

# Some ideas for BGOOD

E. Oset , IFIC . Universidad de Valencia

$N^*(1920)$  from  $K \bar{K} N$  bound state

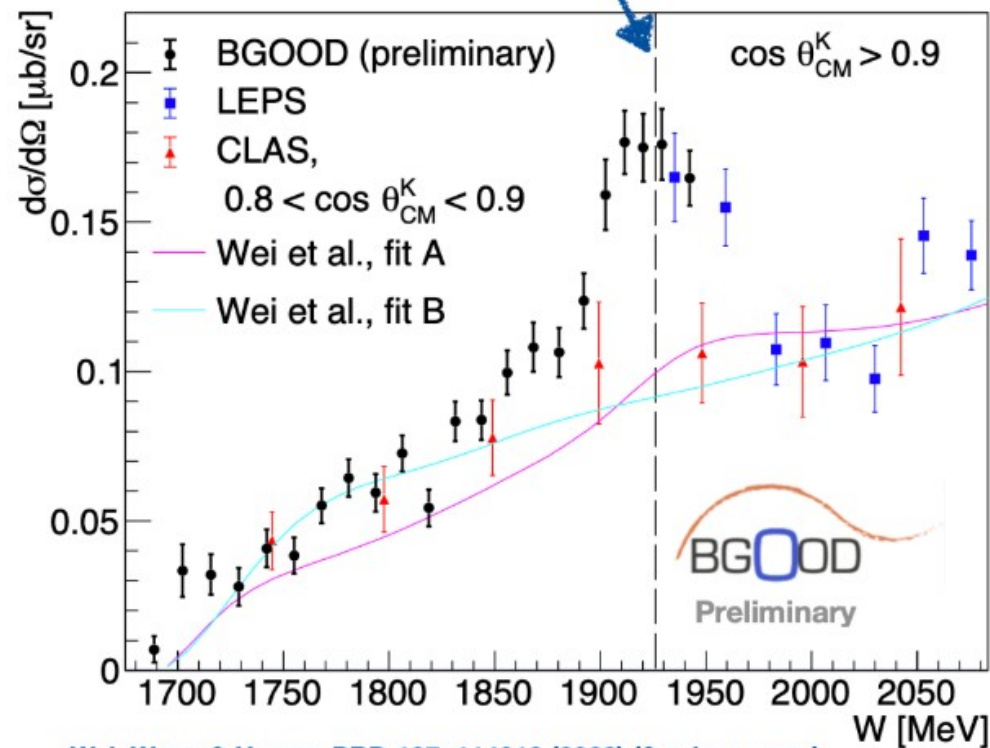
$\Sigma(1430) (1/2^-)$

$\Sigma(1380) (1/2^-) \text{ ?????}$

$N^*(2190)$  from  $p f_1(1285) (p K \bar{K}^*)$

# Forward $K^+\Sigma^-$ photoproduction

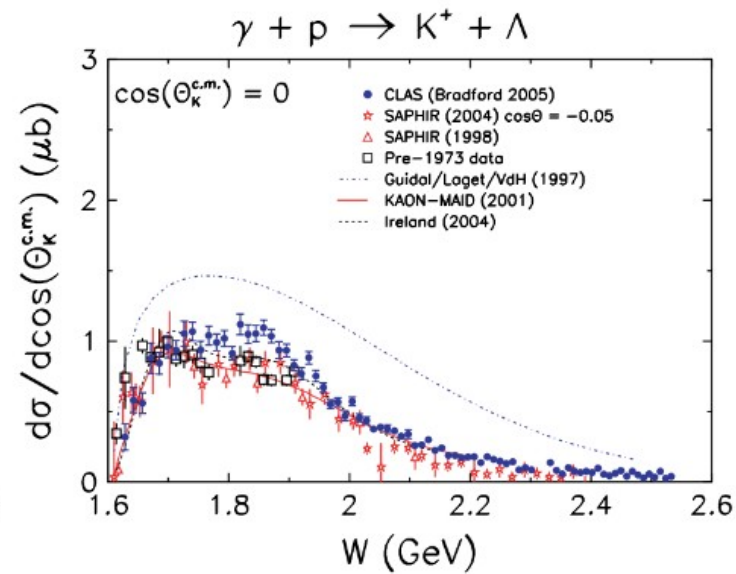
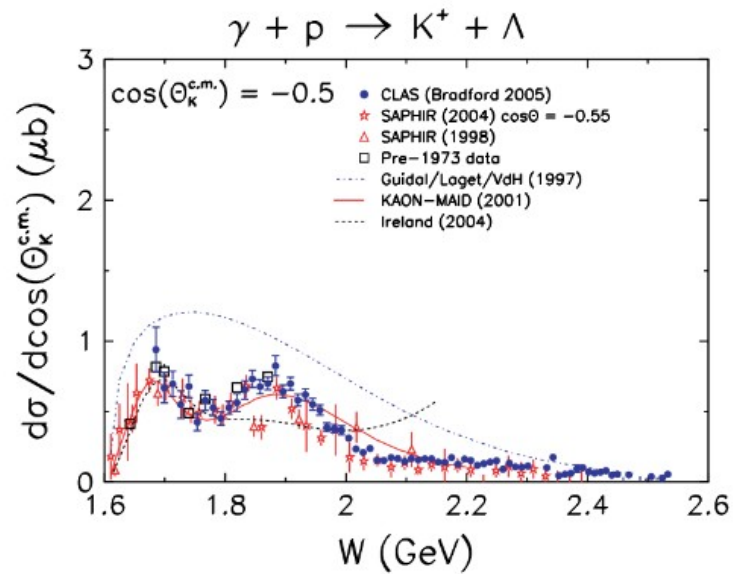
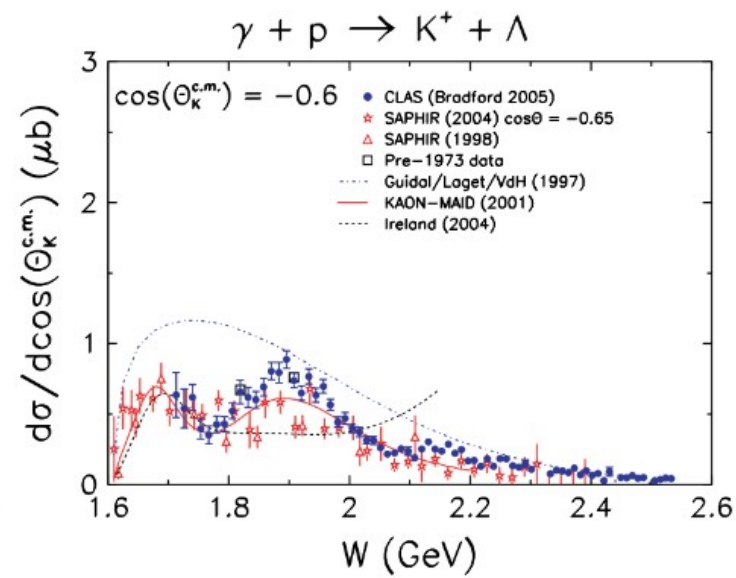
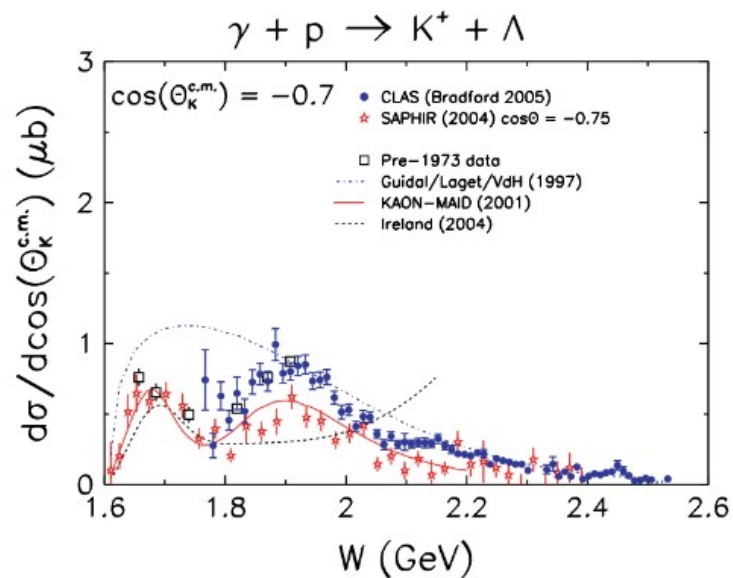
Structure close to  $K^+K^-p$  threshold?

