

Theoretical Aspects of Invisible Light Dark Matter Searches in Fixed Target Experiments

Martin Schürmann^{a*}

M. Schürmann, H. K. Dreiner, and R. Gauld,
Theory Calculations for LDMX and LOHENGRIN beyond Coherent Bethe-Heitler Scattering,
(2026), arXiv: 2606.20327 [hep-ph].

Bechtle, P., Bispin, C., Breton, D. et al.
A proposal for the Lohengrin experiment to search for dark sector particles at the ELSA Accelerator,
Eur. Phys. J. C 85, 600 (2025).

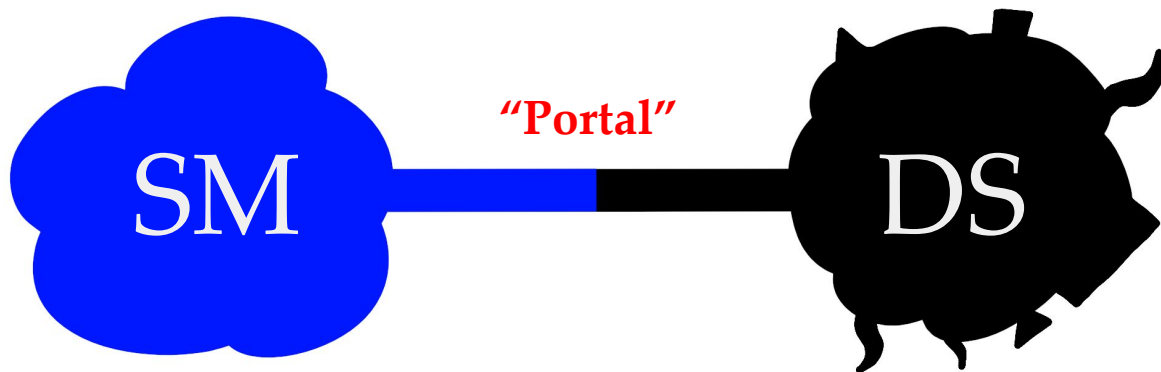
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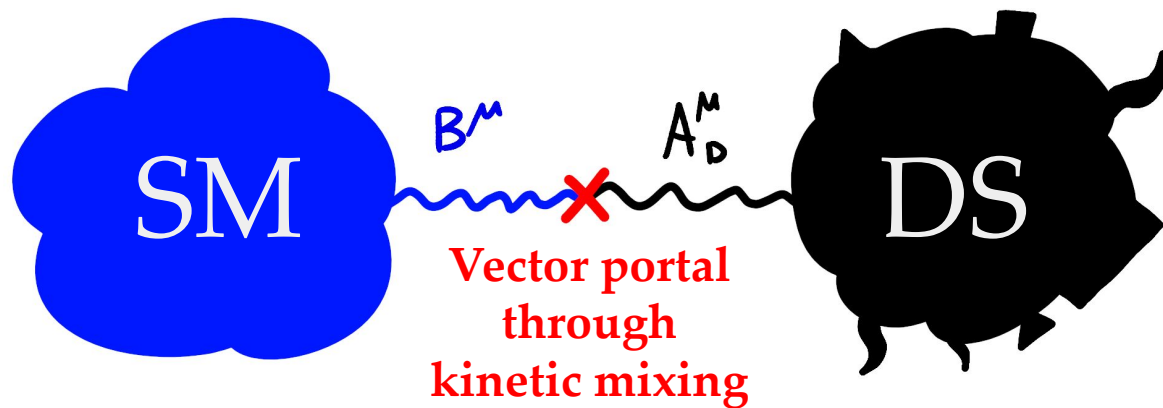
Motivation

- **Dark Matter** (DM) remains an unsolved mystery in the Standard Model (SM)
- Null results from WIMP and axion searches: Consider more exotic options.
 - ➔ **Light Dark Sectors** (DS)



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Dark Photon Theory

- In general, the kinetic mixing of new $U(1)_D$ boson with hypercharge gauge boson,

$$\mathcal{L} \supset -\frac{1}{4}F_{D\mu\nu}F_D^{\mu\nu} + \frac{1}{2}m_{A_D}^2 A_{D\mu}A_D^\mu - \frac{\sin \varepsilon_Y}{2}F_D^{\mu\nu}B_{\mu\nu} - g_D A_{D\mu}J_D^\mu$$

results in mass eigenstate interaction terms of the form:

$$\mathcal{L}_{\text{int}} = -A'_\mu \left(\sum_f g_f^{A'} \bar{f} \gamma^\mu f + g_D^{A'} J_D^\mu \right)$$

$$\text{with e.g. } g_{\ell_L}^{A'} = \frac{e\varepsilon_Y}{2\cos\theta_W} \frac{2m_Z^2 \cos^2\theta_W - m_{A_D}^2}{m_Z^2 - m_{A_D}^2}, \quad g_{\ell_R}^{A'} = \frac{e\varepsilon_Y}{\cos\theta_W} \left(1 - \frac{m_Z^2 \sin^2\theta_W}{m_Z^2 - m_{A_D}^2} \right).$$

Dark Photon Theory

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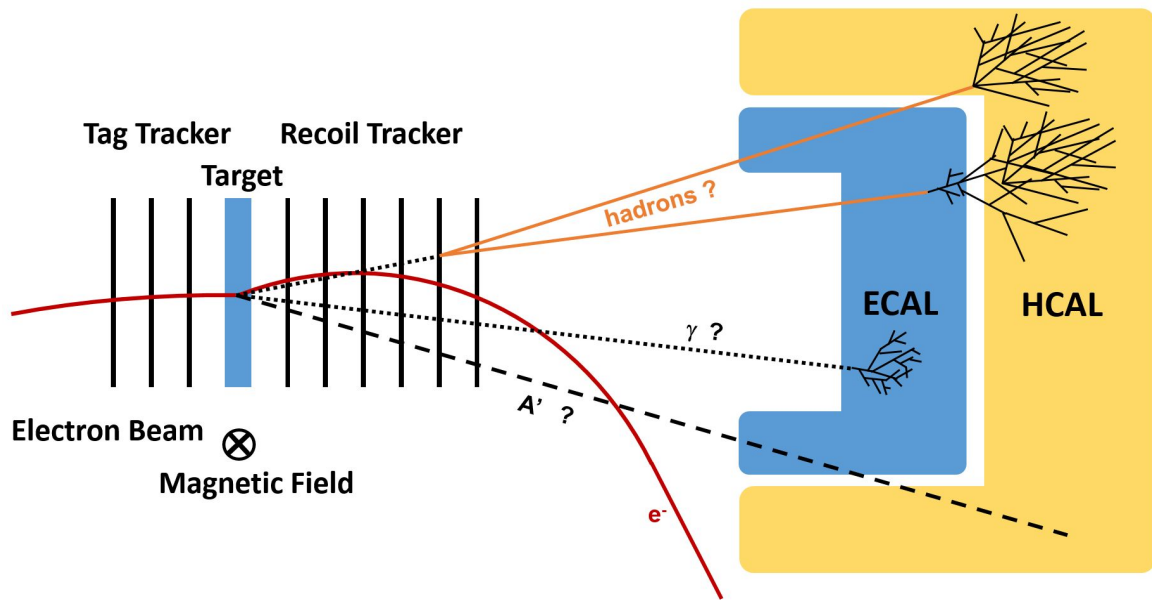
$$\mathcal{L}_{\text{int}} = -A'_\mu \left(\sum_f g_f^{A'} \bar{f} \gamma^\mu f + g_D^{A'} J_D^\mu \right)$$

- For smallish Dark Photon masses, $m_{A'}^2 \simeq m_{A_D}^2 \ll m_Z^2 \propto v^2$,

$$g_f^{A'} \simeq -Q_f e \varepsilon_Y \cos \theta_W \equiv -Q_f e \varepsilon$$

➔ Dark Photon interacts only **electromagnetically**, at high and low energies.

Experimental Idea: Fixed Target Missing Energy



Experiments with similar approach:

LDMX [ldm18, wal25], DarkSHINE [dar22], NA64 [ban19].

Dominant process(es):

$$e^- \mathcal{H} \rightarrow e^- \mathcal{H} \gamma$$

and subsequent photo- and electro-nuclear reactions.

Occasionally:

$$e^- \mathcal{H} \rightarrow e^- \mathcal{H} A'$$

controlled by ε and $m_{A'}$.

