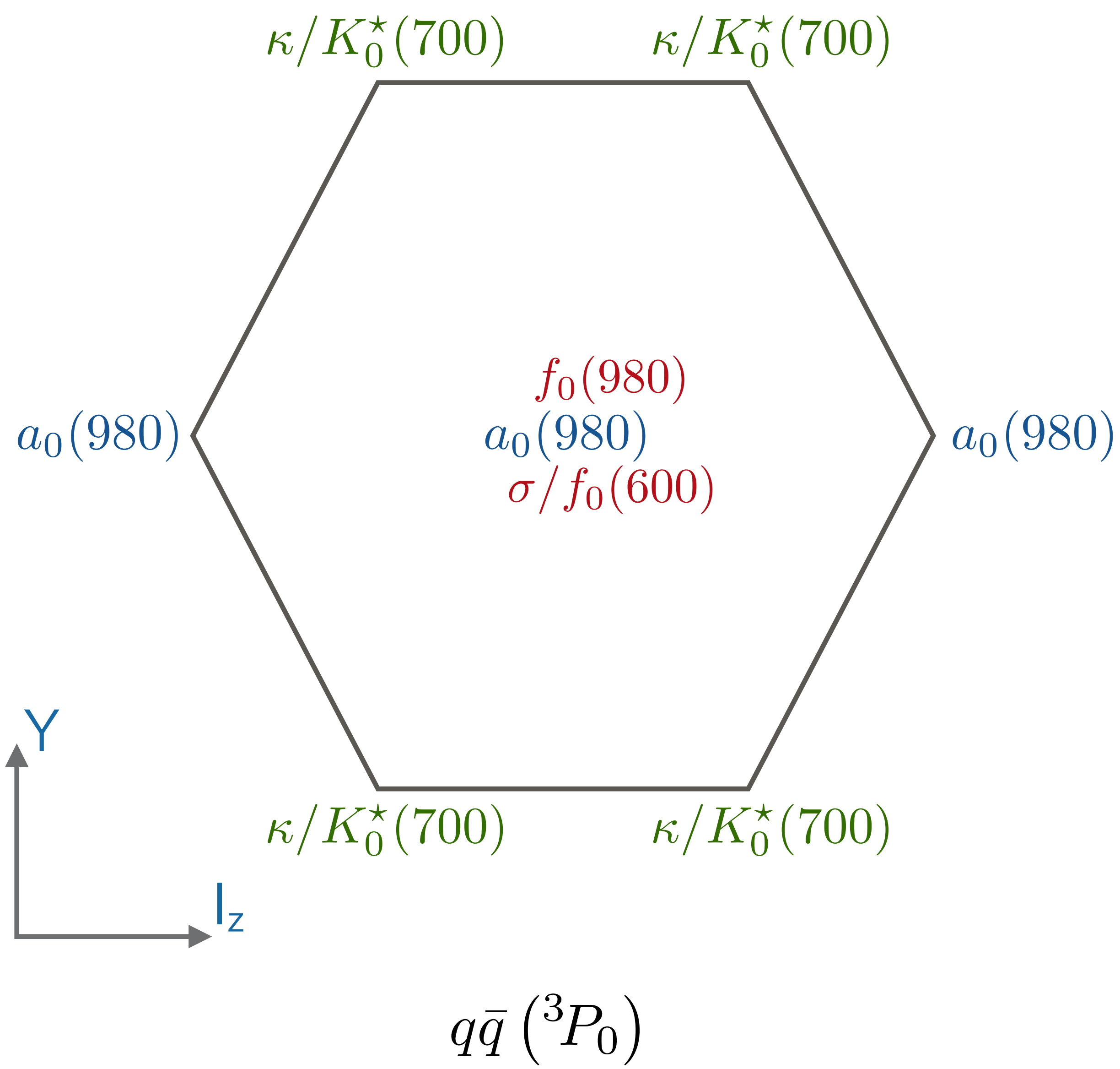


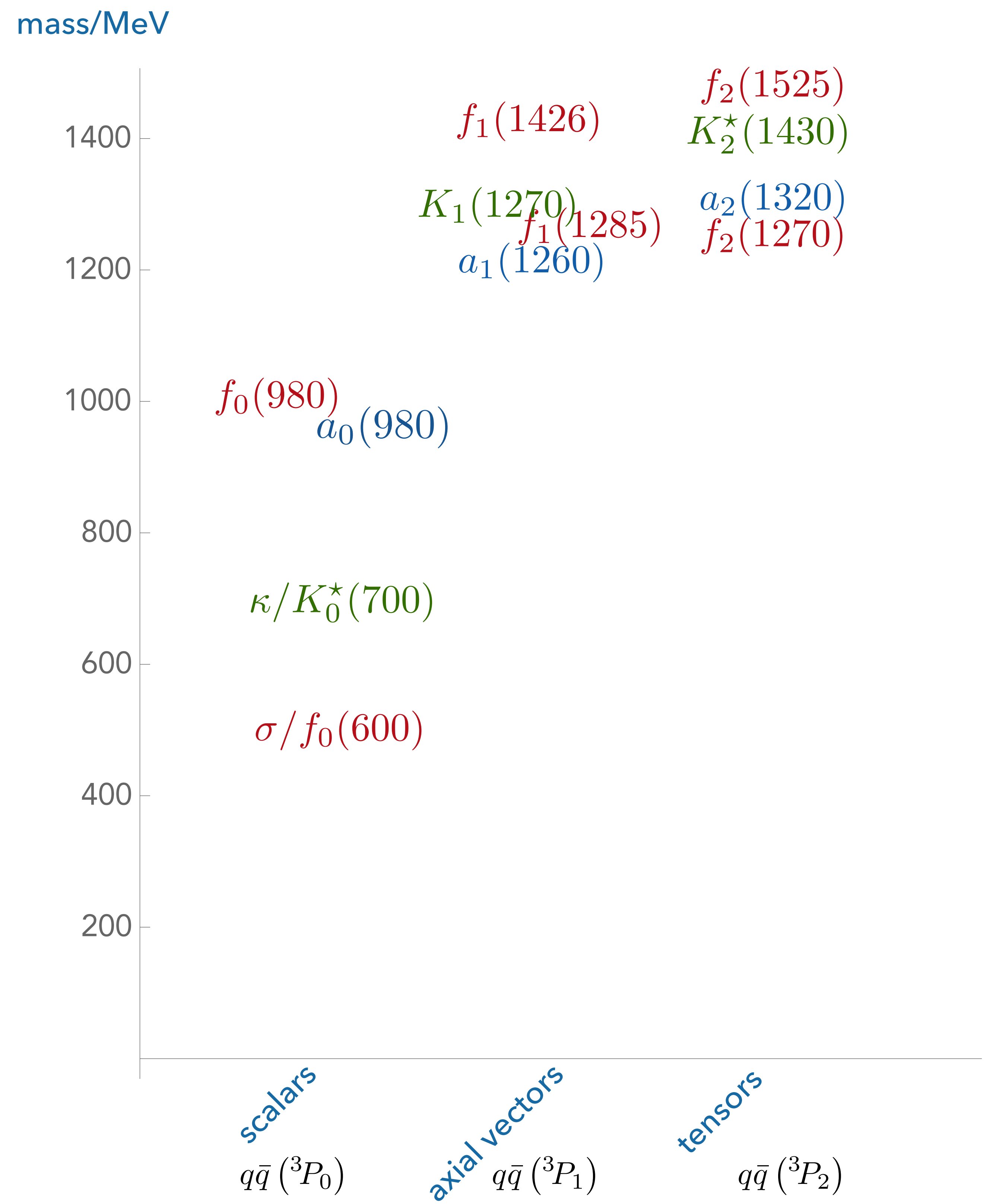
hard to argue for zero at s_A without including left cuts



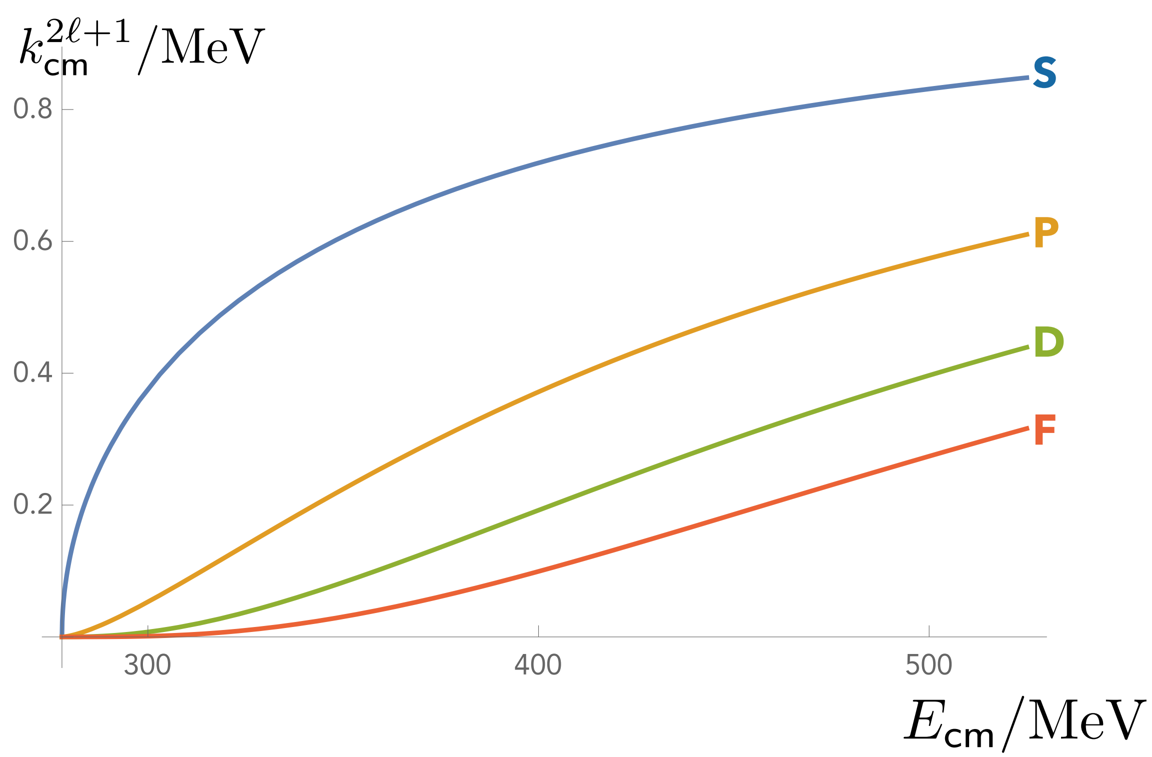
$q\bar{q}\left({}^3P_0\right)$

What about the masses?





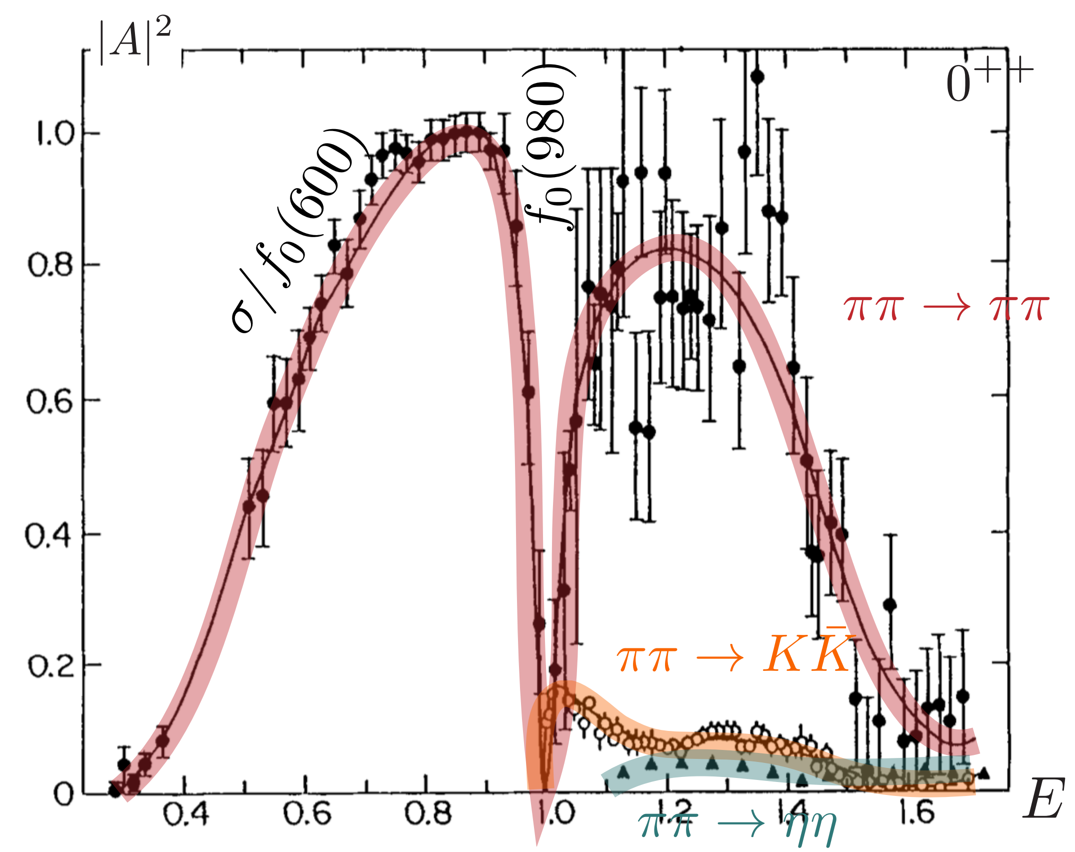
In the scalar sector, amplitudes grow rapidly from threshold:



σ and κ are broad (width ~ mass)

$f_0(980)$ and $a_0(980)$ lie very close to KK threshold

CERN-Munich, ANL, BNL



3 volumes
L=16, 20, 24
anisotropic (3.5 finer spacing in time)
Wilson-Clover

m_π=391MeV

m_K=549MeV

$$\begin{bmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{bmatrix}$$

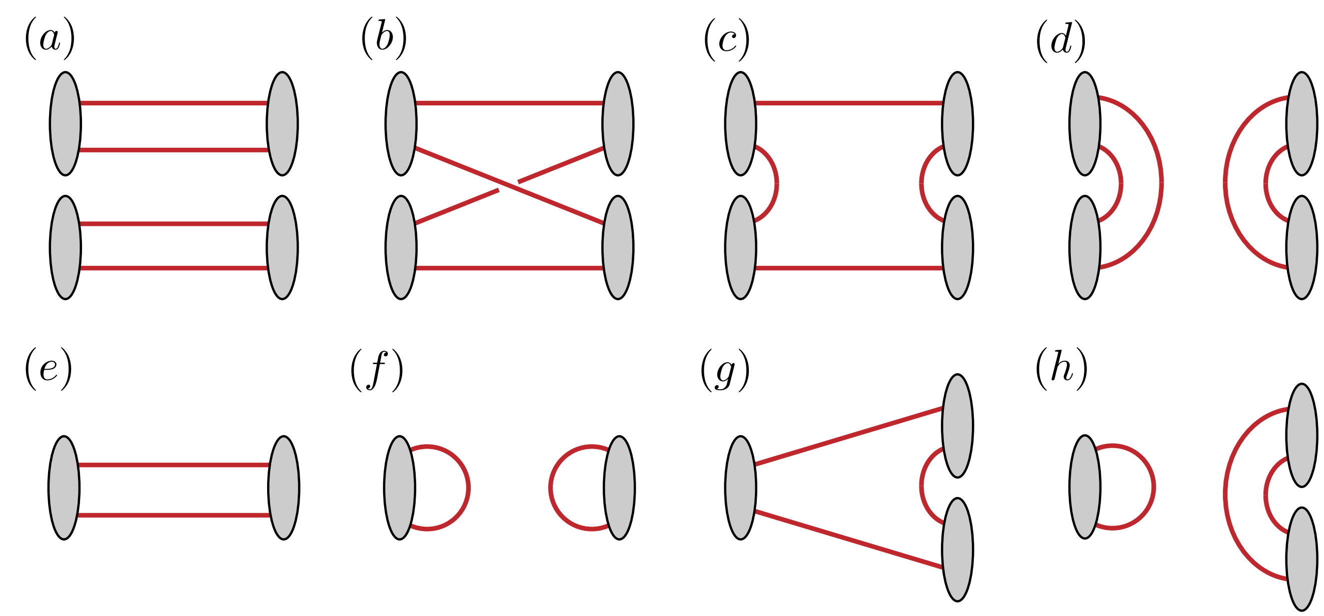
operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$ local qq-like constructions

$$\sum_{\vec{p}_1+\vec{p}_2 \in \vec{p}} C(\vec{p}_1, \vec{p}_2; \vec{p}) \Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$ two-hadron constructions

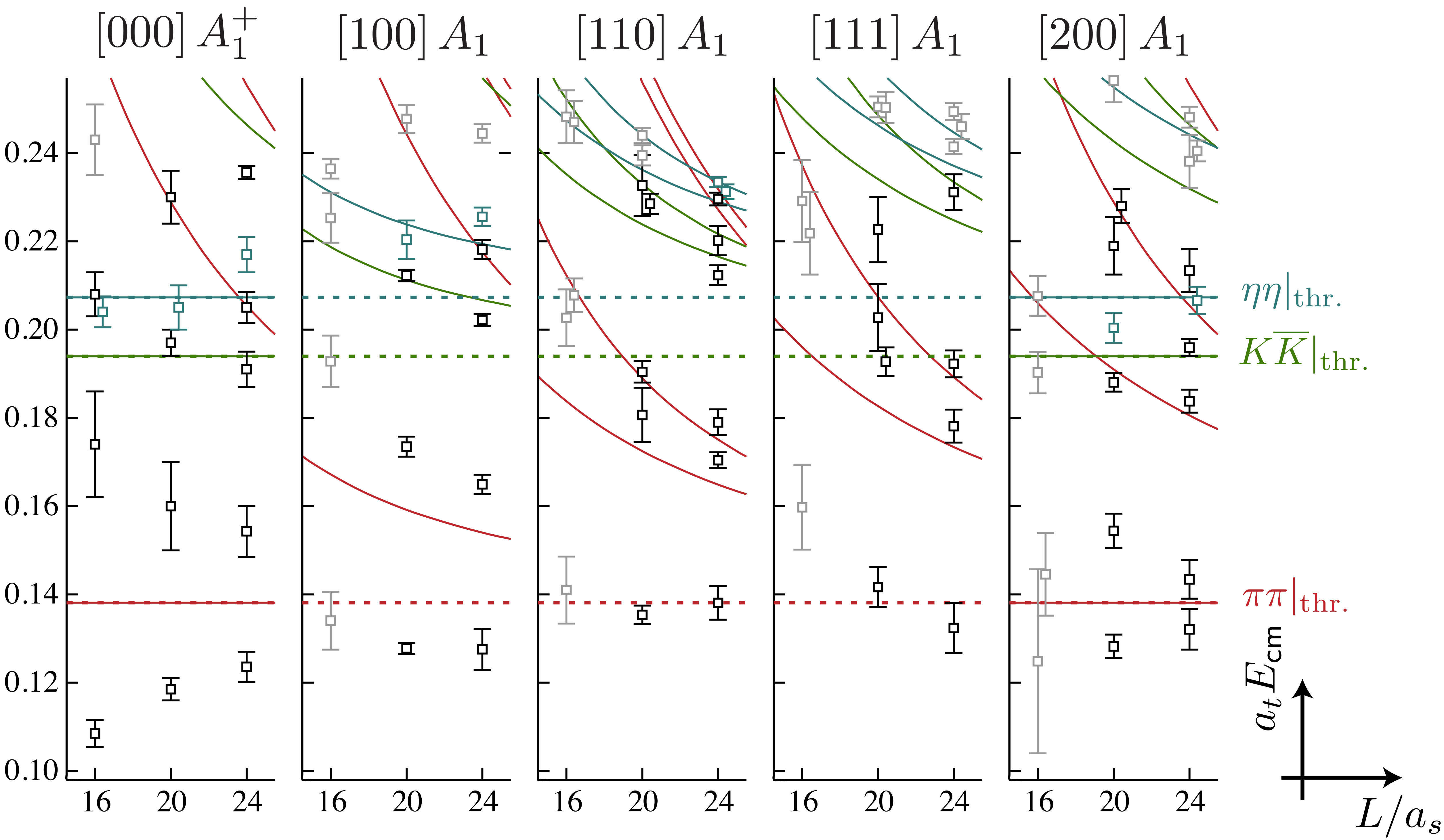
$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$ uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many wick contractions, eg just pi-pi & qq operators:

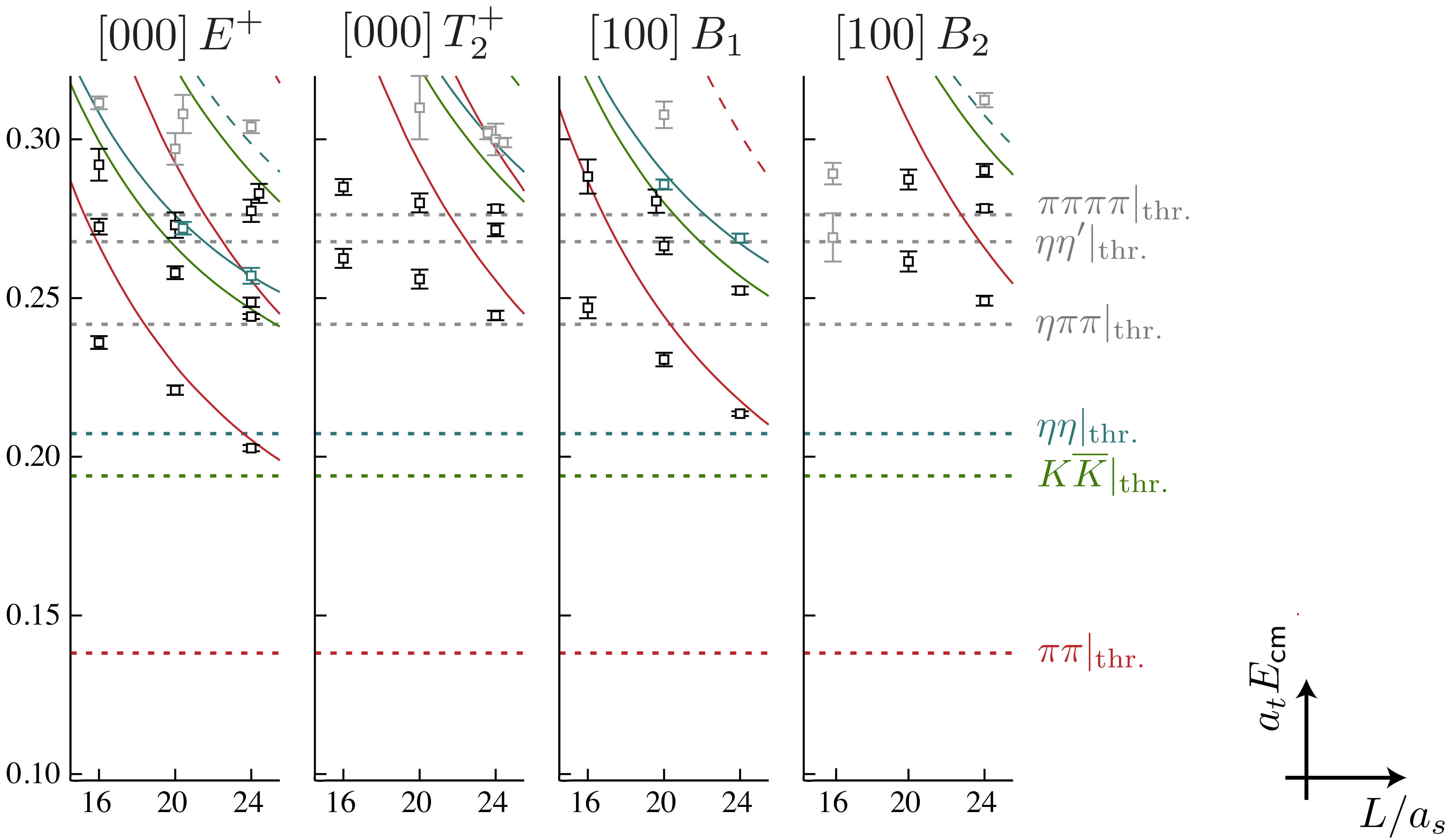


- we compute a large correlation matrix
- then use GEVP to extract energies

Briceno et al, Phys. Rev. D 97, 054513 [Editors' Suggestion]



similar construction of operators: local $q\bar{q}$ & 2-hadron
conservatively 57 energy levels
dominated by S-wave interactions



conservatively 34 energy levels
dominated by D-wave interactions

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

determinant condition:

- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations

- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

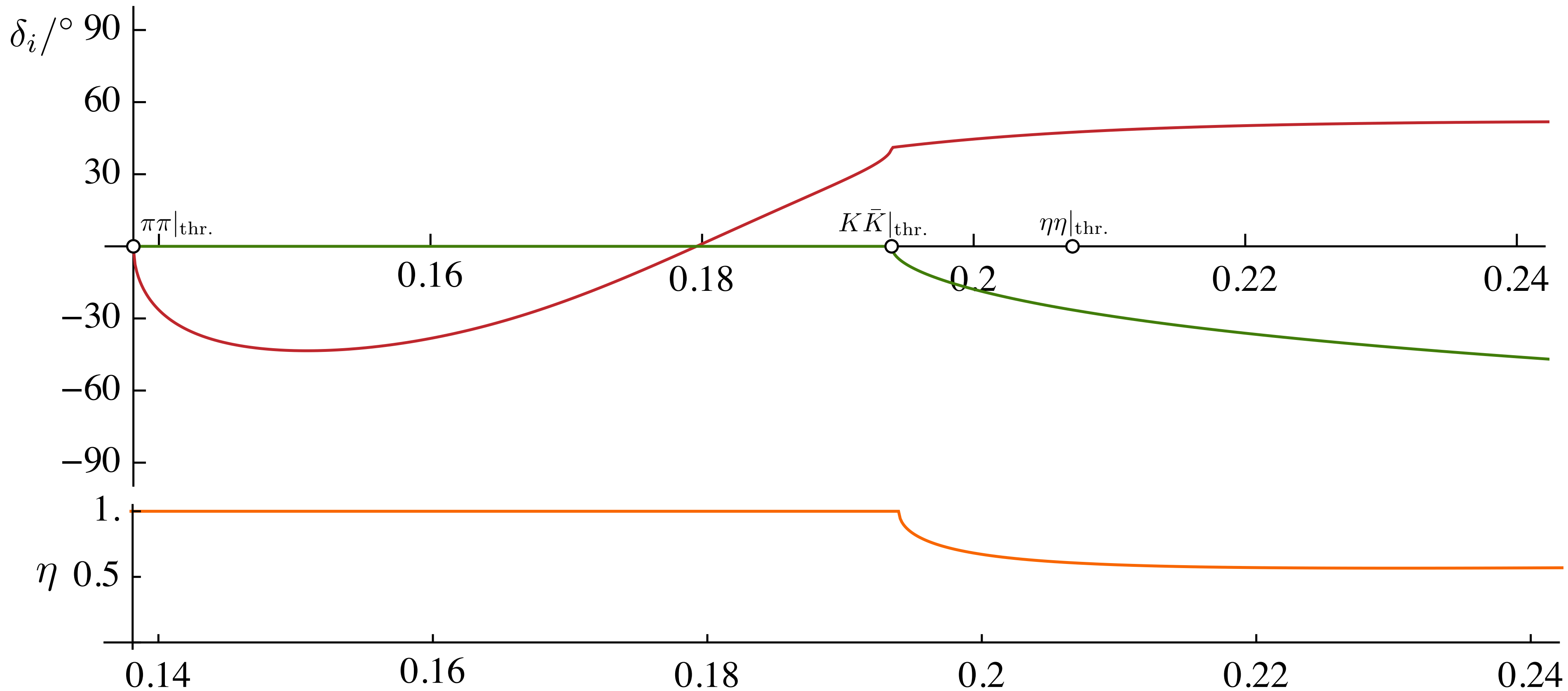
use a dispersion relation to generate a real part from iρ

- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut

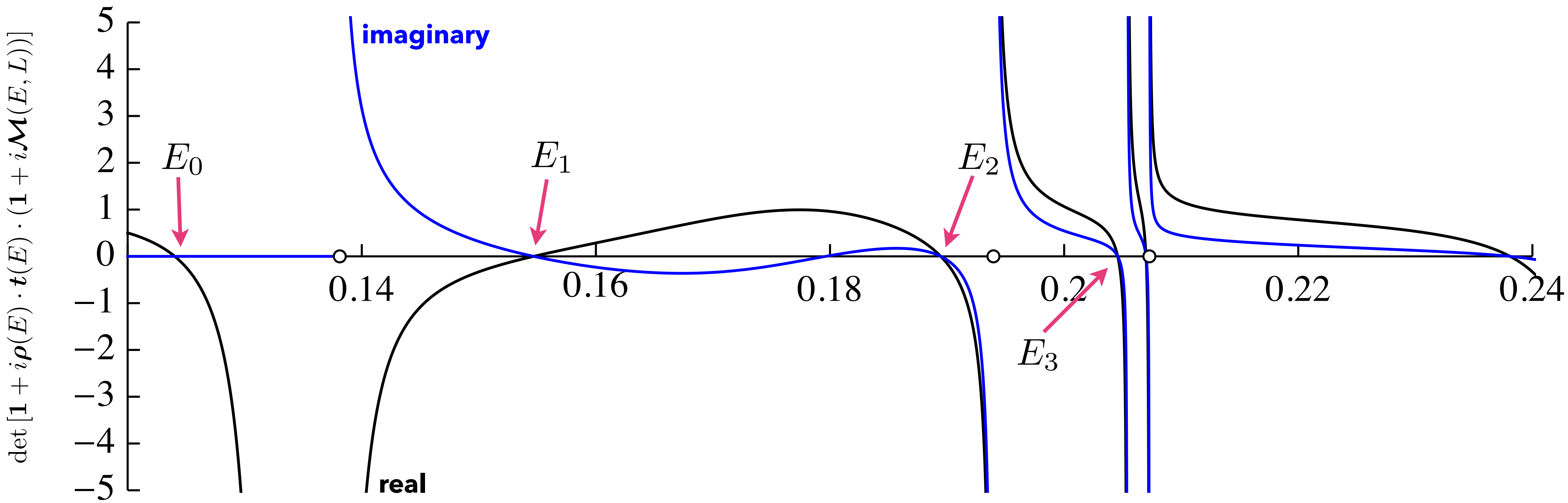
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1 + i\delta_2}$$

$$S_{ii} = \eta e^{2i\delta_i}$$

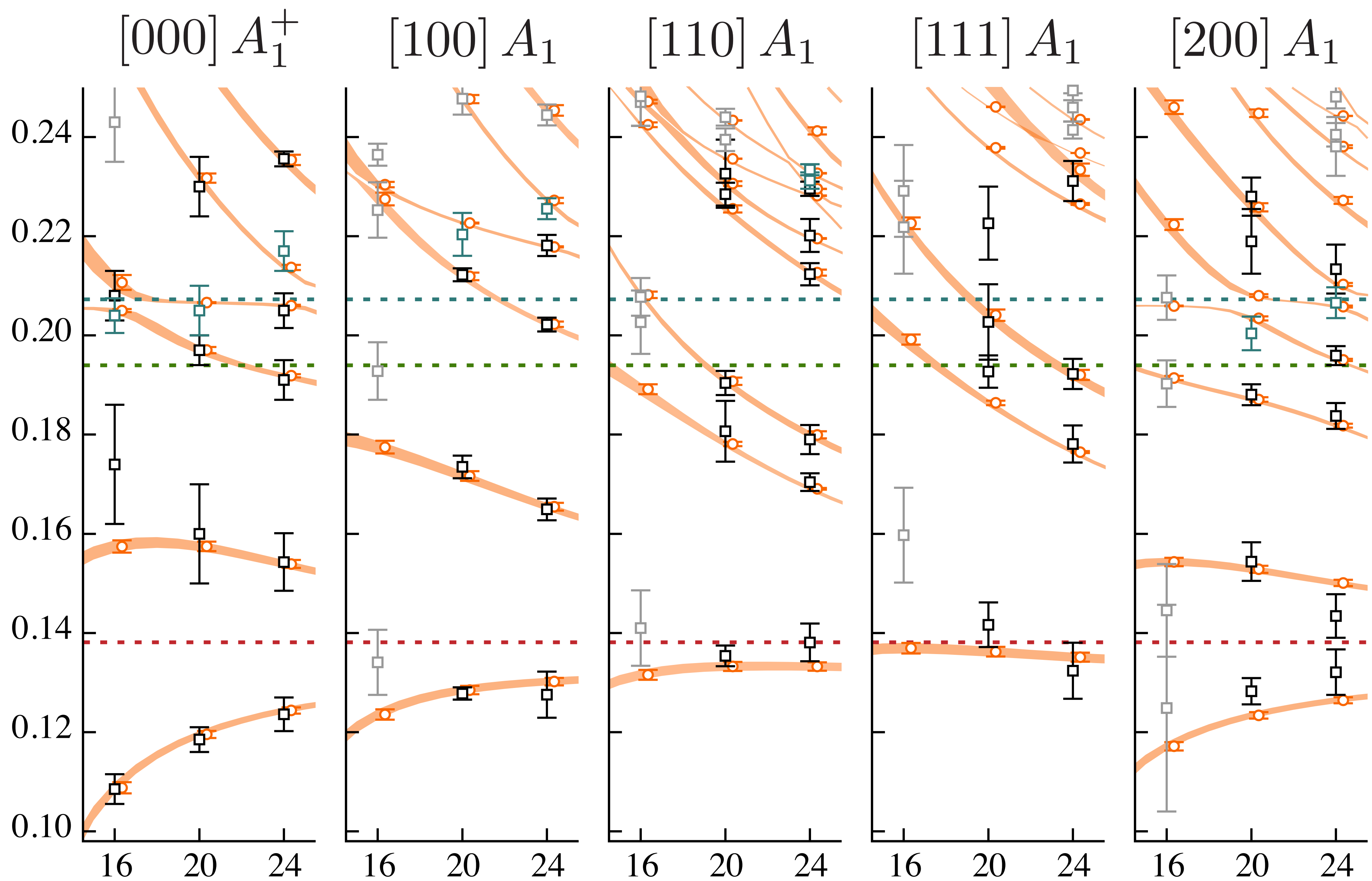


identify the solutions



An example S-wave spectrum fit

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$



$$\chi^2/N_{\text{dof}} = \frac{44.0}{57-8} = 0.90$$

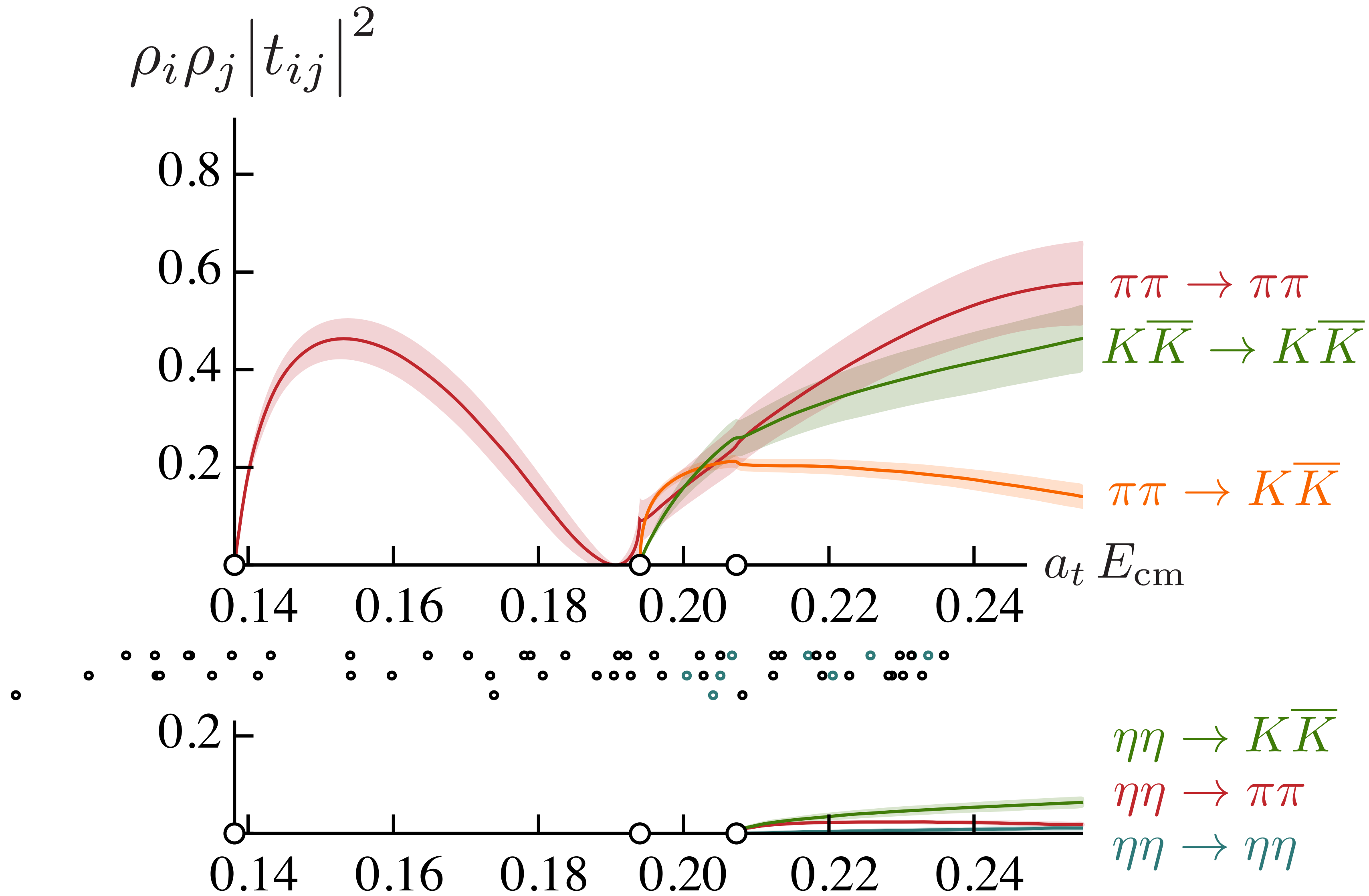
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels



An example S-wave spectrum fit

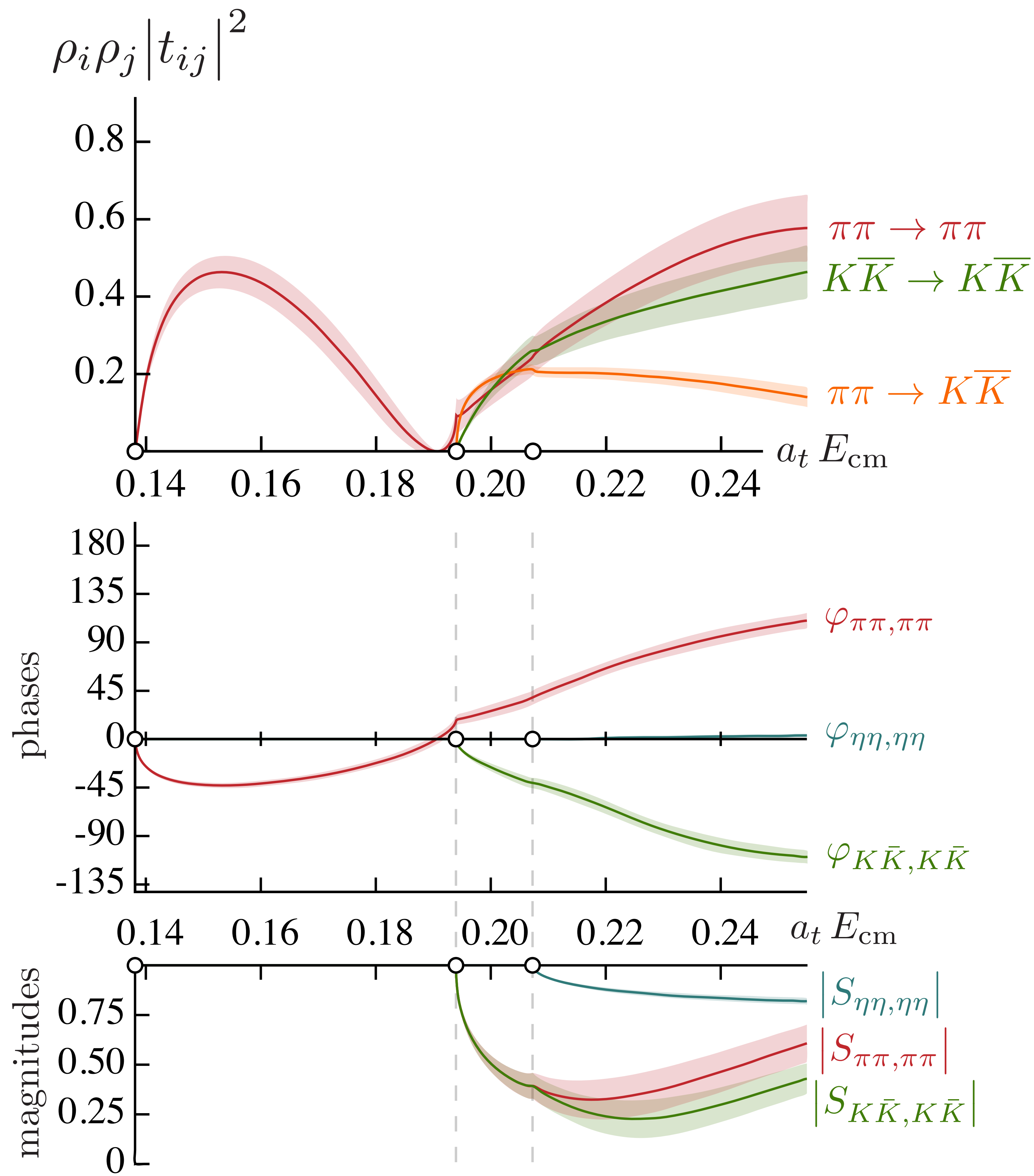
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

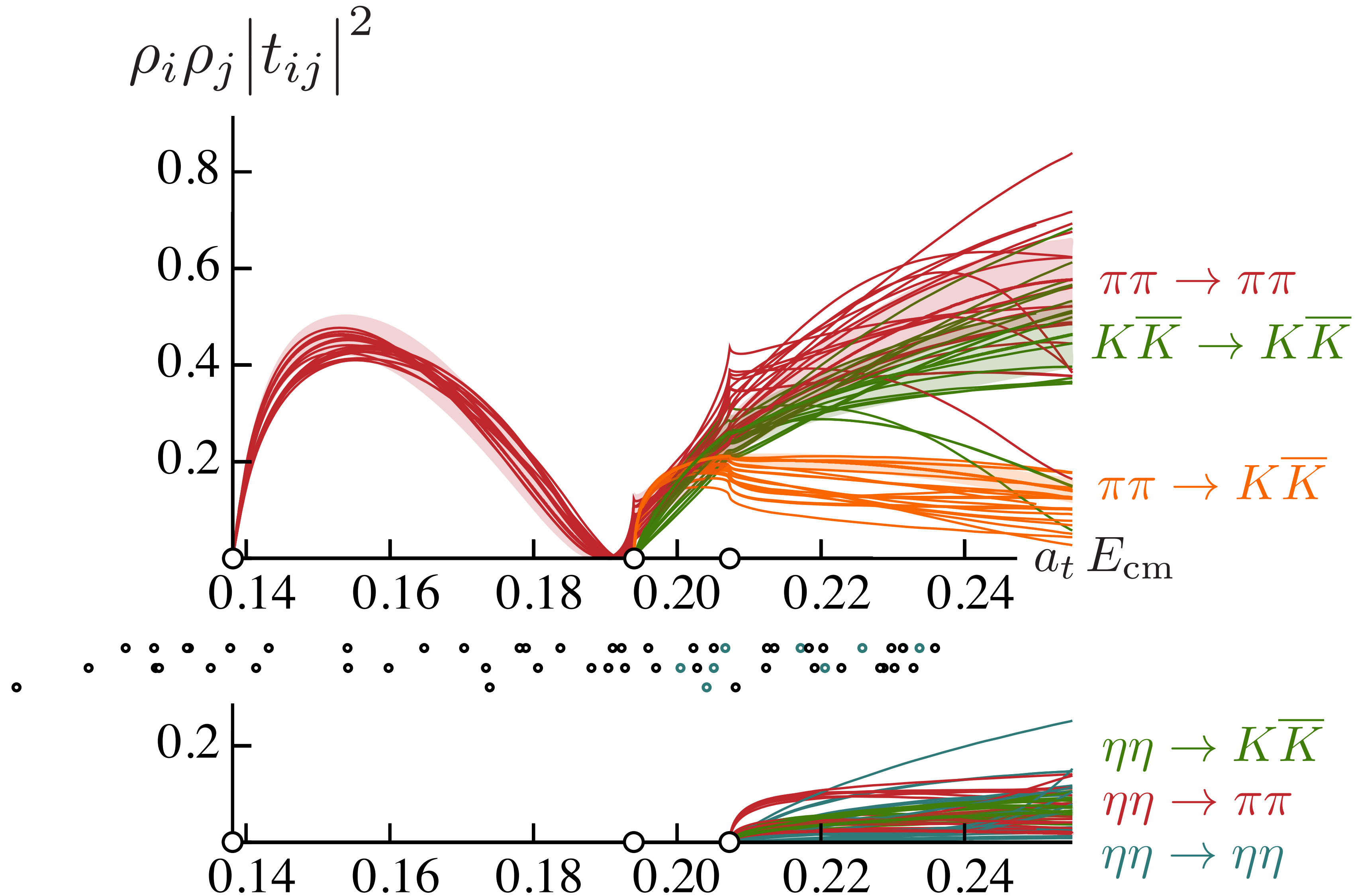
$$S_{ii}(E_{\mathsf{cm}}) = |S_{ii}(E_{\mathsf{cm}})|\, e^{2i\phi_{ii}(E_{\mathsf{cm}})}$$