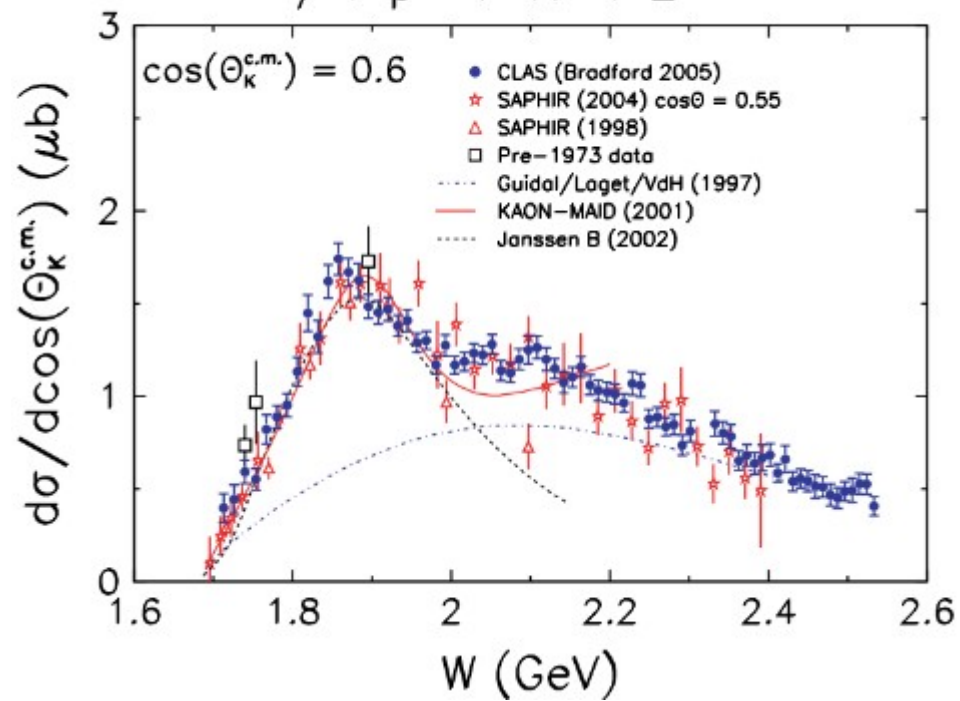
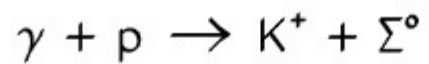


Wei, Wang & Huang, PRD 107, 114018 (2023) (& priv. comm.)

CLAS: Pereira et al., PLB 688 (2010) 289

LEPS: H. Kohri et al., PRL 97 (8 2006) 082003



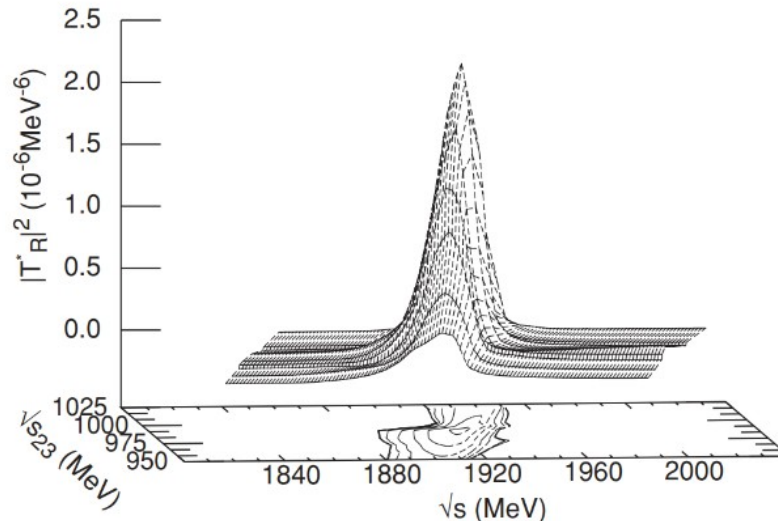


D. Jido, Y. Kanada-En'yo, Phys. Rev. C 78, 035203 (2008)

They find a bound state of  $K \bar{K} N$ , in the  $a_0(980) N$  mode using variational calculations, with mass 1910 MeV

Martinez-Torres, Khemchandani, Oset Phys.Rev.C 79 (2009) 065207

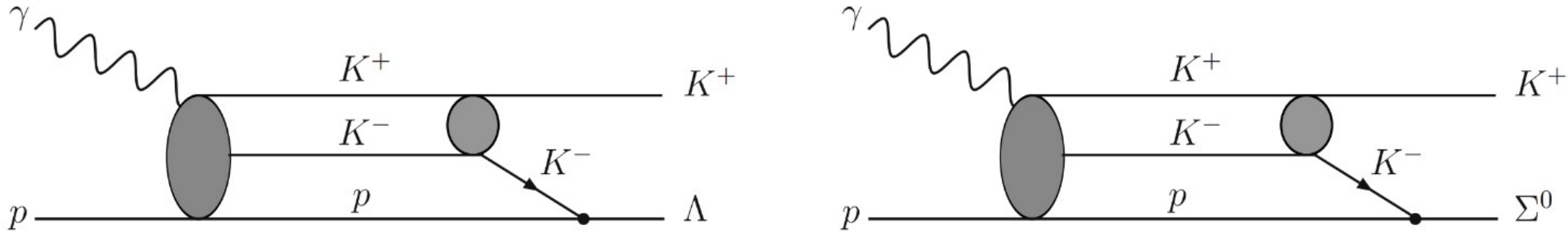
We use Faddeev equations and find with coupled channels,  $\pi \pi N$ ,  $K \bar{K} N$ ,  $\eta \pi N$  a state at 1920 MeV coupled to  $f_0(980)N$  and  $a_0(980)N$



The state has  $I=1/2$

$J^p = 1/2^+$

FIG. 7. A possible  $N^*(1910)$  in the  $NK\bar{K}$  channels.



**Fig. 1.** Diagrams depicting the  $\gamma p \rightarrow K^+ \Lambda$ ,  $\gamma p \rightarrow K^+ \Sigma^0$  processes through the  $1/2^+$   $N^*$   $K^+ K^- N$ -resonance of [16, 21].

$$V_{K^- p \rightarrow \Lambda} = -\frac{2}{\sqrt{3}} \frac{D+F}{2f} + \frac{1}{\sqrt{3}} \frac{D-F}{2f}, \quad -1.26$$

$$V_{K^- p \rightarrow \Sigma^0} = \frac{D-F}{2f} \quad 0.33$$

# Forward $K^+\Sigma^-$ photoproduction

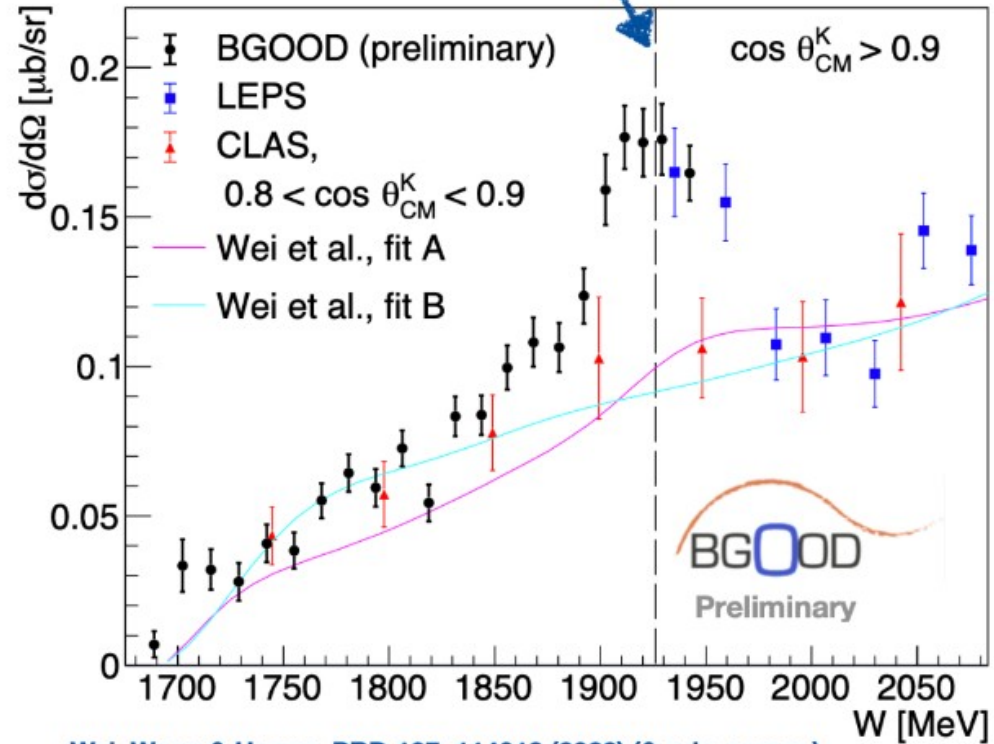
Structure close to  $K^+K^-p$  threshold?

Is that peak related to that state?

I think so.

What can one do experimentally to determine isospin, and  $J^P$  ?

