Majorana Edge Modes

in a spinful particle-conserving model

1) Introduction:
   - Majoranas
   - Kitaev
   - Spinless ladder

2) Our model:
   - Spinful ladder

3) The results:
   - Majorana phase

1) Introduction

Majoranas are half fermionic degree of freedom

Majorana Fermions in HEP:

A fermion that is its *own* antiparticle

Creation = **Anihilation:** $\gamma = \gamma^{\dagger}$

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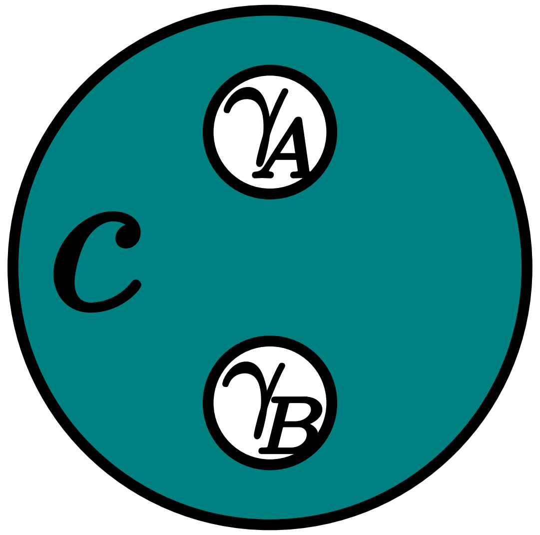
1 Fermion

=

2 Majoranas



Definition:

$$\gamma_A = c + c^\dagger$$
$$i\gamma_B = c - c^\dagger$$

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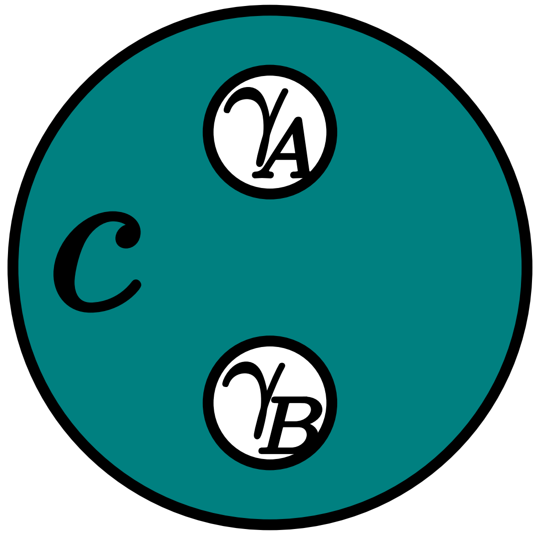


Definition:

$$\gamma_A = c + c^\dagger$$
$$i\gamma_B = c - c^\dagger$$

Majorana occupation?

$\gamma_\alpha^\dagger \gamma_\alpha = 1$ Not very useful!

You cannot occupy a single Majorana mode!!

Majoranas on a chain?

Definition:

$$\gamma_A = c + c^\dagger$$
$$i\gamma_B = c - c^\dagger$$

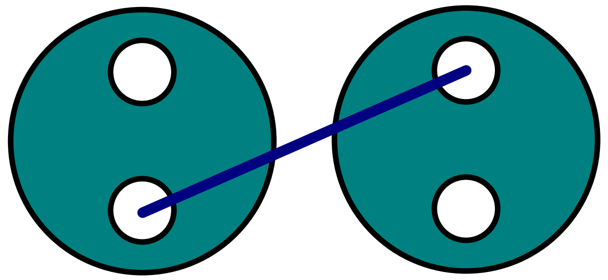
Number operator?

$$n = c^\dagger c = \frac{1}{2}(i\gamma_A\gamma_B + 1)$$

We need **two different** Majoranas
to define an **occupation number!**

$$i\gamma_A\gamma_B = 2c^\dagger c - 1 = \pm 1$$

Different sites?



$$i\gamma_{Bj}\gamma_{Aj+1}$$

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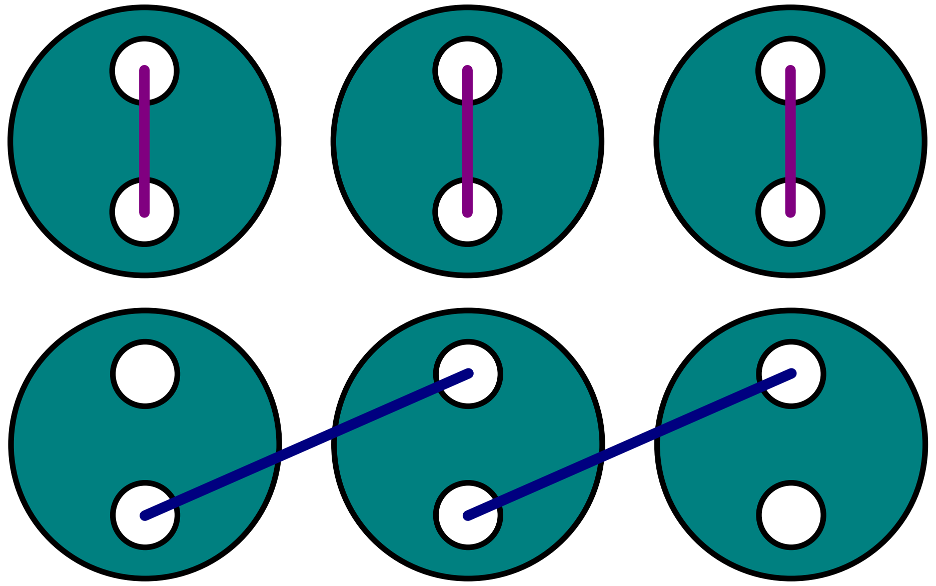
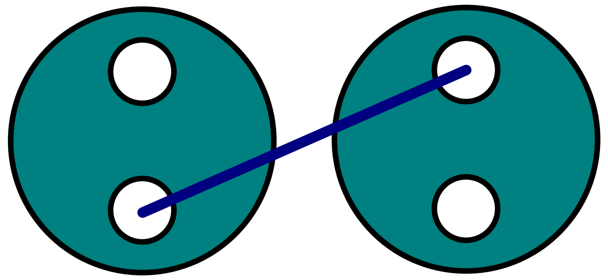


Trivial

Non-local

Different sites?



$$i\gamma_{Bj}\gamma_{Aj+1}$$

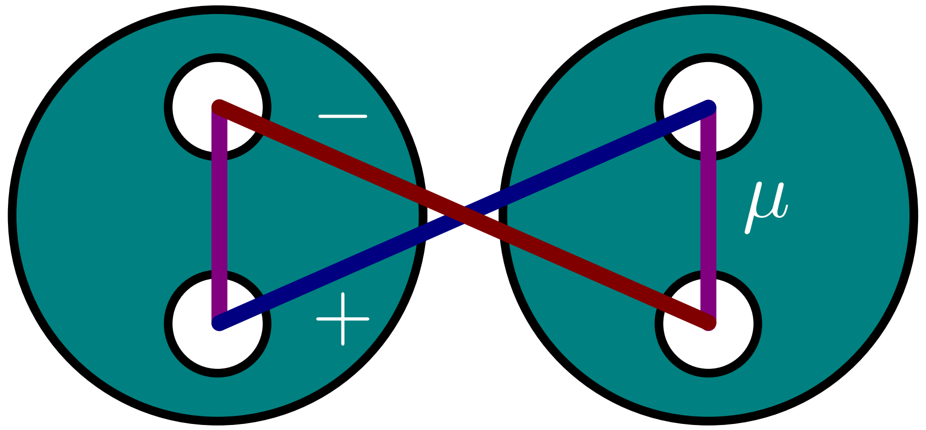
$$H = \sum_j i\gamma_{Bj}\gamma_{Aj+1}$$

Does not conserve **N**, but it conserves **P**!

$$\downarrow$$

$$H = -\sum_j (c_j^\dagger c_{j+1} - c_j c_{j+1} + \text{hc})$$

Topological phase in the Kitaev chain



Kitaev, *Phys. Usp.* **44**, 131 (2001)

$$H = -\sum_j (t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + \mathrm{hc}) - \mu \sum_j n_j$$

- Spinless
- Doesn't conserve **N**

$$H = \frac{i}{2} \sum_j -\mu \gamma_{Aj}\gamma_{Bj} + (\Delta + t)\gamma_{Bj}\gamma_{Aj+1} + (\Delta - t)\gamma_{Aj}\gamma_{Bj+1}$$

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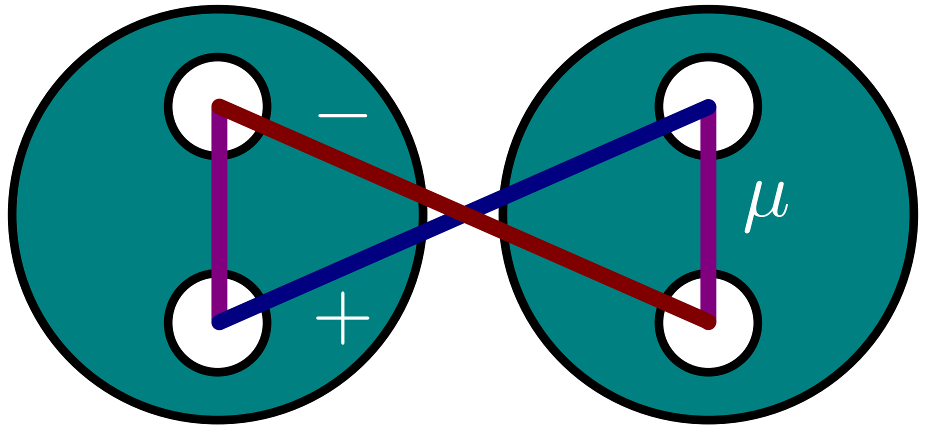

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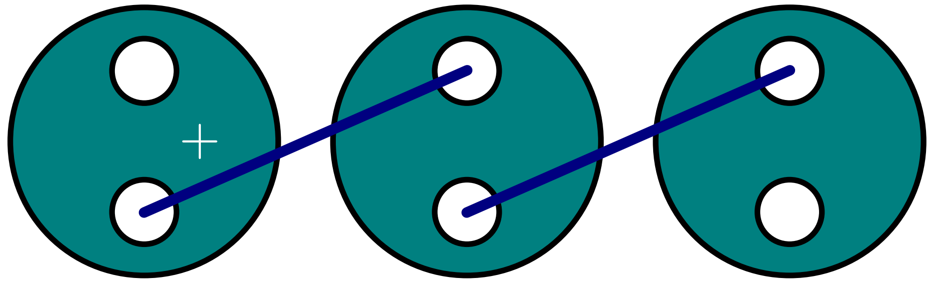
Topological case:

$\Delta = t \quad \mu = 0$



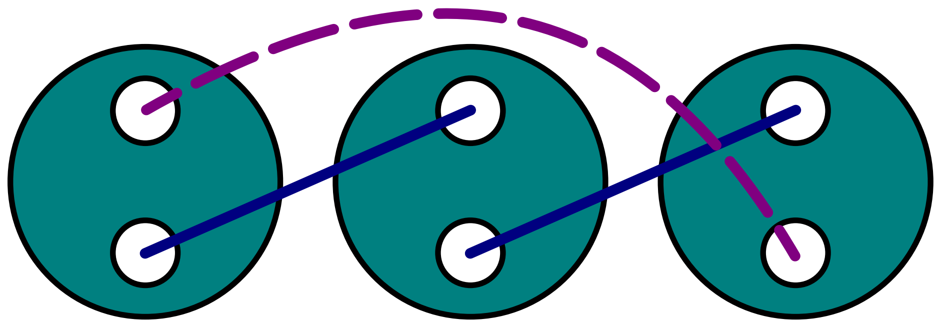
$$H = -it \sum_j \gamma_{Bj}\gamma_{Aj+1}$$

What about the **Majoranas at the ends**?

$$\tilde{c}_M = \frac{1}{2}(\gamma_{A1} + i\gamma_{BL})$$

Non-local zero-energy fermionic state!