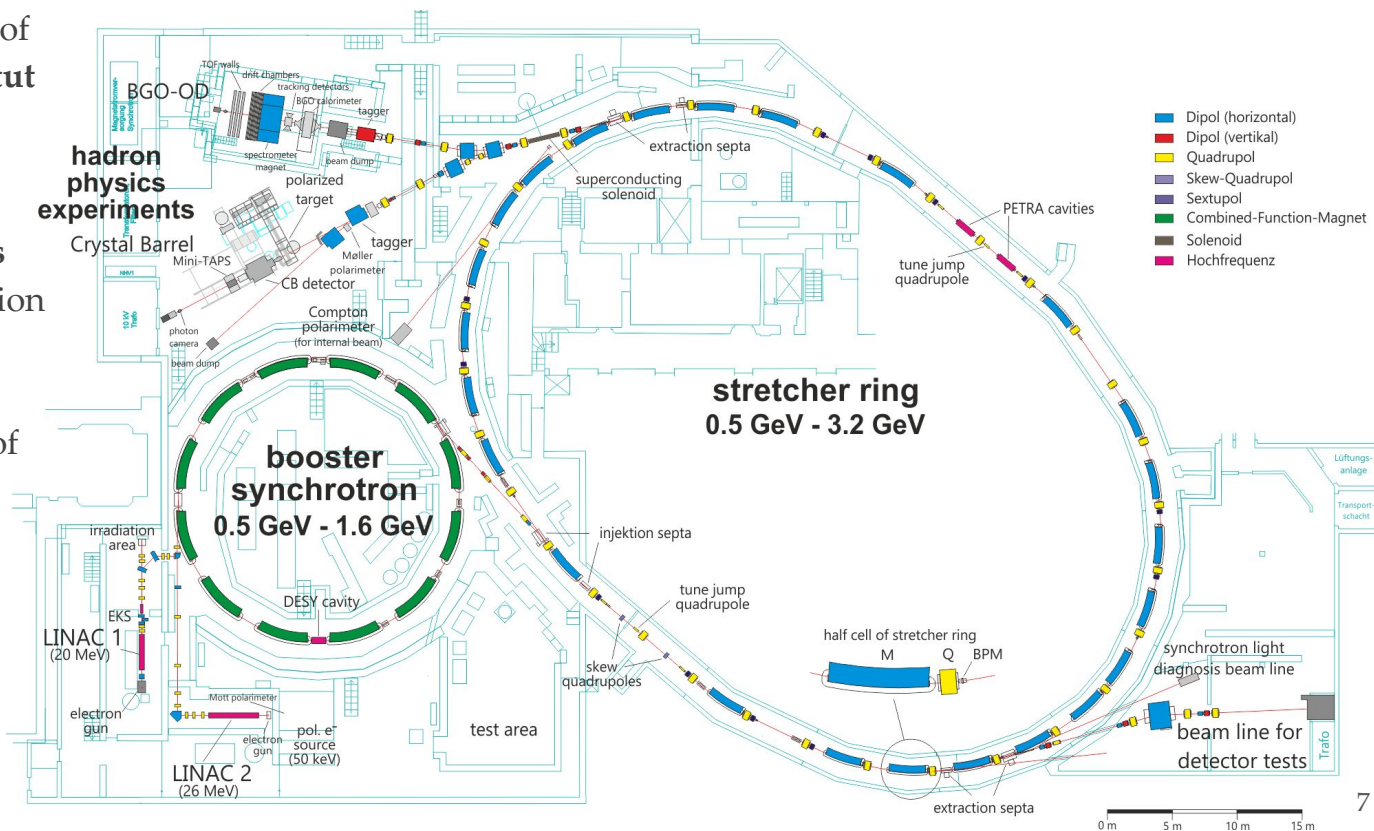
=> Search for **recoiling, low energy solitary electrons** by vetoing backgrounds with ECAL and HCAL!

Electron Stretcher Accelerator (ELSA)

Located in the basement of the **Physikalisches Institut in Bonn**

- **3.2 GeV** beam of (on average) **single electrons** through resonant extraction
- Small energy spread of 0.8‰
- Effective extraction rate of 100 MHz

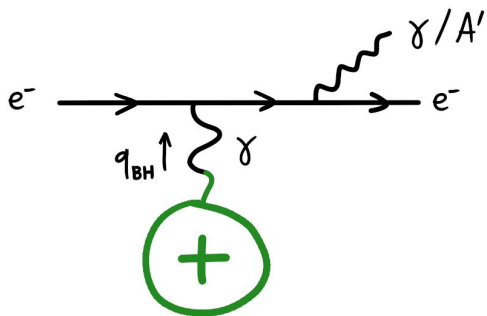
=> $\sim 10^{15}$ EOT within a year



Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

Bethe-Heitler scattering off nuclear charge distribution with one photon exchange:

$$i\mathcal{M}_{\text{coh}}(e^- \Phi^+ \xrightarrow{\gamma} e^- \Phi^+ R) = L_{\text{BH}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{BH}}^2} H_{\text{BH, coh}}^\nu \quad \text{where} \quad R = \gamma, A'$$

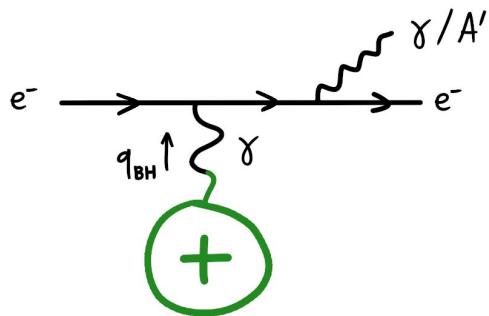


“coherent”: $\mathcal{H} = \Phi^+$

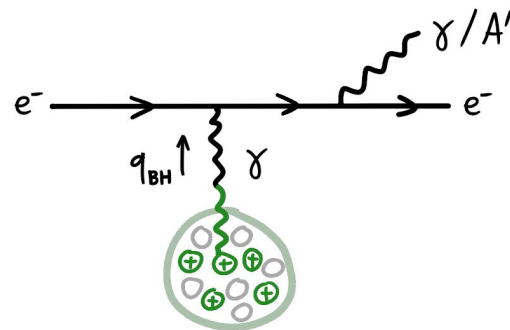
Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

Bethe-Heitler scattering off **generalized hadronic current** with one photon exchange:

$$i\mathcal{M}_{\mathcal{R}}(e^- \mathcal{H} \xrightarrow{\gamma} e^- \mathcal{H} R) = L_{\text{BH}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{BH}}^2} H_{\text{BH}, \mathcal{R}}^\nu$$



“coherent”: $\mathcal{H} = \Phi^+$



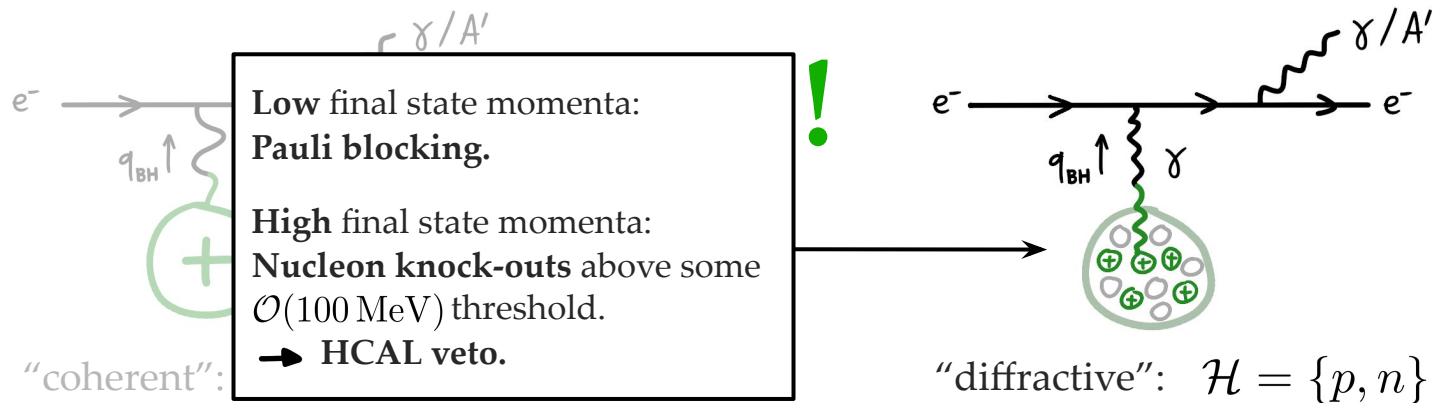
“diffractive”: $\mathcal{H} = \{p, n\}$

$$\Rightarrow d\sigma = d\sigma_{\text{coh}} + d\sigma_{\text{dif}} \quad \text{with} \quad \overline{|\mathcal{M}|^2}_{\text{dif}} = Z \overline{|\mathcal{M}_p|^2} + (A - Z) \overline{|\mathcal{M}_n|^2}.$$

Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

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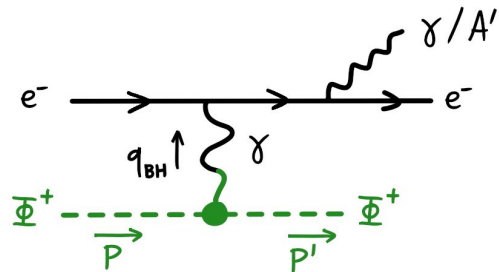


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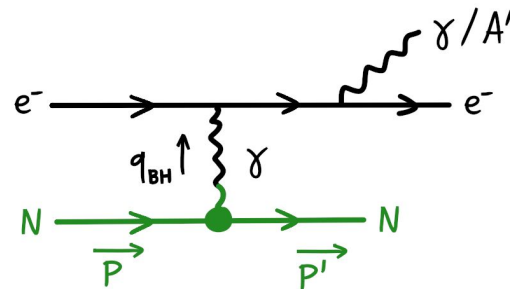
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Bethe-Heitler scattering off **generalized hadronic current** with one photon exchange:

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$$H_{\text{BH}, \text{coh}}^\nu = ieF(q_{\text{BH}}^2) (2P^\nu - q_{\text{BH}}^\nu)$$



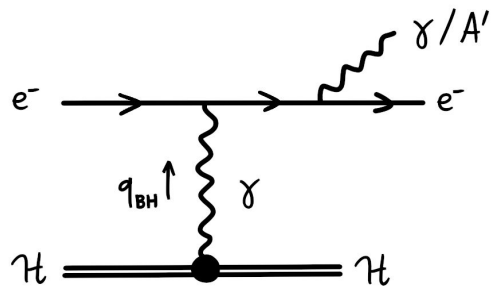
$$\begin{aligned} H_{\text{BH}, \text{dif}}^\nu &= ie \bar{u}_N(P') \left[\gamma^\nu F_1^N(q_{\text{BH}}^2) - \frac{i\sigma^{\nu\kappa} q_{\text{BH}\kappa}}{2m_N} F_2^N(q_{\text{BH}}^2) \right] u_N(P) \\ &= \bar{u}_N(P') \Gamma_N^\nu(q_{\text{BH}}) u_N(P) \end{aligned}$$

$$\Rightarrow d\sigma = d\sigma_{\text{coh}} + d\sigma_{\text{dif}} \quad \text{with} \quad \overline{|\mathcal{M}|^2}_{\text{dif}} = Z \overline{|\mathcal{M}_p|^2} + (A - Z) \overline{|\mathcal{M}_n|^2}.$$

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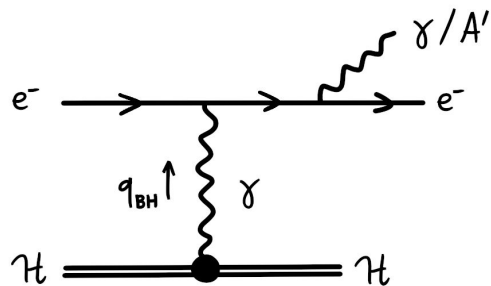


Bethe-Heitler (BH) scattering

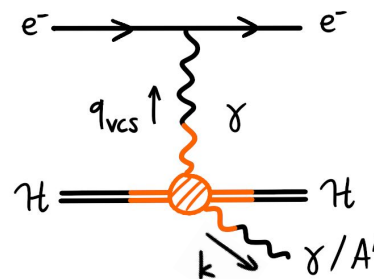
Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

BH and VC scattering off **generalized hadronic current** with one photon exchange:

$$i\mathcal{M}_{\mathcal{R}}(e^- \mathcal{H} \xrightarrow{\gamma} e^- \mathcal{H} R) = L_{\text{BH}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{BH}}^2} H_{\text{BH},\mathcal{R}}^\nu + L_{\text{VCS}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{VCS}}^2} H_{\text{VCS},\mathcal{R}}^\nu \quad \text{where} \quad H_{\text{VCS},\mathcal{R}}^\nu = \varepsilon_\mu^*(k) iC_{\mathcal{R};R,V}^{\mu\nu}$$



Bethe-Heitler (BH) scattering



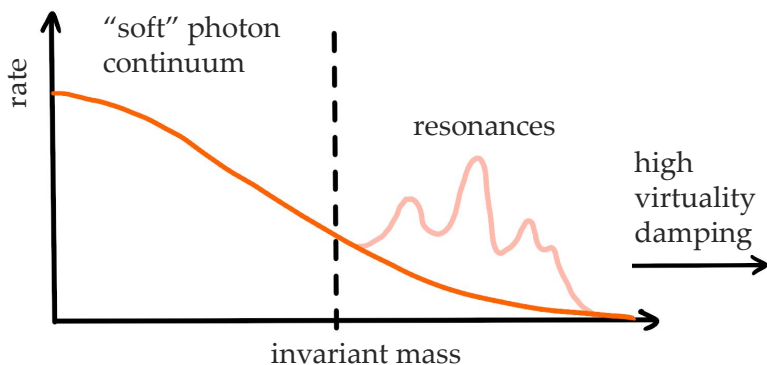
Virtual Compton (VC) scattering

Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

BH and VC scattering off **generalized hadronic current** with one photon exchange:

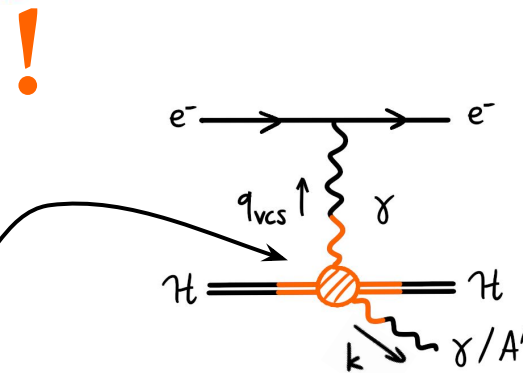
$$i\mathcal{M}_{\mathcal{R}}(e^- \mathcal{H} \xrightarrow{\gamma} e^- \mathcal{H} R) = L_{\text{BH}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{BH}}^2} H_{\text{BH},\mathcal{R}}^\nu + L_{\text{VCS}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{VCS}}^2} H_{\text{VCS},\mathcal{R}}^\nu \quad \text{where} \quad H_{\text{VCS},\mathcal{R}}^\nu = \varepsilon_\mu^*(k) iC_{\mathcal{R};R,V}^{\mu\nu}$$

Composite hadronic systems have resonances:



=> For simplicity, account only for **“soft” photon continuum**:

$$\sqrt{(P' + k)^2} < \text{resonance energy scale}$$



Virtual Compton (VC) scattering

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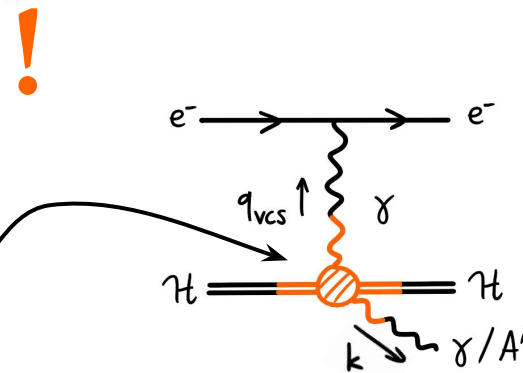
Nuclear resonances lie extremely low, with relative energy of $\mathcal{O}(100 \text{ keV}) \rightarrow$ account for **VCS** only in **diffractive** regime and probe only below Δ resonance, i.e.

$$\sqrt{(P' + k)^2} < 1.2 \text{ GeV}$$

The **soft photon continuum** is described by the following **Compton tensor** [van00, dju08]:

$$C_{\text{dif};\gamma,\gamma}^{\mu\nu} = \bar{u}_N(P') \left[\Gamma_N^\nu(q) \frac{\not{P} - \not{k} + m_N}{(P - k)^2 - m_N^2} \Gamma_N^\mu(k) + \Gamma_N^\mu(k) \frac{\not{P} - \not{q} + m_N}{(P - q)^2 - m_N^2} \Gamma_N^\nu(q) \right] u_N(P)$$

with appropriate $-\varepsilon$ rescaling for A' emission.