Partial wave analysis can help



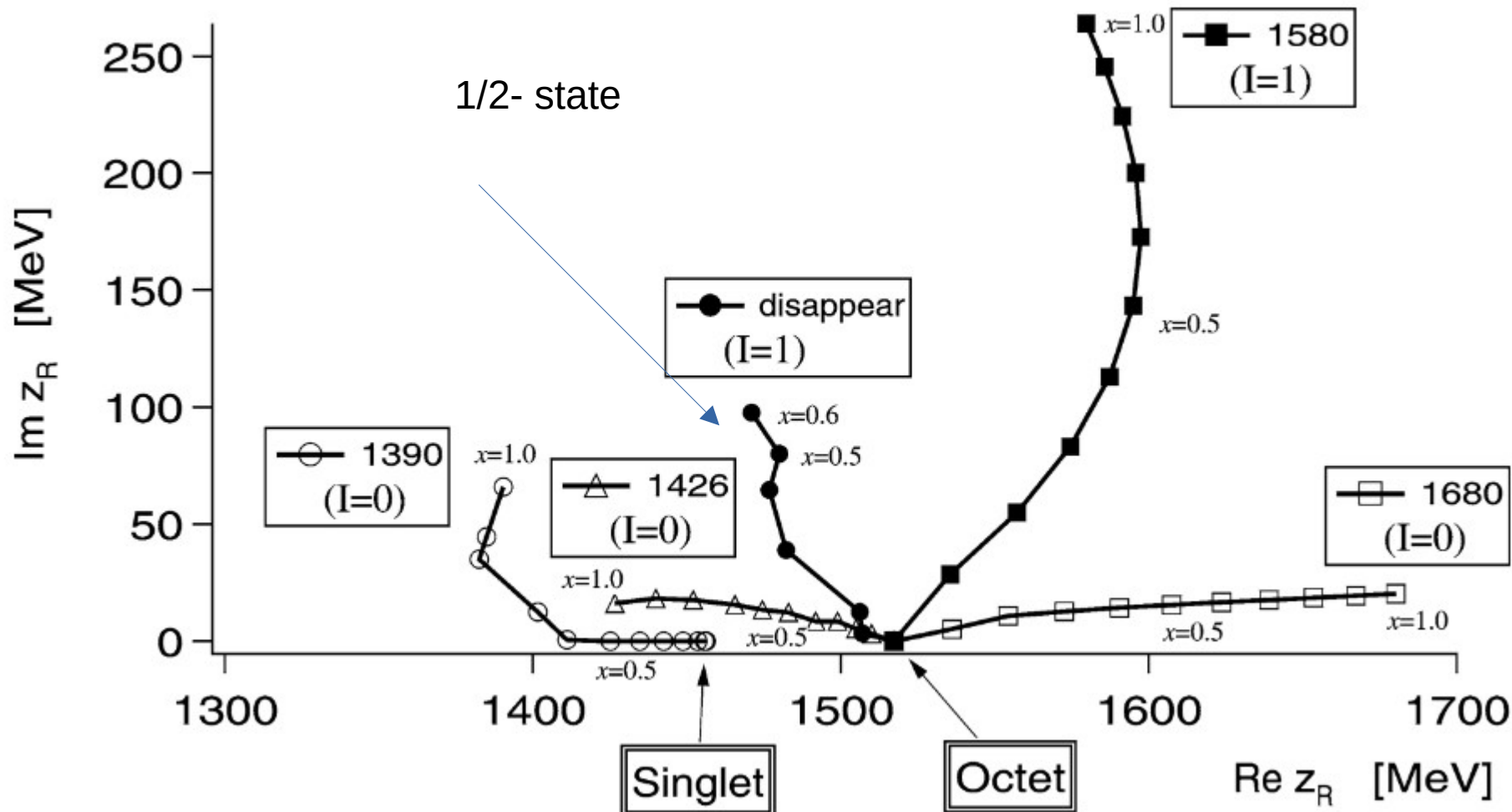
Wei, Wang & Huang, PRD 107, 114018 (2023) (& priv. comm.)

CLAS: Pereira et al., PLB 688 (2010) 289

LEPS: H. Kohri et al., PRL 97 (8 2006) 082003

# The $\Sigma(1430)$ state

D.~Jido, J.~A.~Oller, E.~Oset, A.~Ramos and U.~G.~Meissner, "Chiral dynamics of the two  $\Lambda(1405)$  states," Nucl. Phys. A **725**, 181-200 (2003)



Many works supporting the  
existence and nature of this state

M. F. M. Lutz and E. E. Kolomeitsev, On meson resonances and chiral symmetry, Nucl. Phys. A **730**, 392 (2004), arXiv:nucl-th/0307039.

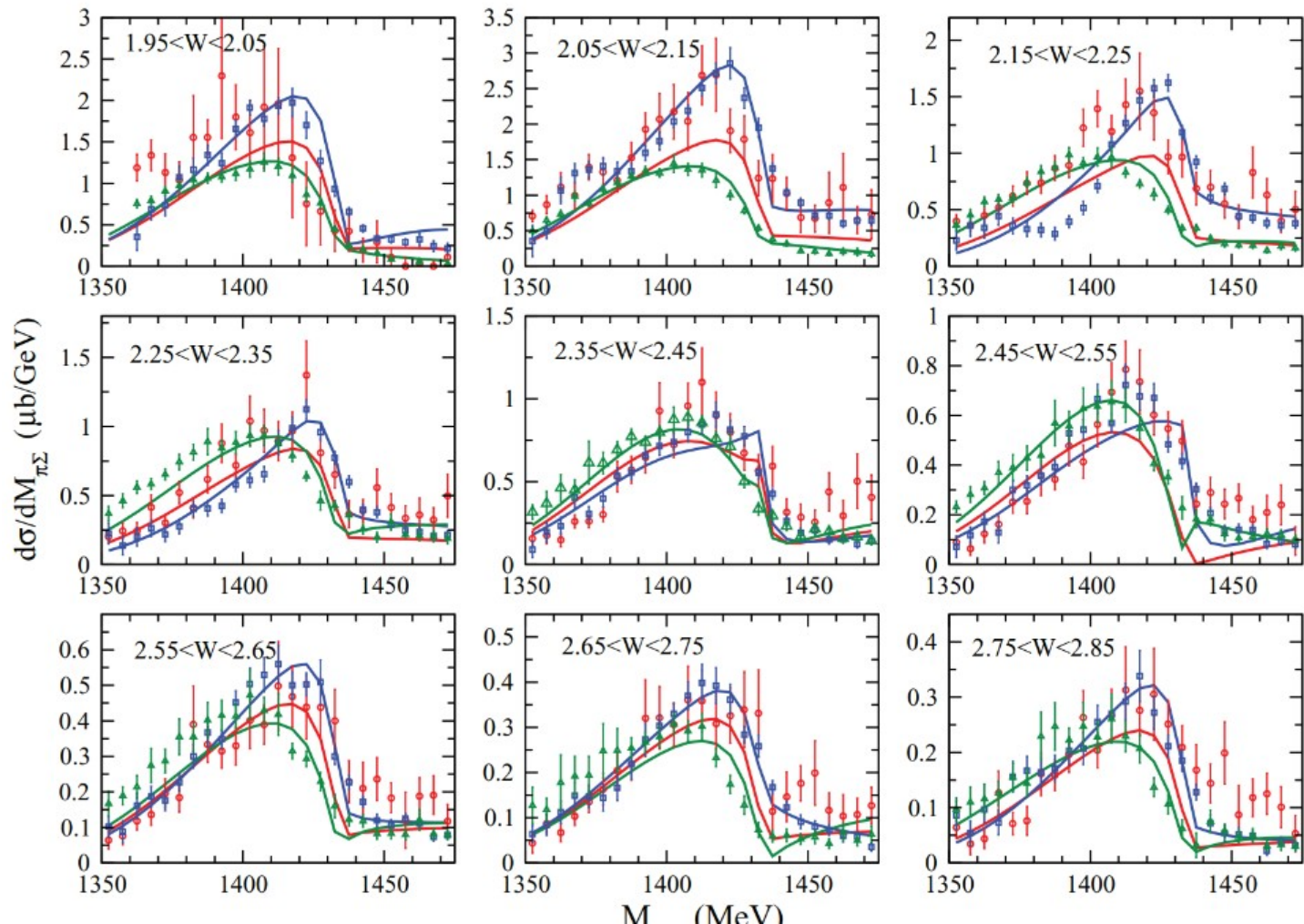
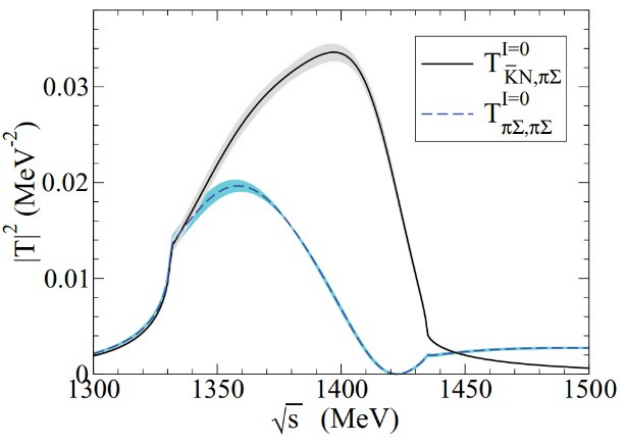
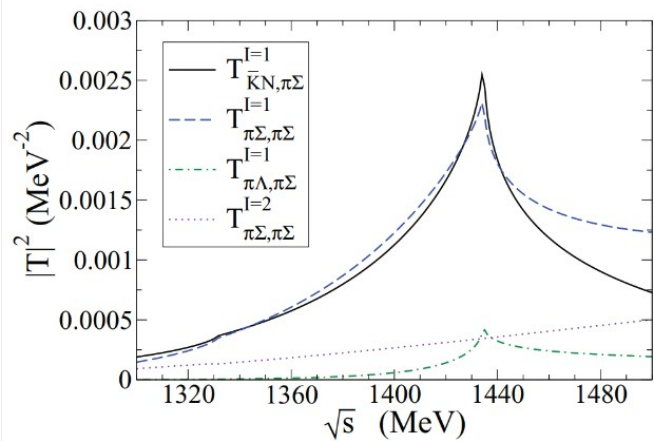
L. Roca, E. Oset, and J. Singh, Low lying axial-vector mesons as dynamically generated resonances, Phys. Rev. D **72**, 014002 (2005), arXiv:hep-ph/0503273.