Characteristics



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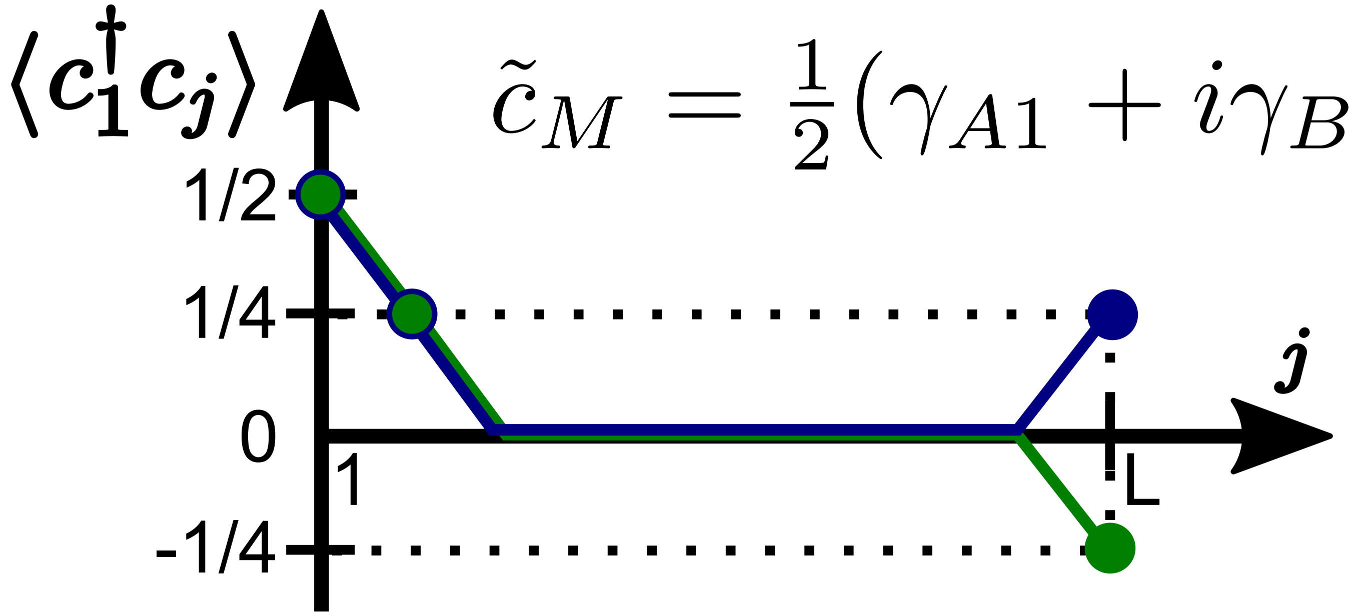
① **Both parity sectors are quasi-degenerate**

E(P=0) = E(P=1) !
Zero-energy mode

② **Non-local end-end correlations**



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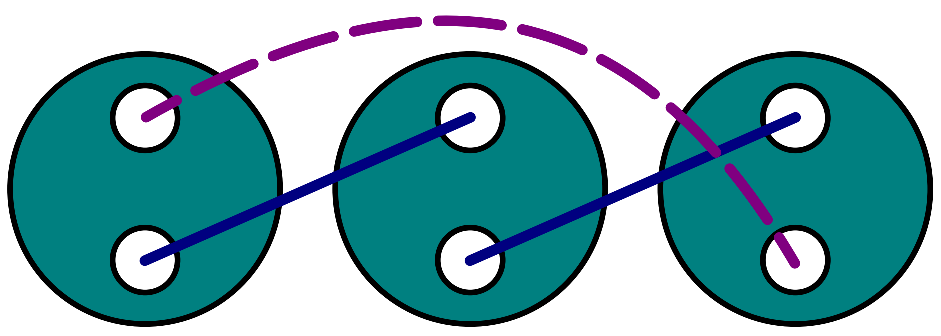

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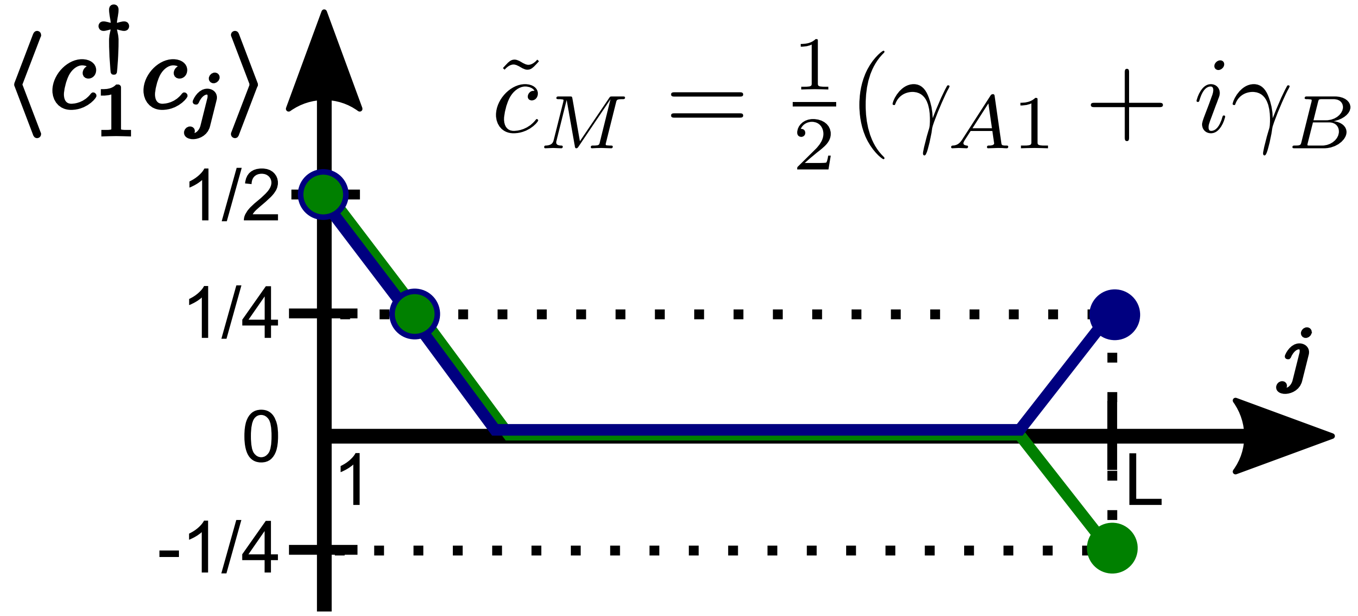

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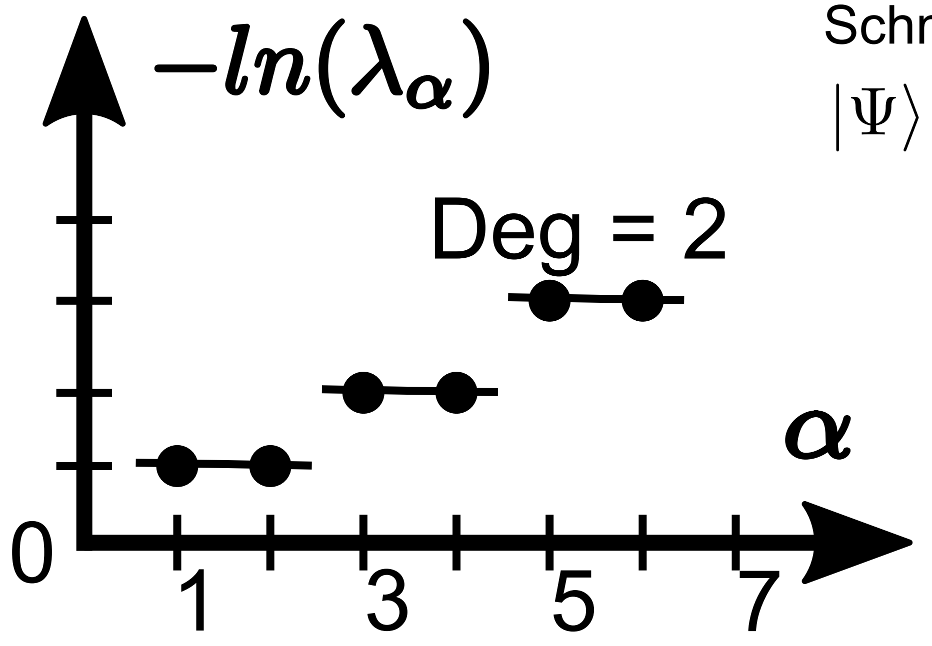
③ Double degeneracy on the Entanglement Spectrum



Schmidt decomposition

$$|\Psi\rangle = \sum_\alpha \underbrace{\sqrt{\lambda_\alpha}}_{\text{sv}} |\alpha\rangle_A |\alpha\rangle_B$$

④ Robust against local noise

A number conserving model

- Works from 2010
- Work from 2013 by Kraus *et al*

Idea:

Two chains coupled only by a pair-hopping

Each one can act as a pair reservoir for the other

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Kraus et al, *PRL* **111**, 173004 (2013)

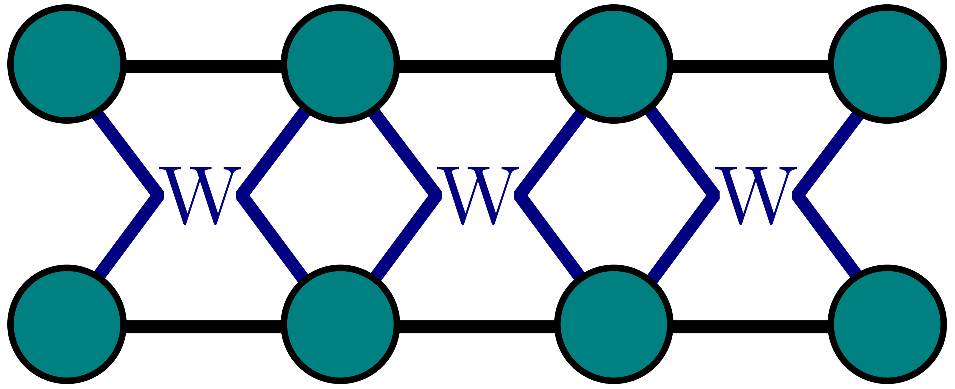
$$H = -t\sum_j (a_j^\dagger a_{j+1} + b_j^\dagger b_{j+1} + hc) + \\ + W\sum_j (a_j^\dagger a_{j+1}^\dagger b_j b_{j+1} + hc)$$

Fixed N!

Idea:

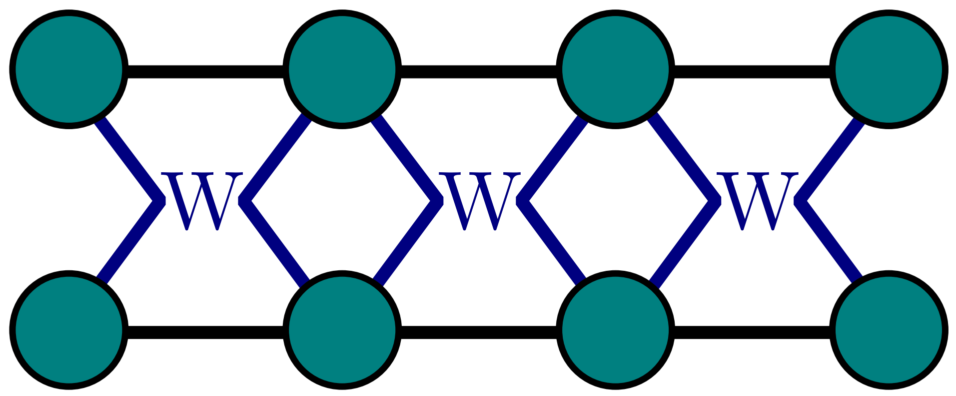
Two chains coupled only by a pair-hopping

Each one can act as a pair reservoir for the other



They find a Majorana phase!