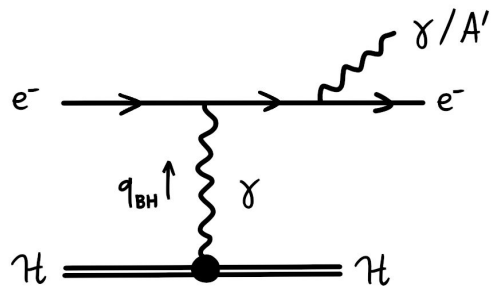


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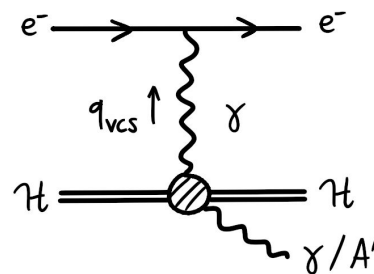
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Bethe-Heitler (BH) scattering

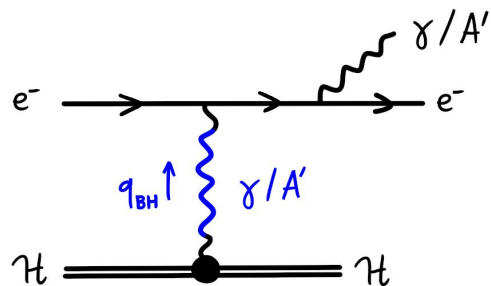


Virtual Compton (VC) scattering

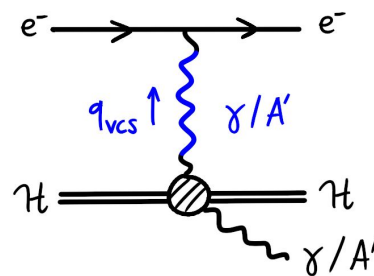
Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

BH and VC scattering off **generalized hadronic current** with **one (dark) photon** exchange:

$$i\mathcal{M}_{\mathcal{R}}(e^- \mathcal{H} \xrightarrow{V=\gamma, A'} e^- \mathcal{H} R) = L_{\text{BH}}^\mu i\Pi_{\mu\nu}^V(q_{\text{BH}}) H_{\text{BH}, \mathcal{R}}^\nu + L_{\text{VCS}}^\mu i\Pi_{\mu\nu}^V(q_{\text{VCS}}) H_{\text{VCS}, \mathcal{R}}^\nu$$



Bethe-Heitler (BH) scattering

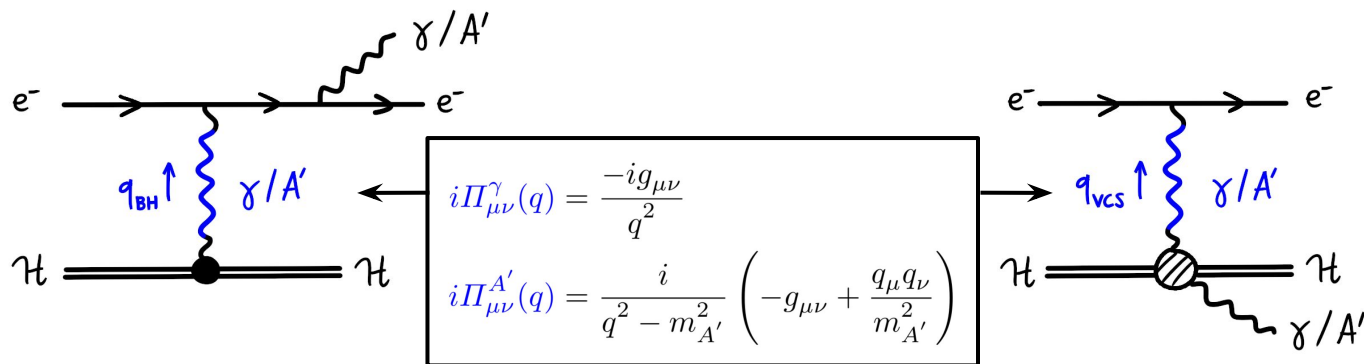


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Bethe-Heitler (BH) scattering

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However, this gives only the **real emission (RE)** component. In general, **RE** at $\mathcal{O}(\alpha^3 \varepsilon^2)$ is accompanied by **virtual corrections** at the same order, $\mathcal{O}(\alpha^3 \varepsilon^2)$.

For simplicity, assume the dark photon **only couples to charged leptons**:

$$|\mathcal{M}|^2(e^- \mathcal{H} \rightarrow e^- \mathcal{H} A'; \alpha^3 \varepsilon^2) = \left| \begin{array}{c} e^- \rightarrow e^- \\ \text{wavy line } \gamma \\ \mathcal{H} \end{array} \begin{array}{c} A' \\ \text{wavy line } A' \end{array} + \begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} \begin{array}{c} A' \\ \text{wavy line } A' \end{array} \right|^2$$

$$|\mathcal{M}|^2(e^- \mathcal{H} \rightarrow e^- \mathcal{H} \quad ; \alpha^3 \varepsilon^2) = 2 \operatorname{Re} \left\{ \left(\begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} \begin{array}{c} A' \\ \text{loop} \end{array} + \begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} \begin{array}{c} \text{cross} \end{array} \right) \begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} + \left(\begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} \begin{array}{c} \text{loop} \end{array} + \begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} \begin{array}{c} \text{cross} \end{array} \right) \begin{array}{c} \text{wavy line } \gamma \\ \mathcal{H} \end{array} \right\}$$

Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

Counterterms determined through **renormalization** of the theory. The Feynman rules are:

$$A_{\mu} \sim \text{wavy line} \otimes A'_{\nu} = -i \frac{\delta Z_{AA'} + \delta Z_{A'A}}{2} (q^2 g_{\mu\nu} - q_{\mu} q_{\nu}) + i g_{\mu\nu} \frac{\delta Z_{A'A}}{2} m_{A'}^2$$

$$\ell \rightarrow \text{wavy line} \otimes \ell = -ie Q_{\ell} \delta Z_1 \gamma^{\mu}$$

where $\delta Z_{A'A} = 0$

$$\delta Z_{AA'} = \frac{2\alpha\varepsilon Q_{\ell}^2}{9\pi m_{A'}^2} \left\{ m_{A'}^2 - 6m_{\ell}^2 + 6A_0(m_{\ell}^2) - 3(m_{A'}^2 + 2m_{\ell}^2) B_0(m_{A'}^2; m_{\ell}^2, m_{\ell}^2) \right\}$$

$$\delta Z_1 = \frac{\alpha\varepsilon^2 Q_{\ell}^2}{4\pi m_{A'}^2 m_{\ell}^2} \left\{ (m_{A'}^2 + m_{\ell}^2) A_0(m_{A'}^2) - m_{A'}^2 A_0(m_{\ell}^2) - m_{A'}^4 B_0(m_{\ell}^2; m_{A'}^2, m_{\ell}^2) \right. \\ \left. + m_{\ell}^2 m_{A'}^2 [1 + 2(m_{A'}^2 + 2m_{\ell}^2) B'_0(m_{\ell}^2; m_{A'}^2, m_{\ell}^2)] \right\}$$

$$|\mathcal{M}|^2(e^{-} \mathcal{H} \longrightarrow e^{-} \mathcal{H} \quad ; \alpha^3 \varepsilon^2) = 2 \operatorname{Re} \left\{ \left(\text{diagram 1} + \text{diagram 2} \right) \text{diagram 3}^{\dagger} + \left(\text{diagram 4} + \text{diagram 5} \right) \text{diagram 6}^{\dagger} \right\}$$

Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

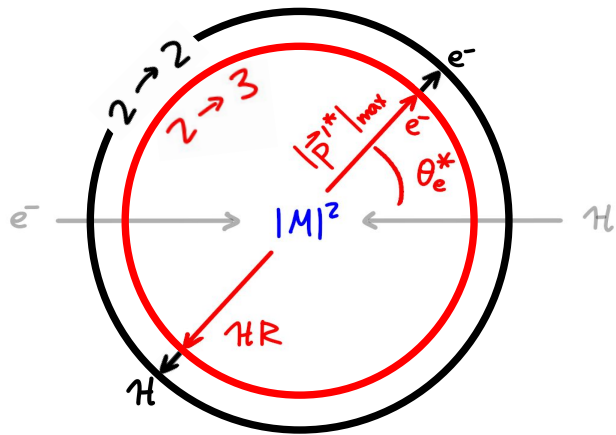
Real emission from **BH** and **VC** scattering off **generalized hadronic current**
with **one (dark) photon** exchange
and
virtual corrections for special case of leptophilic dark photon...

... implemented in dedicated Monte-Carlo code **Lohengrin++** for cross section study.