C. Garcia-Recio, L. S. Geng, J. Nieves, and L. L. Salcedo, Low-lying even parity meson resonances and spin-flavor symmetry, Phys. Rev. D **83**, 016007 (2011), arXiv:1005.0956 [hep-ph].

Y. Zhou, X.-L. Ren, H.-X. Chen, and L.-S. Geng, Pseudoscalar meson and vector meson interactions and dynamically generated axial-vector mesons, Phys. Rev. D **90**, 014020 (2014), arXiv:1404.6847 [nucl-th].

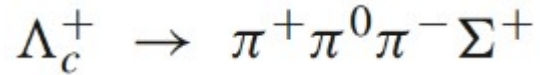
L.-S. Geng, X.-L. Ren, Y. Zhou, H.-X. Chen, and E. Oset,  $S$ -wave  $KK^*$  interactions in a finite volume and the  $f_1(1285)$ , Phys. Rev. D **92**, 014029 (2015), arXiv:1503.06633 [hep-ph].

P.-L. Lü and J. He, Hadronic molecular states from the  $K\bar{K}^*$  interaction, Eur. Phys. J. A **52**, 359 (2016), arXiv:1603.04168 [hep-ph].

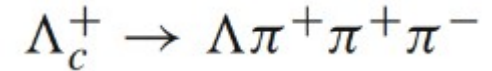


J.-J. Xie, E. Oset, Phys. Lett. B 792, 450 (2019)

studied

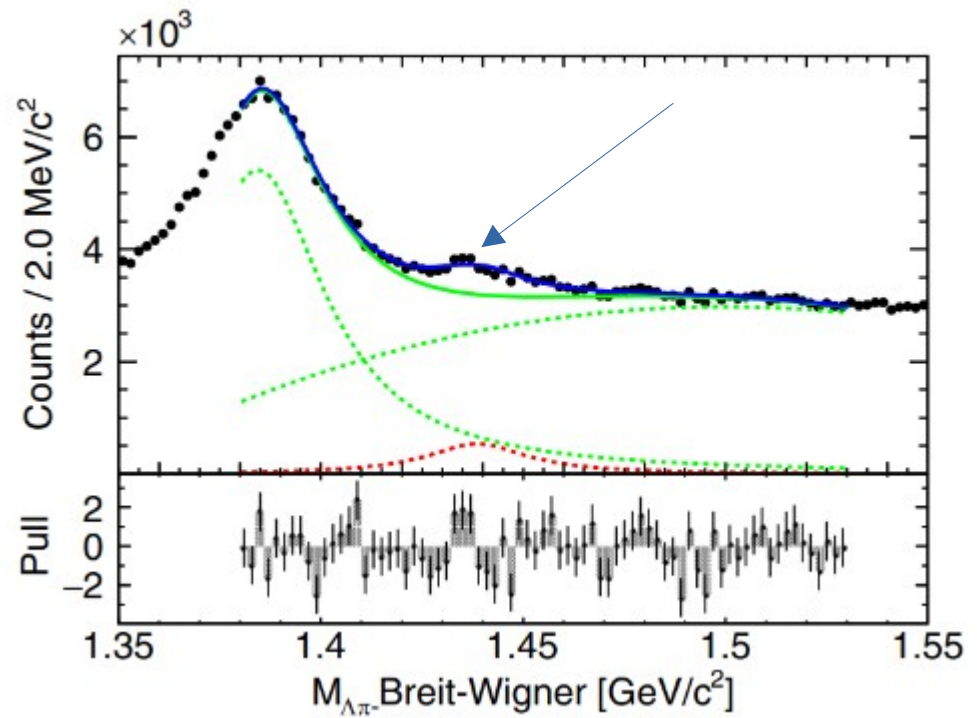
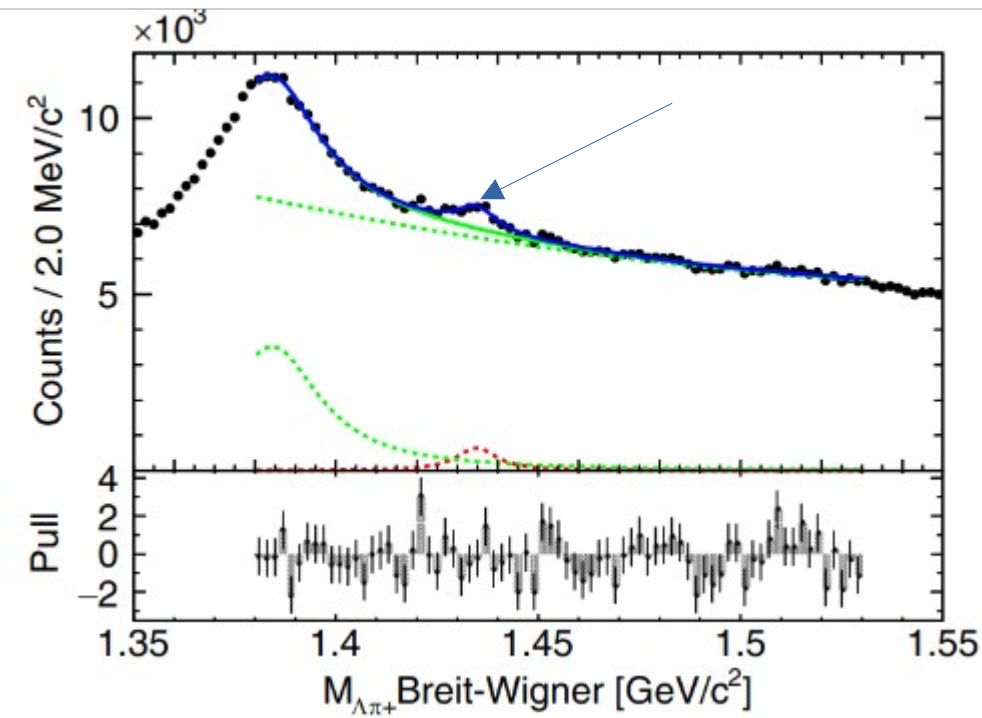


and proposed



The latter reaction has been done by Belle,

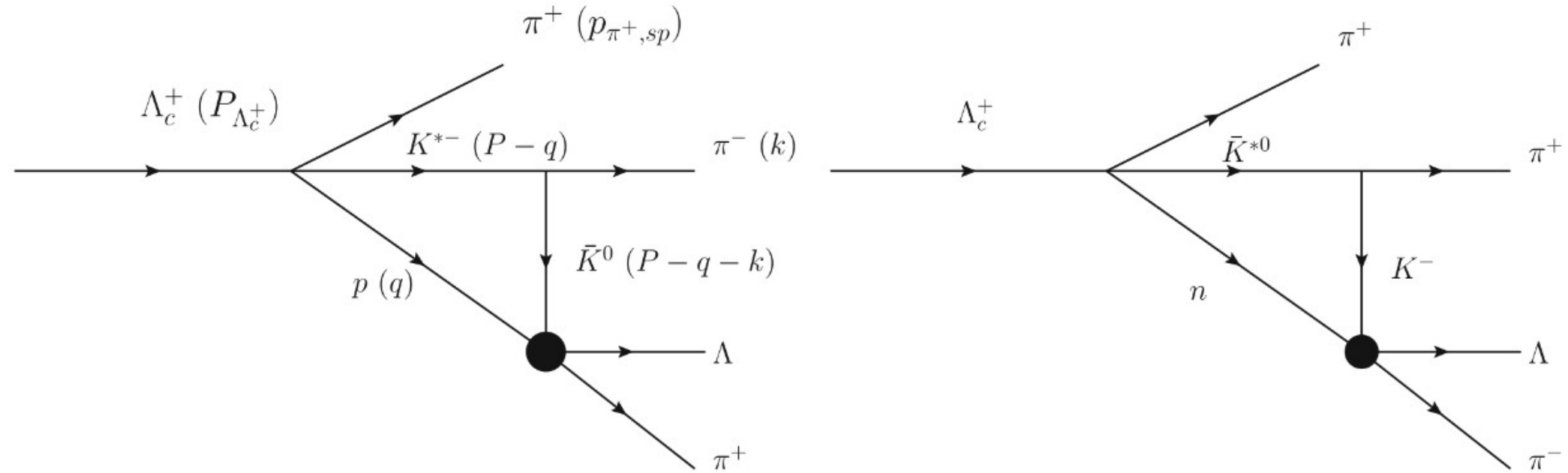
Y. Ma et al. [Belle], Phys. Rev. Lett. 130, 151903 (2023)