A number conserving model



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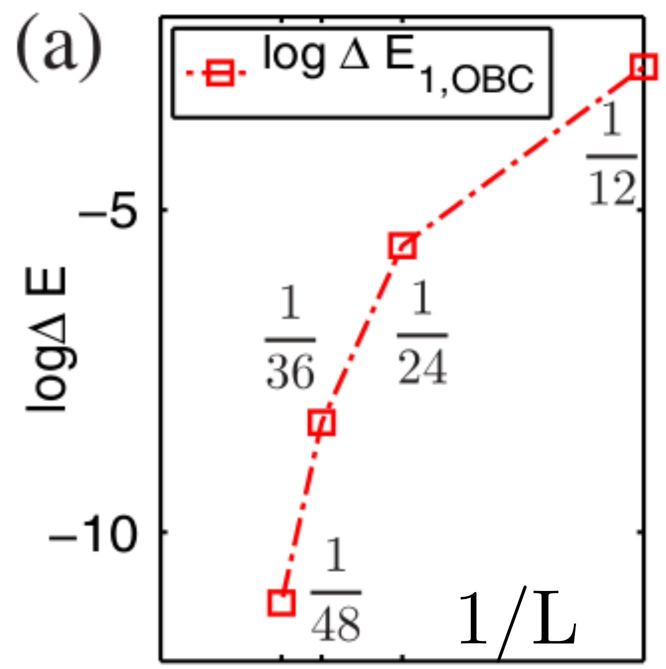
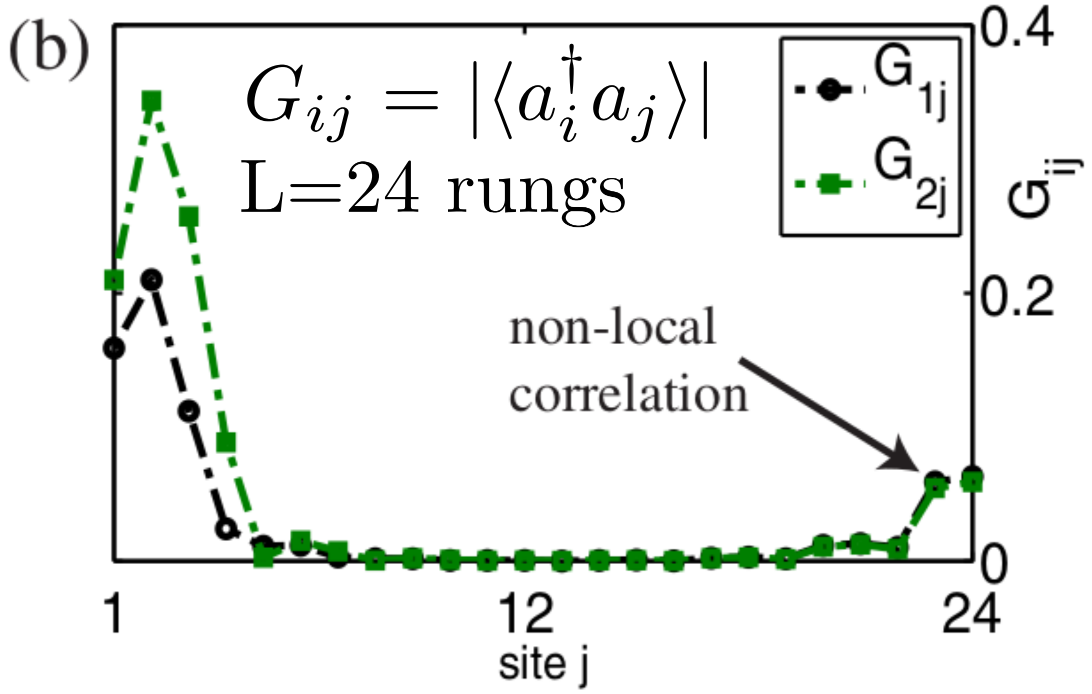


② **Non-local end-end correlations**



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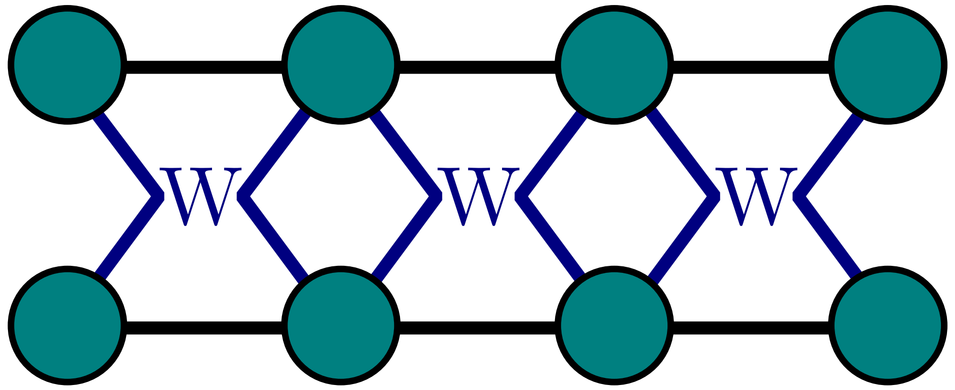

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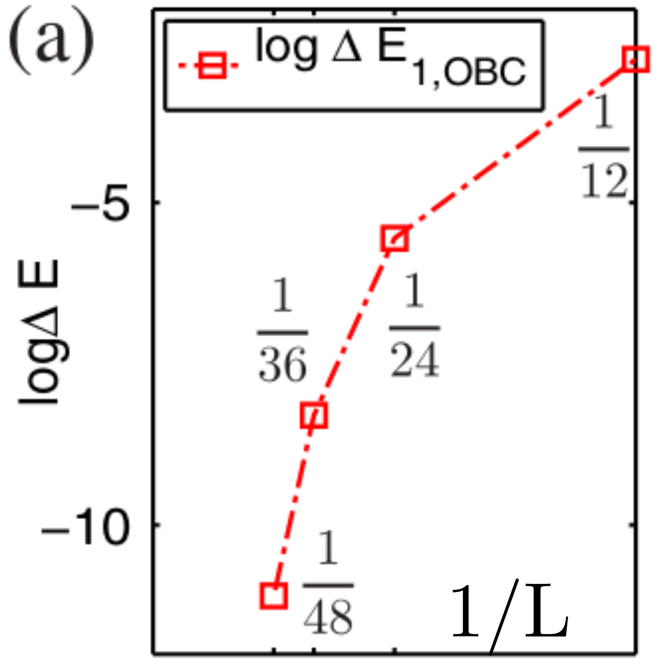

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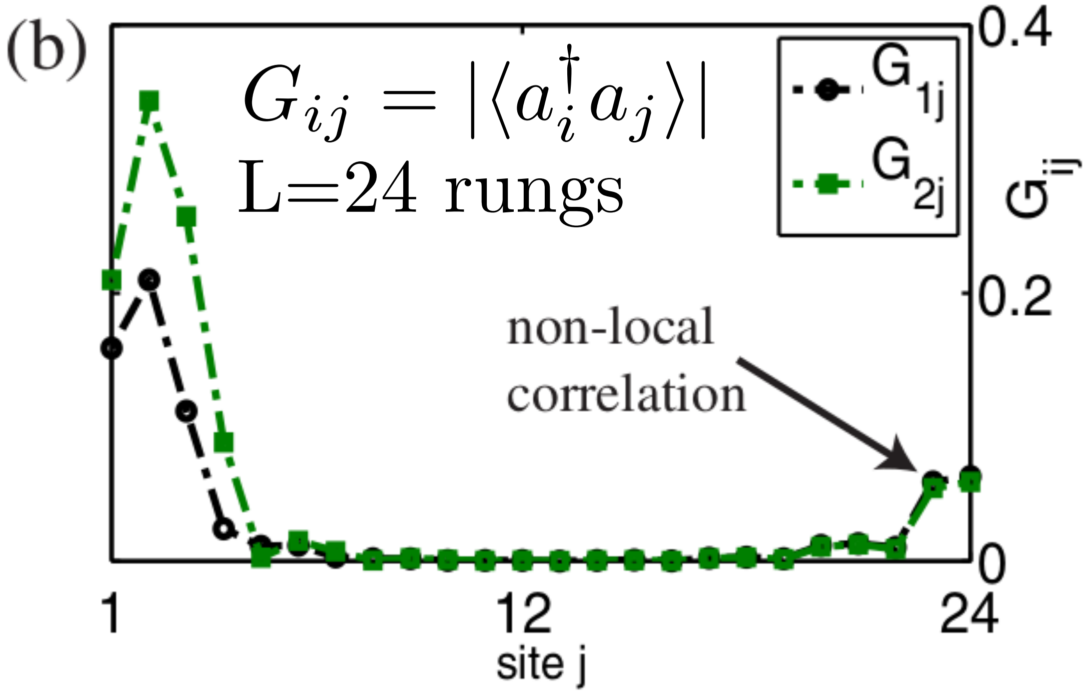
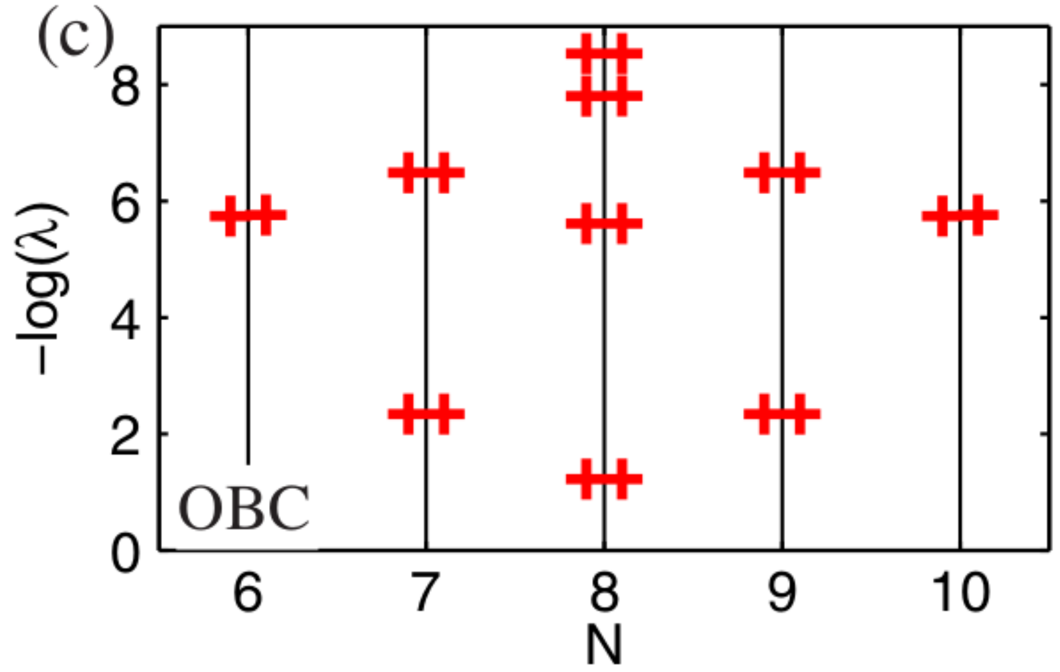