- Written in C++.
- Amplitudes (squared): **FeynRules** [chr08, all13], **FeynArts** [hah00] and **FeynCalc** [mer90, sht16, sht20].
- Phase space integration: Vegas algorithm from Cuba library [hah04].

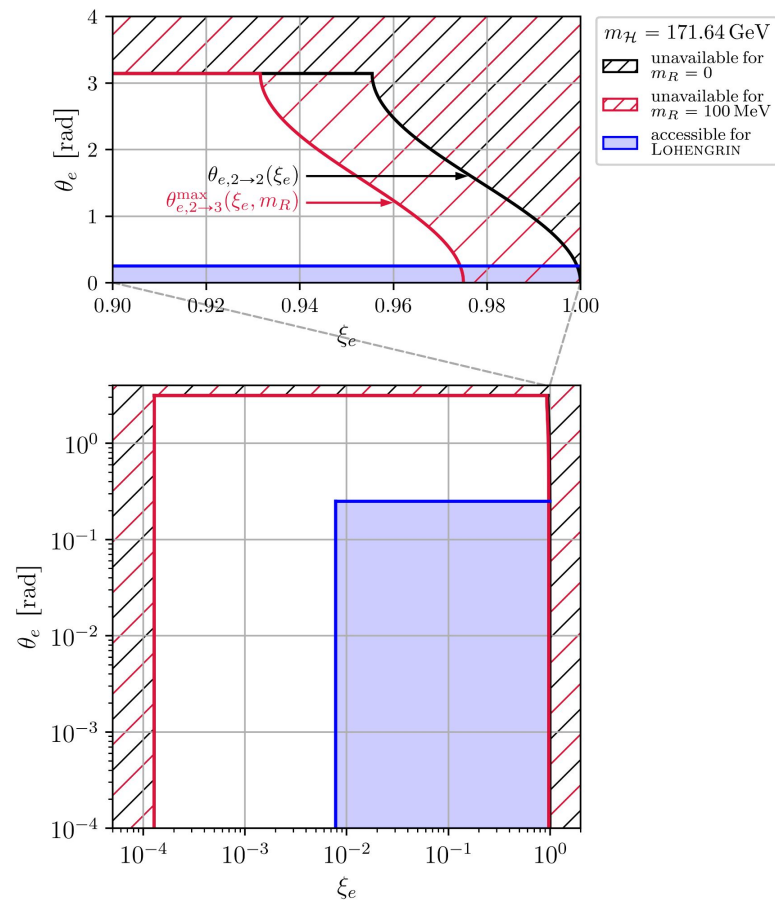
Phase Space

CM frame:

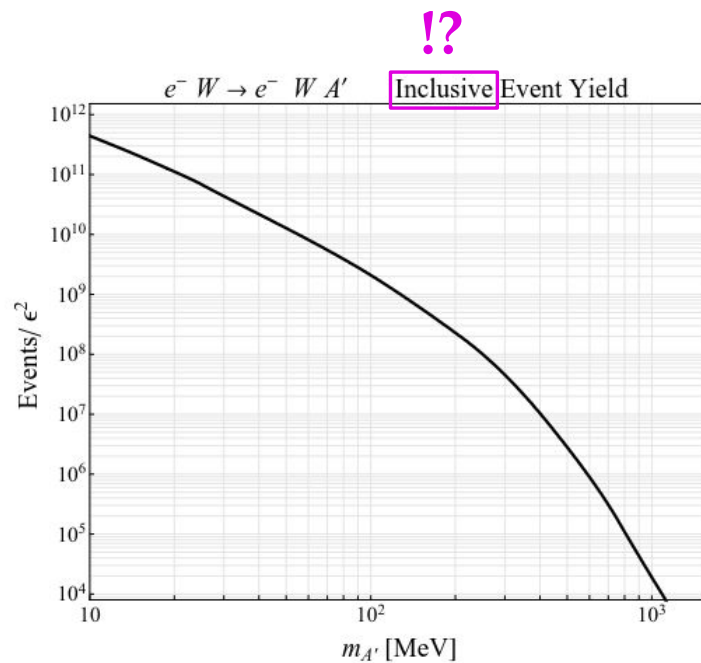


➔ Boost into lab frame!

- Spherical coordinates:
 $(|\vec{p}'|, \theta_e, \varphi_e)$ or $(\xi_e, \theta_e, \varphi_e)$ with $\xi_e = E'/E$.



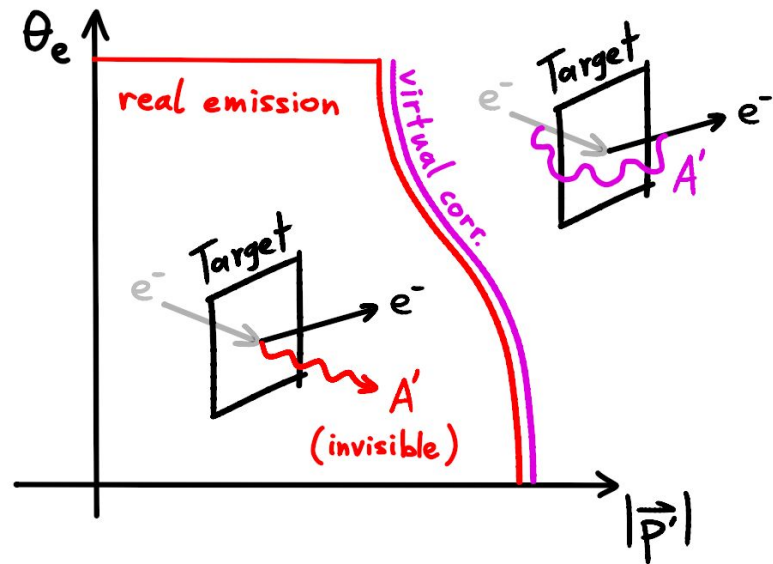
Virtual Correction



Taken from [ldm18].

Virtual Correction

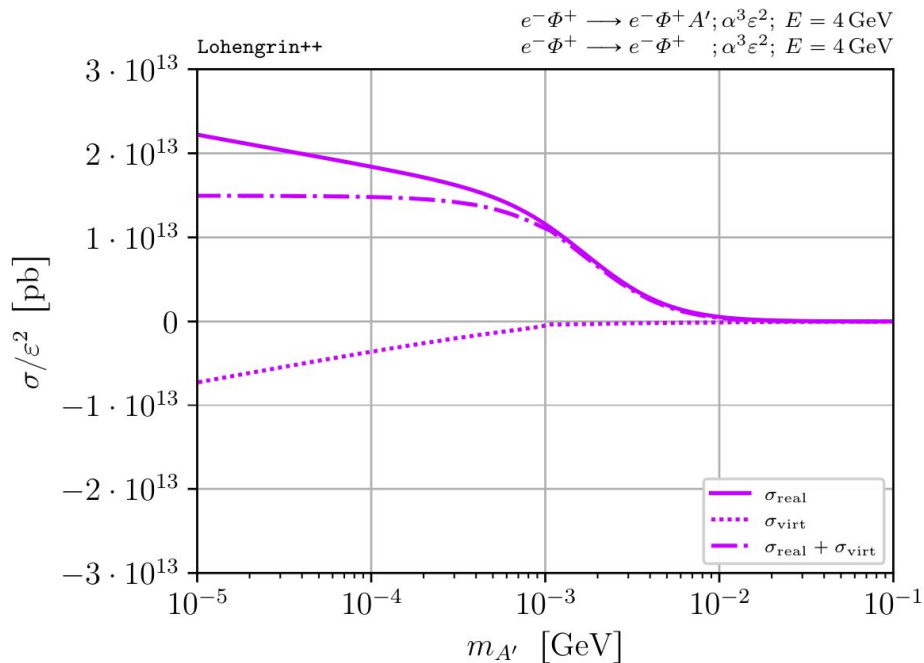
- The **IR divergence** makes the **inclusive RE** cross section ill defined in the limit $m_{A'} \rightarrow 0$.
- **Inclusive RE** cross section potentially overestimates “inclusive event yield” at $\mathcal{O}(\alpha^3 \varepsilon^2)$ due to **infrared growth** of cross section for small $m_{A'}$.



➔ **Inclusive RE** quantities in general **not suitable** for this type of disappearance experiment!

Virtual Correction

- Total cross section is constant in the limit $m_{A'} \rightarrow 0$!
- For relevant masses, $m_{A'} \gtrsim 1 \text{ MeV}$, virtual correction to the inclusive cross section is only a few percent.