Y.~Y.~Li, J.~Song, E.~Oset, W.~H.~Liang and R.~Molina,

"The  $\Lambda_c^+ \rightarrow \Lambda \pi^+ \pi^- \pi^+$  reaction, and a triangle singularity producing the  $\Sigma^*(1430)$  state,"

Eur. Phys. J. C **85**, no.9, 1086 (2025)



One can obtain the properties of the Belle peak and its strength

More experiments to find it would be welcome

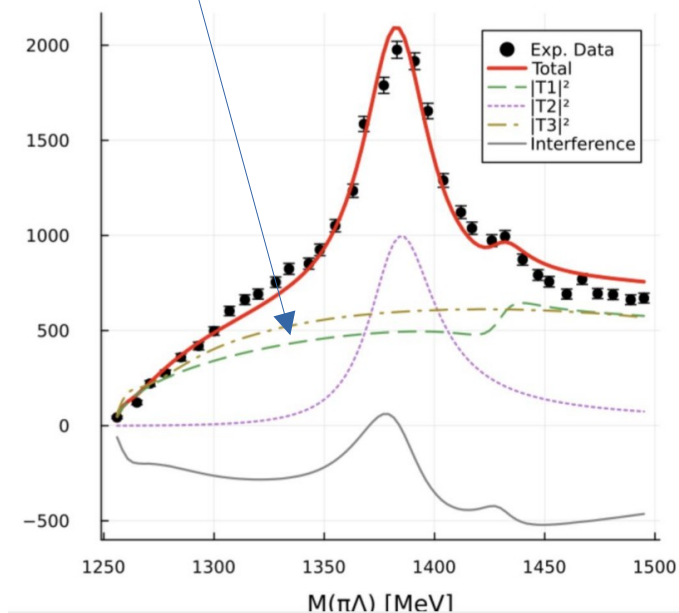
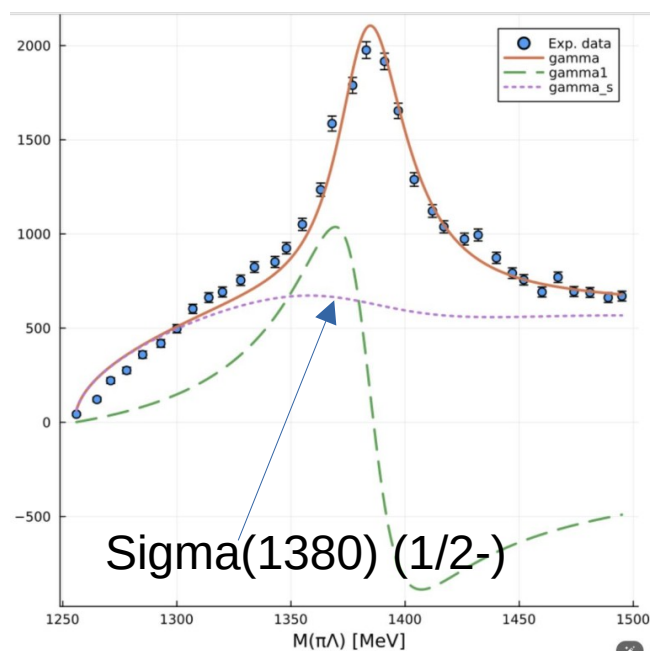
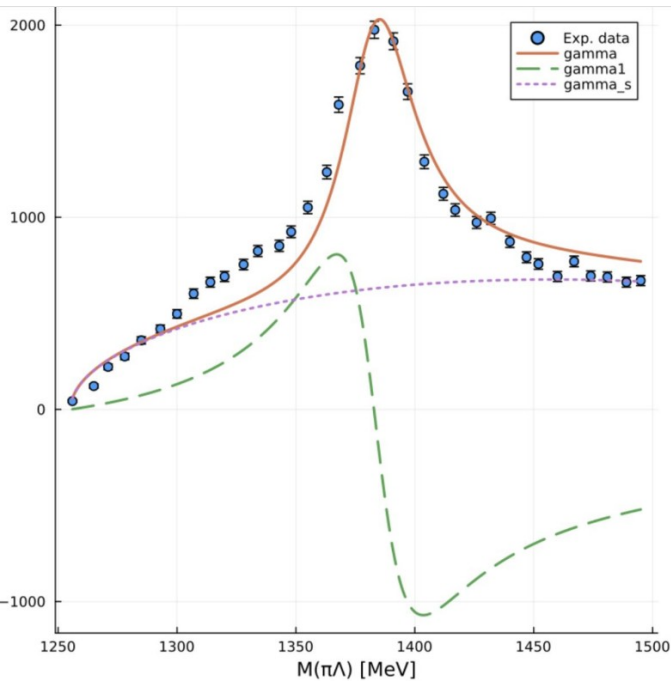
# Identifying $\Sigma(1380)$ and $\Sigma(1430)$ in the $J/\psi \rightarrow \bar{\Lambda}\pi\Sigma$ reaction

Wen-Tao Lyu,<sup>1,2,\*</sup> Eulogio Oset,<sup>2,†</sup> De-Min Li,<sup>1,‡</sup> and En Wang<sup>1,§</sup>

$\Sigma(1385) 3/2^+$

▶ M. Ablikim et al. [BESIII], Phys. Rev. D 108, 112012 (2023)

No  $\Sigma(1380)$   
but  $\pi\Sigma$  interaction



# Scattering observables and correlation function for $p f_1(1285)$ revisited

P. Encarnacion, A. Feijoo and E. Oset,  
"Correlation function for the  $p f_1(1285)$  interaction,"  
Phys. Rev. D **111**, no.11, 114023 (2025)

Arxiv: 2603.09852

$$f_1(1285) = -\frac{1}{\sqrt{2}}(K^* \bar{K})^{I=0} - \frac{1}{\sqrt{2}}(\bar{K}^* K)^{I=0}$$

$I^G(J^PC)=0^+(1^{++})$

For  $p f_1$  interaction we follow the traditional method of **particle-nucleus interaction**:

**Define optical potential and solve the Lippmann Schwinger Equation**

**The optical potential is provided by the Fixed Center Approach** to Faddeev equations, which is particularly suited to this case, since we have a well defined "cluster" the  $f_1$  molecule