③ **Double degeneracy on the ES**

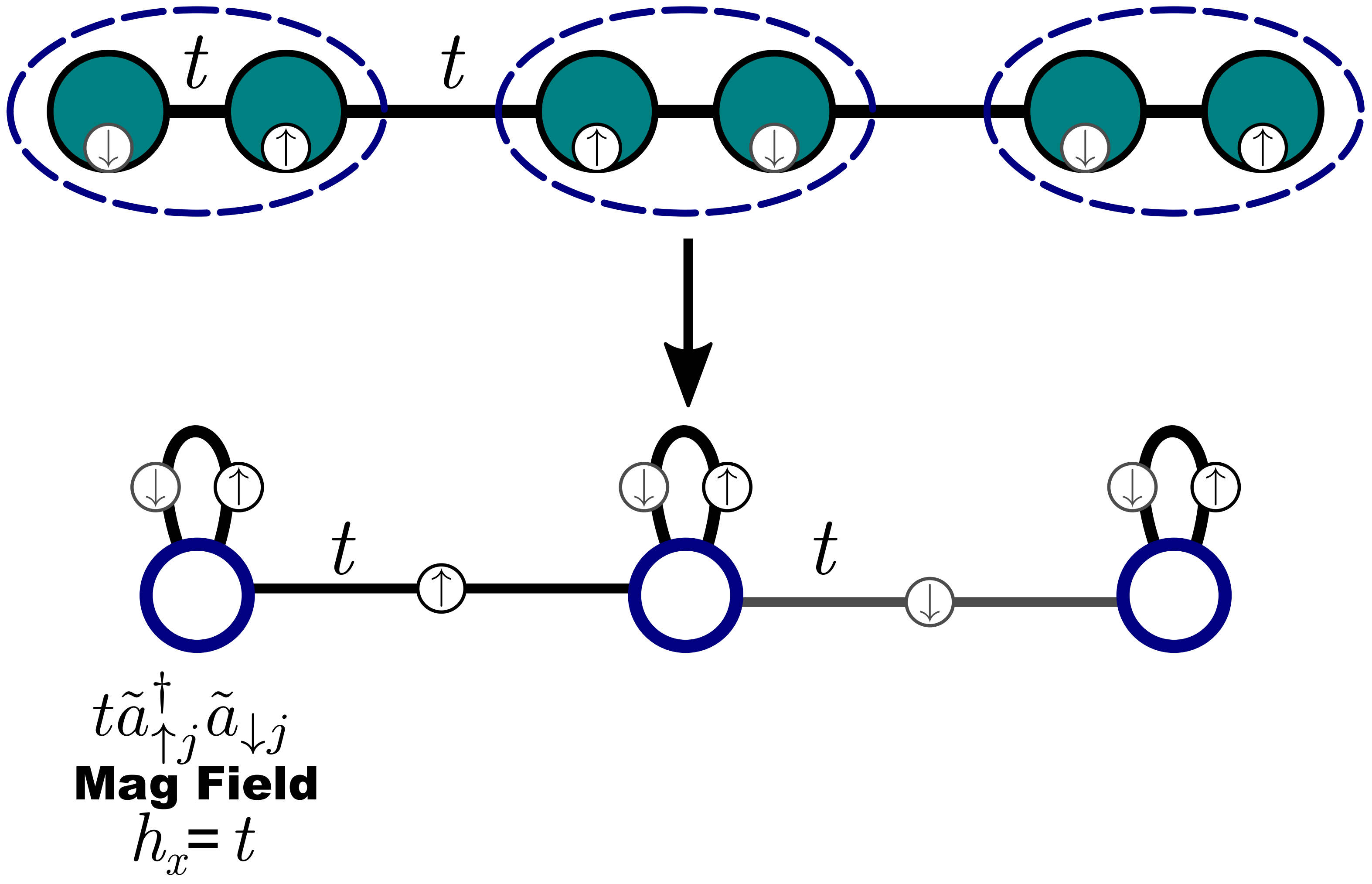


④ **Robust against local noise**

2) Our model

A spinful model?

Single leg:



A spinful model?

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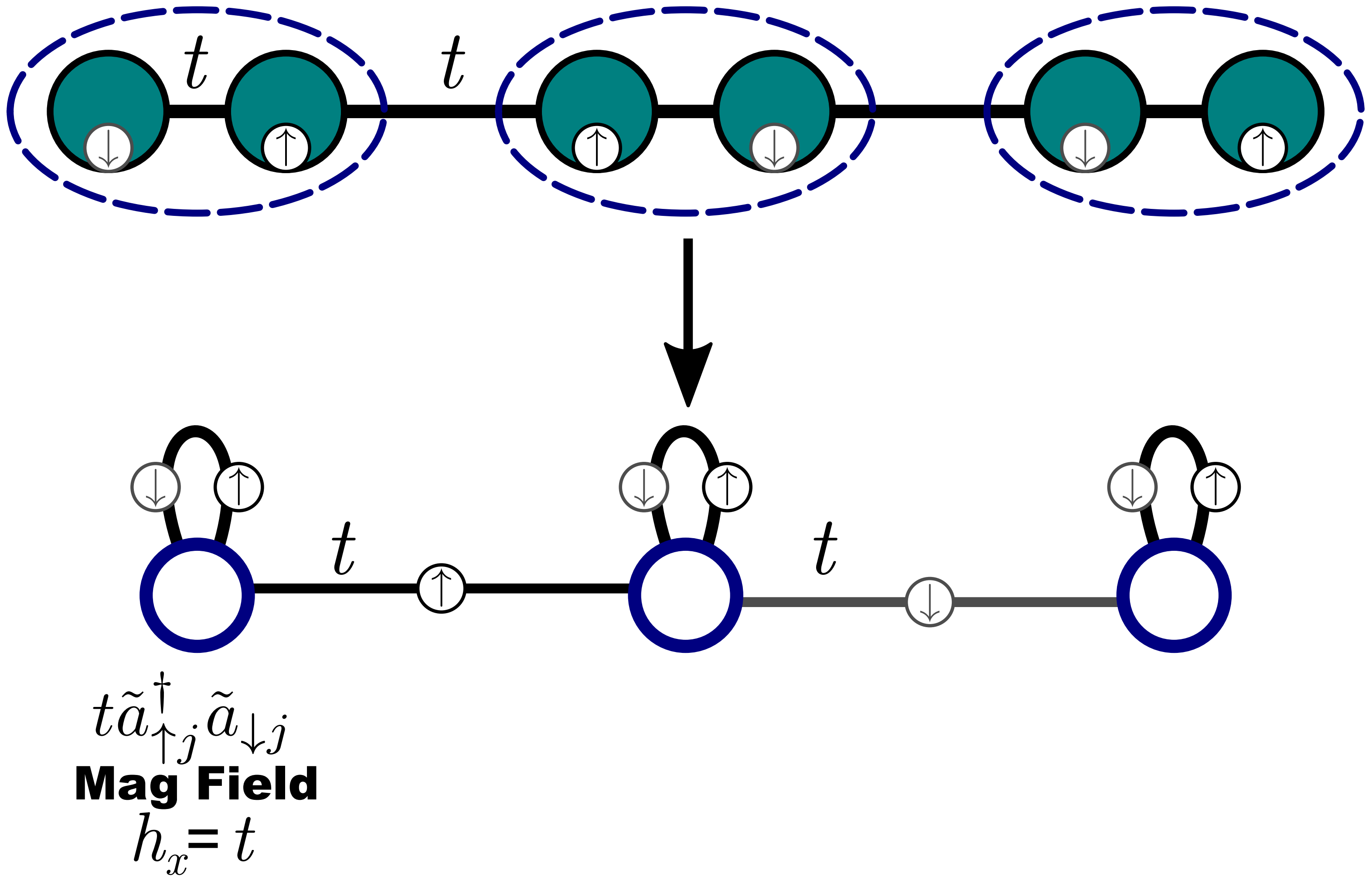
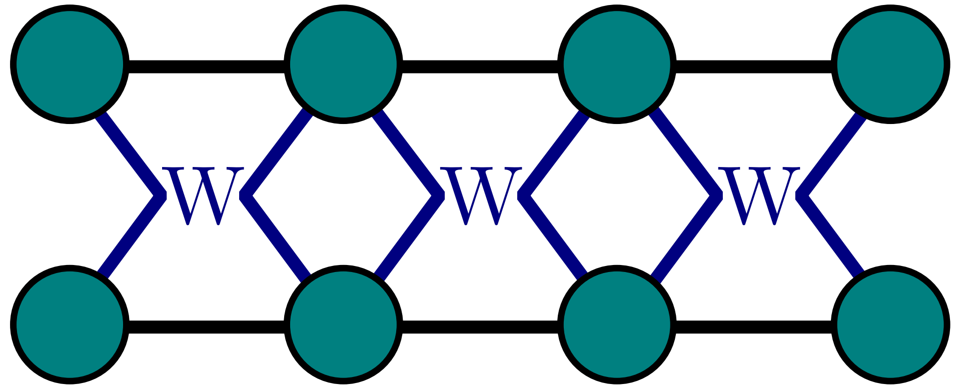
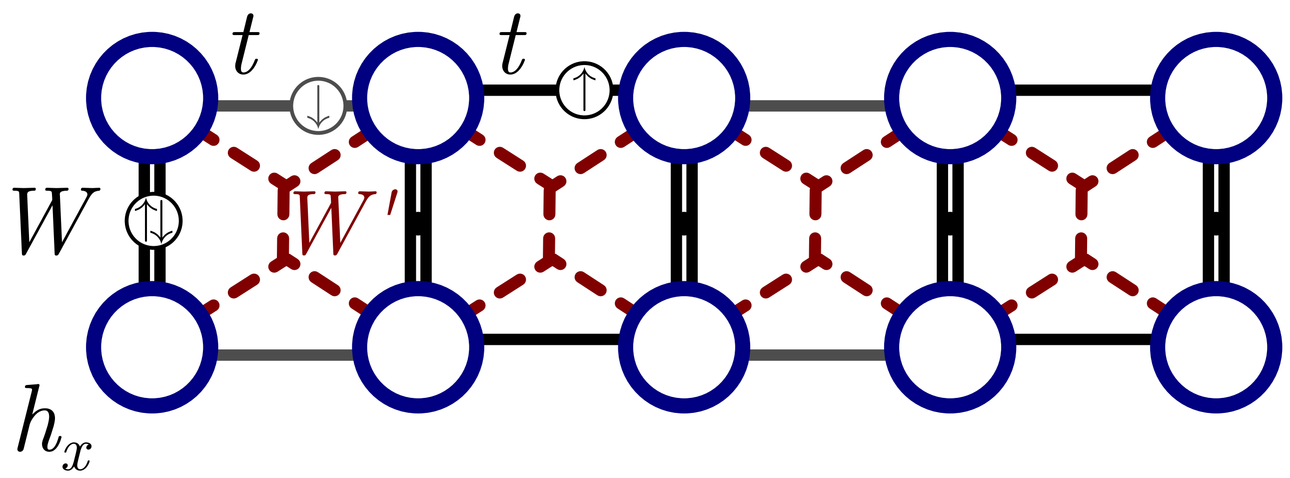


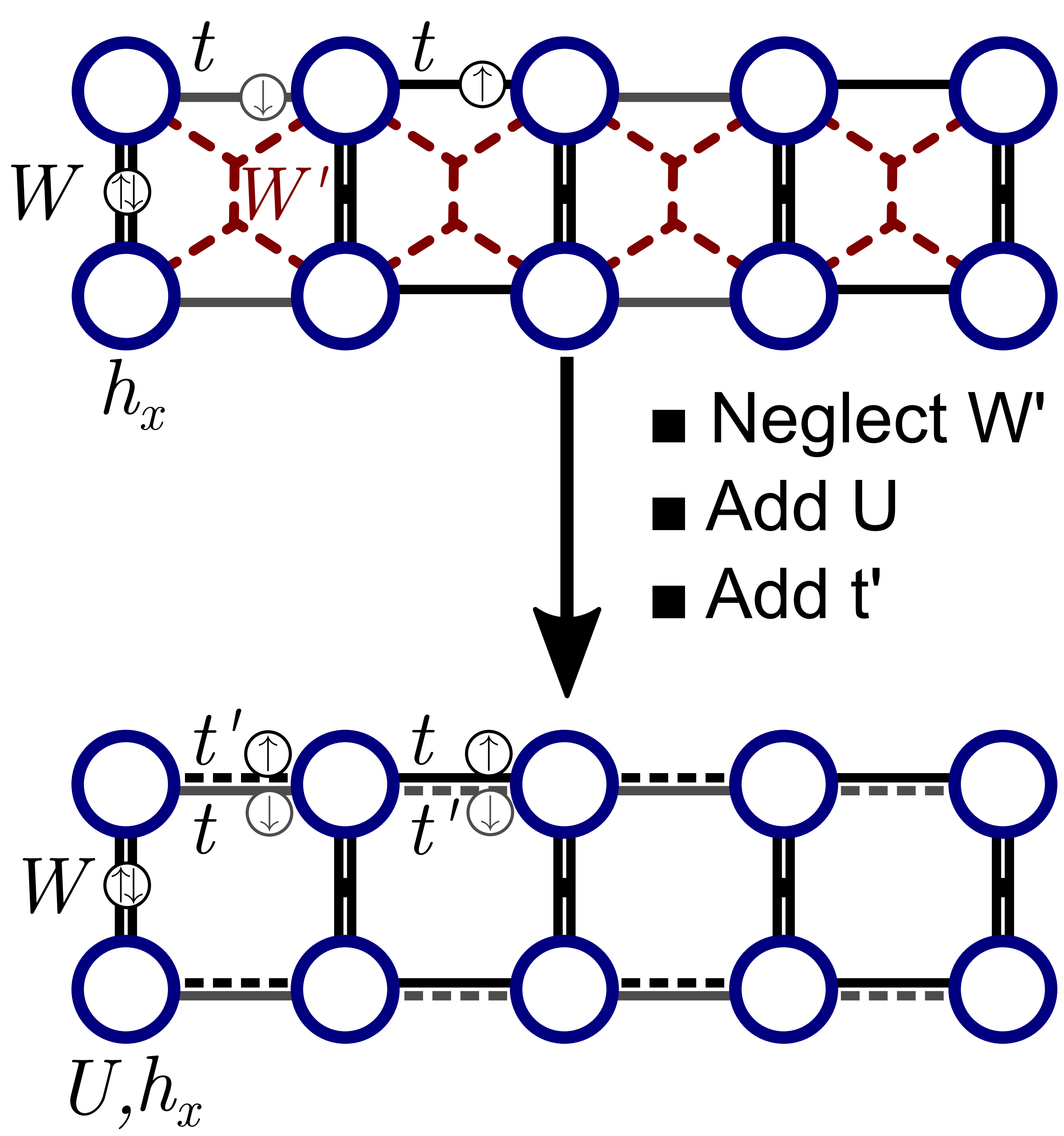
Spinless Ladder:



Spinful Ladder:



Our Model



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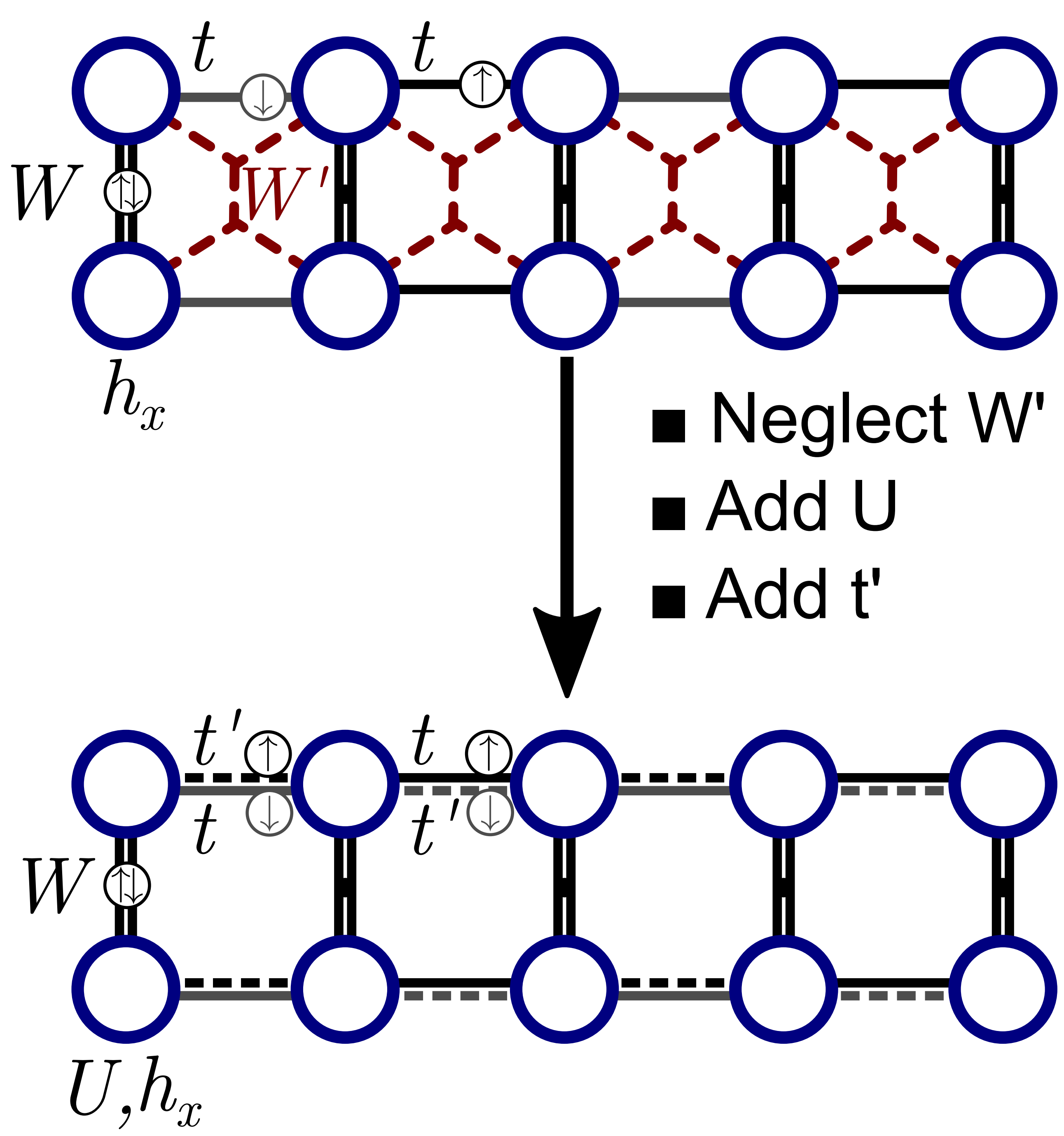


$$H = H_{tt'} + H_x + H_U + H_W$$

- $$H_{tt'} = \sum_{j\alpha} (-t\alpha^\dagger_{\downarrow 2j}\alpha_{\downarrow 2j+1} - t'\alpha^\dagger_{\downarrow 2j+1}\alpha_{\downarrow 2j+2} - t'\alpha^\dagger_{\uparrow 2j}\alpha_{\uparrow 2j+1} - t\alpha^\dagger_{\uparrow 2j+1}\alpha_{\uparrow 2j+2} + hc)$$
- $$H_x = \sum_{j\alpha} (h_x \alpha^\dagger_{\uparrow j}\alpha_{\downarrow j} + hc)$$
- $$H_U = \sum_{j\alpha} U n_{\alpha\uparrow j} n_{\alpha\downarrow j}$$
- $$H_W = \sum_{j} (W a^\dagger_{\uparrow j} a^\dagger_{\downarrow j} b_{\downarrow j} b_{\uparrow j} + hc)$$

- Single leg: P_a conserved (N_a is not)
- Doesn't conserve the **total spin projection** (ΣS_z)!

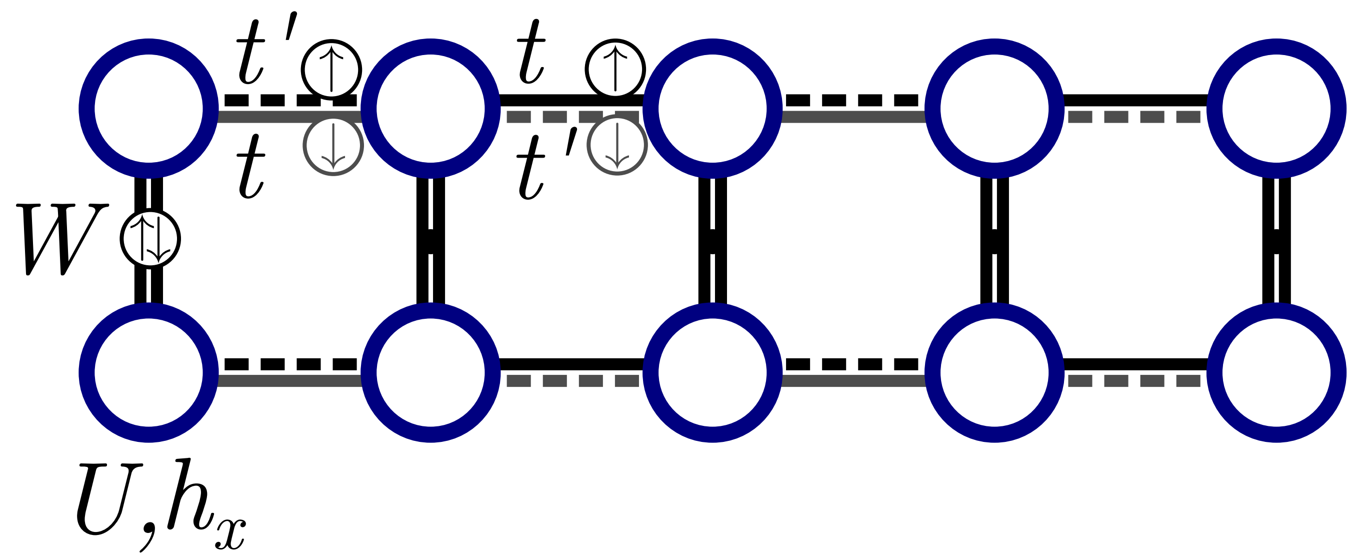
Do we have a **Majorana Phase?**

3) The results

Methods?

Ground state calculations using MPS

Size: up to **150 rungs**
Bond dim: up to **3000**
W = 2.6t
n = N/2L = 0.32 (fixed)



Criteria:

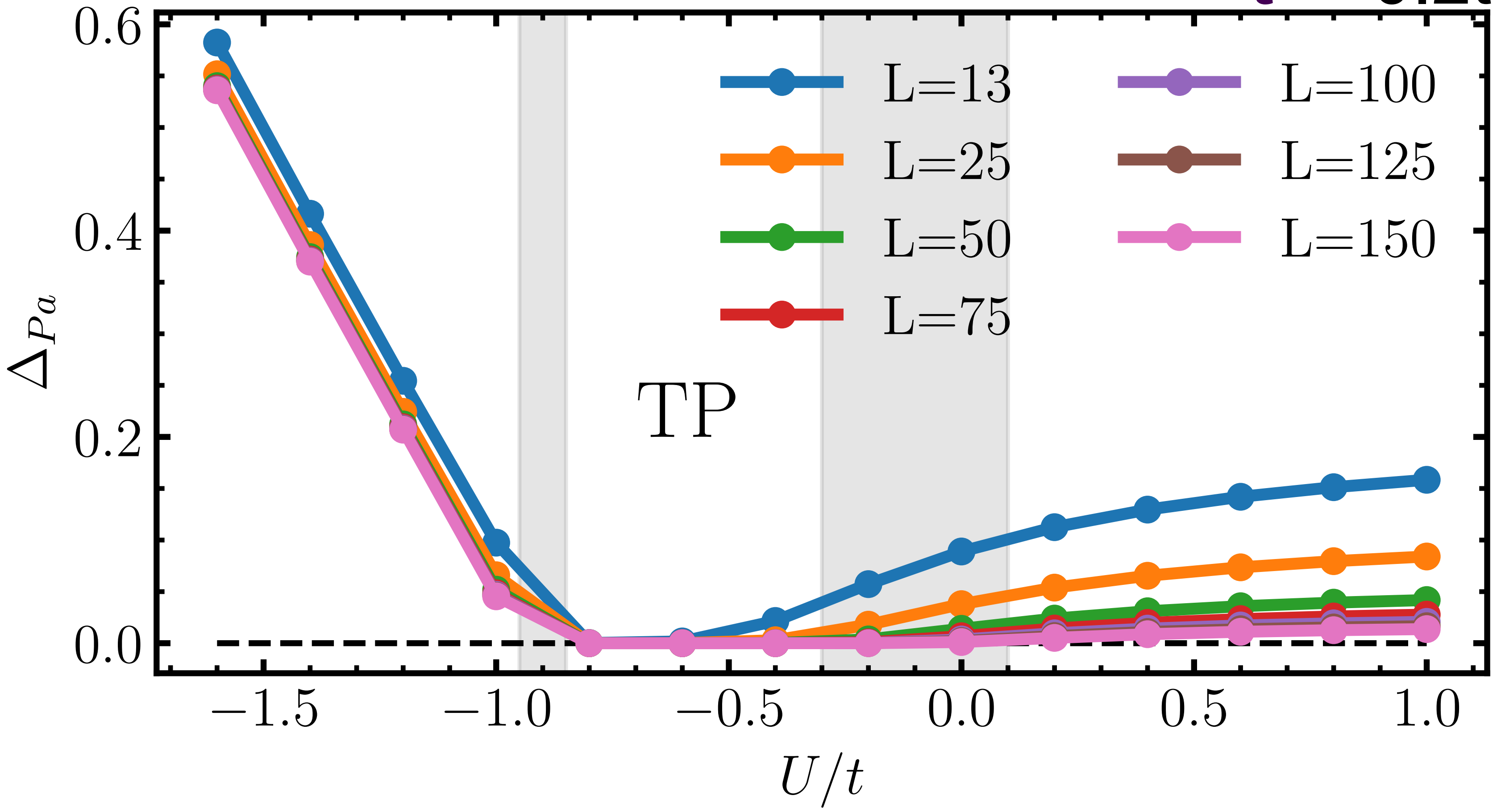
① Both parity sectors have the same E
② We have end-end correlations
③ There is an even Deg in the ES
④ Robust against local noise