20 amplitudes

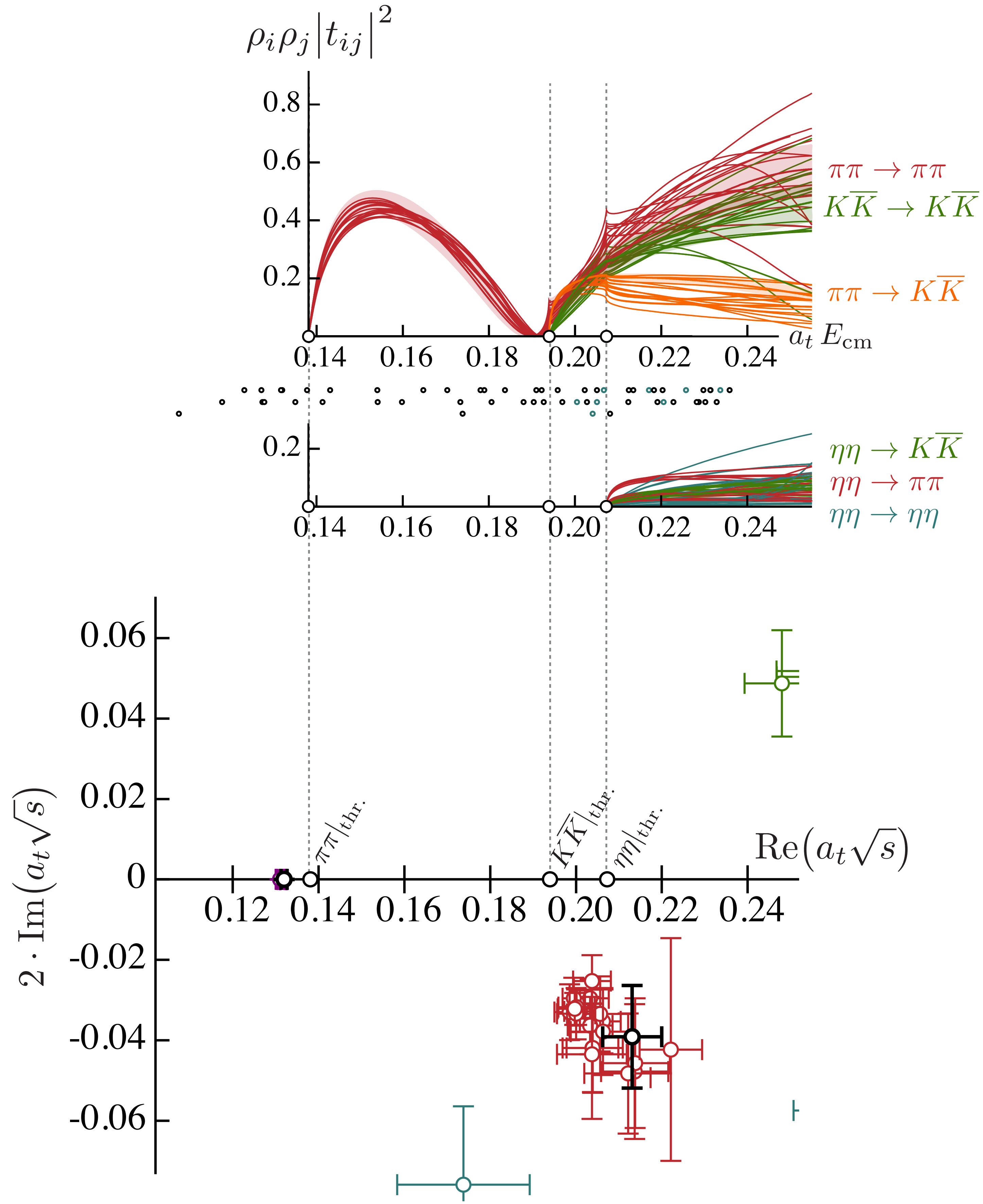
$\chi^2/N_{\mathrm{dof}} < 1.05$

57 energy levels



Near a t-matrix pole

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

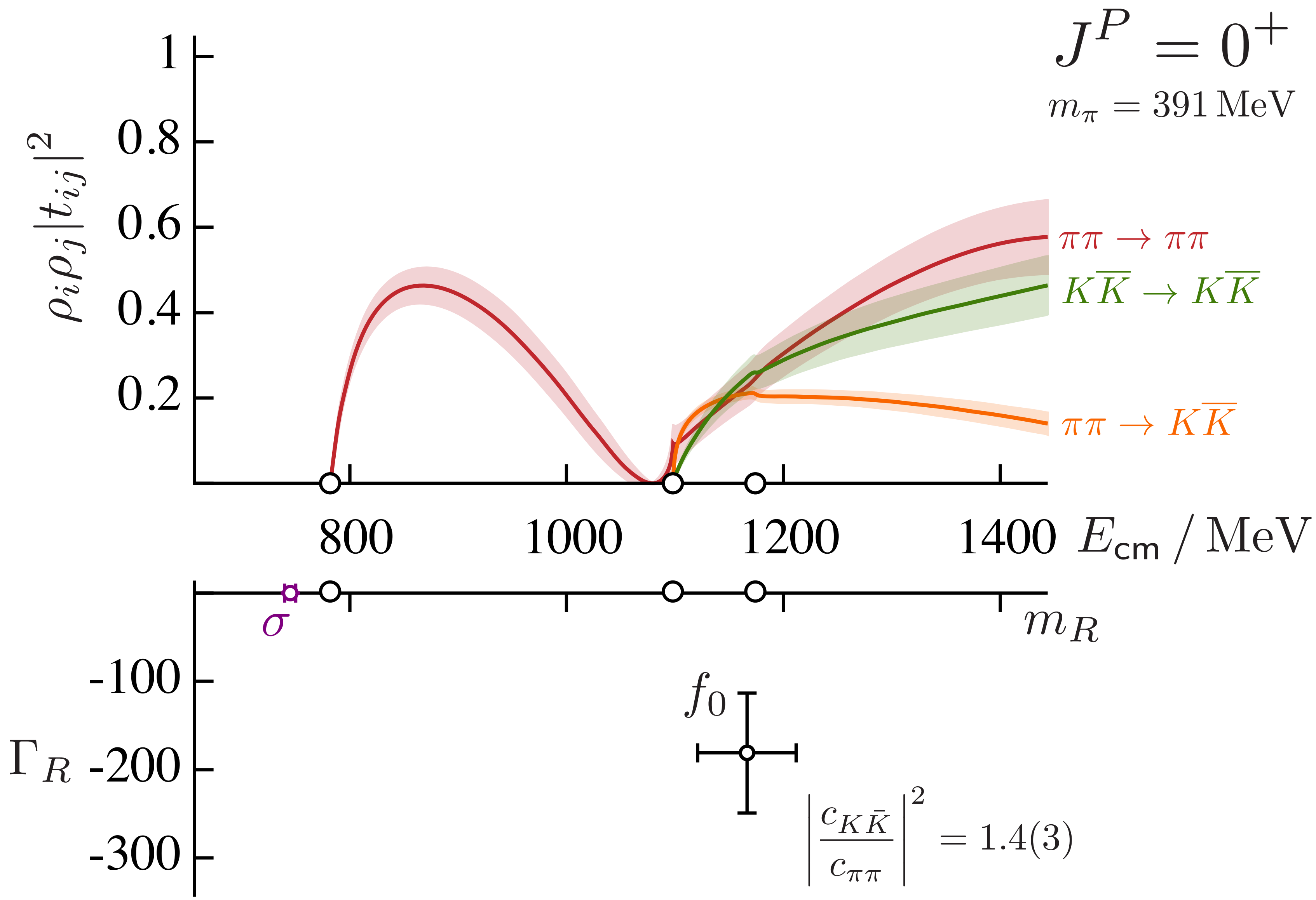


An example S-wave spectrum fit

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

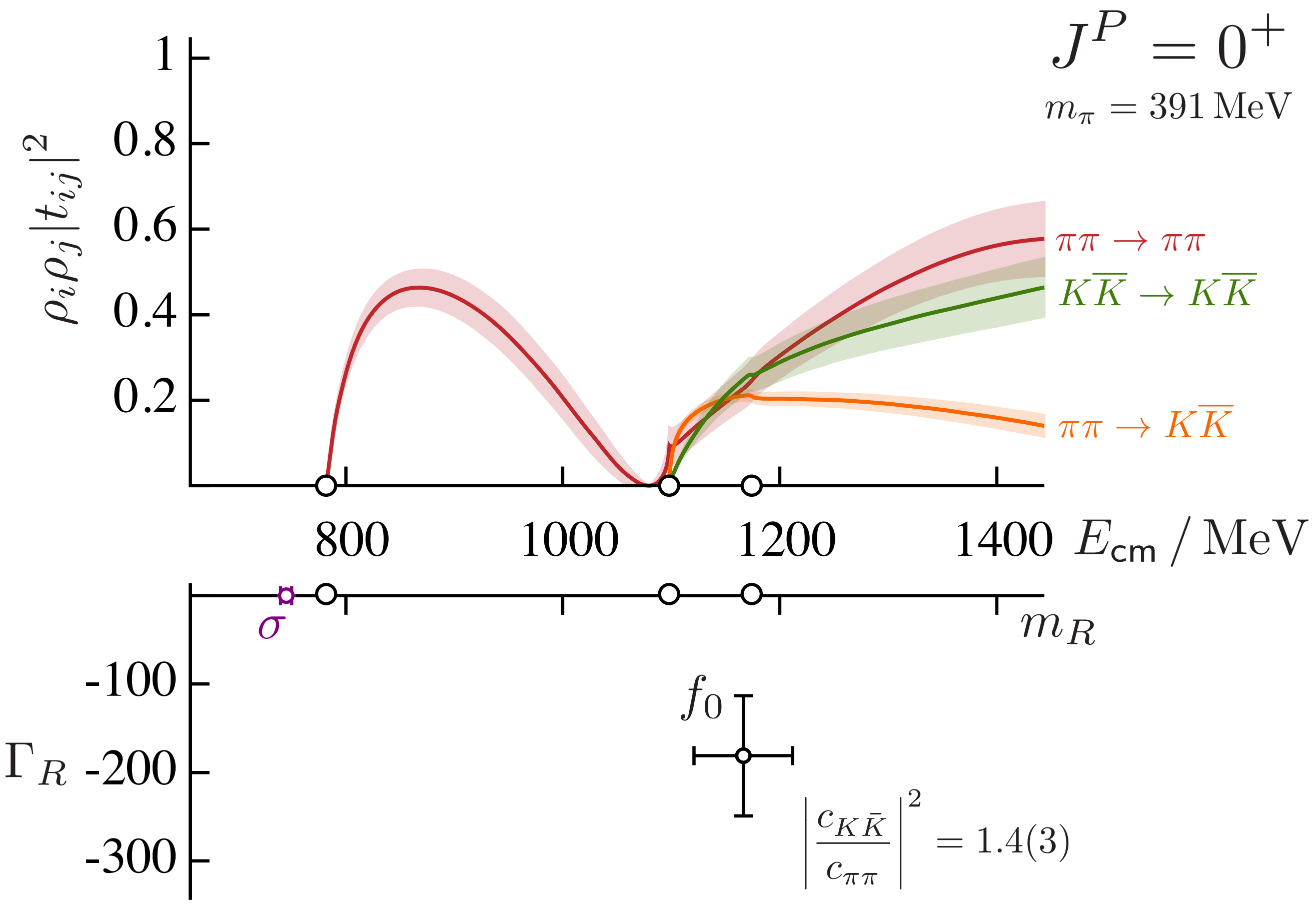
$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

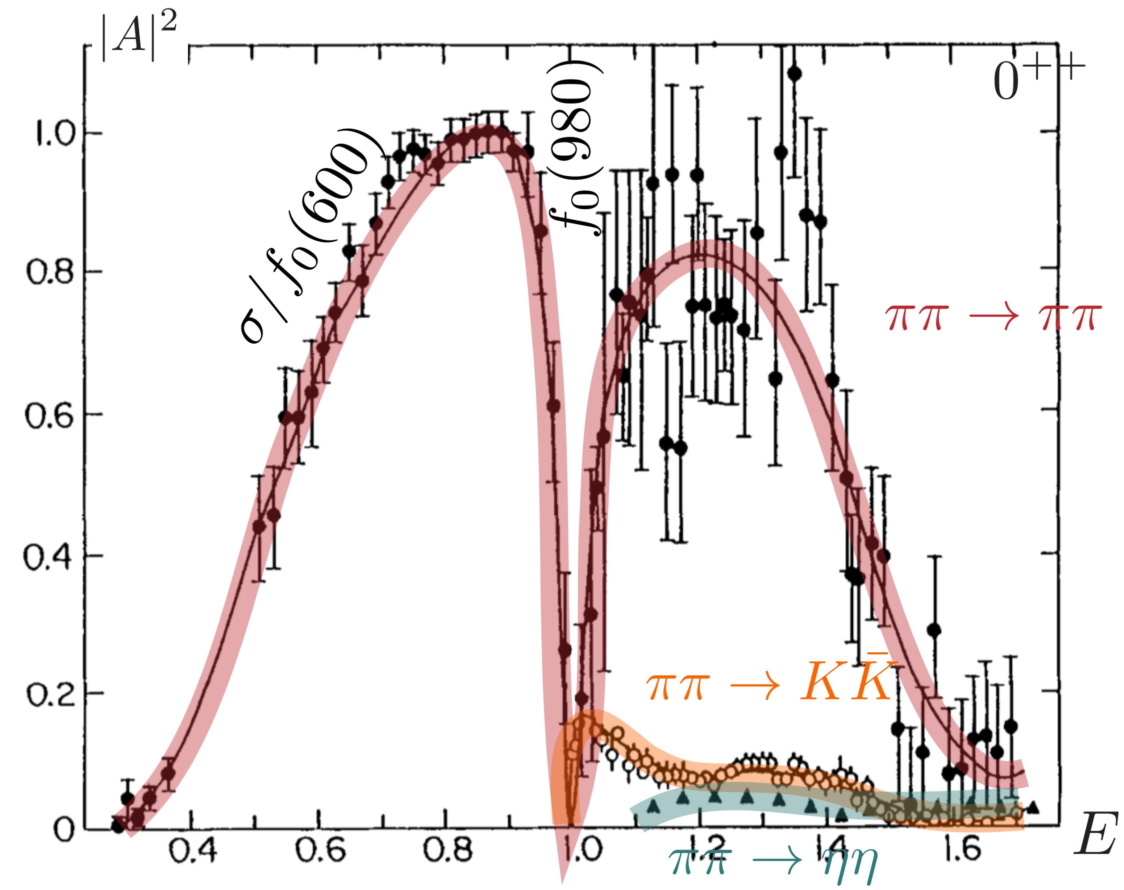


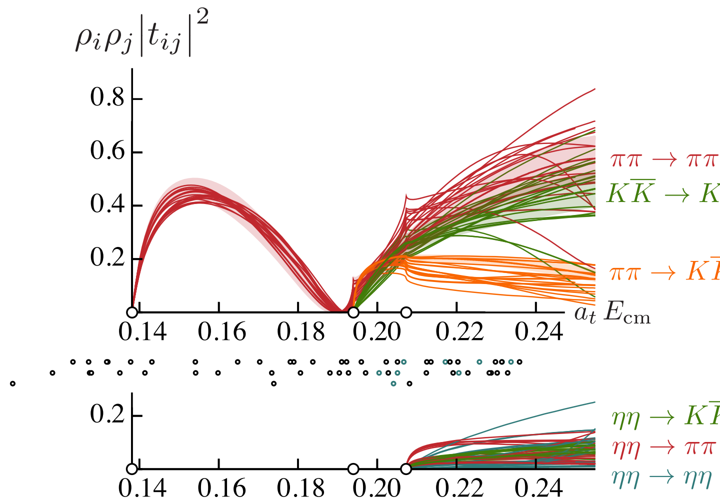
Near a t-matrix pole

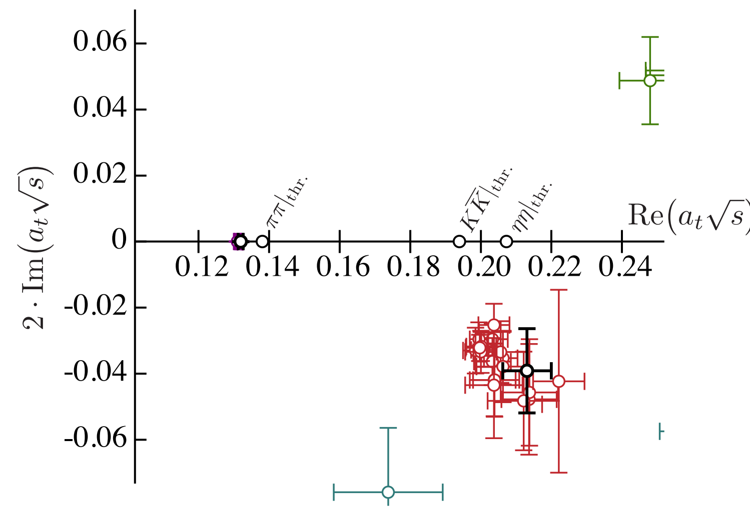
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

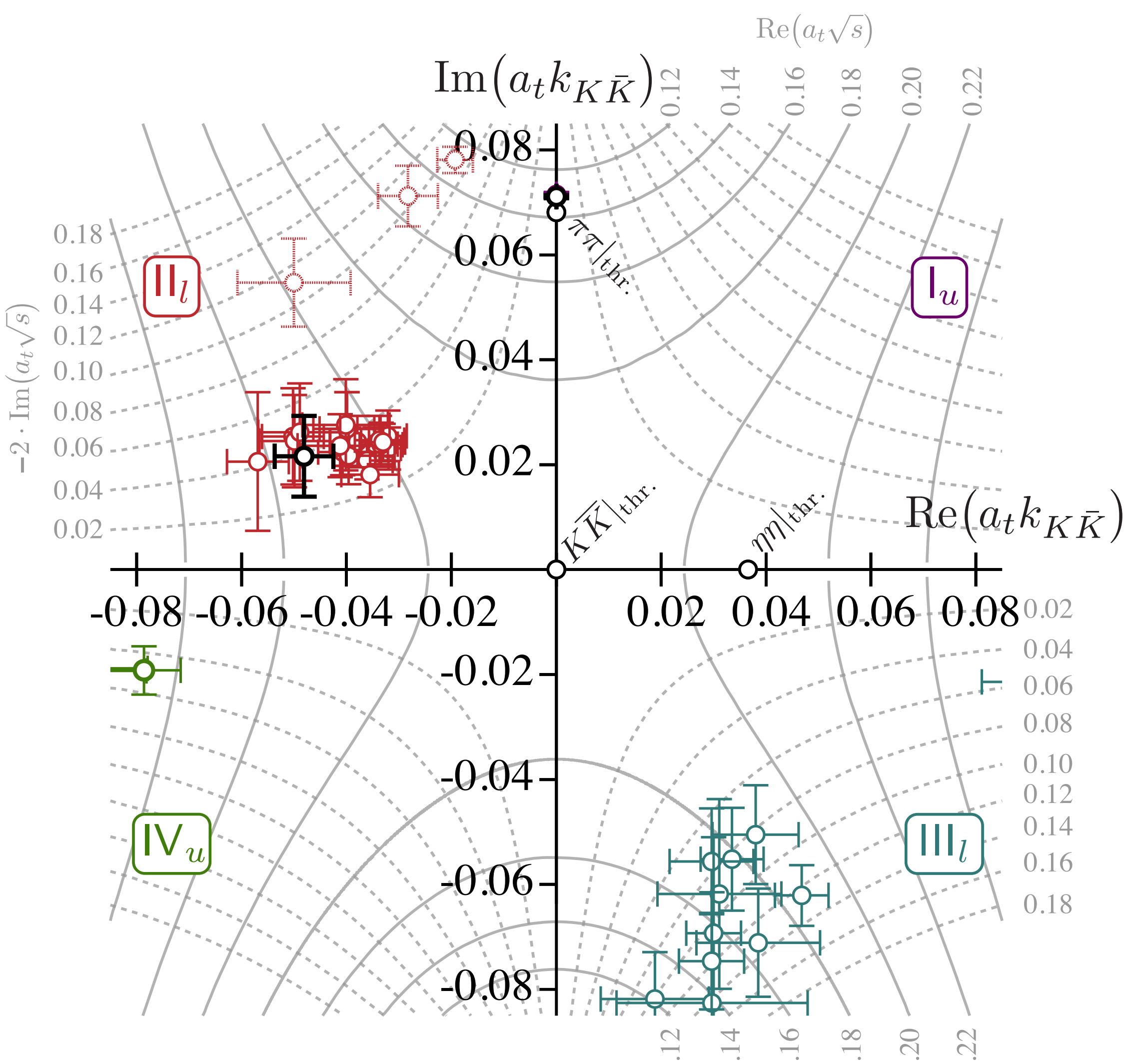


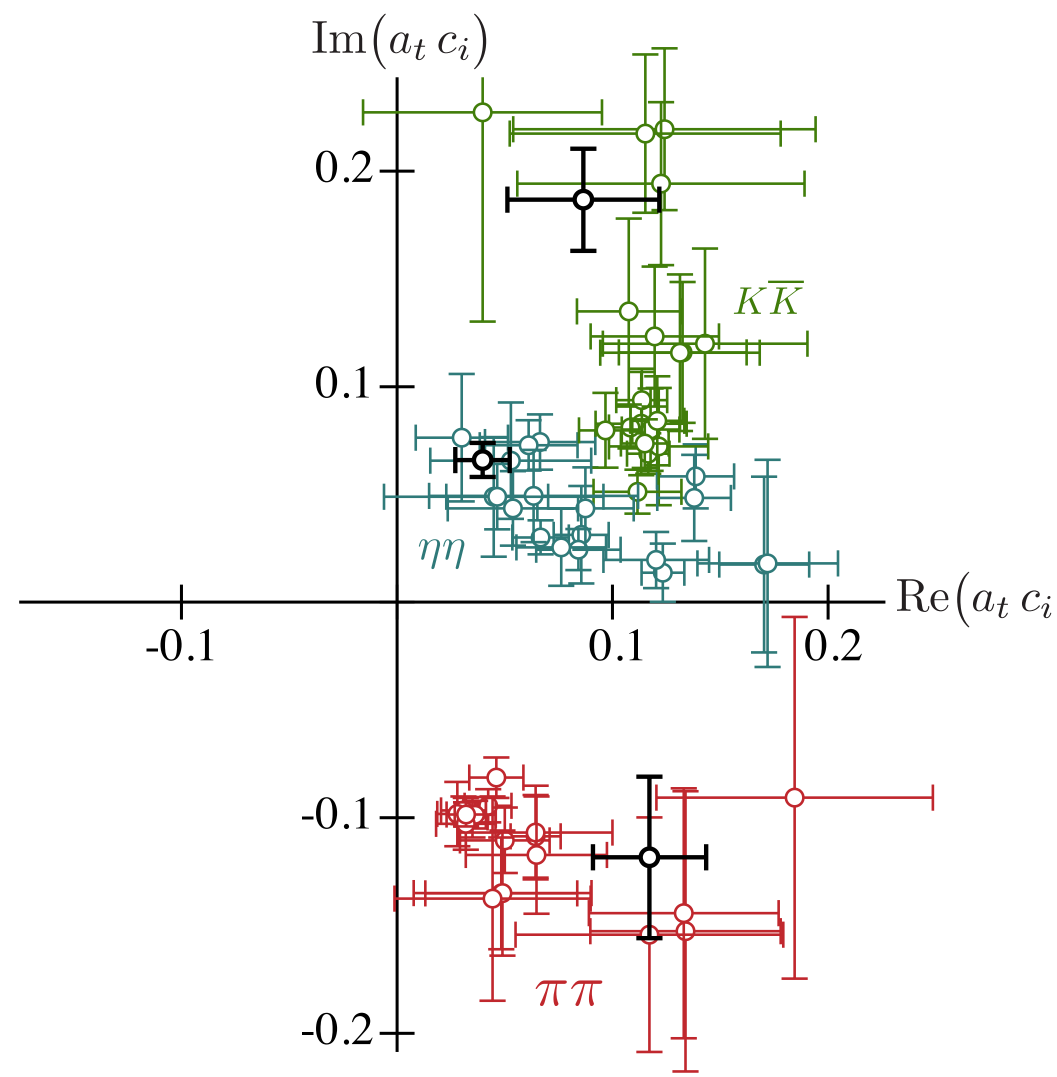
CERN-Munich, ANL, BNL

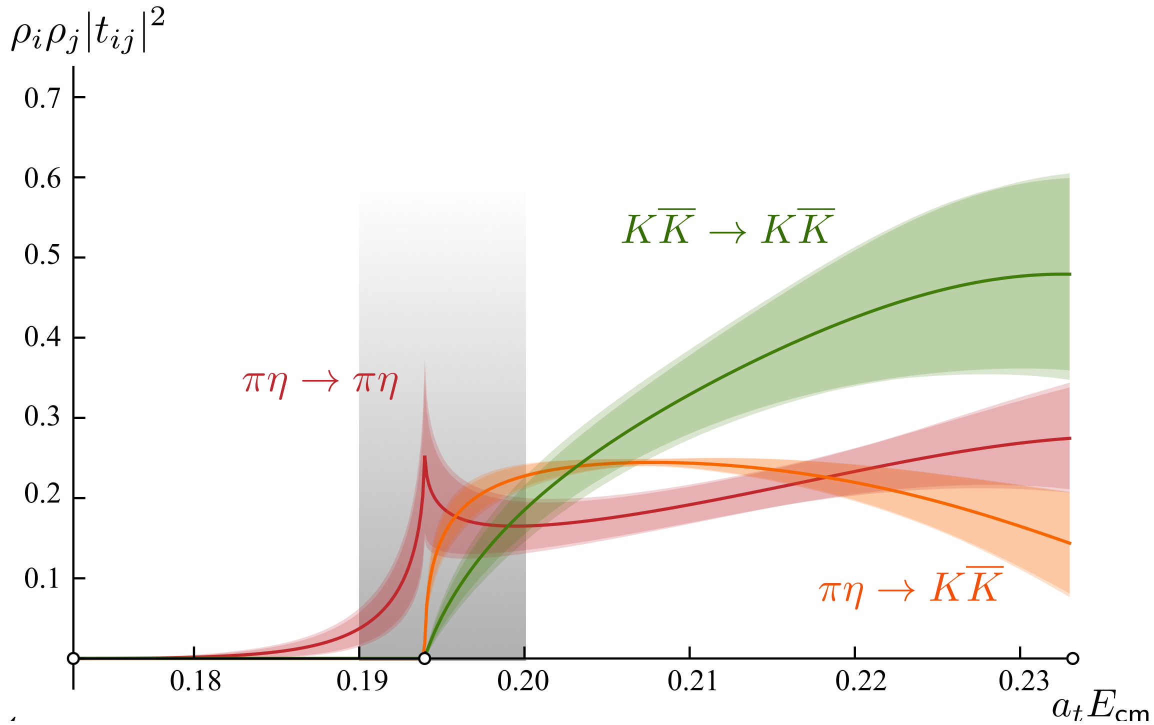


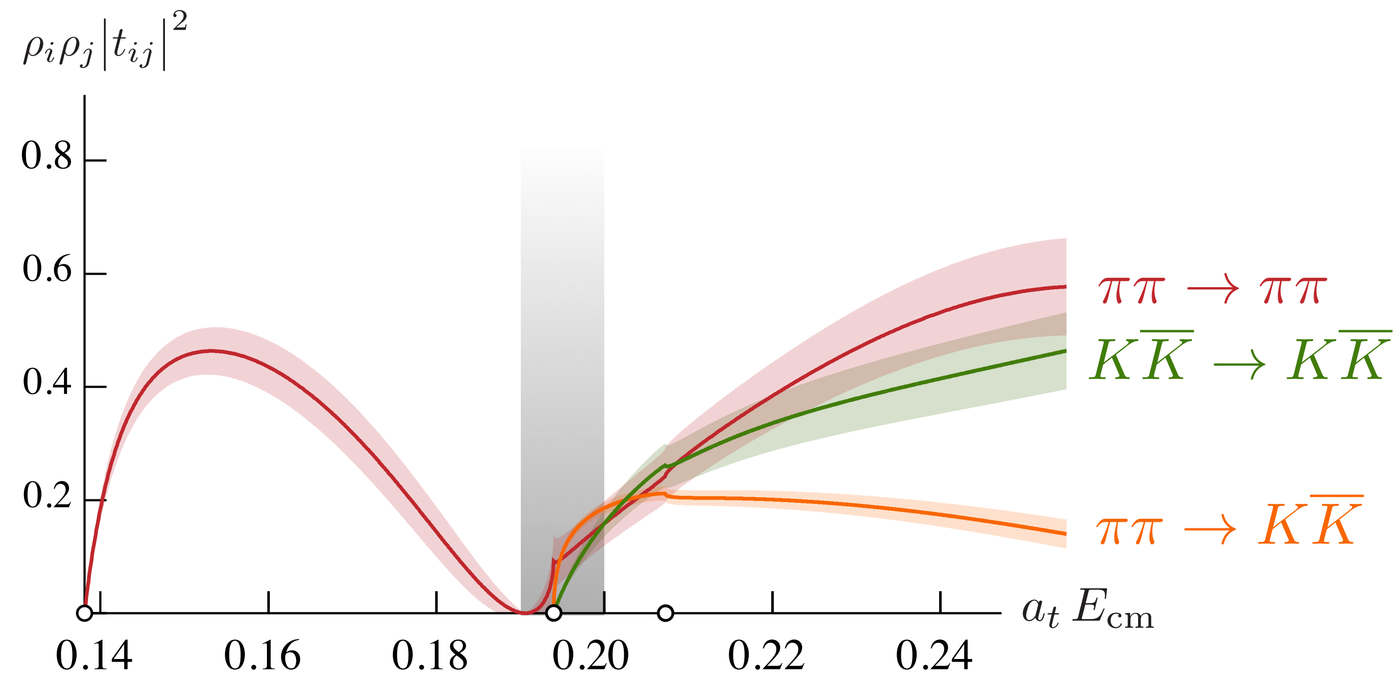

$\rho_i\rho_j|t_{ij}|^2$
$\pi\pi \to \pi\pi$
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
$a_t E_{\rm cm}$
$\eta\eta \to K\overline{K}$
$\eta\eta \to \pi\pi$
$\eta\eta \to \eta\eta$

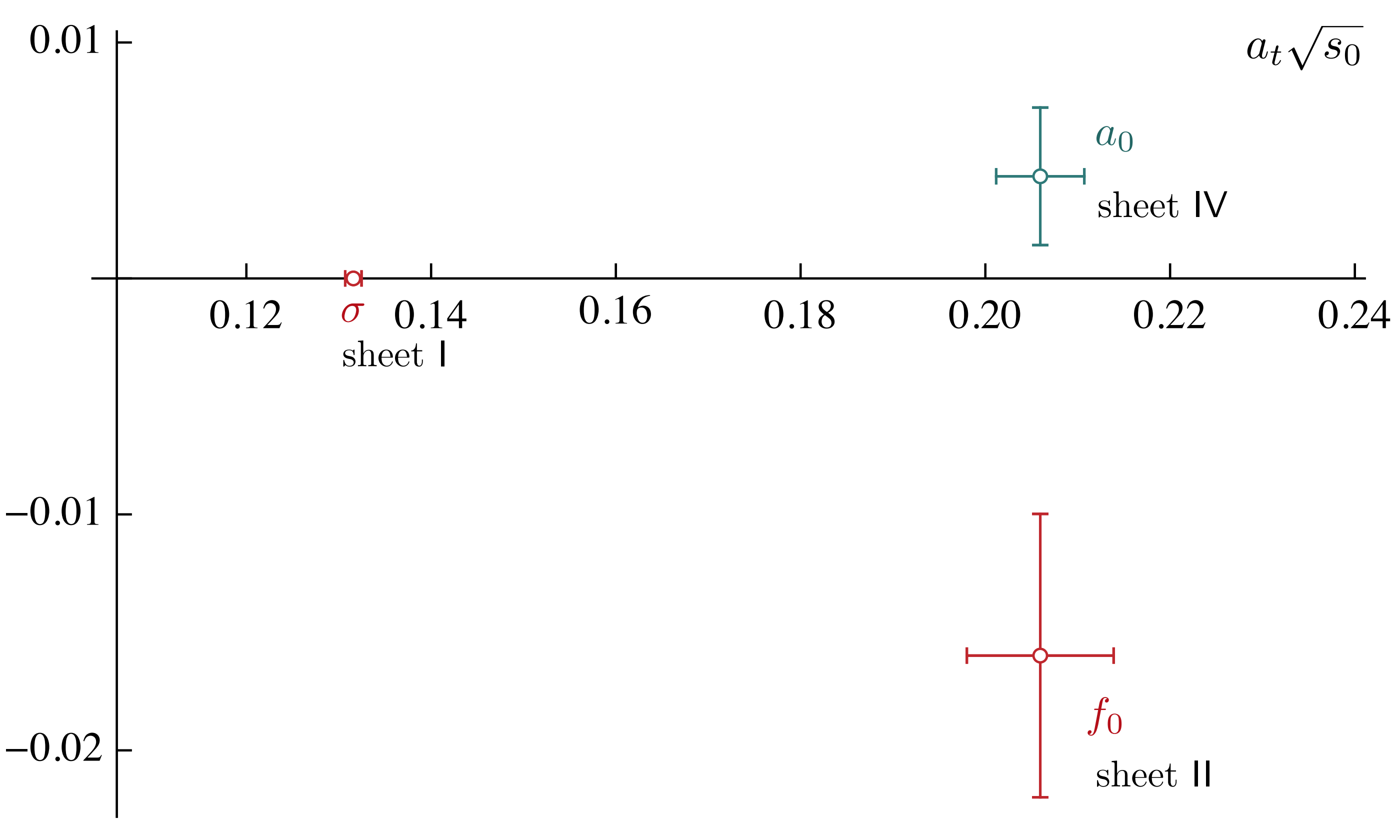

$2 \cdot \mathrm{Im}(a_t\sqrt{s})$
$\mathrm{Re}(a_t\sqrt{s})$
$\pi\pi|_{\rm thr.}$
$K\overline{K}|_{\rm thr.}$
$\eta\eta|_{\rm thr.}$