$$t_1 = \frac{3}{4}t_{pK^*}^{(1)} + \frac{1}{4}t_{pK^*}^{(0)}$$

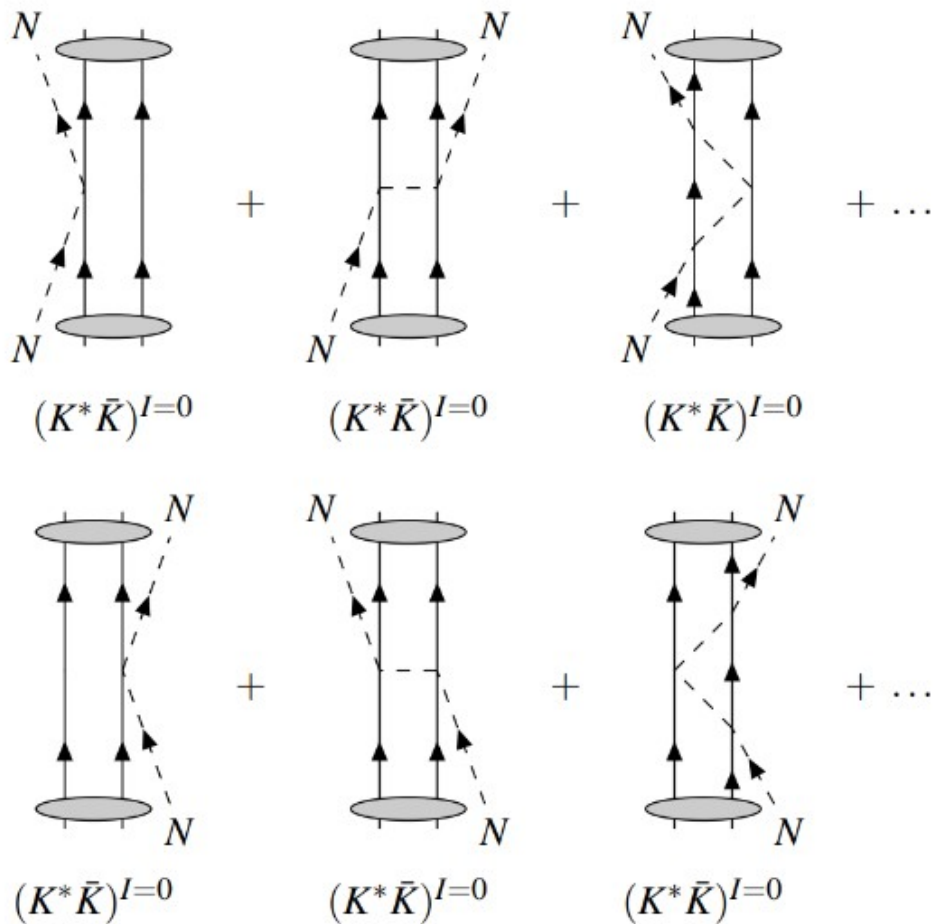
$$t_2 = \frac{3}{4}t_{p\bar{K}}^{(1)} + \frac{1}{4}t_{p\bar{K}}^{(0)}$$

The FCA

$$\tilde{t}_1 = \frac{M_C}{M_{K^*}}t_1 \quad ; \quad \tilde{t}_2 = \frac{M_C}{M_{\bar{K}}}t_2$$

$$\tilde{T}_{11} = \frac{\tilde{t}_1}{1 - \tilde{t}_1\tilde{t}_2G_0^2} \quad ; \quad \tilde{T}_{22} = \frac{\tilde{t}_2}{1 - \tilde{t}_1\tilde{t}_2G_0^2}$$

$$\tilde{T}_{12} = \tilde{T}_{21} = \frac{\tilde{t}_1\tilde{t}_2G_0}{1 - \tilde{t}_1\tilde{t}_2G_0^2}$$



$$G_0(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{2M_N}{2E(q)2\omega_C(q)} \frac{F_C(q)}{\sqrt{s} - E(q) - \omega_C(q) + i\epsilon}$$

$$\times \theta(q_{max}^{(1)} - q_1^*) \theta(q_{max}^{(2)} - q_2^*) \quad (8)$$

$$\tilde{T} = \begin{pmatrix} \tilde{T}_{11} & \tilde{T}_{12} \\ \tilde{T}_{21} & \tilde{T}_{22} \end{pmatrix}$$

$$\tilde{T}' = [1 - \tilde{T}G_C]^{-1}\tilde{T}$$

$$G_C = \begin{pmatrix} G_C^{(1)} & 0 \\ 0 & G_C^{(2)} \end{pmatrix}$$

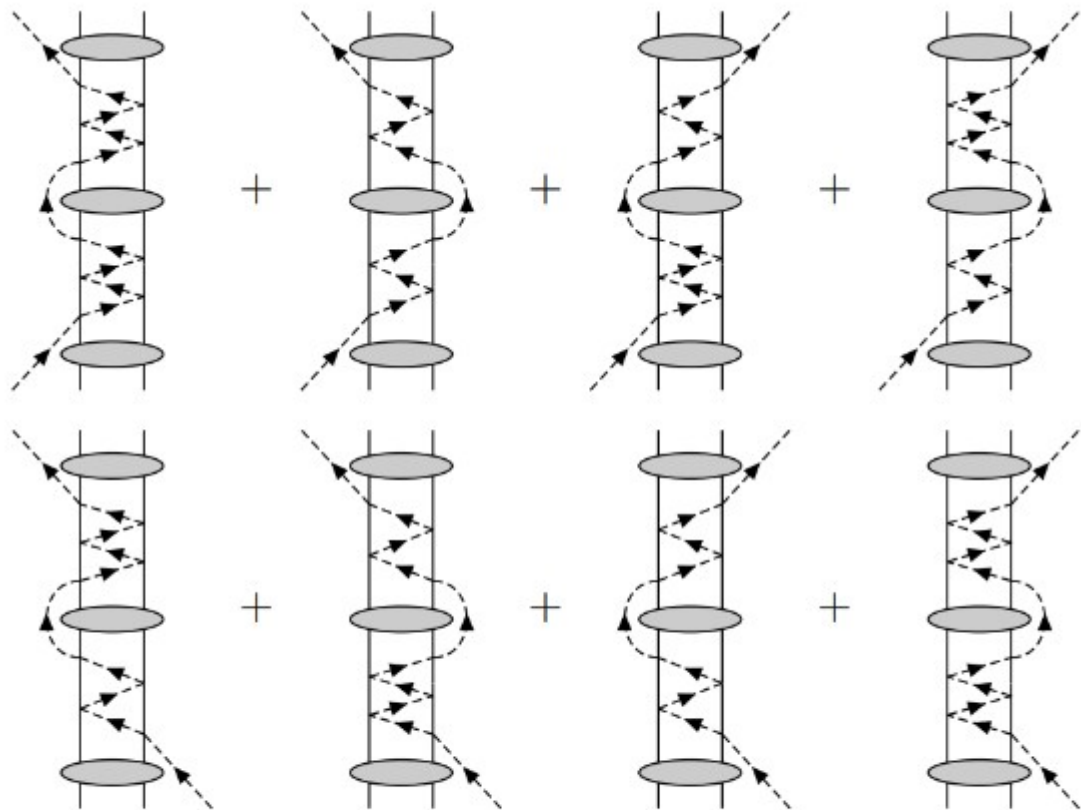


FIG. 2. Diagrams required for the unitarization of the  $pf_1(1285)$

$$G_C^{(i)}(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{2M_N}{2E(q)2\omega_C(q)} \frac{[F_C^{(i)}(q)]^2}{\sqrt{s} - E(q) - \omega_C(q) + i\epsilon} \times \theta(q_{\max}^{(i)} - q_i^*), \quad (15)$$

$$F_C^{(1)}(q) = F_C \left( \frac{m_{\bar{K}}}{m_{K^*} + m_{\bar{K}}} q \right)$$

$$F_C^{(2)}(q) = F_C \left( \frac{m_{K^*}}{m_{K^*} + m_{\bar{K}}} q \right)$$

$$T_{K^*\bar{K}}^{\text{tot}}(\sqrt{s}) = \sum_{i,j} \tilde{T}'_{ij} = \frac{\tilde{t}_1 + \tilde{t}_2 + (2G_0 - G_C^{(1)} - G_C^{(2)})\tilde{t}_1\tilde{t}_2}{1 - G_C^{(1)}\tilde{t}_1 - G_C^{(2)}\tilde{t}_2 - (G_0^2 - G_C^{(1)}G_C^{(2)})\tilde{t}_1\tilde{t}_2} \quad (17)$$

$$-\frac{8\pi\sqrt{s}}{2M_N} (T_{K^*\bar{K}}^{\text{tot}})^{-1} = (f^{QM})^{-1} \simeq -\frac{1}{a_0} + \frac{1}{2}r_0q_{cm}^2 - iq_{cm}$$