Energy difference and Entanlgement spectrum

We compute the lowest-lying energy in each parity sector

$\Delta = E(P_a = 0) - E(P_a = 1)$

t' = 0.2t



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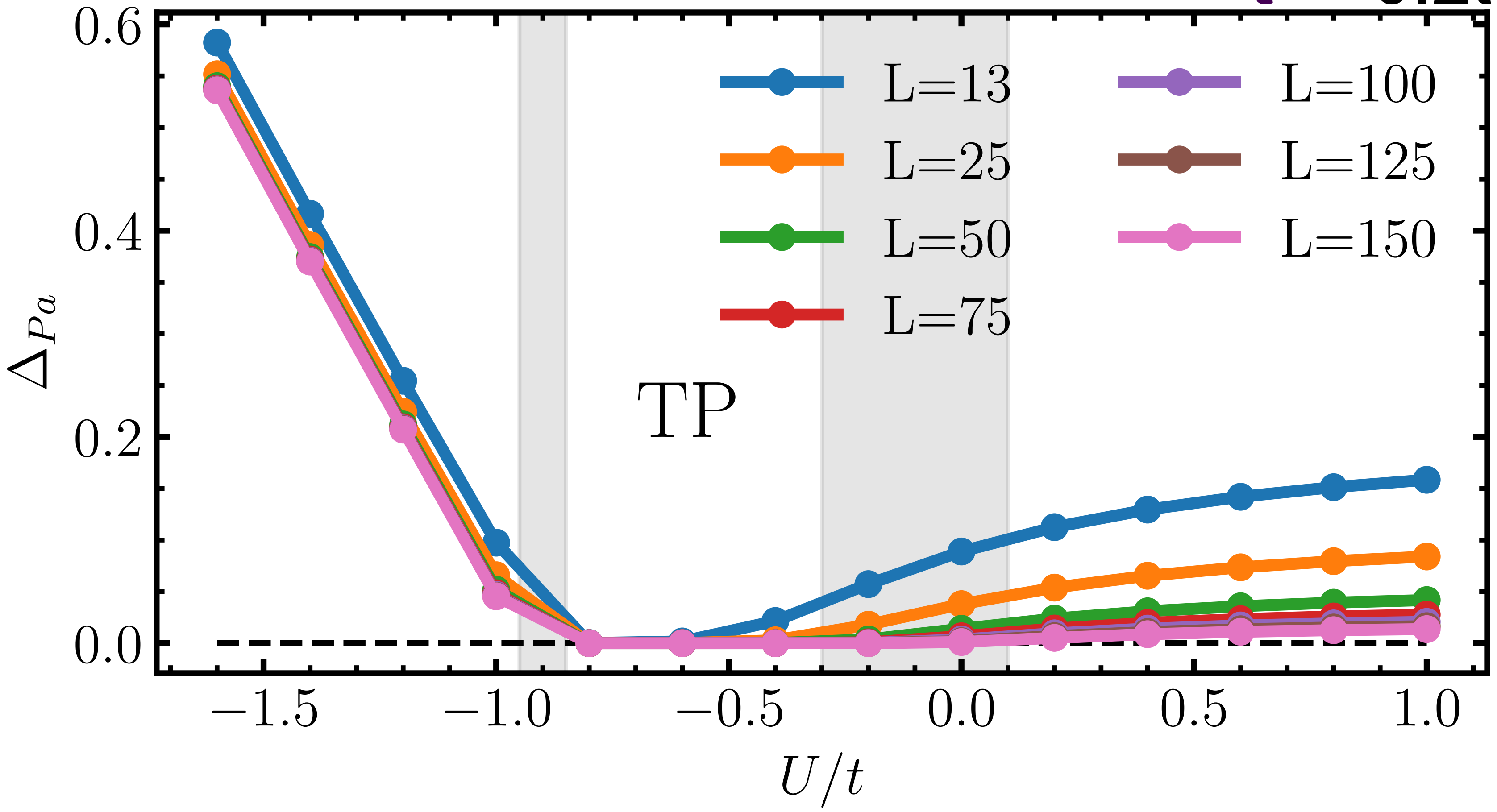
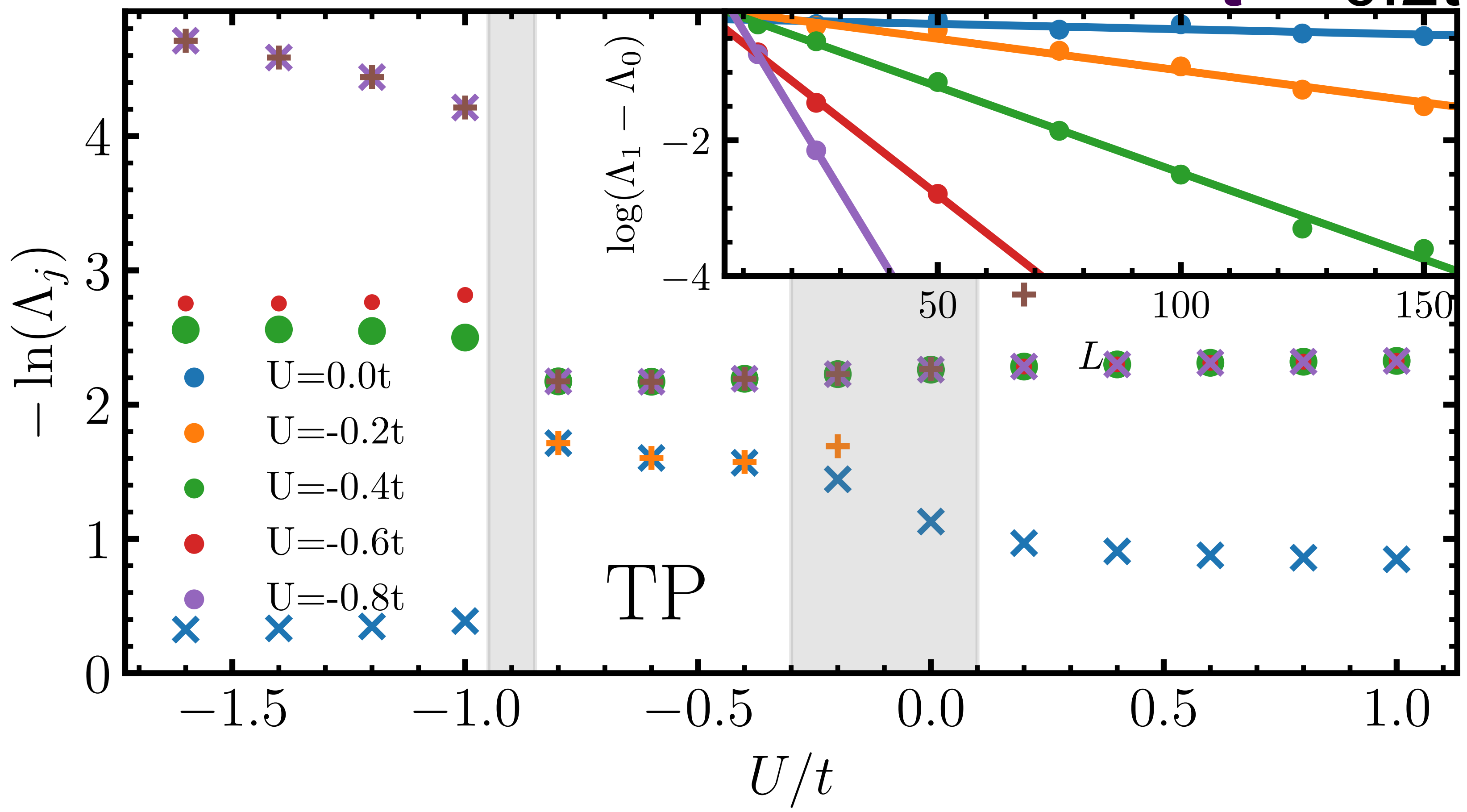


Also the Entanglement Spectrum

t' = 0.2t



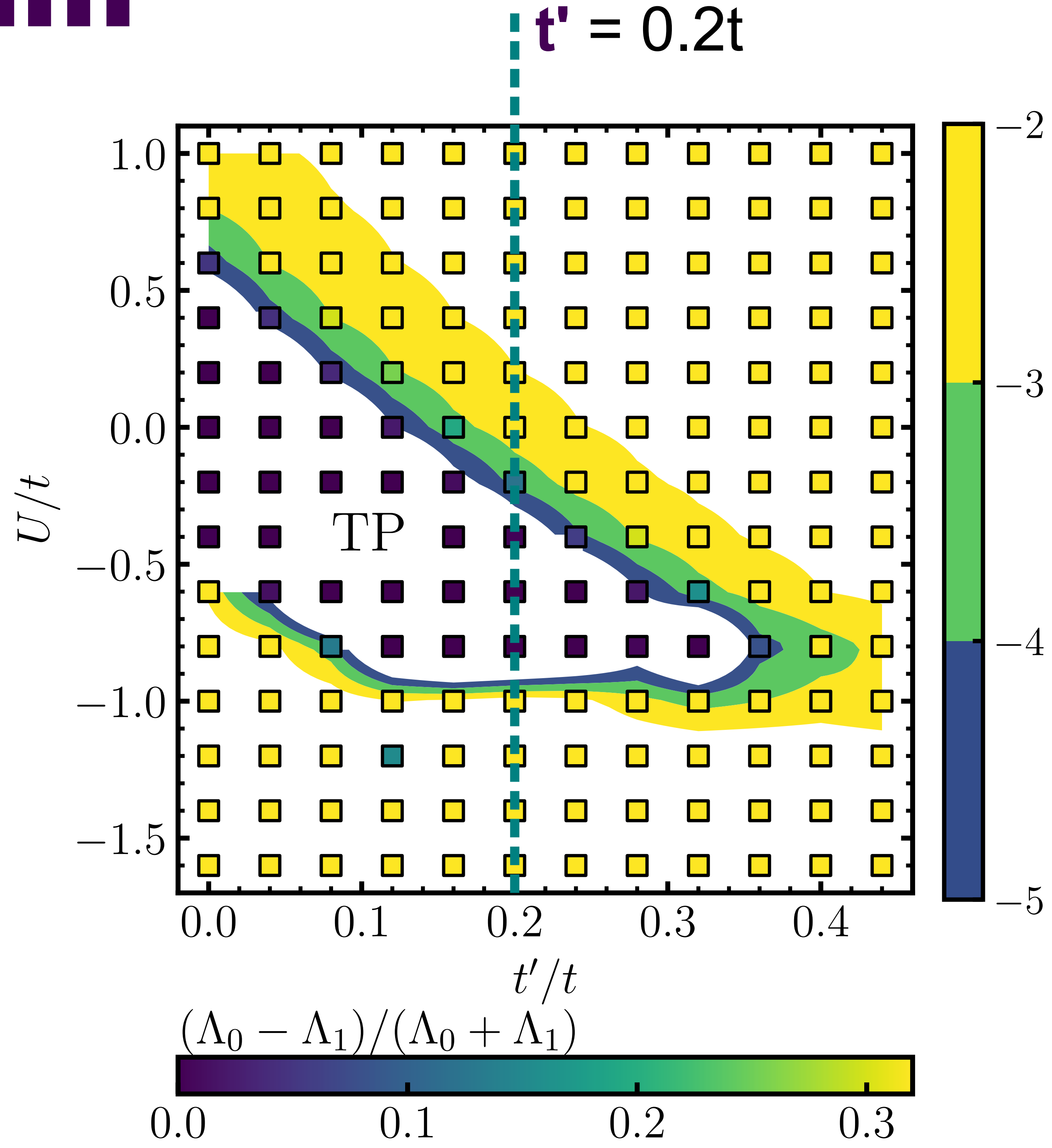
Energy difference and Entanlgement spectrum