$\mathrm{Im}(a_t k_{K\bar{K}})$
$\mathrm{Re}(a_t k_{K\bar{K}})$
$\mathrm{Re}(a_t\sqrt{s})$
$-2 \cdot \mathrm{Im}(a_t\sqrt{s})$
II_l
I_u
IV_u
III_l
$\pi\pi|_{\rm thr.}$
$K\overline{K}|_{\rm thr.}$
$\eta\eta|_{\rm thr.}$

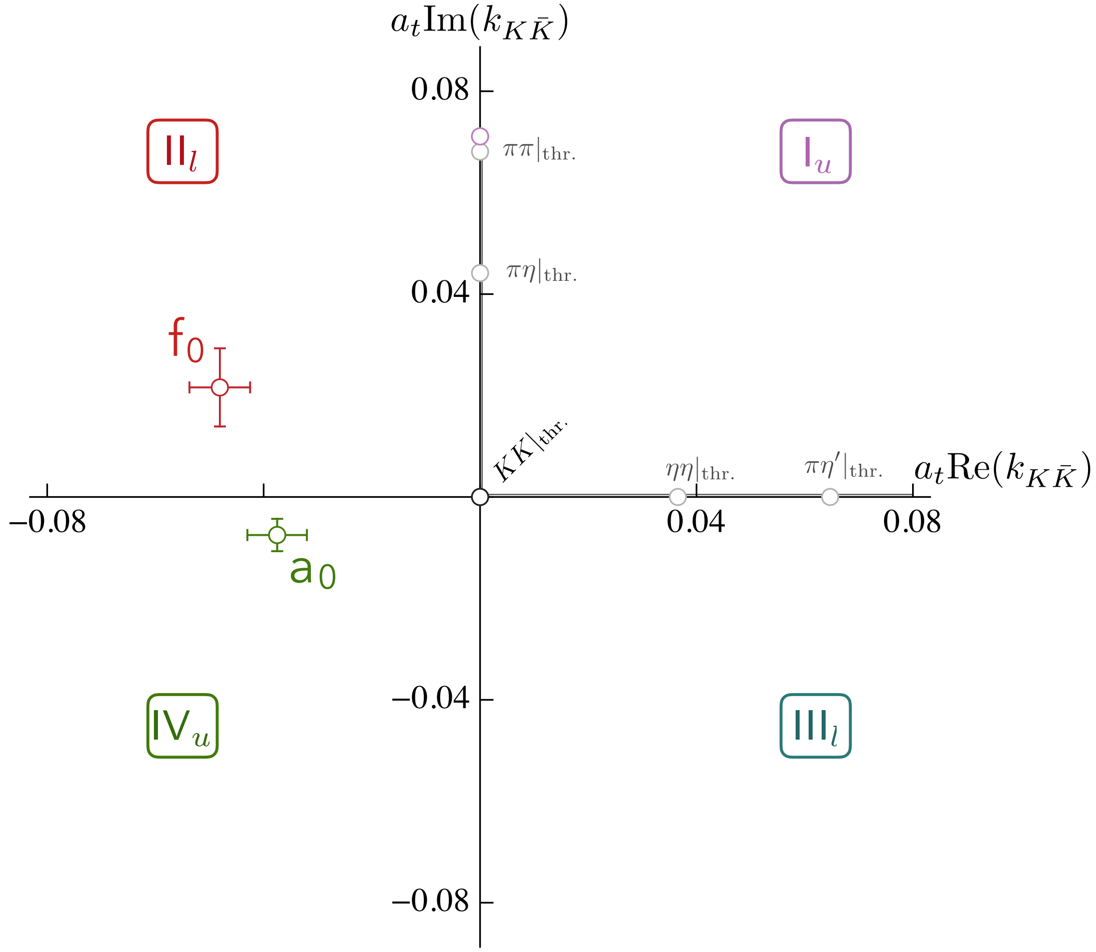

$\mathrm{Im}(a_t\, c_i)$
$\mathrm{Re}(a_t\, c_i)$
$K\overline{K}$
$\eta\eta$
$\pi\pi$







- rapid variation in both amplitudes at $K\bar{K}$ threshold
- both contain a nearby pole with a similar real part



PHYSICAL REVIEW D VOLUME 48, NUMBER 3 1 AUGUST 1993

New data on the $K\bar{K}$ threshold region and the nature of the $f_0(S^*)$

D. Morgan
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 OQX, United Kingdom

M. R. Pennington
Centre for Particle Theory, University of Durham, Durham, DH1 3LE, United Kingdom
(Received 8 January 1993)

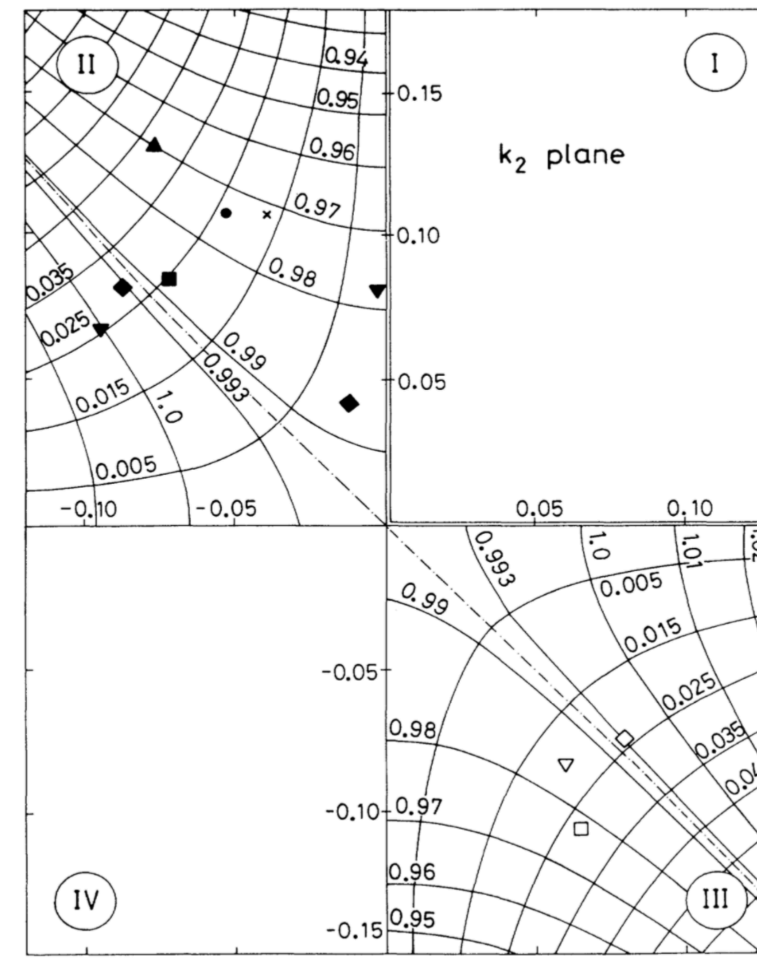
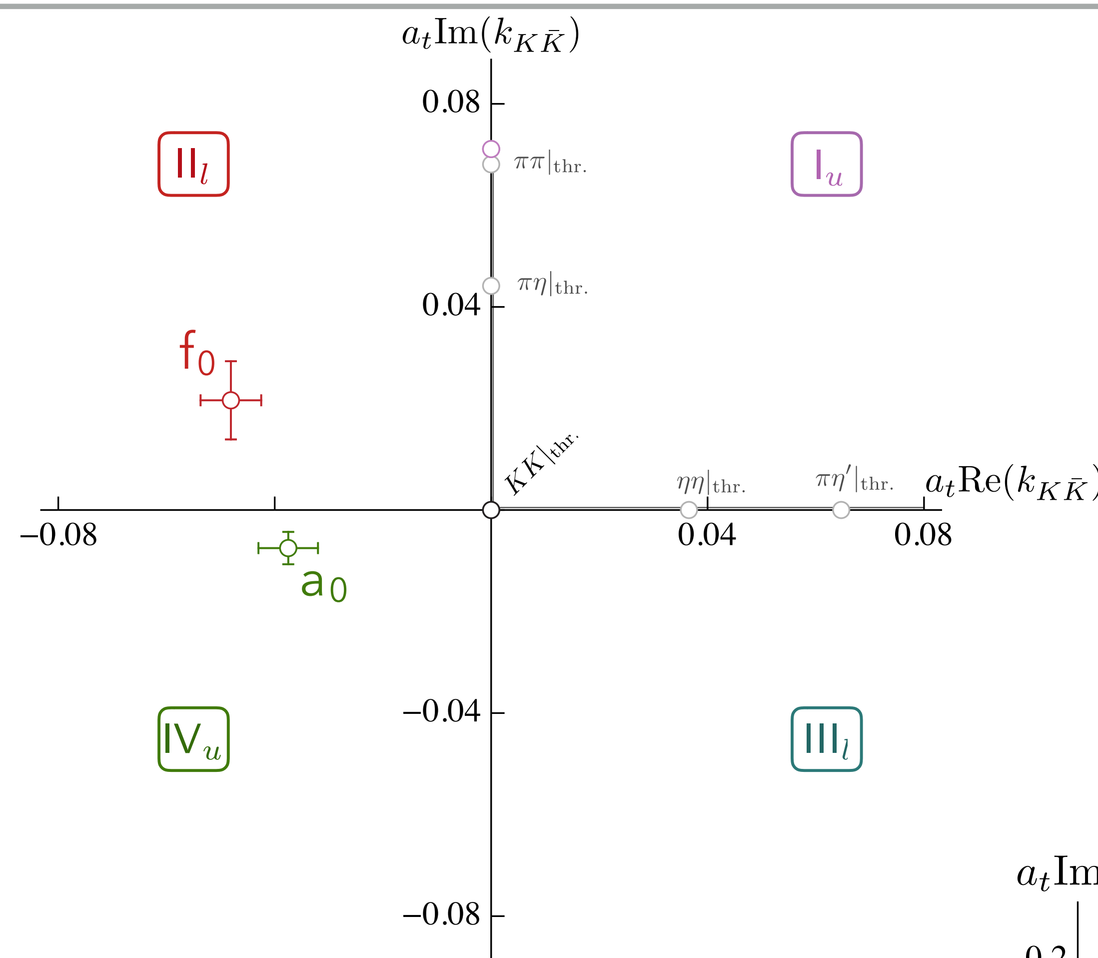
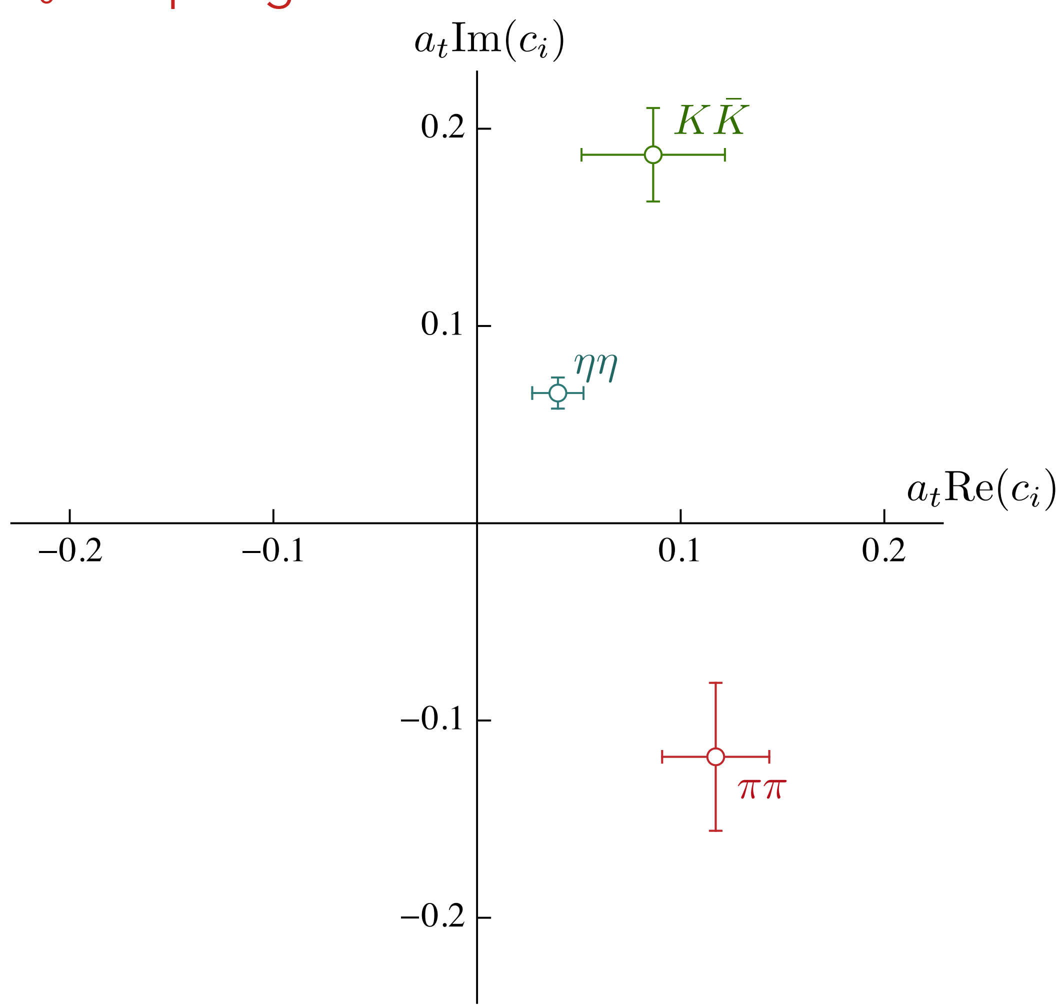


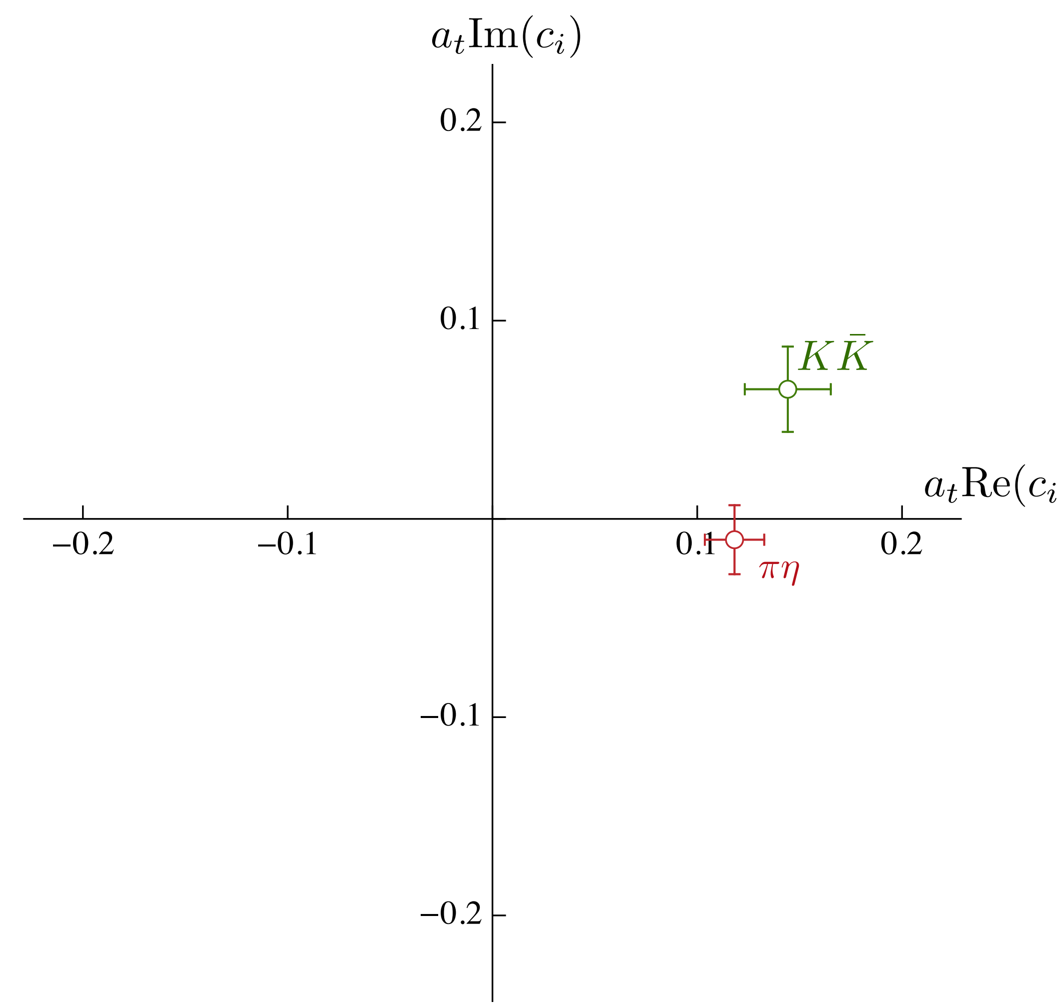
FIG. 11. $f_0(S^*)$ resonance pole positions shown on the k_2 plane (k_2 denotes the $K\bar{K}$ c.m. momentum). Points are labeled as on Fig. 10. The various sectors of the k_2 plane are labeled by the corresponding sheets of the energy plane $E \equiv M - i\Gamma/2 \equiv 2[k_2^2 + m_K^2]^{1/2}$ which are distinguished by the associated signs of $\mathrm{Im}k_1$ and $\mathrm{Im}k_2$: $(++)$ (sheet I), $(-+)$ (sheet II), $(--)$ (sheet III), and $(+-)$ (sheet IV). The families of feint curves superimposed on sheets II and III correspond, respectively, to constant M and $\Gamma/2$ and are labeled in GeV accordingly.



f_0 couplings



a_0 couplings



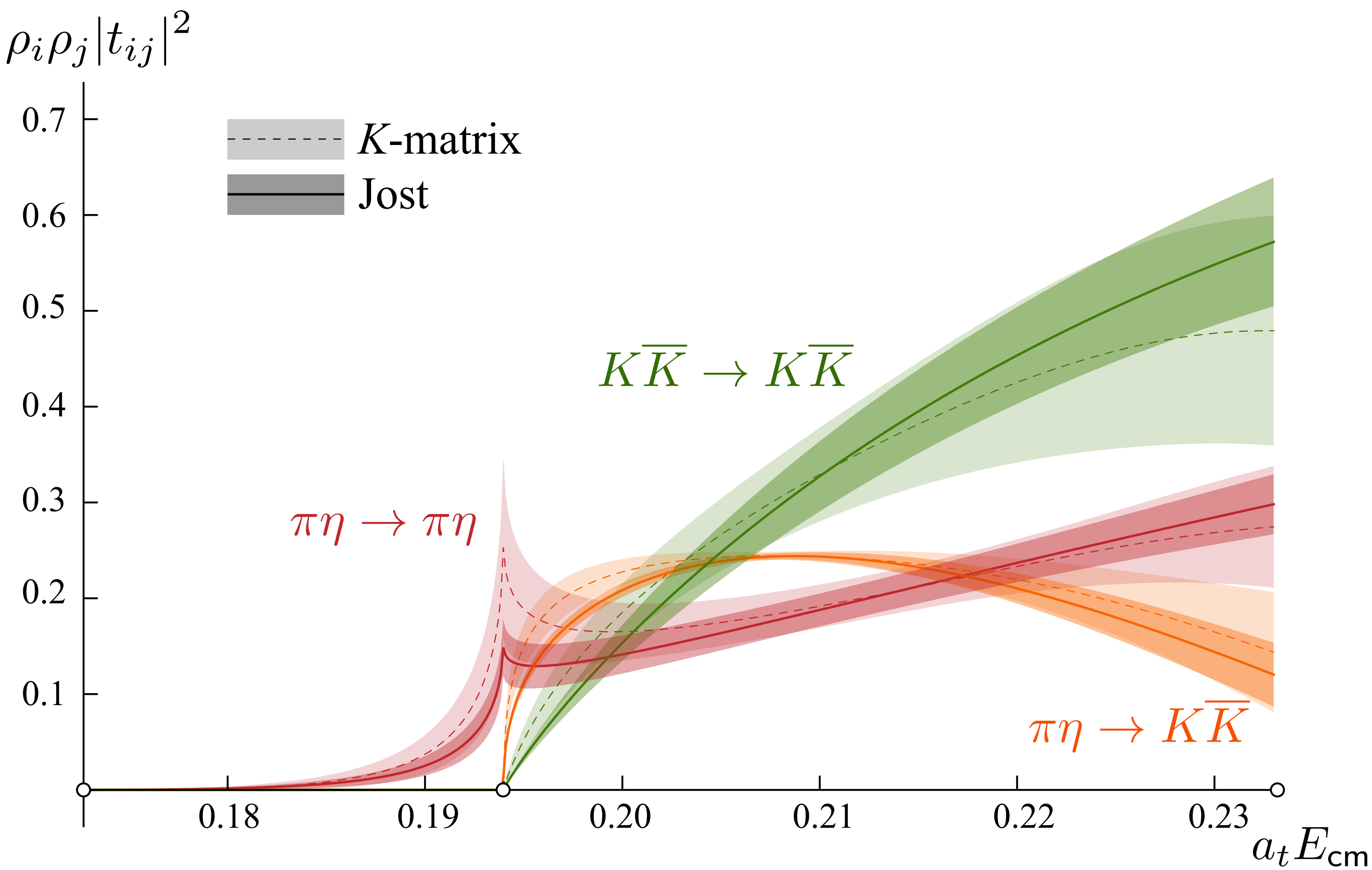
simple functions of scattering momentum

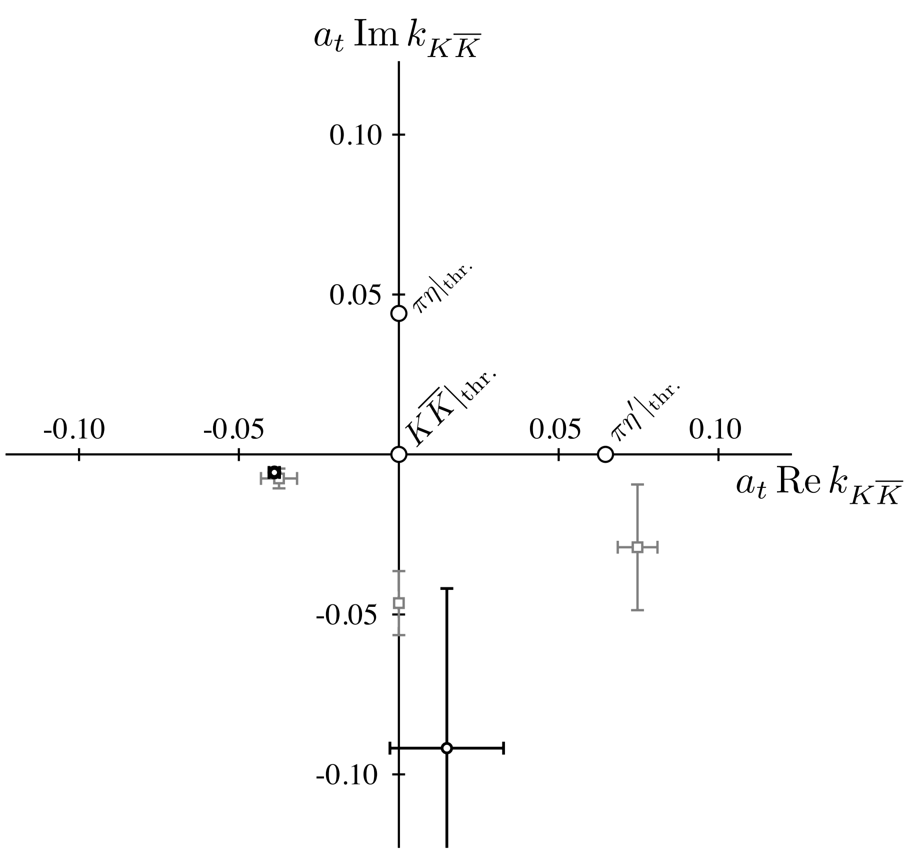
- control of pole positions as a free parameter
- unitarity is not guaranteed (test for a given set of parameters)

$$S_{11} = \frac{\mathfrak{J}(-k_1, k_2)}{\mathfrak{J}(k_1, k_2)}$$

$$S_{22} = \frac{\mathfrak{J}(k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}$$

$$\det \mathbf{S} = \frac{\mathfrak{J}(-k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}.$$