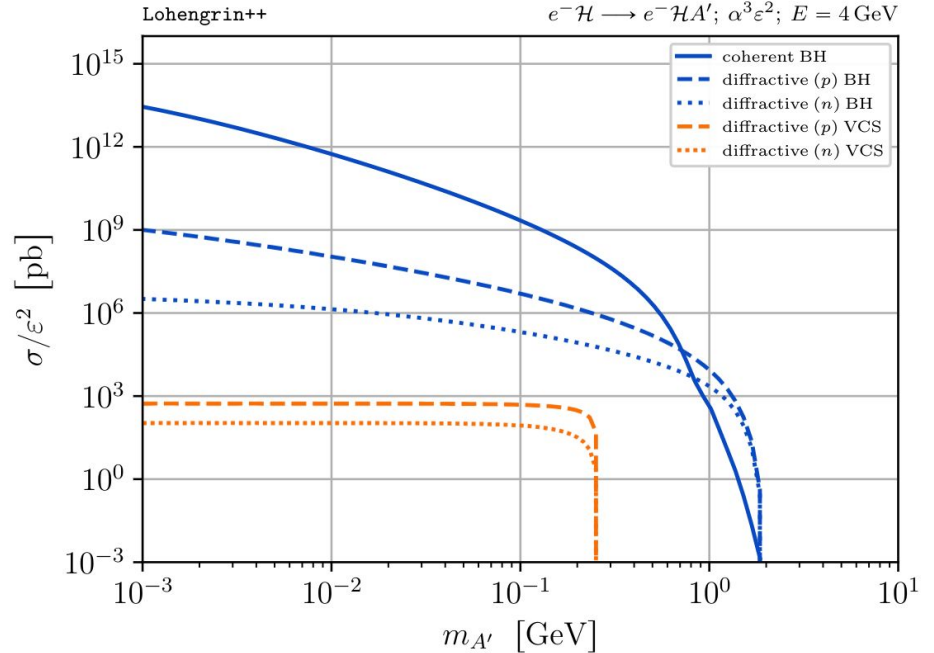


➔ To study only **RE** effects, use some minimal final state selections!

Real Emission: Dark Photon Production at $\mathcal{O}(\alpha^3 \varepsilon^2)$

Final state particle	Minimal real emission selections		
	Energy	Transverse momentum	Angle
e^-	$\xi_e < 0.9$	$p_{T,e} < 0.5 \text{ GeV}$	$\theta_e < \pi/2$
Φ^+, N, A', γ	-	-	-

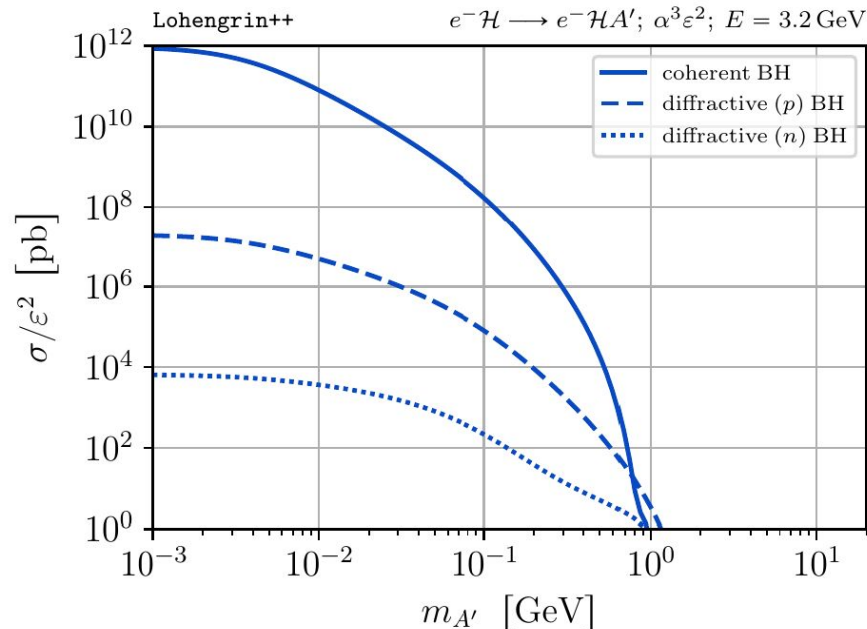
- Diffractive BH scattering relevant for heavy A' .
- Soft dark photon continuum from **VCS** insignificant.
➔ Neglect!



Real Emission: Dark Photon Production at $\mathcal{O}(\alpha^3 \varepsilon^2)$

LOHENGRIN scenario			
Final state particle	Acceptance cuts		
	Momentum		Angle
e^-	$25 \text{ MeV} < \vec{p}' < 75 \text{ MeV}$		$\theta_e < 0.25 \text{ rad}$
ϕ^+	-		-
N	$ \vec{P}' < 123 \text{ MeV}$	OR	$\theta_N > 0.24 \text{ rad}$
A'	-		-
γ	-		$\theta_\gamma > \theta_{\text{ECAL}}$

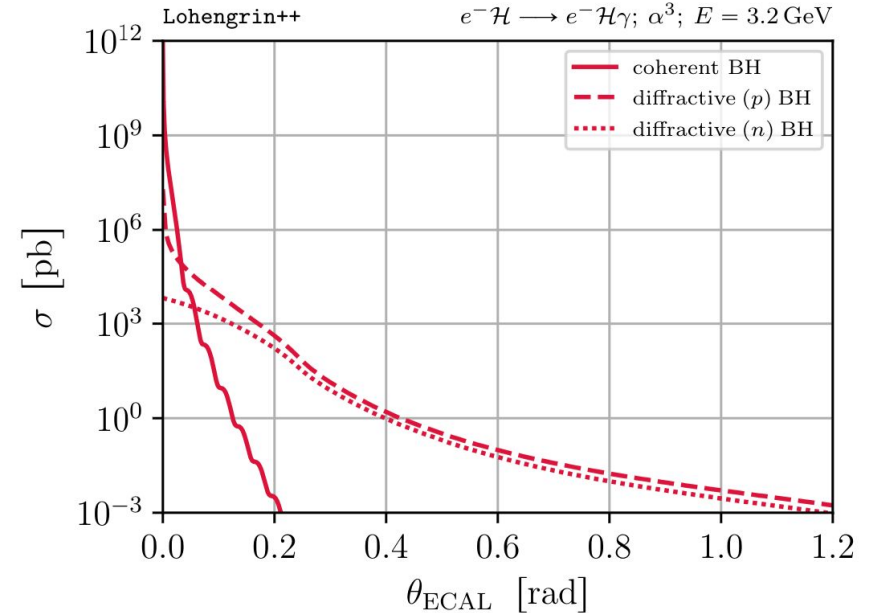
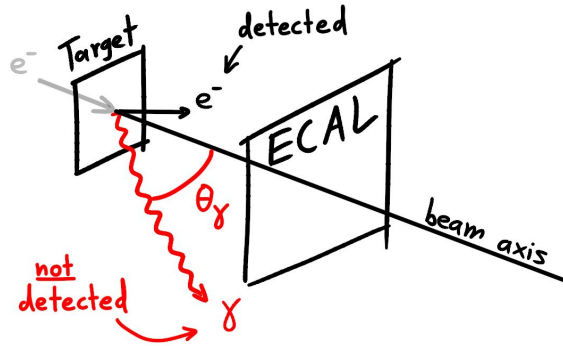
- Diffractive scattering with heavy A' produces nucleons **with large momenta**, i.e. knock-outs. Such events are vetoed by the HCAL.



➔ In SR1, diffractive scattering does not cause a lot of gain in signal events!

Real Emission: Photon Production at $\mathcal{O}(\alpha^3)$

LOHENGRIN scenario			
Final state particle	Acceptance cuts		
	Momentum		Angle
e^-	$25 \text{ MeV} < \vec{p}' < 75 \text{ MeV}$		$\theta_e < 0.25 \text{ rad}$
ϕ^+	-		-
N	$ \vec{P}' < 123 \text{ MeV}$	OR	$\theta_N > 0.24 \text{ rad}$
A'	-		-
γ	-		$\theta_\gamma > \theta_{\text{ECAL}}$



➔ **Increased background** from diffractive scattering. However, nucleons typically carry large momenta and could be vetoed with a bigger HCAL.