unitarity

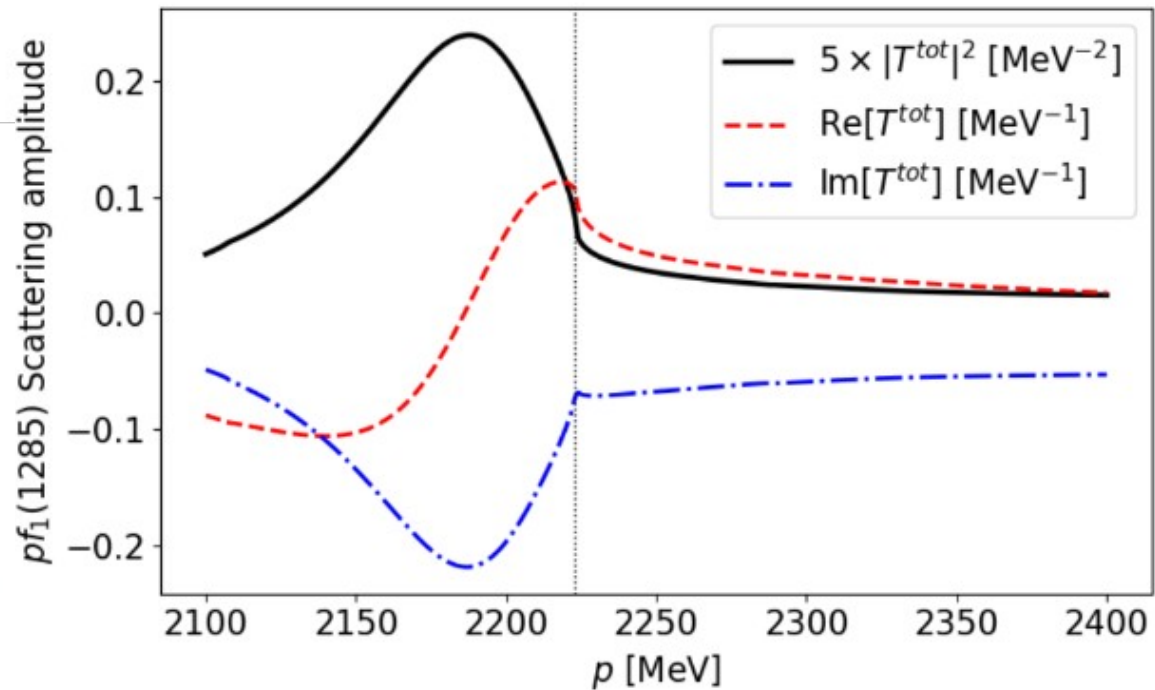
$$(T^{\text{tot}})^{-1} = \frac{1}{2} [(T_{K^*\bar{K}}^{\text{tot}})^{-1} + (T_{\bar{K}^*K}^{\text{tot}})^{-1}]$$

$$M_R = 2189 \pm 10 \text{ MeV},$$

$$\Gamma_R = 53 \pm 12 \text{ MeV},$$

$$a_0 = [0.689 \pm 0.040] - [0.413 \pm 0.084]i \text{ fm},$$

$$r_0 = [-0.025 \pm 0.077] + [0.146 \pm 0.084]i \text{ fm}.$$



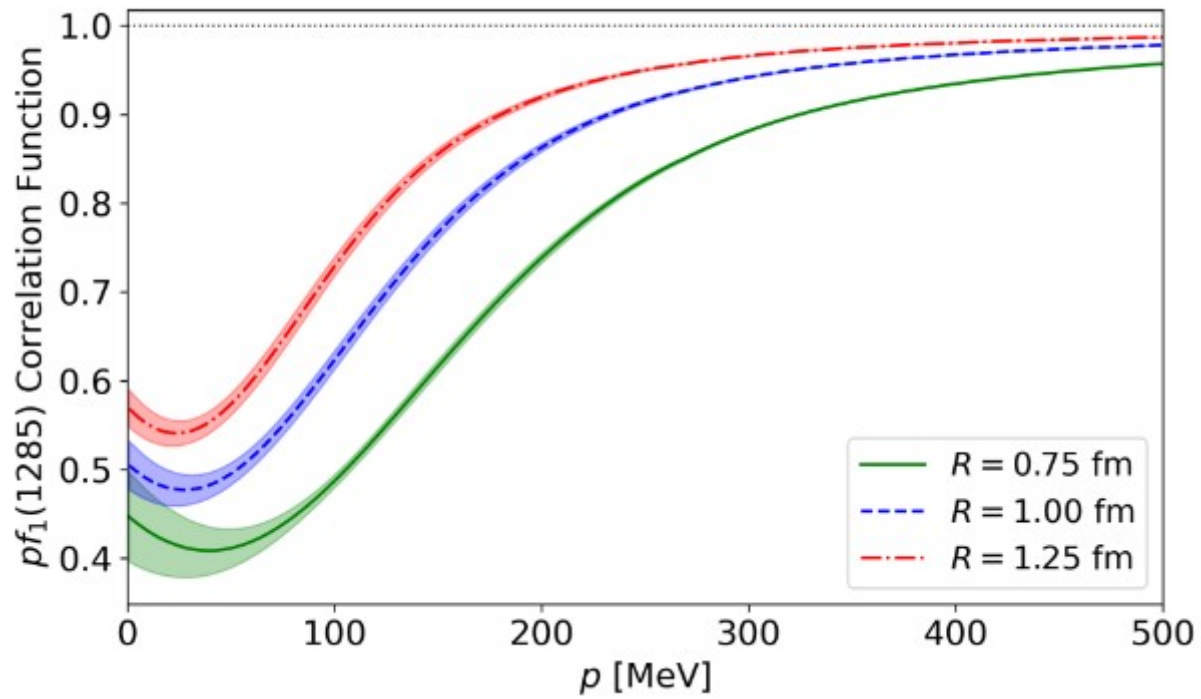
## Correlation function

$$C_{pf_1}(p) = 1 + 4\pi \int_0^\infty dr r^2 S_{12}(r) \times \left\{ |j_0(pr) + TG|^2 - j_0^2(pr) \right\}$$

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}} e^{-r^2/4R^2}$$

$$\begin{aligned} TG &\simeq \left( \tilde{T}'_{11} + \tilde{T}'_{21} + \tilde{T}'_{12} + \tilde{T}'_{22} \right) \frac{1}{2} \left( G_1(\sqrt{s}, r) + G_2(\sqrt{s}, r) \right) \\ &= T^{\text{tot}} \frac{1}{2} \left( G_1(\sqrt{s}, r) + G_2(\sqrt{s}, r) \right) \end{aligned} \quad (25)$$

$$\begin{aligned} G_i(\sqrt{s}, r) &= \int \frac{d^3 q}{(2\pi)^3} \frac{2M_N}{2E(q)2\omega_C(q)} \frac{F_C^{(i)}(q) j_0(qr)}{\sqrt{s} - E(q) - \omega_C(q) + i\epsilon} \\ &\times \theta(q_{\text{max}}^{(i)} - q_i^*). \end{aligned} \quad (24)$$



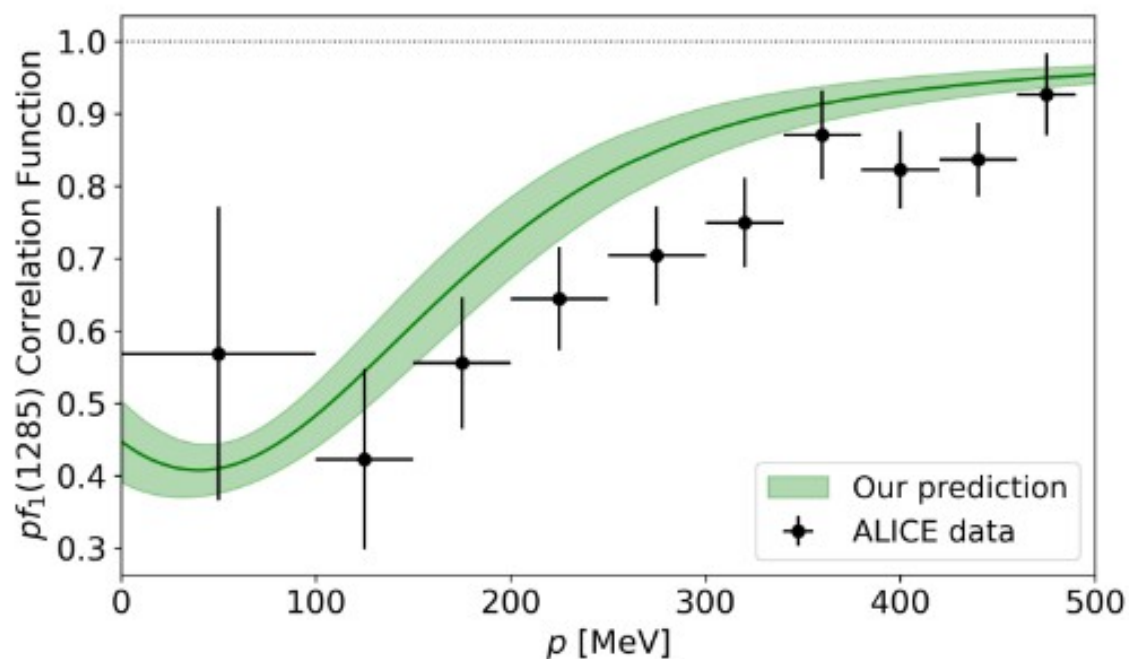


FIG. 5.  $pf_1(1285)$  CF for  $R = 0.75$  fm with the corresponding error band, compared with the ALICE experimental data taken from [54].

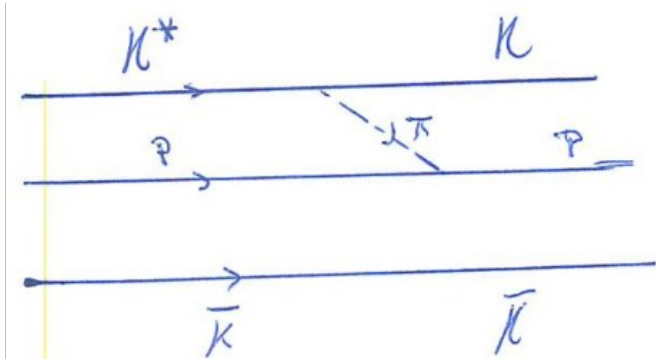
## How to observe the state ?