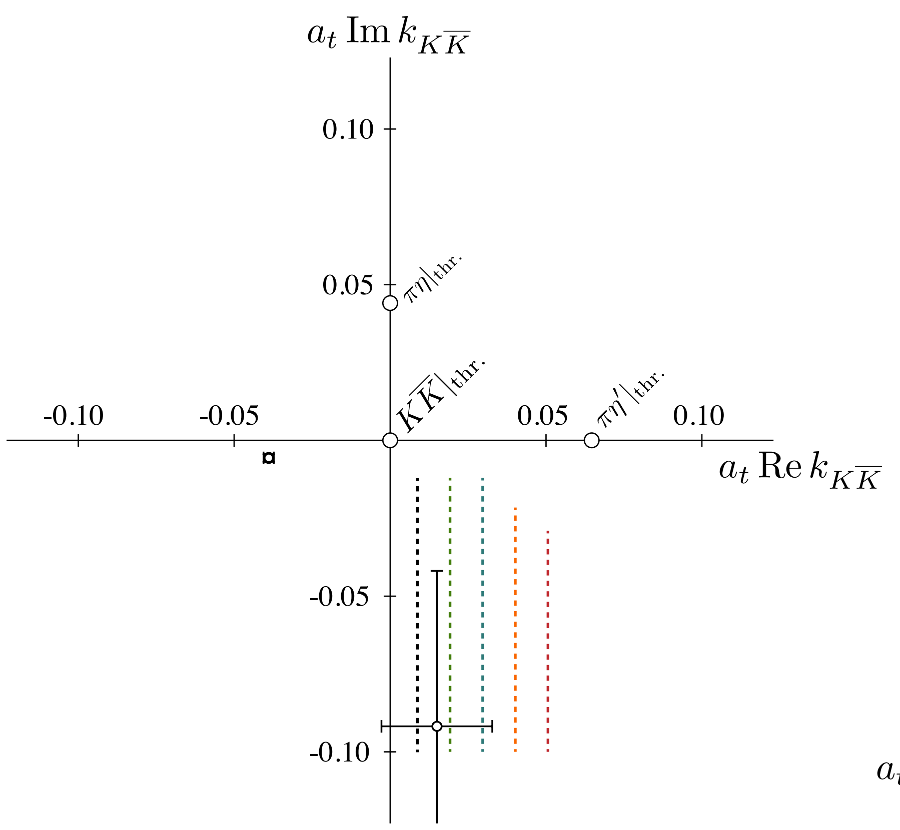
simple functions of scattering momentum

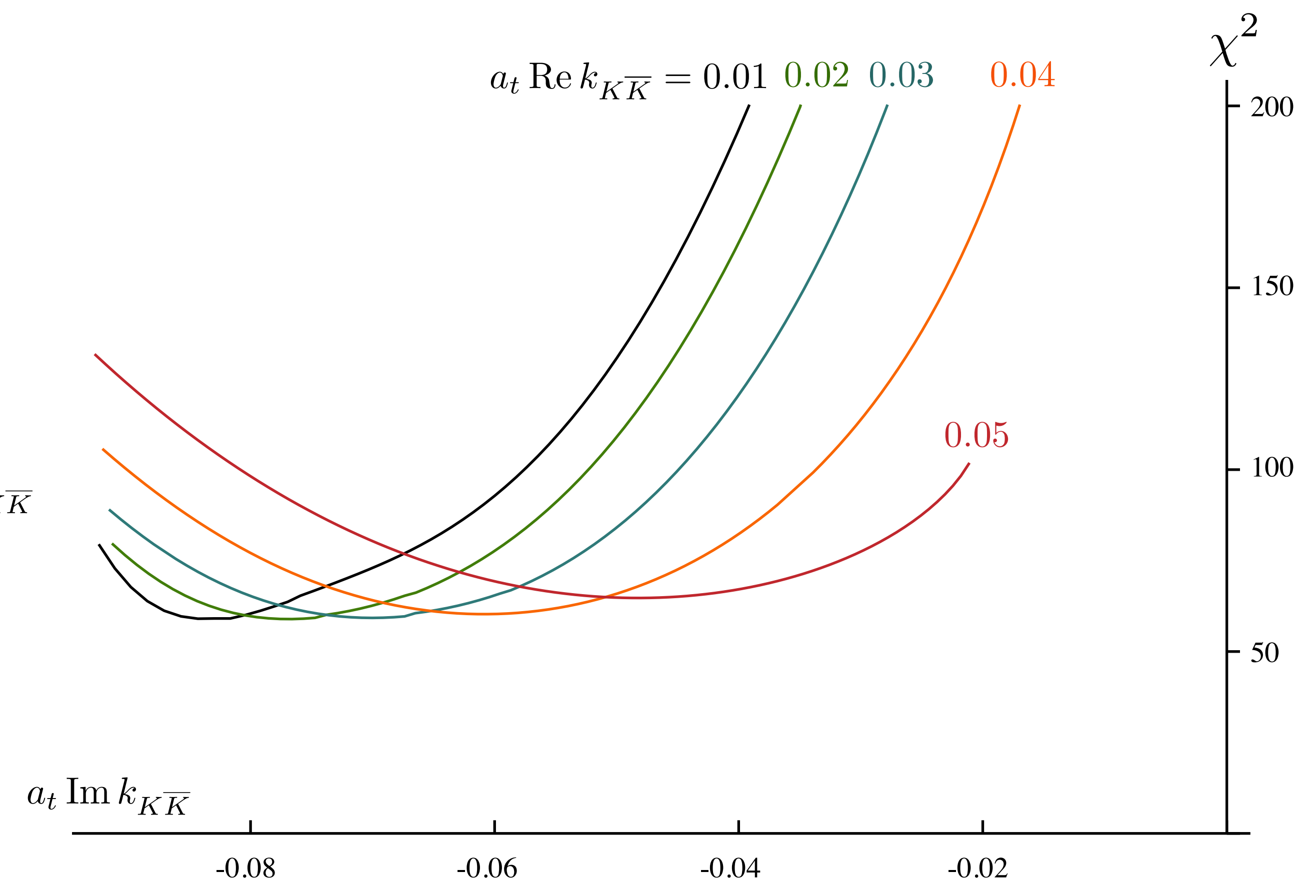
- control of pole positions as a free parameter
- unitarity is not guaranteed (test for a given set of parameters)

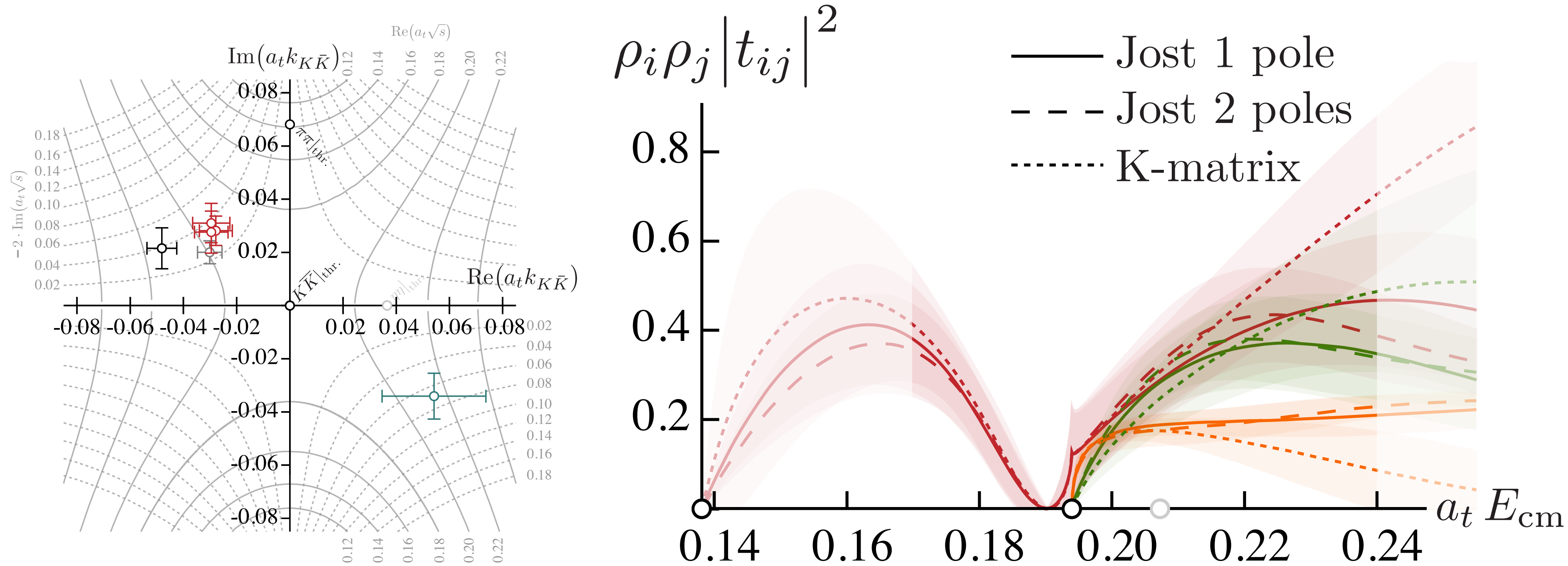
$$S_{11} = \frac{\mathfrak{J}(-k_1, k_2)}{\mathfrak{J}(k_1, k_2)}$$

$$S_{22} = \frac{\mathfrak{J}(k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}$$

$$\det \mathbf{S} = \frac{\mathfrak{J}(-k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}.$$



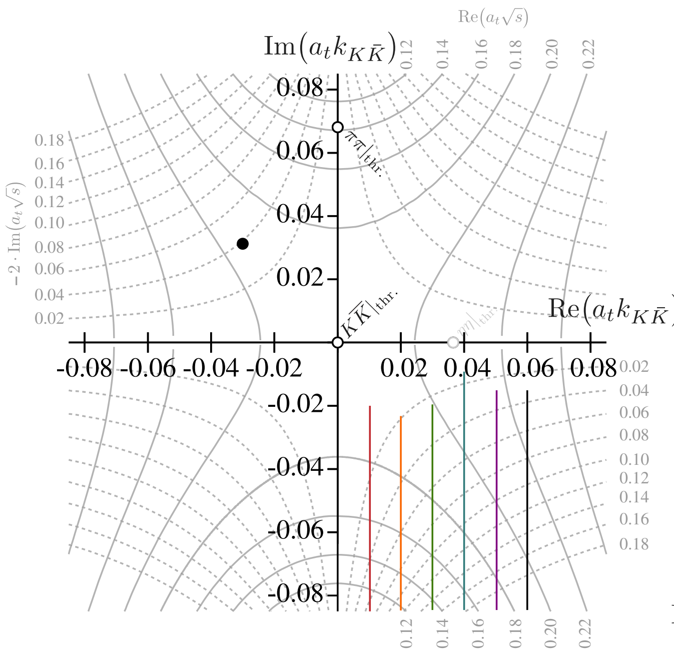


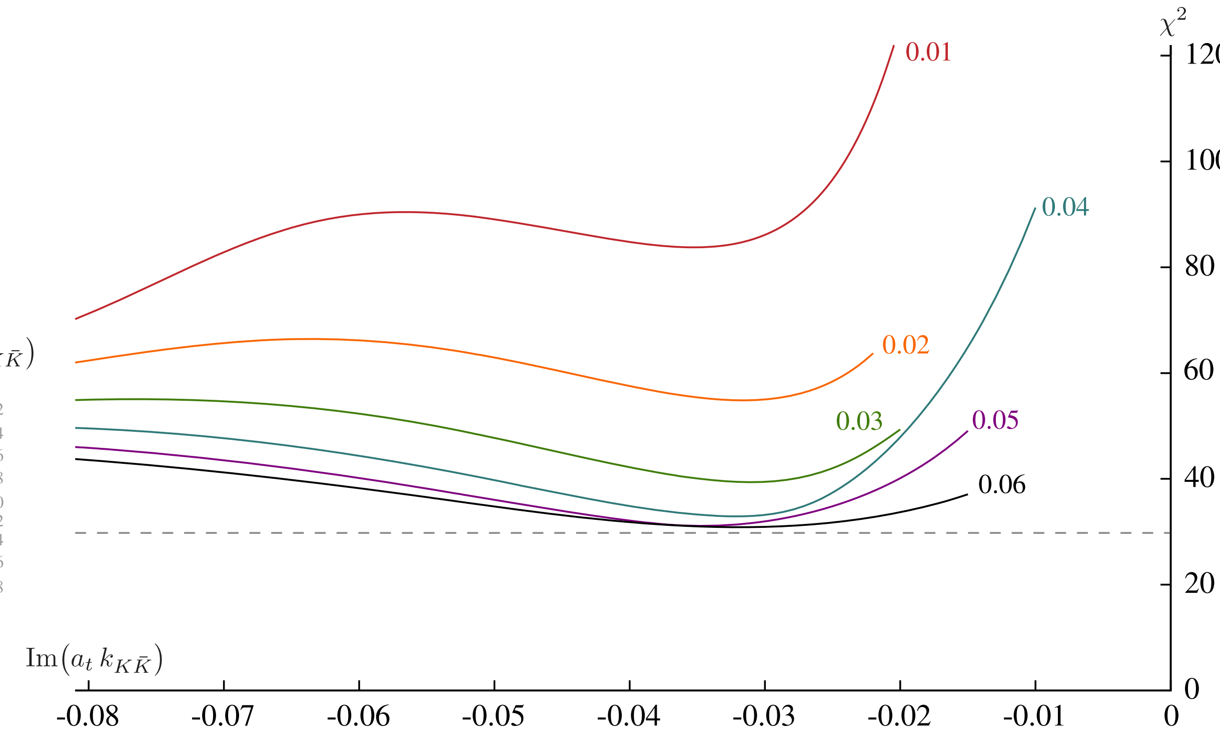


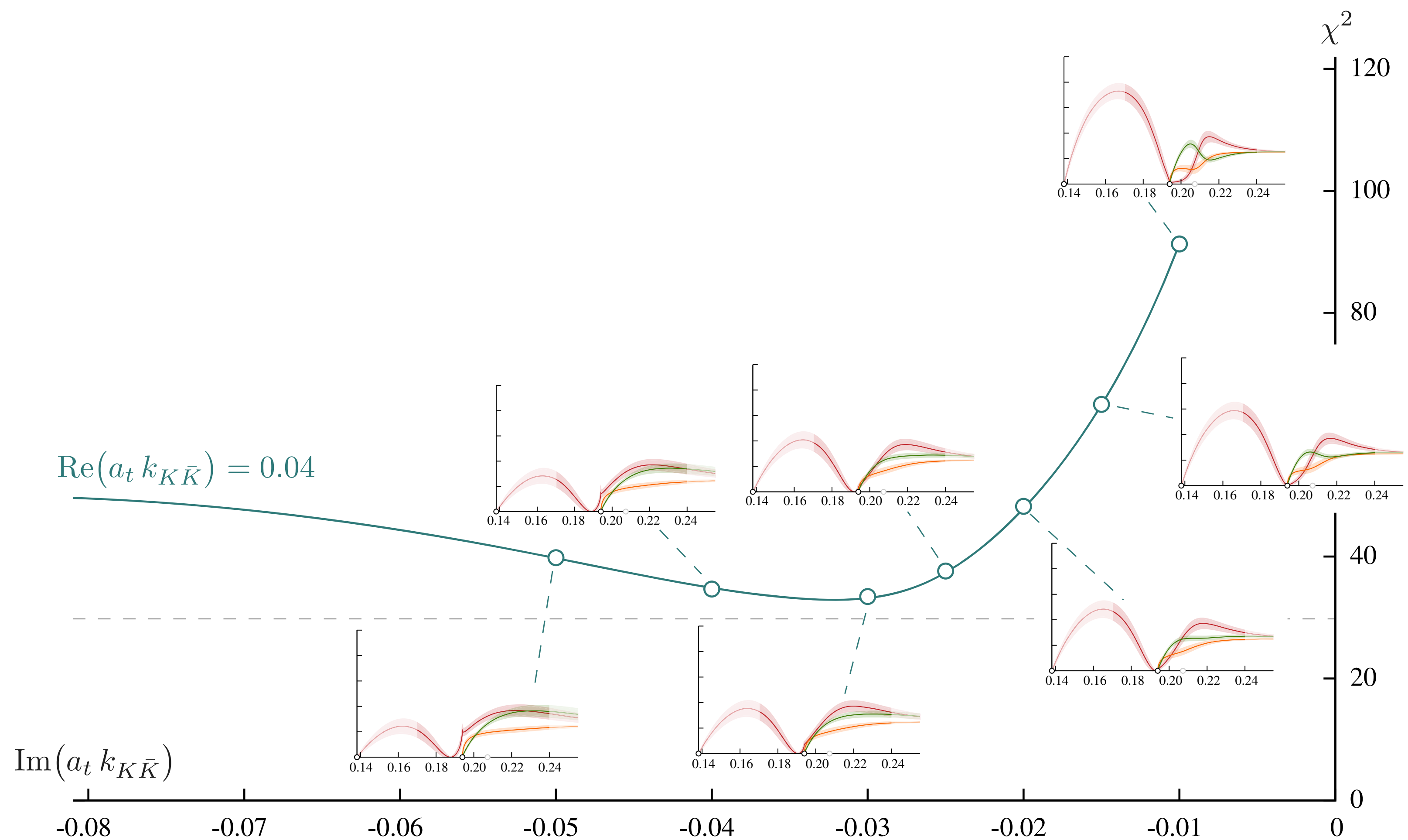
- best used for just a narrow energy region
  à la Morgan & Pennington 1993
- using Jost amps for 2-channel scattering only
- KK threshold region only
- dropping eta-eta
- 1 pole form + smooth background
- 2 pole form

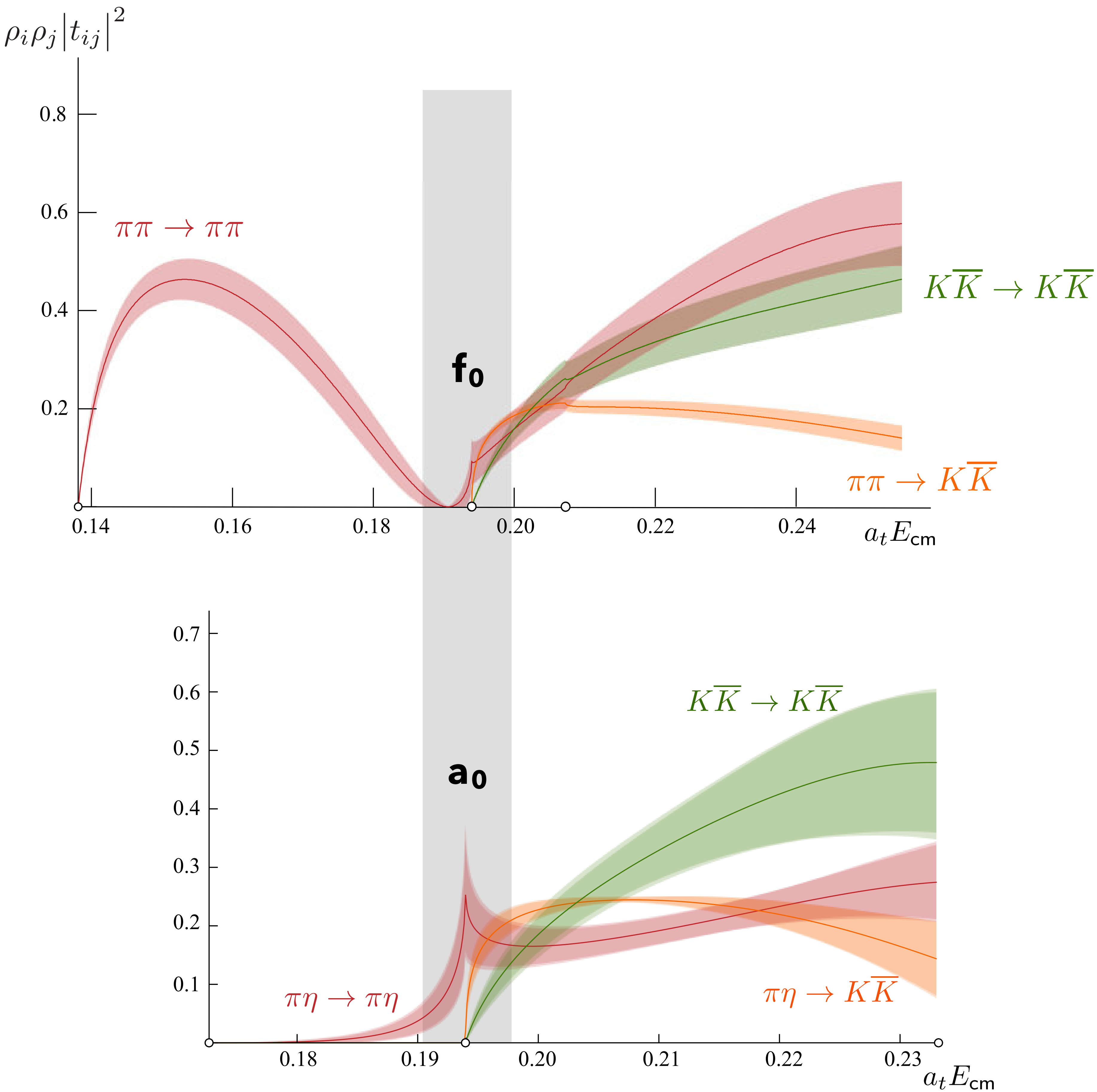
Jost amplitudes - f_0

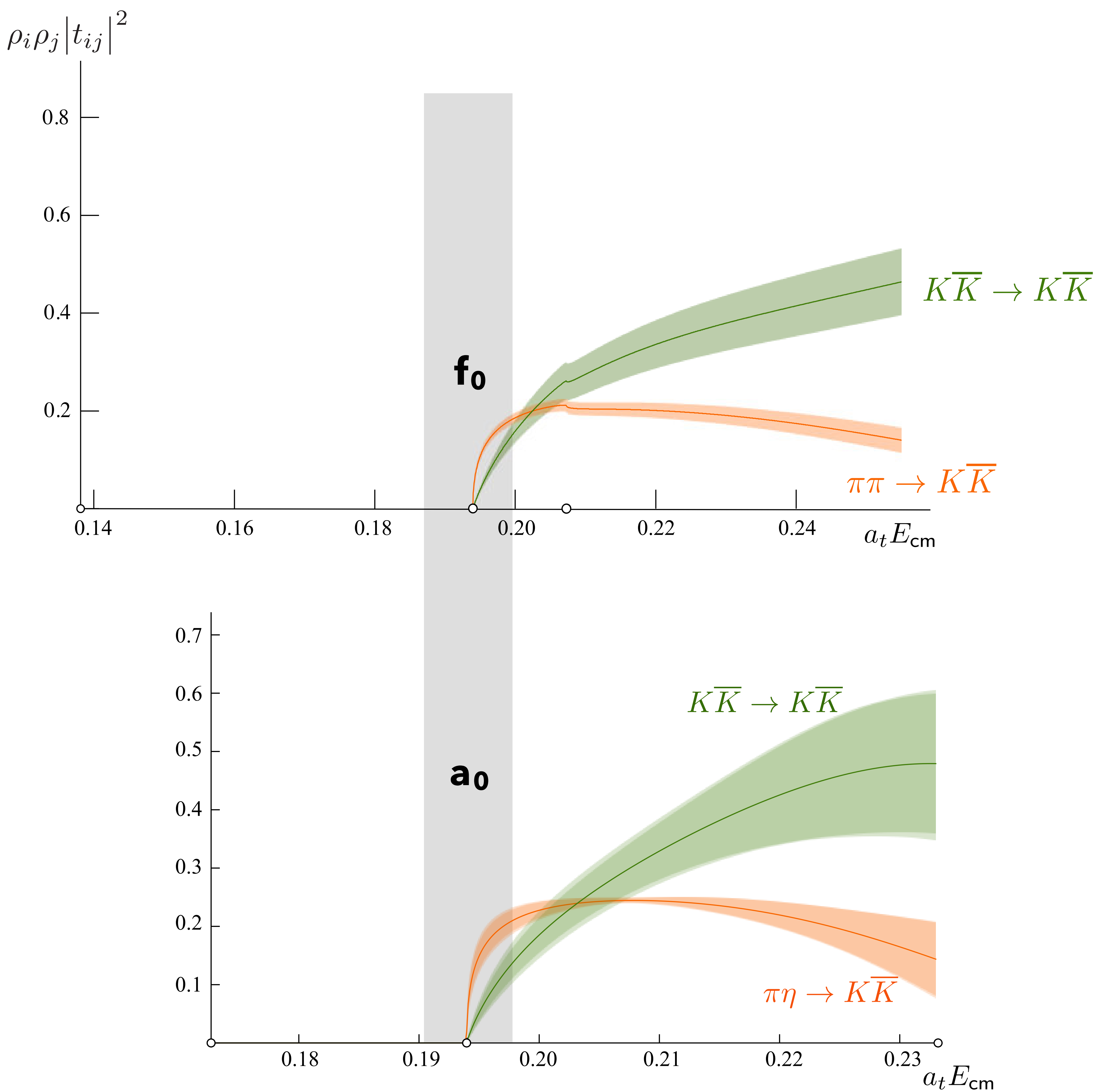
- two pole form,
  investigating second pole








$\rho_i \rho_j |t_{ij}|^2$
0.8
0.6
0.4
0.2
0.14
0.16
0.18
0.20
0.22
0.24
$a_t E_{\mathrm{cm}}$
$\pi\pi \to \pi\pi$
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
f0
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{\mathrm{cm}}$
$K\overline{K} \to K\overline{K}$
a0
$\pi\eta \to \pi\eta$
$\pi\eta \to K\overline{K}$


$\rho_i \rho_j |t_{ij}|^2$
0.8
0.6
0.4
0.2
0.14
0.16
0.18
0.20
0.22
0.24
$a_t E_{\mathsf{cm}}$
f_0
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{\mathsf{cm}}$
a_0
$K\overline{K} \to K\overline{K}$
$\pi\eta \to K\overline{K}$