L = 100 rungs
N = 64

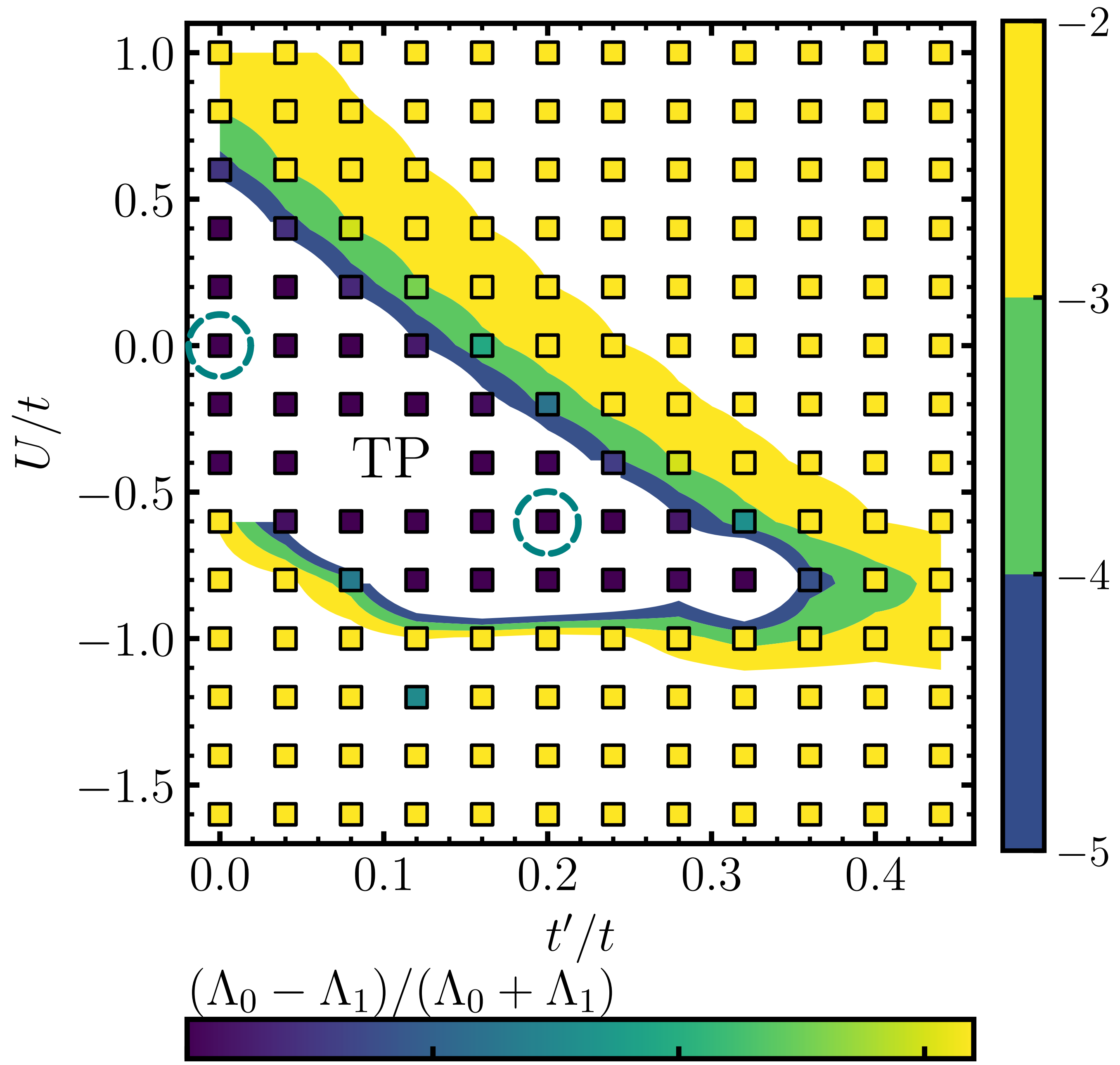
①Both parity sectors have the same E
③There is an even Deg in the ES

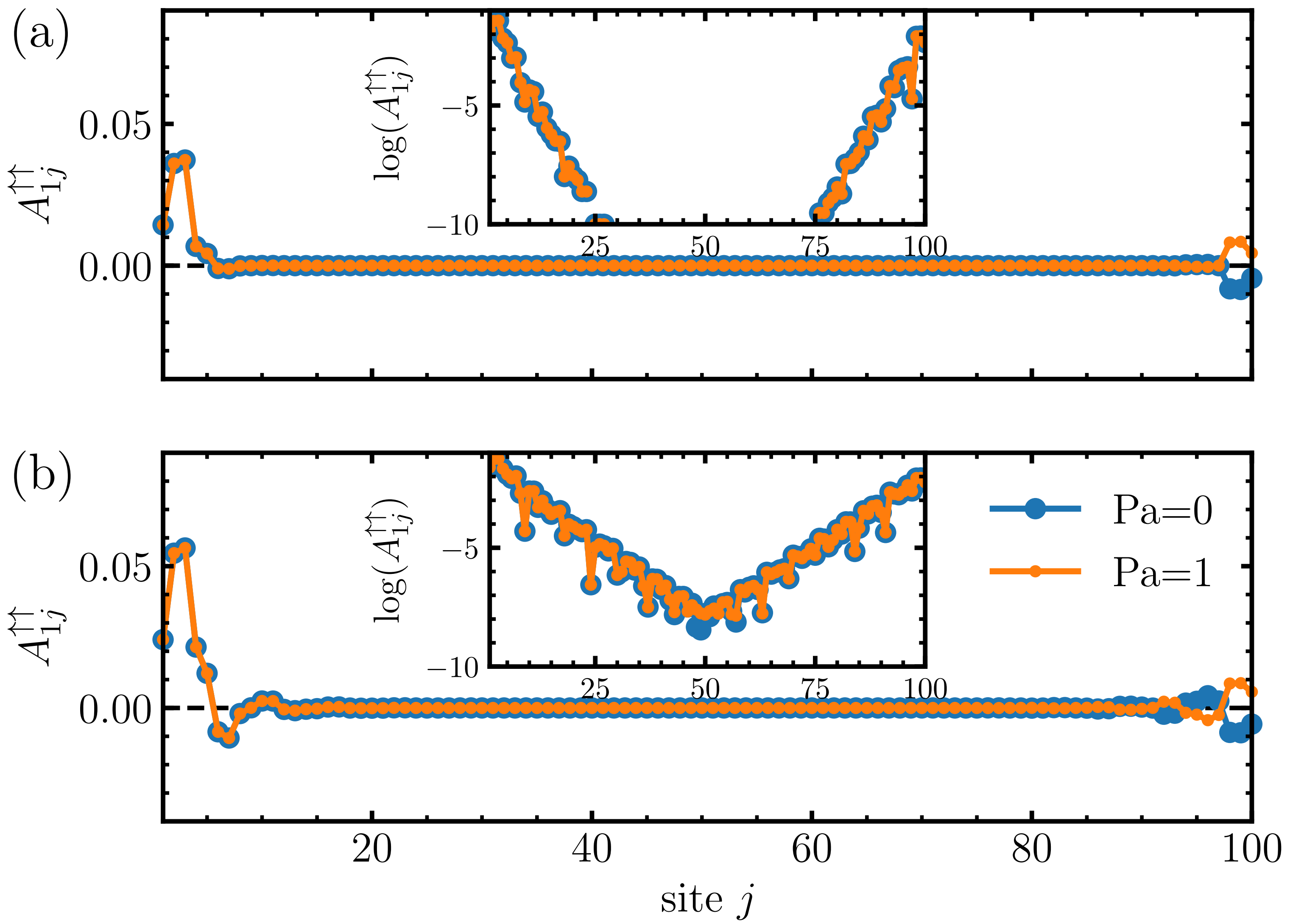
$\Delta = E(P_a = 0) - E(P_a = 1)$
Λ_i: entanglement spectrum eigenvalue

- Contour Plot: $log_{10}(\Delta)$
- Squares: $\frac{\Lambda_0 - \Lambda_1}{\Lambda_0 + \Lambda_1}$



Finite end-end correlations





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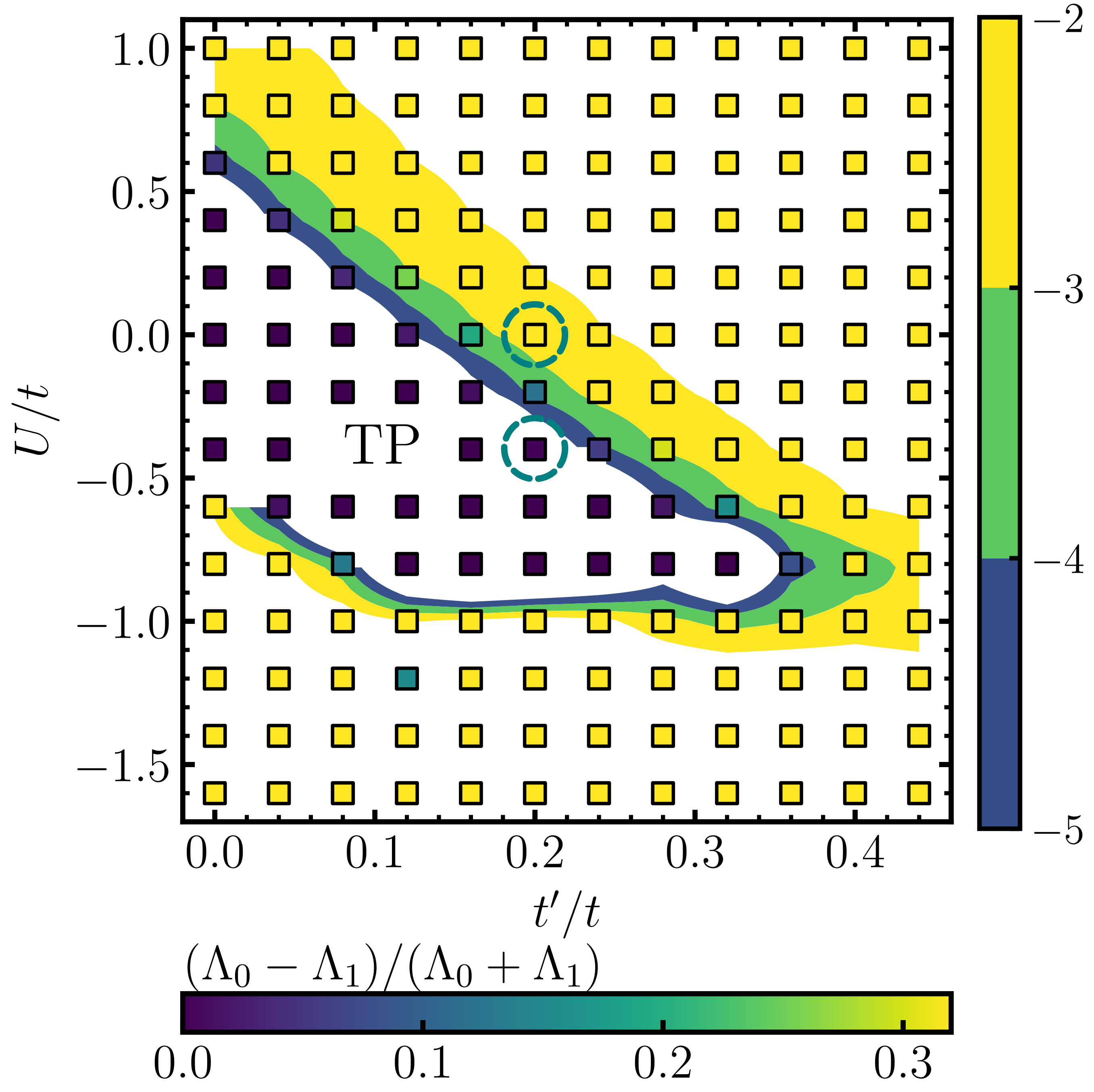

