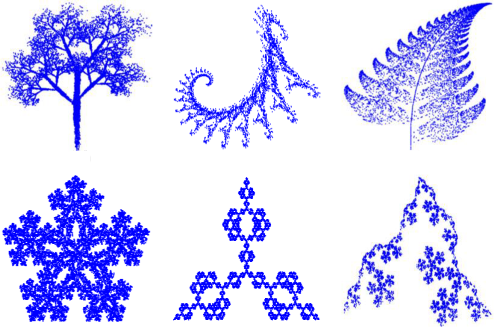
Interacting spins under quasiperiodic drive



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Periodic vs. aperiodic drive

Continuous drive over time

- Time-periodic systems: $\hat{H}(t+nT)=\hat{H}(t)$, $\omega=\frac{2\pi}{T}$, $n\in$ integer, $\omega\rightarrow$ rational number. N. Goldman and J. Dalibard, PRX **4**, 031027 (2015)
- Example: $\hat{H}(t)=\frac{\hat{p}^2}{2m}+\frac{1}{2}m\Omega^2(t)\hat{x}^2$, $\Omega(t)=\Omega_0+\bar{\Omega}\cos\omega t$
- Irrational frequency? Example: $\omega=\beta_{\rm G}:\frac{\sqrt{5}+1}{2}\rightarrow$ golden mean.

Periodic to aperiodic

- Periodic: $\begin{cases}\hat{H}(t,\omega_1) & 0<t\leq T_1,\ T_1=2\pi/\omega_1\\ \hat{H}(t,\omega_2) & T_1<t\leq T_1+T_2,\ T_2=2\pi/\omega_2\end{cases}$

  Unitary operators: $\hat{\mathcal{F}}_1=e^{-i\int_0^{T_1}\hat{H}(t,\omega_1)dt}$, $\hat{\mathcal{F}}_2=e^{-i\int_{T_1}^{T=T_1+T_2}\hat{H}(t,\omega_2)dt}$

  Time evolution: $|\psi(nT)\rangle=\hat{\mathcal{F}}^n|\psi(0)\rangle$, $\hat{\mathcal{F}}=\hat{\mathcal{F}}_2\hat{\mathcal{F}}_1\rightarrow$ Floquet

- Fibonacci: $\hat{\mathcal{U}}^{[m]}=\hat{\mathcal{U}}^{[m-2]}\hat{\mathcal{U}}^{[m-1]}$, $m\equiv F_m=F_{m-1}+F_{m-2}$, $m>2$

  Time evolution: $|\psi(m)\rangle=\hat{\mathcal{U}}^{[m]}|\psi(0)\rangle$, $\hat{\mathcal{U}}^{[1]}=\hat{\mathcal{F}}_1$ & $\hat{\mathcal{U}}^{[2]}=\hat{\mathcal{F}}_2\hat{\mathcal{F}}_1$

Fibonacci sequence

- $\hat{\mathcal{F}}_1 = e^{-i\int_0^{T_1}\hat{H}(t,\omega_1)dt} \to \omega_1$, $\hat{\mathcal{F}}_2 = e^{-i\int_{T_1}^{T=T_1+T_2}\hat{H}(t,\omega_2)dt} \to \omega_2$
- Recursion relation: $\hat{\mathcal{U}}^{[m]} = \hat{\mathcal{U}}^{[m-2]}\hat{\mathcal{U}}^{[m-1]}$, $m > 2$

$$m = 1 \to n = F_1 = 1: \quad \hat{\mathcal{U}}^{[1]} = \hat{\mathcal{F}}_1 := \omega_1$$
$$m = 2 \to n = F_2 = 2: \quad \hat{\mathcal{U}}^{[2]} = \hat{\mathcal{F}}_2\hat{\mathcal{F}}_1 := \omega_2,\ \omega_1$$