An example D-wave spectrum fit

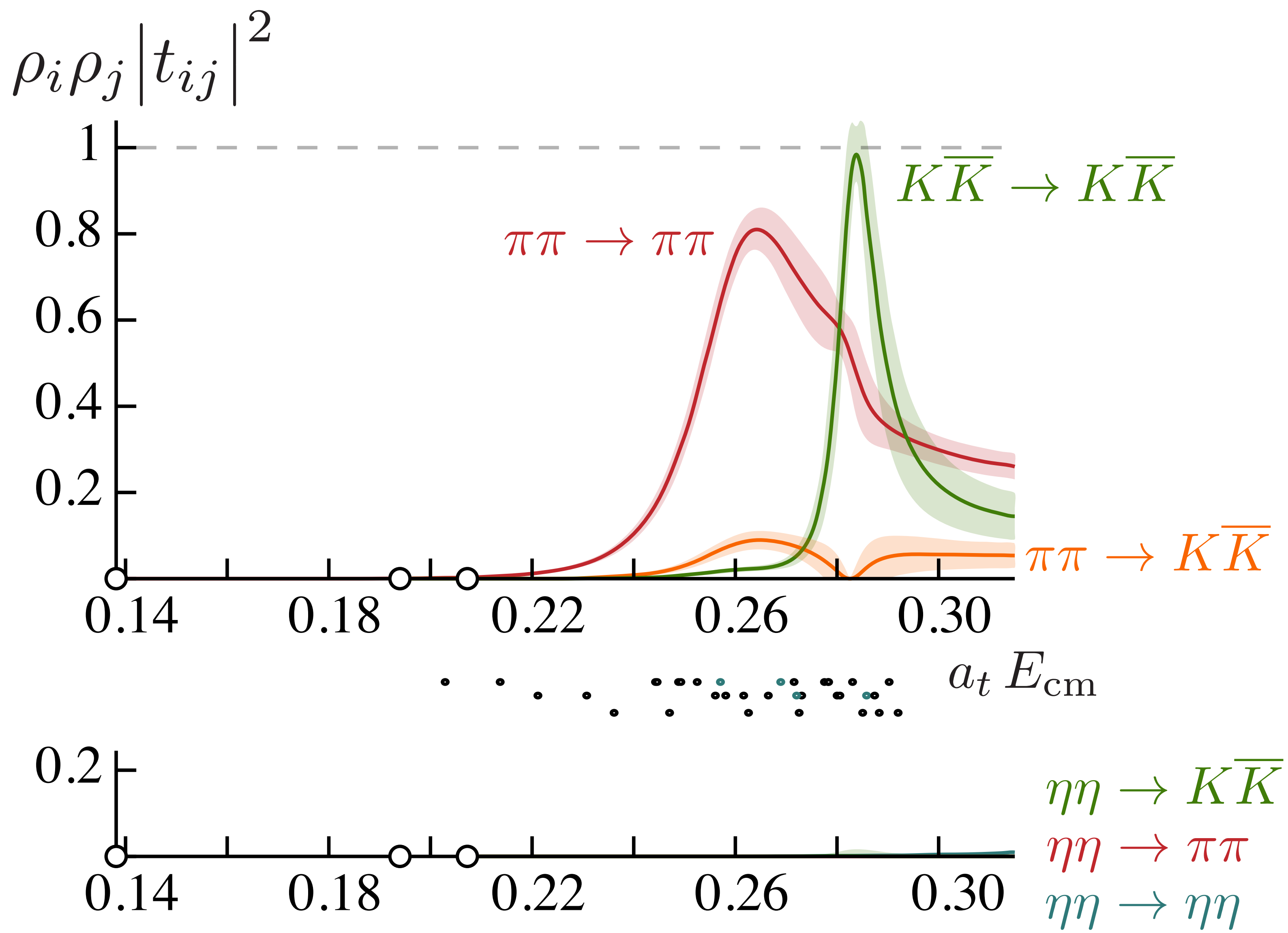
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

$\gamma_{\eta\eta} \neq 0$
$\gamma_{ij} = 0$ otherwise

$$\chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15$$

34 energy levels



An example D-wave spectrum fit

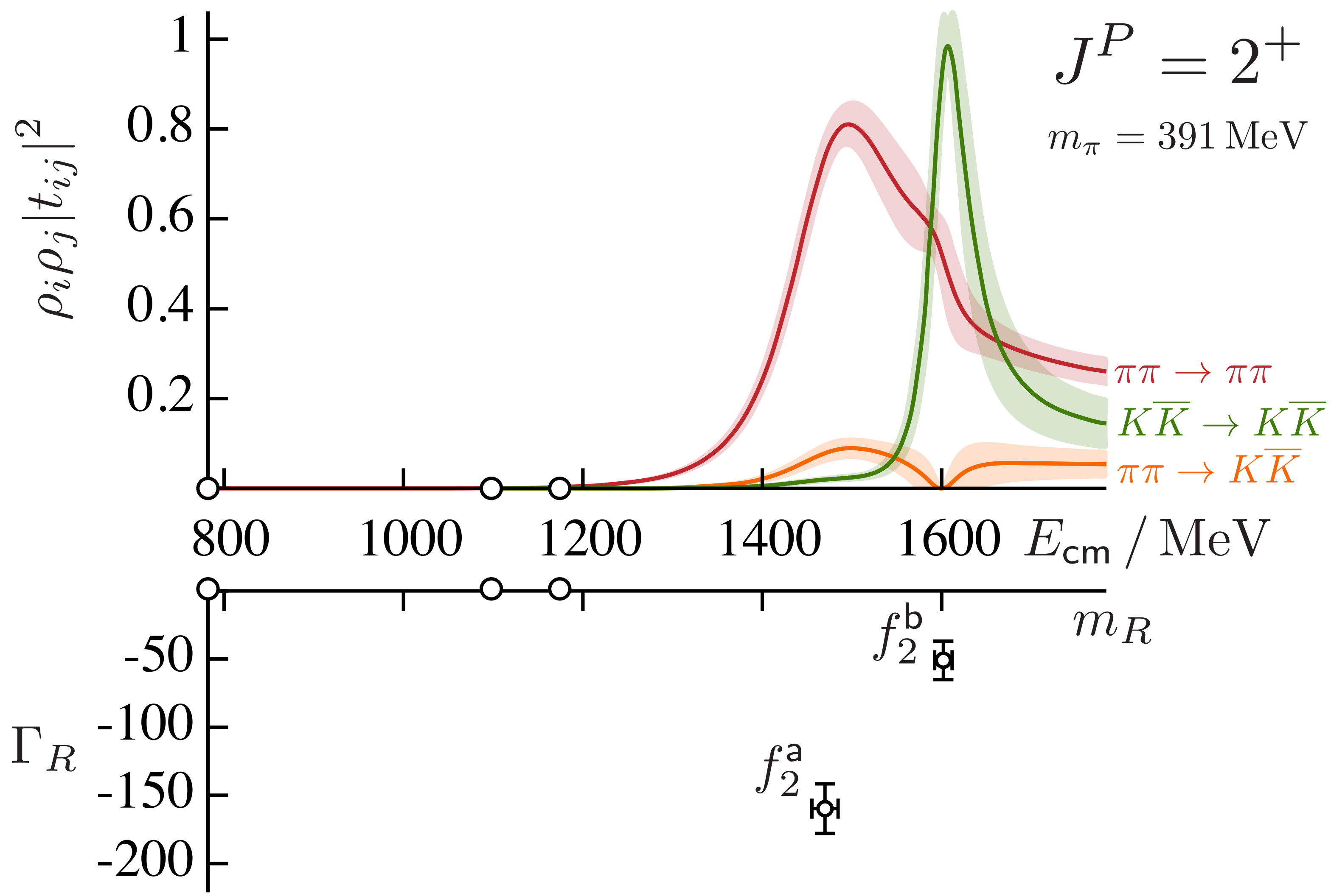
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

$$\gamma_{\eta\eta} \neq 0$$
$$\gamma_{ij} = 0 \quad \text{otherwise}$$

$$\chi^2/N_{\text{dof}} = \frac{28.9}{34-9} = 1.15$$

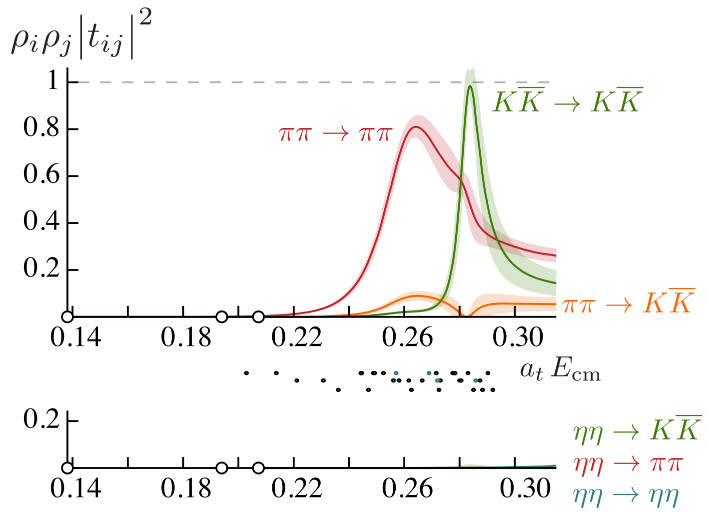
34 energy levels

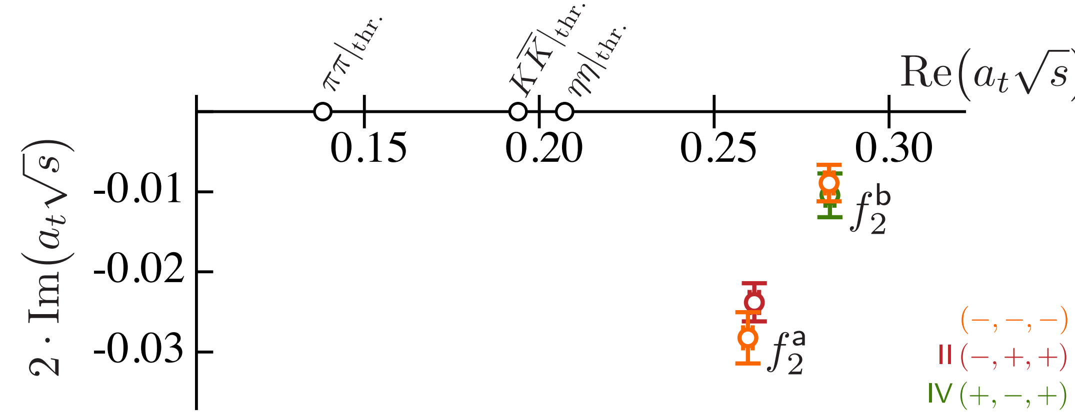


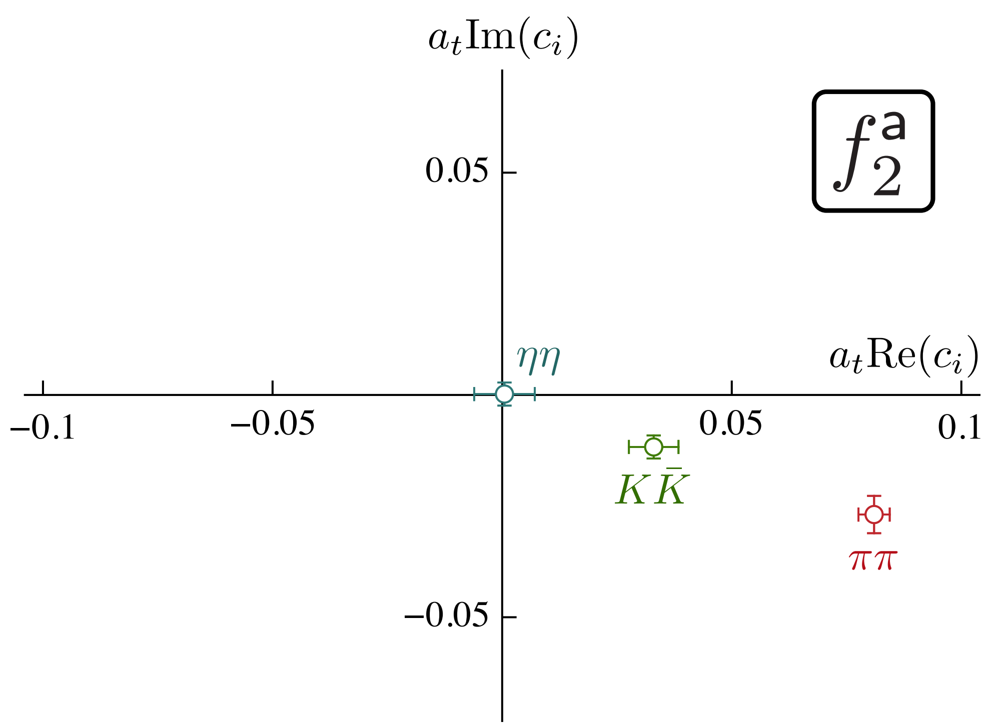
Near a t-matrix pole

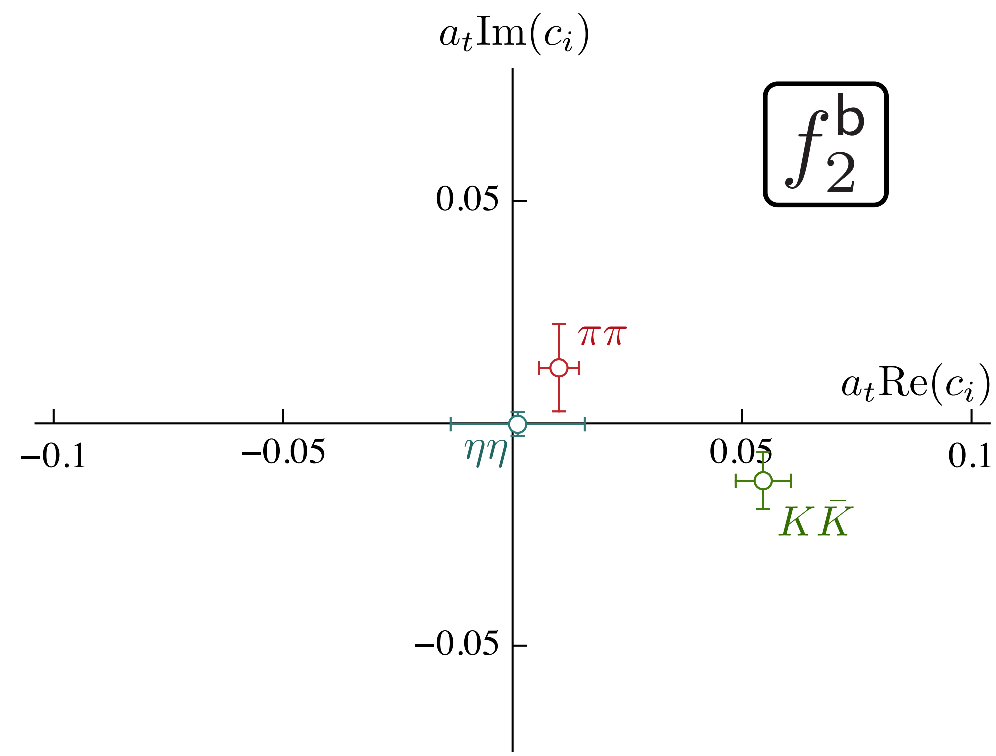
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

Tensor resonance poles







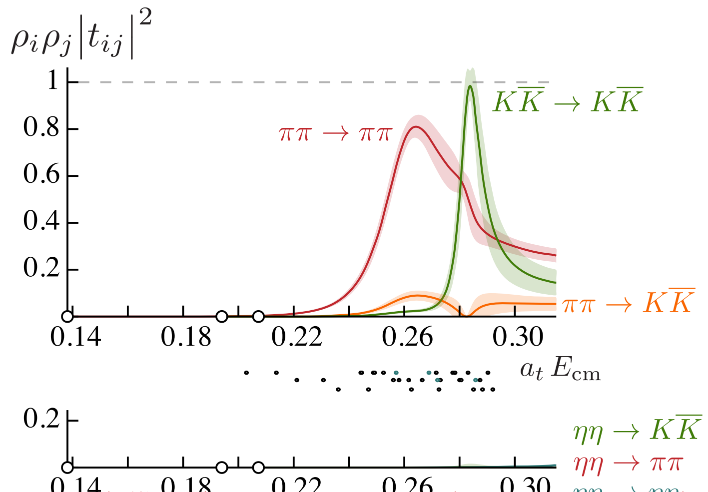


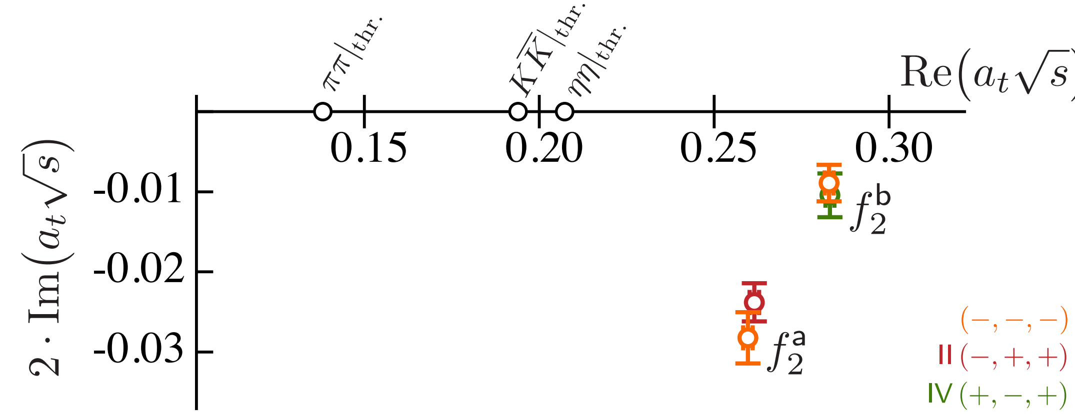
$f_2^{\mathsf{a}}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

$f_2^{\mathsf{b}}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)$ MeV

$\mathrm{Br}(f_2^{\mathsf{b}} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\mathsf{b}} \to K\overline{K}) \sim 92\%$





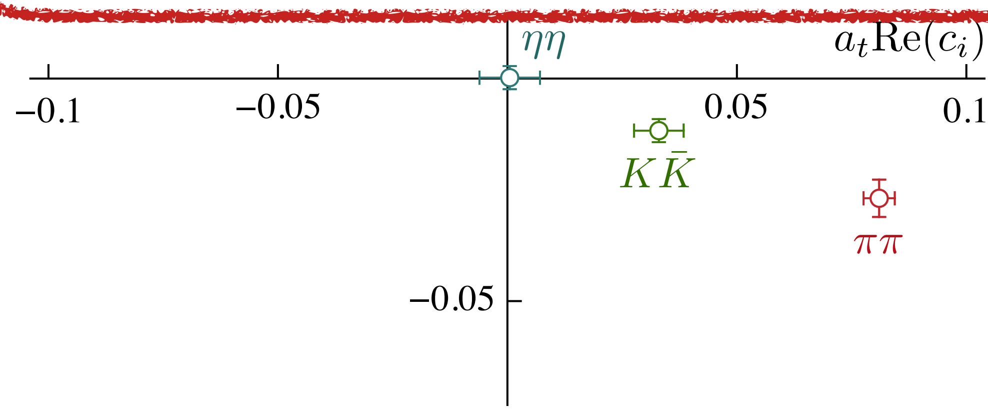
$f_2(1270)$ DECAY MODES

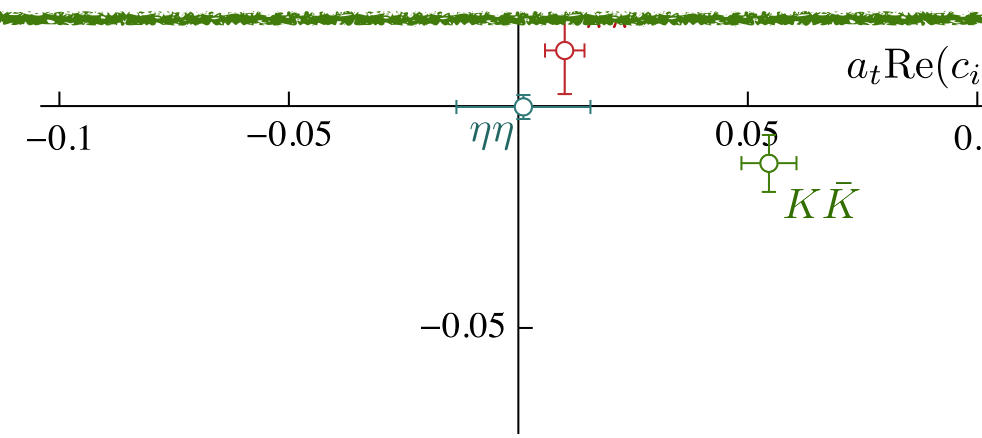
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ ^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(\ 7.7\ ^{+1.1}_{-3.2})$ %
Γ_3	$K\overline{K}$	$(\ 4.6\ ^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	$(\ 2.8\ \pm0.4)$ %

PDG2017

$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}$	$(88.7\ \pm2.2)$ %
Γ_2	$\eta\eta$	$(10.4\ \pm2.2)$ %
Γ_3	$\pi\pi$	$(\ 8.2\ \pm1.5)\times10^{-3}$
Γ_4	$K\overline{K}^*(892)+$ c.c.	



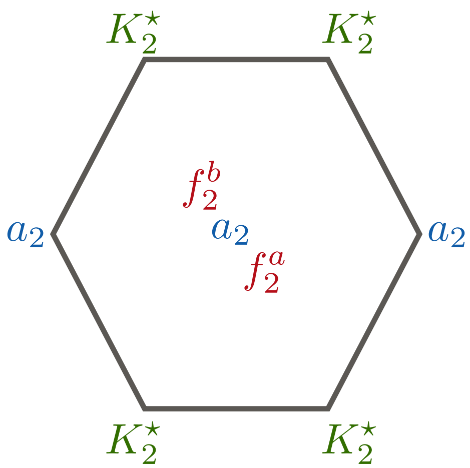


$$f_2^{\rm a}:\quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)\ \mathrm{MeV}$$

$$\mathrm{Br}(f_2^{\rm a}\to\pi\pi)\sim 85\%,\quad \mathrm{Br}(f_2^{\rm a}\to K\overline{K})\sim 12\%$$

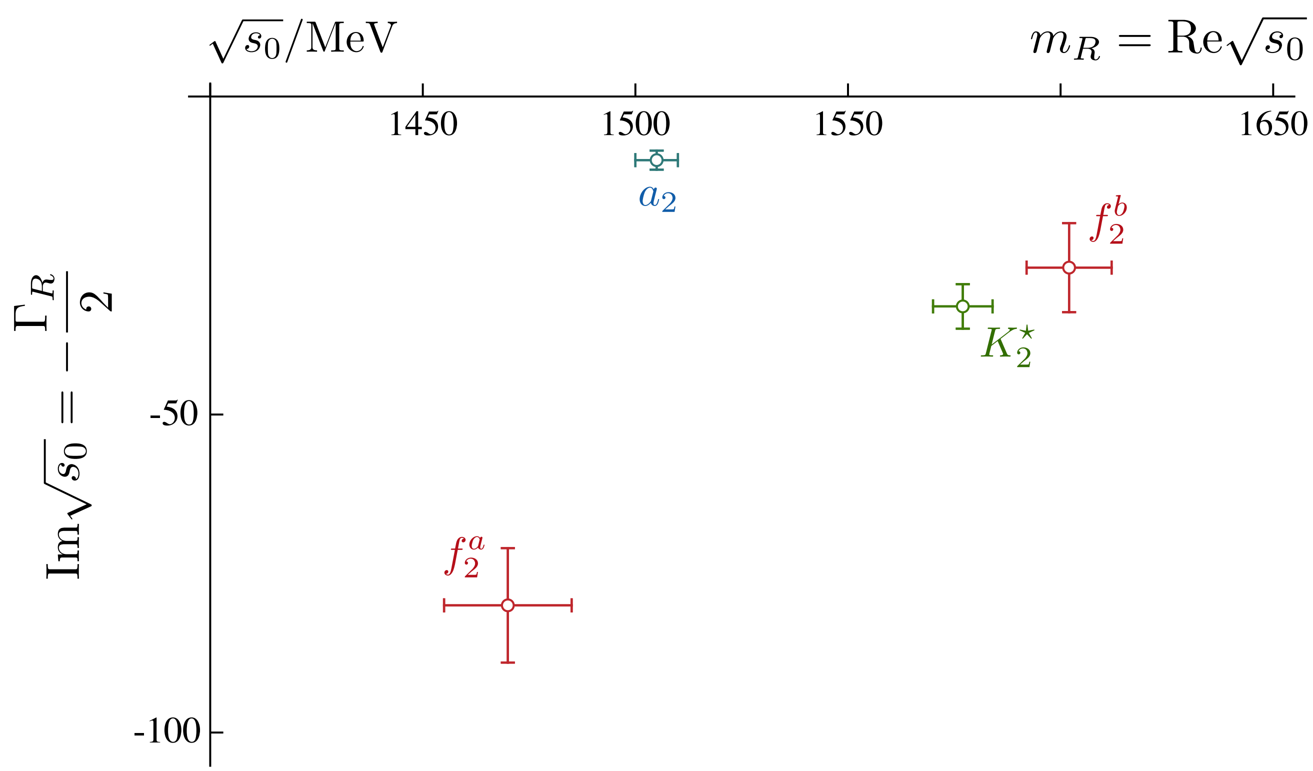
$$f_2^{\rm b}:\quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)\ \mathrm{MeV}$$

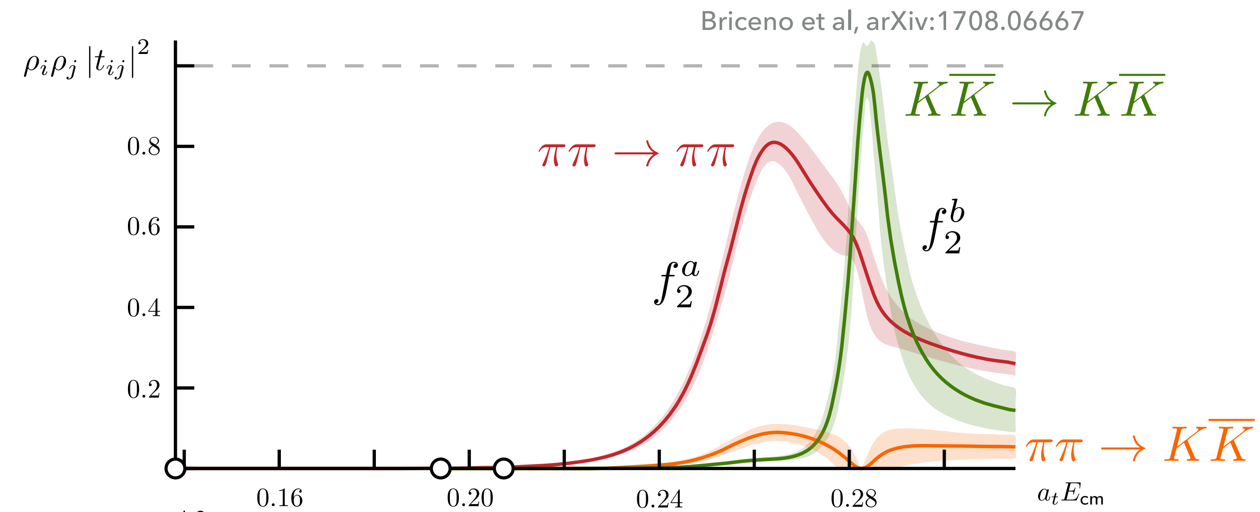
$$\mathrm{Br}(f_2^{\rm b}\to\pi\pi)\sim 8\%,\quad \mathrm{Br}(f_2^{\rm b}\to K\overline{K})\sim 92\%$$