Real Emission: Photon Production at $\mathcal{O}(\alpha^3 \varepsilon^2)$

$$|\mathcal{M}|^2(e^- \mathcal{H} \rightarrow e^- \mathcal{H} \gamma; \alpha^3 \varepsilon^2) = 2 \operatorname{Re} \left\{ \left(\text{diagram 1} + \text{diagram 2} \right) \left(\text{diagram 3} + \text{diagram 4} \right)^\dagger \right\}$$

Since $m_{A'} > 0$, the **dark photon propagator** and **photon propagator** scale differently with q^2 :

$$i\Pi_{\mu\nu}^\gamma(q) = \frac{-ig_{\mu\nu}}{q^2}$$

$$i\Pi_{\mu\nu}^{A'}(q) = \frac{i}{q^2 - m_{A'}^2} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{A'}^2} \right)$$

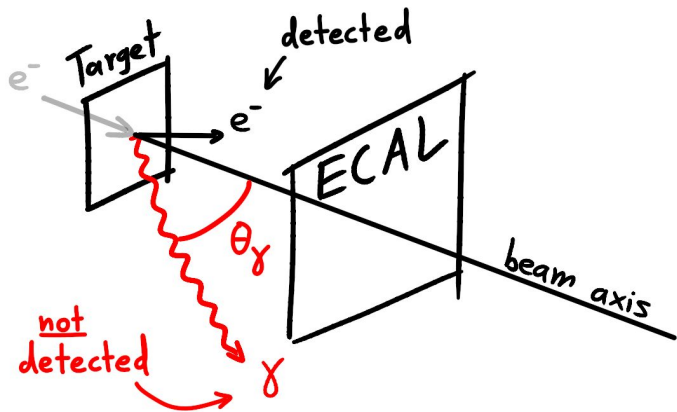
➔ The **interference term** producing photons at $\mathcal{O}(\alpha^3 \varepsilon^2)$ scales differently with q^2 , and thus **the final state electron kinematics**, as compared to the standard photon production at $\mathcal{O}(\alpha^3)$!

Real Emission: Photon Production at $\mathcal{O}(\alpha^3 \varepsilon^2)$

$$|\mathcal{M}|^2(e^- \mathcal{H} \rightarrow e^- \mathcal{H} \gamma; \alpha^3 \varepsilon^2) = 2\text{Re} \left\{ \left(\text{diagram 1} + \text{diagram 2} \right) \left(\text{diagram 3} + \text{diagram 4} \right)^\dagger \right\}$$

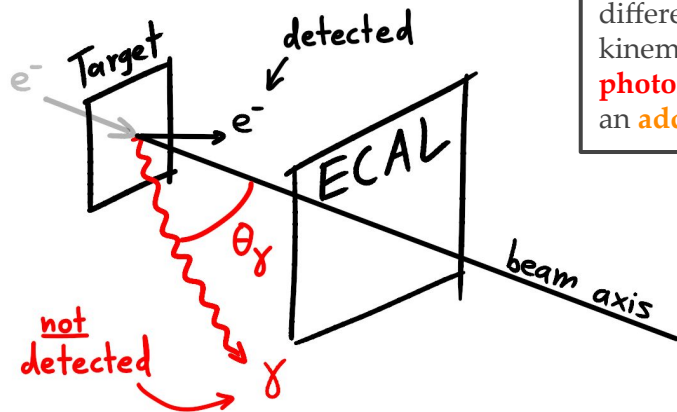
The diagrams represent the following Feynman processes:

- Diagram 1:** An incoming electron (black line) emits a photon (red wavy line) and then interacts with a target (black dot) via a virtual photon (red wavy line).
- Diagram 2:** An incoming electron (black line) interacts with a target (black dot) via a virtual photon (red wavy line) and then emits a photon (red wavy line).
- Diagram 3:** An incoming electron (black line) emits a photon (red wavy line) and then interacts with a target (black dot) via a virtual photon (blue wavy line, labeled A').
- Diagram 4:** An incoming electron (black line) interacts with a target (black dot) via a virtual photon (blue wavy line, labeled A') and then emits a photon (red wavy line).

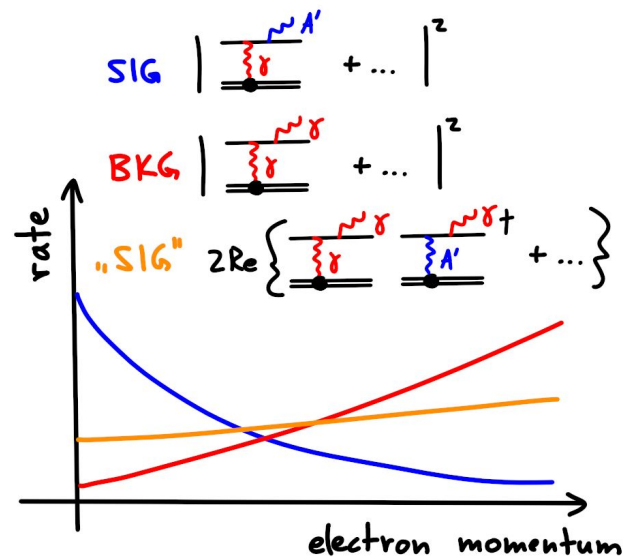
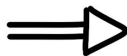


Real Emission: Photon Production at $\mathcal{O}(\alpha^3 \varepsilon^2)$

$$|\mathcal{M}|^2(e^- \mathcal{H} \rightarrow e^- \mathcal{H} \gamma; \alpha^3 \varepsilon^2) = 2\text{Re} \left\{ \left(\begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \end{array} \right) \left(\begin{array}{c} \text{diagram 3} \\ \text{diagram 4} \end{array} \right)^\dagger \right\}$$



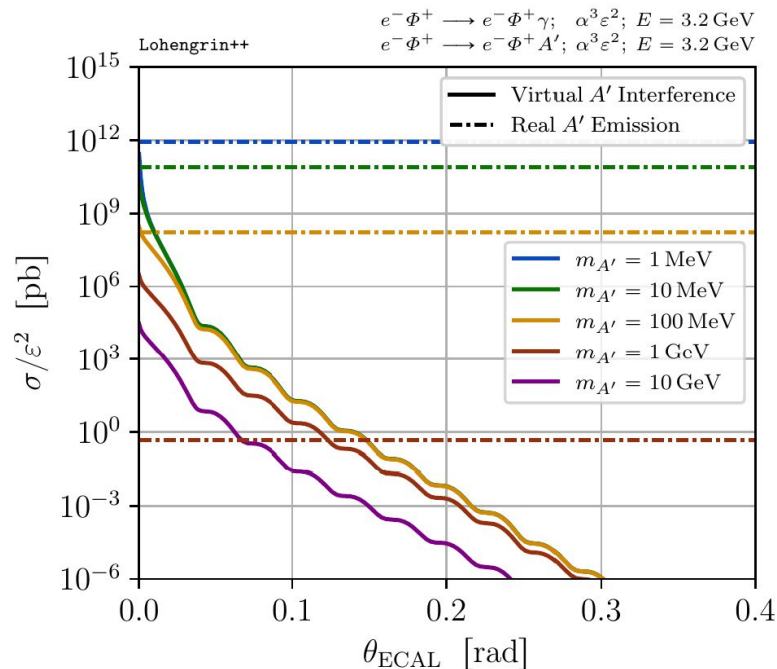
The **interference term** contributes to missing energy events. Since it scales differently with electron kinematics than the leading **photon background**, it yields an **additional signal**!



Real Emission: Photon Production at $\mathcal{O}(\alpha^3 \varepsilon^2)$

LOHENGRIN scenario			
Final state particle	Acceptance cuts		
	Momentum		Angle
e^-	25 MeV < $ \vec{p}' $ < 75 MeV		$\theta_e < 0.25$ rad
Φ^+	-		-
N	$ \vec{P}' < 123$ MeV	OR	$\theta_N > 0.24$ rad
A'	-		-
γ	-		$\theta_\gamma > \theta_{\text{ECAL}}$