$$\begin{aligned}
m = 3 \to n = F_3 = 3: \quad \hat{\mathcal{U}}^{[3]} &= \hat{\mathcal{U}}^{[1]}\hat{\mathcal{U}}^{[2]} := \omega_1,\ \omega_2,\ \omega_1 \\
m = 4 \to n = F_4 = 5: \quad \hat{\mathcal{U}}^{[4]} &= \hat{\mathcal{U}}^{[2]}\hat{\mathcal{U}}^{[3]} := \omega_2,\ \omega_1,\ \omega_1,\ \omega_2,\ \omega_1 \\
&\vdots \\
m \gg 1 \to n \approx \beta_{\mathrm{G}}^m: \quad \hat{\mathcal{U}}^{[m]} &= \cdots\omega_1,\ \omega_2,\ \omega_1,\ \omega_2,\ \omega_1,\ \omega_1,\ \omega_2,\ \omega_1
\end{aligned}$$

- Sutherland invariant: $I_s = x_{m-2}^2 + x_{m-1}^2 + x_m^2 + 2x_{m-2}x_{m-1}x_m - 1$

$x_m = \mathrm{Tr}\,\hat{\mathcal{U}}^{[m]}$

Condition: $\hat{\mathcal{U}}^{[m]} \to$ SU(2) matrices. Bill Sutherland, PRL **57**, 770 (1986)

Driven spin under transverse field

- A spin-S particle under kicking:

$$\hat{H}(t) \;=\; \omega_0 \hat{S}_z \;+\; \lambda \hat{S}_x \sum_{n=-\infty}^{\infty} \delta\left(t - \sum_n T_n\right)$$

$\omega_0 \rightarrow$ magnetic field strength along $\hat{z}$, $\lambda \rightarrow$ kicking from transverse magnetic field along $\hat{x}$.

- Choice of T_n: $T_n = T_0(1 \mp \epsilon) \equiv T_{1(2)}$, $\epsilon = 1 \rightarrow T_{1(2)} = 0 \; (2T_0)$
- Unitary evolution: $|\psi(m)\rangle = \hat{\mathcal{U}}^{[m]}|\psi(0)\rangle$
- Fibonacci recursion: $\hat{\mathcal{U}}^{[m]} = \hat{\mathcal{U}}^{[m-2]}\hat{\mathcal{U}}^{[m-1]}$, $m > 2$

$$\hat{\mathcal{U}}^{[1]} \;=\; e^{-i\lambda \hat{S}_x}, \quad \hat{\mathcal{U}}^{[2]} \;=\; e^{-i2T_0\hat{S}_z} e^{-i\lambda \hat{S}_x}$$

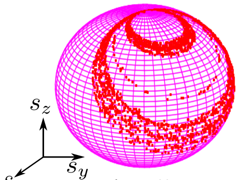
- Stroboscopic steps: $|\psi(n)\rangle = \hat{\mathcal{F}}_n|\psi(0)\rangle$, $\hat{\mathcal{F}}_n \;=\; e^{-iT_n\hat{S}_z} e^{-i\lambda \hat{S}_x}$
- Fibonacci sequence: $T_n := \cdots T_1,\; T_2,\; T_1,\; T_2,\; T_1,\; T_1,\; T_2,\; T_1$

Classical dynamics

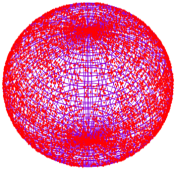
- Map of spin operators: $\hat{A}_{n+1} = \hat{\mathcal{F}}_n^\dagger \hat{A}_n \hat{\mathcal{F}}_n \rightarrow$ Heisenberg eqn. for $\hat{A}$