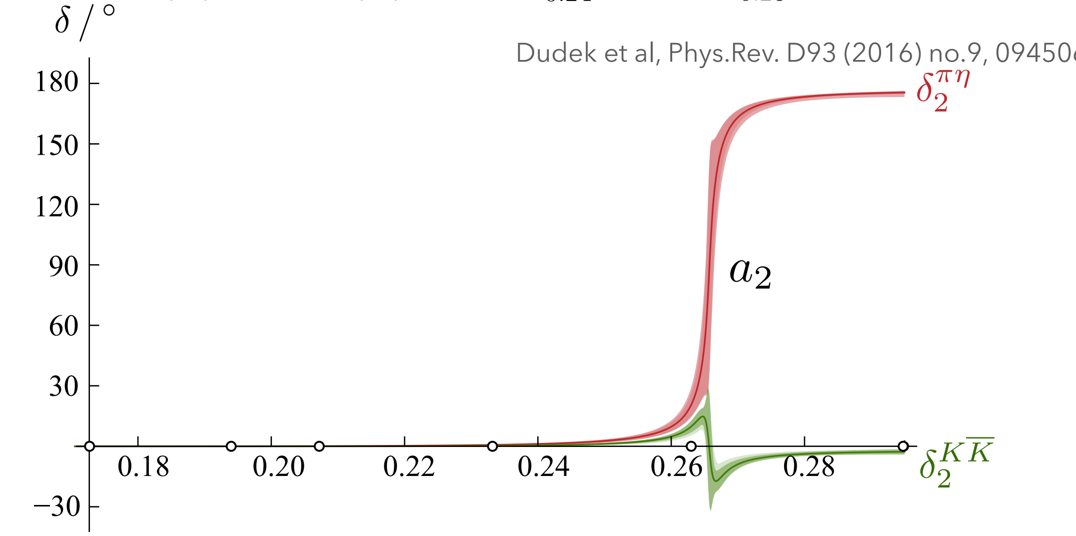


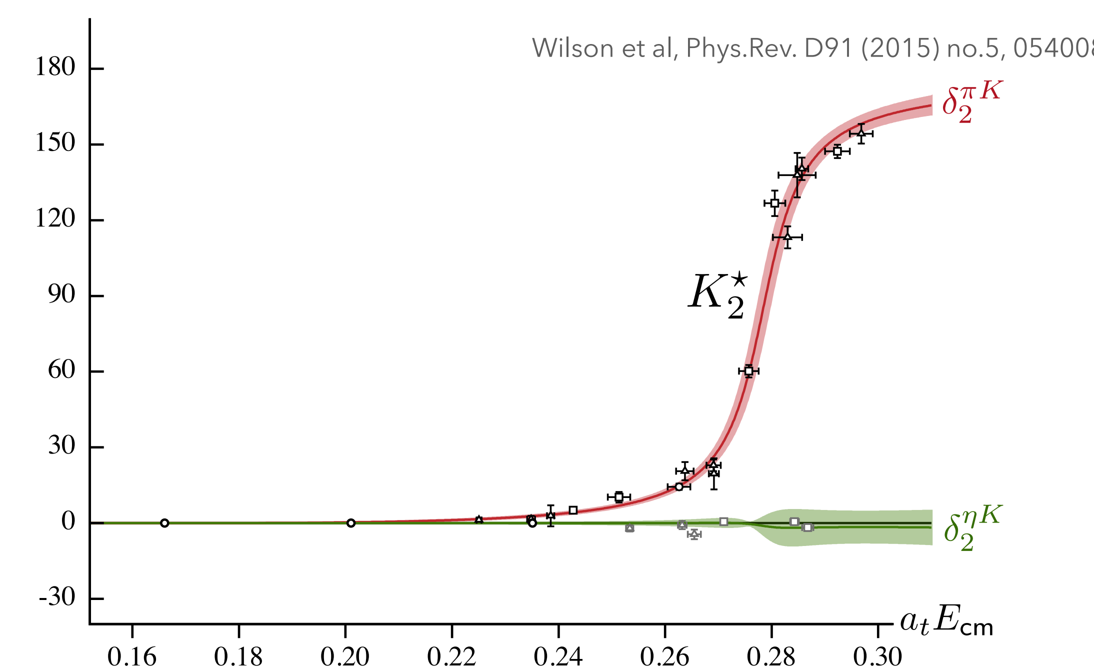
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

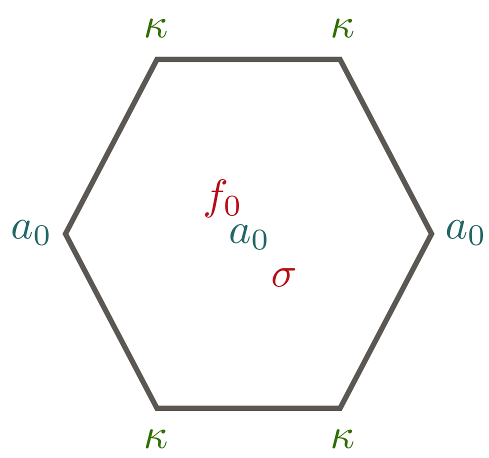
$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$





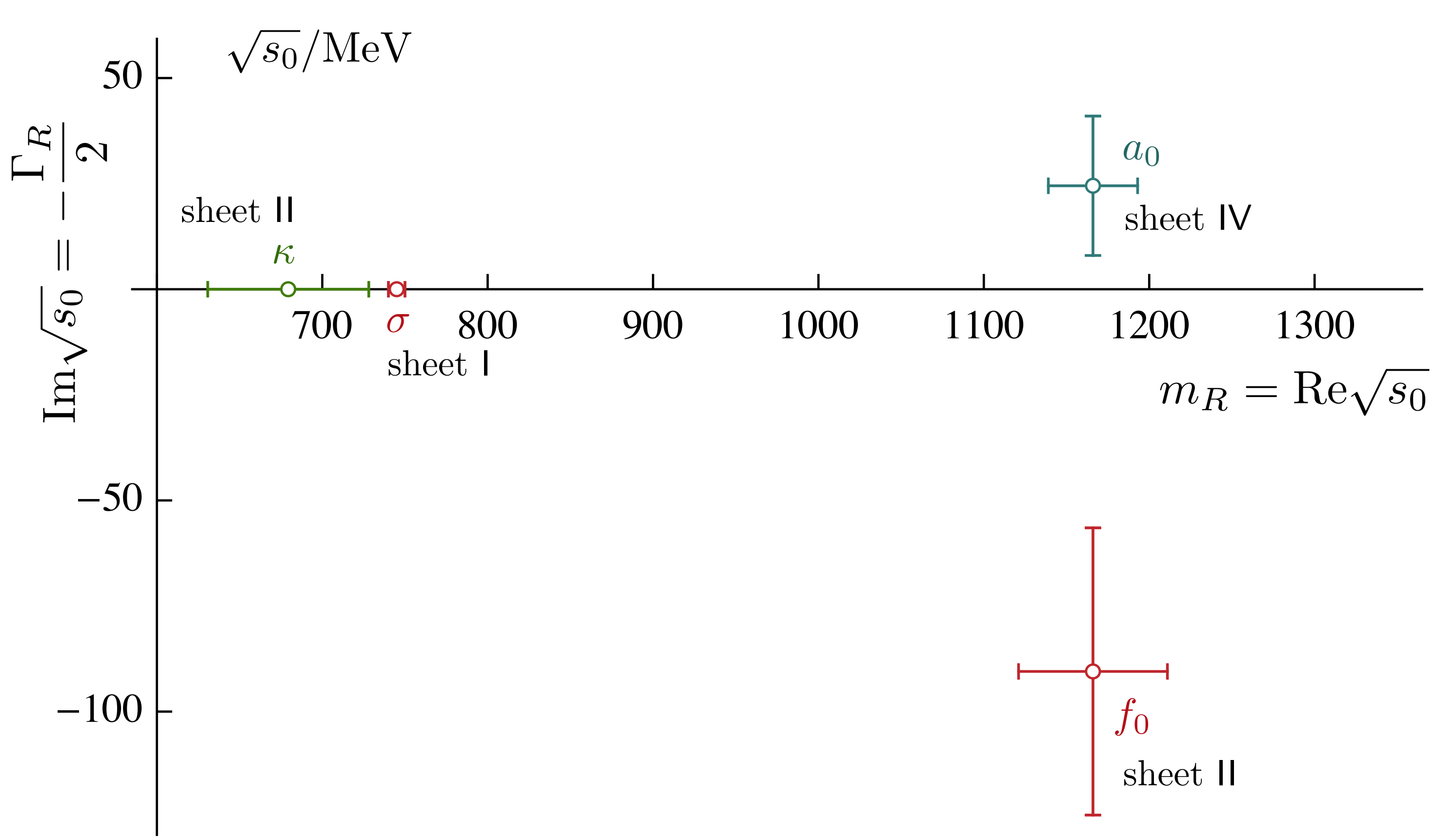


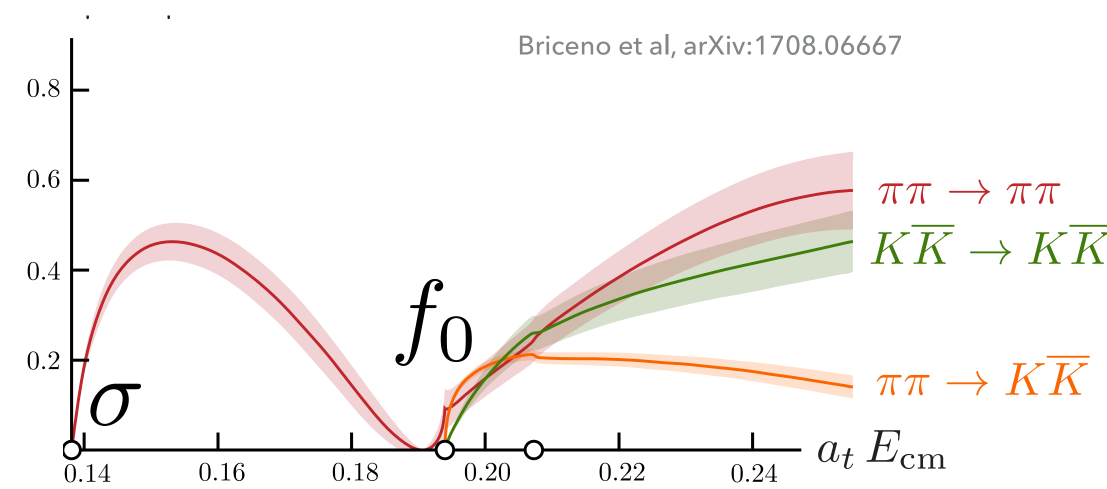


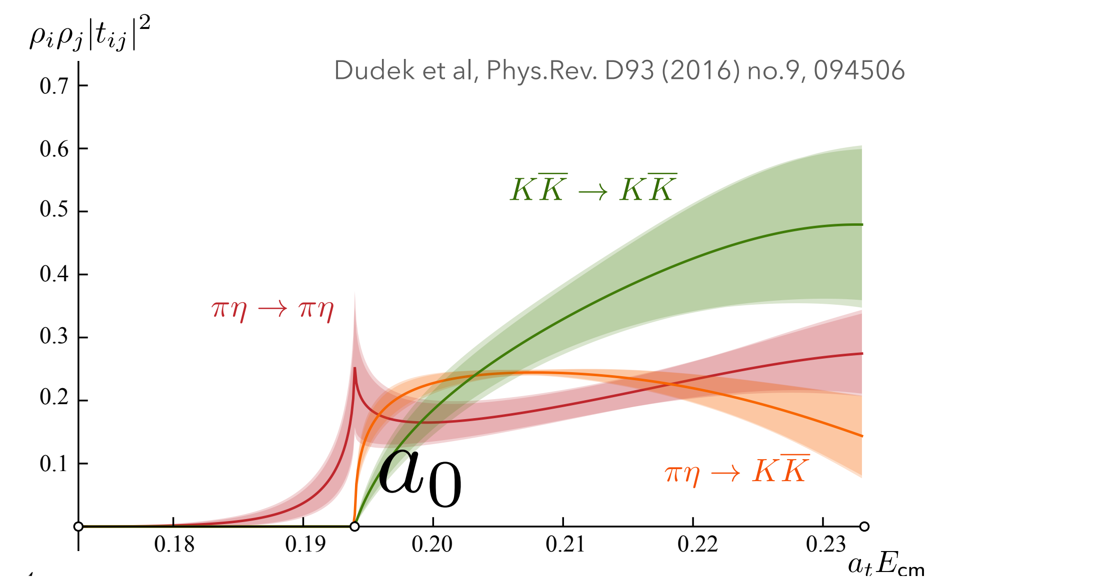


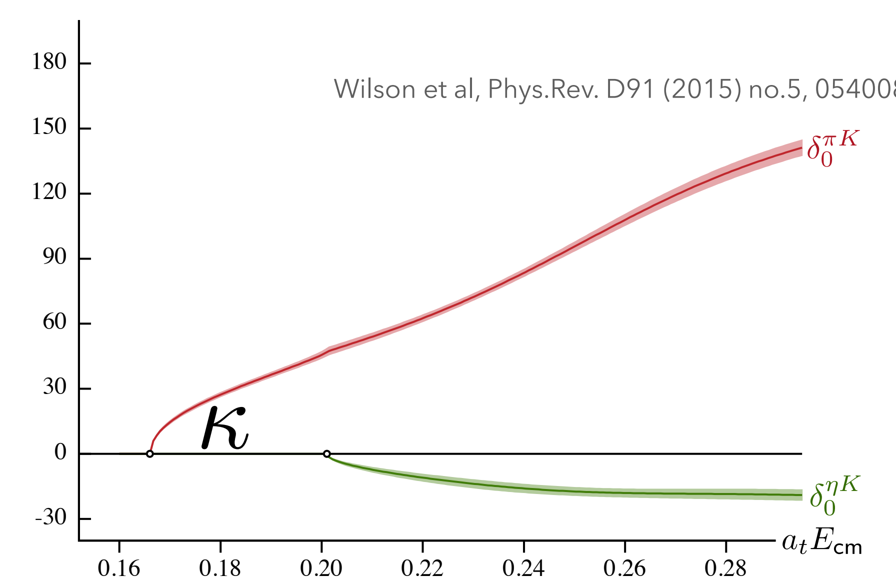
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$







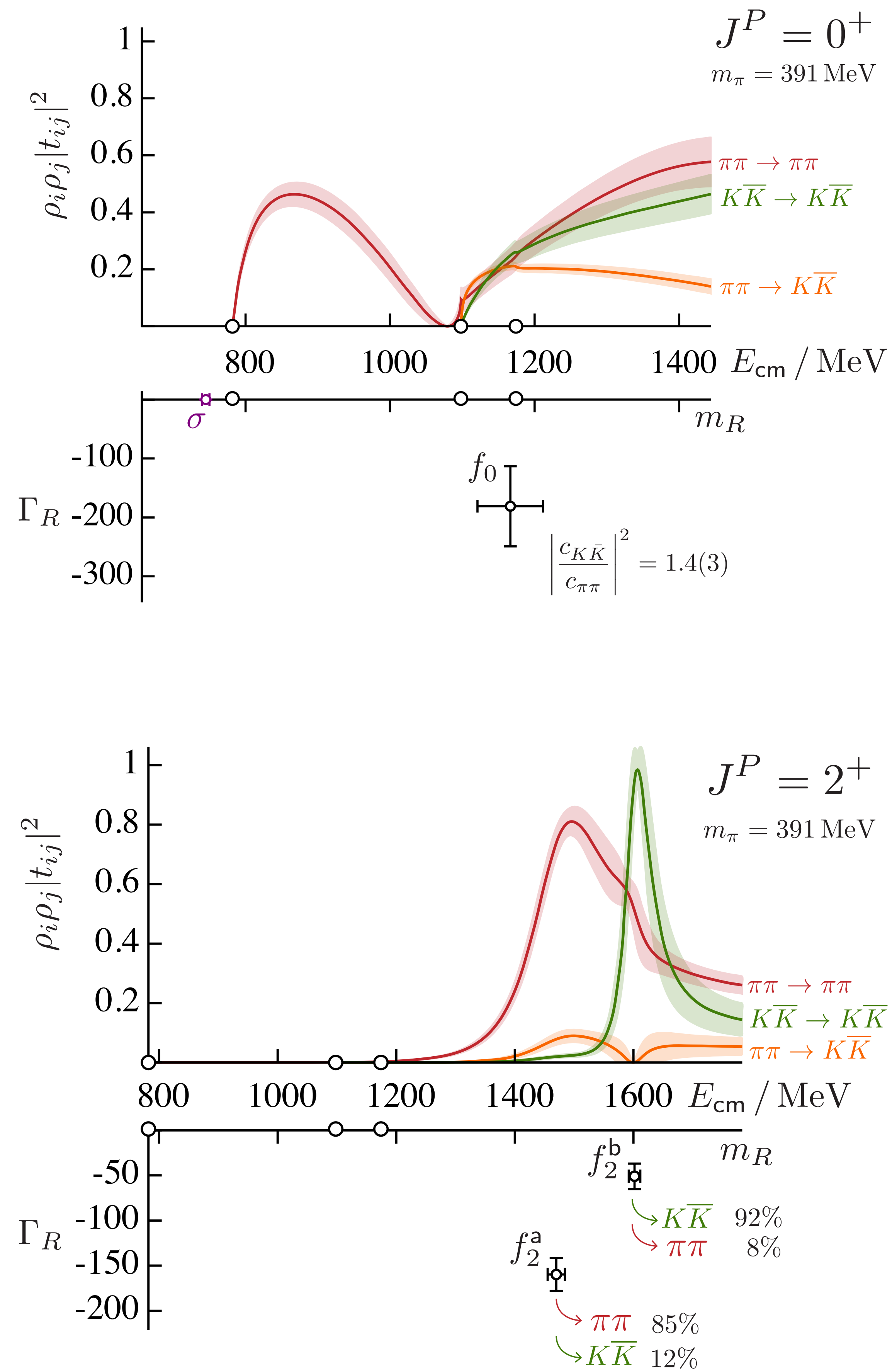


Scattering amplitudes of pairs of pseudo-scalar hadrons can be computed from lattice QCD

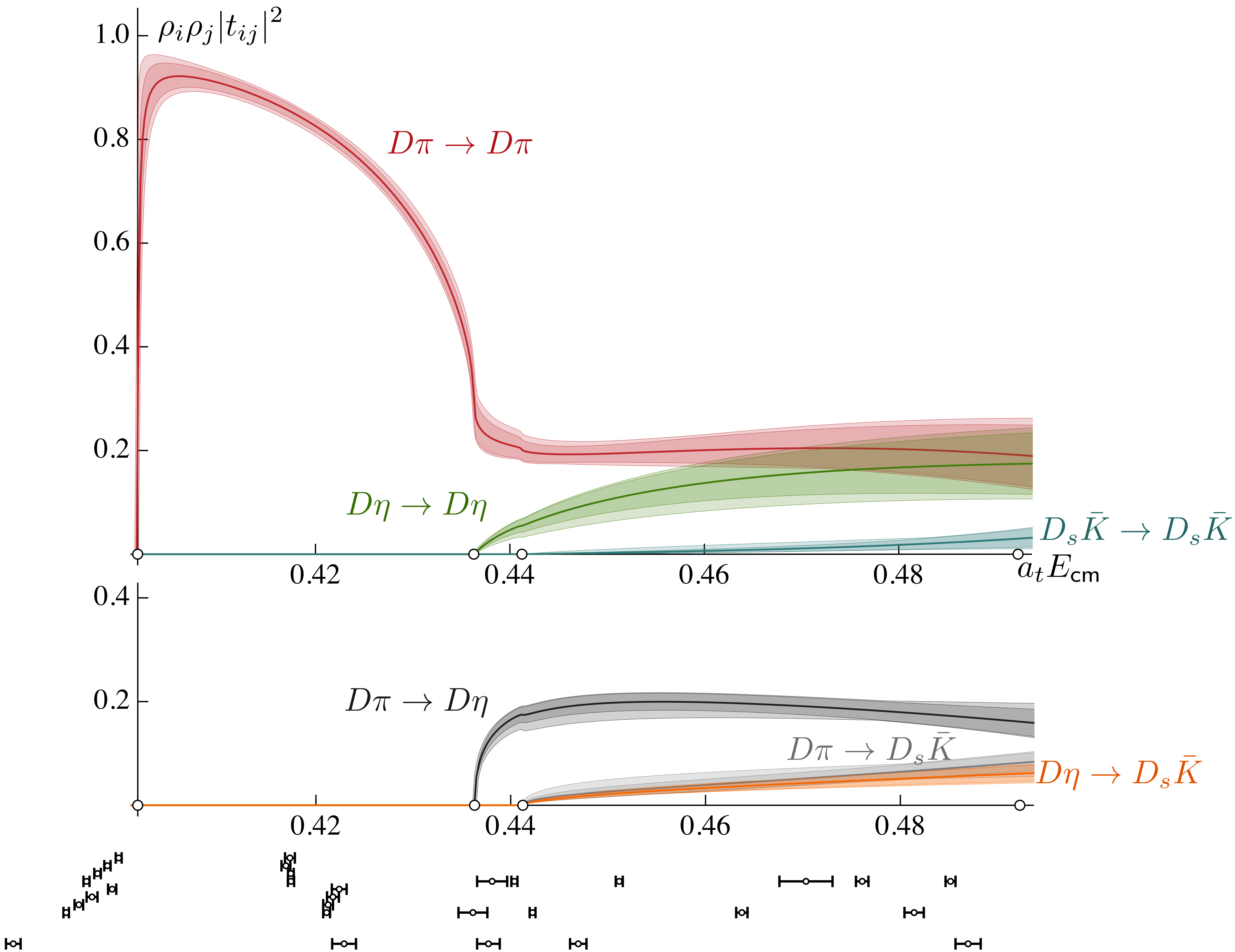
Several channels with scalar, vector and tensor resonances have been computed

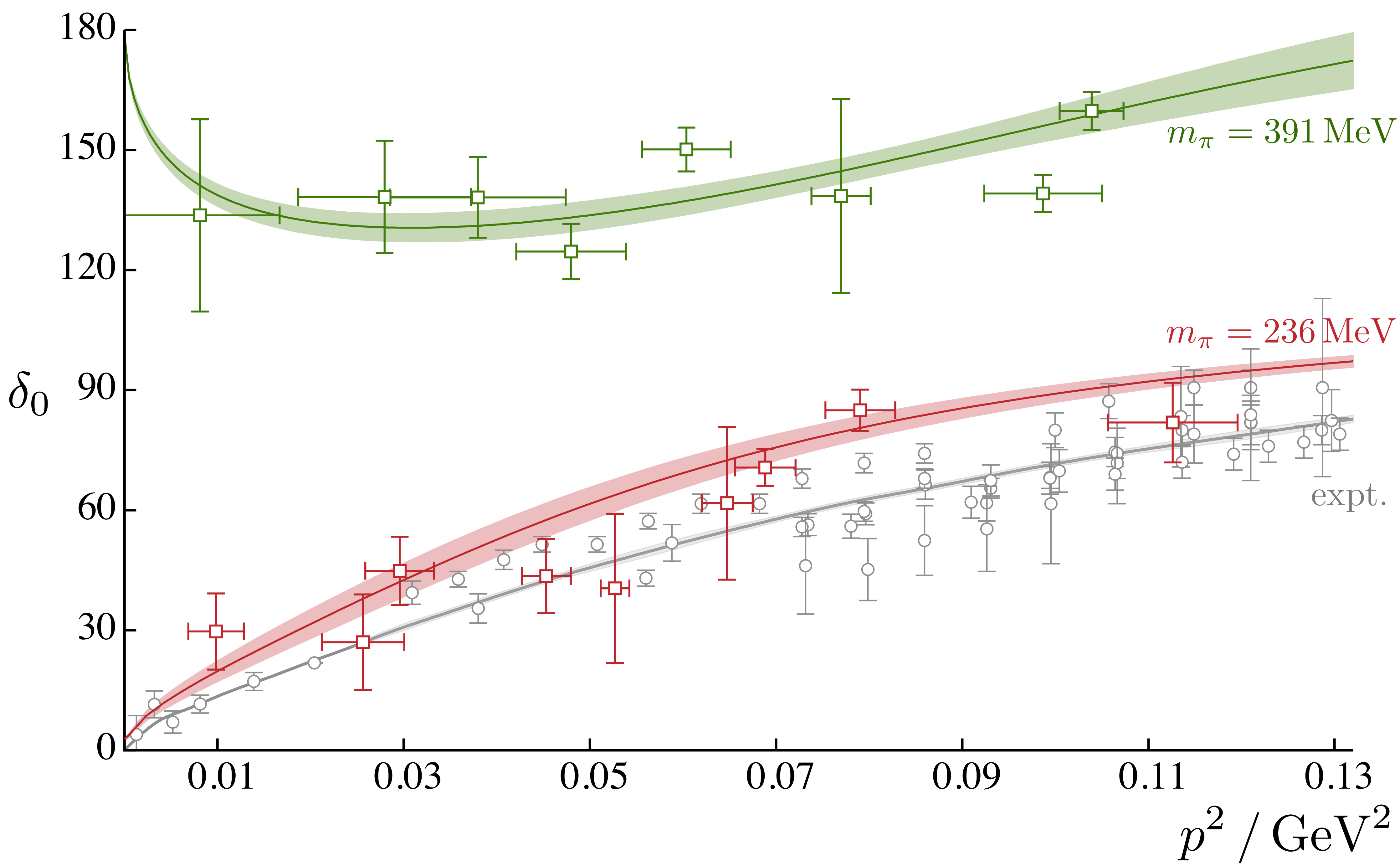
Control of 3+ body effects needed for
- lighter pion masses
- higher mass resonances