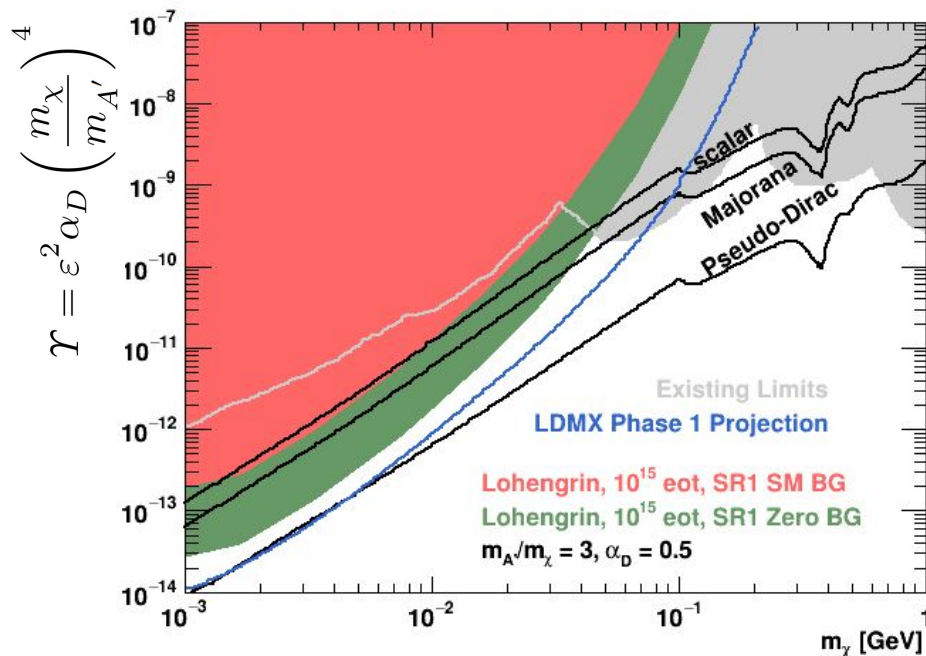
- Real A' emission experiences severe phase space suppression for large $m_{A'}$.
- ➔ Missing energy signature from **virtual A' interference** is **larger** than **real A' emission** for $\theta_{\text{ECAL}} \approx 0.07$ rad and e.g. $m_{A'} = 1$ GeV.



Projected Sensitivity

- ➔ The considered effects are all negligible, except the diffractive photon background.
- ➔ With some further improvements, especially an improved HCAL veto, LOHENGRIN will probe the **scalar and Majorana thermal relic targets** in low mass region.

- Conservative estimate
- Simple cut and count analysis
- 90% CL



Thank you!



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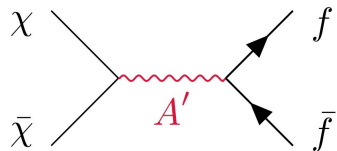
Dark Photon Theory

- For smallish Dark Photon masses, $m_{A'}^2 \simeq m_{A_D}^2 \ll m_Z^2 \propto v^2$,

$$g_f^{A'} \simeq -Q_f e \varepsilon_Y \cos \theta_W \equiv -Q_f e \varepsilon$$

➔ Dark Photon interacts only **electromagnetically**, at high and low energies.

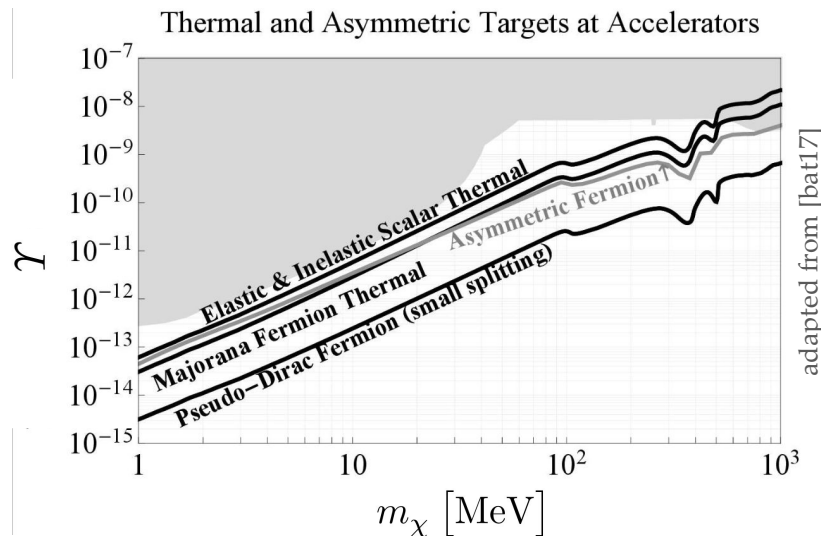
- If $m_{A'} > m_\chi$, DM freezeout through direct annihilation:



with thermal averaged cross section

$$\langle \sigma v \rangle \sim \Upsilon m_\chi^2 \stackrel{!}{\gtrsim} 10^{-9} \text{ GeV}^{-2}$$

where $\Upsilon = \varepsilon^2 \alpha_D \left(\frac{m_\chi}{m_{A'}} \right)^4$.

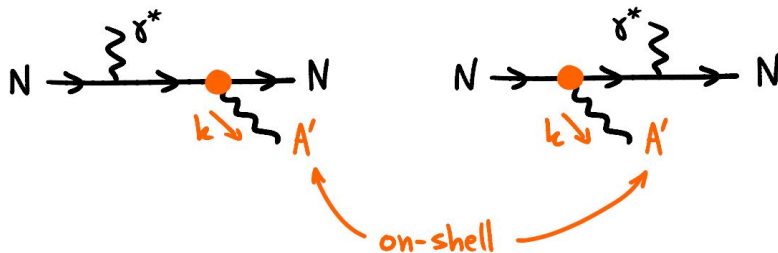


Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

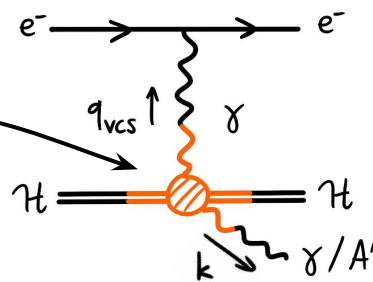
BH and VC scattering off **generalized hadronic current** with one photon exchange:

$$i\mathcal{M}_{\mathcal{R}}(e^- \mathcal{H} \xrightarrow{\gamma} e^- \mathcal{H} R) = L_{\text{BH}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{BH}}^2} H_{\text{BH},\mathcal{R}}^\nu + L_{\text{VCS}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{VCS}}^2} H_{\text{VCS},\mathcal{R}}^\nu \quad \text{where} \quad H_{\text{VCS},\mathcal{R}}^\nu = \varepsilon_\mu^*(k) iC_{\mathcal{R};R,V}^{\mu\nu}$$

For A' emission, requires evaluation of the form factors (FFs) in “unphysical”, **timelike region**: $F_{1,2}^N(k^2), k^2 = m_{A'}^2 > 0$



!



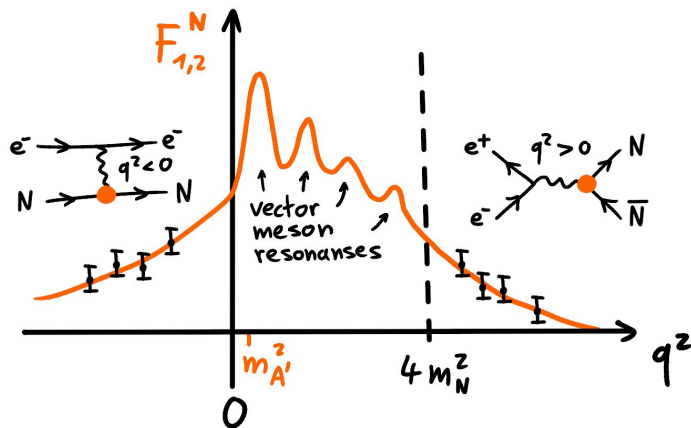
Virtual Compton (VC) scattering

Theory Predictions at $\mathcal{O}(\alpha^3 \varepsilon^2)$

BH and VC scattering off **generalized hadronic current** with one photon exchange:

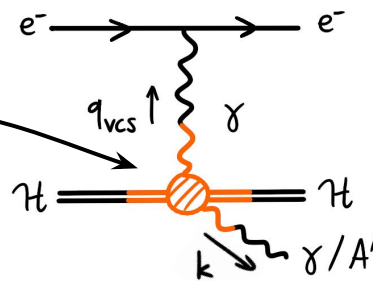
$$i\mathcal{M}_{\mathcal{R}}(e^- \mathcal{H} \xrightarrow{\gamma} e^- \mathcal{H} R) = L_{\text{BH}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{BH}}^2} H_{\text{BH},\mathcal{R}}^\nu + L_{\text{VCS}}^\mu \frac{-ig_{\mu\nu}}{q_{\text{VCS}}^2} H_{\text{VCS},\mathcal{R}}^\nu \quad \text{where} \quad H_{\text{VCS},\mathcal{R}}^\nu = \varepsilon_\mu^*(k) iC_{\mathcal{R};R,V}^{\mu\nu}$$

For A' emission, requires evaluation of the form factors (FFs) in “unphysical”, **timelike region**: $F_{1,2}^N(k^2), k^2 = m_{A'}^2 > 0$



For nucleons, FF models available from F. Kling et al. [kli26]

!



Virtual Compton (VC) scattering