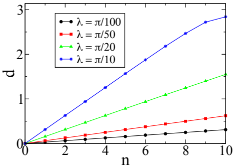
$$\begin{pmatrix} \hat{S}_x^{n+1} \\ \hat{S}_y^{n+1} \\ \hat{S}_x^{n+1} \end{pmatrix} = J_n \begin{pmatrix} \hat{S}_x^{n} \\ \hat{S}_y^{n} \\ \hat{S}_x^{n} \end{pmatrix}, \quad J_n = \begin{pmatrix} \cos T_n & -\sin T_n \cos\lambda & \sin T_n \sin\lambda \\ \sin T_n & \cos T_n \cos\lambda & -\cos T_n \sin\lambda \\ 0 & \sin\lambda & \cos\lambda, \end{pmatrix}$$

- Classical limit: $\hat{s}_i = \hat{S}_i/S \rightarrow [\hat{s}_i, \hat{s}_j] = i\epsilon_{ijk}\hat{s}_k/S$, $S \rightarrow \infty \equiv \vec{s} = \{s_i\}$
- Classical map: $(s_x^{n+1}, s_y^{n+1}, s_z^{n+1})^{\mathrm{T}} = J_n(s_x^n, s_y^n, s_z^n)^{\mathrm{T}}$



λ = π/100

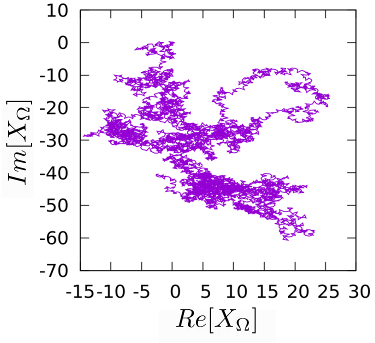

λ = π/10



Eigenvalue of J_n: $1, e^{\pm i\varepsilon}$, $d = \cos^{-1}(\vec{s}_i \cdot \vec{s}_f) \propto n \rightarrow$ Lyapunov exponent $= 0$

Strange non-chaotic attractor

- Power spectrum: $X_\Omega = \sum_{m=1}^{N} x_m e^{i2\pi\Omega m}$, $x_m = s_z$, $\Omega \rightarrow$ frequency.



- Classification of dynamics: $|X_\Omega|^2 \sim N^\beta$, $\boxed{\beta = 1.16,\ \Omega = 1/\beta_G}$

$\beta = 1\,(2) \rightarrow$ random (regular), $1 < \beta < 2 \rightarrow$ fractal path.

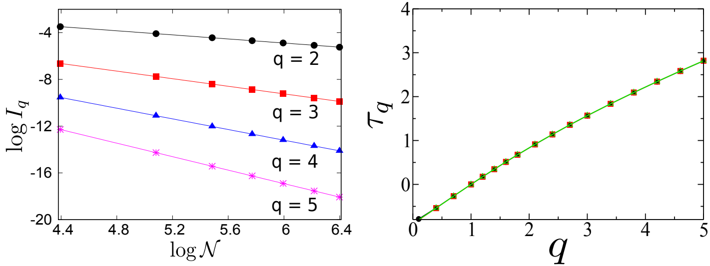
Spectral properties of Floquet operator

- Eigenmodes: $\hat{\mathcal{U}}^{[m]}|\chi_\nu\rangle = e^{i\varepsilon_\nu}|\chi_\nu\rangle$ $(m >> 1)$

  eigenphase: $\varepsilon_\nu \in [-\pi, \pi]$ and eigenvector: $|\chi_\nu\rangle$ of ν-th eigenmode.

  Moments of eigenstates: $I_q = \frac{1}{\mathcal{N}} \sum_\nu \sum_{m_s=-S}^{S} |\chi_\nu(m_s)|^{2q} \sim \mathcal{N}^{-\tau_q}$

  $\chi_\nu(m_s) = \langle \chi_\nu | \alpha_{m_s} \rangle$, $|\alpha_{m_s}\rangle \to$ spin basis, $\mathcal{N} = 2S + 1 \to$ dimension.