Scattering of particles with spin
- see next talk

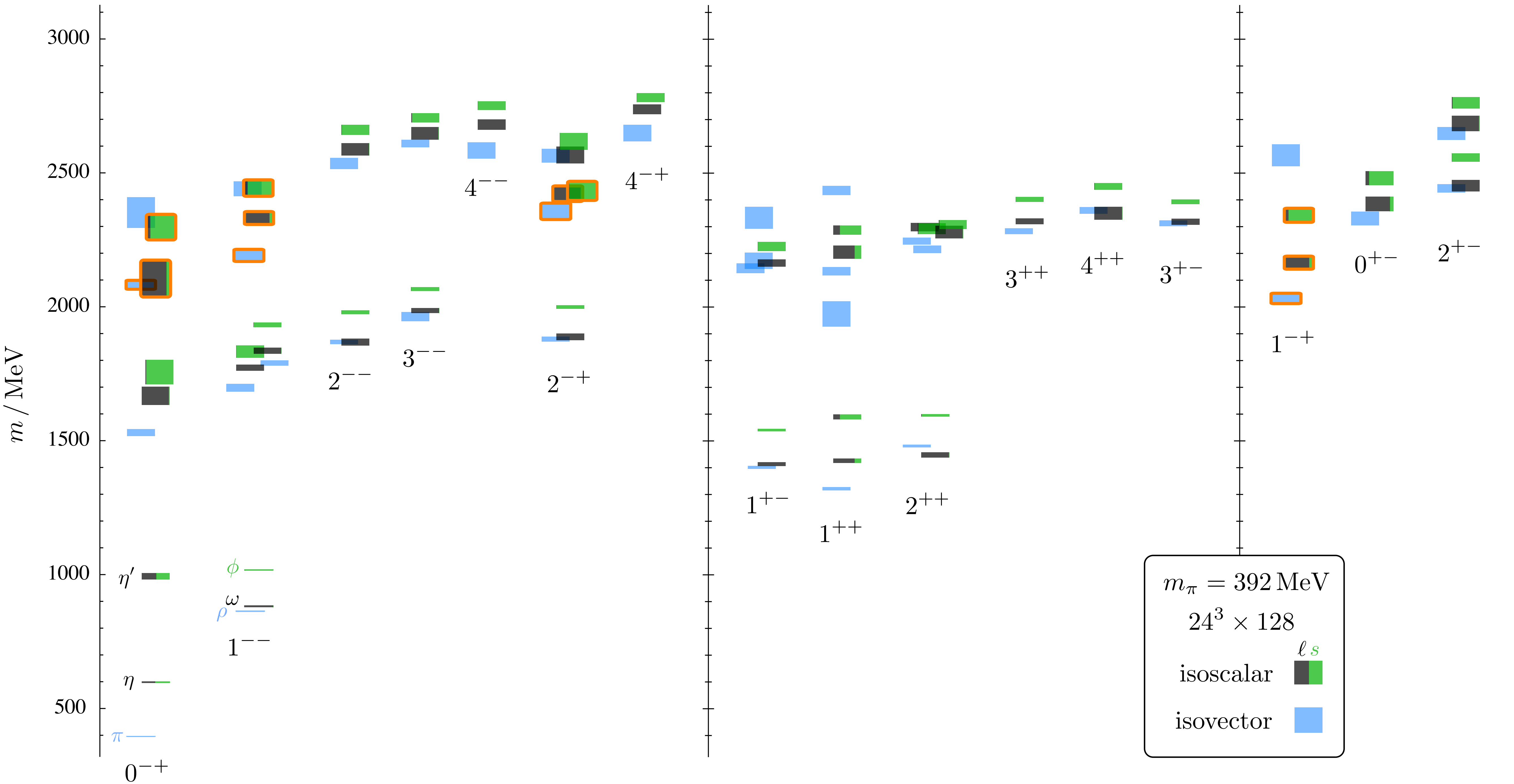


Moir et al, JHEP 1610 (2016) 011

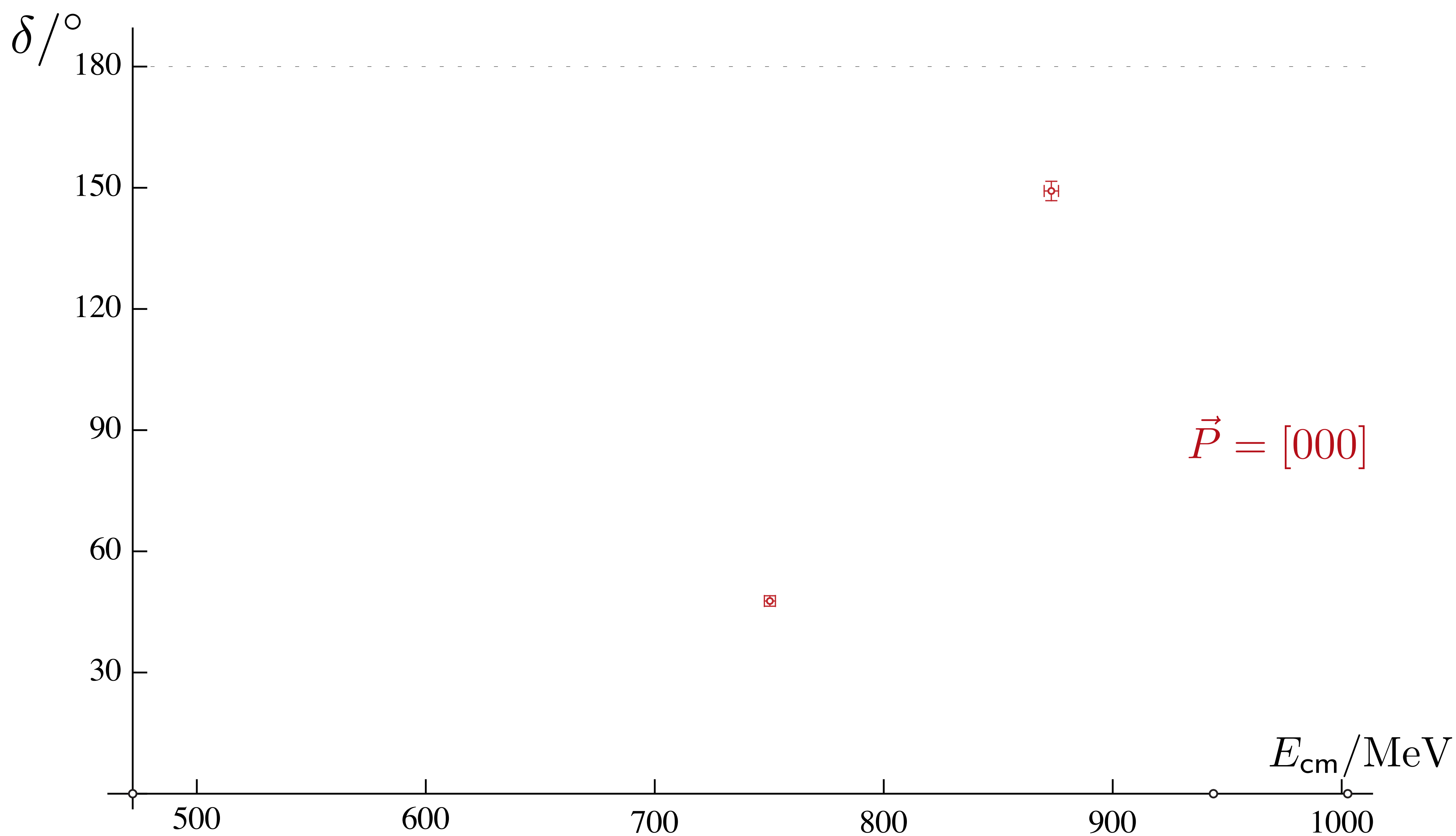



180
150
120
90
60
30
0
δ_0
$m_\pi = 391$ MeV
$m_\pi = 236$ MeV
expt.
0.01
0.03
0.05
0.07
0.09
0.11
0.13
p^2 / GeV^2

from a large basis of local qq-like operators



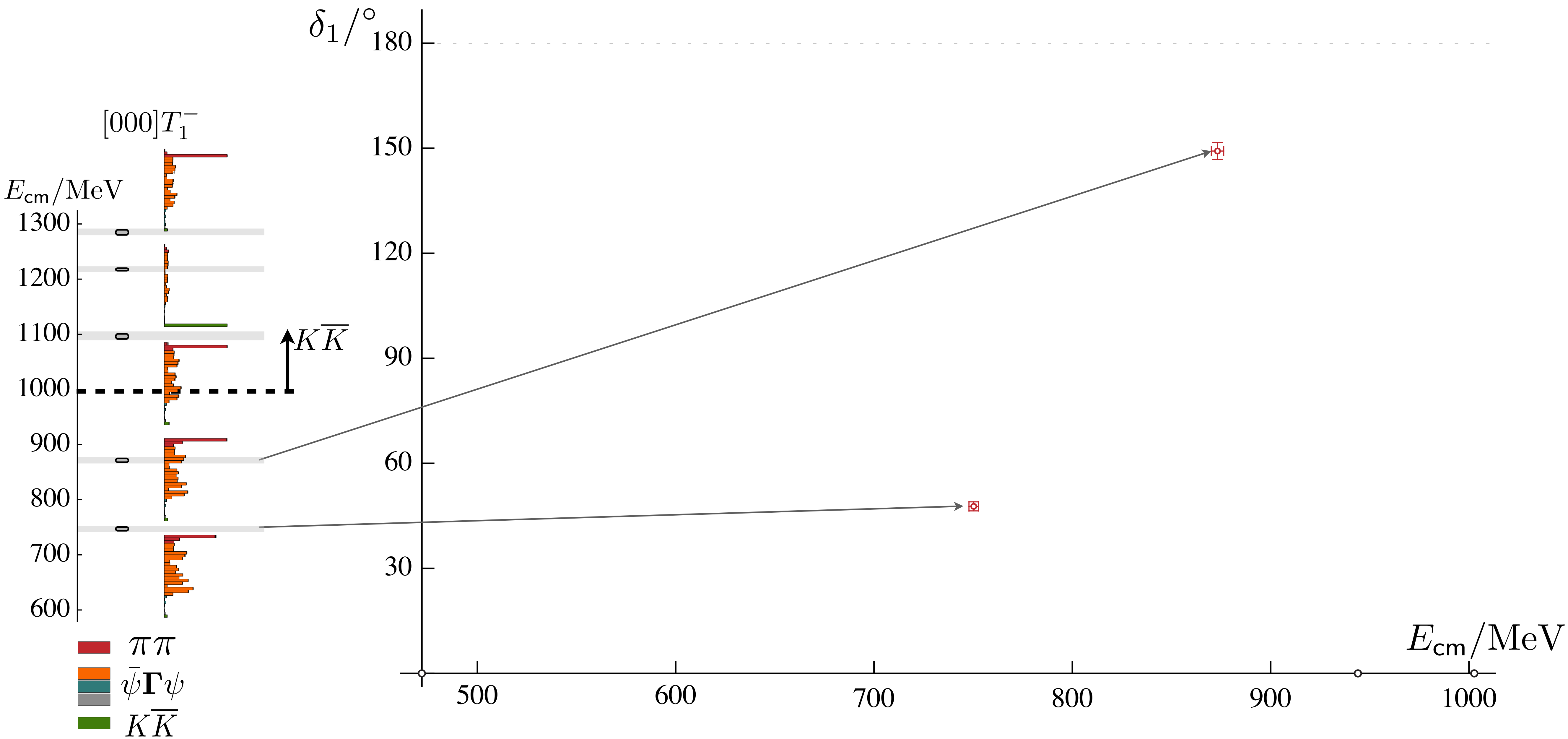
J. J. Dudek, R. G. Edwards, P. Guo and C. E. Thomas, Phys.Rev. D88 (2013) 9, 094505.


$\delta/^\circ$
180
150
120
90
60
30
$\vec{P} = [000]$
$E_{\mathsf{cm}}/\mathrm{MeV}$
500
600
700
800
900
1000

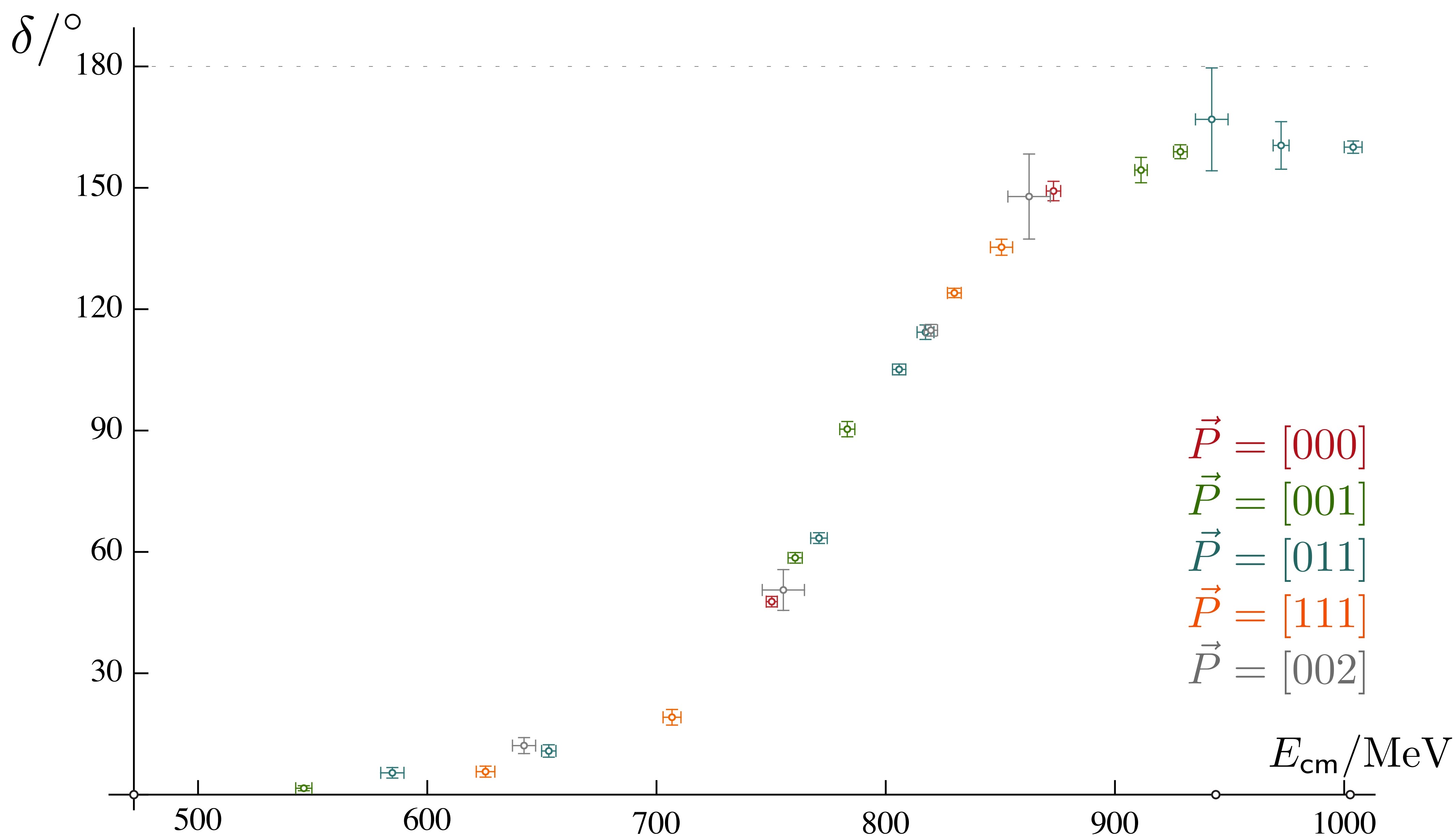
$m_\pi = 236$ MeV

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

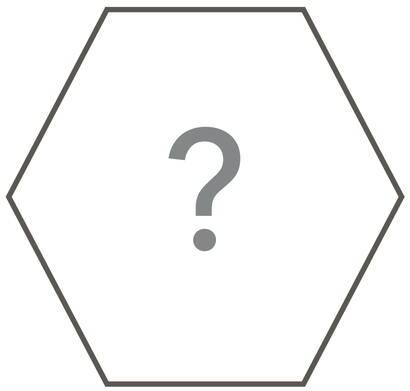
$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$



$m_\pi = 236$ MeV

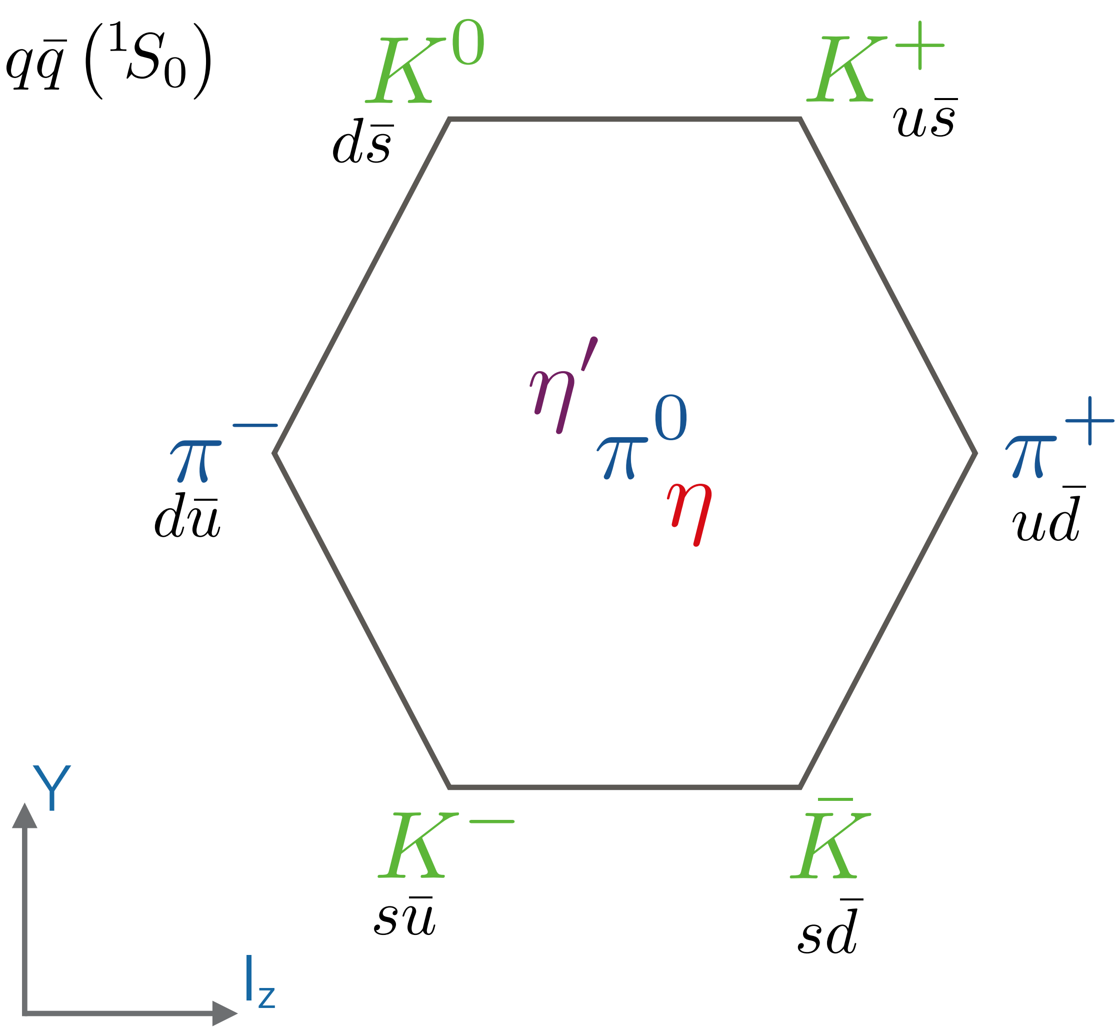

$\delta/^\circ$
180
150
120
90
60
30
500
600
700
800
900
1000
$E_{\rm cm}$/MeV
$\vec{P}=[000]$
$\vec{P}=[001]$
$\vec{P}=[011]$
$\vec{P}=[111]$
$\vec{P}=[002]$

$m_\pi = 236$ MeV

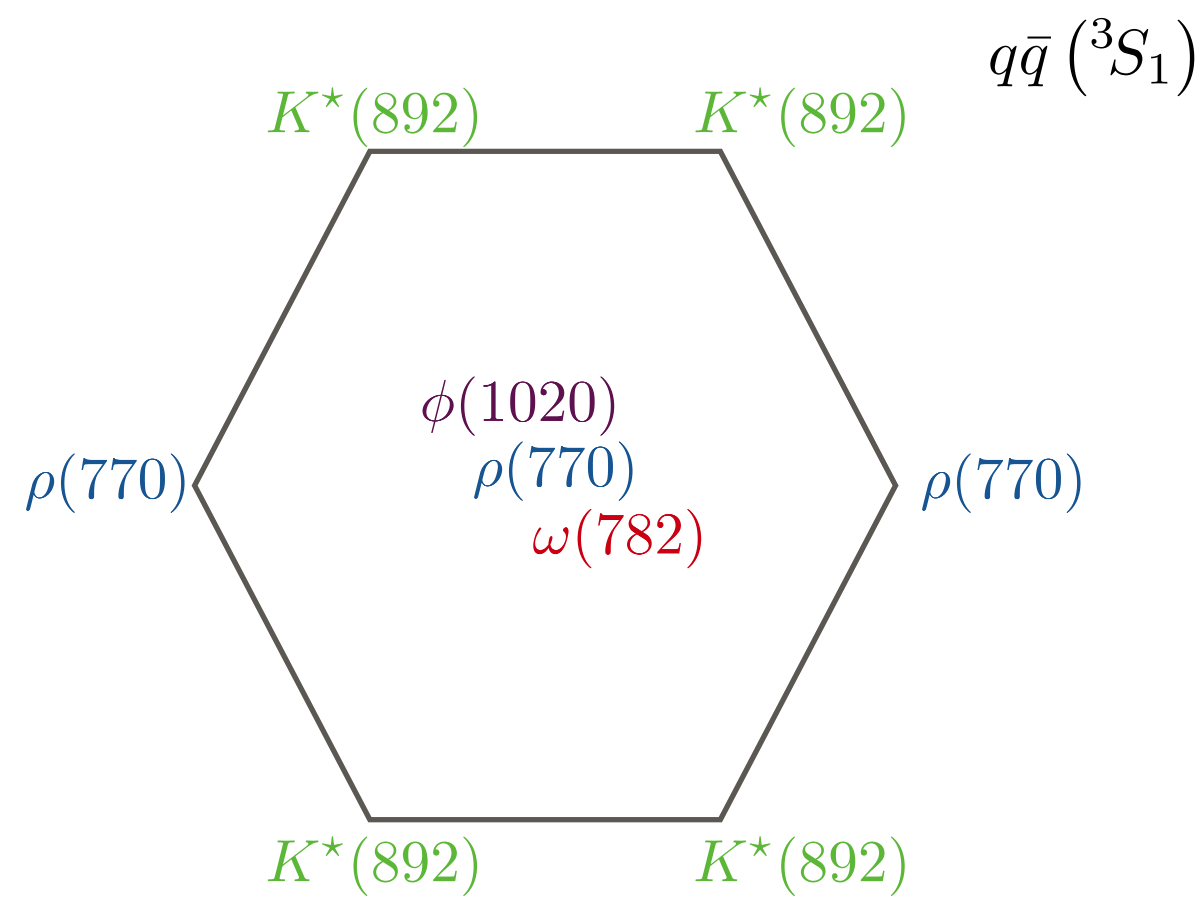


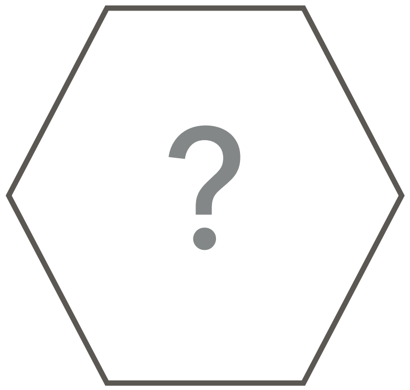
Considering *u,d,s* quarks: $q\bar{q}\,(^{2S+1}L_J)$

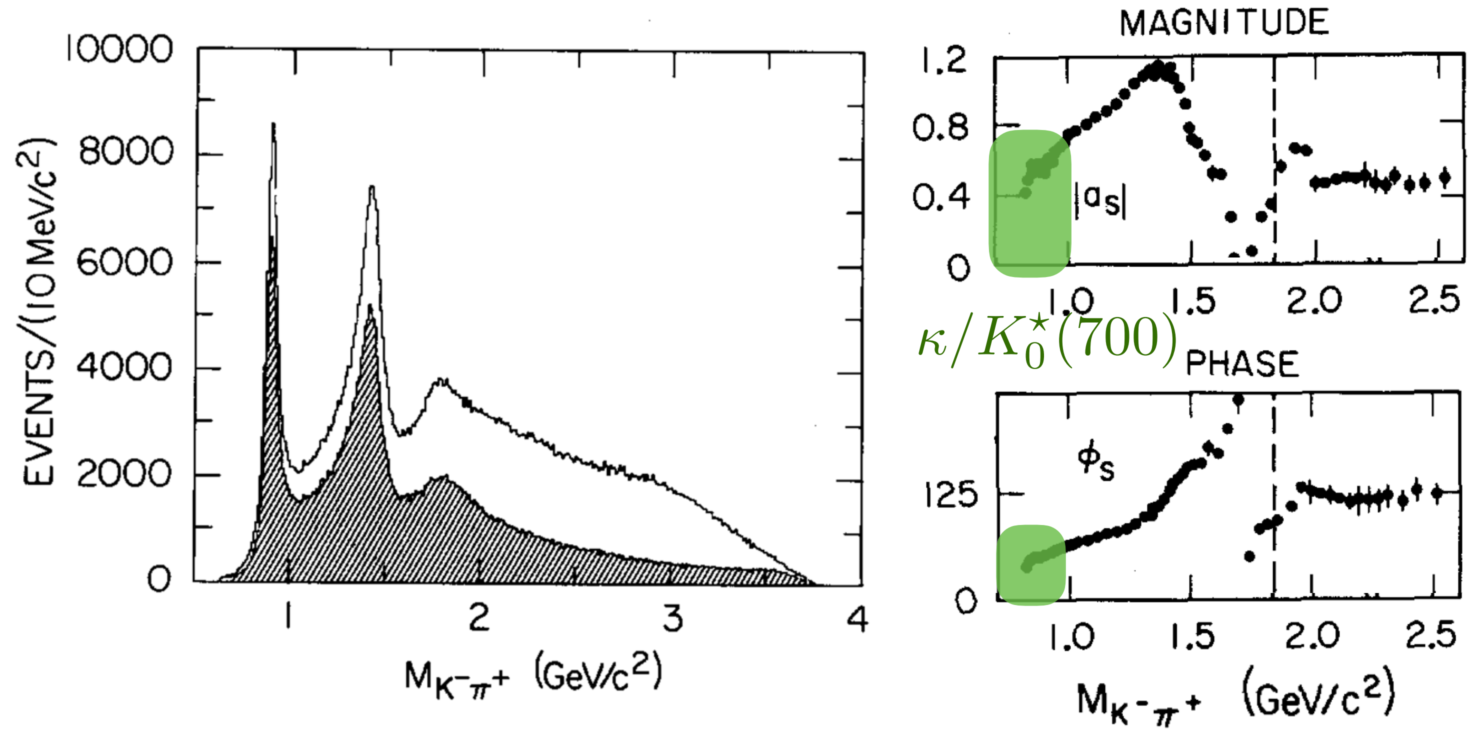
pseudoscalar nonet $J^P=0^-$



vector nonet $J^P=1^-$







LASS experiment at SLAC $E_K = 11$ GeV

GAMS, Alde *et al* PLB 203 397, 1988.