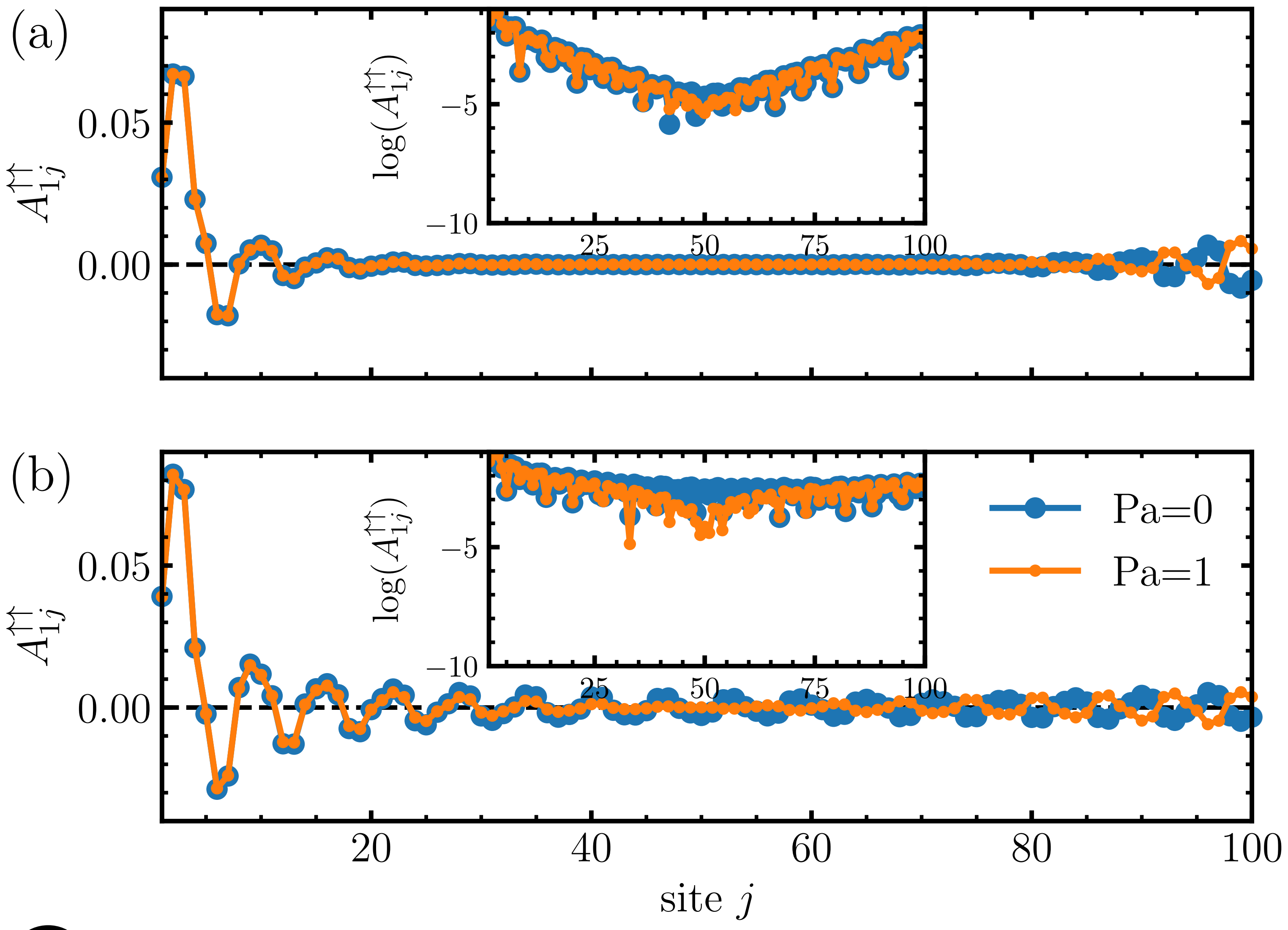

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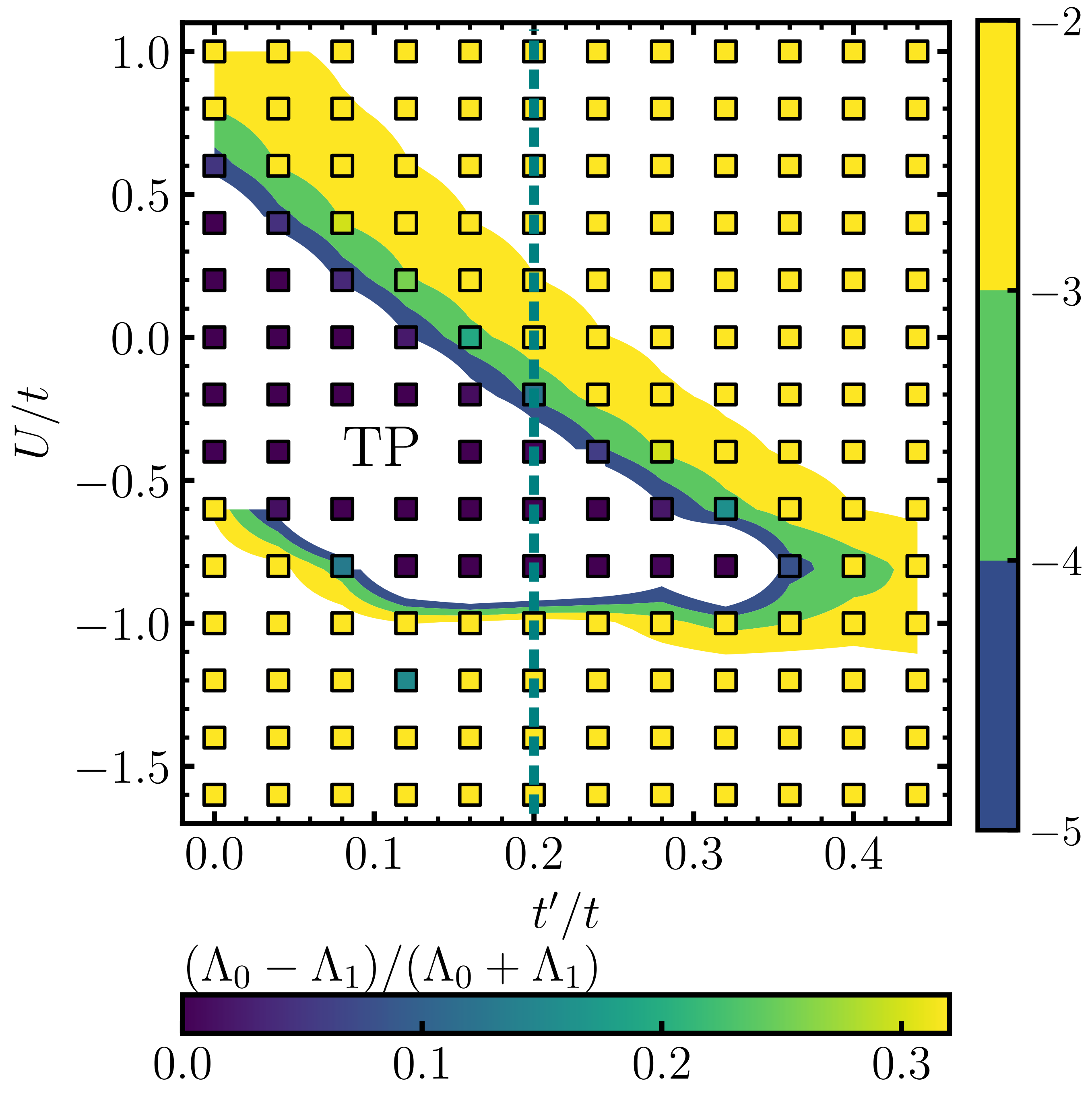
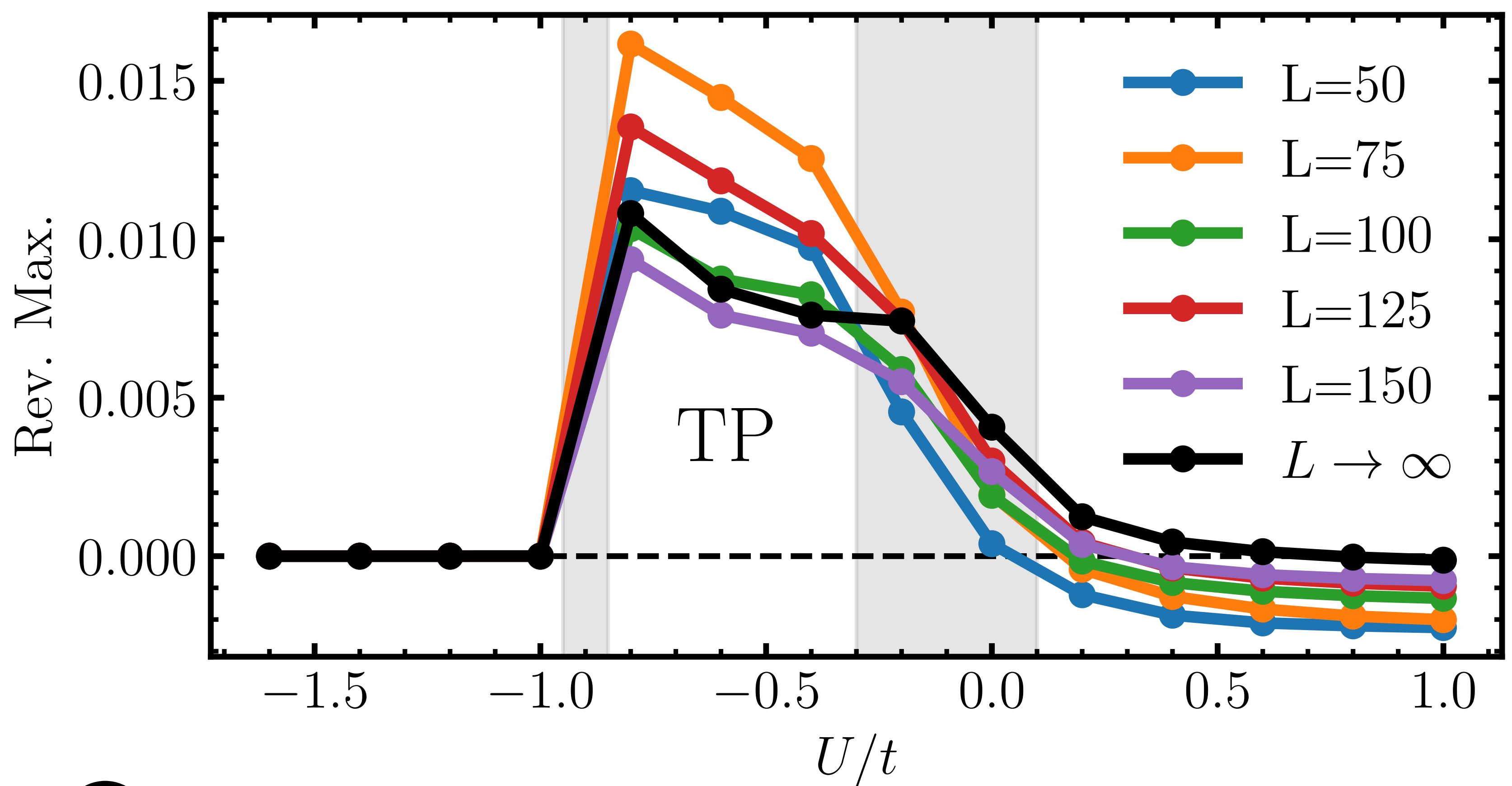


$$\mathrm{MR} = \max_{j\in[\text{L-10,L}]}\left(A_{1j}^{\uparrow\uparrow}\right) - \max_{j\in[\text{L/2-5,L/2+5}]}\left(A_{1j}^{\uparrow\uparrow}\right)$$



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Summary:

❶ Both parity sectors have the same E
❷ We have end-end correlations
❸ There is an even Deg in the ES
❹ Robust against local noise