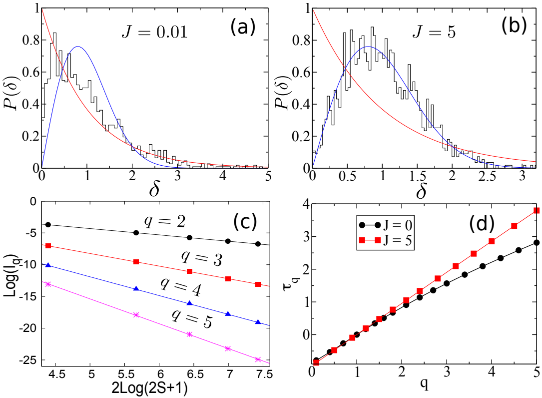
- Fractal dimension D_q: $\tau_q = D_q(q-1)$

  $D_q = 1 \to$ ergodic, $0 < D_q < 1 \to$ non-ergodic extended

Interaction vs. fractality

$$\hat{H}(t) = \hat{H}_0 + \lambda \hat{S}_x^{A/B} \sum_{n=-\infty}^{\infty} \delta\left(t - \sum_n T_n\right), \quad \hat{H}_0 = \hat{S}_z^A + \hat{S}_z^B - J\hat{S}_z^A \hat{S}_z^B$$

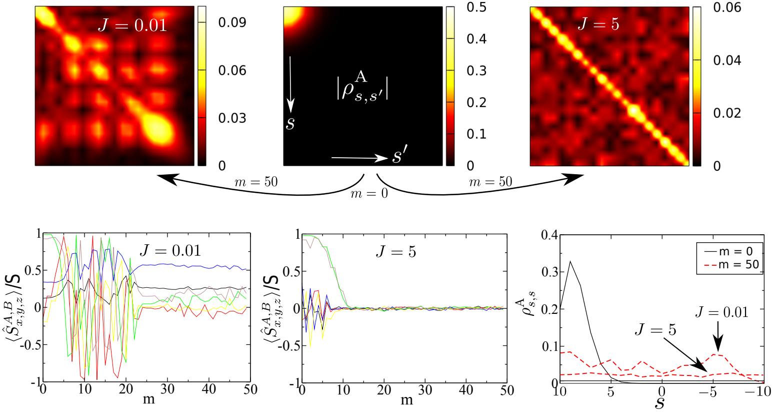
- Floquet operators: $\hat{\mathcal{U}}^{[1]} = e^{-iT\hat{H}_0} e^{-i\lambda \hat{S}_x^A}$, $\hat{\mathcal{U}}^{[2]} = e^{-iT\hat{H}_0} e^{-i\lambda \hat{S}_x^B}$
- Level spacings: $\delta_\nu = \varepsilon_{\nu+1} - \varepsilon_\nu$, $\int P(\delta) d\delta = 1$ and $\int \delta P(\delta) d\delta = 1$



$P(\delta) \to$ Wigner-Surmise, $D_q = 1$: Interaction $\to$ onset of ergodicity!

Microcanonical thermalization

- Initial state: $|\psi_{\rm AB}(0)\rangle = |\Theta, \Phi\rangle_{\rm A} \otimes |\Theta, \Phi\rangle_{\rm B} \rightarrow$ spin coherent state
- Reduced density matrix: $\hat{\rho}^{m}_{A(B)} = \mathrm{Tr}_{\mathrm{B(A)}}|\psi_{AB}(m)\rangle\langle\psi_{AB}(m)|$



- Microcanonical thermalization to infinite temperature

Conclusion and outlook

- Fibonacci driving $\rightarrow$ fractal dynamics, non-ergodic extended states.
- Interaction $\rightarrow$ crossover to ergodicity.

- Other metallic mean vs. fractal dimension $\leftrightarrow$ role of Sutherland invariant.
- Connection between fractal dimension (quantum vs. classical) and SNA spectrum.
- Quasi-periodically driven MBL systems, Quasi-time-crystalline state.

THANK YOU