### $f_1(1285)$ DECAY MODES

$\Gamma_9$	$a_0(980) \pi$ $K \bar{K}$	[ignoring $a_0(980) \rightarrow$	$(38 \pm 4) \%$
$\Gamma_{11}$	$K \bar{K} \pi$		$(9.0 \pm 0.4) \%$

One must look at mass distribution of a proton + these other products

4 body



$$pK\bar{K} \quad I = 1/2, \quad J^P = 1/2^+, 3/2^+$$

3 body final state

The world of **three body bound states** involving mesons and baryons is a new frontier in Hadron Physics. **ELSA** could say something about.



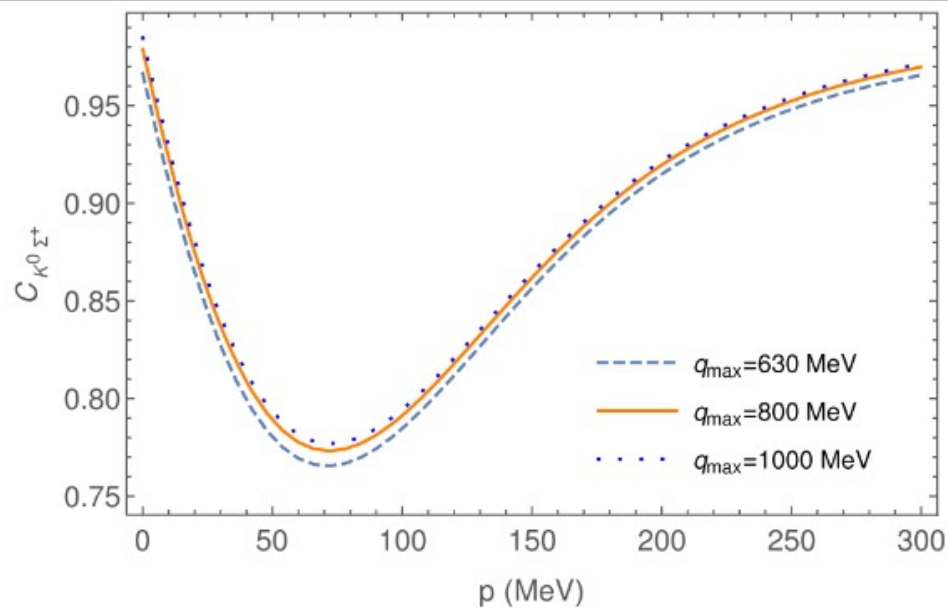


FIG. 1. Correlation function of the  $K^0 \Sigma^+$  channel for different cutoffs and  $R = 1$  fm.

R.~Molina and E.~Oset,  
 "Determination of off-shell ambiguities in  
 correlation functions: Strategies to  
 minimize them,"  
 Phys. Rev. D **112**, no.9, 096006  
 (2025)

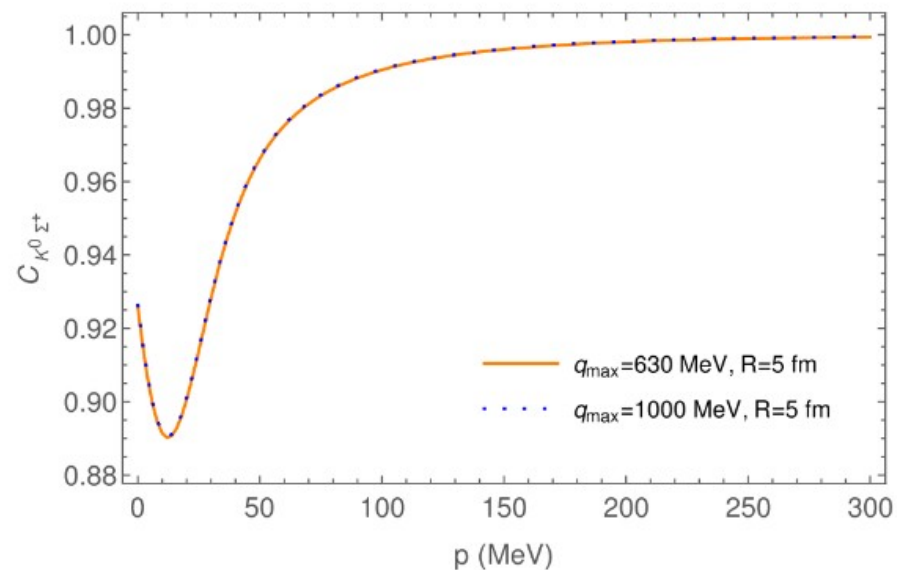


FIG. 5. Correlation function of the  $k^0 \Sigma^+$  channel for  $R = 5$  fm and different cutoffs.

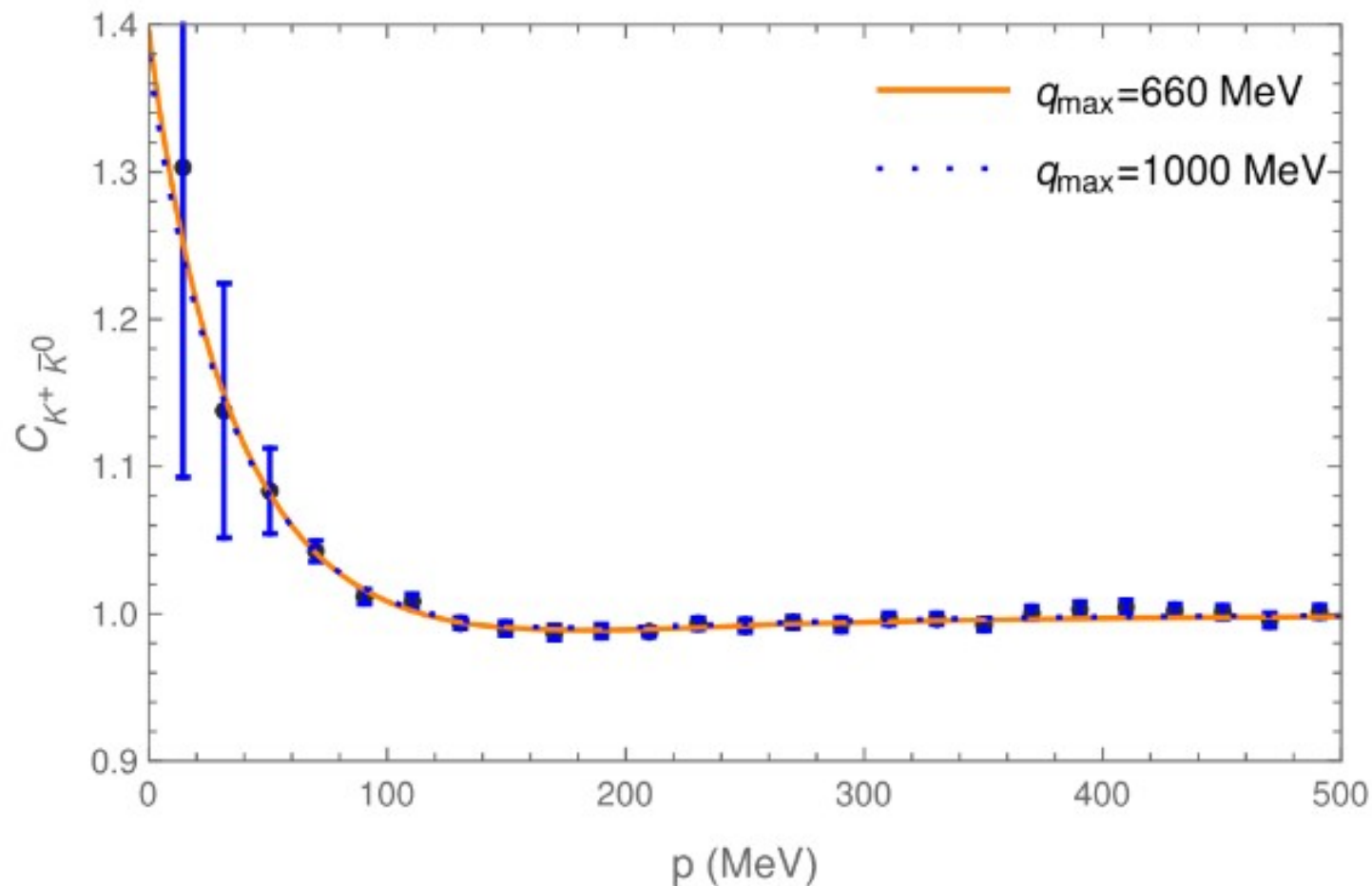


FIG. 6. Correlation function for Fit II in [23] for two different values of the cutoff in  $\tilde{G}$ .